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EFFECTS OF INFLOW DISTORTION PROFILES ON FAN TONE NOISE CALCULATED USING A 3-D THEORY

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TECHNICAL PAPER to be presented at the Fifth Aeroacoustics Conference sponsored by the American Institute of Aeronautics and Astronautics
Seattle, Washington, March 12-14, 1979
Abstract

Calculations of the fan tone acoustic power and modal structure generated by complex distortions in axial inflow are presented. The method used treats the rotor as a rotating three-dimensional cascade and calculates the acoustic field from the distortion-produced dipole distribution on the blades including noncompact source effects. Radial and circumferential distortion effects are synthesized from Fourier-Bessel components representing individual distortion modes. The relation between individual distortion modes and the generated acoustic modes is examined for particular distortion cases. Computational models were carried out and compared for the case of an inflow distortion which was uniform radially and sinusoidal circumferentially. However, real inflow distortions and a quasi three-dimensional model were carried out and compared for the case of an inflow distortion which was uniform radially and sinusoidal circumferentially. Moreover, real inflow distortions are expected to have more complex shapes than the simple sinusoidal circumferential variation. Therefore, the present paper explores the character of the predicted tone generation for more complex distortion types. In particular, the relation between the modal content of the inflow distortion and the generated acoustic field is emphasized; and comparisons between the theoretical and experimental results for distortions produced by wakes from upstream radial rods show that the analysis is a good predictor of acoustic power dependence on disturbance strength.

Introduction

Many papers have shown that the fan noise level differences between static testing and projections to forward flight are due to the inflow distortion and turbulence present during most static test conditions. However, the importance of the inflow distortion contribution to fan noise will depend on the particular fan stage and inflow distortion structure. It is therefore of interest to theoretically study the fan noise due to inflow distortion interactions with a three-dimensional annular blade row and to compare the results of the theory to experimental data. This theoretical study provides the fan noise modal structure which is required for the prediction of far-field radiation patterns and the design of acoustic suppressors where inflow distortion is the known source mechanism.

A previous paper presented the three-dimensional theoretical analysis of pure tone fan noise generated by inflow distortion/rotor interaction. It extended the three-dimensional unsteady lifting surface theory developed by Namba, accepted as input various shapes of inflow distortion, and predicted the forward and aft radiated pure tone power and modal power distribution. Special emphasis was placed upon the clarification of the role of the three-dimensional and noncompact source effects. The numerical calculations for the full three-dimensional model, a two-dimensional model and a quasi three-dimensional model were carried out and compared for the case of an inflow distortion which was uniform radially and sinusoidal circumferentially. However, real inflow distortions are expected to have more complex shapes than the simple sinusoidal circumferential variation. Therefore, the present paper explores the character of the predicted tone generation for more complex distortion types. In particular, the relation between the modal content of the inflow distortion and the generated acoustic field is emphasized; and comparisons between the theoretical and experimental results for distortions produced by wakes from upstream radial rods show that the analysis is a good predictor of acoustic power dependence on disturbance strength.

Analytical Model

Fluctuating Velocity Induced on a Rotor Blade Row

The theoretical model consists of a single three-dimensional annular blade row with N B blades rotating at constant angular velocity ω. In an annular duct, the duct diameter is infinite. Thus, the duct is a three-dimensional annular rigid wall rotating at constant angular velocity ω. The fluid flow is inviscid, of uniform axial velocity We and radial velocity zero. The fluctuating flow field is composed of an undis-
where \( G_{o,k} \) is an acoustic dipole distribution on the blade surfaces and \( K_{T_k} \) is an upwash kernel function. The upwash kernel function contains parameters of \( N ( \text{blade number}) \), \( h (\text{hub-tip ratio}) \), \( q, p (\text{circumferential and radial mode numbers of inflow distortion}) \), \( \omega_r (\text{rotor speed/axial flow speed}) \), \( \sigma (\text{interblade phase angle}) \) and \( L (\text{number of terms in the finite series approximation for } G_{o,k}) \). The detailed expressions for the kernel function \( K_{T_k} \) are given in Ref. 5.

**Inflow Distortion**

In this paper, the inflow distortion is assumed to have only an axial velocity component. (Other components can be considered using the same theoretical methods.) When a Fourier-Bessel analysis of arbitrary shapes of inflow distortion is carried out, the following axial component of external fluctuating velocity is obtained:

\[
\nu_{x,a}(r, \theta, z) = c_a \sum_{q, p} R_q(k, q, p) e^{i(q \theta + p \phi)} \text{ for } q \neq 0
\]

(2)

where \( R_q(k, q, p) \) is a radial eigenfunction of \( q \) order, \( c_a \) denotes a small quantity which is the ratio of the external fluctuating velocity to mean flow velocity and \( R_{0,0} \) are the Fourier coefficients of the inflow distortion. Then, the upwash component of the external fluctuating velocity on a blade surface can be expressed by:

\[
\nu_{x,u}(r, \theta, z) = -i \Omega \nu_{x,a}(r, \theta, z) / \Omega
\]

(3)

where \( \Omega \) is a convenient to suppose that the distortion velocity can be expressed as the product:

\[
\nu_{x,u}(r, \theta, z) = \nu_{x,a}(r, \theta, z) \Omega
\]

then

\[
B_{q,p} = a_q b_p
\]

where

\[
a_q = \frac{1}{2\pi} \int_0^{2\pi} e^{-i\Omega \theta} \Theta_q(\delta) d\theta
\]

(6)

and

\[
b_p = \int_H r^* R_q(k, q, p) F_1(r) dr
\]

(7)

When the inflow distortion can be represented by \( N \) Gaussian profiles (for \( N = 1, 2, \ldots \)) in the circumferential direction, then

\[
\Theta_q(\delta) = \exp \left[ \left( \frac{\delta}{\theta} - \frac{1 + 2\pi j}{2N} \right)^2 \right] \text{ for } j = 0, 1, 2, \ldots, N-1
\]

(8)

**Model of Rod Wakes**

For the specific case of an axial velocity distortion produced by the wake of a cylindrical tube or rod of diameter \( d^* \); the magnitude of the wake defect, \( c_a \), and the Gaussian half-width of the wake profile, \( \sigma \), are given by the following equations based on Prandtl's mixing length hypothesis (cf., Ref. 7, p. 691).

\[
e_a = \frac{1}{4\sqrt{0.0222n(d^*)^{1/2}}}
\]

(10)

where \( \Theta_q(\delta) \) is given by eq. 8 and

\[
\Theta_2 = \frac{0.298}{2\pi \sigma} \sqrt{C_D \sigma^2}
\]

(11)

The \( N \) rods are a distance \( x^* \) upstream of the rotor blade tip leading edge and \( C_D \) is the drag coefficient of a rod.

**Determination of Acoustic Dipole Distribution**

The upwash component of the external fluctuating velocity \( \nu_{x,u} \) must be cancelled by the induced upwash velocity \( \nu_{x,u} \) at the blade surfaces. Then, an integral equation for the unknown acoustic dipole \( G_{o,k}(r, q, p) \) is obtained in the form:

\[
\int_0^{2\pi} e^{-i\Omega \theta} \Theta_q(\delta) d\theta = 0
\]

(4)
The quantity $\tilde{C}_{n,R}q(n,R,q)$ is calculated from eq. (12) by a collocation method.

### Pure Tone Acoustic Power

The dimensionless acoustic power $E_j^1$ with respect to the $j$th pure tone fan harmonic (for example, $j = 1$ corresponds to the fundamental) is given by:

$$E_j^1 = \sum_q \sum_{n=1}^{N_R} \sum_{\ell=0}^{2} E_{j,\ell}^1(n,R,q)$$

and

$$E_{j,\ell}^1 = \left[ H_j^\ell(n,R,q) \right]^2 \frac{2}{\pi} \frac{(n\omega_T + \omega)}{(n\omega_T + \omega)^2} \frac{|n_{1,1}|}{(a_2 + m_\omega + \omega)^2}$$

$E_{j,\ell}^1(n,R,q)$ is the modal component of the dimensionless acoustic power, and is nondimensionalized with $\pi/4 \Omega R^2$. $H_j^\ell(n,R,q)$ denotes the non-dimensional pressure amplitude in the $(n,l)$ acoustic mode, which is given by:

$$H_j^\ell(n,R,q) = \frac{N_R}{4\pi \beta_a} \sum_{k=0}^{\infty} \left[ \frac{\omega_a^2}{\beta_{n,l,k}} \right] \int_{-a_2/2}^{a_2/2} \int_{-c_a/2}^{c_a/2} g_{n,k}(c,q,p) \exp \left[ -i(\alpha_x + m_\omega \eta) \xi \right] d\xi$$

$\alpha_x$, $\eta$, $\xi$ are the axial wave number nondimensionalized by $\nu$. In eqs. (12) to (15) the $+$ and $-$ subscripts refer to downstream and upstream propagation, respectively. For the compact source analysis, $\exp \left[ -i(\alpha_x + m_\omega \eta) \xi \right] = 1$ in eq. (15).

### Numerical Results

The numerical calculations are carried out in this paper for three different rotor configurations. Their overall geometric and aerodynamic parameters are given in Table I and the radial variations of stagger angle, $\gamma$, pitch-chord ratio, $S$, and ratio of blade relative to axial velocity, $Q/W$, are shown in Fig. 2. The numerical results include four cases: (1) the acoustic power in the modal component generated by particular Fourier-Bessel components of inflow distortion in the case of the fan denoted No. 1; (2) the acoustic power and modal content variations with the half-width of a single Gaussian profile representing the local inflow distortion profile in the circumferential direction (fan No. 2); (3) the comparison between the theoretical and experimental tone power for the case of an upstream distortion produced by the wake of a cylindrical tube immersed to varying depths from the duct wall (fan No. 3); and (4) the comparison between numerical and experimental tone powers and the analysis of modal content for the cases of 28 and 41 rod wakes interacting with a 28 blade rotor (fan No. 3).

### Acoustic Modal Power Generated by Particular Fourier-Bessel Inflow Distortion Modes

Figs. 3 and 4 show the fundamental pure tone modal powers as a function of the circumferential lobe number $q$ of the inflow distortion whose radial distribution is assumed to be the 1st (Fig. 3) or 3rd (Fig. 4) radial eigenfunction of order $q$. Figs. 3(a) and 4(a) are for upstream propagation, while Figs. 3(b) and 4(b) are for downstream propagation. The fan used in this calculation has the largest number of blades of the three cases as shown in Table I and, for similar tip speed conditions, generates the largest number of propagating modes. The calculations were limited to $q \leq 50$. The lat radial inflow distortion mode has a maximum amplitude at the rotor tip, while the 3rd radial inflow distortion mode peaks near the middle of the blade span. With the radial distortion mode number $p$ held constant, the radial distribution of inflow distortion velocity changes continuously with circumferential distortion mode number $q$ with more skewing toward the wall as $q$ increases. The maximum amplitudes of all inflow distortion modes used in these calculations are equal to 1. The numerical results show that while the lat radial mode of inflow distortion generates all radial acoustic modes, the lat radial acoustic mode, $n = 0$, dominates for most values of $q$. The 3rd radial mode of inflow distortion also generates all radial acoustic modes, but, over a wide range of $q$, many of them carry more power than $p = 8$ or 2.

In the case of the lat radial mode of inflow distortion (Fig. 3), the lat radial acoustic mode power is larger than other radial acoustic mode powers by more than 5 dB in both upstream and downstream cases, except for the points at $q = 40$ and the downstream values at $q = 50$. At $q = 40$, the circumferential inflow distortion mode number equals the rotor blade number and the circumferential acoustic mode number, $n$, is zero ($n = J_0 - q$). The $(0,0)$ acoustic mode is a plane wave. If one accepts the intuitive notion that the acoustic pressures in the various modes will sum in a manner such that
the net radial variation will be similar to the radial variation of distortion, then it is reasonable to expect a combination of 2nd and 3rd radial acoustic modes will exist at \( q = 20 \) in addition to the plane wave. For the cases where \( n \neq 0 \), the \( k = 0 \) modes apparently provide a reasonable radial match to the \((q,0)\) distortion modes.

In the case of the 3rd radial mode of inflow distortion (Fig. 4), the acoustic powers of the higher order radial modes \((k \neq 2)\) contribute to the total fundamental tone power over the midspan values of \( q \) which correspond to peak total fundamental power generation. The 3rd radial acoustic mode \((k = 2)\) dominates only at lower values of \( q \) near 20. Note that the \( k = 2 \) acoustic mode in Fig. 4(a) shows a distinct minimum at \( n = 10 \). This is probably a point where the resultant acoustic dipole distribution on the blades cannot couple with the \((10,2)\) acoustic mode similar to the case discussed in connection with Fig. 13 of Ref. 5 which applied to rotor-stator interaction. In general, for cases other than the simplest \((p = 0)\) radial variation of inflow distortion, we are not yet able to rationalize the calculated distribution of radial acoustic mode powers produced by a single higher order radial distortion mode.

**Acoustic Power and Modal Content Generated by a Gaussian Circumferential Inflow Distortion Profile**

Fan No. 2 used in this calculation has a low tip speed and small blade number. Therefore, the total number of acoustic modes generated at a particular circumferential distortion mode number is less than for fan No. 1. Fig. 5 shows the acoustic power generated by fan No. 2 interacting with a single inflow distortion represented by a Gaussian profile in the circumferential direction and a radial step function velocity defect that extends from the blade tip down to 72 percent of the span. The power is plotted as a function of the half-width of the Gaussian profile, \( \Theta_2 \). Results are shown for the total, 1st, 2nd, and 3rd harmonic tone powers propagating in both upstream and downstream directions.

Fig. 5 shows that the acoustic power in any harmonic peaks at particular values of the Gaussian half-width. This phenomenon, mentioned in Ref. 6 and discussed in Ref. 1, is related to an interplay between the number of propagating acoustic modes and the Fourier-Bessel inflow distortion harmonic amplitudes which can couple to the acoustic modes. An example of the circumferential Fourier distortion harmonic amplitudes and the corresponding modal amplitudes is shown in Fig. 5. The Fourier harmonics which in couple to propagating acoustic modes decreases and the net acoustic power differences between noncompact and compact results are considerable and varied in magnitude. At small values of \( \Theta_2 \), the number of coupling distortion harmonics increases and therefore, the acoustic power differences among modes are averaged out to small net amounts in the summation over the wide range of coupled \( q \) 's for a particular tone harmonic. The compact source predictions in Fig. 7 tend to underestimate the upstream acoustic power and overestimate downstream acoustic power in the fundamental tone. The trends at the 2nd and 3rd harmonics indicate that a compact source assumption underestimates harmonic power for both propagation directions. These results differ from those of Ref. 9 which considered only trends associated with single sinusoidal modes of inflow distortion entering a two-dimensional cascade. The summation over many distortion modes representing noncompact shapes such as the Gaussian, makes the effects of neglecting noncompactness specific to the particular case considered.

Figs. 6(a) and (b) show the variation of the acoustic mode structure with the half-width of the Gaussian profile, \( \Theta_2 \). Note that the minimum power calculated was \(-70 \text{ dB} \) and that lower values are plotted at \(-70 \). The step radial tip distortion assumed favors 1st radial mode generation similar to the results discussed in connection with Fig. 3. For particular modes, clear generation minima are also evident, e.g., for \( \Theta_2 = 0.0125, k = 0, n = 2 \) in Fig. 6(a). The envelopes of the peak powers of the individual modes follow the trends with \( q \) shown in Fig. 6.

**Comparisons of Theory and Experiment**

**A Single Tube Wake Interacting with a Rotor**

Experiments were conducted in an anechoic wind tunnel in which the variation of upstream radiated fundamental tone power was measured as function of the length of a cylindrical probe tube inserted through inlet wall upstream of rotor No. 2. The wind tunnel was operated with 40 knots velocity and the fan fundamental tone due to rotor-stator interaction was cutoff such that inlet tone noise was dominated by the probe tube wake interacting with the rotor. Fig. 9 shows the measured and calculated tone power (referenced to full immersion) as a func-
Concluding Remarks

The three-dimensional, noncompact source theory has been applied to calculate fan tone power and detailed acoustic mode structure for several assumed inflow distortions of increasing complexity and for experimental situations where upstream radial rods generated the distortions. While generated radial acoustic mode structure can be rationalized for tip distortions (for radial distortion modes that favor radial acoustic modes), the radial acoustic mode content becomes much more complex for higher order radial distortion modes. Circumferential distortion profiles such as the Gaussian used herein simply generate circumferential acoustic mode content corresponding to the circumferential Fourier harmonic content of the distortion. However, individual acoustic mode levels are subject to a coupling constraint associated with the way the dipole on the blade surfaces couple to a particular acoustic mode. Poor coupling results in very low values of generated mode power. The use of this minimum generation property to practically reduce fan noise would require that the mode minimized be the single mode responsible for the particular tone harmonic power generation at the given fan operating condition. The presence of other modes not obeying the particular decoupling constraint would defeat the tone power minimization. Generalizations regarding the magnitude and sign of the differences between compact and noncompact source results are not available for complex inflow distortions since the net tone powers are the result of summations over many individual modes, each having varied degrees of noncompact dependence. The fact that the analysis is able to predict the trend in tone power as a function of the immersion depth of an upstream rod disturbance is encouraging. An important task remaining is to link acoustic mode content at the fan face to far-field directivity patterns which can be compared with experimental results.

References


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**TABLE I. - FAN PARAMETERS**

<table>
<thead>
<tr>
<th>Fan number</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotord blade number, ( N_b )</td>
<td>40</td>
<td>15</td>
<td>28</td>
</tr>
<tr>
<td>Dimensionless axial tip chord length, ( C_a )</td>
<td>0.055</td>
<td>0.258</td>
<td>0.094</td>
</tr>
<tr>
<td>Rotord tip speed/axial flow speed, ( \omega_c )</td>
<td>2.474</td>
<td>1.052</td>
<td>3.011</td>
</tr>
<tr>
<td>Rotord relative Mach number, ( M_r )</td>
<td>0.934</td>
<td>0.865</td>
<td>0.913</td>
</tr>
<tr>
<td>Axial flow Mach number, ( M )</td>
<td>0.350</td>
<td>0.596</td>
<td>0.288</td>
</tr>
<tr>
<td>Hub/tip ratio, ( h )</td>
<td>0.40</td>
<td>0.46</td>
<td>0.40</td>
</tr>
</tbody>
</table>
Figure 1. - Nomenclature for analytical model.
Figure 2. - Radial variations of stagger angle $\gamma$, pitch-chord ratio $S$ and velocity ratio $Q/W_a$ of the rotor.
Figure 3. - Fundamental pure tone power variation with radial inflow distortion given by the Fourier-Bessel 1st radial mode of order $q$, fan no. 1, maximum amplitude = 1.
Figure 4. Fundamental pure tone power variation with radial inflow distortion given by the Fourier-Bessel 3rd radial mode of order $q$, fan no. 1, maximum amplitude $= 1$. 

(a) UPSTREAM PROPAGATION.

(b) DOWNSTREAM PROPAGATION.
h = 0.46 \quad N_B = 15 \quad M_B = 0.596 \quad M_T = 0.865 \quad C_a = 0.258

- TOTAL
- Δ FUNDAMENTAL
- □ 2nd HARMONIC
- ○ 3rd HARMONIC

OPEN - UPSTREAM
SOLID - DOWNSTREAM

Figure 5. - Acoustic power variation with half width of a single Gaussian circumferential inflow distortion profile, fan no. 2, noncompact, 28% tip radial distortion.

Figure 6. - Fourier coefficients of circumferential distortion as a function of single Gaussian distortion width (fan no. 2).
Figure 7. - Comparison of compact and noncompact source acoustic power prediction for a single Gaussian circumferential distortion, fan no. 2, 28% tip radial distortion.
Figure 8. Variation of the fundamental tone modal structure with half width of Gaussian circumferential distortion profile, 28% tip radial distortion, fan no. 2.
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Figure 10. - Variation of fundamental tone modal structure with probe length immersed in a fan inlet, fan no. 2, upstream propagation.
Figure 11. - Modal structure of the fundamental pure tone power generated by rod wakes interacting with a fan, fan no. 3, upstream propagation.