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MIE DISDROMETER FOR IN SITU MEASUREMENT OF DROP SIZE DISTRIBUTIONS

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ABSTRACT

Test results are shown for a disdrometer breadboard which uses Mie scattering and incoherent optical correlation for in situ measurement of drop size distributions in a cloud chamber.

INTRODUCTION

Certain expansion cloud chamber experiments planned for the Atmospheric Cloud Physics Laboratory (ACPL) payload for Spacelab III require in situ measurement of drop size distributions inside the chamber. With joint ONR-NASA MSFC support, an instrument has been breadboarded and tested which may provide the drop size distribution measurement capability required for the ACPL, as well as for other settings.

The instrument processes in parallel fashion the light scattered by all drops in a sizeable portion of the illuminating collimated light beam, and utilizes the details of the scattered light intensity versus scattering angle in a technique which combines incoherent optical correlation by means of spatial filters with digital data processing. Besides providing a proof of concept, the breadboard was built and tested in order to obtain experimental information about certain noise features of the instrument, so that these could be compared with theoretical results in an effort to check our understanding of the signal to noise degradation suffered inthe data reduction process. Such understanding is indispensable for successful design of a practical instrument of this type.

The statistical noise measurements require that repeated measurements be made on the same cloud. Therefore, we chose as test object a cloud of latex spheres, which are small enough so that gravitational settling does not present a problem. Repeated measurements were made on monodispersed as well as polydispersed clouds, using latex spheres of .30, .36, .40, .55, and 1.01 micrometers radius.

Description of the instrument breadboard

A two-color collimated light beam is used to illuminate part of the cloud in the chamber. Light scattered by the cloud drops in a certain range of angles passes through a chamber window and is processed by the detector which is mounted outside the chamber. The detector consists of a wide-angle 35mm camera lens, in the focal plane of which there is a color transparency which acts as a spatial filter; the light which passes through the color slide falls on a photomultiplier tube. The PMT signal is measured with a number of different filters in place; in the present breadboard, 11 filters are arranged on a disc which can be turned by a stepper motor. The 12th "filter" is opague and serves to measure the dc offset to be subtracted. Each of the filters is a color slide with green and red transmittances which vary with location on the slide. The PMT signal is proportional to the sum of the correlations, in the green and in the red, of the light intensity function incident on the filter, and the filter transmittance function. Hence, in the linear regime of the PMT, the subtracted PMT outputs are linear functionals of the drop size distribution. If the drop size distribution is a linear combination of at most 11 basis functions, the drop size distribution can be calculated by executing a fixed linear transformation on the 11- dimensional subtracted PMT output vector, and by using the resulting numbers as coefficients in a linear combination of basis distribution functions. This data reduction is done digitally. Near shot-noise limited operation is obtained by following each measurement (for each filter) by a calibration, using an LED. This makes it possible to eliminate the effects of all gain drifts in the system, except for the drift due to temperature change of the differential spectral quantum efficiency of the PMT photocathode. The breadboard is completely controlled by an HP 9825A desc calculator, and data acquisition is done by an HP 3437A system voltmeter which interfaces with the HP 9825A. The scattering angle ranges from 20 to 80 degrees. The 11 filters used in the breadboard are rather crude and are far from optimal; consequently, the signal to noise ratio degradation in the data reduction is much larger than need be. A process to produce better filters is under development. For simplicity, we used as basis functions the very sharply peaked size distributions of Dow-Chemical latex spheres of radii .30, .36, .40, .55, and 1.01 microns. The background scattering in the chamber is considered as a separate scattering object, whose intensity is allocated a separate axis in size space.

Noise considerations

Let $\vec{n}$ be the vector whose components are the expansion coefficients of the drop size distribution in terms of the basis size distribution functions; $\vec{n}$ is here called a size vector and the space spanned by the size vectors is called the size space. Let $\vec{s}$ be the vector whose components are the (dc offset-subtracted) PMT outputs with the different filters in place. For linear PMT operation and incoherent superposition of the light scattered by the different drops one has

$$\vec{s} = M \vec{n}, \tag{1}$$

where M is a linear transformation, represented by a matrix. M is here called the Mie mapping or the forward mapping. If M is not singular, there exists an inverse mapping

$$\vec{n}=C\vec{s}, \tag{2}$$

such that either $\vec{n}$ of (2) is a solution of (1) or else the forward map (1) applied to $\vec{n}$ of (2) gives a result $\vec{s}$ which differs minimally from the measured data vector. These cases occur respectively if the data space dimension is equal to or larger than the size space dimension."Minimal difference" is meant here as minimal Euclidean norm of the difference vector, and is equivalent to a least squares fit. The matrix C is determined once and for all, and is stored in the instrument microcomputer (here, the HP9825A). The measured data vector has a noise part $\vec{\Delta s}$, which through (2) gives rise to a noise part

$$\vec{\Delta n}=C\vec{\Delta s} \tag{3}$$

in the size vector.The ratio

$$\beta\frac{|\vec{\Delta n}|/|\vec{n}|}{|\vec{\Delta s}|/|\vec{s}|}=\frac{|C\vec{\Delta s}|}{|\vec{\Delta s}|}\cdot\frac{|\vec{s}|}{|C\vec{s}|} \tag{4}$$

depends on the directions of $\vec{\Delta s}$ and $\vec{s}$.$\vec{s}$ lies in the image of the non-negative cone in size space, whereas the direction of $\vec{\Delta s}$ is unrestricted. In practice, we are interested in some appropriate average $\langle\beta\rangle$, which expresses the degradation of signal to noise ratio suffered in the data reduction (2). For $\vec{\Delta s}$ in the direction of $\vec{s}$, one has $\beta=1$; the associated $\vec{\Delta n}$ is in the direction of $\vec{n}$ and therefore causes an apparent fluctuation in cloud density. $\vec{\Delta s}$ perpendicular to $\vec{s}$ causes a vector $\vec{\Delta n}$ with components $\vec{\Delta n}_{\parallel}$ and $\vec{\Delta n}_{\perp}$, respectively in the direction of $\vec{n}$ and perpendicular to $\vec{n}$. $\vec{\Delta n}_{\perp}$ causes a change in the direction of $\vec{n}$, resulting in an apparent fluctuation of the normalized measured size distribution. In practice, the "angle noise" $|\vec{\Delta n}_{\perp}|/|\vec{n}|$ of the size vectors sets the instrument resolution. There is angle noise $|\vec{\Delta s}_{\perp}|/|\vec{s}|$ in data space as well, and the ratio

$$\gamma=\frac{\langle|\vec{\Delta n}_{\perp}|/|\vec{n}|\rangle}{\langle|\vec{\Delta s}_{\perp}|/|\vec{s}|\rangle} \tag{5}$$

of average angle noises in size space and data space is a useful expression for the degradation due to data reduction.

There are contributions to $\vec{\Delta s}$ from "counting"errors in the sensitive experimental volume (SEV), from nonuniformities of the drop densities in the SEV, from quantum effects in the scattering, from PMT shot noise, from changes in the quantum efficiency of the PMT photocathode, and from post-detection noise. For the better runs the measurement turns out to be nearly PMT-shot-noise limited . For shot noise, the standard deviation of the vector component $(\vec{\Delta s})_j$ is proportional to $\sqrt{s_j}$, and therefore, the surfaces of constant noise probability in data space are hyperellipsoids with an aspect ratio and an orientation which depends on the data vector $\vec{s}$. In the noise calculation, a conservative simplification was made, in which these shot noise ellipsoids are replaced by spheres with a radius equal to the rms value of the shot noise vector components. For an ensemble of normalized noise vectors $\vec{\Delta s}$ with uniformly random directions, β of (4) was calculated, together with the maximum value of β, the directions $\vec{\Delta s}$ and $\vec{s}$ for the maximum, and the ratio of rms values

$$\langle\beta\rangle = \frac{\text{rms} \mid C\vec{\Delta s} \mid / \mid \vec{\Delta s} \mid}{\text{rms} \mid C\vec{s} \mid / \mid \vec{s} \mid} , \qquad (6)$$

where $\vec{s}$ is determined from (1) and $\vec{n}$ runs over an ensemble of normalized vectors with non-negative components but otherwise uniformly random directions. The degradation $\langle\beta\rangle$ of (6) depends on the range of scattering angles seen by the detector, the basis size distribution functions, the number of instrument channels, and the filter transmittance functions. Choosing these items in such a manner that the degradation $\langle\beta\rangle$ has an acceptable value is one of the main concerns in the instrument design.

Measurements

We measured 33 clouds consisting of latex spheres of .30, .36, .40, .55, and 1.01 microns radius, with either a single particle size or mixtures. For each cloud, 20 complete measurements were made (a full revolution of the filter wheel constitutes one complete measurement). Concentrations ranged from about 30 to several hundred particles per cm^3. For each ensemble of 20 measurements the ratio of noise to shot noise, the average data vector, and the rms angle noise in the data vector were calculated . For each data vector the corresponding size vector was calculated, using an inverse mapping computed from the forward mapping obtained from the ensemble average of measured forward mappings; from these results, the average size vector for the ensemble and the rms angle noise of the size vector was obtained. From the rms angle noises the degradation γ of (5) was calculated. Also, to provide further insight into the mapping M, the angles between data vectors in a few data subspaces were calculated for several monodispersed clouds.

Results

19 to 25 of the 33 runs had a ratio of noise to shot noise below 2, depending on the choice of basis and data subspace. Poor runs resulted from large powerline fluctuations which sometime plague our laboratory.

The rms angle noise in the data vector and in the size vector , as well as the ratio of these two angle noises has been calculated from the data for the forward mapping M, for two choices A and B of basis and data subspace. Choice A has been made to investigate the instrument resolution at the small particle end; we use as basis for the mapping M the background, and monodispersed clouds with spheres of .30, .36, and .40 microns radii, together with a 5-dimensional data subspace. In choice B, the .36 micron particle is dropped in favor of two particles of .55 and 1.01 micron radius, and a 6-dimensional data subspace is used. Table I shows the rms value $\langle\gamma\rangle$ of the ratio γ of rms angle noise of size vector and data vector, for the collection of all runs, including the bad runs with noise / shot noise ≥ 2, for the choices

A and B, together with the theoretical results for the maximum value of β, and the average value $\langle\beta\rangle$ of (6), calculated from an ensemble of random vectors in the manner discussed above.

	choice A	choice B
measured:		
$\langle\gamma\rangle$	5.1	10.9
standard deviation in γ	2.1	8.1
theoretical:		
β_{max}	19.7	100.0
$\langle\beta\rangle$	8.7	43.0

Table I
Comparison of measured and theoretical results for the degradation suffered in the data reduction process.

It is seen that, as expected, the calculated degradation $\langle\beta\rangle$ is conservative for both choices A and B, but much more so for B. The theoretical overestimation of the degradation is due to the simplification discussed above, in which the shot noise ellipsoids in data space are replaced by spheres. The very large overestimation of the degradation in case B suggests that for the design of a practical shot-noise-limited instrument with an appreciable number of channels the noise analysis should be refined to take into account the ellipsoidal shape of the surfaces of constant probability in data space.

For monodispersed clouds, the instrument capability to recognize clouds of single-size particles among the radii of .30, .40, .55, and 1.01 microns may be expressed in terms of the total electric energy W supplied to the illuminating lamp during the measurement, and the visible range R of the cloud. From the data it follows that correct identification of a .40 micron particle cloud with a confidence level of 99.7 % requires

$$W/R \geq 1.0\times10^{-3} \text{ Joules/Meter.} \qquad (7)$$

Instrument resolution for polydispersed clouds may be expressed in terms of W, R, and the rms angle noise $\Delta\phi$ in size space. From the data we find for a polydispersed cloud consisting of particles of radii .30, .36, and .40 microns

$$\Delta\phi = A\sqrt{R/W} \quad \text{degrees,} \qquad (8)$$

where A=.46 for the worst run, and A= 8.5×10^{-2} for the best run. In (8), W is the electric energy to the lamp in Joules, and R is the visible range in Meters. The results (7) and (8) hold as long as measurements are shot-noise-limited, and for serial operation. For parallel operation, the right hand sides must be divided by the number of instrument channels.