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EFFECTS OF INFLOW DISTORTION PROFILES
ON FAN TONE NOISE CALCULATED
USING A 3-D THEORY

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TECHNICAL PAPER to be presented at the
Fifth Aeroacoustics Conference
sponsored by the American Institute of
Aeronautics and Astronautics
Seattle, Washington, March 12-14, 1979
EFFECTS OF INFLOW DISTORTION PROFILES ON FAN TONE NOISE CALCULATED USING A 3-D THEORY

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Abstract

Calculations of the fan tone acoustic power and modal structure generated by complex distortions in axial inflow are presented. Theoretical results are obtained using an inviscid, noncompact theory and shown to be a good predictor of acoustic power dependence on disturbance strength. The modal content of the inflow distortion and the generated acoustic field is emphasized; and comparisons between the theoretical and experimental results for distortions produced by wakes from upstream radial rods show that the analysis is a good predictor of acoustic power dependence on disturbance strength.

Introduction

Many papers have shown that the fan noise level differences between static testing and projections to forward flight are due to the inflow distortion and turbulence present during most static test conditions. However, the importance of the inflow distortion contribution to fan noise will depend on the particular fan stage and inflow distortion structure. It is therefore of interest to theoretically study the fan noise due to inflow distortion interactions with a three-dimensional annular blade row and to compare the results of the theory to experimental data. This theoretical study provides the fan noise modal structure which is required for the prediction of far-field radiation patterns and the design of acoustic suppressors where inflow distortion is the known source mechanism.

A previous paper presented the three-dimensional theoretical analysis of pure tone fan noise generated by inflow distortion/rotor interaction. It extended the three-dimensional unsteady lifting surface theory developed by Namba, accepted as input various shapes of inflow distortion, and predicted the forward and aft radiated pure tone power and modal power distribution. Special emphasis was placed upon the clarification of the role of the three-dimensional and noncompact source effects. The numerical calculations for the full three-dimensional model, a two-dimensional model and a quasi three-dimensional model were carried out and compared for the case of an inflow distortion which was uniform radially and sinusoidal circumferentially. The real inflow distortions are expected to have more complex shapes than the simple sinusoidal circumferential variation. Therefore, the present paper explores the character of the predicted tone generation for more complex distortion types. In particular, the relation between...
\[
q_1(r,\theta,z) = -\frac{1}{\rho_0} \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} \frac{N_B}{2\pi} \int_{-\infty}^{\infty} G_0, k_2 \left( r, q_1, p \right) K_T \left( r, \theta, z/n, q, p \right) d\theta (1)
\]

where \( G_0, k \) is an acoustic dipole distribution on the blade surfaces and \( K_T \) is an upwash kernel function. The upwash kernel function contains parameters of \( N_B \) (blade number), \( h \) (hub-tip ratio), \( q \) and \( p \) (circumferential and radial mode numbers of inflow distortion), \( \omega_r \) (rotor speed/axial flow speed), \( \sigma \) (interblade phase angle) and \( L \) (number of terms in the finite series approximation for \( G_0, k \)). The detailed expressions for the kernel function \( K_T \) are given in Ref. 5.

**Inflow Distortion**

In this paper, the inflow distortion is assumed to have only an axial velocity component. (Other components can be considered using the same theoretical method.) When a Fourier-Bessel analysis of arbitrary shapes of inflow distortion is carried out, the following axial component of external fluctuating velocity is obtained:

\[
\nu_{a,n}(r,\theta) = e_a \sum_{q,p} B_{q,p} R_0, k_2 \left( r, q_1, p \right) e^{-iq_2(\theta-\theta_0)} (2)
\]

where \( R_0, k_2 \) is a radial eigenfunction of \( q \) order, \( e_a \) denotes a small quantity which is the ratio of the external fluctuating velocity to mean flow velocity and \( B_{q,p} \) are the Fourier coefficients of the inflow distortion. Then, the upwash component of the external fluctuating velocity on a blade surface can be expressed by:

\[
\nu_{a,n}(r,\theta) = -\nu_{e,n} e^{-iq_2(\theta-\theta_0)} \frac{\omega_r}{\sqrt{1 + \omega_r^2}} (3)
\]

It is convenient to suppose that the distortion velocity can be expressed as the product:

\[
\nu_{a,n}(r,\theta) = P_1(r) \Theta_1(\theta) \Theta_2(\theta) (4)
\]

then

\[
B_{q,p} = a_q b_p (5)
\]

where

\[
a_q = \frac{1}{2\pi} \int_0^{2\pi} e^{-i\theta_2(\theta)} d\theta (6)
\]

and

\[
b_p = \int_0^\infty r^2 R_q(k_2, p) F_1(r) dr (7)
\]

When the inflow distortion can be represented by \( N \) Gaussian profiles (for \( N = 1, 2, \ldots \)) in the circumferential direction, then

\[
\Theta_1(\theta) = \exp \left[ \frac{1}{2} \left( \frac{\theta - 1 + 2\pi j}{2\pi} N - 1 \right)^2 \right] \quad \text{for } j = 0, 1, 2, \ldots, N-1 (8)
\]

and we find that:

\[
a_q = \left\{ \begin{array}{ll}
\frac{-i\pi}{\sqrt{2\pi}N^2} e^{-\pi^2 q_2^2} & \text{for } q = 0, \pm N, \pm 2N, \\
0 & \text{otherwise, for } q \neq 0, \pm N, \pm 2N, \ldots
\end{array} \right. (9)
\]

**Model of Rod Wakes**

For the specific case of an axial velocity distortion produced by the wake of a cylindrical tube or rod of diameter \( d^* \); the magnitude of the wake defect, \( \epsilon_a \), and the Gaussian half-width of the wake profile, \( \epsilon_2 \), are given by the following equations based on Prandtl's mixing length hypothesis (cf., Ref. 7, p. 691).

\[
\epsilon_a = \frac{1}{4\sqrt{0.0222m_0 d^*}} \Theta_1(\theta) (10)
\]

where \( \Theta_1(\theta) \) is given by eq. 8 and

\[
\epsilon_2 = \frac{0.298}{\pi} \sqrt{\frac{C_D}{\pi}} (11)
\]

The \( N \) rods are a distance \( \pi^2 d^* \) upstream of the rotor blade tip leading edge and \( C_D \) is the drag coefficient of a rod.

**Determination of Acoustic Dipole Distribution**

The upwash component of the external fluctuating velocity \( q_1, w \) must be cancelled by the induced upwash velocity \( q_{1,W}(r,\theta, z, t) \) at the blade surfaces. Then, an integral equation for the unknown acoustic dipole \( G_0, k_2(r, q_1, p) \) is obtained in the form:

\[
b_p = \int_0^\infty r^2 R_q(k_2, p) F_1(r) dr (7)
\]
The quantity $\tilde{B}_{n,m}(n,R,q)$ is calculated from eq. (12) by a collocation method.

### Pure Tone Acoustic Power

The dimensionless acoustic power $E_j^q$ with respect to the $j$th pure tone fan harmonic (for example, $j = 1$ corresponds to the fundamental) is given by:

$$E_j^q = \sum_q \sum_{n=0}^{n_{NB}-q} \sum_{\ell=0}^{n} E_{j}^q(n,\ell,q)$$

(13)

and

$$E_{j}^q = \frac{|H_{p_{E}^q}(n,\ell,q)|^2}{(\omega_n + \omega_p - \omega)^2}
\int_{-(n,\ell,q)}^{(n,\ell,q)} \frac{C_{n,\ell,q}}{2} \int_{-C_{n,\ell,q}/2}^{C_{n,\ell,q}/2} G_{a,k}(c,q,p) \exp\left[-i(\omega_n + \omega_p)\xi\right]d\xi$$

(14)

E_{j}^q(n,\ell,q) is the modal component of the dimensionless acoustic power, and is nondimensionalized with $n/4 \sigma_0 \omega_3^2 \omega_2^2$. $E_{j}^q(n,\ell,q)$ denotes the nondimensional pressure amplitude in the $(n,\ell)$ acoustic mode, which is given by:

$$E_j^q = \sum_q \sum_{n=0}^{n_{NB}-q} \sum_{\ell=0}^{n} E_{j}^q(n,\ell,q)$$

(13)

and

$$E_{j}^q = \frac{|H_{p_{E}^q}(n,\ell,q)|^2}{(\omega_n + \omega_p - \omega)^2}
\int_{-(n,\ell,q)}^{(n,\ell,q)} \frac{C_{n,\ell,q}}{2} \int_{-C_{n,\ell,q}/2}^{C_{n,\ell,q}/2} G_{a,k}(c,q,p) \exp\left[-i(\omega_n + \omega_p)\xi\right]d\xi$$

(14)

### Numerical Results

The numerical calculations are carried out in this paper for three different rotor configurations. Their overall geometric and aerodynamic parameters are given in Table I and the radial variations of number $q$ of the inflow distortion whose radial distribution is assumed to be the 1st radial mode of inflow distortion (Fig. 3) or 3rd radial eigenfunction of order $q$. Figs. 3(a) and 4(a) are for upstream propagation, while Figs. 3(b) and 4(b) are for downstream propagation. The fan was in this calculation the largest number of blades of the three cases as shown in Table I and, for similar tip speed conditions, generate the largest number of propagating modes. The calculations were limited to $q \leq 50$. The 3rd radial inflow distortion mode has a maximum amplitude at the rotor tip, while the 3rd radial inflow distortion mode has peaks near the middle of the blade span. With the radial distortion mode number $p$ held constant, the radial distribution of inflow distortion velocity changes continuously with circumferential distortion mode number $q$ with more skewing toward the wall as $q$ increases. The maximum amplitudes of all inflow distortion modes used in these calculations are equal to 1. The numerical results show that while the 1st radial mode of inflow distortion generates all radial acoustic modes, the 3rd radial acoustic mode, $k = 0$, dominates for most values of $q$. The 3rd radial mode of inflow distortion also generates all radial acoustic modes; but, over a wide range of $q$, many of them carry more power than $p = \ell = 2$.

In the case of the 1st radial mode of inflow distortion (Fig. 3), the 1st radial acoustic mode power is larger than other radial acoustic mode powers by more than 5 dB in both upstream and downstream cases, except for the points at $q = 40$ and the downstream value at $q = 50$. At $q = 40$, the circumferential inflow distortion mode number equals the rotor blade number and the circumferential acoustic mode number, $n$, is zero $(n = n_{NB} - q)$. The $(0,0)$ acoustic mode is a plane wave. If one accepts the intuitive notion that the acoustic pressures in the various modes will sum in a manner such that
the net radial variation will be similar to the radiative variation of distortion, then it is reasonable to expect a combination of 2nd and 3rd radial acoustic modes will exist at \( q = 40 \) in addition to the plane wave. For the cases where \( n \neq 0 \), the \( k = 0 \) modes apparently provide a reasonable radial match to the \((q,0)\) distortion modes.

In the case of the 3rd radial mode of inflow distortion (Fig. 4), the acoustic powers of the higher order radial modes \((k > 2)\) contribute to the total fundamental tone power over the midrange values of \( q \) which correspond to peak total fundamental power generation. The 3rd radial acoustic mode \((k = 2)\) dominates only at lower values of \( q \) near 20. Note that the \( k = 2 \) acoustic mode in Fig. 4(a) shows a distinct minimum at \( q = 10 \). This is probably a point where the resultant acoustic dipole distribution on the blades cannot couple with the \((10,2)\) acoustic mode similar to the case discussed in connection with Fig. 13 of Ref. 9 which applied to rotor-stator interaction. In general, for case other than the simplest \((p = 0)\) radial variation of inflow distortion, we are not yet able to rationalize the calculated distribution of radial acoustic mode powers produced by a single higher order radial distortion mode.

**Acoustic Power and Modal Content Generated by a Gaussian Circumferential Inflow Distortion Profile**

Fan No. 2 used in this calculation has a low tip speed and small blade number. Therefore, the total number of acoustic modes generated at a particular circumferential distortion mode number is less than for fan No. 1. Fig. 5 shows the acoustic power generated by fan No. 2 interacting with a single inflow distortion represented by a Gaussian profile in the circumferential direction and a radial step function velocity defect that extends from the blade tip down to 72 percent of the span. The power is plotted as a function of the half-width of the Gaussian profile, \( \sigma \). Results are shown for the total, 1st, 2nd, and 3rd harmonic tone powers propagating in both upstream and downstream directions.

Fig. 5 shows that the acoustic power in any harmonic peaks at particular values of the Gaussian half-width. This phenomenon, mentioned in Ref. 6 and discussed in Ref. 1, is related to an interplay between the number of propagating acoustic modes and the Fourier-Bessel inflow distortion harmonic amplitudes which can couple to the acoustic modes. An example of the circumferential Fourier distortion harmonics can yield significant differences for the cases considered.

The results discussed in connection with Fig. 3. For particular modes, clear generation minima are also evident, e.g., for \( n = 2 \) and \( q = 0 \), the power is shown in Fig. 6 the envelopes of the peak powers of the individual modes follow the trends with \( q \).

**Comparisons of Theory and Experiment**

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**A Single Tube Wake Interacting with a Rotor**

Experiments were conducted in an anechoic wind tunnel in which the variation of upstream radiated fundamental tone power was measured as function of the length of a cylindrical probe tube inserted through inlet wall upstream of rotor No. 2. The wind tunnel was operated with 40 knots velocity and the fan fundamental tone due to rotor-stator interaction was cut off such that inlet tone noise was dominated by the probe tube wake interacting with the rotor. Fig. 9 shows the measured and calculated tone power (referenced to full immersion) as a func-

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Figs. 7(a) and (b) compare compact source predictions with the noncompact calculations of Fig. 5 for upstream and downstream propagation, respectively. These figures indicate that the acoustic power differences between corresponding harmonics are considerably for the fundamental and 3rd harmonic propagating upstream and for all harmonics propagating downstream, particularly in the range of \( \theta_2 \) around the peak acoustic power generation.
of probe immersion. The detailed shape of the probe tip is illustrated in the upper right corner of Fig. 9. The probe tube wake was modeled with Eqs. (9) to (11) as having a Gaussian profile in the circumferential direction. A step profile corresponding to immersion length was used to represent the radial distortion variation. The theoretical results show good agreement with the measured increase in fundamental tone power per length increment of probe immersion. For full immersion, L = 11 cm, the calculated tone power is 130 dB compared with the measured value of 133 dB. The absolute amplitude calculated is subject to some uncertainty associated with the wake modeling; e.g., the effective length corresponding to immersion length was used to represent the complex probe tip. Of more significance is the agreement in the measured power trend with increasing probe immersion.

Fig. 10 shows the variation of acoustic mode powers contributing to the fundamental tone as a function of probe tube immersion. The differences in mode power between 6.7 cm and 11 cm immersion are seen to be small for both the 1st and 2nd radial modes echoing the overall power variation shown in Fig. 9 and emphasizing that the generated power is dominated by the fan tip region as previously shown in Ref. 5.

Multiple Rod Wake Interacting with a Rotor

A JT15D turbofan engine was operated on an outdoor test stand with two separate sets of disturbance rods (28 and 41) as a means of studying the transmission characteristics of inflow control devices. The inflow control greatly reduced fan noise sources associated with inflow disturbances and left the dominant rod wake - rotor interaction as the main source of inlet noise. The inflow and operating condition are described in Table I as fan No. 3. The 28 rods were 0.635 cm in diameter and extended from the wall 57.8 percent of the blade span, and the 41 rods were 0.476 cm in diameter and extended from the wall 62.5 percent span. The centerline of both rod sets was located 10.3 cm upstream of the rotor tip. Fig. 11 compares the calculated and measured fundamental tone power and shows the calculated distribution of modal powers for the two cases. The modal powers are roughly equal for the 28 rod case with the exception of the low 3rd radial which appears to be another example of weak coupling between blade dipoles and the (0,2) acoustic mode. The agreement between measured and calculated tone powers is rather good in each case, but such agreements of isolated powers are subject to some skepticism. The quantity of more interest is the comparison of far-field directivity between measurements and predictions from the calculated modal content. However, several links in the analytical chain relating source modes in an annulus to far-field directivity are missing. The modal scattering in the transition from an annular to a circular duct would be described numerically. The solution as does the detailed radiation from an untagged inlet duct to the far-field with a superimposed inlet potential flow field. Ref. 12 looks at this same rod disturbance data from the standpoint of inferring source modal content from the far-field directivity. A reconciliation of those results with the present calculation requires additional analysis.

Concluding Remarks

The three-dimensional, noncompact source theory has been applied to calculate fan tone power and detailed acoustic mode structure for several assumed inflow distortions of increasing complexity and for experimental situations where upstream radial rods generated the distortions. While generated radial acoustic mode structure can be rationalized for tip distortions (1st radial disk having modes appear to favor 1st radial acoustic modes), the radial acoustic mode content becomes much more complex for higher order radial distortion modes. Circumferential distortion profiles such as the Gaussian used herein simply generate circumferential acoustic mode content corresponding to the circumferential Fourier harmonic content of the distortion. However, individual acoustic mode levels are subject to a coupling constraint associated with the way the dipoles on the blade surfaces couple to a particular acoustic mode. Poor coupling results in very low values of generated mode power. The use of this minimum generation property to practically reduce fan noise would require that the mode minimized be the solo mode responsible for the particular tone harmonic power generation at the given fan operating condition. The presence of other modes not obeying the particular decoupling constraint would defeat the tone power minimization. Generalizations regarding the magnitude and sign of the differences between compact and the noncompact source results are not available for complex inflow distortions since the net tone powers are the result of summations over many individual modes, each having varied degrees of noncompact dependence. The fact that the analysis is able to predict the trend in tone power as a function of the immersion depth of an upstream rod disturbance is encouraging. An important task remaining is to link acoustic mode content at the fan face to far-field directivity patterns which can be compared with experimental results.

References


<table>
<thead>
<tr>
<th>TABLE I. - FAN PARAMETERS</th>
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<tr>
<td>Fan number</td>
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<tr>
<td>---------------------------</td>
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<tr>
<td>Rotor blade number, N_b</td>
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<tr>
<td>Dimensionless axial tip chord length, C_a</td>
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<tr>
<td>Rotor tip speed/axial flow speed, C_f</td>
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<tr>
<td>Rotor relative Mach number, M_r</td>
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<td>Axial flow Mach number, M_A</td>
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</table>
(a) FAN GEOMETRY AND COORDINATE SYSTEM.

(b) INFLOW DISTORTION AND BLADE ROW PARAMETERS AT A CONSTANT RADIUS.

Figure 1. - Nomenclature for analytical model.
Figure 2. - Radial variations of stagger angle $\gamma$, pitch-chord ratio $S$ and velocity ratio $Q/W_a$ of the rotor.
Figure 3. Fundamental pure tone power variation with radial inflow distortion given by the Fourier-Bessel 1st radial mode of order q, fan no. 1, maximum amplitude = 1.
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