

NONLINEAR INITIALIZATION OF THE GLAS MODEL

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ABSTRACT

A simplified version of the GLAS model is linearized and the normal modes extracted. These modes show the necessary separation for nonlinear initialization.

INTRODUCTION

The finite difference form of the GLAS model is exceptionally difficult to put into normal mode form. A procedure is outlined which will allow a simplification of the model and still incorporate the essential ingredients for nonlinear initialization so that the high frequency components may be slowed. A small version of the simplified system is presented, linearized, and the normal modes described. Some form of this system will ultimately be initialized and tested in the GLAS model.

INITIALIZATION

The procedure for nonlinear initialization has been presented by Baer and Tribbia (1977). The system equations should be written in the form,

$$\frac{dx}{dt} + Ax = \epsilon G(x, x) \quad (1)$$

where x is a vector of dependent variables (grid point values of the flow field and physical variables, for example), A is a matrix of fixed quantities dependent on the system, and ϵ is small parameter, usually the Rossby number. Since most systems of this sort are exceptionally complicated, as in the GLAS model, let us rewrite (1) in the form

$$\frac{dx}{dt} + \hat{A}x = \epsilon G + (\hat{A}-A)x \quad (2)$$

where \hat{A} is a "simpler" matrix, and $\|S^{-1}(\hat{A}-A)S\|$ is of order ϵ . Here S is the modal matrix which diagonalises A .

We shall find the modes of \hat{A} , a simple version of the GLAS model so that we may separate the frequencies into fast and slow, a procedure essential to the nonlinear initialization scheme. Thus we will find,

$$\hat{A} = \underline{S} \begin{pmatrix} \varepsilon \Lambda & 0 \\ 0 & \Lambda_y \end{pmatrix} \underline{S}^{-1} \quad (3)$$

where clearly the scaled frequencies $\varepsilon \lambda_x \ll \lambda_y$.

GLAS MODEL NORMAL MODES

We begin the simplification of the GLAS model from the non-forced version (see Somerville, et. al., 1974) described as follows:

$$\begin{aligned} \frac{\partial \mathbb{W}}{\partial t} + \mathbb{W} \cdot \nabla \mathbb{W} + \hat{f} k_x \mathbb{W} &= - \nabla_{\sigma} \phi + \sigma \alpha \nabla_{\sigma} \pi \\ \nabla_{\sigma} \cdot \mathbb{W} + \frac{\partial \dot{\sigma}}{\partial \sigma} &= - \frac{d}{dt} \ln \left(\rho \frac{\partial \phi}{\partial \sigma} \right) \\ \frac{\partial \phi}{\partial \sigma} &= - \alpha \pi \\ \frac{\partial \ln \theta}{\partial t} + \mathbb{W} \cdot \nabla \ln \theta + \dot{\sigma} \frac{\partial \ln \theta}{\partial \sigma} &= 0 \\ P &= P_t + \sigma \pi, \quad \pi = P_s - P_t \end{aligned} \quad (4)$$

Symbol definitions may be found in the above reference. We now linearize this system about a state of rest, let $\pi \rightarrow \pi(x, y, t) + \Pi$ and $\alpha \rightarrow \alpha(x, y, \sigma, t) + A(\sigma)$ where $\pi \ll \Pi$ and $\alpha \ll A$. We include the following definition where ϕ is the perturbation of Φ ,

$$\psi \equiv \phi + \sigma A \pi, \quad \omega \equiv \dot{\sigma} + \frac{\sigma}{\Pi} \frac{\sigma \pi}{\partial t}, \quad \mathbb{W} = (u, v). \quad (5)$$

The resulting linearized system then becomes,

$$\begin{aligned} \frac{\partial \mathbb{W}}{\partial t} + \hat{f} k_x \mathbb{W} + \nabla_{\sigma} \psi &= 0 \\ \frac{\partial}{\partial t} \psi_{\sigma} + N^2(\sigma) \omega &= 0 \\ \nabla_{\sigma} \cdot \mathbb{W} + \omega_{\sigma} &= 0 \end{aligned} \quad (6)$$

If we now assume a solution in longitude of periodic form, e^{ikx} , we get,

$$\begin{aligned} u_t - fv + ik\psi &= 0 \\ fu + v_t + \psi_y &= 0 \\ iku + v_y + \omega_{\sigma} &= 0 \\ \psi_{\sigma t} + N^2 \omega &= 0 \end{aligned} \quad (7)$$

Now to separate the vertical dependence from the latitudinal, let

$$(u, v, \psi) = G(\sigma) (\hat{u}, \hat{v}, \hat{\psi}) (y, t) \quad (8)$$

$$\omega = H(\sigma) \hat{\omega}(y, t)$$

Substitution of (8) into (7) yields the result that

$$\hat{\psi}_t = C^2 (i k \hat{u} + \hat{v}_y) \quad (9)$$

$$H_{\sigma\sigma} = \frac{N^2}{C^2} H ; G_{\sigma\sigma} = (\ell n N^2)_\sigma G_\sigma + \frac{N^2}{C^2} G \quad (10)$$

and application of the boundary conditions that $\hat{\sigma} = 0$ at $\sigma = 0, 1$, gives the conditions that $H(\sigma=0) = 0$ and $H_\sigma(\sigma=1) = -(\Pi|C^2)A(1)H(1)$. These conditions applied to the solution of (10) yield the required separation constants C^2 , which are also denoted as "equivalent depths".

The first two equations of (7), together with (9) allow for a solution of the latitudinal structure and generate the normal modes and frequencies. We have,

$$\frac{\partial \chi}{\partial t} + i \hat{A} \chi = 0 ; \chi^T \equiv (\hat{u}, i \hat{v}, \hat{\psi}) \quad (11)$$

$$\hat{A} = \begin{pmatrix} 0 & f & k \\ f & 0 & \frac{\partial}{\partial y} \\ -kC^2 & C^2 \frac{\partial}{\partial y} & 0 \end{pmatrix}$$

To better understand these modes in a simplified framework, let us solve (11) in a channel extending from $-60^\circ \leq \theta \leq 60^\circ$ latitude with rigid walls so that $\hat{v}(+60) = 0$. Equations (11) also show that $\hat{v}_{yy}(+60) = 0$ and give equations on the boundaries for ψ and \hat{u} . For additional simplification, we assume the channel to be broken into $2N + 1$ equally spaced grid-points such that $Y = N\Delta y$ and $-Y \leq y \leq Y$, with $y = n\Delta y$. We also use centered differences to replace derivatives. In the finite-difference form, system (11) becomes,

$$\frac{d\chi}{dt} + i \hat{A} \chi = 0 \quad \chi^T = (U, V, \Phi)$$

$$U^T = (\hat{u}_{N-1} \cdots \hat{u}_0 \cdots \hat{u}_{1-N}) \quad (2N-1) \text{ values} \quad (12)$$

$$V^T = i(\hat{v}_{N-1} \cdots \hat{v}_{1-N})$$

$$\psi^T = (\hat{\psi}_{N-1} \cdots \hat{\psi}_{1-N})$$

and \hat{A} is a $3(2N-1)$ square matrix. The roots of \hat{A} represent the frequencies of the reduced system as specified in (3) and will represent an approximation of the GLAS model.

STRUCTURE OF MODES AND FREQUENCIES

For pilot experiments, we have chosen to break up the atmosphere into three equal σ -layers. Finite difference solution of H from (10) then yields the following three equivalent depths in units of geopotential height;

$$C_1^2 = - 8.402 \cdot 10^4$$

$$C_2^2 = - 4.235 \cdot 10^3$$

$$C_3^2 = - 8.012 \cdot 10^2$$

The corresponding structures of vectors H_1, H_2 and H_3 may be seen on Figure 1. They have the expected distribution for this resolution. For the latitudinal structure, we have taken $\Delta y = 5^\circ$ and $\Delta y = 10^\circ$. For the former case, we generate 69 modes for each equivalent depth (C^2) and for each planetary wave (k). For the latter case, the number reduces to 33 modes. In either case, many of the modes are computational and presenting their structures would be overwhelming. Fig. 2 gives some indication of the structure of the latitudinal modes. We have plotted the 49th vector for the case of $\Delta y = 5^\circ$, showing the distribution for waves 1, 6, 15 and 30 for both the external and first internal modes. Note that the vector for wave one and the external mode shows some truncation properties which are not apparent for the internal mode. The other vectors all have a very similar structure. These vectors reflect Rossby type oscillations with periods in excess of 40 hours.

In Table 1 we describe the distribution of eigenvalues for the $\Delta y = 10^\circ$ truncation. We denote the number of modes which fall into the "high" frequency as compared to the "low" frequency for selected wave numbers and the three vertical structures. Note that all but wave number one have very strong separation between the gravity and rotational modes; i.e., there are twenty two "fast" modes and eleven "slow" ones. Separation in wave one, although not pronounced, is apparent. This difficulty with long waves has been seen in other experiments.

SUMMARY

The data presented indicate that normal modes may be extracted from a simplified version of the GLAS model, and mode separation is evident to allow for nonlinear initialization. Tests must now be performed which will determine how important the approximations to the true numerical model are, and how effective initialization with the simplified model will be.

An interesting observation of the study to date shows that some of the modes are strongly affected by truncation. Devices for removing the amplitudes of these modes (if they have slow frequency) are under investigation. The high frequency computational modes may be removed by the nonlinear initialization technique.

References

- Baer, F., J. T. Tribbia, 1977: On complete filtering of gravity modes through nonlinear initialization. Mon. Wea. Rev., **105**, 1536-1539.

Somerville, R. C. J., et. al., 1974: The GISS model of the global atmosphere. J. Atmos. Sci., 31, 84-117.

TABLE 1: Periods in hours for all frequencies for model with $\Delta y = 10^\circ$ and for selected wave numbers. Values in parentheses indicate the number of modes in the specified range.

<u>WAVE NUMBER</u>	<u>EXT. MODE</u>	<u>FIRST INT.</u>	<u>SECOND INT.</u>
1	$6 \leq T \leq 16$ (19)	$15 < T < 56$ (22)	$15 < T < 60$ (20)
	$23 \leq T \leq 45$ (7)	$T > 170$ (11)	$T = 102$ (2)
	$T > 189$ (7)		$T > 393$ (11)
6	$4.1 < T < 6.5$ (22)	$13 < T < 29$ (22)	$15 < T < 43$ (20)
	$T = 18.2$ (2)	$T > 103$ (11)	$T = 65.6$ (2)
	$T > 85$ (9)		$T > 140$ (11)
15	$2.3 < T < 2.6$ (22)	$8.9 < T \leq 11.4$ (22)	$13 < T < 26.3$ (22)
	$T = 31.9$ (2)	$T > 201$ (11)	$T > 238$ (11)
	$T > 198$ (9)		
30	$1.24 < T \leq 1.28$ (22)	$5.3 \leq T \leq 5.7$ (22)	$10 < T < 13.2$ (22)
	$T = 58.8$ (2)	$T > 312$ (11)	$T > 420$ (11)
	$T > 390$ (9)		

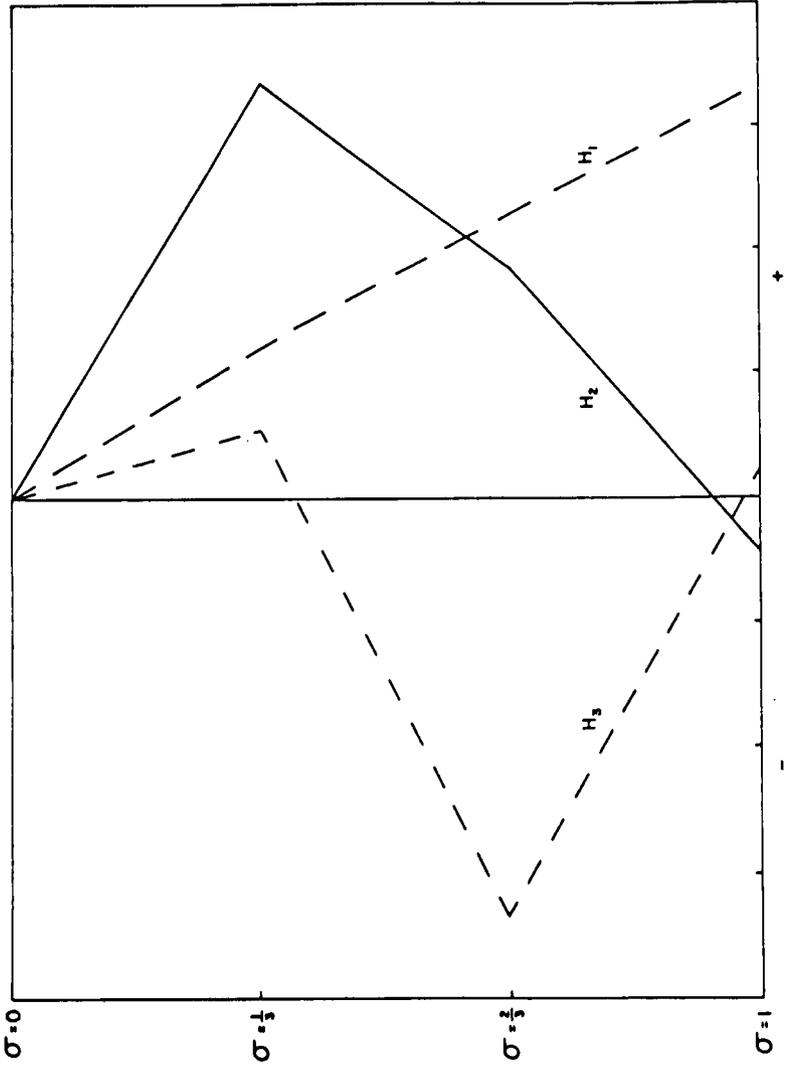
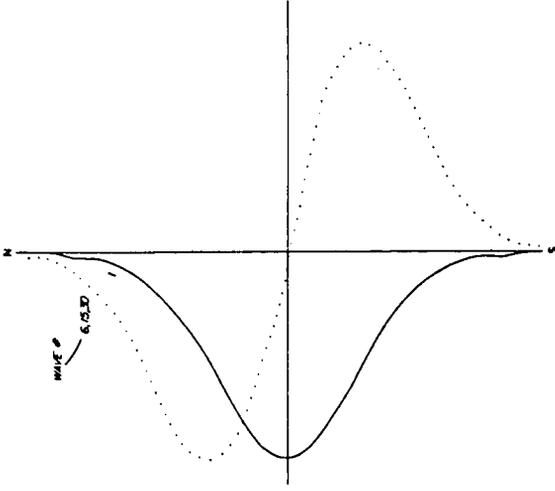


Figure 1: Vertical structures for the external and two internal modes of the simplified model. The vectors are not normalized.

FIRST INTERNAL MODE, VECTOR 49



EXTERNAL MODE, VECTOR 49

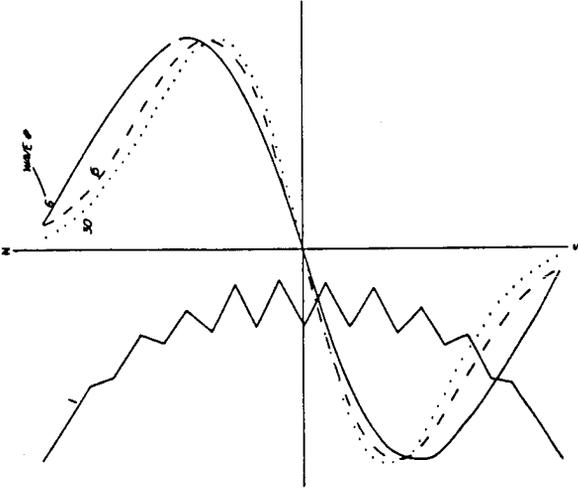


Figure 2: Latitudinal structure of the 49th vector (a Rossby mode) of the stream function for the external and first internal mode, for wave numbers 1, 6, 15 and 30. The ordinate represents the N-S width of the channel and the vectors are not normalized.