PETRI NETS AS A MODELING TOOL FOR DISCRETE CONCURRENT TASKS
OF THE HUMAN OPERATOR

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SUMMARY

Petri nets have been developed as a fundamental model of technical systems with concurrent discrete events. The major use of Petri nets has been the modeling of hardware systems and software concepts of computers. After a very brief introduction to their basic concepts, the use of Petri nets is proposed for modeling the human operator dealing with concurrent discrete tasks. Their properties useful in modeling the human operator are discussed and practical examples are given. By means of an experimental investigation of binary concurrent tasks which are presented in a serial manner it is shown how human behavior may be represented by Petri nets.

INTRODUCTION

In different application areas the human operator's role in man-machine systems is changing from that of a continuous controller to that of a monitor. This change is happening in control rooms of industrial plants and in aircraft piloting, where dispatching of concurrent demands becomes an essential feature of the human operator's task. Furthermore the multiple task situation is also given in automobile driving. Especially in high density traffic situations the driver has to deal with many concurrent demands originating from other road users, from traffic regulation, and from his own vehicle.

In general, concurrent tasks are imposed on the human operator by displays or by real events, their service requires a response from the human operator which is specified by the task. There are continuous demands like the control error of a continuous control loop or there are discrete events, which require continuous or discrete actions respectively. These demands compete for the human operator's attention, if they arrive in such an intensity, that the human operator's capacity is at least temporarily exceeded. In this case demands which cannot be dispatched immediately have to be stored in his memory, otherwise they are lost.
For the design of such man-machine systems, i.e. of their
dynamic properties, displays, and controls, the human strategies
in dispatching concurrent demands have to be described by means
of experimental investigations and resulting quantitative models.
Notions of queueing theory are suitable for the formulation of
this task of the human operator (refs. 1, 2, 3), however instead
of analytical solutions human behavior is often studied by simu-
lation.

In this paper Petri nets are discussed as a modeling tool
for the human operator dealing with concurrent demands. As a
practical example the application of Petri nets for modeling
human strategies is shown. These strategies have been evaluated
by means of an experimental investigation of binary concurrent
tasks displayed in a serial manner.

**PETRI NETS**

In the following a brief introduction to the Petri net is
given; a more detailed presentation is contained in ref. 4.

A Petri net is an abstract, formal model of the information
flow in systems with discrete sequential or parallel events. Its
pictorial representation is a directed graph, for which an
example is shown in Fig. 1. The graph consists of two types of
nodes: places $p_i$ (represented by circles) and transitions $t_j$
(represented by bars). These nodes are connected by directed
arcs from places to transitions and from transitions to places.
If an arc is directed from node $i$ to node $j$, then $i$ is an input
to $j$, and $j$ is an output of $i$. In Fig. 1, e.g., place $p_1$ is an
input to transition $t_2$, while places $p_3$ and $p_4$ are outputs of
transition $t_3$. The nodes and arcs describe the static properties
of a Petri net, its dynamic characteristics are represented by
the movement of tokens (represented by black dots within the
places). The distribution of tokens in a Petri net defines the
state of the net and is called marking $\mu$. For each marking $\mu$ a
new marking $\mu'$ is defined by the following rules:

1. A transition is called enabled, if each of its input places
   has at least one token in it (e.g. transition $t_2$ in Fig. 1 is
   enabled).

2. Each transition which is enabled may fire.

3. A transition fires by removing one token from each of its in-
   put places and by adding one token to each of its output
   places (e.g. in Fig. 1, firing of transition $t_2$ results in
   two tokens in place $p_1$, zero token in $p_4$, and one token in
   place $p_2$).
The formal description of a Petri net is defined as a four-tuple of sets

\[ C = (P, T, I, O) \]

with \( P \) as a set of places, \( T \) as a set of transitions, \( I \) as the input function, and \( O \) as the output function. The input function \( I \) defines for each transition \( t \) the set of input places \( I(t) \). The output function \( O(t) \) is defined correspondingly. For the example shown in Fig. 1, there are the following sets

\[ P = \{p_1, p_2, p_3, p_4\}, \]
\[ T = \{t_1, t_2, t_3, t_4\}, \]
\[ I(t_2) = \{p_1, p_4\}, \]
\[ I(t_3) = \{p_2\}, \]
\[ I(t_4) = \{p_3\}, \]
\[ O(t_1) = \{p_1\}, \]
\[ O(t_2) = \{p_2\}, \]
\[ O(t_3) = \{p_3, p_4\}. \]

The vector \( \mu = (\mu_1, \mu_2, \ldots, \mu_n) \) gives, for each of the \( n \) places in the net, the number of tokens in that place. A Petri net \( C = (P, T, I, O) \) with the marking \( \mu \) becomes the marked Petri net \( C^* = (P, T, I, O, \mu) \).

An important tool for analysis of systems modelled by a Petri net is the reachability tree. It consists of a tree, whose nodes represent markings of the Petri net, and whose arcs represent the transitions which are enabled. The Petri net shown in Fig. 1 has the following degenerated reachability tree:

\[ (3, 0, 0, 1) \]
\[ \downarrow t_2 \]
\[ (2, 1, 0, 0) \]
\[ \downarrow t_3 \]
\[ (2, 0, 1, 1). \]
PROPERTIES OF PETRI NETS WITH REGARD TO THE DESCRIPTION OF THE HUMAN OPERATOR

In the following the properties of Petri nets are summarized with regard to the description of the human operator dealing with discrete concurrent demands (Table 1).

- Description of sequential and parallel processes
- Description of interactions between parallel processes
- Interpreted and uninterpreted modeling
- Hierarchical modeling
- Description of temporal order
- Description of deterministic and stochastic processes
- Modeling of priority systems
- Formal and graphic description

Table 1: Properties of Petri Nets with regard to the Modeling of the Human Operator

Petri nets are suitable for the description of sequential and parallel concurrent demands of the human operator as well. A net having only one token at the same time is describing a sequential process. The position of the token represents the state of the 'sequence control register' of the process. The graphic representation of such a net containing only transitions with one input and one output corresponds to the usual flow-chart. Fig. 2 shows an example the observation of a traffic light represented by a flow-chart and by a Petri net.

A net having more than one token at any time describes a non-sequential process. Several tokens may result from a transition with several output places or they are an initial marking. Fig. 1 shows a Petri net with an initial marking of four tokens. Therefore it describes a system of partially parallel activities. It may be interpreted as a general model of the dispatching of concurrent demands, presented in a serial manner to the human operator. Concurrency is given by the fact, that during the service of one demand other demands are waiting or arriving. Then places p₁ to p₄ of Fig. 1 have the following interpretations:
$p_1$: number of demands waiting for service (represented by the corresponding number of tokens),

$p_2$: one demand is being served,

$p_3$: one demand has been served,

$p_4$: the human operator is ready for the next service.

The description of parallel activities by Petri nets can be applied to the modeling of interactions between different stages of human information processing and between the tasks of several operators as well. Fig. 3 shows as an example the crossing of two automobile drivers were two conflicting transitions without indication of priority exist. Modeling of priority systems is considered later on.

The interpretation of the Petri net shown in Fig. 1 may be specified with regard to practical applications. E.g. the demands may represent traffic signs which have to be observed by the driver or they are alarms of an industrial plant presented on displays in the control room. On the other hand Petri nets exist as uninterpreted models of which the abstract properties may be investigated.

Another valuable feature of Petri nets is their ability to model a system hierarchically. This means that parts of the human information processing may be represented by a single place or transition in order to have a more abstract model. Conversely places and transitions may be specified by subnets in order to get a more detailed description. The example of Fig. 1 is an extremely abstract model of the human operator, which will be specified by means of experimental results in chapter 4.

Petri nets describe the possible sequences of events, they do not reflect the variable amounts of time required by the different events. Because of this property Petri nets give no information about the duration of information processing of the human operator.

Petri nets are suitable for the modeling of deterministic and of stochastic sequences of events. Deterministic sequences ever have one transition being enabled, while stochastic sequences lead to situations in which more than one transition is enabled. The choice of the next transition to be fired occurs randomly. Fig. 4 shows two types of stochastic firing of transitions. Concurrent transitions may fire in either order whereas in conflicting transitions the firing of one will disable the other.
In order to model priority systems Petri nets were extended by inhibition arcs represented by an arc with a small circle instead of an arrowhead. An inhibition arc from place $p_1$ to transition $t_1$ enables the transition only to fire if the place $p_1$ has zero tokens in it. Fig. 5 shows as an example the crossing of two drivers (compare Fig. 3) with priority of driver 1, described by an inhibition arc.

Furthermore Petri nets may be described in a formal as well as in a graphic manner. Especially the graphic representation seems to be a useful tool in describing complex information processing of the human operator.

MODELING OF HUMAN STRATEGIES IN DISPATCHING CONCURRENT DEMANDS BY PETRI NETS

Experimental Set-up

In order to investigate the human behavior in dispatching concurrent demands a simulator for the generation of the demands and for their service has been established. Fig. 6 shows the block diagram of the experimental set-up. There are 8 streams of binary demands presented by the numbers 1 to 8 on a common numeric display to the operator. The arrival pattern of each stream is given by the Poisson distribution. The service of each demand consists in pressing a corresponding push-button during a fixed lapse of time which is indicated by a service time lamp. The traffic intensity $\rho$, i.e. the ratio of the service time and the mean interarrival time of the demands, varied in the range $0.8 \leq \rho \leq 1.6$. The service of the demands had to be done in the order of arrival. The experimental sessions consisted of five trials of 200 s duration each. After each trial the traffic intensity was increased by an amount of 0.2, beginning with the value of $\rho = 0.8$. For $\rho \geq 1$ the human operator is unable to deal with the demands as fast as they arrive. Consequently the demands have to queue up in the operator's short term memory. Because of its limited capacity demands may be lost. By recording the service activities of the subjects the strategies in dealing with concurrent demands could be evaluated.

In the following the results from one experiment are presented, further investigations with this experimental set-up are described in ref. 5.

Experimental Results

In order to analyse the human strategies the contents of
the memory were evaluated; it is called waiting-room diagram. Fig. 7 shows a typical waiting-room diagram, where the number of demands in waiting-room 1, i.e., the length of the queue, is plotted as a function of time. At the arrival of a demand the length of the queue increases by one, at the beginning of a service the plot decreases by one. The upper plot shows the ideal waiting-room diagram, which is based on the assumption that there is an infinite waiting-room. All arriving demands are waiting for service, no demand is lost. The lower plot shows the minimal waiting-room diagram, with the assumption that lost demands did never enter the waiting-room. Demands which are not served are marked by a "N". By asking the subjects to communicate the contents of their memory at certain time instants, it could be shown that the real waiting-room diagram corresponds largely to the minimal waiting-room diagram.

By the analysis of the waiting-room diagram and supported by statements of the subjects two strategies for the service of the concurrent demands can be specified:

- Waiting-room with permanent access
  The demands enter into the waiting-room and queue up until a maximum length of the queue ($l_{\text{max}} = 3$) is reached. Then if one demand is served another may enter.

- Waiting-room with intermittent access
  The demands enter the waiting-room and queue up. By certain triggering events all arriving demands are rejected until the length of the queue is reduced to a low value. Triggering events are the reaching of a maximum length of the queue or the arrival of several demands with short interarrival times. In this case the maximum length of the queue is higher ($l_{\text{max}} = 5$) than with the strategy of permanent access.

The described strategies are extreme forms of behavior, in reality they occur in an approximate and mixed form. The strategy with intermittent access can be observed more often (factor 1.5) than the strategy with permanent access. Fig. 7 shows the waiting-room diagram in the case of intermittent access. This strategy is less efficient, because with permanent access there is a higher utilization of the waiting-room, i.e., the mean length of the queue is increased.

Modeling of the Human Strategies by Petri Nets

The human operator's activities in dealing with concurrent tasks are the input of information, the storage of information in his memory (waiting-room), and the service of demands by reactions. Fig. 8 shows the Petri net of the strategy with permanent access divided into these three parts. For simplicity the waiting-
room is assumed to have a capacity of three demands.

Complementary places are labelled by p and p'. For example the interpretation of place p2 is "information input is idle" and of place p2' "information input is busy". At the arrival of a demand it depends on the state of the information input whether the demand is lost (place p2' marked) or not (place p2 marked). In the latter case transition t1 is fired and the demand is stored in that position of the waiting-room (p4 ... p6), which is free and has the lowest number. By the entrance of the demand into the waiting-room (firing of transition t4, t5 or t6) the information input is reset by firing transition t11. Demands arriving when the input is busy are lost by firing of transition t2. Also if the waiting-room is completely occupied, i.e. the place p6 is marked, the arriving demand is rejected and lost by firing of transition t3.

If the waiting-room is empty, the arriving demand is stored in the first place of the waiting-room (marking of p6'). If the service mechanism is idle (place p2' marked), then the service of this demand may be carried out by firing of transition t7 and by setting free the waiting-room place. If there are further demands in the waiting-room they advance one step by means of the transitions t8 and t9.

Fig. 9 shows the Petri net of the strategy with intermittent access. Compared with permanent access there are modifications especially in the input part of the model. The states of the places p12 and p12' determine whether arriving demands are rejected or not. The rejection state (p12' is marked) is triggered if the waiting-room is completely occupied, i.e. p6' is marked. Then all demands are rejected and lost until the waiting-room is empty and the transition t12 is fired. Then arriving demands again have access to the waiting-room.

The strategies described have been simulated by means of a digital computer. Simulation data showed close agreement with experimental results. The description of the human strategies by means of Petri nets turned out to be a valuable tool for the analysis of human information processing.

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REFERENCES


Fig. 1: A Petri net graph

Fig. 2: Observation of a traffic light represented by a flow-chart and a Petri net
Fig. 3: Petri net of the crossing of two drivers

Fig. 4: a) Concurrent and b) conflicting transitions $t_i$ and $t_j$

Fig. 5: Petri net of the crossing of two drivers with priority of driver 1
Fig. 6: Block diagram of the experimental set-up
Fig. 8: Waiting-room with permanent access
Fig. 9: Waiting-room with intermittent access