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FLOW FRICTION OF THE TURBULENT COOLANT FLOW IN CRYOGENIC POROUS CABLES

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ABSTRACT

Considered are cryogenic power transmission cables with porous cores. Calculations of the turbulent coolant flow with injection or suction through the porous wall are presented within the framework of a two-layer model. Universal velocity profiles have been obtained for the viscous sublayer and flow core. Integrating the velocity profile, the law of flow friction in the pipe with injection has been derived for the case when there is a tangential injection velocity component. The effect of tangential velocity on the relative law of flow friction is analysed. The applicability of the Prandtl model to the problem under study is discussed. It is shown that the error due to the acceptance of the model increases with the injection parameter and at lower Reynolds numbers; under these circumstances, the influence of convective terms in the turbulent energy equation on the mechanism of turbulent transport should be taken into account.
INTRODUCTION

One of the best techniques for cooling cryogenic power transmission lines consists in the utilization of cores permeable for the coolant. When cables of this type are involved, the turbulent coolant flow in a long pipe with injection or suction of the coolant through the porous wall should be considered in order to predict thermal and hydraulic behaviors.

Previous experimental and theoretical investigations have been mainly focused on boundary layer flows while flows with injection or suction have received inadequate attention in literature. It is a common practice to estimate parameters of turbulent flows in channels with injection or suction using procedures developed for the boundary layer (for instance, ref. 1). Besides, numerical calculations for flows with injection or suction have been reported (refs. 2 and 3).

However, for practical purposes, it is of great interest to develop calculation procedures applicable to channels with injection or suction and involving relatively simple analytical equations. Such a procedure is proposed in this paper.

The flow of an incompressible fluid with constant physical properties through a circular pipe is described by the following equation

\[
\frac{\partial}{\partial x} ru^2 + \frac{\partial}{\partial r} ruv = - \frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{1}{\rho} \frac{\partial}{\partial r} \tau \quad (1)
\]

When normal and tangential injection velocities are taken into account, the integration of equation (1) over the pipe cross-section yields

\[
\frac{d}{dx} \beta u^2 + \frac{2u v w}{R} = - \frac{1}{\rho} \frac{dP}{dx} - \frac{2\tau w}{\rho} \quad (2)
\]

where \( u \equiv (2/R^2) \int_0^R u^2 r dr \) is the mean mass velocity, \( \beta \equiv (2/R^2 u^2) \int_0^R u^2 r dr \) is the coefficient of impulse flow.

Since within the turbulent flow core the velocity profile is nearly uniform, it may be approximately assumed that for the entire cross-section of the pipe exclusive of the region near the wall

\[
\frac{\partial}{\partial x} u^2 + \frac{1}{\rho} \frac{\partial P}{\partial x} = \frac{d}{dx} \beta u^2 + \frac{1}{\rho} \frac{dP}{dx} \quad (3)
\]
Using equations (2) and (3), equation (1) can be rewritten as follows

\[
\frac{\partial}{\partial x} \left( \nu \frac{\partial u}{\partial x} \right) = \frac{2 \tau}{\rho R} - \frac{1}{\rho} \frac{\partial}{\partial x} \left( \rho \frac{\partial u}{\partial x} \right)
\]

(4)

Below, in accordance with the assumption of local similarity (ref. 2), \( \bar{u}_+ \) and \( \bar{r}_+ \) are presumed to be dependent only upon the transverse coordinate \( \bar{r} \).

Moreover, bearing in mind experimental results obtained with injection and given in reference 4 as well as numerical calculations carried out for suction in reference 2, we assume the linear dependence between the radial velocity and:

\[
v = v_w \bar{r}
\]

(4a)

Now the solution of equation (4) is

\[
\bar{r}_+ = \bar{r} \left[ 1 + v_w \left( \bar{u}_+ - u_w \right) \right]
\]

(5)

To describe the turbulent impulse transport, the Prandtl model may be employed

\[
\tau_+ = \frac{\kappa}{2} \left( \frac{d u_+}{d \bar{r}} \right)^2
\]

(6)

where \( \kappa = \kappa f(y) \), \( f(y) \) at \( \bar{y} \to 0 \)

Considering equation (5), the integration of equation (6) gives the velocity profile

\[
\frac{2}{v_w} \left[ \sqrt{1 + v_w \left( \bar{u}_+ - u_w \right)} - 1 \right] = \frac{1}{\kappa} \int_{\bar{y}_0}^{\bar{y}} \frac{\sqrt{1 - \bar{y}}}{f(\bar{y})} d\bar{y}
\]

(7)

As in reference 5, \( \bar{y}_0 \) is a distance from the wall to where
the velocity equals to \( u_y \).

Subtracting from equation (7) its expression at \( y = 1 \), one obtains

\[
\frac{2}{V_+} \left[ \sqrt{1 + \frac{V_+(u_y - u_w)}{V_+}} - \sqrt{1 + \frac{u_y - u_w}{V_+}} \right] = \frac{1}{\kappa} \int y^2 \left( \frac{1 - y^2}{y^2} \right)^{1/2} dy = F(y)
\]

The above equation gives the velocity defect for pipes with injection/suction.

According to reference 5, the explicit function \( F(y) \) may be given as follows

\[
F(y) = Aln(\frac{1}{y}), \quad A = \frac{1}{\kappa} = 2.5
\]

From equations (8) and (9), it may be concluded that for the entire pipe cross-section the following logarithmic dependence is valid and equation (7) becomes

\[
\frac{2}{V_+} \left[ \sqrt{1 + \frac{V_+(u_y - u_w)}{V_+}} - 1 \right] = Aln_{y_+} + B
\]

where \( B = 5.5 \) - constant independent of injection.

With normal injection, equation (10) for the region near the wall is well known (cf. refs. 1 and 6) while for the flow core it can be easily found from equations (8) and (9).

The explicit expression for the velocity profile can be derived from equation (10)

\[
u_+ = u_{w_+} + Aln_{y_+} + B + \frac{V_+}{4} (Aln_{y_+} + B)^2
\]
The presence of a tangential velocity, the velocity profile within the viscous sublayer of turbulent flow with injection is

\[ u_+ = u_{\nu+} + \frac{\exp(V_+\nu_+) - 1}{V_+} \]  

(12)

Within the framework of two-layer flow model, the most natural manner for determining the viscous sublayer thickness consists in setting the right-hand sides of both velocity expressions (eqs. (11) and (12)) equal to each other

\[ \text{Aln}\delta_+ + B + \frac{V_+}{4}(\text{Aln}\delta_+ + B)^2 = \frac{\exp(V_+\delta_+) - 1}{V_+} \]  

(13)

Equation (13) shows that the thickness \( \delta_+ \) is independent of the tangential velocity \( u_{\nu+} \).

Figure 1 presents the calculated dependence between \( \delta_+ \) and \( V_+ \). Moreover, the experimental data from reference 8 along with the viscous sublayer thickness obtained in references 9 and 10 are also shown in figure 1.

\[ \delta_+ = \frac{1}{V_+} \ln(1 + \sigma_0 V_+) \]  

(14)

where \( \sigma_0 = 13.1 \) according to reference 9 and \( \sigma_0 = 11.9 \) according to reference 10; \( V_+ \) is greater than \( \frac{1}{\sigma_0} \).

It should be noted that a measure of stability, \( \text{Re}_\delta \) including parameters calculated from equations (12) and (13) for the viscous sublayer boundary, reduces as the injection velocity grows and it increases with the suction velocity. This is in agreement with the physical picture of the injection influence on the region near the wall (refs. 8 and 10).

Employing the relation \( U_+ = \sqrt{\text{Re}_\delta} \), the friction resistance in the pipe can be derived. \( U_+ \) is given by equation (12) for the viscous sublayer and by equation (11)
for the remainder of the flow.

The friction resistance in a pipe with slanted injection (suction) can be found by integrating the velocity profile to give:

\[
\frac{\sqrt{B}}{\lambda} = u_+ + \frac{2}{V_+} \left[ \frac{\exp(V_+ \delta_+)}{V_+ R_+^2} + \frac{\exp(V_+ \delta_+)}{V_+ R_+} \left( 1 - \frac{\delta_+}{R_+} \right) \right]
\]

\[
- \frac{1}{V_+^2 R_+^2} - \frac{\delta_+}{R_+} \left( 1 - \frac{\delta_+}{2R_+} \right) - \frac{1}{V_+ R_+^2} + 2 A \left( \frac{\ln R_+}{2} - \frac{3}{4} \right)
\]

\[
- \frac{\delta_+}{R_+} \left( \ln \delta_+ + \frac{\delta_+}{R_+} \right) + B \left( \frac{1}{2} - \frac{\delta_+}{R_+} \right) + V_+ \left( \frac{1}{2} \left( \frac{\ln^2 R_+}{2} \right) \right)
\]

\[
- \frac{3}{2} \ln R_+ - 7 - \frac{\delta_+}{R_+} \ln \delta_+ + 2 \frac{\delta_+}{R_+} \ln \delta_+ - 2 \frac{\delta_+}{R_+} +
\]

\[
+ 2AB \left( \frac{\ln R_+}{2} - \frac{3}{4} - \frac{\delta_+}{R_+} \ln \delta_+ + \frac{\delta_+}{R_+} \right) + B^2 \left( \frac{1}{2} - \frac{\delta_+}{R_+} \right)
\]  (15)

where

\[
R_+ = \frac{Re}{2} \sqrt{\frac{\lambda}{B}}
\]

The terms of an order of \( \delta_+^2/R_+^2 \) are omitted in equation (15) since \( \delta_+/R_+ \ll 1 \).

In the absence of injection or suction, equation (15) gives the flow friction for a round tube

\[
\sqrt{\frac{B'}{\lambda_0}} = A \ln R_+ + B - \frac{3}{2} A + \frac{\delta_+}{R_+} \left( \delta_+ + 2A - 2A \ln \delta_+ - 2B \right)
\]  (16)
Solving equation (16) by the method of integration at high Re numbers, the explicit expression for $\lambda(Re)$ can be obtained.

Four iterations produce

$$\sqrt{\frac{N}{\lambda_0}} = \left(1 + \frac{2N}{Re}\right) \left(Aln \frac{Re}{2} + D - D \ln \left[Aln \frac{Re}{2} + D - D \ln (Aln \frac{Re}{2})\right]\right)$$

where

$$N = \delta_+ (\delta_+ + 2A - 2Aln \delta_+ - 2B)$$
$$D = B - \frac{3A}{2}$$

It is convenient to present calculations as a dependence between the relative flow friction $\lambda/\lambda_0$ and injection or suction parameter $b = \delta v_w/\delta U_0$ which is a common practice in the boundary layer theory (ref. 1).

When Re tends to infinity, it follows from equations (15) and (16) that

$$\lim_{Re \to \infty} \left(\frac{\lambda}{\lambda_0} = \gamma\right) = (1 - \omega_w)^2 \left(1 - \frac{b}{b_{kp}}\right)^2$$

where

$$b_{kp} = -4(1 - \omega_w), \quad \omega_w = \frac{u_w}{U}$$

It can be readily seen that in the absence of the tangential injection velocity, equation (18) coincides with the limit friction dependence suggested by Kutateladze-Leontyev for turbulent boundary layers.

Figure 2 shows $\Psi(b)$ calculated from equations (15) and (17).

The contribution of the viscous sublayer to the flow friction appears to be insignificant even at Re $= 10^4$ and it decreases further as the Reynolds number grows.

When there is an injected flow in the pipe, the flow friction decreases with Re, similarly to the case with the boundary layer (ref. 1), while in the presence of suction flow, the Reynolds number influence becomes insignificant. The latter is in agreement with reference 2 and the simplified equation of Wallis (ref. 2-Discussion).

Figure 2 indicates that the results obtained are in satisfactory agreement with other experiments and calculations carried out for injection and suction conditions at $\omega_w = 0$. 

where

$$Re_+ = \frac{Re}{2} \sqrt{\frac{\lambda_0}{\delta_+}}, \quad \delta_+ = 11.64$$
Figure 3 illustrates the influence of the tangential velocity on the relative flow friction in a pipe with injection or suction. The skin friction stress $\tau$ decreases smoothly as the tangential velocity component $u_w$, directed along the flow, increases; while $\tau$ increases when $u_w$, directed against the flow, increases.

The procedure proposed here is based on certain assumptions the main of which is the acceptance of the Brandt model (equation (6)). To estimate the region wherein equation (6) holds, it is necessary to consider the equation of turbulence energy balance without the diffusion term but containing terms characterizing convection, generation, and dissipation (ref. 15)

$$u \frac{\partial E}{\partial x} + v \frac{\partial E}{\partial r} = -\frac{1}{\rho} \frac{\partial u}{\partial r} - \frac{cE^{3/2}}{\bar{x}}$$  \hspace{1cm} (19)

where $\bar{x}$ - turbulence scale assumed to be equal to the Nikuradse mixing region length

$$\bar{x} = 0.14 - 0.08r^2 - 0.06r^4$$  \hspace{1cm} (20)

Similarly to the assumption of local similarity made in above in respect to $\tau$ and $u$, the turbulent energy, $E$, is assumed to be a function only of the transverse coordinate, $E = E(r)$.

To determine the Reynolds stresses, the Kolmogorov correlation can be used (ref. 15)

$$\tau_+ = c_1/2 \frac{du_+}{dy}$$  \hspace{1cm} (21)

If the convective terms are absent in equation (19), the turbulent tangential stress and energy can be obtained from equations (19) to (21)

$$\tau_+ = \frac{a}{c} \frac{3/2}{2} \frac{du_+}{dy}^2$$  \hspace{1cm} (22)

$$E_+ = \frac{a}{c} \frac{2}{3} \left( \frac{du_+}{dy} \right)^2$$  \hspace{1cm} (23)
The constants $a$ and $c$ may be determined from equations (22) and (6), $\sqrt{\alpha a/c} = 1$, and, since in the region near the wall, the ratio between the turbulent tangential stress and turbulent energy remains constant, this parameter, according to reference 16, can be taken to be $\tau_*/E_* = (ac)^{1/2} \approx 0.26$.

Consequently, $a \approx 0.51$, $c \approx 0.13$.

The assumption of turbulent energy local similarity makes it possible to define the term $u \partial E/\partial x$ in equation (19)

\[
\frac{\partial E}{\partial x} = uE_+ \frac{d^2 u}{d\bar{x}^2} = 2u_* E_+ \sqrt{\frac{\alpha}{8}} \frac{dU}{dx}
\]

or, taking into account the continuity equation, one obtains

\[
\frac{\partial E}{\partial x} = -\frac{4v_u}{R} u_* \sqrt{\frac{\alpha}{8}} E_+ \tag{24}
\]

Using equations (4a), (21), and (24), equation (19) can be written as follows

\[
\frac{cE_+}{\bar{x}} - \alpha \left(\frac{du_+}{dy}\right)^2 = -2v_+ \left[ 2 \sqrt{\frac{\alpha}{8}} u_+ E_+^{1/2} + (1 - \bar{y}) \frac{dE_+^{1/2}}{dy} \right] \tag{25}
\]

where the velocity profile $u_+$ is determined from equation (11). The iteration procedure has been employed to solve equation (25). Considering the right-hand side as a disturbing factor, the solution of equation (25) in the first and second approximations may be written

\[
E_{+1} = \frac{a}{c} \bar{x}^2 \left(\frac{du_+}{dy}\right)^2 \tag{26}
\]

\[
E_{+2} = \frac{a}{c} \bar{x}^2 \left(\frac{du_+}{dy}\right)^2 - 2v_+ \frac{\bar{x}}{c} \left[ 2 \sqrt{\frac{\alpha}{8}} u_+ E_+^{1/2} + (1 - \bar{y}) \frac{dE_+^{1/2}}{dy} \right] \tag{27}
\]
The approximation \( E_1 \) is justified under the assumption that the turbulence energy generation and dissipation are equal, that is, when the Prandtl model is valid.

Calculated values of \( E_1 \) and \( E_2 \) at \( Re = 10^4 \) and \( \omega_w = 0 \) normed on \( u_\tau^* \) are plotted in figure 4.

It is evident that the maximum of turbulence increases and moves away from the wall as the injection flow grows. This is found to be in qualitative agreement with experiments reported in reference 17 for a rectangular duct with the injection flow entering the duct from one of the walls. At high parameters of suction, the flow in the pipe center tends to become laminar.

Discrepancy between \( E_1 \) and \( E_2 \) due to the influence of convective terms on the turbulent transport mechanism, characterizes the region of applicability of results on skin friction in pipes with injection or suction.

As the injection parameter increases (b < -4) and Re decreases, the error due to the Prandtl model grows and one should consider the influence of convective terms in the turbulent energy equation on the turbulent transport mechanism.

**NOMENCLATURE**

- \( x, r \) axial and radial coordinates
- \( u, v \) axial and radial velocity correspondingly
- \( p \) pressure
- \( \rho \) density
- \( \tau \) shearing stress
- \( R \) pipe radius
- \( \delta \) viscous sublayer thickness
- \( \nu \) kinematic viscosity
- \( u_* = \sqrt{\tau_w / \rho} \) shear velocity
- \( \lambda \) skin-friction coefficient

Dimensionless variables:

\[
\bar{r} = \frac{r}{R}
\]

\[
\bar{y} = 1 - \bar{r}
\]

\[
\bar{\tau} = \frac{\tau}{\tau_w}
\]

\[
\bar{u}_+ = \frac{u}{u_*}
\]

\[
\bar{v}_+ = \frac{v}{u_*}
\]
\[
\begin{align*}
y_+ &= \frac{yu^*}{v} \\
\delta_+ &= \frac{\delta u^*}{v} \\
U_+ &= \frac{U}{u^*} \\
Re &= \frac{2RU}{v} \\
R_+ &= \frac{Re}{2} \sqrt{\frac{A}{\delta}} \\
E_+ &= \frac{E}{u^*} \\
\end{align*}
\]

Subscripts:

W \quad \text{at the wall}
0 \quad \text{without injection or suction}

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REFERENCES


FRICION DE L'ECOULEMENT TURBULENT EN CABLE POREUX CRYOGENIQUE

SUMMAIRE: Ici sort considérés les cables de puissance cryogénique avec injection ou aspiration à travers la muraille poreuse, est présenté dans le cas d'un modèle avec deux couches. Le profil de vitesse universelle est dérivé pour la couche visqueuse et le centre d'écoulement. Avec l'injection du profil de vitesse, ont arrivé à la loi qui dérègne la friction de l'écoulement dans un tube avec injection pour le cas où il y a un composant de vitesse d'injection tangentielle. L'effet de la vitesse tangentielle sur la loi relative à l'écoulement de friction est analysé. L'application du modèle de Prandtl au problème ici est discuté. Il est montré que l'erreur du modèle est augmentée par le paramètre d'injection pour les nombres Reynolds plus bas; sous ces circonstances, l'influence des termes convectifs dans l'équation d'énergie turbulente devrait être pris en compte.
1 CALCULATION FROM REF. 10
2 CALCULATION BY THE METHOD PROPOSED
O EXPERTIMENTAL DATA FROM REF. 8

Figure 1. - Viscous sublayer thickness versus injection or suction velocity.

1 Re = 0
2, 3 Re = 10^4
4 Re = 10^5
5 Re = 10^6
6 FROM REF. 4
7 FROM REF. 11
8 FROM REF. 12
9 FROM REF. 13
10 FROM REF. 14
1A REF. 2 AND WALLIS REF. 2: 1 + 0.7b - \phi SUCTION ONLY

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INJECTION

SUCTION

\( \varphi \)

Figure 2. - The influence of injection or suction parameter on relative flow friction.
Figure 3. - Injection or suction tangential velocity versus flow friction.

Figure 4. - Distribution of turbulent energy in the pipe cross-section.