The Influence of Earth Tides on Earth's Coordinates

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The importance of Earth tides on Earth coordinates has been considered only recently for several reasons: 1) the precision that we are obtaining nowadays for Earth's coordinates shows that the effects of Earth tides appear on the values obtained for the coordinates; 2) the possibility of determining, by observations, the values of Earth tides; 3) the consideration of theoretical models that can compute the values of Earth tides.

We are going to examine briefly some of these reasons. First of all, it should be pointed out that we have to be careful about the definitions of what we mean by Earth coordinates.

When it was only possible to obtain a precision of a few meters for the values of the Earth coordinates, some of the systems of reference adopted in the dynamics of the Earth rotation could not be distinguished within that precision. But when we are aiming at precisions of a few centimeters, it is very important to define carefully the systems of reference employed for our coordinates on the Earth.

We can consider, for instance, astronomical and geodetic coordinates of a point at the Earth's surface.

The astronomical coordinates are referred to the astronomical local vertical, and therefore, to the instantaneous axis of rotation of the Earth, briefly called the axis of rotation. The intersection of this axis with the Earth's surface defines the geographic poles. The complement of the acute angle between the astronomical vertical and the axis of rotation of the Earth is called the astronomical latitude. For the definition of astronomical longitude we need a reference meridian passing by the geographic poles [Woolard and Clemence, 1966].

Considering the case of the real Earth, there are known irregularities in the direction of the astronomical vertical from place to place over the Earth, and, therefore, the astronomical meridians, parallels of latitude and the equator are irregular curves of double curvature, but they do not depart very far from plane curves.

The instruments (visual zenith telescopes (VZT), photographic zenith telescopes (PZT) and astrolabes), employed in the classical techniques of determining polar motion, refer their observations to the astronomical vertical.

Depending on the adopted pole and reference meridian, we can have different definitions of the coordinates of a point on the Earth's surface. The danger, nowadays, is that different people speak about different poles and, therefore, the coordinates they are speaking about are not the same.

The simplest Earth model considers the Earth as a solid rigid body. Even this simple model shows the very important distinction between free and forced motions, originating periodic displacements of the axes employed in the dynamics of the Earth rotation. They have been called nutations because the displacements are periodic.

The equations of motion can be represented by the vector equation

$$\frac{d\vec{H}}{dt} = \vec{G}$$

showing that the time derivative of the angular momentum vector $\vec{H}$ around the centre of mass is equal to the vector moment $\vec{G}$ of the external forces, mainly due to the Sun and Moon.

When there are no external forces acting on the Earth $\vec{G}=0$, therefore, $\vec{H}$ is constant and fixed in space. We have the so-called free nutation because this type of motion exists even without external forces. The important consequence is the fact that any free nutation changes the Earth coordinates showing, for instance, variations of latitude, but it does not affect star places on the celestial sphere. The name wobble has, unfortunately, been recently employed to designate the free nutations.

When there are external forces acting on the Earth, $\vec{G}\neq 0$, and the position of $\vec{H}$ is not fixed in space. We obtain the forced nutations that are so important in any problem dealing with the transformation of reference systems from one epoch to another because they affect star places.

One aspect of the actions of the Sun and the Moon on the Earth concerns the tidal attractions of the luni-solar forces, which deform the Earth, giving rise to the tides of the solid Earth.

Different components of the Earth tide produce the nutations that are important in astronomy, and the forced nutations correspond to diurnal tides, so there is a close connection between nutations and tides.

It is fortunate that all the studies concerned with the tides of the solid Earth employ the same standard, that is, the bodily tide numbers h, k and ℓ are defined considering a statical theory applied to an elastic solid Earth with spherical symmetry, and the disturbing tidal potential is a spherical harmonic of the second degree [Jeffreys, 1976].

The value of k is related to the period of the free Eulerian nutation, and the value of $\Lambda=1+k-\ell$ is influenced by disturbances that affect the position of the astronomical vertical of a place on the Earth, that is, there are variations in the geographic coordinates of a place with periods depending on the periods of the tidal forces.

The actions of the Sun and Moon deflect the position of the astronomical vertical, and the maximum deviation is of the order of 0.″05; they are the most regular of the variations in direction of the astronomical vertical. There are remaining variations due to geophysical causes, not yet very well known, and the tidal variations are very much modified by local geophysical factors.

In practice, the determination of the values of the bodily tide numbers is more difficult because there are local corrections.

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TABLE 1. Diurnal Waves

	Jeffreys-Vicente 1957		Molodensky 1961	Shen-Mansinha 1976	Observations
	Central Particle	Roche	2	β_o	
			$\delta=1+h-\frac{3}{2}k$		
K_1	1.183	1.185	1.151	1.1380	1.1406 A.M. 1.1507 W.M.
P_1	1.209	1.172	1.161	1.1539	1.1699 A.M. 1.1664 W.M.
O_1	1.221	1.211	1.166	1.1596	1.1522 A.M. 1.1676 W.M.
			$\gamma=1+k-h$		
K_1	0.714	0.693	0.721	0.7340	0.7422 A.M. 0.7501 W.M.
P_1	0.676	0.696	0.695	0.7019	0.7068 A.M. 0.7167 W.M.
O_1	0.658	0.658	0.686	0.6895	0.6785 A.M. 0.6752 W.M.
			$\Lambda=1+k-\ell$		
K_1	1.120	1.162		1.1689	
P_1	1.149	1.180		1.2033	
O_1	1.160	1.183		1.2055	

A.M. Arithmetic Mean
W.M. Weighted Mean

It is convenient to have the possibility of determining the values of k and ℓ by processes that do not depend on any geophysical assumptions. This fact shows the importance of having good determinations of the period of the free Eulerian nutation, called the Chandler period, that permit the determination of the value of k, and the advantage of careful analysis of the observations of variation of latitude that give the possibility of computing $(1+k-\ell)$. Some of the modern techniques that can observe polar motion have an important role to play in these studies.

The observations of Earth tides are interpreted in amplitude ratio form (observed to theoretical amplitude), and we can consider, for instance, the following linear combinations for the bodily tide numbers:

for the horizontal component $=1-h+k$
for the vertical component $=1+h-\frac{3}{2}k$

The computed values of Earth tides show that the maximum effect will correspond to a radial deformation of about 30 cm. We can see the importance of Earth tides for the present day goal of achieving precisions of a few centimeters on the Earth's coordinates.

It has been demonstrated that the application of a statical theory for the semi-diurnal, fortnightly and semi-annual tides does not introduce appreciable errors in theoretical models [Jeffreys, 1949]. But for diurnal tides we have to consider a dynamical theory and the importance of the liquid core of the Earth has been proved. The main waves are:

Doodson's code number	Tidal component	Nutation component
165.555	K_1	Precession
163.555	P_1	Semiannual
145.555	O_1	Fortnightly

The observations of Earth tides show a very irregular distribution over the Earth. The majority of stations are concentrated in Europe and North America. Local effects have been detected, for instance, due to the proximity of oceans and big rivers.

It is fortunate that there is an "International Centre for Earth Tides" where all the observations are collected and the computations performed, therefore, we have a consistent treatment of the data which is very important.

Some results of theoretical and observed values for δ, γ and Λ are indicated in Table 1.

The observations correspond to 21 stations with very irregular distribution on the Earth [Melchior, 1971]. We can see that the consideration of arithmetic mean or weighted mean even alters some values. This is a good example about the difficulties encountered in the combination of observations which become important for the precision we are trying to achieve.

The consideration of the fourth decimal place has probably no physical meaning at the present time.

The theoretical models indicated correspond to different approximations that were made. The values computed by Jeffreys-Vicente [1957] and Molodensky [1961] were based on models of the internal structure of the Earth that appeared about 25 years ago, taking account of the liquid core.

Jeffreys-Vicente employed two models (central particle and Roche) which were considered as representing extreme cases of the possible behaviour of the core. This way of looking at the problem corresponds to setting up an upper and a lower bound to the behaviour of the core, considering that the knowledge about the structure of the core was not very detailed at the time. More recent models, based on a better knowledge of the structure of the core, have confirmed the general trends found by the central particle and Roche models.

Molodensky model 2 considers an inner core. It should be pointed out that the existence of the inner core introduces difficulties, and it has been shown that the partial differential equations to be satisfied are hyperbolic and the boundary conditions have to be considered carefully [Jeffreys and Vicente, 1966, p. 24].

Shen-Mansinha [1976] employed a model β_o corresponding to an elliptical rotating Earth with a liquid outer core. The consideration of the other models, designated by $\beta=-0.2$ and $\beta=+0.2$, does not alter the conclusions.

The theoretical values obtained for the vertical component δ, written on Table 1, show that model β_o presents slightly smaller values for all waves in comparison with the other models.

Considering the wave O_1, we see that the vertical component δ is more affected by different types of models than the horizontal component γ.

For the horizontal component γ, the model β_o shows slightly greater values than Molodensky model 2, being more in agreement with the observations.

A few theoretical values of Λ are computed and there is no comparison with the observations because it is very difficult, at the present time, to obtain reliable observational values for these waves.

The models show general agreement with the observations because we cannot rely too much on the geophysical meaning of the third and fourth decimal places. The differences among the theoretical models are within five per cent.

Another important aspect is the frequency dependence of the bodily tide numbers which was revealed by the earlier investigations [Jeffreys and Vicente, 1957]. This result means that the behaviour of the Earth is very much conditioned, for the diurnal tides, by the period of the waves considered, as it is shown by the values written on Table 1.

The existence of damping and some recent determinations of Q values, depending on the period of the motion considered, show some of the difficulties encountered in the researches about Earth tides.

The global values of the bodily tide numbers have been employed to allow for the influence of Earth tides on the coordinates of a point on the Earth, but the improvement in the precision of the determination of coordinates would benefit from local observations of Earth tides by the appropriate techniques.

The waves with greater amplitudes appear in the following sequence: K_1 (due to the Moon and Sun), O_1 (due to the Moon) and P_1 (due to the Sun and Moon); and the frequencies are well defined by the theory of the diurnal tides. The great advantage of the bodily tides is the fact of their well-known periodicities.

The analyses of the observations, made by some of the modern techniques, should reveal these periodicities if they correspond to an adequate interval of time. It is nevertheless necessary that the observational programme is well planned and does not suffer from discontinuities. The modern techniques, employing artificial satellites, the Moon and radio interferometry, offer the advantage of providing global values.

We are attempting to obtain precisions of the order of a few centimeters and that raises the question if we are nowadays approaching the noise level of the system. In this case, we mean by noise level of the system, the possible range of values for the coordinates of a point on the Earth which are affected by numerous phenomena, some of them not very well known, and corresponding to the more general case of an Earth model considered as a collection of particles subject to so many forces.

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