FINAL REPORT

"DESIGN, FABRICATION AND DELIVERY OF A PROTOTYPE Saturator FOR ACPL"

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CONTRACT TITLE: PRELIMINARY DESIGN AND DEVELOPMENT OF THE ACPL SATURATOR

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>1-1</td>
</tr>
<tr>
<td>II.</td>
<td>2-1</td>
</tr>
<tr>
<td>III.</td>
<td>3-1</td>
</tr>
<tr>
<td>IV.</td>
<td>4-1</td>
</tr>
<tr>
<td>V.</td>
<td>5-1</td>
</tr>
<tr>
<td>VI.</td>
<td>6-1</td>
</tr>
</tbody>
</table>

APPENDIX I - Saturator Calibration by a Gravitational Method

APPENDIX II - Notes on Saturator Design

1. General Considerations
2. The Reheater Channel
3. Major Requirements
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PERSONNEL:
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Mechanical Engineering: G. Keyser
Testing: F. Rogers
I.- INTRODUCTION

The following report is a summary of the design configuration and performance characteristics of a Saturator designed and constructed for the NASA. The hardware was developed to provide ground-based simulation for some of the experiments for ACPL-1 first flights of Spacelab. The design shared concepts and technology developed somewhat earlier for the Continuous Flow Diffusion Chamber (CFD)[5]. Some of the difficulties encountered with the apparatus during this project are discussed, and recommendations concerning testing of this type of instrument are presented in the text as well as in Appendix I. A theoretical study, undertaken before construction of the instrument, is presented in Appendix II. (Part of Appendix II appeared in earlier progress reports of this Contract).

Two Saturators were constructed; one instrument was integrated into a common control system with the CFD, and the second unit was delivered to the NASA with a comparatively simple manual control system. Some minor improvements were incorporated into the second Saturator based upon experience with the first device.

The new Saturators provide a means of accurately [3] fixing the water vapor mixing ratio of an aerosol sample. Dew point temperatures from almost freezing to ambient room temperatures can be attained with high precision. The instruments can accommodate aerosol flow rates approaching 1000 cc/s. Provisions were made to inject aerosols upstream of these Saturators, although downstream injection can be accomplished as well.

A device of this type will be used in the ACPL-1 to condition various aerosols delivered concurrently to a CFD, Expansion Chamber, and Static Diffusion Chamber used in zero gravity cloud-forming experiments. The Saturator was designed to meet the requirements projected for the flight instrument as specified in SVS-9800 [3].
II. DESCRIPTION OF THE INSTRUMENT

The Saturator incorporates a flow passage consisting of two closely spaced parallel plates maintained at a stable and well-known temperature. The plates are maintained wet; water is supplied to the plates through a capillary duct connecting the plates to a water storage sump. The length of the flow passage is such that approximately ten time constants are available for diffusion of water vapor into the moving airstream at flow rates less than about 1000 cc/s.

Temperature control of the plates is accomplished by circulating a coolant through channels machined in the thermal plates. This coolant is separated from the chamber space by appropriate seals. The thermal plate is constructed of aluminum plate; the coolant consists of a 4 to 1 mixture of distilled water and Prestone antifreeze. Dye present in the antifreeze solution provides a reliable means of determining if coolant leaks exist in the chamber space.

A stagnant air region approximately 2.5 cm wide is provided around both longitudinal edges of the chamber space in order to isolate the aerosol from thermal edge effects. Ample insulation is provided over the thermal plates to minimize thermal losses to the environment. Chamber dimensions are shown in Figure 1. A plastic body fitted with 0.13 mm diameter stainless steel wires stretched transversely across the channel of flow is mounted just downstream of the wetted surfaces of the thermal plates. These wires serve as electrical heater elements and are provided for the purpose of raising the temperature of the sample aerosol to prevent condensation downstream of the Saturator. The heater body is constructed of thermoplastic (ABS) with a low thermal conductivity in order to restrict heat conduction from the
CHAMBER DIMENSIONS

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate Width</td>
<td>38 cm</td>
</tr>
<tr>
<td>Chamber Space Width</td>
<td>33 cm</td>
</tr>
<tr>
<td>Plate Spacing</td>
<td>0.15 cm</td>
</tr>
<tr>
<td>Wetted Length</td>
<td>27 cm</td>
</tr>
<tr>
<td>Post Heater Body Length</td>
<td>7.0 cm</td>
</tr>
</tbody>
</table>

LEGEND:

1. Air Inlet Manifold
2. Thermal Plate
3. Stainless Steel Wicking Surface
4. Copper Coated ABS Post Heater Body
5. Post Heater Elements
6. Exhaust Manifold

Figure 1. Cross-Sectional Schematic of Saturator
post heater body to the thermal plates. Radiation heat transfer from the wires to the thermal plates is not significant because the transverse wires operate at just a few degrees above the temperature of the Saturator, and because of the small surface area of the wires. The ABS post heater body is electroplated with a thin layer of copper in order to prevent electrostatic charging of these surfaces.

Large metal manifolds are fitted to the inlet and outlet of the Saturator to provide a plenum effect for the air entering and leaving the Saturator. Aerosol is introduced using turbulent mixing at the entrance of the upstream manifold. A pressure port is fitted in the exit manifold.

Temperature measurements are taken on the back side of one thermal plate at downstream locations near the coolant inlet manifold. Holes are milled in the heavy aluminum thermal plate to accept closely fitting brass holders coated with thermal grease. Glass bead thermistors were potted into the holders using thermally conductive epoxy. During calibration testing of the instrument, a larger hole was machined into the thermal plate to accept a quartz thermometer probe, as discussed in Appendix I.
III. TEMPERATURE CONTROL

As mentioned earlier, the first Saturator shares the control system developed concurrently for the CFD. The system is shown in Figure 2. Control hardware incorporated an INTEL 8010 microprocessor.

A flow schematic for the first Saturator is shown in Figure 3. Coolant is circulated through the thermal plates and a thermoelectric powered heat exchanger. Circulation is provided by a small centrifugal pump at a rate of about 30 cm$^3$/s to each plate. Just upstream of the circulation pump a small resistance Servo heater is fitted. The coolant is also circulated over a small glass bead thermistor just downstream of the pump; this method of temperature measurement provides an accurate estimate of the bulk coolant temperature because of the thorough mixing provided by the pump. The sensitivity and short time constant available when using the thermistor provides an accurate measurement of the rate of change of the bulk coolant temperature. This information is used to provide feedback for the immersion heater circuitry. Temperature stability better than ±0.01°C has been achieved using this system. More detailed discussions of temperature measurement performance and temperature calibration with these new systems are provided in Reference 5.

The thermoelectric powered heat exchangers originally failed to perform satisfactorily because of inadequate clamping between the heat exchanger and TEM's; this problem was solved by redesigning the clamping method. Additional cooling performance was obtained by reducing the hot side temperature of the TEM's. These problems and solutions are discussed in more detail in Reference 5.

The second Saturator is provided with a Haake Model FK laboratory circulator instead of the system described above.
Figure 2a. Photo of Saturator

Figure 2b. Front View of CFD and Saturator control rack

1. Air flow control panel
2. Royco 225 mainframe
3. Computer and power supply
4. 28 VDC supply
5. TEM waste heat radiators
6. Teletype
IV. WETTABLE METAL SURFACES

The wettable metal surfaces used in the Saturator consist of a random, diffusionally bonded mesh of micronic-sized stainless steel fibers. The media is 0.32 mm thick and is purchased as Type 304 stainless steel, Special Media No. 86 from Fluid Dynamics of DeLand, Florida. The media is subjected to a special Fluid Dynamics process to reduce carbon contact to 0.04%.

The media is diffusionally bonded to a stainless steel sheet in a hydrogen-fired oven at a temperature of 980 to 1090°C. During the bonding process, the stainless media is in contact with the substrate and is under some pressure to assure uniform adhesion. In order to reduce corrosion in the finished wicking surface and substrate, the part undergoes a second heat treatment after bonding: in a hydrogen oven fixtured to allow rapid cooling, the temperature of the part is raised to 1090°C and then rapidly quenched in N₂, such that the time of exposure in the temperature range of 425 to 815°C is below about 5 minutes.

Part distortion was experienced using an 0.8 mm thick substrate. Substrates 1.6 mm thick were acceptably flat after processing. Care must be exercised during the diffusion bonding process not to stretch the substrate.

These surfaces perform extremely well in the Saturator and CFD instruments. Good wettability is achieved with the supply sumps located at the bottom of the instrument in the down position. During the Saturator evaluation testing, all tests were run with the sump in the "down" position. Furthermore, dry air was run through the instrument for a period of about one hour before taking data to make sure the rate of flow of distilled water to the wicking surfaces was adequate.
A problem has been experienced with the sump design just after filling that is not fully explained; just after filling (perhaps overfilling) the sump, some liquid water can sometimes be drained from the Saturator (a few cm³). After the initial draining, the problem does not recur.

At an air flow of 500 cc/sec, the Saturator can be run over four hours before it is necessary to refill the sump. The sump is packed with spun glass and has dimensions of 7.1 cm diameter x 30 cm length.

The wicking surfaces are cleaned using the glow discharge cleaning process. Based upon experiences with the CFD and Saturator, it appears that several months of use can be obtained from these instruments before cleaning is necessary if adequate filtering is present. Our filters consist of activated charcoal and were designed to remove any long chain hydrocarbons.

Handling of the wicking surfaces is kept to a minimum during cleaning and assembly. All leak testing is performed before cleaning, and assembly of the unit is completed as far as possible before the start of the cleaning operation. Using these concepts, satisfactorily clean surfaces can be obtained [5].
V. CORROSION PROBLEMS

Initially the post heater body was plated with chrome over a base of nickel. Severe corrosion took place on this surface during use in the Saturator. This surface was stripped off on the first unit. The post heater body on both instruments is now coated only with a thin layer of copper. This seems to have eliminated the corrosion problem encountered with the first unit.

Corrosion pitting of the aluminum thermal plate was eliminated by using a 4:1 mixture of distilled water and Prestone antifreeze. Aluminum and copper samples soaking in this mixture for over a period of a year have shown no sign of corrosion. No pitting has been seen on the thermal plates. Distilled water alone will cause severe pitting on aluminum in just a short period of time if any heavy metal ions are in solution.

Some pitting has been experienced where aluminum is in close contact with stainless steel and is exposed to the distilled water supplied to the wicking surfaces.
VI. REFERENCES


APPENDIX I

SATURATOR CALIBRATION BY A GRAVIMETRIC METHOD

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SATURATOR CALIBRATION BY
A GRAVIMETRIC METHOD

Dr. F. Rogers

I. PHILOSOPHY

A measured mass of dry air was passed through the saturator at a flow rate of approximately 500 cm$^3$ sec$^{-1}$, while the output of the saturator passed through sealed dessicant canisters. The canisters were weighed before and after each experiment to give the weight of trapped water. Two simple checks were routinely utilized to determine whether or not the dessicant traps were completely efficient. The measured mixing ratio was then compared with theory.

II. METHOD

A. Weighing of Dry Air

It was originally intended to complete all tests with dry air supplied by the Matheson Company and certified to have a dew point temperature no higher than -60°C. Tests 1 through 4 were conducted with this air, but it was later necessary to use locally-available, dried, compressed air when the decision was made to continue with tests 5 through 8. In the former case, the amount of water vapor present in the gas presents an estimable but negligible error in the measurement of the mass of dry air passed through the saturator. In the latter case, it was necessary to measure the water vapor content of the locally-obtained air.

For all experiments, the air was supplied from pressure bottles weighing about 13.7 Kg. An additional 5.4 Kg was added by the pressure regulator and connections necessary to interface between the bottle and the saturator. Fortunately, it was possible to obtain an excellent high-capacity balance.
the Ohaus Model 1119D Heavy Duty Solution Balance, of total capacity 20 kg and sensitivity 1 g. The sensitivity was verified by loading the balance with a typical air bottle, then adding a 1 g weight after the balance had been zeroed; a detectable deflection was always found to result. In actual use, the balance, loaded with air bottle plus regulator, was zeroed before each experiment. At the end of each run, masses calibrated on Mettler balances were added to the pan to re-establish the zero - the poises supplied with the balance were not touched - and the mass of air expended was given by the calibrated masses. Therefore, the mass of air expended, usually around 900 g, was considered to be measured with an error equal to the sensitivity of the balance, or about ± 1 g (the repeatability of the balance seemed to be better than ± 1 g). Care was taken to ensure that the hose connecting the air bottle to the Saturator did not cause spurious weight changes because of movement of the hose. Photo #1 shows the Ohaus balance zeroed with an air bottle in position on the pan, while mounted in a wooden cradle.

The weight of water vapor contained in the gas must be subtracted from the weight loss of the cylinder to give the weight of the dry air. In the case of runs 1 through 4, using Matheson "dry air" of dewpoint ≤-60°C, the maximum error caused in the measured mixing ratio would be less than 0.1%, since the saturation vapor density at -60°C is about 0.025 gms/m³. In the case of runs 5 through 8, the amount of water vapor present in the locally-supplied "dry air" was measured, using a bag sample and one of the canisters described in Section C.1. The mixing ratio, to an accuracy of only ±90%, was found to be 0.5 g/kg. However, the resulting error in the dry air mass measurement was only about 0.04%, as typically about 900 g dry air were expended. In runs 5, 6 and 7, canisters of dessicant were inserted between the air source and the Saturator to intercept this moisture and allow its weight to be subtracted from the weight of the air expended, but this attempt was abandoned in run 8 due to continuing problems with leaks.
B. Saturator Support Systems

1. Temperature Control

For all the runs (1 through 8), the Haake thermostatted circulating bath, to be supplied with the unit, was used for plate temperature control, and was left at the same setting, arbitrarily about 19.6°C. In all cases, fluctuations greater than about ±0.01°C were not observed.

2. Temperature Measurement

A special well was drilled in one of the saturator plates to receive the probe from a Hewlett-Packard Model 2801A Quartz Thermometer, with absolute accuracy ±0.05°C and resolution 0.001°C (see Photo #2). In the vicinity of 20°C, and at about 900 mb total pressure, 0.05°C corresponds to an error of about 0.3% in mixing ratio.

3. Pressure Measurement

The absolute pressure inside the saturator was measured with a Setra Systems, Inc., Model 204 pressure transducer. This device operates over the range zero to 1700 mb, with the major error contributions, according to factory specs, being non-linearity (0.1%) and hysteresis (0.05%). The original factory calibration graph was obtained, and showed that, for this particular instrument, hysteresis was limited to about 0.02% in the operating region of interest. A far worse error was observed at this laboratory; the zero of the device, checked during and after the calibration runs, seemed to drift by about 2 mb, or about 0.2% of the typical value measured in the saturator. Therefore, the pressure measurement error, estimated by the "root sum-of-squares" of the non-linearity, hysteresis, and zero-drift errors, was about 0.2%. This error contributes to the error of calculated mixing ratio, as discussed in Section III.A.
4. Water Supply to Wicking Surfaces

The saturator sump was filled with distilled and deionized water. After run #4, it was found that the sump was nearly dry, after an estimated five hours of saturator operation at 19.6°C on the dry air discussed.

5. Saturator Warm-up and Flush

Starting with run #3, the practice was adopted of allowing ordinary, compressed, "breathing quality" air (relative humidity about 15%) to circulate through the saturator for at least one hour before the actual calibration run began. In runs 4 through 8, no more than two minutes elapsed between shutting off this "flushing" flow and beginning the calibration flow. The temperature control and measurement devices were always turned on at least two hours before calibration began.

C. Measurement of Water Trapped Downstream of Saturator

1. Dessicant Canisters

Three 1000 ml "Naigene" bottles, arranged in parallel, received the saturator output. These were filled with indicating "Drierite", and efficient dessicant which, in equilibrium, reduces the dewpoint of air to approximately -80°C. These canisters (see Photo #3 and #4) were fitted with inlet and outlet tygon tubing connections, manifolds to distribute incoming air into the Drierite packs, and were leak-tested several times at an overpressure of about 2 psi. The flow resistance of these canisters and of a flow meter also placed downstream of the saturator (on the canister exhaust) caused the saturator pressure to be elevated about 1 psi above ambient. The saturator was previously leak tested at about 5 psi. These canisters were separately weighed before and after each run on an ordinary 2-pan laboratory balance (Ohaus Model 1600) sensitive to about 0.03 g. Masses calibrated on a Mettler H51AR balance were used for the measurement, but it was not possible to use the Mettler balance for the calibration itself. The calibration procedure would improve considerably if all
three canisters could be weighed at the same time, say on a top-loading balance of better sensitivity.

Let $B_1$ be the weight of canister #1 before a calibration run, and $A_1$ be its weight afterward. Then, the weight gain of the canister is given by $G_1$,

$$B_1 \pm 0.03 \text{ g}$$

$$A_1 \pm 0.03 \text{ g}$$

$$G_1 \pm \sqrt{0.03^2 + 0.03^2} \text{ g},$$

or, the propagated error in $G_1$ is about $\pm 0.04 \text{ g}$. The total water trapped by all three canisters is given by

$$\frac{3}{2} \sum_i G_i \pm \sqrt{3} (0.04)^2,$$

or the total propagated error is about $\pm 0.07 \text{ g}$. Since about 14 g water were typically measured, the total error contribution to the measured mixing ratio is about $\pm 0.5\%$.

2. Checks of Canister Efficiency

Loose Drierite in a container can be subject to "channeling"; that is, air can sometimes find preferred channels of relatively low flow resistance rather than evenly permeating the entire dessicant pack. Indicating Drierite changes from blue to red in color when water absorption has taken place, and the first indication that channeling had occurred, or that the amount of dessicant in a given canister was too small, would be a red zone penetrating to the outlet end of the canister. This visual check of canister efficiency always indicated that, in the first approximation, air coming from the saturator at about 500 cm$^3$sec$^{-1}$ had more than adequate residence time in the dessicant canisters for equilibrium with the Drierite. As a second check, however, a
bag sample was taken downstream of the canisters (and of a flow meter used for rough monitoring of the system air flow rate). The bag, of about 200 L capacity, could usually be "on-line" for about 8 minutes out of the total run length of about 30 minutes; it was not attached until each calibration run had been in progress about 5 minutes. Following the run, about two-thirds to three-quarters of the bag contents could effectively be delivered to one of the Drierite canisters; to give a measure of how much, if any, water had passed the three primary traps. Again, masses of water less than about .04 g could escape detection by this procedure (see Section C.1). Furthermore, since the bag procedure represented only about (2/3 x 8 =) 5 minutes out of the total run time of around 30 minutes, it was necessary to multiply the moisture content of the bag by a factor of about 6. Therefore, if .04 g were present in the bag, the total moisture escaping the traps would be estimated to be around 0.2 g, or some 1.5% of the total water collected in any one run. Therefore, this procedure represents the largest potential error source of the entire calibration effort - but the adjective "potential" is used because it seems unlikely, in the light of the first check mentioned above, that 0.2 g escaped detection every time. In future efforts, however, it is certainly desirable to improve this check-procedure, for example, by utilizing a bag of large enough capacity to collect the exhaust air during the entire time of an experiment - none such were available for the present effort.
III. RESULTS

A. Total Error in Calculated Mixing Ratio

The calculated mixing ratio, $X_c$, was computed from the usual formula,

$$ X_c = 0.622 \frac{e_s(T)}{p - e_s(T)} $$

where $e_s(T)$ is the saturation vapor pressure at the temperature $T$ of the saturator, and $p$ is the total gas pressure in the saturator. The values of $e_s(T)$ were taken from the Smithsonian Meteorological Tables, using the temperature indicated by the quartz thermometer, as discussed in Section II.B.2. The total pressure measurement has been discussed in Section II.B.3. These two measurement errors were treated as random and independent and were propagated according to the usual "root sum-of-squares", giving a total percentage error estimate for the calculated mixing ratio of ±0.4%.

B. Total Error in Measured Mixing Ratio

The measured mixing ratio was simply the mass of water collected, divided by the mass of dry air expanded. The former, however, had to be corrected as discussed in Section II.C.2. The latter had to be corrected as discussed in Section II.A.1. The uncertainty in the measurements of collected water were about ±0.07 g, from canister weighing procedures, and about ±0.2 g, from the bag sample check procedure. The root sum-of-squares of these two errors is 0.21 g, or about 1.6% of a total collected water mass of 13 g.

The dry air mass measurement had very small error by comparison. In the worst cases, those involving the use of the locally-supplied compressed dry air, the total uncertainty in the measurement was about ±0.04 g, from the error in the estimate of the original mixing ratio of that air, and about 1 g,
due to the Ohaus 1119D balance. The root sum-of-squares of these values would be very close to 1 g, giving a 0.11% error estimate for a dry air mass measurement of 900 g.

In the case of a division operation, such as dividing the dry air mass into the collected water mass, the percentage errors of the divisor and dividend are propagated by a root sum-of-squares rule if they are independent and random, giving for the total percentage error of the measured mixing ratio the value 1.6%. As previously discussed, this error estimate is probably rather conservative due to the conservative way in which its major contributor, the bag sample procedure error, was estimated.

C. Discussion

Results are given in Table 1. Comments which generally summarize important problems or changes are given in Table 2. Note that Runs 1, 4, and 5 had problems so serious as to completely eliminate their data from consideration. Also, it was suspected in Runs 6 and 7 that the upstream canisters were leaking (though it was never possible to detect the leak among the many junctions and seals involved); eliminating those canisters led to the rather good agreement of Run 8.

Since the calculated mixing ratio could be given to 0.4%, and the measured value to only 1.6%, Runs 2, 3 and 8 would indicate that the saturator is performing without detectable errors. Furthermore, the agreement in Runs 6 and 7 is good enough, given both the known and suspected errors, that no malfunction of the saturator would be indicated.

The most obvious ways to improve this calibration technique have already been discussed; namely, improvement of the canister design and weighing procedure (provide, for example, one large canister and a top-loading balance of high enough capacity that the trapped water can be weighed with two instead of six operations), and improvement of the bag sample check procedure.
<table>
<thead>
<tr>
<th>Run No.</th>
<th>Saturator Mean T°C</th>
<th>Saturator Mean Pmb</th>
<th>Calculated Mixing Ratio</th>
<th>Length of Run</th>
<th>Weight of Dry Air, g</th>
<th>Weight of Collected Water, g</th>
<th>7.5 x Water in Bag, g</th>
<th>Measured Mixing Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19.58</td>
<td>920.5</td>
<td>.0158</td>
<td>38 min.</td>
<td>≈ 749 gm</td>
<td>11.30</td>
<td>≈ 1 g</td>
<td>.0164</td>
</tr>
<tr>
<td>2</td>
<td>19.60</td>
<td>966.1</td>
<td>.0150</td>
<td>27 min.</td>
<td>858</td>
<td>13.00</td>
<td>U.D.</td>
<td>.0152</td>
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<tr>
<td>3</td>
<td>19.62</td>
<td>966.8</td>
<td>.0150</td>
<td>30 min.</td>
<td>891</td>
<td>13.40</td>
<td>U.D.</td>
<td>.0150</td>
</tr>
<tr>
<td>4</td>
<td>19.59</td>
<td>965.0</td>
<td>.0150</td>
<td>30 min.</td>
<td>894</td>
<td>11.95</td>
<td>.38</td>
<td>.0138</td>
</tr>
<tr>
<td>5</td>
<td>Ruined by failure of upstream canister seals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>19.59</td>
<td>957.0</td>
<td>.0152</td>
<td>30 min.</td>
<td>935</td>
<td>13.80</td>
<td>---</td>
<td>.0148</td>
</tr>
<tr>
<td>7</td>
<td>19.59</td>
<td>973.6</td>
<td>.0149</td>
<td>29 min.</td>
<td>937</td>
<td>13.55</td>
<td>U.D.</td>
<td>.0145</td>
</tr>
<tr>
<td>8</td>
<td>19.59</td>
<td>964.3</td>
<td>.0151</td>
<td>29 min.</td>
<td>933</td>
<td>13.90</td>
<td>.11</td>
<td>.0150</td>
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U.D. = Undetectable
<table>
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<th>Run No</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Moisture in bag probably influenced by previous use on room air. No flushing flow. This run mainly to familiarize workers with procedures.</td>
</tr>
<tr>
<td>2</td>
<td>Measured mixing ratio 0.8% higher than predicted. No flushing flow.</td>
</tr>
<tr>
<td>3</td>
<td>Five minute delay between flushing flow and calibration flow. No detectable disagreement between measured, predicted mixing ratio.</td>
</tr>
<tr>
<td>4</td>
<td>Sump allowed to nearly dry out.</td>
</tr>
<tr>
<td>5</td>
<td>Added upstream dessicant canisters - failure of seals at beginning.</td>
</tr>
<tr>
<td>6</td>
<td>Bag seal accidentally failed. Calculated mixing ratio 2.8% higher than measured.</td>
</tr>
<tr>
<td>7</td>
<td>Calculated mixing ratio 2.7% higher than measured.</td>
</tr>
<tr>
<td>8</td>
<td>Removed upstream dessicant canisters. Calculated mixing ratio 0.7% higher than measured.</td>
</tr>
</tbody>
</table>
1. Air bottle in place on Ohaus 1119D balance.

Open Saturator
Covered with Insulation

Quartz Thermometer Probe Atop Carbon Air Filter Provided with Saturator.

2. General view of saturator, filter canister, and quartz thermometer probe.
3. Drierite canisters and flowmeter.

4. Drierite canisters connected to saturator.
NOTES ON SATURATION DESIGN - 1: General Considerations
Dr. P. Squires

1. INTRODUCTION

The purpose of the saturator is to produce a flow of air of order a liter a second with an accurately known mixing ratio, by exposing the air sufficiently to wet surfaces at a known temperature.

2. MAJOR DESIGN ASPECTS

A. Plate Spacing (h)

If the air is to be exposed for (say) 10 time constants, the total volume of the channels is about $5Fh^2$, independent of the number of such channels (N). Thus it is desirable to keep $h$ small. The volume of the plates is $10NFht$ where $N$ is the number of air channels and $t$ is the plate thickness. It is therefore desirable also to keep $N$ small (preferably, $N = 1$).

(see pg. 1 of "Notes on the Saturator Design - 3", in this Appendix, Part 3, for this derivation).

B. Plate Contamination

Contamination of the wet surfaces by aerosol particles does not appear to be a significant factor, but if surfactant materials are present and accumulate in the critical region, they may cause deleterious effects, e.g. reducing the rate at which water vapor molecules pass through the surface.

With a design in which $N$ and $h$ are small (in order to economize volume), the drag of the air flow on the surface of the water in the plates is so large that this surface will move downstream with the air, if it is free to do so. This would appear to indicate that either the capillary surface of the plates should be in the form of transverse grooves, or else the supply of water should be introduced at the upstream end and some surplus water
drained off at the downstream end, beyond the critical region.

If transverse grooves are used, and the transverse water flow is expected to transport the surfactants to one side of the plates (beyond the effective working humidification region), the question remains — how rapidly will the surfactant layer accumulate and build back into the working region? At present, there seems to be no way to design such a system in such a way as to ensure the required performance, unless the surfactant layer is continuously removed on the "far" side of the plates.

Thus either of the possible designs seems to require that so much water be supplied to the plates that there will be an overflow to be drained away (either at the "far" side, or at the downstream end), and that this water be drained off in such a way that the surfactant layer is carried with it. This latter requirement may prove troublesome: it seems not too likely that a capillary wick surface into which the water could drain would readily accept the surfactant layer. One method which probably would achieve the desired result would be to extract the water from the ends of the grooves by means of a rapidly-moving air stream, since the stress would be applied directly to the surfactant layer. This however would represent a very considerable complication of the overall design. Furthermore, at the downstream end, with closely spaced plates, it is doubtful if there would be space available to install the required "extraction" devices, unless the grooves could be continued around a corner on the plate so that the groove ends were of order a centimeter.
from the saturator air flow. In the case of transverse grooves, there probably would be more space available for the extraction operation along the "far" side of the saturator.

A more radical approach would be to keep the plates wet by condensing water vapor onto them from the air stream. It seems reasonable to assume that by the use of activated charcoal filters, the long-chain molecules which form surfactant layers could be effectively stripped from the air stream. It may be necessary to pass the entire saturator flow (F ≈ 10^3 cm^3 sec^-1) through such filters so that surfactant type molecules originating in various parts of the system do not accumulate in the recycling air. If this is practicable, the only remaining source of surfactant would be the water supply to the saturator. If the water surface on the plates were being continuously replaced by condensation, while the surplus water were being drained away, the risk of contamination would seem to be lessened. If the contamination remained small enough, it would be possible to make use of capillary wicks to drain the plates, to a capillary reservoir similar to those which have been planned as sources of water. With this design, it would seem to be necessary to use a wicking surface consisting of longitudinal grooves, since condensation could not be relied upon to keep the plates thoroughly wet in the critical downstream region, where the air stream is very close to saturation. The capillary drain would then be located at the downstream end.
It is clear that the concern about the effects of surfactant materials is a major driver in the overall design. Provided that arrangements can be made to start with the plates thoroughly clean, and that evaporation is never allowed to occur from the plates, even when the plate temperatures are being changed, the continued renewal of the water surface in a "condensing saturator" would appear to be attractive. Its practicability depends on the design of a "vapor generator" which would add water vapor to the air stream to raise its dew point above the main plate temperature. This generator would need to be flexible enough to be able to cope with transition situations; for example, if the plates were being warmed from 5°C to 20°C, unless water vapor could be added to the circulating air fairly rapidly, evaporation would occur from the plates, causing a reverse flow from the capillary reservoir, and increasing the risk of surfactant contamination.

In connection with a condensing saturator design, it would probably be possible to design this in such a way that, with relatively minor changes, it could be operated as an evaporating saturator, using the capillary reservoir as a source instead of as a sink.

C. The Main Plates

As discussed above, a single air channel design with closely spaced main plates is desirable, since it economizes volume.

However, as the plate spacing (h) is reduced, the pressure drop in the saturator increases, as does the frictional drag of
the air on the water surfaces. This could result in difficulty in ensuring the proper irrigation of the plates. Furthermore, too large a gradient of pressure in the air stream will result in an error in the mixing ratio, which depends on the total gas pressure as well as on the partial pressure of water vapor.

Some economy of volume may be achieved if the plates, instead of being parallel, diverge slightly downstream. In this way, it may be possible to some extent to combine the advantages of close plate spacing (at the upstream, non-critical end) with satisfactorily low values of the air velocity and pressure gradient, (in the critical region at the downstream end). If it is desired to mechanically reinforce the plates by placing spacers between them, it would probably be necessary in this case to make these in the form of tapered rails rather than washers.

D. The Reheater

The saturator must include a reheater to raise the temperature of the saturated air stream to a level where it can be allowed to flow to other components of the ACPL without risk of condensation. The reheater may be in two sections; one at the end of the main plates to raise the temperature by a small amount, and a second one downstream to raise the temperature further, to any desired level.

Designs which have been discussed so far feature a reheater consisting of parallel plates maintained at a temperature above that of the main plates. An insulating section is used to restrict the flow of heat back into the main plates. A more attractive
approach may be to heat the air directly by means of electrically heated wires stretched across the stream. This would appear to be simpler, and would utilize the air stream itself as an insulator, possibly leading to a more compact design.

3. CONCLUSIONS

The choices discussed above depend critically on whether it is possible to design an effective and compact vapor generator and direct electrical reheater.
APPENDIX II
2: THE REHEATER CHANNEL
NOTES ON THE SATURATOR DESIGN - 2: The Reheater Channel

1. INTRODUCTION

The simplest reheater design would appear to be one in which the channel between parallel main plates (of depth \( h \) cm) is continued between insulating walls (possibly metallized to avoid electrostatic effects), and heat is added to the stream electrically, using taut transverse wires; some distance beyond the heater assembly, the channel opens into a transverse duct which collects the whole air flow. The material used for the heater channel walls is assumed to have a thermal conductivity \( k_w \) about 10 times that of air (i.e. \( 6 \times 10^{-4} \) cal cm\(^{-1}\) deg\(^{-1}\) sec\(^{-1}\)). A capillary barrier may be included at the downstream end of the main plates.

2. HEATER DESIGN

In order to avoid all risk of thermal modification of aerosol particles, it is desirable that the heater wires not be too hot - say less than 10°C above the air temperature. It is assumed that, at this stage, it will be sufficient to raise the air stream temperature by \( \Delta T = 2°C \) (further heating may be added in a less critical location downstream if desirable). A 2°C temperature rise will require the dissipation of about 2 watts (1/2 cal sec\(^{-1}\)), since the mass flux of air is about 1 g sec\(^{-1}\).
For reasons discussed in Section 5 of "Notes on a Saturator Design - 3", it is desired that the pressure drop through the reheater channel (caused by wall drag and wire drag) should not be too large; as shown there, this requirement can be met if a design is chosen in which \( b = 30 \text{ cm}, \ h = 0.15 \text{ cm} \). The present purpose is to discuss the design of a reheater channel of the same dimensions, with parallel walls; it is assumed here that these walls are effectively insulated from environmental influences.

3. HEATER CHANNEL DESIGN

A. Upstream of the Heater

To design this channel, it is necessary to evaluate the longitudinal flow of the heat in the heater channel walls back to the main plates. Beyond the heater, the walls will be heated by the air, and by the metal duct which is attached to the downstream end of the walls and will no doubt be close to the actual heater air temperature (\( T + \Delta T \)). This question is treated here as a one-dimensional problem, as if the thickness of the channel wall were small compared with its length.

Considering first the region upstream of the heater, let the temperature of the main plates be \( T \), and that of the channel walls opposite the heater assembly, \( T + \delta T \). Heat will be conducted back to the main plates, but will also be removed from the reheater channel wall by the air flow as it approaches the heater. Let the
total effective cross-sectional area of the material in each channel wall be \( A = b \, w \), where \( w \) is the thickness of the channel wall) and the thermal conductivity of the wall material, \( k_w \).

Clearly, the thinner the wall the better, from a thermal point of view. It may be found advantageous to stiffen the wall structure by fabricating it with ribs. If so, the cross-section of longitudinal ribs should be included in \( A \). It is assumed that the insulation material (with a thermal conductivity about equal to that of air) used to isolate the channel walls from environmental influences will not contribute significantly to the longitudinal transfer of heat in the walls.

Let the origin of \( x \) be at the upstream end of the channel wall, i.e., at the end of the main plate. The solution due to Polhausen for heat transfer from a flat plate at temperature \( T_w \) (e.g., Goldstein, vol. 2, 1965, p. 627) gives an expression for the total heat transfer:

\[
Q = 0.686 \frac{1}{\sigma^3} k_a b (T_w - T) \sqrt{\frac{u_0 x}{v}}
\]

where \( b \) is the breadth of the plate, \( T \) the air temperature at infinity, \( x \) the plate length, \( k_a \) the thermal conductivity of air and \( u_0 \) the air velocity of the free stream.

Differentiating, one derives an expression for the rate of transfer per unit length at the downstream end:

\[
\frac{dQ}{dx} = 0.343 \frac{1}{\sigma^3} k_a b (T_w - T) \sqrt{\frac{u_0}{v x}}
\]
This expression may be used to give an estimate for the transfer of heat from the channel walls. It tends to overestimate the local rate of heat transfer because the gas stream is of finite depth only. On the other hand, it tends to underestimate the local heat transfer because the gas-wall temperature contrast increases downstream. Further studies are needed to more clearly define the nature of this approximate estimate. (Underestimating heat loss to the air is conservative, because it will result in exaggerating the heat flux to the main plate. As discussed in Section B below, downstream of the heater assembly, the two errors cooperate to ensure that the treatment is conservative).

Considering conduction in the material of the wall,

\[ Ak_w \frac{d^2 T_w}{dx^2} = 0.343 \sigma^2 k_a b (T_w - T) \sqrt{\frac{u_0}{v}} \]

This may be written

\[ \frac{d^2 y}{dx^2} = a y \times \frac{1}{2} \]

where \( y = T_w - T \) and \( a = \frac{0.343 \sigma^2 k_a b}{Ak_w} \sqrt{\frac{u_0}{v}} \).

Taking \( u_0 = \frac{3F}{2b} \), the velocity in the center of the channel; \( u_0 = 333 \text{ cm sec}^{-1} \) and with \( k_w = 10 \text{ k}_a \), \( w = 1 \text{ cm} \) \( (A = 30 \text{ cm}^2) \), it results that \( a = 1.4 \).

The boundary conditions are: \( y = 0 \) at \( x = 0 \), \( y = \delta T \) at \( x = l, \) where \( l \) is the distance from the main plate to the heater. Considering the boundary condition at \( x = 0 \), it is found convenient to write a solution in the form:
\[
y = C_1 \alpha \times f(z) = C_1 \frac{2}{3} z^\frac{2}{3} f(z)
\]

where \( C_1 \) is an arbitrary constant, \( z = \alpha x^2 \), and \( f(z) \) is a function which may be expanded as

\[
f(z) = 1 + a_1 z + a_2 z^2 + \ldots + a_n z^n + \ldots
\]

Then,

\[
y' = C_1 \alpha^\frac{2}{3} f(z) + \frac{3}{2} C_1 \alpha^\frac{5}{3} x^\frac{3}{2} f'(z)
\]

\[
y'' = \frac{15}{4} C_1 \alpha^\frac{5}{3} x^\frac{3}{2} f'(z) + \frac{9}{4} C_1 \alpha^\frac{5}{3} x^2 f''(z)
\]

so that the required solution for \( f(z) \) is that for:

\[
9z f''(z) + 15 f'(z) - 4f(z) = 0
\]

where

\[
f(z) = 1 + a_1 z + a_2 z^2 + \ldots + a_n z^n + \ldots
\]

and

\[
f'(z) = a_1 + 2a_2 z + 3a_3 z^2 + \ldots + (n+1) a_{n+1} z^n + \ldots
\]

\[
f''(z) = (2x1) a_2 + (3x2) a_3 z + \ldots + n(n+1)a_{n+1} z^{n-1} + \ldots
\]

Thus, the coefficients \( a_n \) may be found as follows:

\[
15 a_1 = 4,
\]

\[
9(2x1) a_2 + 15x2 a_2 = 4a_1
\]

5
and in general
\[ 9n (n+1)^2 a_{n+1} + 15(n+1) a_n = 4a_n \]

Clearly, \( a_1 = \frac{4}{15} \), while for \( n \geq 1 \),
\[ a_{n+1} = \frac{4a_n}{3(3n+5)(n+1)} \]

for example,
\[
\begin{align*}
a_2 &= \frac{4}{48} a_1, \quad a_3 = \frac{4}{99} a_2, \quad \ldots \quad a_6 = \frac{4}{360} a_5, \quad \ldots \quad \text{etc.} \\
\end{align*}
\]

Evaluating the coefficients in turn gives:
\[
\begin{align*}
a_1 &= 0.26667, \quad a_2 = 2.2222 \times 10^{-2}, \quad a_3 = 8.9787 \times 10^{-4} \\
a_4 &= 2.1378 \times 10^{-5}, \quad a_5 = 3.3534 \times 10^{-7}, \quad a_6 = 3.726 \times 10^{-9}
\end{align*}
\]

The value of \( C_1 \) is determined by evaluating the solution at \( x = \lambda_1 \), \( z = z_1 = \alpha \lambda_1^{\frac{3}{2}} \) (where \( y = \delta T \)); values are given for various values of \( \alpha \lambda_1^{\frac{3}{2}} \) in Table I.

<table>
<thead>
<tr>
<th>( z_1 = \alpha \lambda_1^{\frac{3}{2}} )</th>
<th>( f(z_1) )</th>
<th>( z_1^{\frac{2}{3}} f(z_1) )</th>
<th>( \frac{1}{z_1^{\frac{2}{3}} f(z_1)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.28981</td>
<td>1.28981</td>
<td>0.77531</td>
</tr>
<tr>
<td>2</td>
<td>1.62976</td>
<td>2.5871</td>
<td>0.38654</td>
</tr>
<tr>
<td>3</td>
<td>2.0261</td>
<td>4.2144</td>
<td>0.23728</td>
</tr>
<tr>
<td>5</td>
<td>3.0156</td>
<td>8.8177</td>
<td>0.13341</td>
</tr>
<tr>
<td>10</td>
<td>7.0378</td>
<td>32.667</td>
<td>0.03061</td>
</tr>
</tbody>
</table>

The value of the first derivative of \( y \) with respect to \( x \) at \( x = 0 \), is \( C_1 \alpha^{2/3} \), so that the flux of heat \( (H) \) from the heater channel wall into the main plate is:
\[ H = A k_w C_1 \alpha^\frac{2}{3} = \frac{A k_w \alpha^\frac{2}{3}}{z_1^\frac{2}{3} f(z_1)} \delta T \]

Thus, for \( k_w = 6 \times 10^{-4} \text{ cal cm}^{-1} \text{sec}^{-1} \text{deg}^{-1} \), \( A = bw = 30 \text{ cm}^2 \), \( u_o = \frac{3F}{2bh} = 333 \text{ cm sec}^{-1} \) (\( \alpha = 1.4 \), \( \delta T = 1 \), this flux is

\[ \frac{2.3 \times 10^{-2}}{z_1^\frac{2}{3} f(z_1)} \]

For example, if \( z_1 = 3 \), this amounts to \( 5.4 \times 10^{-3} \text{ cal sec}^{-1} \).
B. **Downstream of the Heater**

In the section of the heater channel downstream of the heater wires, a flux of heat passes from the air (now raised to temperature $T + \Delta T$) to the walls. Another source of heat will be the transverse duct used to collect the air from the whole width of the saturator, which will be attached to the downstream ends of the heater channel walls. This duct is expected to be close to $T + \Delta T$; and it will be assumed that the outermost ends of the channel walls are maintained at this temperature by conduction from the duct.

As noted earlier, the approximate procedure of differentiating the Polhausen solution for a plate at constant temperature may or may not result in an underestimate of the local rate of heat transfer from the wall to the air in the region upstream of the heater, which would ensure that the treatment is conservative. However, downstream of the heater, it certainly results in an overestimate of the heat transfer from the air to the wall both because the gas is of finite depth and because the gas-wall contrast in temperature decreases downstream; this is conservative for the present purpose, which is to estimate the flux of heat to the main plate.

It is convenient in this instance to locate the origin of $x$ at the plane of the heater, with $x$ increasing downstream. The heat transfer to the walls from the air is then given by:
\[ \frac{dQ}{dx} = 0.343 \frac{1}{3} k \beta (T + \Delta T - T_w) \sqrt{\frac{u}{\nu x}} \]

This influx of heat must be balanced by an increase in the flux carried by conduction within the channel wall towards negative x (i.e. upstream).

Thus:

\[ \frac{d^2 T_w}{dx^2} = \alpha \left( T_w - T - \Delta T \right) x \frac{1}{2} \]

and writing

\[ Y = T_w - T - \Delta T, \]

\[ \frac{d^2 Y}{dx^2} = \alpha Y x \frac{1}{2} \]

Three boundary conditions designated below as (a), (b), (c) must be satisfied, in the manner described below:

(a) \( Y = \delta T - \Delta T \) at \( x = 0 \)

(b) \( Y = 0 \) at \( x = l_2 \) where \( l_2 \) is the length of the channel downstream of the heater.

(c) \( \frac{dY}{dx} \bigg|_{x=0} = \left( \frac{dY}{dx} \right) \bigg|_{x=l_1} \), to ensure that the gradient of \( T_w \) is continuous at the plane of the heater. (Note that the origin of \( x \) is different on the two sides of this equation).

The solution utilized in the previous section vanishes at the origin, and therefore cannot meet condition (a); it must be generalized by the addition of an independent solution. The third degree of freedom is then handled by taking account of the fact that physically, \( \delta T \) is not an independent boundary condition, but depends on \( \Delta T \).
The required independent solution may be found in the form:

\[ Y = g(z) \]
\[ \frac{3}{2} \]
\[ z = \alpha x^2 \]

Thus, it is required that

\[ Y' = \frac{3}{2} \alpha x^{\frac{1}{2}} g'(z) \]
\[ Y'' = \frac{3}{4} \alpha x^{\frac{1}{2}} g'(z) + \frac{9}{4} \alpha^2 x^2 g''(z) \]

It is therefore necessary to solve:

\[ 9z g''(z) + 3 g'(z) - 4 g(z) = 0 \]

Writing:

\[ g(z) = 1 + b_1 z + b_2 z^2 + \ldots + b_n z^n + \ldots \]
\[ g'(z) = b_1 + 2b_2 z + 3b_3 z^2 + \ldots + (n+1)b_{n+1} z^n + \ldots \]
\[ g''(z) = (2x1)b_2 + (3x2)b_3 z + \ldots + n(n+1)b_{n+1} z^{n-1} + \ldots \]

it is seen that:

\[ 3b_1 = 4 \]
\[ 9(2x1)b_2 + 3(2)b_2 = 4b_1, \text{ etc.} \]

and in general, for \( n \geq 1 \),

\[ 9n(n+1)b_{n+1} + 3(n+1)b_{n+1} = 4b_n \]

so that

\[ b_{n+1} = \frac{4b_n}{3(3n+1)(n+1)} \]
For example, \( b_2 = \frac{4}{24} b_1, b_3 = \frac{4}{63} b_2 \)
\( b_4 = \frac{4}{120} b_3, b_5 = \frac{4}{195} b_4, b_6 = \frac{4}{288} b_5, b_7 = \frac{4}{399} b_6 \)

Evaluating the coefficients in turn gives:
\( b_1 = 1.3333, b_2 = 0.22222, b_3 = 1.4109 \times 10^{-2} \)
\( b_4 = 4.7031 \times 10^{-4}, b_5 = 9.6474 \times 10^{-6}, b_6 = 1.3399 \times 10^{-7} \)
\( b_7 = 1.3433 \times 10^{-9} \)

Thus a general solution is:
\[ y = C_2 z^3 f(z) + C_3 g(z) \]

where \( f(z) = 1 + a_1 z + a_2 z^2 \ldots \)
\( g(z) = 1 + b_1 z + b_2 z^2 \ldots \)

Given \( \delta T \), \( C_1 \) is known (Table 1), and thus from boundary condition (c),
\[ C_2 = \alpha^{-\frac{2}{3}} \left( \frac{dy}{dx} \right) x = z_1 \]
\[ = C_1 h(z_1) \]

where \( h(z) = f(z) + \frac{3}{2} z f'(z) \)

Hence,
\[ C_2 = \frac{h(z_1) \delta T}{z_1^3 f(z_1)} \]
From condition (b),

\[ C_3 = \frac{-z_2^2 f(z_2) C_2}{g(z_2)} \]

\[ = \frac{-z_2^2 f(z_2) h(z_1) \delta T}{z_1^3 f(z_1) g(z_2)} \]

From condition (a)

\[ C_3 = \delta T - \Delta T \]

Hence,

\[ \frac{\Delta T}{\delta T} = 1 + \frac{\frac{2}{3} f(z_2) h(z_1)}{z_2^2 \frac{2}{3} f(z_1) g(z_2)} \]

Consequently, the heat flux to the main plate (H) is:

\[ H = \frac{A k_w a^3 \delta T}{z_1^3 f(z_1)} = \frac{A k_w a^3 g(z_2) \Delta T}{\left( \frac{2}{z_1^3 f(z_1) g(z_2)} + \frac{2}{z_2^3 f(z_2) h(z_1)} \right)} \]

\[ = \frac{A k_w a^3 \Delta T}{\left( \frac{2}{z_1^3 f(z_1)} + \frac{2}{z_2^3 f(z_2)} h(z_1) \right) g(z_2)} \]

It will be seen that, if \( z_2 = 0 \), so that \( \delta T = \Delta T \), this result is consistent with that derived at the end paragraph A above. (\( H = A k_w a^3 \frac{\delta T}{z_1^3 f(z_1)} \)). The second term in the denominator represents the beneficial effect in reducing H which is gained by continuing the heater channel walls beyond the heater.
It is easily shown that
\[ h(z) = f(z) + \frac{3}{2} zf'(z) \]
\[ = 1 + \frac{5}{2} a_1 z + \frac{8}{2} a_2 z^2 + \ldots + \frac{(3n+2)}{2} a_n z^n + \ldots \]

where the coefficients of \( z^n \) in \( h(z) \) are

\[ \begin{array}{c|c}
  n & \frac{(3n+2)}{2} a_n \\
  \hline
  1 & 6.6667 \times 10^{-1} \\
  2 & 8.8889 \times 10^{-2} \\
  3 & 4.9383 \times 10^{-3} \\
  4 & 1.4965 \times 10^{-4} \\
  5 & 2.8504 \times 10^{-6} \\
  6 & 3.726 \times 10^{-8} \\
\end{array} \]

Values of \( h(z) \) are given in Table 2.

<table>
<thead>
<tr>
<th>z</th>
<th>h(z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.7606</td>
</tr>
<tr>
<td>2</td>
<td>2.7309</td>
</tr>
<tr>
<td>.3</td>
<td>3.9462</td>
</tr>
<tr>
<td>.5</td>
<td>7.2759</td>
</tr>
<tr>
<td>10(^2)</td>
<td>23.313</td>
</tr>
</tbody>
</table>

Values of \( g(z) \) and of \( z^{3/2} f(z)/g(z) \) are given in Table 3.

(The latter function appears in the expression for \( H \)).
TABLE 3

<table>
<thead>
<tr>
<th>z</th>
<th>g(z)</th>
<th>(\frac{2}{z^3 f(z)/g(z)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5702</td>
<td>0.50183</td>
</tr>
<tr>
<td>2</td>
<td>4.6763</td>
<td>0.55324</td>
</tr>
<tr>
<td>3</td>
<td>7.4215</td>
<td>0.56786</td>
</tr>
<tr>
<td>5</td>
<td>15.312</td>
<td>0.57586</td>
</tr>
<tr>
<td>10</td>
<td>56.480</td>
<td>0.57838</td>
</tr>
</tbody>
</table>

From Table 3, it can be seen that provided \(z_2\) is large enough (say, \(z_2 > 2\)), \(H\) is essentially independent of \(z_2\) and one may write

\[
H = \frac{\frac{2}{z^3 f(z)/g(z)}}{D(z_1)} = \frac{2}{D(z_1)} \left( \frac{2}{z^3 f(z_2)/g(z_2)} \right) x h(z_1)
\]

where \(D(z_1) = z_1^3 f(z_1) + \left( \frac{2}{z^3 f(z_2)/g(z_2)} \right)_{\text{const}}\).

(Large \(z_2\) corresponds to the situation where the effect of the heat flux from the heated air into the wall downstream of the heater dominates that due to heat conducted from the transverse duct to the downstream ends of the channel walls - which has been assumed here to maintain the downstream ends (\(x = z_2\)) at a temperature of \((T + \Delta T)\). Substituting a constant value of 0.57 for \(\frac{2}{z^3 f(z_2)/g(z_2)}\) yields approximate values of \(H/(\alpha \Delta T)\) as a function of \(z_2\), alone as shown in Table 4.)
TABLE 4

Approximate values of $\frac{H}{\frac{2}{3}} = \frac{1}{D(z_1)}$ for $z_2 > 2$

<table>
<thead>
<tr>
<th>$z_1$</th>
<th>$\frac{1}{D(z_1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.436</td>
</tr>
<tr>
<td>2</td>
<td>0.241</td>
</tr>
<tr>
<td>3</td>
<td>0.155</td>
</tr>
<tr>
<td>5</td>
<td>0.077</td>
</tr>
<tr>
<td>10</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Now, since $u_o = \frac{3F}{2bh}$,

$$Ak_w \alpha^3 = \left(\frac{Ak_w b}{3}\right)^{\frac{1}{3}} \left(0.343 \frac{1}{3} k_a \right)^{\frac{2}{3}} \left(\frac{3F}{2h\nu}\right)^{\frac{1}{3}}$$

Substituting $\sigma = 0.73$, $k_a = 6 \times 10^{-5}$ cal cm$^{-1}$ sec$^{-1}$ deg$^{-1}$,

$$F = 10^3 \text{ cm}^3 \text{sec}^{-1}, h = 0.15 \text{ cm}, \nu = 0.16,$$

$$Ak_w \alpha^3 = 2.78 \times 10^{-2} \left(Ak_w b\right)^{\frac{1}{3}}$$

Writing $A = bw$, where $w$ is the thickness of the heater channel walls,

$$H = \frac{2.78 \times 10^{-2}b^{\frac{2}{3}} (w k_w)^{\frac{1}{3}} \Delta T}{D(z_1)}$$

For $w = 1 \text{ cm}$, $b = 30 \text{ cm}$, $k_w = 6 \times 10^{-4}$ cal cm$^{-1}$ sec$^{-1}$ deg$^{-1}$ this gives:

$$H = \frac{2.26 \times 10^{-2} \Delta T}{D(z_1)}$$
It is clear from Table 3 that there is little or nothing to be gained by increasing $z_2$ above 3. With the design parameter values mentioned above (for which $a = 1.4$) this implies $l_2 \approx 2$ cm. Thus, if an overall channel length of 6 cm is considered acceptable, $l_1$ may be chosen as large as 4 cm, so that with $a = 1.4$, $z_1 \approx 11$, so that $\frac{1}{D(z_1)} \approx 0.02$. For $\Delta T = 2^\circ C$, this would yield $H = 9 \times 10^{-4}$ cal sec$^{-1}$.

Even if none of this flux were removed by the circulating water within the main plate, but the whole flux to the two plates ($2 \times 10^{-3}$ cal sec$^{-1}$) were used to heat the air stream and to evaporate additional water into the stream, the resulting increase in the mixing ratio of the air would be only about $2 \times 10^{-6}$ g g$^{-1}$ (about half of the heat would be used for evaporation and half for heating the air stream). At the minimum temperature (5°C), where the saturation mixing ratio at 1 atmosphere is about $5 \times 10^{-3}$ g g$^{-1}$, this would result in an increase in the mixing ratio of only 0.04%, which is negligible.

(In fact, the error will probably be much less than this estimate, most of the heat flux from the channel wall being removed by the circulating water within the main plate).
4. OTHER PERTURBING EFFECTS

A. Radiation

The radiative effects will be overestimated if both the heater wires and the main plates are treated as black bodies. If they were both at temperature $T$, the radiative flux passing through the entrance to the heater channel from the wires to the main plates would be of order \( \frac{h}{2\pi d} n^2 \sigma T^4 \) where $n$ is the number of wires and $d$ is their diameters; the flux returning from the main plate to the wires would be the same. When therefore the wires are hotter than the main plates, being at temperature $T+\delta T$, the net flux from the wires to the plates is of order \( (2\sigma T^3 n h b d \delta T/l_1) \text{ cal sec}^{-1} \).

With the design parameters discussed in Notes on the Saturator Design - 3, \((n = 5, h = 0.15 \text{ cm}, b = 30 \text{ cm}, d = 1.3 \times 10^2 \text{ cm}, \delta T = 6^\circ \text{C}, l_1 = 4 \text{ cm})\), this flux amounts to about $3 \times 10^{-5} \text{ cal sec}^{-1}$. This is equivalent to less than 0.02 times that estimated as due to conduction from the two heater channel walls, and is therefore negligible.

Radiative heating of the two channel walls by the heater wires, treating both as black bodies, is of order $4\pi b d \sigma T^3 \delta T \text{ cal sec}^{-1}$ which, for the values quoted above, amounts to $5 \times 10^{-3} \text{ cal sec}^{-1}$, at the most; a rough estimate can be made of the resulting error as follows. Most of the radiant heat will reach the walls close to the heater wires. If it is assumed that all of it is deposited directly opposite the heater, the heat flux so introduced may be compared with that flowing through the wall towards each main plate of this point \((H')\), according to the analysis given in Section 3 above.
Thus, \( \frac{H'}{A_k} = \frac{dV}{dx} x = 0 = C_1 a^{2/3} h(z_1) \)

whereas \( \frac{H'}{A_k} = C_1 a^{2/3} \)

Thus, \( H' = h(z_1) H \). With the design discussed above, \( z_1 = 11 \), and \( h(z_1) \approx 25 \) (Table 2). Hence (in each wall) \( H' \approx 2.5 \times 10^{-2} \) cal sec\(^{-1} \), whereas the additional flux deposited by radiation is about \( 2.5 \times 10^{-3} \) cal sec\(^{-1} \). It would therefore appear that radiative heating of the channel walls would probably result in an increase in \( H \) by about 10%. With the saturator operating at 5°C, this could cause the mixing ratio of the air to increase by a further 0.004% only. These estimates are likely to be quite exaggerated, since they are based on the assumption that none of the heat leaking from the channel walls to the main plates will be carried away by the temperature controlling water flow within the latter.

B. Upstream Diffusion of Heat in the Air

This may be neglected provided \( Re Pr > 10^2 \). With the design parameters discussed above, this condition is met:

\[
Re Pr = \frac{Eh \nu}{b_h \nu} \approx \frac{E}{b} \approx 170.
\]
CONCLUSIONS

The proposed reheater design, with 5 transverse wires raised 6°C above the temperature of the air stream, has walls only 6 cm long, and dispenses with the need for a temperature control system for the reheater walls. The total energy dissipation is about 2 watts, and the leakage of heat back to the main plates is trivial; the insulating properties of the air flow itself are helpful in this regard.

As discussed in Notes on the Saturator Design - 3, the pressure drop through the reheater channel \((b = 30 \text{ cm}, h = 0.15 \text{ cm})\) is acceptable.

REFERENCES

APPENDIX II

3: MAJOR REQUIREMENTS
NOTES ON THE SATURATOR DESIGN - 3: Major Requirements

1. Introduction

The purpose of this section of these notes is to explore major features of the design of the saturator, assuming that the longitudinal gradient of temperature in the saturator plates is small enough to be neglected. Preliminary investigations having indicated that the advantages gained by using slightly diverging plates are not sufficiently great to compensate for the added complexity of construction, the discussion assumes parallel plates. The Reynolds number of the flow between parallel main plates is

\[ \frac{F}{bV} \]

where \( F \) (cm\(^3\) sec\(^{-1}\)) is the flux of air (10\(^3\) cm\(^3\) sec\(^{-1}\)) and \( b \) is the breadth of the main plates. Exploratory calculations indicate that the overall length of a single-channel saturator will probably be of order 50 cm; it would appear convenient therefore for packaging purposes if \( b \) were somewhat less. Provided \( b < 20 \) cm, \( Re < 350 \), and the flow will be laminar. Also, provided \( b < 50 \) cm, \( Re Pr = \frac{F}{bD} \) will exceed 10\(^2\), so that longitudinal diffusion of heat may be neglected. Similarly, if \( b < 40 \) cm, the analogous parameter \( \frac{F}{bD} \) will exceed 10\(^2\), and longitudinal diffusion of water vapor may also be neglected. It will be assumed in the following discussion that a value of \( b \) will be chosen lying between 20 and 40 cm. It is also assumed that the plate spacing (\( h \)) will be not less than 0.1 cm. Using spacers between the plates to help maintain uniformity of spacing, this would seem to approach the practical limit beyond which variations in the plate spacing might cause significant channeling of the air flow, with some degradation of the saturator performance.
In these circumstances, the approach of the temperature and vapor density to equilibrium with the main plates may (far enough downstream) be estimated by means of an exponential formula in which the residence time is calculated from the mean velocity \( v = F/bh \), and the time constant is \( \tau_h = h^2/\pi^2 \kappa \) for temperature, and \( \tau_w = h^2/\pi^2 D \) for vapor density. Since \( D \approx 0.24, \kappa \approx 0.20, \tau_h > \tau_w \), and the saturator design may be discussed conservatively in terms of \( \tau = \tau_h \).

If the air is to spend \( n(=10) \) time constants between the main plates, their length \( L \) must be given by:

\[
L = \frac{n h F^2}{\pi^2 \kappa b}
\]

Thus the area of the plates is proportional to \( h \). From this point of view therefore, it is advantageous if \( h \) can be kept small.

2. The Direct Effects of Pressure Gradient on the Mixing Ratio

The longitudinal gradient of pressure increases rapidly as \( h \) decreases:

\[
\frac{dp}{dx} = -\frac{12nF}{bh^3}
\]

This pressure fall downstream affects the value of the mixing ratio \( (w) \). If the vapor pressure is in equilibrium with the channel walls, as a result of the downstream pressure fall alone,

\[
\frac{dw}{dx} = \frac{12nFw}{bh^3 p}
\]
At the downstream end, therefore, the mixing ratio would be elevated above its equilibrium value by $\delta w$, where

$$\frac{\delta w}{w} = \frac{\gamma v}{w} \frac{dw}{dx} = \frac{12\eta F^2}{\pi^2 kb^2 h^2 p}$$

Substituting $\eta = 1.6 \times 10^{-4}$ (cgs), $F = 10^3 \text{ cm}^3 \text{ sec}^{-1}$, $\kappa = 0.20$, $p = 10^6 \text{ dynes \ cm}^{-2}$,

$$\frac{\delta w}{w} = \frac{9.7 \times 10^{-4}}{b^2 h^2}$$

Provided $b > 20 \text{ cm}$, $h > 0.1 \text{ cm}$, the proportional error in $w$ caused directly by the downstream fall of pressure will be less than $0.03\%$, which is negligible.

3. The Irrigation of the Main Plates

A. Using Longitudinal Grooves

If the wicking surface consists of square longitudinal grooves of side $s$, spaced with their centers $2s$ apart, the longitudinal forces acting on the strip of water in a groove are those due to

(a) the downstream decrease of air pressure, i.e.

$$-Ls^2 \frac{dp}{dx} = \frac{12\eta F^2 \eta s^2}{\pi^2 kb^2 h^2}$$

(b) the drag of the air on the water surface $Ls\eta \frac{dv}{dy} = \frac{6Fs\eta L}{bh^2} = \frac{6\eta s F^2}{\pi^2 kb^2 h}$, and (c) the drag exerted by the walls of the groove on the
water moving through it. Force (c) is small compared with (b); the total flow of water in the grooves, \(F_w\), is expected to be of order \(10^{-3} g \text{ sec}^{-1}\) at the most. This flow is divided between \(\frac{b}{2s}\) grooves.

The velocity gradient at the groove wall is therefore of order \(\frac{12F_w}{bs^2}\). The total force acting on the water in a groove due to wall drag on three sides is therefore of order \(\frac{36F_w \eta L}{bs}\). The ratio of force (c) to force (b) is therefore

\[
\frac{6F_w \eta h^2}{F \eta s^2}.
\]

Provided \(h \leq 0.3 \text{ cm, } s \leq 2.5 \times 10^{-2} \text{ cm, this ratio is less than } 10\%\), and force (c) may be neglected. (In fact, as will be seen later, there seems no reason why \(h\) should exceed 0.15 cm).

Forces (a) and (b) (the effect of which are additive) must be dominated by the capillary stress exerted by the groove, which can exert a pressure of \(\frac{4\sigma}{s}\), corresponding to a force on the strip of water of \(4\sigma s\).

Thus, the condition to be met is:

\[
\frac{6nF^2 \eta}{\pi^2 kb^2 h} (1 + \frac{2s}{h}) \ll 4\sigma
\]

Thus, provided \(s < \frac{h}{2}\), as seems certain to be the case, the actual magnitude of \(s\) is not of dominant importance in this connection.
If then the maximum capillary stress which can be exerted by the grooves is to dominate the disturbing influence by a factor of 5, it is required that

\[ b^2 h > \frac{15nF^2\eta}{\eta^2 \kappa \sigma} \approx 170 \]

for \( n = 10, \ F = 10^3 \text{ cm}^3\text{ sec}^{-1} \).

This requirement specifies minimum values of \( h \) for any value of \( b \), and consequently, determines also minimum values of \( L \).

\( \left( = \frac{nhF}{\pi^2 \kappa b} \right) \). Values of \( h_{\min} \) and \( L_{\min} \) are given as a function of \( b \) in Table 1 (\( n = 10, \ F = 10^3 \text{ cm}^3\text{ sec}^{-1} \) for \( 20 < b < 40 \).

<table>
<thead>
<tr>
<th>( b ) (cm)</th>
<th>( h_{\min} ) (cm)</th>
<th>( L_{\min} ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.43</td>
<td>109</td>
</tr>
<tr>
<td>25</td>
<td>0.27</td>
<td>55</td>
</tr>
<tr>
<td>30</td>
<td>0.19</td>
<td>32</td>
</tr>
<tr>
<td>35</td>
<td>0.14</td>
<td>20</td>
</tr>
<tr>
<td>40</td>
<td>0.11</td>
<td>14</td>
</tr>
</tbody>
</table>

B. Using a Sintered Metal Wicking Surface

If the wicking surface is a sintered capillary material, the drag exerted by the air will be resisted locally by the surface. The capillary forces need therefore resist only the pressure gradient, and the condition to be fulfilled becomes:
\[
\frac{12nF^2n\varepsilon}{\pi^2kb^2h^2} \ll 4\gamma
\]

where \(\varepsilon\) is the pore diameter of the wicking material. If capillary forces are to dominate by a factor of 5,

\[
\frac{b^2h^2}{\varepsilon} > \frac{15nF^n}{\pi^2k\gamma} \approx 170
\]

Since \(\varepsilon\) can be as small as \(10^{-3}\) cm, this condition places no significant limitations on the design parameters \(b\) and \(h\). The restrictions on the design indicated in Table I may therefore be avoided by using a sintered capillary material instead of grooves; the choice of \(h\) is then effectively independent of that of \(b\), and the length is given by \(L = \frac{nhF}{\pi^2kb}\). With \(n = 10\), this results in the designs shown in Table 2.

**TABLE 2**

Possible Designs Using a Sintered Metal Wicking Surface

<table>
<thead>
<tr>
<th>(b) (cm)</th>
<th>(h = 0.1) cm</th>
<th>(h = 0.15) cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>25</td>
<td>38</td>
</tr>
<tr>
<td>30</td>
<td>17</td>
<td>25</td>
</tr>
<tr>
<td>40</td>
<td>13</td>
<td>19</td>
</tr>
</tbody>
</table>
4. Energy Consumption and Dissipation

The energy dissipated per second in forcing the air flow through the main plates is

\[
F \Delta p = \frac{12nF^3 \eta}{\pi^2 k b^2 h^2} = \frac{9.7 \times 10^6}{b^2 h^2} \text{ (erg sec}^{-1})
\]

The minimum value of \( b^2 h^2 \) occurring in the sample designs shown in Tables 1 and 2 is 4 cm\(^4\). Therefore the maximum energy dissipation is 2.4 \times 10^6 \text{ erg sec}^{-1}, or 0.24 watt. This heat will eventually be deposited in the water flow used to control the plate temperatures, which will be of order 10^2 g sec\(^{-1}\). The resulting temperature rise of the water as it leaves the plates will therefore be of order 6 \times 10^{-4} \text{°C}, which is negligible.

The heating of the air stream by the viscous dissipation of energy is relevant only to the degree that it results in an increase in the temperature of the water surface, since this could perturb the vapor pressure. After several time constants, the energy flux density to each wall from the air amounts to:

\[
\frac{F \Delta p}{2 b h} = \frac{6F^2 \eta}{b^2 h^3} = \frac{9.6 \times 10^2}{b^2 h^3} \text{ erg cm}^{-2} \text{ sec}^{-1}
\]

\[= \frac{2.3 \times 10^{-5}}{b^2 h^3} \text{ cal cm}^{-2} \text{ sec}^{-1}\]
The minimum value of \( b^2h^3 \) in Tables I and II is 0.4 \( \text{cm}^5 \); hence the maximum flux density occurring in any of the sample designs is \( 6 \times 10^{-5} \text{ cal cm}^{-2} \text{ sec}^{-1} \). Even if the thermal resistance between the wall surface and the temperature controlling water flow is equivalent to that of 1 cm of material of thermal conductivity 0.05 cal cm\(^{-1}\) sec\(^{-1}\), (such as stainless steel), the resulting temperature rise of the wall surface is only \( 10^{-3} \text{C} \), which is negligible.

Thus, the viscous dissipation of energy within the air stream does not seem likely to restrict the design.

5. The Measurement of Pressure

At the end of the main plates, the vapor pressure \( (e) \) should be accurately equal to the equilibrium vapor pressure at saturation at the plate temperature. In order to deduce the mixing ratio, both \( e \) and \( p \), the total pressure, must be known at the same point.

The mean velocity of the air between the main plates is \( \frac{F}{Bh} \), which for the designs shown in Tables 1 and 2 has a maximum value of 500 cm sec\(^{-1}\), corresponding to a dynamic pressure head of only about 0.1 mb. Thus the measurement of pressure at this point does not present any problem from this point of view.

Nevertheless, it would seem desirable to make the measurement downstream of the reheater where the air will be unsaturated, in order to avoid any possibility that the measurement might be perturbed by water creeping into, or condensing in, the pressure
sampling tube. If this is to be done, it is essential that the pressure change through the reheater be small, so that the total pressure at the end of the main plates can be accurately estimated.

Preliminary studies of a rather compact and simple "direct" reheater are described in "Notes on the Saturator Design - 2." It consists of electrically heated transverse wires (of length b), and it appears that a total reheater channel length of 10 cm would be more sufficient. In such a reheater, pressure drop would be caused by drag exerted by the walls, and by the transverse wires. The pressure gradient due to the drag exerted by reheater channel walls spaced h cm apart is:

\[
\frac{-dp}{dx} = \frac{12\eta F}{bh^3}
\]

Since the minimum value of \(bh^3\) in Tables 1 and 2 (b = 20 cm, h = 0.1 cm) is 0.02, the pressure gradient in these cases does not exceed 600 \(\eta F\). Thus, for a channel 10 cm long, the pressure change will not exceed \(10^3\) dynes \(cm^{-2}\) (1 mb). Since this pressure change is relatively constant, it could probably be estimated to an accuracy of order 10%, reducing the error to 0.1 mb. However, a design such as that in the second line of Table 2, in which b = 30 cm, h = 0.15 cm so that \(bh^3 = 0.10\) and the total pressure fall is only 0.2 mb would appear to be preferable.
Assuming that the transverse heater wires are located centrally and spaced so that they interact independently with the air stream, both the heat transfer to the air and the drag depend in known ways on the Reynolds number of the flow about the wires \( R_e' \), as shown in Fig. 1 by the uppermost full curves.

If the acceptable temperature rise of \( N \) parallel transverse wires is \( \Delta T \), and it is desired to raise the temperature of the air stream by \( \Delta T \), the Nusselt number \( \frac{H}{k} \) (where \( H \) is the heat transfer per unit length per degree per second) must be such that:

\[
\frac{H}{k} = \frac{F \rho c_p \delta T}{N b k \Delta T}
\]

Also, the pressure decrease caused by the drag of the wire is:

\[
\delta p = \frac{9 N d \rho F^2 c_D}{8 b^2 h^3}
\]

where \( d \) is the diameter of the transverse wires.

Assuming for the moment that these relations can be satisfied with a suitably small integral value of \( N \) (the number of transverse wires), \( N \) may be eliminated to give:

\[
\delta p = \frac{9 F^2 c_D^3 d \delta T}{8 k h^3 b^3 \Delta T} \frac{kc_D}{H}
\]

In Fig. 1, values of \( \log_{10} \frac{c_D k}{H} \) are plotted as isolated heavy points, and the (dashed) line:

\[
\log_{10} \frac{c_D k}{H} = 0.62 - 0.88 \log_{10} R_e'
\]

has been fitted by eye to them in the region \( 0 < \log_{10} R_e' < 2 \), with an accuracy of about 15% in \( \frac{c_D k}{H} \). Thus, to a sufficient accuracy, we may write.
\( \frac{C}{D} \) and \( \frac{H}{k} \) for a Cylinder

\( (H = \text{heat transfer coefficient per unit length,} \ k = \text{thermal conductivity of fluid}) \)

Sources: Goldstein and Roberts, respectively

Fitted line:

\[
\log_{10} \left( \frac{C_D k}{H} \right) = 0.62 - 0.88 \log_{10} R_e
\]

Figure 1
\[ \frac{C_D k}{H} = 4.2 \left( R_e \right)^{-0.88} \]

over this Reynolds number interval.

Thus,

\[ \delta p = 4.7 \frac{\rho^2 c_p F^3 d \delta T}{k h^3 b^3 4 \delta T} \left( \frac{bhv}{F d} \right)^{0.88} = 4.7 \frac{\rho^2 c_p d 0.12 \delta T v 0.88}{k \Delta T} \left( \frac{F}{hb} \right)^{2.12} \]

Clearly, to minimize \( \delta p \), it is somewhat advantageous if \( d \) is chosen as small as possible; a convenient minimum size is five thousandths of an inch (1.3x10\(^{-2}\) cm).

Then, assuming reasonable values such as \( \Delta T = 6 \) C, \( \delta T = 2 \) C,

\[ \delta p = 1.7 \times 10^{-4} \left( \frac{F}{bh} \right)^{2.12} \]

In Tables 1 and 2, the minimum value of bh is 2, and hence \( \frac{F}{bh} \) does not exceed 500 cm sec\(^{-1}\). Hence, \( \delta p < 0.6 \) mb. In a design with \( h = 0.15 \) cm, \( b = 30 \) cm, which, as pointed out above, seems preferable, \( \delta p = 0.1 \) mb; assuming that this pressure change can be estimated to 10\%, the residual error (~0.01 mb) is negligible.

In the case \( b = 30 \) cm, \( h = 0.15 \) cm, \( 3F = 333 \) cm sec\(^{-1}\)

\[ R_e' = \frac{333 \times 1.3 \times 10^{-2}}{v} = 27 \]

The corresponding value of \( \frac{H}{R} \) is about 10. Thus, the required value of \( N \) is:

\[ N = \frac{F \rho c \delta T}{kb \Delta T} \left( \frac{H}{R} \right) \approx 5 \]

In summary, for \( h \geq 0.1 \) cm, the pressure change caused by wire drag does not seem to be a serious problem. However, that
caused by wall and wire drag can be somewhat larger than is desirable, and it appears advisable to adopt a design such as \( b = 30 \text{ cm}, \ h = 0.15 \text{ cm} \), leading to a pressure drop of less than 0.3 mb, which could probably be estimated to within 0.03 mb.

6. Aerosol Behavior

If the aerosol is injected upstream of the saturator, haze droplets will form on the particles as they pass between the main plates, so that the total water content of the gas passing out of the saturator will slightly exceed the saturation value.

It is assumed here that the aerosol consists of a soluble salt such as NaCl, and that cumulative distribution of critical supersaturations is given by \( N = cS_C^k \), the range of \( S_C \) extending from a minimum value(s) up to values exceeding \( 10^{-2} \% \). The value of \( s \) is taken as that corresponding to a maximum dry particle radius \( r_m = 10^{-5} \text{ cm} \). For an aerosol of NaCl particles, this implies:

\[
S^2 = \frac{A}{r_m^3}
\]

where

\[
A = \frac{32 M M_p^2 \sigma}{27 i \rho_p \rho_o R^3 T}
\]

where \( r \) is cm, \( s \) is in absolute units, and \( M \) is the molecular weight of NaCl (57), \( M_p \) that of water, \( \sigma \) the surface tension, \( \rho_p \) the density of the dry particles (2.16 g cm\(^{-3}\)), \( \rho_o \) that of water, and \( i \) the van't Hoff factor (taken to be 2). The value of \( A \) depends somewhat on
temperature; at 20C, $A \approx 1.4 \times 10^{-22}$, and at -20C, $A \approx 2.6 \times 10^{-22}$. Hence, over this temperature range, $s$ varies from $3.7 \times 10^{-4}$ (0.04%) to $5.1 \times 10^{-4}$ (0.05%).

If sufficient time were available, the haze droplets formed on the aerosol particles would grow close to their equilibrium radii $(r_o)$, given by:

$$ r_o = \frac{B}{S} $$

where $B = \frac{4M_o \sigma}{3\sqrt{3} \pi R_m T}$

The value of $B$ is also somewhat dependent on temperature; at 20C, $B \approx 4.2 \times 10^{-8}$, and at -20C, $B \approx 5.2 \times 10^{-8}$.

The total density of water substance (g cm$^{-3}$ of air) which is present in the form of haze droplets ($\rho_h$) would then be:

$$ \rho_h = \frac{4 \pi c_k r_o B^3}{3} \int_{s_m}^{S_m} \frac{s^{(k-4)}}{S_c^{(k-4)}} ds_c $$

$$ = \frac{4 \pi c_k r_o B^3}{3} \cdot \frac{k}{(3-k)} s^{(k-3)} $$

since $s_m^{k-3} \ll s^{k-3}$ for values of $k$ which are likely to occur ($k < 1$).

The term $B^3 s^{(k-3)}$ is temperature dependent, varying as

$$ \frac{3}{2} (k-1) $$
Since $\frac{\sigma}{T}$ is a decreasing function of $T$, for $k < 1$, $\rho_h$ is larger at higher temperatures. It is sufficient therefore to evaluate $\rho_h$ at 20°C, where:

$$\rho_h = 3.1 \times 10^{-22} \frac{c k}{(3-k)} \cdot s^{k-3}$$

For an aerosol in which $N = 10^3 \text{cm}^{-3}$ at $s_0 = 10^{-2}$, $c = 10^{3+2k}$, and

$$\rho_h = 3.1 \times 10^{-13} \frac{k}{(k-3)} \left(10^2 s\right)^{k-3}$$

Substituting $s = 3.7 \times 10^{-4}$, $\rho_h$ is found to be about $10^{-10} \text{g cm}^{-3}$ if $k = 1$, and $2.5 \times 10^{-10} \text{g cm}^{-3}$ if $k = 0.5$ (at 20°C).

The density of saturation water vapor at -20°C is about $9 \times 10^{-7} \text{g cm}^{-3}$. Thus, even at such a low temperature, the water in the form of haze droplets cannot represent an increment in the total mixing ratio of as much as 0.03%, which is negligible. Thus, it is not necessary to investigate whether the haze droplets have sufficient time to grow to sizes close to equilibrium sizes at saturation.

This would not be the case if the aerosol size distribution extended to much larger sizes than $r_p = 10^{-5} \text{cm}$. Thus, if the spectrum extended to $r_p = 10^{-4} \text{cm}$, $s$ would be decreased by a factor of $3/2$, and $\rho_h$ by a factor of $10^3$ for $k = 1$. In that case, it would be important to carefully evaluate the growth of haze droplets during their passage through the saturator, and the need to know the value of $\rho_h$ with appropriate accuracy might well place serious restrictions on the saturator design.
7. Entrainment of Liquid Water by the Flowing Air

Water will not be entrained from the wet walls if the forces exerted by the air flow on an "elementary droplet" or nascent wave are small compared with those due to surface tension. Let the diameter of the elementary droplet be \( d_1 \) cm. Then (the gradient of velocity at the wall being \( 6 \nu/h \)), the aerodynamic stress exerted on it is of order

\[
\frac{6\eta v d_1^2}{h} = \frac{6\eta F d_1^2}{bh^2}
\]

The capillary forces are of order \( \sigma d_1 \). Hence, entrainment would not be expected to occur if

\[
\frac{6\eta F d_1^2}{bh^2} < 1
\]

The minimum value of \( bh^2 \) in Tables 1 and 2 is 0.2, so that

\[
\frac{6\eta F d_1}{bh^2 \sigma} < 7 \times 10^{-2} d_1
\]

Since \( d_1 \) is comparable with the dimensions of the characteristic elements making up the wall surface, it can be of order \( 10^{-3} \) cm, so that

\[
\frac{6\eta F d_1}{bh^2 \sigma} < 10^{-4}
\]

There seems therefore no danger that water droplets could be entrained from the wall surface.
8. Conclusions

The overall saturator size depends chiefly on the plate spacing, h. For values of h > 0.1 cm, there appears to be no problem arising from the direct effect of the pressure gradient \( \frac{dp}{dx} \) on the water vapor mixing ratio; or in connection with the irrigation of the plates, provided a suitable capillary wicking material is used; or from the dissipation of energy within the air stream. However, if longitudinal grooves are used as wicking, there will be certain limitations on the design in connection with satisfactory irrigation of the plates.

In these circumstances, it seems questionable whether the use of longitudinal grooves is advisable; in addition, any surfactant accumulating on the water surface would probably accumulate in the critical downstream portions of the grooves, which may be undesirable. If a suitable wicking material is used instead, surfactants will probably be immobilized. With such a surface, the choice of h (> 0.10 cm) seems to be unrestricted, and then the overall dimensions may be determined so as to provide a reasonable aspect ratio for packaging such as 3:2. If the total length of the entry and reheater sections is \( L_a \), it would then be desired that

\[
\frac{2}{(L+L_a)} = \frac{3}{2} b
\]

Since \( L = \frac{nhF}{\pi \kappa b} \), this leads to a quadratic equation, the relevant solution of which is:

\[
b = \frac{1}{3} L_a + \left( \frac{L_a^2}{9} + \frac{2nhF}{3\pi^2 \kappa} \right)^{1/2}
\]
Another significant consideration for the choice of a design is that it would appear to be desirable to make the measurement of total pressure downstream of the reheater channel. As discussed at the end of Section 5, this suggests the choice of parameter values $b = 30$ cm, $h = 0.15$ cm, which correspond to main plates of length $L = 25$ cm (Table 2). Assuming, as seems likely, that $L_a \approx 20$ cm, the resulting aspect ratio of the whole apparatus is then 45:30, or 3:2.

This design therefore appears to meet all the requirements discussed.