VEHICLE STEERING CONTROL: A MODEL OF LEARNING

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SUMMARY

A hierarchy of strategies were postulated to describe the process of learning steering control. Vehicle motion and steering control data were recorded for twelve novices who drove an instrumented car twice a week during and after a driver training course. Car-driver describing functions were calculated, the probable control structure determined, and the driver-alone transfer function modelled. The data suggested that the largest changes in steering control with learning were in the way the driver used the lateral position cue.

INTRODUCTION

Various aspects of driver behavior have been studied using manual control theory. To date, most, if not all, of this research has used experienced drivers. The research to be described in this paper used inexperienced drivers in order to study the changes in the driver describing function as a novice learns to steer a car.

The mathematical model used to describe the driver is the crossover model, described in Reference 1. Though the model was developed using single-loop, compensatory tracking tasks, it has been successfully used to describe car driving where two loops are involved. The basic tenet of the crossover model is that the human adapts to each controlled element so that the open loop man-machine transfer function always has the form:

\[ Y_p(jw)Y_c(jw) = \frac{w e^{-j\phi}}{jw} \]  

(1)

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where \( w_c \) is the system crossover frequency, and \( \tau \) is the effective time delay (incorporating delays due both to the operator and the control device), \( Y_p \) is the operator describing function, \( Y_c \) is the transfer function describing the control device dynamics, and \( jw \) is the complex frequency variable.

Weir and McRuer (Reference 2) applied this model to automobile lane-keeping steering tasks in order to determine which of the available visual cues would yield good performance without great effort on the part of the driver. From previous studies with the crossover model it has been shown that the human operator selects from the possible cues or feedbacks those that minimize his/her equalization requirements. In other words, the operator prefers to act as a simple gain and time delay rather than as a single or double differentiator, and selects cues so that s/he can do this. The car dynamics in lateral position are such that the use of lateral error as a cue would require the operator to act as a differentiator. \( (Y_p(jw) = jwKe^{-jw\tau}) \) from the crossover model.

This eliminates lateral error as a dominant cue for the experienced driver. Heading angle and rate, path angle and rate, and time-advanced lateral deviation were studied (Reference 2) as possible cues. As heading rate control allows a fairly large lag and produces a high crossover frequency, it appears to be the best cue to use. As its use is associated with high frequency control movements, heading angle (an intermediate frequency cue) is a more probable cue in less demanding situations. Control is unlikely to be purely directional since drifts in lateral position will occur which, if uncorrected, may result in the car going out of the lane. Therefore, it was suggested that a probable structure for an experienced driver is an outer loop controlling lateral position and an inner loop controlling heading angle or rate. The heading angle inner loop provides the path damping necessary for a stable, well-behaved closed loop system - and thereby avoids the necessity of the operator differentiating the input (which would difficult because it must be done at low frequencies as well as high) which would be needed to stabilize the outer loop, if it were the only loop closed. Though a single loop structure of time advanced lateral deviation had also been suggested in Reference 2, the time advance (preview time) necessary for such a control loop to work was in the order of 5 to 10 seconds. Below these values the lead generated by using predicted future lateral deviation 'would not compensate sufficiently for the inherent lags in the driver/vehicle system'. In Reference 3 a survey is presented of the research on estimated preview times used by experienced drivers. Only when the driver viewed the road through a narrow slit were preview times in the range needed for good use of time advanced lateral deviation as a control loop. This suggests that such a control loop is an unlikely possibility under normal driving conditions. The reader must be cautioned at this point that statements about which cues are used in driving in no way imply that these cues are directly perceived by the driver. For example, the driver may perceive heading angle directly or may perceive some function of heading angle. The mathematical analysis cannot differentiate between two such dependent variables.
Hypotheses About Learning Steering Control

Perceptual-motor learning studies, eye movement studies of novice drivers and anecdotal information obtained from driver instructors were used to generate hypotheses about the stages in the learning of steering control. In the first stage it was postulated that the driver controls lateral position (ψ), the most obvious cue. In reference 4 it was shown through a study of the eye movements of novice drivers that novices tended to look closer in front of the vehicle than experienced drivers, suggesting they were looking for lateral position cues. As was pointed out earlier, lateral position is a difficult cue to control so this stage was not expected to last long (see Fig. 1).

With experience the novice begins to look further ahead of the car. This is necessary in order to better monitor the environment but also allows the driver to pick up heading angle (Ψ) movements more easily. The car's dynamics in heading angle (ψ0w) are rate dynamics, so that the driver's control may be modelled by a simple gain and time delay. Thus the second stage is that the driver will use heading angle as the dominant cue, but will still control lateral position directly (as in the first structure), with corrections being made when a significant lateral position error has accumulated. An analogous strategy was used by subjects in an experiment described in reference 5, where subjects using an oscilloscope centered a target on crosshairs by sequentially pressing two keys, one causing target acceleration to the right and the other, to the left. The response pattern suggested that some subjects modified their responses on the basis of feedback i.e. after drifting off target they made a single, long duration corrective movement, while other subjects, who maintained a higher rate of responding and were consistently better in overall performance, used a more efficient strategy. These latter subjects 'when the target drifted off center to the left...maintained a high rate of responding but at the same time gradually increased the length of time the right key was active relative to the left key, so that over a series of responses the target was made to drift back towards the center'. It was postulated that at an intermediate stage, learning drivers would be using a strategy similar to Pew's first group of subjects, which would be represented by an alternating operation on lateral position and heading angle as shown in Fig. 2.

In the final stage of learning, it was postulated that the driver would begin to use dual loop control, where heading angle is the dominant cue, controlled by an inner loop, and lateral position is controlled with an outer loop. In this way lateral position may be controlled by heading angle corrections i.e. using a simple gain (Ψ' = Kψ) rather than having to estimate rate of change of lateral position. The operators control of heading angle was modelled by a gain, Kψ, a lead term (1 + Tψjω), and a time delay (e⁻ʲωt). (i.e. Ψ = Kψ(1 + Tψjω)e⁻ʲωt). The lead term is needed to offset a lag in vehicle response at higher frequencies. For the experimental car the break frequency of this lag occurred at Tψ = 9.4 rad./sec., therefore the same value was assumed for Tψ when the driver-alone transfer function was modelled. In reference 6 it was shown that this structure satisfied the crossover model.
and appeared to provide a reasonable fit to experimental data. Such a form of control is analogous to that used by the subjects using the more efficient strategy in the experiment described in reference 5.

Theoretical Analysis

The driver-car transfer function for the control structures postulated as stages in the learning process will now be derived.

Using Fig. 1, the following relationship may be obtained:

\[ \delta_w = \delta_d n G_s - \delta_w \delta_s y_s n G_s \delta_w y_s \]  

(2)

![Diagram](image-url)

Fig. 1. Single-loop control of lateral position (\(\delta_s\)swa-steering wheel angle, \(\delta_s\)front tire angle, \(G_s\)-steering gain, other definitions in text)

If each variable is cross-correlated (see reference 7 for a description of these techniques) with the input disturbance, \(\delta_d\), the following is obtained:

\[ \phi_{\delta_d \delta_w} = \phi_{\delta_d \delta_d} + \phi_{\delta_d n} G_s - \phi_{\delta_d \delta_w} \delta_s y_s - \phi_{\delta_d n} G_s \]  

(3)

The remnant, \(n\), is by definition that part of the drivers output which is uncorrelated with the input, so that \(\phi_{\delta_d n}\) may be considered to be zero. Because \(\delta_d\) is such designed so that it is much larger than \(n\), \(\phi_{\delta_d n}\) will be negligible in comparison with \(\phi_{\delta_d \delta_w}\) and \(\phi_{\delta_d \delta_d}\). Equation (3) is then reduced to:

\[ \frac{\phi_{\delta_d \delta_d} - \phi_{\delta_d \delta_w}}{\phi_{\delta_d \delta_d}} = \frac{G_s y_s}{G_s \delta_w} \]  

(4)

For structure 2 this expression is equal to \(G_s y_s\) \(\delta_w^\psi\) or \(G_s y_s\) \(\delta_w^\psi\) depending on which loop is in use. For the dual-loop structure 3 this expression becomes:

\[ \frac{\phi_{\delta_d \delta_d} - \phi_{\delta_d \delta_w}}{\phi_{\delta_d \delta_d}} = \frac{G_s y_s}{G_s \delta_w} \]  

(5)
Fig. 2. Parallel loop control of heading angle and lateral position

Fig. 3. Dual loop control of heading angle and lateral position

Thus, no matter which structure the driver is using, the same cross spectral expression is calculated to obtain the car-driver transfer function. However, as will now be shown, the form of the transfer function obtained differs depending on which structure is in use.

In the first two structures, either heading angle or lateral position is being controlled at any one time. Thus the first two structures are single control loops which, in the frequency range used in this study, may be expected to conform closely to the crossover model. Therefore, using equation (1), the car-driver transfer will have the form $w e^{-j\omega t/jw}$. When this function is plotted on a Bode plot (amplitude and phase vs. frequency) the amplitude slope is 20db per frequency decade (see Fig. 4).

The values assumed for the driver's transfer functions in the third structure were such that the car-driver transfer function could be modelled at mid- and high frequencies (i.e., near crossover frequency) by the crossover model, as in the first two structures. However, the presence of the outer loop operating on $y$, affects the expected amplitude slope of the Bode plot. Using equation (5), as frequency increases, the ratio $G_y^1/G_y^0$ decreases rapidly so that the main effect of the $Y_y$ term is at low
frequencies, where it causes an increase in the amplitude slope as seen in Figure 4. Thus structure 3 may be distinguished from structures 1 and 2 by the presence of an increased slope in the Bode amplitude plot of the car-driver transfer function. Structures 1 and 2 must be distinguished from each other by more subtle cues, however. Because of the change in operator requirements, a change from control of lateral position to dominant control of heading angle would result in a jump in crossover frequency and an increased phase angle at low frequencies. (The difficulty of generating the low frequency lead needed for lateral position control results in a pronounced phase droop at low frequencies and a lower crossover frequency.)

The considerations discussed above were used to help determine the control structure used by the subjects.

EXPERIMENTAL PROCEDURE

Subjects

To test the hypotheses about changes in steering control with learning, twelve novice drivers participated in an experiment using an instrumented car over a five week period. The subjects were all high school students who at the start of the experimental test period were beginning a three week intensive driver training program. They were selected on the basis of having had minimal experience of driving. Three subjects had never driven a car before being recorded driving the instrumented car and the other subjects had driven on at most five previous occasions. The subjects were tested on nine separate occasions over the five week period.

Equipment

The instrumented car driven by the subjects was capable of recording driver control measures, vehicle motion variables and vehicle lane position, and was built by Systems Technology Inc., Los Angeles, and lent to this author by the U.S. National Highway Traffic Safety Administration. The car is described in detail in an STI technical manual (Klein et al., 1976). Two features of particular interest, though, are the lateral position detector and the servo control.

The lateral position detector was developed by the Institute for Perception in the Netherlands. It consists of a position transducer and a control unit. The position transducer uses a rotating prism to scan the intensity of reflected light in a lateral plan across the road and reflect the light in a photoamplifier. Any marker which sufficiently contrasts with its surroundings is taken as being part of the reference line by the lane tracker. For the experiment a 2.5 inch wide strip was laid down as a center lane marker to be picked up by the position detector.

The servo control allows for application of steering inputs to the front wheels independently of the driver's steering inputs. This is accomplished by
hooking up an analogue tape recorder containing a taped disturbance which is played back and passed by means of an electro-mechanical device through the steering linkage to the front wheels. This provides a means of measuring the closed loop dynamic behavior of the driver by insertion of a known input or disturbance function into the loop. The disturbance function used in the experiment was a sum of nine sinusoids - .377, .503, .754, 1.257, 1.634, 2.765, 4.271, 5.781 and 10.801 rad./sec. Each of these input frequencies has an integral number of cycles in a 50 second run length. The advantage of using a sum of sines input is that while the remnant is spread out over many frequencies, the input is concentrated at discrete frequencies. Thus, at these discrete frequencies, where the driver car-driver transfer function is measured, the remnant is swamped by that part of the output signal which is correlated with the input, so that relatively clean estimates of the correlated output are obtained.

The variables recorded during the subject runs were: steering wheel angle, front tire angle, heading angle, lateral acceleration, lateral position, forward velocity and the disturbance signal input.

Procedure

Each of the twelve subjects came to the test site twice a week for five weeks. On the first test day it was determined from the first two subjects that the novices could manage a speed of 40 k.p.h. This determined the speed which was used for all the test runs. Runs were made up and down two marked lanes on a half mile stretch of an unused runway. In total 200 seconds of data were collected for each subject on each day.

RESULTS

Changes in the Car-Driver Transfer Function with Learning

Table 1 summarizes the one factor, repeated measures, analyses of variance which were carried out for the amplitude and phase angle values in the car-driver transfer function, using twelve subjects and nine (treatment) days. Analysis of the power spectrum of steering wheel angle showed that the driver's input at frequencies above 2.765 rad./sec. was negligible (< 1% of total input). Also, at these frequencies the signal to noise ratio is high and therefore the estimates are less reliable. Consequently changes at the first six frequency points (in the disturbance signal car-driver transfer function) are of greatest interest.

Table 1 and Figure 5 show that a significant increase in amplitude of the car-driver transfer function occurred over the test period at the first four frequency points. However, the amplitude at the first frequency point showed the most dramatic change. While the means of the first two days were approximately equal, the mean increased by 40% on the third day and fluctuated about this value for the last six days. As this large increase did not occur at frequency points adjoining .377 rad./sec., a change in slope of the amplitude plot of the car-driver transfer function is indicated. When individual subject
TABLE 1
Summary Anova Results: Amplitude and Phase of the Car Driver Transfer Function
Nine (treatment) Days: 12 Subjects

<table>
<thead>
<tr>
<th>Frequency (rad./sec.)</th>
<th>Amplitude Differences Between Days (F, .05 level)</th>
<th>Phase Differences Between Days (F, .05 level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.377</td>
<td>2.242+ Day 1 &lt; 3-9</td>
<td>1.622</td>
</tr>
<tr>
<td>.503</td>
<td>3.158++ Day 1 &lt; 5,6,9; Day 2 &lt; 9</td>
<td>1.550</td>
</tr>
<tr>
<td>.754</td>
<td>4.725+++ Day 1 &lt; 3-9; Day 2 &lt; 4,9</td>
<td>0.930</td>
</tr>
<tr>
<td>1.257</td>
<td>2.651 Day 1 &lt; 3-9</td>
<td>4.216+++ Day 2&gt;5; Day 8&gt;5; Day 9&gt;5-7</td>
</tr>
<tr>
<td>1.634</td>
<td>0.845</td>
<td>8.646+++ Day 1-6 &lt; 8,9</td>
</tr>
<tr>
<td>2.765</td>
<td>1.607</td>
<td>21.234+++ Day 1,2&lt;4-9; Day 3-6&lt;7-9</td>
</tr>
</tbody>
</table>

level of significance: + .05, ++ .01, +++ .001

plots were examined it was found that, for half of the subjects, the amplitude of the .377 rad./sec. point showed a sharper increase over the first three days than did the amplitudes at other frequencies, while, for the other half, the whole amplitude slope increased. As will be shown in the section on modelling, an increase in the amplitude slope, particularly at low frequencies, is a result of the way subjects used the lateral position cue.

When the phase angle (of the car-driver transfer function) drops below -180°, the car-driver system becomes unstable so that an input generates an exponentially increasing output. Therefore, large phase angles (> -180°) are to be desired around crossover. (Phase angles at frequencies further from crossover have little effect on system stability.) Figure 6 shows that at the frequencies surrounding the crossover the phase angle increases gradually, though a little erratically, between days one and nine, indicating that the subjects improved their stability of control.

The changes in amplitude and phase angle of the car-driver transfer function over the test period were reflected in improved tracking performance, with the largest improvements occurring during the first three days.

For all the variables studied, the changes that took place over the last six days were much less dramatic, and much more erratic, than those that occurred over the first three days. If measures on day 3 are compared with those for days 8 and 9, no changes are significant, but the following trends were noted: an increase in the amplitude of the car-driver transfer function at .503, .754, and 1.275 rad./sec., an increase in phase margin, and reduced heading angle deviation.

Modelling the Driver-Alone Describing Function

In the first two structures postulated, the driver adapts to each set of controlled mechanics in such a manner that the overall car-driver transfer function has the same form (see Fig. 4). However, as was noted previously,
Fig. 5. Change in amplitude (12 subjects)

Fig. 6. Phase angle changes around crossover (12 subjects)

experimental data shows that a switch from proportional control (i.e., control of lateral position) to rate control (i.e., control of heading angle) results in a large improvement in crossover frequency. Although there was mean increase in crossover frequency in this study, from 1.696 to 1.929 rad./sec., the increase was small, occurred gradually, and was statistically insignificant. Only two of the twelve subjects showed large changes in crossover frequency. Further examination of the data suggested that reasons other than a change in control structure were responsible for the increase.

Another factor which aids in deciding upon the control structure in use is the percent of high frequency area (%HFA) in the power spectrum of the steering wheel angle. A car's dynamics are such that at the higher frequencies it shows a greater response in heading angle than it does in lateral position. Therefore, a driver who controls lateral position most use lower frequency inputs to get a reasonable response from the car. Consequently, one would expect that %HFA would be lower for a driver controlling lateral position than it would be for a driver controlling heading angle. The data showed that the %HFA was higher rather than lower, though not significantly so, in the first days of the experiment than in the last. This is another indication that the subjects were probably not using the first postulated structure where lateral position was the primary cue for control.

Though this assumption will be used in determining how the driver transfer functions will be modelled, it must be stressed that the structure of a system with only one input, with which to identify two operator transfer functions, can only be inferred; it cannot be known with certainty.

If the first structure can be eliminated as a mode of control, the modelling of the driver-alone transfer function is simplified.
Let us consider the third control structure. As was discussed previously, the form used for \( Y_y \), the driver's operation on functions of lateral position, was to be a simple gain \( K_y \). If the data are fitted to this third structure, but have, in fact, been generated by the subject's using the second structure, the term \( K_y \) will be zero. Consequently, equation (5) will reduce to an equation which describes the second control structure. Consequently the value of \( K_y \) will indicate which structure was probably in use.

The effective driver-alone transfer function for the third structure was derived by removing \( G^p_{w} \), the car's dynamics in heading, and modelled using:

\[
Y_p = K_\psi (1 + T'_r jw)e^{-j\omega t} \left[ 1 + \frac{G^y(w)}{G^p_jw} \right]
\]

(5)

Table 2 shows the values derived for \( K_\psi \), \( \tau \), \( T'_r \), and \( K_y \) for selected test days, averaged over twelve novice drivers (see also Figure 7).

<table>
<thead>
<tr>
<th>Day</th>
<th>( K^p_\psi ) deg./deg.</th>
<th>( K'_y ) rad./sec.</th>
<th>( \tau ) sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.590</td>
<td>0.20</td>
<td>.42</td>
</tr>
<tr>
<td>2</td>
<td>0.655</td>
<td>0.20</td>
<td>.42</td>
</tr>
<tr>
<td>3</td>
<td>0.595</td>
<td>0.44</td>
<td>.36</td>
</tr>
<tr>
<td>6</td>
<td>0.615</td>
<td>0.71</td>
<td>.37</td>
</tr>
<tr>
<td>9</td>
<td>0.630</td>
<td>0.82</td>
<td>.25</td>
</tr>
</tbody>
</table>

(since \( T'_r = T_r \) (= 9.4 rad./sec. for the test car), \( T'_r = T_r \) was assumed),

\( K'_y = K_y U_0 \) where \( U_0 \) is the forward velocity

**Discussion of Modelling Results**

Data, from experienced drivers, that (in reference 8) was fitted to the third control structure show the amplitude fit to be good across all frequencies measured and the phase fit to be best nearest the crossover frequency. This same type of model fit was obtained with the experimental data. Goodness of fit parameters were calculated using the distance from the modelled to the actual data point, relative to the standard deviation at that point. For the experimental car, the response lag which is offset by the driver's use of heading rate (vs. heading angle) begins to have effect at 9.4 rad./sec. (the break frequency). Though \( 1/T'_r \) is expected to be approximately equal to 9.4 rad./sec., and because this value is far enough outside the measurement frequency range to have little effect on the model anyway, 9.4 rad./sec. was used for the value of \( 1/T'_r \) for all days. It is evident from the fit parameters
in Table 2 that the largest changes occurred in the value of $K_y'$. The smaller changes in the $K_y$ parameter indicate that heading angle was controlled in much the same way on the first day as on the last. In contrast, the value of $K_y'$ doubled between days two and three, moving within the measurement frequency range (i.e., > .377 rad./sec.), and doubled again between days three and nine. This change reflects the large increase in the amplitude of the car driver transfer function at .377 rad./sec. between days two and three. The increases in the value of $K_y'$ point to the increased control of lateral position as defined in the third structure. The phase fits were so poor that very little faith can be placed in the time delay values. However, they do conform to the findings of other researchers that the time delay decreases with learning (Reference 1). A large improvement in the model fit to the phase data occurred over the learning period as the low frequency phase droop became less noticeable (as illustrated in Fig. 7).

CONCLUSIONS

In summary, though the fit parameters do not indicate a sharp division between days one and two and day three, enough of a change in $K_y'$ is indicated to suggest that on day three and thereafter the drivers' control structure bore more resemblance to structure three, where an outer loop controlled lateral position, than to structure two, while on days one and two, the reverse was true.

Other experimenters, using laboratory tracking tasks (Reference 9) have not found changes in strategy with the learning of tracking control but did note improvements in gain and crossover frequency. Using a more complex tracking task, steering a car, such a change in strategy was found to occur, as well as the previously noted change in gain. Phase margin rather than crossover frequency was found to improve with learning indicating that the subjects opted for an improvement in stability of control over improved system response.
REFERENCES


