The Inverse Problem: Ocean Tides Derived from Earth Tide Observations

John T. Kuo
Lamont-Doherty Geological Observatory, Columbia University
Palisades, New York 10964

Summary. Evidence has been accumulated to the point that open ocean tides can indeed be mapped by solving the inverse problem using land-based and island-based tidal gravity observations, supplemented by shore and island ocean tide-gauge observations and a few deep ocean-bottom observations. Past efforts have been towards a better understanding of open ocean tides both through numerical integration of Laplace's tidal equations and through direct observations of tides in the open oceans. The co-amplitude and co-phase tidal charts principally of the tidal constituent M2 calculated by numerical integration of Laplace's tidal equations generally fail to agree among themselves, and with the tidal observations on mid-ocean islands. The problem of open ocean tides remains open.

During the last few years, we have indirectly mapped both the M2 and Q1 ocean tides in the northeastern Pacific Ocean, based on the inverse scheme of "Linear Programming." A comparison between the inversion results of the M2 and Q1 ocean tides and the three ocean-bottom observations made by Scripps, which were not included in the inversion scheme, gives an excellent agreement and assures that the proposed technique of indirect mapping of open ocean tides by means of tidal gravity observations can contribute significantly to attack the classic problem of open ocean tides.

Ever since we proposed the possibility of indirectly mapping ocean tides by means of land- and island-based tidal gravity measurements (Kuo et al., 1970), skepticism has been raised principally by physical oceanographers, concerning its actual feasibility, typically such as the recent one by Zetler (1978):

"It has been demonstrated that land-based earth tides observed on a gravimeter near an ocean are modified by the ocean tidal loading. It is more controversial, however, whether it is possible to map the open ocean tides by solving the inverse problem using land-based gravity measurements, shore constraints, and a few ocean-bottom stations. Certainly it can be done with an infinite of stations of perfect precision; the number and precision of land and ocean measurements necessary to achieve required accuracy have not yet been determined."

Evidence has been accumulated to the point that open ocean tides can indeed be mapped by solving the inverse problem using land- and island-based tidal gravity measurements, coupled with shore and island ocean tidal measurements and a few ocean-bottom measurements. There is definitely no need to have an infinite of stations of perfect precision. The degree of precision of tidal gravity measurements, from our experience, must be 1% or better, which is perfectly achievable.

Much effort has been directed toward a better understanding of open ocean tides, both through numerical integration of Laplace's tidal equations, (Pekeris and Accad, 1969; Zahel, 1970; Hendershott and Munk, 1970; and others) and through direct measurements of tides in the deep oceans (Nowroozi et al., 1969; Filloux, 1971, Munk et al., 1970; and others). The co-amplitude and co-phase tidal charts principally of the tidal constituent M2 calculated by numerical integration of Laplace's tidal equations are apparently quite sensitive to the boundaries of the ocean basins and the law of friction, and generally fail to give close agreement with the tidal observations on mid-ocean islands. Moreover, the agreement among various tidal charts is still poor as shown in Figures 1 and 2, comparing the tidal charts for example, the M2 and Q1 in the northeastern Pacific Ocean.

During the last few years, we have indirectly mapped both the M2 and Q1 ocean tides in the northeastern Pacific Ocean, based on the inversion scheme of "Linear Programming." A comparison between the inversion results of the M2 and Q1 ocean tides and of the three ocean-bottom observations made by Munk et al. (1970) and Irish et al. (1971), which were not included in the inversion scheme, gives an excellent agreement.

The basic data used in the inversion are principally from the tidal gravity observations made on North America and on the islands in the northeastern Pacific Ocean and from the tide gauge observations on the coasts and islands. A total of 17 tidal gravity stations and 62 coastal and island tide gauge stations was used for the northeastern Pacific Ocean. The distributions of the tidal gravity stations and the tide gauge stations of the contiguous continents and islands of the northeastern Pacific used in the inversion are shown in Figure 3.

The M2 and Q1 worldwide tide maps of Tiron et al. (1967), designated as Tiron M2 Map and Tiron Q1 Map, respectively, as shown in Figures 4 and 5, were adopted as the starting models and were digitized at 2° by 2° spacings. As a matter of fact, Kuo and Jachens (1977) have shown that a starting model is not of crucial importance in the inversion. For oceanic regions within 2.5° of the tidal gravity observational stations, these maps were modified to conform with coastal observations. The predicted tidal gravity effects arising solely from ocean tides were calculated by numerical convolution of Tiron M2 and Q1 Maps with linear combination of a mass loading Green's function for an oceanic crust model (Farrell, 1972) and, in addition, of a Newtonian attraction Green's function for a density coating layer on the surface of a sphere, which is not accounted for in the formulation of the loading Green's function by Farrell (1972).

A fourth order two-dimensional polynomial was selected as the highest order polynomial surface

https://ntrs.nasa.gov/search.jsp?R=19790013335 2019-05-27T00:40:34+00:00Z
Figure 1. The $M_2$ ocean tidal constituent as derived by empirical (Dietrich, 1944) and Laplace tidal equations by various authors.

Figure 2. The $O_1$ ocean tidal constituent as derived by empirical (Dietrich, 1977) and Laplace tidal equations by various authors.

correction applied to Tiron $M_2$ and $O_1$ Maps. The limited distribution of the available tidal gravity data does not warrant a surface of order higher than fourth.

The inversion scheme is based on "Linear Programming." The reason of choosing this Linear Programming Inversion is very simple. Linear programming is concerned with the optimum operation of interdependent variables, that is the minimization of a linear objective function, whose variables satisfy a system of linear inequalities (Danzig, 1977). Unlike the other numerical solutions of minimization problems by iterative procedures, the solution to a linear programming problem, if exists, is unique and gives a true minimum for the entire system, i.e. a global minimum. Thus, linear programming procedure can be ideally used to seek the optimal inverse solution subject to constraints imposed by the tidal gravity and ocean tide gauge observations.

Figures 6 and 7 give the new $M_2$ and $O_1$ tide maps for the northeastern Pacific Ocean, bounded on the west by 160°W on the south by 15°N, and on the north and east by North America, resulting from the application of the linear programming inversion. The most prominent feature of the new $M_2$ map is the existence of a single amphidromic system in the region, centered at approximately 26°N latitude, 137°W longitude and rotates counterclockwise. The amplitude contours display a low amplitude trough of less than 20 cm. The amplitude increases smoothly north of the trough attaining magnitudes.
greater than 120 cm near the head of the Gulf of Alaska. The prominent feature of the new $O_1$ map is the modification of both the coamplitude and cophase lines from that of the Tiron $O_1$ Map. The new $O_1$ map shows a delay of the cophase lines, compared with the cophase lines of the Tiron $O_1$ Map. Except that the coamplitude line of 25 cm remains nearly the same as Tiron $O_1$ Map, all the other coamplitude lines of 10, 15, and 20 cm are curved somewhat to conform with the west coast of North America.

The goodness of the inversion results must be critically tested to insure the validity of the inversion procedure. It can be best accomplished by comparing the inversion results with observations. The observations from the three ocean-bottom sites, Kathy, Filloux, and Josie II, shown as solid circles in Figure 8 were not included as the basic data in the inversion procedure, and are far removed from possible local perturbation introduced by islands or submarine topographic features. The comparison between the values obtained by the various tidal charts, the inversion procedure and these corresponding observed values is given in Tables I and II.

The agreement between the inversion results and the observations at all three sites is better than 6 cm in amplitude and better than 5° in phase for $M_2$. Although there are no available observed data of $O_1$ for the station Josie II, the agreement for $O_1$ at the stations, Kathy and Filloux is better than 1 cm in amplitude and 1° in phase. The inversion results appear to be quite insensitive to uncertainties in knowledge of worldwide ocean tidal distributions and to possible biases in tidal gravimeter calibration. The agreement at Kathy and Filloux for both $M_2$ and $O_1$ is embarrassingly good, probably somewhat fortuitous, since the starting
Figure 4. Tiron et al., (1967), Original $M_2$ Tidal Map.

<table>
<thead>
<tr>
<th>STATION</th>
<th>OBSERVED (O)</th>
<th>AVAILABLE COTIDAL CHARTS A/G</th>
<th>INVERSION A/G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kathy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>124°25.8'W, 27°45.0'N</td>
<td>$M_2$ 28.6/128.0</td>
<td>-115/115, 31/180, 41/172, 75/305, 35/305, 60/160</td>
<td>28.3/132.9</td>
</tr>
<tr>
<td>129°01.1'W, 24°46.9'N</td>
<td>$O_1$ 17.5/199.0</td>
<td>-190/190, 15/219, 21/206</td>
<td>18.0/199.0</td>
</tr>
<tr>
<td>Filloux</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>124°59.7'W, 34°00.3'N</td>
<td>$O_1$ Not determined</td>
<td>-210/210, 15/245, 25/235</td>
<td>17.0/224.0</td>
</tr>
</tbody>
</table>

*O = Observed (Hurik et al., 1970);  
*D = Dietrich, 1977;  
*T = Tiron et al., 1967;  
*B = Bogdanov et al., 1968  
P&A = Pekeris & Accad, 1969;  
*Z = Zahel, 1970;  
Figure 5. Tiron et al., (1967), Original $O_1$ Tidal Map.

TABLE II
DIFFERENCES

<table>
<thead>
<tr>
<th>STATION</th>
<th>D-O</th>
<th>T-O</th>
<th>B-O</th>
<th>P&amp;A - O</th>
<th>I-O</th>
<th>H-O</th>
<th>INV-O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kathy $M_2$</td>
<td>-/-13</td>
<td>+2.4/+52</td>
<td>+12.4/+44</td>
<td>+46.4/+177</td>
<td>+6.4/+177</td>
<td>+31.4/+32</td>
<td>-0.3/+4.9</td>
</tr>
<tr>
<td>Kathy $O_1$</td>
<td>-/-9</td>
<td>-2.4/-20</td>
<td>+3.5/+9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+0.5/-0</td>
</tr>
<tr>
<td>Filloux $M_2$</td>
<td>-/-15</td>
<td>+1.2/+79</td>
<td>+16.2/+65</td>
<td>+33.2/+193</td>
<td>+13.2/+213</td>
<td>+36.2/+50</td>
<td>-1.0/-2.5</td>
</tr>
<tr>
<td>Filloux $O_1$</td>
<td>-/-11.3</td>
<td>-3.6/-18.7</td>
<td>6.4/-8.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.1/-1.3</td>
</tr>
<tr>
<td>Josie II $M_2$</td>
<td>-/-192</td>
<td>0.4/-6</td>
<td>-11.6/-35</td>
<td>-1.6/-5</td>
<td>+8.4/+3</td>
<td>+3.4/-70</td>
<td>-6.6/+3.0</td>
</tr>
<tr>
<td>Josie II $O_1$</td>
<td>Not Observed</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
</tr>
</tbody>
</table>

model could not be read reliably to one degree and one cm.

The crucial difference between the present procedure and those of past workers is that land-based and island-based tidal gravity observations were used as the bases for interpolating between widely spaced direct ocean tidal observations rather than the traditional empirical procedures based on the numerical integration Laplace tidal equations. For comparison, Figure 9 shows a map...
Figure 6. \(M_2\) obtained from inversion for the northeastern Pacific Ocean.

derived by Munk et al. (1970) based on ocean-bottom tidal measurements and empirical inference.

The inversion results of the \(M_2\) and \(O_1\) tides in the northeastern Pacific Ocean assure that earth tidal gravity can contribute significantly to attack one of the remaining classic geodynamic problems, - the open ocean tides. The worldwide open ocean tides can be mapped by means of solving the inverse problem of ocean tides for a series of tidal gravity observations, complemented by coastal, shelf, and island tidal gauge observations and a limited number of ocean-bottom tidal observations.

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References


Figure 7. $O_1$ obtained from inversion for the northeastern Pacific Ocean.

Figure 8. Three ocean-bottom observations (Munk et al. 1970), not included in the inversion.
Figure 9. The $M_2$ ocean tide as derived by Munk et al. (1970) based on ocean-bottom measurements.


