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INTERNAL GRAVITY WAVES IN THE UPPER ATMOSPHERE, GENERATED BY
TROPOSPHERIC JET STREAMS

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(NASA-TM-75407) INTERNAL GRAVITY WAVES IN
THE UPPER ATMOSPHERE, GENERATED BY
TROPOSPHERIC JET STREAMS (National
Aeronautics and Space Administration) 12 p
Unclas
HC A02/MF A01
CSCL 04A G3/46 17097
INTERNAL GRAVITY WAVES IN THE UPPER ATMOSPHERE, GENERATED BY TROPOSPHERIC JET STREAMS

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A mechanism of internal gravity wave generation by jet streams in the troposphere is considered. Evaluations of the energy and pulse of internal gravity waves emitted into the upper atmosphere are given. The obtained values of flows can influence the thermal and dynamic regime of these layers.
INTERNAL GRAVITY WAVES IN THE UPPER ATMOSPHERE, GENERATED BY TROPOSPHERIC JET STREAMS

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On synoptic maps, related to the upper troposphere (500 millibars, 300 millibars), regions of strong winds are often traced. These zones often accompany atmospheric fronts and cyclones. These layers, which are several kilometers thick, extend several hundred, and sometimes even up to several thousand, kilometers in a horizontal direction. In other words, the movement of air masses bears the nature of jet streams. As a rule, these flows are strongly turbulent, which is indicated by the results of aircraft measurements. The turbulent vortices of different sizes, "frozen" into a jet stream, i.e., carried by it, also serve as a cause of the generation of internal gravity waves (VGV). Actually, we imagine that, in an atmosphere which is, on the whole, calm, a periodic structure, with a spatial period \( \lambda \), rushes through at some velocity. Then, with a velocity of movement of this discontinuity equal to \( C \), the generation of internal gravity waves is possible in the atmosphere, with the condition that the frequency of the pulsations of air particles in the calm atmosphere \( \omega = \frac{2\pi - C}{\lambda} \) is less than the Brant-Weissel frequency \( N, \omega < N \). The spectrum of the occurring waves is clearly associated with the spectrum of turbulence of the jet stream.

In order to obtain quantitative dependences, we will examine a model of a jet stream, extending infinitely along the horizontal, and several kilometers thick. The velocity of the stream is constant, and equal to \( C \), and decreases to

*Numbers in the margin indicate pagination in the foreign text.
zero in bounds at the boundaries. In addition, the model is two-dimensional, i.e., it does not depend on the coordinate in the lateral direction. Generalization to the case of a wind which depends on the altitude \( z \) will be given below.

We will first assume that the atmosphere outside of the jet stream is in a state of calm and hydrostatic equilibrium. We will further assume that \( N(z) \) changes little at a distance on the order of the vertical wavelength. We will note that a similar problem is solved for internal gravity waves which propagate from the jet stream, and are observed at the Earth's surface [1]. We are interested in the spectrum of waves which are radiated into the upper atmosphere.

If the periodic perturbation in the jet stream is conveyed with a flow velocity \( C \), then, in a stationary atmosphere, all of the hydrodynamic fields will be proportional to \( \exp i(\alpha x - \omega t) \), where \( \alpha = 2\pi/\lambda \). It is common knowledge from the theory of internal gravity waves that, for example, the vertical component of velocity \( \omega h(z) \Bbb{P}^{-1/2} \exp i(\alpha x - \omega t) \), where \( h(z) \) is the function which describes the vertical structure of the wave, and \( \Bbb{P}(z) \) is the unperturbed density of the atmosphere. If it is assumed that the horizontal component of velocity in the wave is associated with \( \omega \) by the condition of incompressibility

\[
\nabla U = -w, 
\]

then the function \( h(z) \) satisfies the equation

\[
\frac{d^2h}{dz^2} + \gamma^2 h = 0, \tag{1}
\]

where

\[
\gamma^2 = z^2 \left( \frac{N^2}{\omega^2} - 1 \right) - 1 \left( \frac{d}{dz} \ln \Bbb{P} \right)^2 - \frac{1}{2} \frac{d^2}{dz^2} \ln \Bbb{P}.
\]
The assumption of a smooth change in \(N(z)\) (this is correct up to an altitude of \(\sim 100\) km) makes it possible to utilize the following as solutions \((1)\) of internal gravity wave approximation

\[
h = \gamma^{-1/2} \exp \left| \psi(z) \right|, \tag{2}
\]

where

\[
\psi(z) = \int_{z_0}^{z} \frac{\gamma}{d\gamma} dz.
\]

The index "0", here and subsequently, designates all of the variables at the upper boundary of the jet stream. For waves travelling upward, we will use the "minus" sign. In the case when \(\gamma^2 < 0\), internal gravity waves are not generated, and

\[
h = \left| \gamma \right|^{-1/2} u^{-\gamma(z)},
\]

where

\[
\gamma = \int_{z_0}^{z} \left| \gamma \right| dz.
\]

Then:

\[
U(z) = U_0 \sqrt{\frac{\rho \gamma}{\rho \gamma_0}} \exp \left[ \gamma(z) + ax - \omega t \right].
\]

The magnitude of the pulsations of the temperature \(T\) is found from the approximation of the adiabatic nature for mesoscale waves

\[
i \omega T + (\nabla_v - \nabla T) w = 0,
\]

or
\[ \frac{1}{C} \frac{\partial T}{\partial z} \left( \Gamma_a - \Gamma \right) w = 0, \]

where \( \Gamma_a \) is the adiabatic temperature gradient, and \( \Gamma \) is the actual temperature gradient.

Making use of the fact that
\[ \omega = -\frac{\alpha}{\Gamma(z)} U, \]

we obtain
\[ T = \left( \Gamma - \Gamma_a \right) U_0 \sqrt{\frac{\beta_0}{\beta_0}} \exp \left[ \psi(z) + \frac{\pi}{2} - \omega t \right]. \]

We will square the real part of this expression, and average it according to the period of the oscillations. We will obtain the connection of the spectrum of temperature pulsations \( \langle T^2 \rangle_k \) with the spectrum \( U_0 - S(k) \):
\[ \langle T^2 \rangle_k = \frac{1}{2} \frac{(\Gamma - \Gamma_a)^2 \beta_0}{C \Gamma(z) U_0} S(k). \]

We will note that this formula is obtained with \( \gamma^2 > 0 \). Otherwise (\( \gamma^2 < 0 \)), the internal gravity waves, as has already been noted, would not be generated. As an example, with \( C = 20 \) m/sec, the high-frequency portion of the spectrum \( S(k) \), beginning with \( \lambda' = 2\pi/a'N \) 400 m, is precluded from the generation by the condition \( \gamma^2 \eta \frac{N^2}{C} - a^2 < 0 \). Thus, the internal gravity waves with wavelengths less than \( \lambda' \) are not generated.

Formula (3) in its natural form is generalized to the case when there is a wind \( V(Z) \) changing smoothly with altitude \( (V(Z_0) = 0) \). For this purpose, it is necessary to replace \( C \)
in formula (3) with \( C - V(z) \) [1], and

\[
\gamma^2(z) = \frac{N^2(z)}{|C - V(z)|^2} - a^2.
\]

As is common knowledge [2], at those levels where the phasic velocity of the wave is exactly equal to the velocity of the flow ("critical" level), there occurs disruption of the wave picture, and above these levels, the wave is not propagated. At these altitudes, expression (3) is not applicable. However, situations are possible when the noted singularity in (3) is absent, and the internal gravity waves will penetrate into the upper atmosphere. Then, the spectrum of these waves, according to (3), is similar to the spectrum of the pulsations of the horizontal velocity of the wind in the jet stream.

The following problem consists of the determination of the energy flows of internal gravity waves, generated in jet streams in the atmosphere. As is common knowledge [3], the spectrum of the horizontal component of velocity in the jet stream has a relative maximum (figure). This attests to the fact that the vortices in the given range of wave numbers possess the greatest energy. Evidently, the turbulent movements of these scales are responsible for the generation of the internal gravity waves observed in the upper atmosphere. In the areas to the left and right of the indicated area, there takes place a normal cascade transmission of energy into the high-frequency range, which is not accompanied by any energy losses whatsoever. The physical mechanism of generation of internal gravity waves becomes clearest if the entire examination is carried out in a system of coordinates which is associated with the jet stream. Then, the problem is completely analogous to the problem of flowing of an air
flow around a statistically uneven surface [2]. The velocity of the wake of the waves is equal to $C$ (several dozen $m/sec$), and it follows from the dispersion characteristics of the internal gravity waves that the phasic velocity is directed at a small angle to the vertical, and the waves are short.

The frequency of a single harmonic is

$$\omega = Ck,$$

where $k$ is the wave number along the horizontal. On the other hand, it follows from the approximated dispersion ratio

$$\omega = Nk; k_2,$$

that $k_2 \leq N/C$. It is common knowledge that the grouped velocity of internal gravity waves, in the given case, is orthogonal to the phasic velocity, and, posing the condition of irradiation for a flow of energy, we adopt $k_2$ with a "minus" sign. The energy flow is directed at a small angle to the horizon. The spectral components of the energy flow have the form:

$$I^k = <p_k \omega_k> = \frac{\rho_0 C^2 k}{N} U_h^2,$$

$$P^k = <p_k U_h> = \rho_0 C U_h^2,$$

where $k \equiv k$, and we utilized the formula of continuity, which
associates the vertical and horizontal components of velocity in the wave:

\[ w_h = \frac{kC}{N} U_{k_h} \]

\[ U_k^2 = S(k) \] is the spectrum of the horizontal component of velocity in the jet stream, and the brackets indicate the averaging according to the period of the wave:

\[ |F^k| = \sqrt{(F_x^k)^2 + (F_y^k)^2} = \rho_0 CS(k) \sqrt{\frac{C^2 + N^2}{N^2} + 1} \approx \rho_0 CS(k). \]

By integrating expression (1) according to the indicated area of wave numbers \( \Delta k \), we obtain the total input of internal gravity waves into the energy flow, from the entire spectral interval:

\[
F_z = \int_{\Delta k}^{} F_x^k dk = \frac{\rho_0 C^2}{N} \int_{\Delta k}^{} kS(k) dk, \\
F_x = \int_{\Delta k}^{} F_y^k dk = \rho_0 C \int_{\Delta k}^{} S(k) dk.
\]

The vertical component of the energy flow is much less than the horizontal component. In actuality,

\[
\frac{F_z}{F_x} = \frac{C}{N} \int_{\Delta k}^{} kS(k) dk = \frac{C}{N} \bar{k},
\]

where \( \bar{k} \) is the average wave number in the interval \( \Delta k \) of generation of internal gravity waves. In this case, the
spectrum of $S(k)$ is the weight of averaging. From here, it is evident that $F_z/F_x \ll 1$, i.e., the energy is propagated basically in the direction of the velocity of the jet stream. If we take the characteristic values $p_o(10 \text{ km}) = 0.4 \text{ kg/m}^3$, $C = 40 \text{ m/sec}$, we obtain

$$F_z \approx 16 \text{ W/m}^2, \quad F_x \approx 0.3 \text{ W/m}^2.$$ 

The characteristic periods of similar internal gravity waves in the upper atmosphere are 1.5-2 hours. The rates of their propagation are several dozen m/sec.

If one thinks, as before, that there is an average wind $V(Z)$, which changes smoothly with altitude, then the formulas given above for the energy flow change in the following manner:

$$F_x = \frac{[p_o(C - V(Z))]^2}{N} \int S(k) dk,$$

$$F_z = p_o |C - V(Z)| \int S(k) dk.$$

In the case when $V(Z)$ makes some angle with the direction of the velocity of the jet stream, $V(Z)$ should be construed as the projection of the wind velocity in this direction. The angle $\theta$, at which the energy flow is directed to the horizontal surface at an altitude $Z$, is determined by the relationship:

$$\sin \theta \approx \frac{C - V(Z)}{N} \frac{\int kS(k) dk}{\int S(k) dk}.$$

We will again note that all of the expressions are correct with $|V(Z)| \ll C$. Thus, the beam may intersect the given level at an angle, which differs from the angle at which the energy flow is directed at the altitude of the jet stream. With a
tail wind, this angle decreases, which leads to a decrease in $P_{z_0}$. Conversely, with a head wind, the energy flow into the upper atmosphere may increase substantially. For the conduct of concrete evaluations, it is necessary to bear in mind the data of the magnitudes of $C$ and $V(z)$.

The energy flows, entering the upper atmosphere and dissipating in these areas, may lead to variations in the thermal conditions. The vertical flows of a horizontal pulse affect the dynamics of these layers:

$$I_z = \rho_0 \cdot Uw = \rho_0 \int_{w}^\infty U_k w_k dk = \rho_0 \frac{C}{Nv} \int_{k}^\infty kS(k)dk, \quad \text{for}, \quad \text{F, C}.$$

Utilizing the previous evaluations of $F$, we obtain $I_x \approx 0.1$ dynes/cm². In the problem of flowing of the wind around the statistically uneven Earth's surface [2], $I_x = 4$ dynes/cm², and, consequently, the air flows which flow around mountains evoke a more intense generation of energy flows and a pulse, as compared with jet streams.
REFERENCES

