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EFFECTS OF INFLOW DISTORTION PROFILES ON FAN TONE NOISE CALCULATED USING A 3-D THEORY

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Effects of Inflow Distortion Profiles on Fan Tone Noise Calculated Using a 3-D Theory

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Abstract
Calculations of the fan tone acoustic power and modal structure generated by complex distortions in axial inflow were presented in a previous paper. This work extends the three-dimensional model used in that work to treat the rotor as a rotating three-dimensional cascade and calculates the acoustic field from the distortion-produced dipole distribution on the blades including noncompact source effects. Radial and modal power distribution shapes were synthesized from Fourier-Bessel components representing individual distortion modes. The relation between individual distortion modes and the generated acoustic modes is examined for particular distortion cases. Comparisons between theoretical and experimental results for distortions produced by wakes from upstream radial rods show that the analysis is a good predictor of acoustic power dependence on disturbance strength.

Introduction
Many papers have shown that the fan noise level differences between static testing and projections to forward flight are due to the inflow distortion and turbulence present during most static test conditions. However, the importance of the inflow distortion contribution to fan noise will depend on the particular fan stage and inflow distortion structure. It is therefore of interest to theoretically study the fan noise due to inflow distortion interactions with a three-dimensional annular blade row and to compare the results of the theory to existing experimental data. This theoretical study provides the fan noise modal structure which is required for the prediction of far-field radiation pattern and the design of acoustic suppressors where inflow distortion is the known source mechanism.

A previous paper presented the three-dimensional theoretical analysis of pure tone fan noise generated by inflow distortion/rotor interaction. It extended the three-dimensional unsteady lifting surface theory developed by Nambu to include various shapes of inflow distortion, and predicted the forward and aft radiated pure tone power and modal power distribution. Special emphasis was placed upon the clarification of the role of the three-dimensional and noncompact source effects. The numerical calculations for the full three-dimensional model, a two-dimensional model, and a quasi three-dimensional model were carried out and compared for the case of an inflow distortion which was uniform radially and sinusoidal circumferentially. However, real inflow distortions are expected to have more complex shapes than the simple sinusoidal circumferential variation. Therefore, the present paper explores the character of the predicted tone generation for more complex distortion types. In particular, the relation between the modal content of the inflow distortion and the generated acoustic field is emphasized; and comparisons between the theoretical and experimental results for distortions produced by wakes from upstream radial rods are made.

Analytical Model
Fluctuating Velocity Induced on a Rotor Blade Row
The theoretical model consists of a single three-dimensional annular blade row with \( N_B \) blades rotating at constant angular velocity \( \omega \) in an annular rigid-walled duct of infinite axial extent as shown in Fig. 1(a). (Note that an asterisk means the quantity is dimensional and lengths are non-dimensionalized by \( r_f \), the duct radius.) Thus the duct and reflection, the effect of upstream or down stream blade rows, and the effects of duct area variation are not considered. But, these effects, except for the area variation could be analyzed using the solution procedure of the present study. The fluid flow (see Fig. 1(b)) is composed of an undisturbed flow with a uniform axial velocity \( W_e \) and small fluctuating flows \( \eta_{r,\theta,z} \) due to the inflow distortion. The flow is inviscid, of uniform entropy and has no thermal conductivity. The fluctuations induced on the rotor by the inflow distortion convected by the mean flow are assumed to be isentropic and small compared with the undisturbed flow. It is also assumed that the fluid velocity relative to the blade is subsonic along the whole span and that the blades have no steady load, i.e., a cascade of flat plate airfoils is analyzed.

The fluctuating velocity \( \eta_{r,\theta,z} \) induced by the rotor blade row/distortion interaction can be obtained by integrating the linearized Euler's equation of motion and the upwash component on the blade surface can be expressed in the following form:
\[ q_1(\tau, \theta, z) = -\frac{1}{\rho_0 \omega_a x^2} \sum_{q=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} \int_{-a/2}^{a/2} C_{0,k}(\tau, q, p) K_T(\tau, \theta, z) e^{iq\theta} d\theta \] (1)

where \( C_{0,k} \) is an acoustic dipole distribution on the blade surfaces and \( K_T \) is an upwash kernel function. The upwash kernel function contains parameters of \( N_B \) (blade number), \( h \) (hub-tip ratio), \( q \) and \( p \) (circumferential and radial mode numbers of inflow distortion), \( \omega_T \) (rotor speed/axial flow speed), \( \sigma \) (interblade phase angle) and \( N \) (number of terms in the finite series approximation for \( C_{0,k} \)). The detailed expressions for the kernel function \( K_T \) are given in Ref. 5.

**Inflow Distortion**

In this paper, the inflow distortion is assumed to have only an axial velocity component. (Other components can be considered using the same theoretical methods.) When a Fourier-Bessel analysis of arbitrary shapes of inflow distortion is carried out, the following axial component of external fluctuating velocity is obtained:

\[ \psi_{ea}(\tau, \theta, z, t) = e_a \sum_{q} E_{1,2} R_q \overline{R_{q,p}} e^{iq\theta} \] (2)

where \( R_{p}(k_p, r) \) is a radial eigenfunction of \( q \) order, \( e_a \) denotes a small quantity which is the ratio of the external fluctuating velocity to mean flow velocity and \( E_{1,2} \) are the Fourier coefficients of the inflow distortion. Then, the upwash component of the external fluctuating velocity on a blade surface can be expressed by:

\[ [\psi_{u}, \psi_{w}] = -\psi_{e,a} e^{i\omega_a x} \frac{\omega_a x}{\sqrt{1 + \omega_a^2 x^2}} \] (3)

It is convenient to suppose that the distortion velocity can be expressed as the product:

\[ \psi_{ea} = \psi_{e,a} e^{i\omega_a x} \] (4)

then

\[ B_{q,p} = a_q b_p \] (5)

where

\[ a_q = \frac{1}{2\pi} \int_{0}^{2\pi} e^{-i\phi} \theta_1(\phi) d\phi \] (6)

and

\[ b_p = \int_{h}^{\infty} r \cdot B_q(k_p, r) F_1(r) dr \] (7)

When the inflow distortion can be represented by \( N \) Gaussian profiles (for \( N = 1, 2, \ldots \)) in the circumferential direction, then

\[ \theta_1(\phi) = \exp \left[ -\frac{(\phi - \theta) \cdot \left(1 + \frac{1}{2N} \right)}{\theta_2} \right] \] (8)

and we find that:

\[ a_q = \begin{cases} \frac{-i\pi}{\sqrt{N\pi}} e^{-\pi^2 q^2 \theta_2} e^{i\pi q \theta_2} \left[ \text{Re} \left( \frac{1}{2N} + i\pi q \theta_2 \right) \right] & \text{for } q = 0, \pm 1, \pm 2N, \ldots \\ 0 & \text{otherwise} \end{cases} \] (9)

**Model of Rod Wakes**

For the specific case of an axial velocity distortion produced by the wake of a cylindrical tube or rod of diameter \( d^* \), the magnitude of the wake defect, \( c_a \), and the Gaussian half-width of the wake profile, \( \theta_2 \), are given by the following equations based on Prandtl's mixing length hypothesis (cf., Ref. 7, p. 691).

\[ c_a = \frac{1}{4\sqrt{0.0222}(d^*)^{1/2}} \theta_2 \] (10)

where \( \theta_1(\phi) \) is given by eq. 8 and

\[ \theta_2 = \frac{0.298}{2\pi^{1/2}} \sqrt{\frac{c_a x^*}{\theta_2}} \] (11)

The \( N \) rods are a distance \( x^* \) upstream of the rotor blade tip leading edge and \( C_D \) is the drag coefficient of a rod.

**Determination of Acoustic Dipole Distribution**

The upwash component of the external fluctuating velocity \( q_1, w \) must be cancelled by the induced upwash velocity \( q_1(\tau, \theta, z, t) e^{i\omega_T t} \) at the blade surfaces. Then, an integral equation for the unknown acoustic dipole \( C_{0,k}(\tau, q, p) \) is obtained in the form:
Pure Tone Acoustic Power

The dimensionless acoustic power $E^p$ with respect to the $j$th pure tone fan harmonic (for example, $j = 1$ corresponds to the fundamental) is given by:

$$E^p_j = \sum q \sum_{n=1}^{n_B} \sum_{k=0}^{k_B} \text{EI}_j(n,k,q)$$

and

$$\text{EI}_j(n,k,q) = \left[ \omega_n \left( n \pi q + \omega \right) \right] \left( a_2 + a_4 \right)^2$$

$E^p_j(n,k,q)$ is the modal component of the dimensionless acoustic power, and is nondimensionalized with $\pi/4 \sigma_0 \rho_0 \alpha_0 \beta_0 = 0.2^2$. $\text{EI}_j(n,k,q)$ denotes the nondimensional pressure amplitude in the $(n,k)$ acoustic mode, which is given by:

$$\text{EI}_j(n,k,q) = \frac{\omega_n \left( n \pi q + \omega \right) \left( a_2 + a_4 \right)^2}{\left( a_2 + a_4 \right)^2}$$

$\omega_n = \text{n-th order}$ of inflow distortion whose mode number $q$ with more skewing toward the wall as $q$ increases. The maximum amplitudes of all inflow distortion modes used in these calculations are equal to 1. The numerical results show that while the 1st radial inflow distortion generates the largest number of propagating modes, the calculations were limited to $q \leq 50$. The 1st radial inflow distortion mode has a maximum amplitude at the rotor tip, while the 3rd radial inflow distortion mode peaks near the middle of the blade span. With the radial distortion mode number $p$ held constant, the radial distribution of inflow distortion velocity changes continuously with circumferential distortion mode number $q$, many of which carry more power than $p = 0$. Over a wide range of $q$, many of them carry more power than $p = 2$.

In the case of the 1st radial mode of inflow distortion (Fig. 3), the 1st radial acoustic mode power is larger than other radial acoustic mode powers by more than 5 dB in both upstream and downstream cases, except for the points at $q = 40$ and the downstream values at $q = 50$. At $q = 40$, the circumferential inflow distortion mode number equals the rotor blade number and the circumferential acoustic mode number, $n$, is zero ($n = n_B - q$). The $(0,0)$ acoustic mode is a plane wave. If one accepts the intuitive notion that the acoustic pressures in the various modes will sum in a manner such that
the net radial variation will be similar to the radial variation of distortion, then it is reasonable to expect a combination of 2nd and 3rd radial acoustic modes will exist at \( q = 40 \) in addition to the plane wave. For the cases where \( n \neq 0 \), the \( l = 0 \) modes apparently provide a reasonable radial match to the \((q,0)\) distortion modes.

In the case of the 3rd radial mode of inflow distortion (Fig. 4), the acoustic powers of the higher order radial modes \((l > 2)\) contribute to the total fundamental tone power over the mid-range values of \( q \) which correspond to peak total fundamental power generation. The 3rd radial acoustic mode \((l = 2)\) dominates only at lower values of \( q \) near 20. Note that the \( l = 2 \) acoustic mode in Fig. 4(a) shows a distinct minimum at \( n = 10 \). This is probably a point where the resultant acoustic dipole distribution on the blades cannot couple with the \((10,2)\) acoustic mode similar to the case discussed in connection with Fig. 13 of Ref. 9, which applies to rotor-stator interaction. In general, for cases other than the simplest \((q = 0)\) radial variation of inflow distortion, we are not yet able to rationalize the calculated distribution of radial acoustic mode powers produced by a single higher order radial distortion mode.

**Acoustic Power and Modal Content Generated by a Gaussian Circumferential Inflow Distortion Profile**

Fan No. 2 used in this calculation has a low tip speed and small blade number. Therefore, the total number of acoustic modes generated at a particular circumferential distortion mode number is less than for fan No. 1. Fig. 5 shows the acoustic power generated by Fan No. 2 interacting with a single inflow distortion represented by a Gaussian profile in the circumferential direction and a radial step function velocity defect that extends from the blade tip down to 72 percent of the span. The power is plotted as a function of the half-width of the Gaussian profile, \( \delta_2 \). Results are shown for the total, 1st, 2nd, and 3rd harmonic tone powers propagating in both upstream and downstream directions.

Fig. 5 shows that the acoustic power in any harmonic peaks at particular values of the Gaussian half-width. This phenomenon, mentioned in Ref. 6 and discussed in Ref. 1, is related to an interplay between the number of propagating acoustic modes and the Fourier-Bessel inflow distortion harmonic amplitudes which can couple to the acoustic modes. An example of the circumferential Fourier distortion harmonic can readily be compared to the conditions from the blade profile down to 72 percent of the span. The power is plotted as a function of the half-width of the Gaussian profile, \( \delta_2 \). Results are shown for the total, 1st, 2nd, and 3rd harmonic tone powers propagating in both upstream and downstream directions.

For values of \( \delta_2 \) greater than about 0.02, the downstream propagating acoustic power in the fundamental tone is higher than the upstream power. For the 2nd and 3rd harmonics and the total power, the downstream power is always greater than the upstream power. Of course, in a real fan, downstream radiated power must pass through a stator blade row and is subject to modification.

Figs. 7(a) and (b) compare compact source predictions with the noncompact calculations of Fig. 5 for upstream and downstream propagation, respectively. These figures indicate that the acoustic power differences between corresponding harmonics are considerable for the fundamental and 3rd harmonic propagating upstream and for all harmonics propagating downstream, particularly in the range of \( \delta_2 \) around the peak acoustic power generation.

An explanation for this behavior is as follows: The results in Fig. 7 correspond to summations over the inflow distortion circumferential mode number \( q \). As \( q \) is varied, the noncompact acoustic power in the fundamental tone is sometimes higher and sometimes lower than that based upon the compact prediction (see Fig. 7 in Ref. 5). At large values of \( \delta_2 \), the number of circumferential distortion harmonics which couple to propagating acoustic modes decreases and the net acoustic power differences between noncompact and compact results are considerable and varied in magnitude. At small values of \( \delta_2 \), the number of propagating distortion harmonics increases and the acoustic power differences among modes are averaged out to small net amounts in the summation over the wide range of coupled \( q \)'s for a particular tone harmonic. The compact source predictions in Fig. 7 tend to underestimate the upstream acoustic power and overestimate downstream acoustic power in the fundamental tone. The trends at the 2nd and 3rd harmonics indicate that a compact source assumption underestimates harmonic power for both propagation directions. These results differ from those of Ref. 9 which considered only trends associated with single sinusoidal modes of inflow distortion entering a two-dimensional cascade. The summation over many distortion modes representing more complex shapes such as the Gaussian, makes the effects of neglecting noncompactness specific to the particular case considered.

Figs. 8(a) and (b) show the variation of the acoustic mode structure with the half-width of the Gaussian profile, \( \delta_2 \). Note that the minimum power calculated is 70 dB and that lower values are plotted at 70. The step radial tip distortion assumed favors 1st radial mode generation similar to the results discussed in connection with Fig. 3. For particular modes, clear generation minima are also evident, e.g., for \( \delta_2 = 0.0125 \), \( \delta = 0 \), \( n = 2 \) in Fig. 8(a). The envelopes of the peak powers of the individual modes follow the trends with \( q \) shown in Fig. 6.

**Comparisons of Theory and Experiment**

A Single Tube Wake Interacting with a Rotor

Experiments were conducted in an anechoic wind tunnel in which the variation of upstream radiated fundamental tone power was measured as function of the length of a cylindrical probe tube inserted through inlet wall upstream of rotor No. 2. The wind tunnel was operated with 40 knots velocity and the fan fundamental tone due to rotor-stator interaction was cutoff such that inlet tone noise was dominated by the probe tube wake interacting with the rotor. Fig. 9 shows the measured and calculated tone power (referenced to full immersion) as a func-
...tion of probe immersion. The detailed shape of the probe tip is illustrated in the upper right corner of Fig. 9. The probe tube wake was modeled with Eqs. (9) to (11) as having a Gaussian profile in the circumferential direction. A step profile corresponding to immersion length was used to represent the radial distortion variation. The theoretical results show good agreement with the measured increase in fundamental tone power per length increment of probe immersion. For full immersion, L = 11 cm, the calculated tone power is 130 dB compared with the measured value of 133 dB. The absolute amplitude calculation is subject to some uncertainty associated with the wake modeling; e.g., the effective source region for the complete probe tip. Of more significance is the agreement in the measured power trend with increasing probe immersion.

Fig. 10 shows the variation of acoustic mode powers contributing to the fundamental tone as a function of probe tube immersion. The differences in mode power between 6.7 cm and 11 cm immersion are seen to be small for both the 1st and 2nd radial modes and that the overall power variation shown in Fig. 9 and emphasizing that the generated power is dominated by the fan tip region as previously shown in Ref. 5.

Multiple Rod Wakes Interacting with a Rotor

A JT15D turbofan engine was operated on an outdoor test stand with two separate sets of distortion rods (28 and 41) as means of studying the transmission characteristics of inflow control devices.11 The inflow control greatly reduced fan noise sources associated with inflow disturbances and left the dominant rod wake - rotor interaction as the main source of inlet noise. The fan parameters and operating condition are described in Table I as fan No. 3. The 28 rods were 0.635 cm in diameter and extended from the wall 57.8 percent of the blade span, and the 41 rods were 0.476 cm in diameter and extended from the wall 22.6 percent span. The element of both rod sets was located 10.3 cm upstream of the rotor tip. Fig. 11 compares the calculated and measured fundamental tone power and shows the calculated distribution of modal powers for the two cases. The modal powers are roughly equal for the 28 rod case with the exception of the low 3rd radial which appears to be another example of weak coupling between blade dipoles and the (0,2) acoustic mode. The agreement between measured and calculated tone powers is rather good in each case, but such agreements of isolated powers are subject to some skepticism. The quantity of more interest is the comparison of far-field directivity between measurements and predictions from the calculated modal content. However, several links in the analytical chain relating source modes in an annulus to far-field directivity are missing. The modal scattering in the transition from an annular to a circular duct would be detailed numerical solution as does the detailed radiation from an unbladed inlet duct to the far-field with a superimposed inlet potential flow field. Ref. 12 looks at this same rod disturbance data from the standpoint of inferring source modal content from the far-field directivity. A reconciliation of those results with the present calculation requires additional analysis.

Concluding Remarks

The three-dimensional, noncompact source theory has been applied to calculate fan tone power and detailed acoustic mode structure for several assumed inflow distortions of increasing complexity and for experimental situations where upstream radial rods generated the distortions. While generated radial acoustic mode structure can be rationalized for tip distortions (1st radial disk having modes appear to favor 1st radial acoustic modes), the radial acoustic mode content becomes much more complex for higher order radial distortion modes. Circumferential distortion profiles such as the Gaussian used herein simply generate circumferential acoustic mode content corresponding to the circumferential Fourier harmonic content of the distortion. However, individual acoustic mode levels are subject to a coupling constraint associated with the way the dipoles on the blade surfaces couple to a particular acoustic mode. Poor coupling results in very low values of generated mode power. The use of this minimum generation property to practically reduce fan noise would require that the mode minimized be the sole mode responsible for the particular tone harmonic power generation at the given fan operating condition. The presence of other modes not obeying the particular decoupling constraint would defeat the tone power minimization. Generalizations regarding the magnitude and sign of the differences between compact and the noncompact source results are not available for complex inflow distortions since the net tone powers are the result of summations over many individual modes, each having varied degrees of noncompact dependence. The fact that the analysis is able to predict the trend in tone power as a function of the immersion depth of an upstream rod disturbance is encouraging. An important task remaining is to link acoustic mode content at the fan face to far-field directivity patterns which can be compared with experimental results.

References


### TABLE I. - FAN PARAMETERS

<table>
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<td>Hub/tip ratio, h</td>
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<td>.46</td>
<td>.40</td>
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Figure 1. - Nomenclature for analytical model.
Figure 2. - Radial variations of stagger angle $\gamma$, pitch-chord ratio $S$ and velocity ratio $Q/W_a$ of the rotor.
Figure 3. - Fundamental pure tone power variation with radial inflow distortion given by the Fourier-Bessel 1st radial mode of order \( q \), fan no. 1, maximum amplitude = 1.

(a) UPSTREAM PROPAGATION.

(b) DOWNSTREAM PROPAGATION.
Figure 4. - Fundamental pure tone power variation with radial inflow distortion given by the Fourier-Bessel 3rd radial mode of order q, fan no. 1, maximum amplitude = 1.
h = 0.46  \( N_b = 15 \)  \( M_d = 0.596 \)  \( M_t = 0.865 \)  \( C_a = 0.258 \)

- **TOTAL**
- **FUNDAMENTAL**
- **2nd HARMONIC**
- **3rd HARMONIC**

OPEN - UPSTREAM
SOLID - DOWNSTREAM

Figure 5. - Acoustic power variation with half width of a single Gaussian circumferential inflow distortion profile, fan no. 2, noncompact, 28% tip radial distortion.

Figure 6. - Fourier coefficients of circumferential distortion as a function of single Gaussian distortion width (fan no. 2).
Figure 7. - Comparison of compact and noncompact source acoustic power prediction for a single Gaussian circumferential distortion, fan no. 2, 28% tip radial distortion.
Figure 8. - Variation of the fundamental tone modal structure with half width of Gaussian circumferential distortion profile, 28% tip radial distortion, fan no. 2.
Figure 9. - Fundamental tone acoustic power generated by rotor-probe tube wake interaction, fan n° 2, upstream propagation.
Figure 10. - Variation of fundamental tone modal structure with probe length immersed in a fan inlet, fan no. 2, upstream propagation.
Figure 11. - Modal structure of the fundamental pure tone power generated by rod wakes interacting with a fan, fan no. 3, upstream propagation.