

Final Report  
**Vol. II Multispectral Scanner System  
Parameter Study and Analysis  
Software System Description**

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16. Abstract This report represents the culmination of three major pieces of research effort in defining and evaluating the performance of the next generation of multispectral scanners within the framework of a set of analytic analysis packages. The integration of the available methods provides the analyst with the Unified Scanner Analysis Package (USAP), the flexibility and versatility of which is superior to many previous integrated techniques. USAP consists of three main subsystems, (a) a spatial path, (b) a spectral path and (c) a set of analytic classification accuracy estimators which evaluate the system performance. The spatial path consists of satellite and/or aircraft data, data spatial correlation analyzer, scanner IFOV and random noise model. The output of the spatial path is fed into the Analytic Classification Accuracy Predictor (ACAP). The spectral path consists of laboratory and/or field spectral data, EXOSYS data retrieval, optimum spectral function calculation, data transformation and statistics calculation. The output of the spectral path is fed into the Stratified Posterior Performance Estimator (SPEST). A brief theoretical exposition of the USAP individual building blocks are presented and example outputs produced. References are provided for a more complete coverage of the algorithms. Each building block carries with it at least one software unit. The programming provides a complete input-output compatibility among these units. One test case starting from the raw data base is carried through the system and the performance figures in terms of a set of classification accuracies are produced. A listing of the underlying software is provided in Appendix I.			
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Cl. Multispectral Scanner System Parameter Study and  
Analysis Software System Description

1. INTRODUCTION

The utilization of sensors on earth orbiting platforms as the main element of an Earth Observational System has undergone substantial growth in recent years. ERTS-1 (Landsat-1) followed by Landsat-2 and -3 have proven exceptionally successful in collecting data to help monitor the Earth's resources.

The principal data collection unit aboard the first three Landsats is the multispectral scanner known as MSS. Although this scanner has been providing data with a quality which exceeded most prelaunch expectations, it has been clear from the beginning that MSS does not represent the ultimate in multispectral instruments; more advanced instruments providing greater detail would be needed as the user community begins to become familiar with the use of such space data.

The design of a multispectral scanner is a very complex matter; many different, interacting factors must be properly taken into account. Currently operational systems such as MSS have been designed primarily using subjective judgements based upon experience with experimental data. In designing a scanner the use of empirical methods, at least in part, is essential. Each of the large collections of scenes which a given scanner will be used upon is a very complex information source; not enough is known to make a simple (or even a complex) model of it by which to make the design of a scanner a simple straightforward exercise of a mathematical procedure.

And yet, more is known than when MSS was designed, and it is important to be able to carry out future designs on a more objective basis than in the past. Thus the purpose of the present work is the development of appropriate mathematical design machinery within a theoretical framework to allow: (a) formulation of an optimum multispectral scanner

\* The work in this report was done under Task 2.2C1 Multisensor Parametric Evaluation and Radiometric Correction Model.

system according to defined conditions of optimality and (b) an ability for convenient manipulation of candidate system parameters so as to permit comparison of the theoretically optimum designs with that of practical approximations to it.

In order to deal with the complexity of the design situation, the first step is to determine a suitable set of parameters which adequately characterize it but is not so large as to be unmanageable. It has been observed [1] that there are five major categories of parameters which are significant to the representation of information in remotely sensed data. They are:

1. The spatial sampling scheme
2. The spectral sampling scheme
3. The signal-to-noise ratio
4. The ancillary data type and amount
5. The informational classes desired

Thus, it is necessary to have present in the design machinery, some means for evaluating the impact of change in parameter values in each of these five categories.

Such a scanner design tool has been assembled in the form of a software package for a general purpose computer. Each of the parts of this package, called Unified Scanner Analysis Package (USAP) has been carefully devised and the theory related to it fully documented [2, 3, 4, 5]. The goal of this report is to provide a documentation and description of the software. In constructing this documentation it was assumed that this package will be useful for sometime into the future, however it was also assumed that it will only be used by a small number of highly knowledgeable scientists.

Section 2 recaps the theoretical concepts behind some of the primary components of USAP. These are divided into (a) scanner spatial characteristics modeling and noise effects, (b) optimum spectral basis function calculations, (c) analytical classification accuracy predictions

(d) stratified posterior classification estimation and (e) an information theory approach to band selection. Although (e) is not a part of the USAP system, the results from this approach are helpful in understanding the scanner design problem.

Section 3 shows the integration of the above modules into the software system. Section 4 is the user's guide to USAP describing the required inputs and the available output products. A listing of all programs is provided in the appendix.

The work which led to USAP was immediately preceded by a simulation study of possible parametric values for the Thematic Mapper, a new scanner now being constructed for launch on Landsat-D in 1981. The purpose of this simulation was to compare the performance for several proposed sets of parameters. We will conclude this introductory section by briefly describing this work because it provides useful background and serves well to illustrate the problem. A more complete description of this simulation study is contained in [1, 6].

The general scheme used was to simulate the desired spaceborne scanner parameter sets by linearly combining pixels and bands from (higher resolution) airborne scanner data to form simulated pixels, adding noise as needed to simulate the desired S/N; the data so constructed was then classified using a Gaussian maximum likelihood classifier and the performance measured. The problem was viewed as a search of the five dimensional parameter space defined above with the study localized around the proposed Thematic Mapper parameters. The scope of the investigation was primarily limited to three parameters (a) spatial resolution, (b) noise level and (c) spectral bands. Probability of correct classification and per cent correct area proportion estimation for each class were the performance criteria used. The major conclusions from the study are as follows:

1. There was a very small but consistent increase in identification accuracy as the IFOV was enlarged. This is presumed to stem primarily from the small increase in signal-to-noise ratio with increasing IFOV, Figure 1.
2. There was a more significant decrease in the mensuration accuracy as the IFOV was enlarged, Figure 2.
3. The noise parameter study proved somewhat inconclusive due to the greater amount of noise present in the original data than desired. For example, viewing Figure 3 moving from right to left, it is seen that the classification performance continues to improve as the amount of noise added is decreased until the point is reached where the noise added approximately equals that already initially present.\* Thus, it is difficult to say for what signal-to-noise ratio a point of diminishing return would have been reached had the initial noise not been present.
4. The result of the spectral band classification studies may also be clouded by the noise originally present in the data. The relative amount of that change in performance due to using different combinations of the .45-.52  $\mu\text{m}$ , .74-.80  $\mu\text{m}$ , .80-.91  $\mu\text{m}$  and .74-.91  $\mu\text{m}$  bands is slight but there appears to be a slight preference for the .45-.52  $\mu\text{m}$  band. The performance improvement of the Thematic Mapper channels over those approximating Landsat-1 and -2 is clear however.
5. Using spectrometer data it was verified that the .74-.80  $\mu\text{m}$  and .80-.91  $\mu\text{m}$  bands are highly correlated.
6. Correlation studies also showed that the range from 1.0-1.3  $\mu\text{m}$  is likely to be an important area in discriminating between earth surface features. Further, it is noted that the absolute calibration procedure described above results in a global atmosphere correction of a linear type in that assuming a uniform atmosphere over the test site, the calibration procedure permits a digital count number at the airborne scanner output to be related directly to the present reflectance of a scene element.

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\* The noise level in the original A/C data was equivalent to about .005 NE $\Delta\rho$  on the abscissa. See Reference [1].

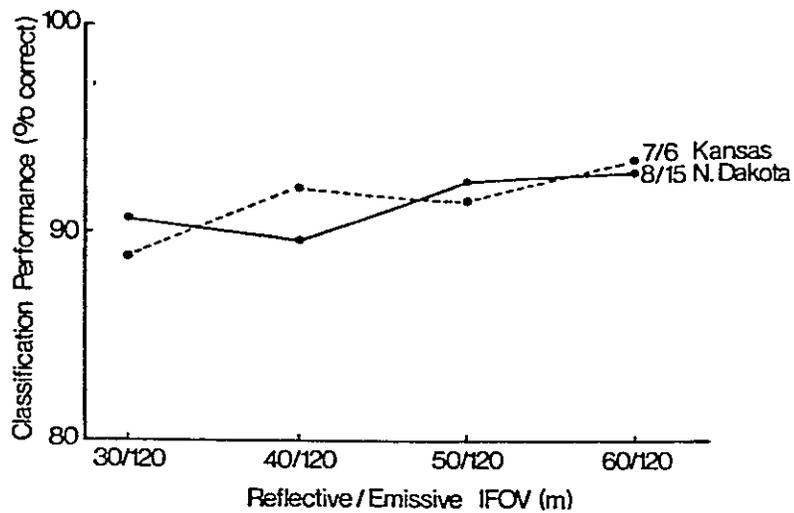


Figure 1. Classification Performance vs. Spatial Resolution.

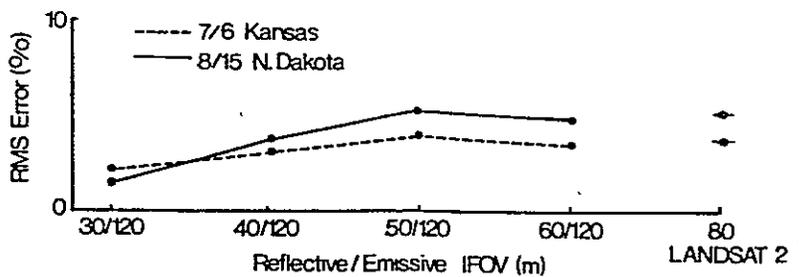
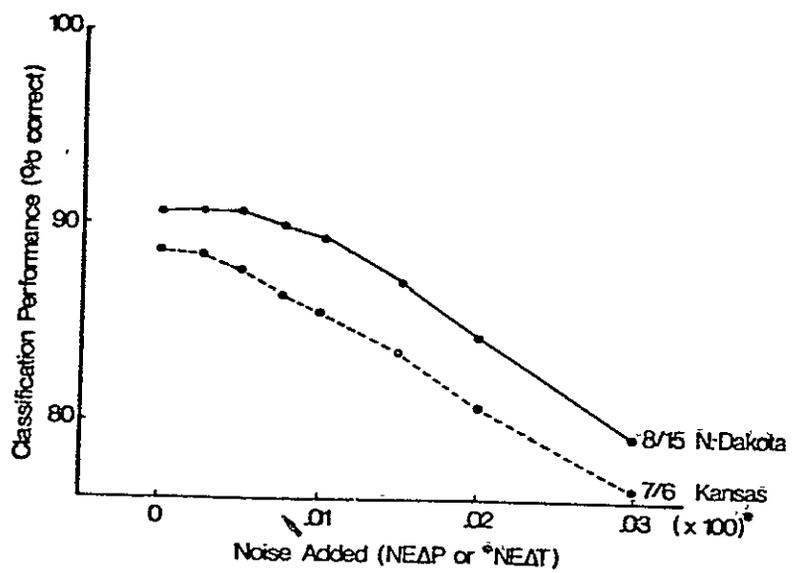


Figure 2. RMS Error of Proportion Estimates vs. Spatial Resolution.



## 2. SCANNER PARAMETERS ANALYSIS TECHNIQUES

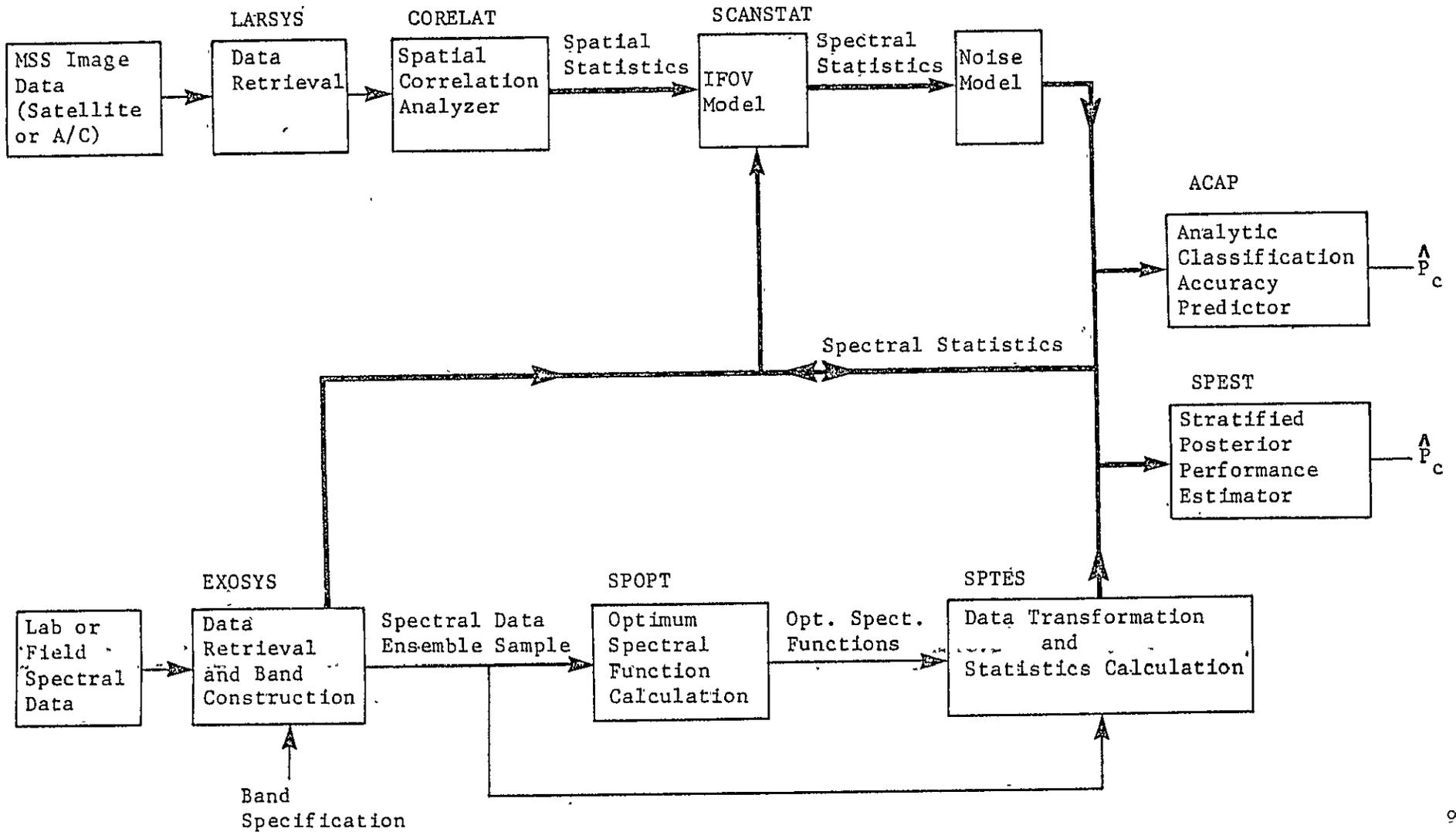
Based upon the parametric approach introduced above, the development of a parametric scanner model must give explicit concern for the spatial, spectral and noise characteristics of the systems. This is what has been done in the Unified Scanner Analysis Package (USAP) shown in Figure 4. USAP is composed of two distinct subsystems. The spatial aspect of it contains (a) a data spatial correlation analyzer, (b) a scanner IFOV model and (c) a random noise model. The spectral techniques are capable of producing an optimum spectral representation by modeling the scene as a random process as a function of wavelength, followed by the determination of optimum generalized spectral basis functions. Conventional spectral bands can also be generated. Also studied was an information theory approach using maximization of the mutual information between the reflected and received (noisy) energy. The effect of noise in the data can be simulated in the spectral and spatial characteristics. Two different data bases are used in the system. The spectral techniques require field spectral data while the spatial techniques require MSS generated data, aircraft and/or satellite. The system performance, defined in terms of the classification accuracy, is evaluated by two parametric algorithms. A detailed system description and user's guide is presented in Sections 3 and 4. In the following, the theoretical ideas behind the five major elements of USAP are discussed.

### 2.1 Analytical Classification Accuracy Prediction

Throughout the analysis of remotely sensed data, the probability of correct classification has ranked high among the set of performance indices available to the analyst. This is particularly true in a scanner system modeling where generally the optimization of various system parameters has as its prime objective the maximization of the classification accuracy of various classes present in the data set.

The estimation of the classification accuracy is fairly straightforward if Monte-Carlo type methods are employed. In system simulation and modeling however, such approaches are generally a handicap due to their

Figure 4. Block Diagram of the Unified Scanner Analysis Package (USAP).



 Implies path for spectral/spatial statistics  
 Implies path for data or spectral functions

heavy dependence on an experimental data base, the availability of which can be limited due to a variety of reasons. What is required, therefore, is a parametric classification accuracy estimator for a multiclass, multidimensional Gaussian Bayes classifier. This procedure should require the class statistics, mean vectors and covariance matrices, as its only input and produce a set of probabilities of correct classification. This technique has been developed, tested, implemented and comprehensively reported [2]. The following is a summary of the method and some results.

The probability of Error as an N-Tuple Integral.

The classification of a multidimensional observation vector into one of M populations is conceptually identical to the binary case. Let  $\Omega$ , M and N be the feature space, number of classes and the dimensionality of  $\Omega$  respectively. The procedure is to divide  $\Omega$  into M mutually disjoint sets,  $Z_i$ , and to assign each feature vector to a set in accordance with an appropriate rule. This is illustrated in Figure 5. Let  $Z_i$ ,  $i = 1, 2, \dots, M$  partition  $\Omega$  in  $R^N$ . The Bayes risk is defined as

$$R = \sum_{i=1}^M \int_{Z_i} \sum_{j=1}^M P(\omega_j) C_{ij} f(\underline{X}|\omega_j) d\underline{X} \quad (1)$$

where  $C_{ij}$  is the cost of deciding  $\omega_i$  where  $\omega_j$  is true. In the case where  $C_{ij}=0$  for  $i=j$  and  $C_{ij}=1$  for  $i \neq j$ . R is the probability of error.

Among all possible choices of  $Z_i$  the Bayes rule partitions  $\Omega$  into  $Z_i = Z_i^*$  such that  $R=R^*$  is the the minimum probability of error. Assuming that the population statistics follows a multivariate normal law, the optimum Bayes rule is as follows [7].

$$\underline{X} \in \omega_i \text{ if } W_i < W_j \quad \forall i \neq j = 1, 2, \dots, M$$

where

$$W_i = (\underline{X} - \underline{\mu}_i)^T \sum_{i=1}^{-1} (\underline{X} - \underline{\mu}_i) + \ln |\sum_{i=1}| - 2 \ln P(\omega_i)$$

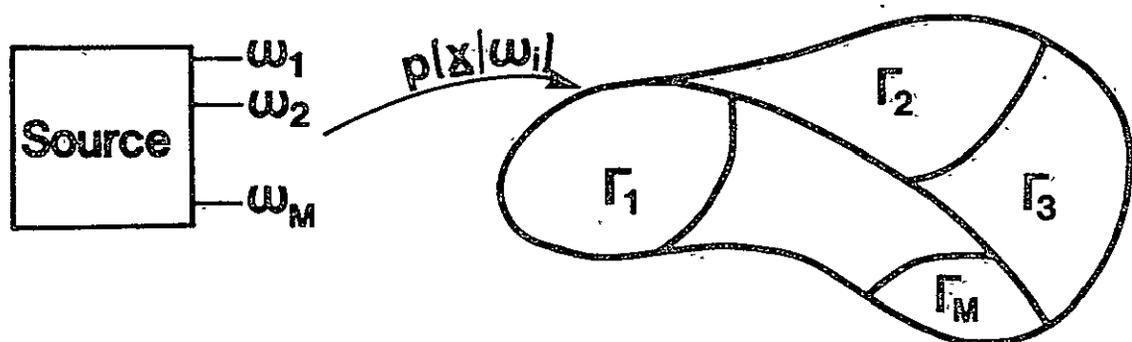


Figure 5. Allocation of a Measurement Vector  $\underline{X}$  to an Appropriate Partition of the Feature Space.

with

$$\begin{aligned}
 \underline{X} &= \text{observation vector} \\
 \underline{\mu}_i &= \text{mean vector for class } \omega_i \\
 \underline{\Sigma}_i &= \text{covariance matrix for class } \omega_i \\
 P(\omega_i) &= \text{apriori probability for } \omega_i
 \end{aligned}
 \tag{2}$$

The error estimate based on direct evaluation of Eq. (1) exhibits all the desired properties outlined previously.

The evaluation of multiple integrals bears little resemblance to their one dimensional counterparts, mainly due to the vastly different domains of integration. Whereas there are three distinct regions in one dimension; finite, singly infinite, and double infinite; in an N dimensional space there can potentially be an infinite variation of domains. The established one dimensional integration techniques, therefore, do not carry over to N dimensions in general. Hence, it is not surprising that no systematic technique exists for the evaluation of multivariate integrals except for the case of special integrands and domains [8]. The major complicating factor is the decision boundaries defined by Eq. (2).  $\Gamma_i$  is defined by a set of intersecting hyperquadratics. Any attempt to solve for the coordinates of intersection and their use as the integration limits will be frustrated if not due to the cumbersome mathematics, because of impractically complicated results.

In order to alleviate the need for the precise knowledge of boundary locations and reduce the dimensionality of the integral, a coordinate transformation followed by a feature space sampling technique is adopted. The purpose of the initial orthogonal transformation of the coordinates is an N to 1 dimensionality reduction such that the N-tuple integral is reduced to a product of N one dimensional integrals. Let the conditional classification accuracy estimate,  $\hat{P}_c | \omega_j$  be the desired quantity. Then the transformed class  $\omega_j$  statistics is given by

$$\begin{aligned}
\mu_j' &= \mu_j - \mu_i \\
M_j &= \phi_i^T \mu_j' \quad j = 1, 2, \dots, M \\
S_j &= \phi_i^T \sum_j \phi
\end{aligned} \tag{3}$$

where  $\phi_j$  is the eigenvector matrix derived from  $\sum_i$ . Naturally, in each transformed space,  $T_i(\Omega)$ ,  $\omega_i$  has a null mean vector and a diagonal covariance matrix.

The discrete feature space approach is capable of eliminating the need for the simultaneous solution of M quadratic forms. If  $\Omega$  is the continuous probability space, a transformation  $T_i$  is required such that in  $T_i(\Omega)$ ,  $\Gamma_i$  can be completely described in a nonparametric form, thereby bypassing the requirement for an algebraic representation of  $\Gamma_i$ . This desired transformation would sample  $\Omega$  into a grid of N-dimensional hypercubes. Since the multispectral data is generally modeled by a multivariate normal random process, a discrete equivalent of normal random variables that would exhibit desirable limiting properties is required. Let  $y_n \sim \text{Bi}(n, p)$  be a binomial random variable with parameters n and p. The  $x_n$  defined by

$$x_n = \frac{y_n - np}{\sqrt{np(1-p)}} \quad y_n = 0, 1, 2, \dots, n \tag{4}$$

converges to  $x \sim N(0, 1)$  in distribution [9], i.e.,

$$\lim F_n(x) \longrightarrow F(x)$$

The convergence is most rapid for  $p=1/2$ , then

$$x_n = \frac{(y_n - n/2)2}{\sqrt{n}} \tag{5}$$

The variance of  $x_n$  is set equal to the eigenvalues of the transformed  $\underline{\sum}_i$  by incorporating a multiplicative factor in Eq. (5).

The segmentation of  $\Omega$  by a union of elementary hypercubes makes nonparametric representation of  $\Gamma_i$  and its contours feasible. Following the orthonormal transformation on  $\omega_i$  and sampling of  $\Omega$  accordingly, each cell's coordinate is assigned to an appropriate partition of  $\Gamma$ . This process is carried out exhaustively, therefore  $\Gamma_i$  can be defined as a set such that

$$\Gamma_i = \{UX_n : X_n \in \Gamma_i\} \quad (6)$$

once the exhaustive process of assignment is completed, the integral of  $f(\underline{X}|\omega_i)$  over  $\Gamma_i$  is represented by the sum of hypervolumes over the elementary cells within  $\Gamma_i$ . The elementary unit of probability is given by

$$\int_{C_i} f(\underline{X}|\omega_i) = \int_{\frac{-\delta_1}{2}}^{\frac{\delta_1}{2}} f(x_1|\omega_i) dx_1 \int_{\frac{-\delta_2}{2}}^{\frac{\delta_2}{2}} f(x_2|\omega_i) dx_2 \dots \int_{\frac{-\delta_N}{2}}^{\frac{\delta_N}{2}} f(x_N|\omega_i) dx_N \quad (7)$$

where  $C_i$  is the domain of a sampling cell centered at the origin and  $\delta_i$  is the width of a cell along the  $i$ th feature axis. The conditional probability of correct classification is therefore given by

$$\hat{P}_c|\omega_i = \sum_{C \in \Omega} \int_{c_1 - \frac{\delta_1}{2}}^{c_1 + \frac{\delta_1}{2}} f(x_1|\omega_i) I_i(C) dx_1 \int_{c_2 - \frac{\delta_2}{2}}^{c_2 + \frac{\delta_2}{2}} f(x_2|\omega_i) I_i(C) dx_2 \quad (8)$$

$$\dots \int_{c_n - \frac{\delta_n}{2}}^{c_n + \frac{\delta_n}{2}} f(x_n|\omega_i) I_i(C) dx_n$$

with overall classification accuracy given by

$$\hat{P}_c = \sum_{i=1}^M P(\omega_i) \hat{P}_c | \omega_i \quad (9)$$

where

$$I_i(C) = \begin{cases} 1 & \text{if } C \in \Gamma_i \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

C = The domain of an elementary cell

Figure 6 is a geometrical representation of Eq. (8).

### Experimental Results.

The analytic classification accuracy prediction (ACAP) has been thoroughly tested and documented [2]. Two examples are repeated here. The first experiment investigates the performance of the estimator vs. grid size i.e., number of cells per axis, n. Small to moderate range of n is required if computation time is to remain realistic. Figure 7 shows the variation of  $\hat{P}_c | \omega_i$  vs. n for three classes having some hypothetical statistics in 3 dimensions. The main property of the estimator is its rapid convergence toward a steady state value thereby alleviating the need for excessively fine grids and hence high computation costs.

The data collected over Graham County, Kansas is used to perform a comparison between the ACAP algorithm and a ratio estimator such as LARSYS. The results are tabulated in Table 1.

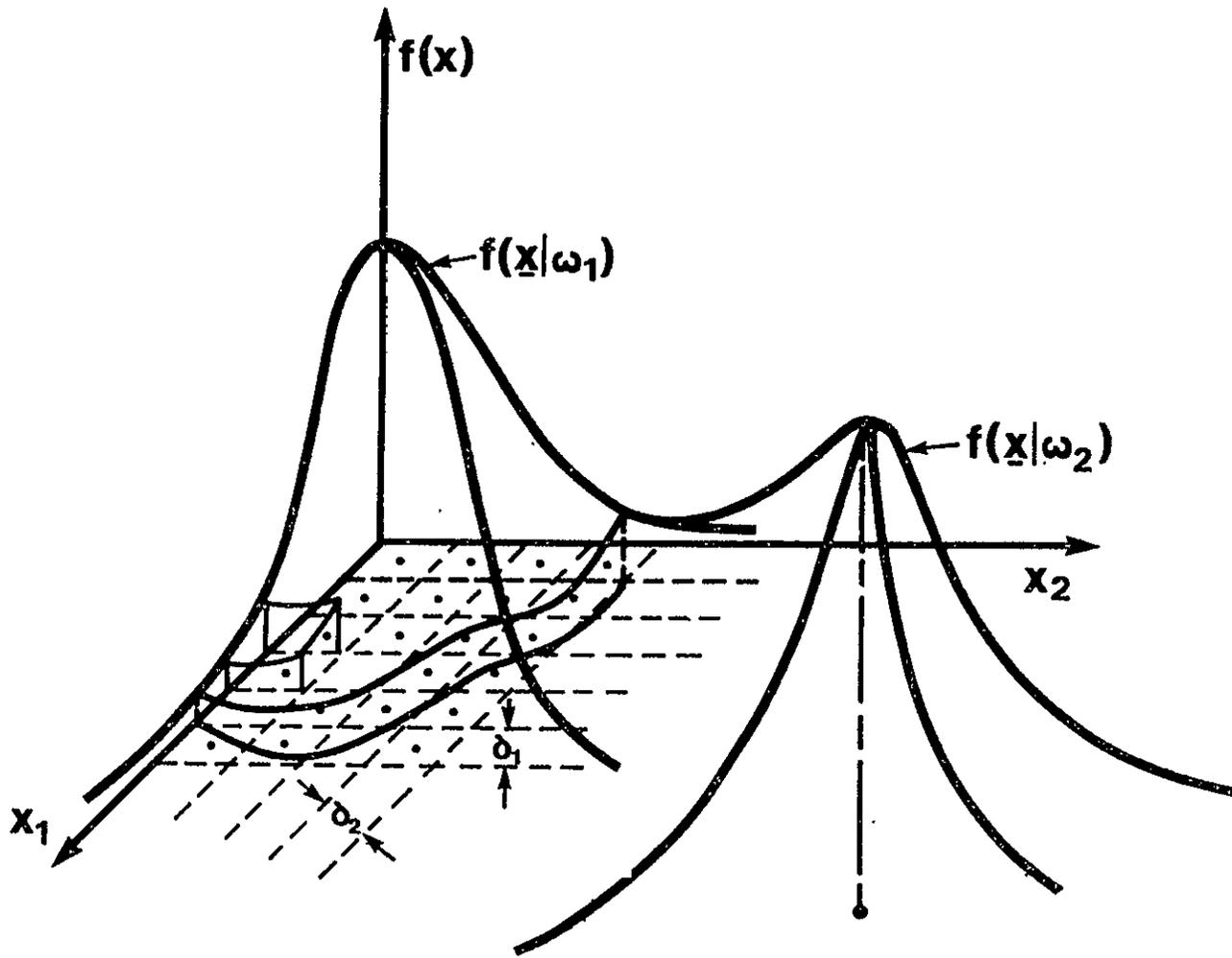


Figure 6. A Conceptual Illustration of the ACAP Error Estimation Technique Using Two Features.

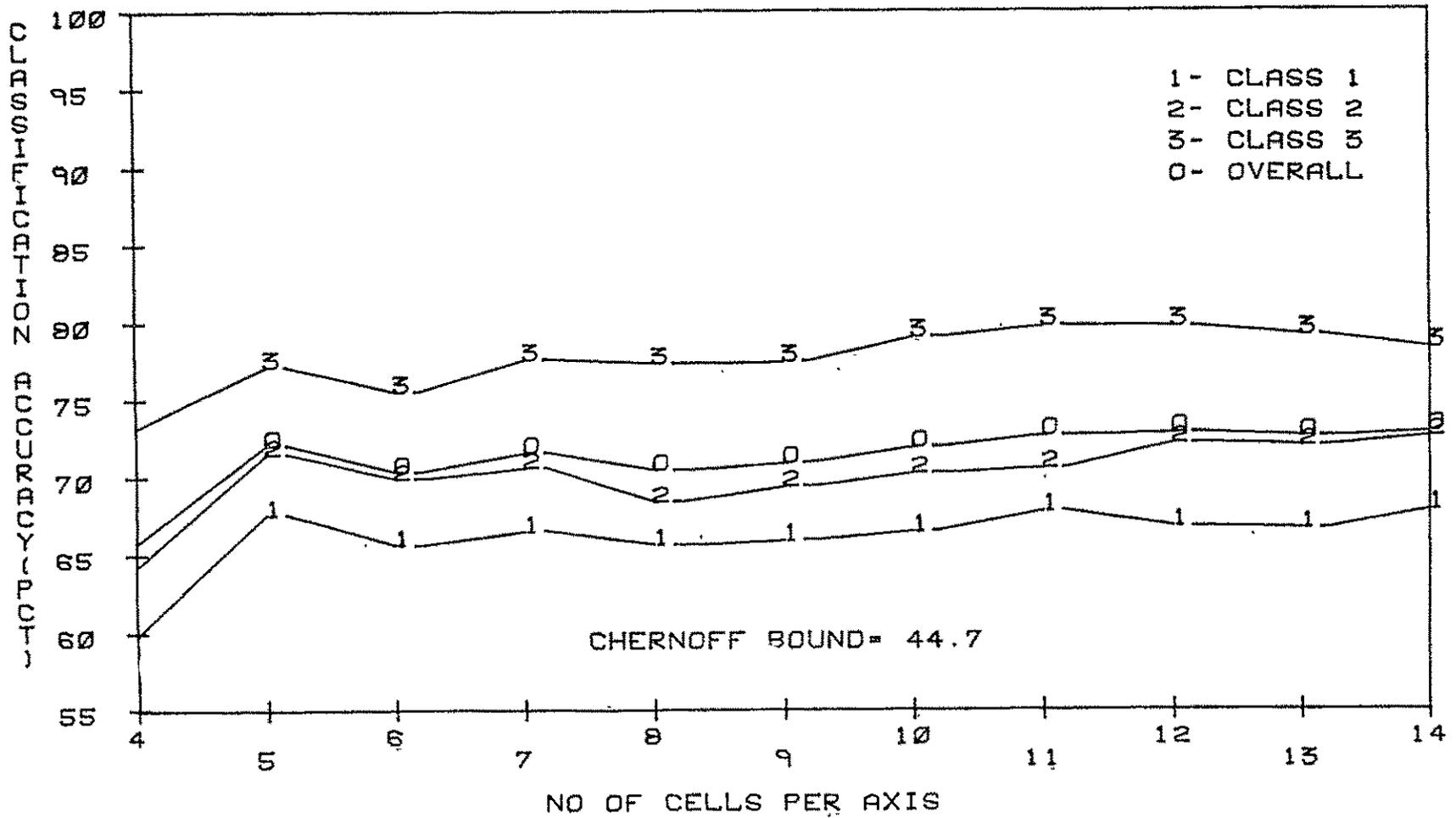


Figure 7. ACAP Classification Accuracy Estimate vs. Grid Size for the Test Population Data.

Table 1. ACAP and LARSYS Performance Comparison.

Class	No. of Points	LARSYS	ACAP
Bare Soil	443	65.9	78.3
Corn/Sorghum	99	89.9	91.0
Pasture	1376	98.4	95.0
Wheat	459	94.8	93.9
Overall	2377	87.2	89.6

The comparison of ACAP and LARSYS results from Table 1 is inconclusive. In some cases the difference is negligible (corn) and in some, significant (bare soil). Examination of the data statistics revealed that the assumption of normality does not hold throughout the populations statistics. This problem can be rectified by simulating random Gaussian data having identical statistics with the real data, hence assuring the normality assumption. Repeating the LARSYS and ACAP procedures produces a new set of classification accuracies, Table 2.

Table 2. ACAP and LARSYS Performance Comparison Simulated Data.

Class	LARSYS%	ACAP%	Accuracy Difference%
Bare Soil	77.8	78.3	0.5
Corn	91.2	91.0	0.2
Pasture	95.3	95.1	0.2
Wheat	94.2	93.9	0.2
Overall	89.6	89.6	0

The differential between ACAP and LARSYS results has been narrowed considerably, ranging from a high of 0.5% for bare soil to 0% for the overall classification accuracy. Two conclusions can be drawn from the results of this experiment. First, the ACAP and Monte Carlo type classifiers produce practically identical results if the underlying assumptions are satisfied (e.g., normality of the statistics). Second and more revealing is the fact that the results of the ACAP processor indicated an upper bound for the classifiability of bare soil had

its statistics been Gaussian. This result is a direct property of ACAP's data independence. Figure 8 is the ACAP estimator vs.  $n$  for Graham Co. data.

The above selected experiments and others reported in the bibliography establishes ACAP as a viable and necessary tool in any analytical remote sensing data collection system modeling and simulation when the performance index is defined as the probability of correct classification.

## 2.2 Stratified Posterior Classification Performance Estimator.

The second classification accuracy estimator to be presented here (SPEST) is based on the maximization of the a posteriori probability associated with each sample. This formulation is closely related to the maximum likelihood principle used in the ACAP. The distinction arises in the determination of integration domains. Where in ACAP a "deterministic" grid was set up to sample the feature space, SPEST uses an internally generated random data base and assigns the feature vector to the appropriate class via the maximum a posteriori principle. Due to the different approaches adopted, the statistical properties of the estimators could be substantially different although no major study has yet been carried out. It has been observed however, that the SPEST algorithm is somewhat faster than the ACAP in selected cases. The approach here is similar to that described in Moore, Whitsitt, and Landgrebe [10].

Let  $X$  be an observation from one of  $M$  classes  $\omega_i$ ,  $i = 1, 2, 3, \dots, M$ , with a priori probabilities  $P_i$ . The maximum likelihood decision rule can be stated as follows: Assign  $X$  to the class  $\omega_k$  if

$$P(\omega_k | X) = \max_i \{P(\omega_i | X)\}$$

This rule partitions the observation space  $\Omega$  into subregions  $\Gamma_1, \Gamma_2, \dots, \Gamma_M$ , corresponding to the classes  $\omega_1, \omega_2, \dots, \omega_M$ , respectively. Define the indicator function as

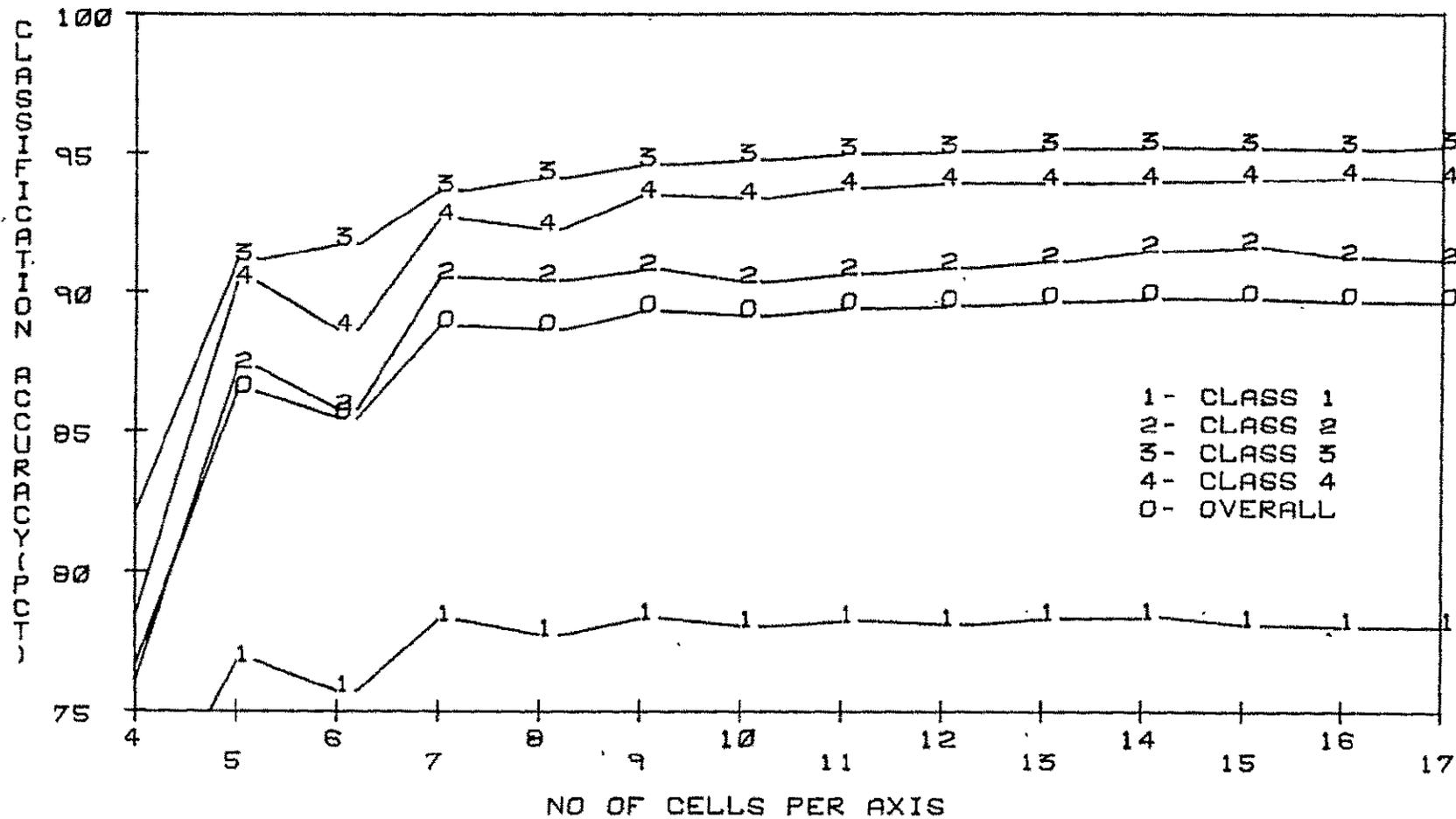


Figure 8. ACAP Classification Accuracy Estimate vs. Grid Size for Graham Co., Kansas Data.

$$I_i(x) = \begin{cases} 1 & x \in \Omega_i \\ 0 & x \notin \Omega_i \end{cases}$$

The probability of correct classification integral is given by

$$P_c = \int_{\Omega} \sum_{i=1}^M P_i I_i(x) p(x|\omega_i) dx \quad (11)$$

It is desirable to evaluate the probability of correct classification for each class as well as the overall probability. The performance probability for the  $i$ th class is

$$P_{ci} = \int_{\Omega} I_i(x) p(x|\omega_i) dx \quad (12)$$

The overall performance, then, is

$$P_c = \sum_{i=1}^M P_i P_{ci} \quad (13)$$

From Bayes' rule

$$p(x|\omega_i) = \frac{P(\omega_i|x) p(x)}{P_i}$$

hence,

$$P_{ci} = \int_{\Omega} I_i(x) \frac{P(\omega_i|x)}{P_i} p(x) dx$$

$p(x)$  is a mixture density

$$p(x) = \sum_{j=1}^M P_j p(x|\omega_j)$$

Therefore,

$$P_{c,i} = \sum_{j=1}^M \frac{P_j}{P_i} \int_{\Omega} I_i(x) P(\omega_i | x) p(x | \omega_j) dx \quad (14)$$

Define

$$Q_i(x) = I_i(x) P(\omega_i | x)$$

Then

$$\int_{\Omega} Q_i(x) p(x | \omega_j) dx$$

is the conditional expected value of  $Q_i(x)$  given that  $X$  comes from the class  $C_j$ . The estimate

$$\hat{P}_{ci} = \sum_{j=1}^M \frac{P_j}{P_i} \frac{1}{N_j} \sum_{k=1}^{N_j} Q_i(x_k) \quad (15)$$

is unbiased. This estimator is similar to the stratified posterior estimator described by Whitsitt [10].

To do this a pseudo-random sequence of uniformly distributed random digits is generated by the power-residue method and is transformed by the inverse cumulative-distribution-function method to obtain nearly Gaussian samples. These samples are used to fill the elements of the data vector  $\underline{Y}$ . Each vector  $\underline{Y}$ , then, has expected value  $\underline{0}$  and covariance matrix  $\underline{I}$ .

By performing the transformation

$$\underline{X} = \Phi_j \frac{1}{\Gamma_j} \underline{Y} + \underline{m}_j \quad (16)$$

on the vectors  $\underline{Y}$ , the random vectors for class  $j$  are obtained, where  $\underline{\Phi}_j$  is the matrix of eigenvectors required to diagonalize the covariance matrix of class  $j$ ,  $\underline{\Gamma}_j$  is the diagonal matrix of eigenvalues and  $\underline{m}_j$  is the desired mean vector. These random vectors are used to evaluate the estimators in Eqs. (15) and (13).

The term that must be evaluated from Eq. (14) is

$$P(C_j | \underline{x}) = \frac{P_j p(\underline{x} | \omega_j)}{\sum_k P_k p(\underline{x} | \omega_k)}$$

To evaluate this probability compute  $P_j p(\underline{x} | \omega_j)$  for each class  $\omega_j$ . Choose the largest value of the product  $P_j p(\underline{x} | \omega_j)$  and divide by the sum  $\sum_k P_k p(\underline{x} | \omega_k)$

$$p(\underline{x} | \omega_k) = \frac{1}{(2\pi)^{N/2} |\underline{K}_k|^{1/2}} \exp \{ -\frac{1}{2} (\underline{x} - \underline{m}_k)^T \underline{K}_k^{-1} (\underline{x} - \underline{m}_k) \} \quad (17)$$

$\underline{m}_k$  and  $\underline{K}_k$  are the mean vector and covariance matrix respectively for class  $k$ . Substituting Eq. (16) into (17),

$$p(\underline{x} | \omega_k) = \frac{1}{(2\pi)^{N/2} |\underline{K}_k|^{1/2}} \exp \{ -\frac{1}{2} [ \underline{Y}^T \underline{\Gamma}_j^{-1} \underline{\Phi}_j^T \underline{K}_k^{-1} \underline{\Phi}_j \underline{\Gamma}_j^{-1} \underline{Y} + 2 \underline{Y}^T \underline{\Gamma}_j^{-1} \underline{\Phi}_j^T \underline{K}_k^{-1} (\underline{m}_j - \underline{m}_k) + (\underline{m}_j - \underline{m}_k)^T \underline{K}_k^{-1} (\underline{m}_j - \underline{m}_k) ] \} \quad (18)$$

In this form it is not necessary to perform the intermediate computational step of transforming the data. We need only to generate  $M$  sets of random vectors  $\underline{Y}$  with mean vector  $\underline{0}$  and covariance matrix  $\underline{I}$  and use them in the Eq. (18).

Estimator Evaluation.

A subroutine program was written to evaluate classification performance by the above method. To test the method a three class problem was constructed. The mean vectors for the classes were

$$\begin{aligned}\underline{M}_1 &= [-1, -1, \dots, -1]^T \\ \underline{M}_2 &= [0, 0, \dots, 0]^T \\ \underline{M}_3 &= [1, 1, \dots, 1]^T\end{aligned}$$

The covariance for each class was the identity matrix. The number of random vectors generated for each class was 1000. The exact classification accuracy as a function of the dimensionality can be evaluated for this case

$$\begin{aligned}P_{c1} &= 1 - \operatorname{erfc}(\sqrt{N}/2) \\ P_{c2} &= 1 - 2 \operatorname{erfc}(\sqrt{N}/2) \\ P_{c3} &= 1 - \operatorname{erfc}(\sqrt{N}/2) \\ P_c &= 1 - 4/3 \operatorname{erfc}(\sqrt{N}/2)\end{aligned}$$

$$\text{where } \operatorname{erfc}(a) = \int_a^\infty \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx$$

and  $n$  is the number of dimensions. Table 3 contains the results of evaluating the class conditional performance and overall performance from one to ten dimensions.

To evaluate the variance of the estimates different starting values for the random number generator were used. Twenty trials were used to evaluate the maximum bound and the standard deviation from the true value. These results are presented in Table 4.

For the overall accuracy the estimate is within .005 of the true value. This is certainly sufficient for performance estimation. The

Table 3. Test of the SPEST Error Estimate.

	$P_{c_1}$	$P_{c_2}$	$P_{c_3}$	$P_c$	$\hat{P}_{c_1}$	$\hat{P}_{c_2}$	$\hat{P}_{c_3}$	$\hat{P}_c$
1	0.6915	0.3829	0.6915	0.5886	0.6859	0.3793	0.7001	0.5884
2	0.7602	0.5205	0.7602	0.6803	0.7671	0.5116	0.7700	0.6829
3	0.8068	0.6135	0.8068	0.7423	0.8037	0.6202	0.8081	0.7440
4	0.8413	0.6827	0.8413	0.7885	0.8283	0.6852	0.8550	0.7895
5	0.8682	0.7364	0.8682	0.8243	0.8642	0.7425	0.8703	0.8256
6	0.8897	0.7793	0.8897	0.8529	0.8767	0.7939	0.8787	0.8498
7	0.9071	0.8141	0.9071	0.8761	0.8993	0.8242	0.9065	0.8766
8	0.9214	0.8427	0.9214	0.8951	0.9129	0.8472	0.9240	0.8947
9	0.9332	0.8664	0.9332	0.9109	0.9193	0.8809	0.9360	0.9120
10	0.9431	0.8862	0.9431	0.9241	0.9209	0.9012	0.9481	0.9234

Table 4. Variation of Estimates.

	$P_{c1}$	$P_{c2}$	$P_{c3}$	$P_c$	
1	.016 .033	.010 .019	.017 .049	.003 .005	$\sigma$ Bound
2	.018 .036	.010 .018	.014 .027	.002 .005	$\sigma$ Bound
3	.016 .046	.017 .031	.017 .055	.003 .007	$\sigma$ Bound
4	.011 .025	.016 .029	.015 .029	.003 .005	$\sigma$ Bound
5	.015 .031	.014 .033	.012 .026	.002 .004	$\sigma$ Bound
6	.014 .026	.014 .023	.010 .022	.003 .006	$\sigma$ Bound
7	.009 .027	.016 .033	.012 .027	.003 .005	$\sigma$ Bound
8	.013 .025	.013 .036	.012 .023	.003 .006	$\sigma$ Bound
9	.013 .026	.014 .031	.012 .021	.002 .004	$\sigma$ Bound
10	.009 .016	.012 .024	.009 .019	.002 .005	$\sigma$ Bound

$\sigma$  = standard deviation

Bound = maximum difference between estimate and true value over  
20 trials

class conditional estimates are less reliable but are sufficient to observe trend in the performance due to individual classes.

### 2.3 Scanner Spatial Characteristics Modeling

The multispectral scanner represents the most important element in a remote sensing data gathering system. Therefore, an understanding of the signal flow through this subsystem is essential. As data is processed through the scanner, its statistical properties undergo a transformation. This in turn will alter the population separabilities and hence the classification accuracies. The comparison of this quantity at the scanner input and output and observation of its variation with the system parameters sheds considerable light on the overall system design. Since the Bayes spectral classifier depends solely on the population of spectral statistics, methods need to be developed that relate the scanner's input and output statistics. A complete derivation of such relationship is given in Appendix A of [2]. A summary follows:

#### Scanner Characteristic Function.

Figure 9 is a basic block diagram of the scanner spatial model where  $f_1$  through  $f_N$  are  $N$  stochastic processes corresponding to  $N$  spectral bands and  $h(x,y)$  is a two dimensional PSF. In particular where the Landsat scanner is concerned, the assumption of a Gaussian shaped IFOV has been widespread. Let  $f(x,y)$ ,  $g(x,y)$  and  $h(x,y)$  denote the input and output random processes associated with any two matching bands and the scanner PSF respectively. It is well known that the above quantities are related by a convolution integral.

$$g(x,y) = \int \int f(x-\lambda_1, y-\lambda_2) h(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 \quad (19)$$

it follows that

$$S_{\sigma\sigma}(u,v) = S_{ff}(u,v) |H(u,v)|^2 \quad (20)$$

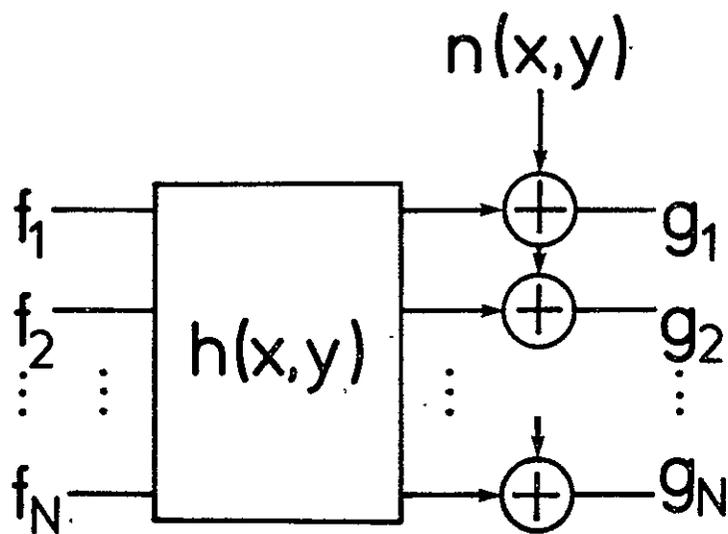


Figure 9. Scanner Spatial Model as a Linear System.

where  $S(u,v)$  is the spectral density of the appropriate random process and  $H(u,v)$  is the two dimensional Fourier transform of the scanner PSF. Let  $M(u,v) = |H(u,v)|^2$ , and  $m(\tau,\eta)$  its inverse transform. Then the output, spatial autocorrelation function is given by

$$R_{gg}(\tau,\eta) = R_{ff}(\tau,\eta) * m(\tau,\eta) \quad (21)$$

In order to obtain specific results, the following assumptions are invoked; (a) exponential data spatial correlation, (b) Gaussian IFOV,

$$R_{ff}(\tau,\eta) = \rho_x^{|\tau|} \rho_y^{|\eta|} \quad \tau,\eta = 0, 1, 2, \dots \quad (22)$$

$$h(x,y) = c_1 e^{-\frac{x^2}{2r_o^2}} e^{-\frac{y^2}{2r_o^2}}$$

where  $\rho_x = e^{-a}$  and  $\rho_y = e^{-b}$  are the adjacent sample and line correlation coefficients respectively,  $r_o$  is the scanner PSF characteristic length in pixels and  $c_1$  a constant providing unity filter gain. Using the separability property of the functions involved:

$$R_{gg}(\tau,\eta) = \int R_{ff}(\tau-x) m(x) dx \int R_{ff}(\eta-y) m(y) dy$$

where

$$m(\tau,\eta) = \frac{\pi c_1^2 r_o^2}{2} e^{-\frac{\tau^2}{2r_o^2}} e^{-\frac{\eta^2}{2r_o^2}} \quad (23)$$

carrying out the integration, the scanner characteristics function is given by

$$W_s(\tau, \eta, a, b) = \left[ \frac{a^2 r_o^2}{e^2} - a\tau \quad Q\left(ar_o - \frac{\tau}{r_o}\right) + \frac{a^2 r_o^2}{e^2} + a\tau \quad Q\left(ar_o + \frac{\tau}{r_o}\right) \right] \times$$

$$\left[ \frac{b^2 r_o^2}{e^2} - b\eta \quad Q\left(br_o - \frac{\eta}{r_o}\right) + \frac{b^2 r_o^2}{e^2} + b\eta \quad Q\left(br_o + \frac{\eta}{r_o}\right) \right] \quad (24)$$

where

$$Q(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{\xi^2}{2}} d\xi$$

The spectral statistics is a subset of the spatial statistics hence  $W_s(0,0,a,b)$  defines the ratio of the variance at the scanner output to the corresponding input quantity.

The output crosscovariance terms can be similarly derived. Let the crosscorrelation function between bands  $i$  and  $j$  be defined as

$$R_{f_i f_j}(\tau, \eta) = r_{f_i f_j} \sigma_{f_i} \sigma_{f_j} e^{-a_{ij}|\tau|} e^{-b_{ij}|\eta|} \quad (25)$$

where  $r_{f_i f_j}$  is the spectral crosscorrelation coefficient at the input such that  $|r_{f_i f_j}| \leq 1$ .  $a_{ij}$  and  $b_{ij}$  are defined similar to  $a$  and  $b$  with the additional channel specification. Following the previous technique it follows that the crosscovariance term between channels  $i$  and  $j$  is given by

$$R_{g_i g_j}(0,0) = r_{f_i f_j} \sigma_{f_i} \sigma_{f_j} W_s(0,0,a_{ij},b_{ij}) \quad (26)$$

The corresponding crosscorrelation coefficient follows:

$$r_{g_i g_j} = \frac{W_s(0,0,a_{ij},b_{ij})}{W_s^{1/2}(0,0,a_{ii},b_{ii}) W_s^{1/2}(0,0,a_{jj},b_{jj})} r_{f_i f_j} \quad (27)$$

Therefore, the band-to-band correlation coefficients are identical at the scanner input and output provided the spatial auto and crosscorrelation function at the input are equivalent, i.e.,  $a_{ii}=a_{ij}$ ,  $b_{ii}=b_{ij}$ . A closed expression can also be obtained for a rectangular PSF defined by

$$h(x,y) = \begin{cases} \frac{1}{r_o} & |x|, |y| \leq r_o/2 \\ 0 & \text{elsewhere} \end{cases} \quad (28)$$

the corresponding characteristic function is given by

$$W_s(0,0,a_{ii},b_{ii}) = \frac{2}{a_{ii}r_o} \left(1 - \frac{1-e^{-a_{ii}r_o}}{a_{ii}r_o}\right) \times \frac{2}{b_{ii}r_o} \left(1 - \frac{1-e^{-b_{ii}r_o}}{b_{ii}r_o}\right) \quad (29)$$

Eq. (24) and Eq. (29) are plotted in Figures 10 through 13 for different scene correlations with the IFOV =  $r_o$  as a running parameter. The universal property exhibited by  $W_s$  of either PSF is the increasing output variance reduction as IFOV is increased. This property has been widely verified experimentally. Comparison of Figures 11 and 13 indicates that for the same IFOV, a Gaussian shaped IFOV causes a heavier variance reduction in a spectral band than that of a rectangular PSF. This property can potentially produce a higher separability among the populations as the signal is processed through the scanner electronics.

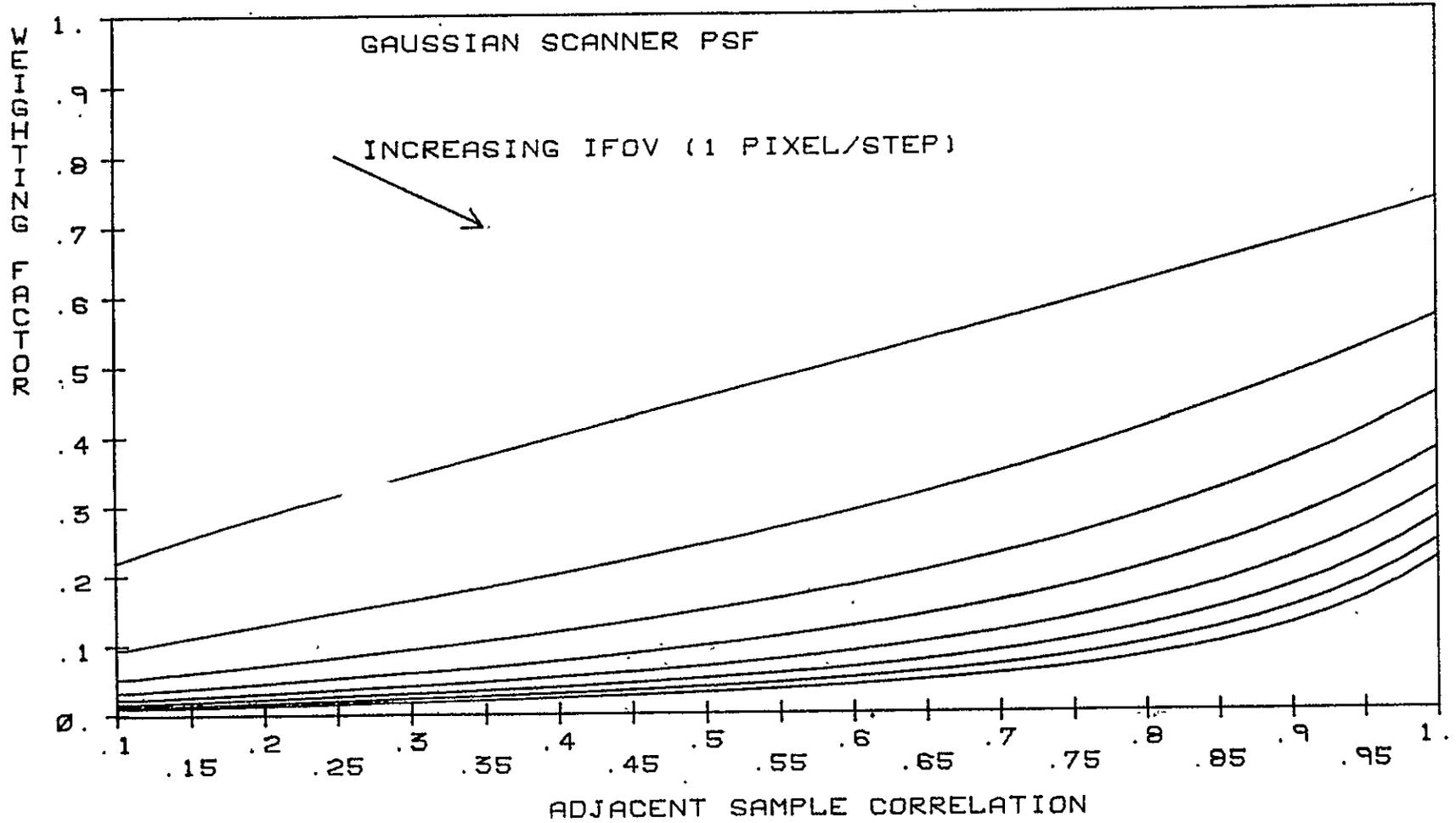


Figure 10. Scanner Characteristic Function vs. Scene Correlation.  
Adjacent Line Correlation = .65.

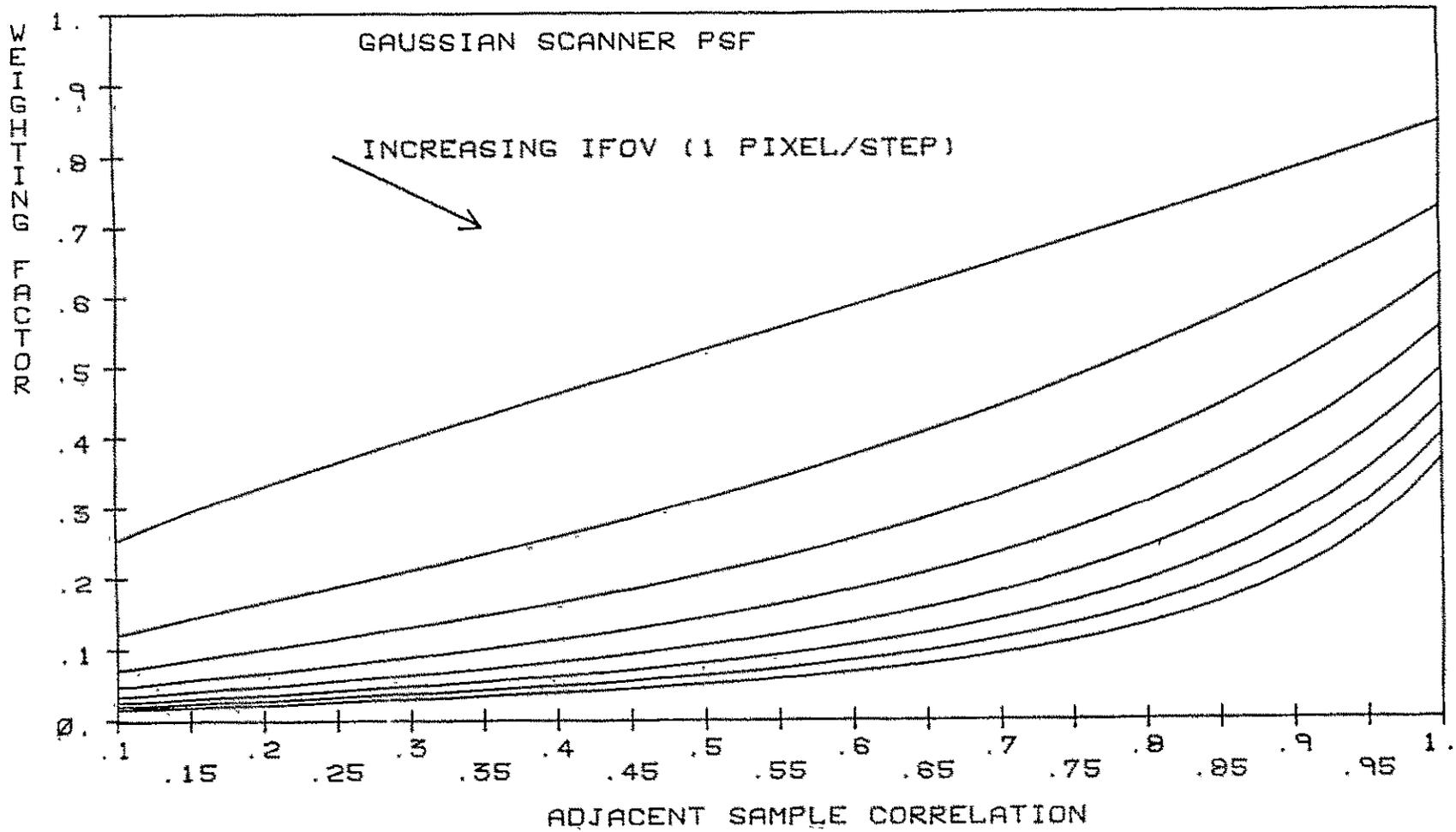


Figure 11. Scanner Characteristic Function vs. Scene Correlation.  
Adjacent Line Correlation = .8.

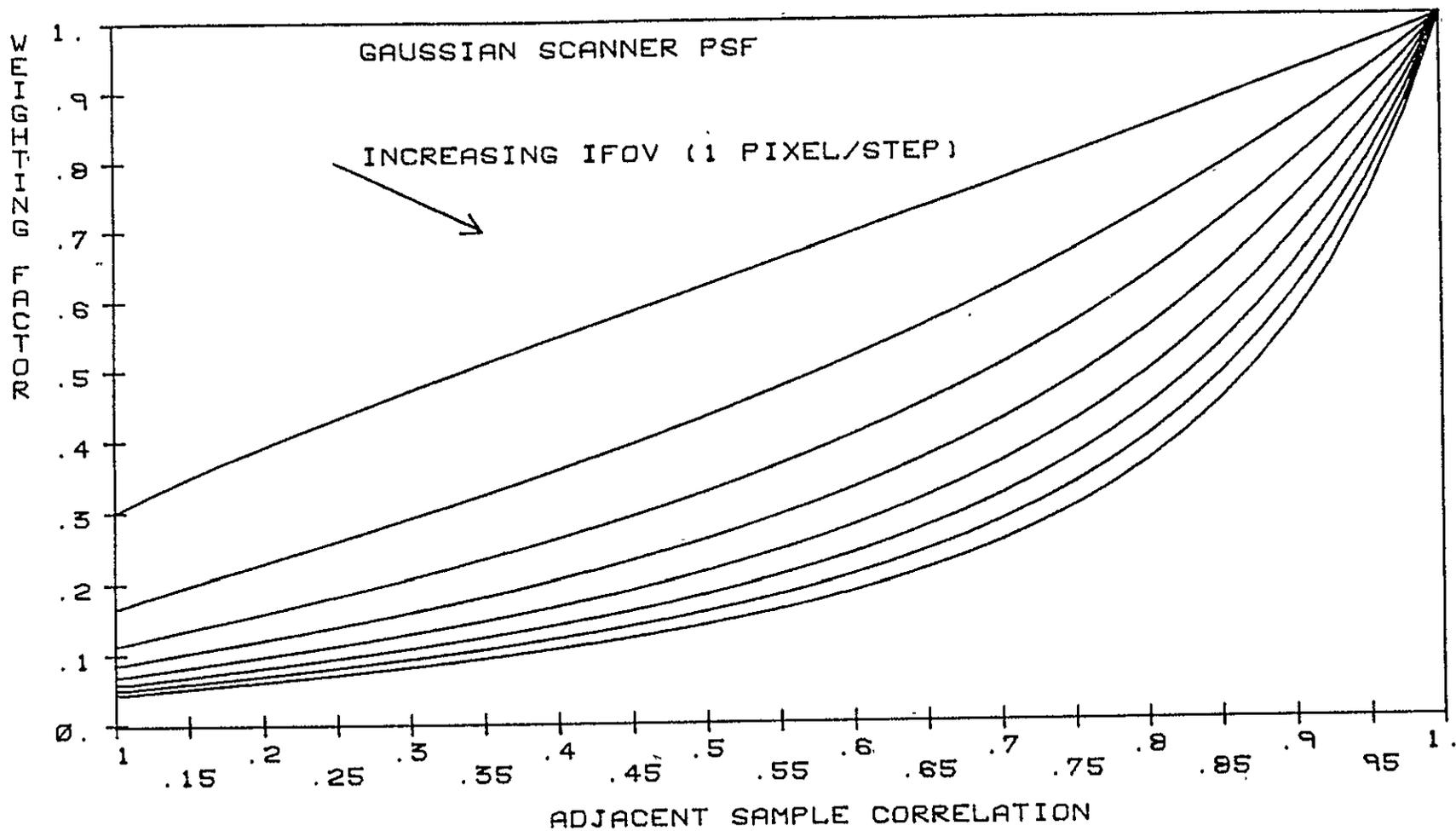


Figure 12. Scanner Characteristic Function vs. Scene Correlation.  
Adjacent Line Correlation = 1.

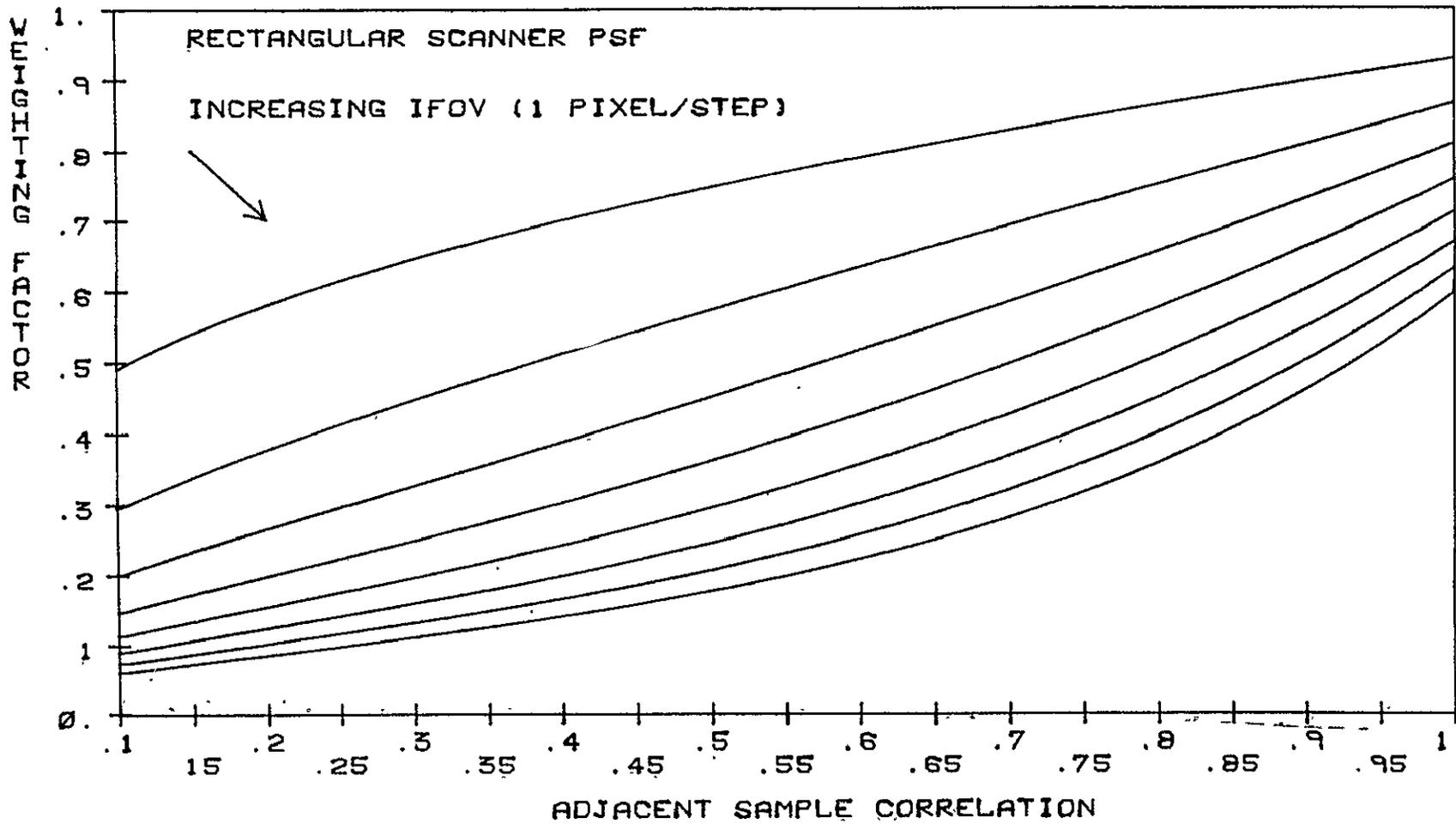


Figure 13. Scanner Characteristic Function vs. Scene Correlation.  
Adjacent Line Correlation = .8.

The input-output statistical relationship just developed along with the analytic classification accuracy predictor, provide the two basic tools required for a parametric evaluation of the MSS performance under varying operating conditions. As an illustration, three hypothetical classes with some prescribed statistics are specified at the scanner input. Three different scenes with adjacent sample correlations of 0.5, 0.85 and 0.95 are considered. The scanner characteristic function produced a set of transformed statistics at the output followed by the estimation of the conditional classification accuracies using ACAP, for 8 different IFOV's. The results are plotted in Figures 14 through 16.

Two main properties stand out. First is the improvement in class separability as the IFOV is enlarged. This is true in all the cases. The rate of improvement however, is strongly correlated with the scene spatial correlation. The lower the input scene correlation, the higher the classification accuracy improvement per IFOV step. This property is brought about by the characteristics of  $W_s$  where one step increase in IFOV size produces a greater variance reduction for a low scene spatial correlation than a similar increase would cause in a highly correlated scene.

The scene spatial correlation plays a significant role in the overall system performance which is not readily obvious. One of the well known properties of linear systems with random inputs is the reduction of the output variance/input variance ratio as the point spread function (PSF) is widened. In this section it has been shown theoretically that a third factor in this reduction is the spatial correlation structure of the input stochastic process. Specifically, with everything else fixed, a process having a moderate scene correlation will undergo a tighter clustering around its mean than an otherwise identical process with a highly correlated spatial characteristic. On the extreme side of the correlation scale with a small pixel-to-pixel correlation, the ratio of the output variance to the corresponding input quantity is very negligible. Consider a bandlimited white noise process with a spectral density, shown in Figure 17, where  $W$  is the bandwidth and  $N_0/2$  the two sided spec-

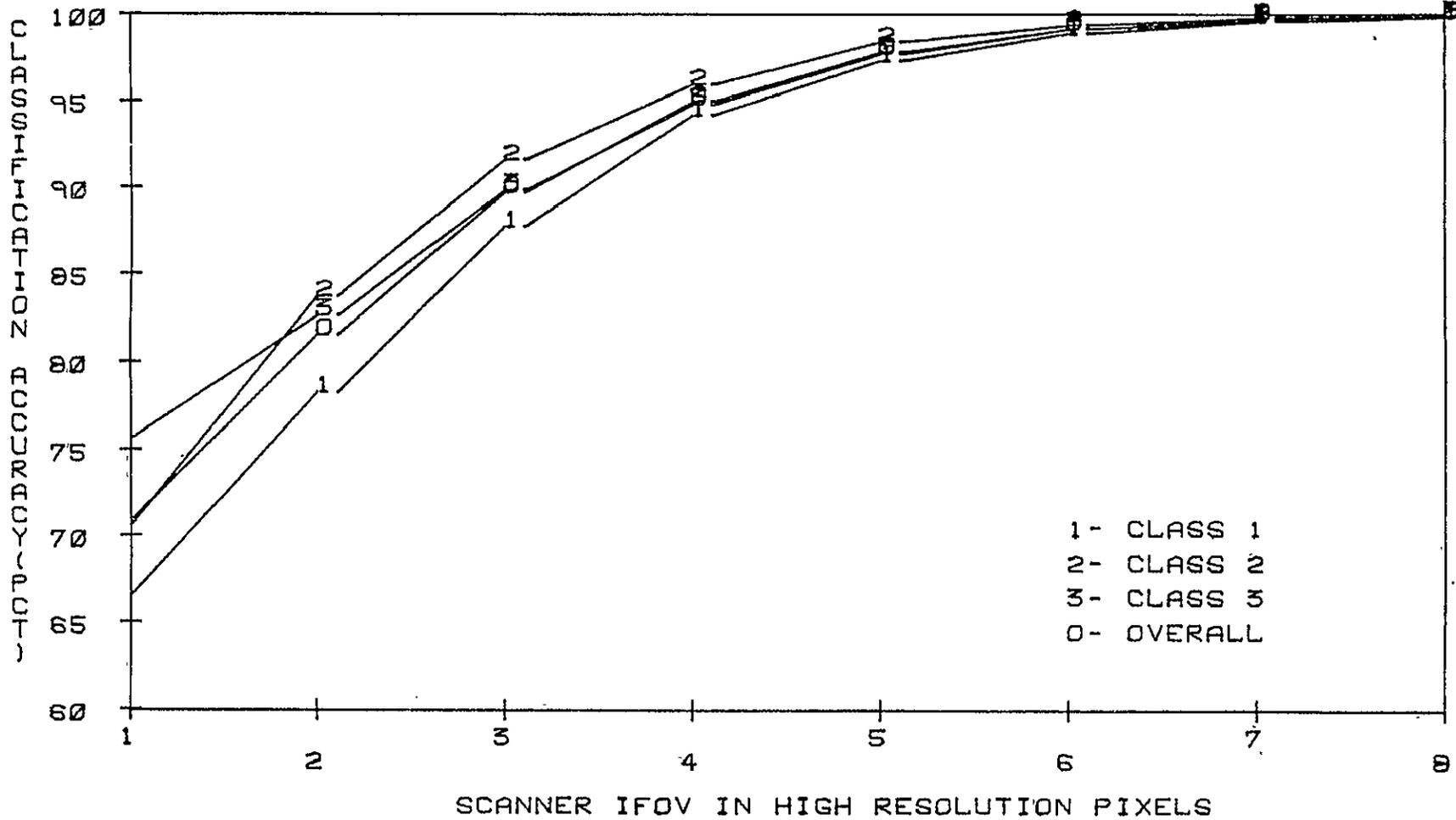


Figure 14. Scanner Output Classification Accuracy vs. IFOV.  
Adjacent Sample Correlation = .55.

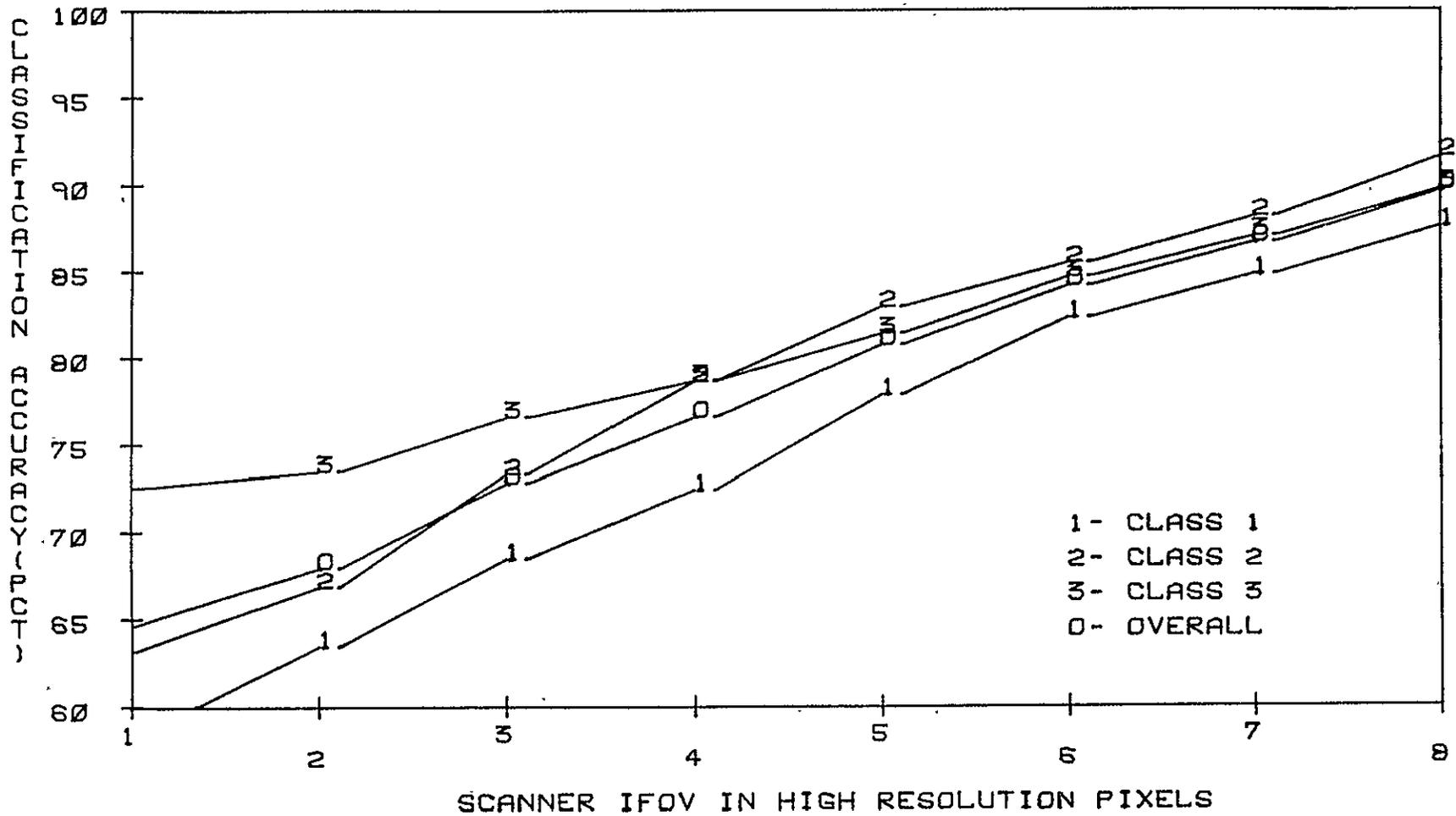


Figure 15. Scanner Output Classification Accuracy vs. IFOV.  
Adjacent Sample Correlation = .8.

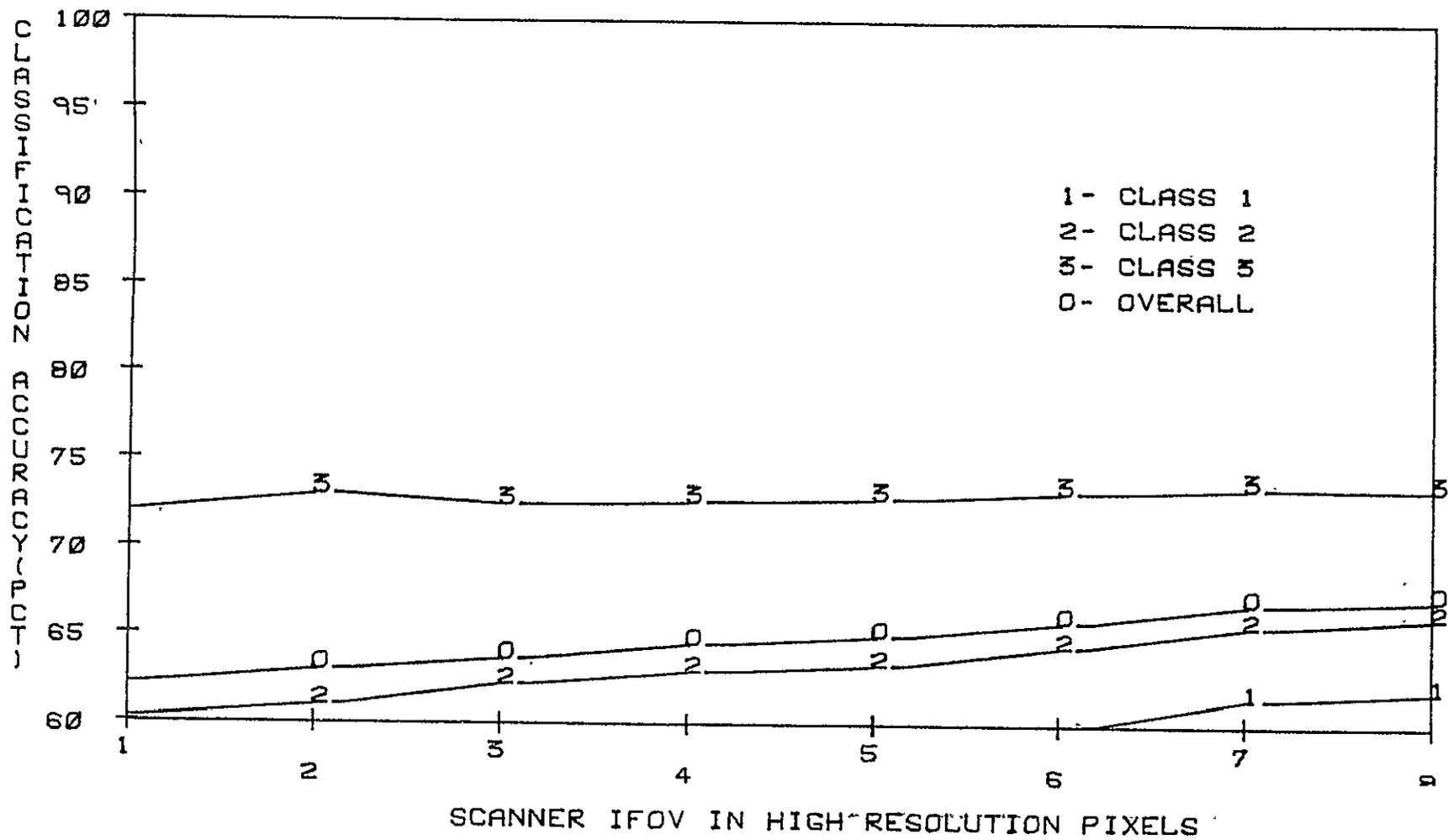


Figure 16. Scanner Output Classification Accuracy vs. IFOV.  
Adjacent Sample Correlation = .95.

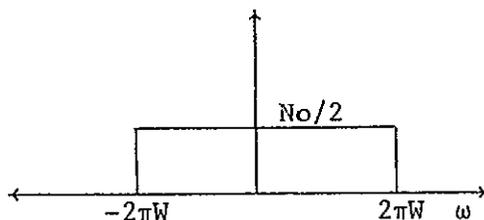


Figure 17. Bandlimited White Noise Spectral Density.

tral density. As  $W$  increases the adjacent pixel correlation in the scene decreases. The increase in  $W$ , however, is accompanied by a decrease in  $No/2$  if this process is to remain physically realizable (finite energy). Under a finite energy constraint, therefore, as  $W \rightarrow \infty$ ,  $No/2 \rightarrow 0$ . In the limit the energy content of the output random process will be nil.

#### Random Noise.

Additive random noise entered at various stages of a scanner system can degrade the overall system performance substantially. The noise can be classified into two broad categories: external and internal. A major source of external noise is atmospheric in nature mainly due to absorption (e.g., water vapor) and scattering. The detector and quantization noise comprise the major component of the internal noise sources. From a system analysis point of view, the latter represents a more tractable and better understood component of the random noise [11], while the former still awaits further experimental documentation. The purpose of this work is not so much the exploration of the various noise sources but the integration of its effect within an analytic analysis package once its characteristics and origin has been determined.

From the theoretical results obtained it can be stated that atmospheric noise, in the uplink path at least, has negligible degrading factor compared with the detector and quantization noise. Let  $f(x,y)$ ,  $N_f(x,y)$ ,  $f'(x,y)$  and  $N_{f'}(x,y)$  be the input random process, input additive white

noise, the output random process and the noise component of the output signal respectively, then

$$f'(x,y) = f(x,y) * h(x,y) \quad (30)$$

$$N'_{f'}(x,y) = N_{f'}(x,y) * h(x,y) \quad (31)$$

Define

$$(SNR)_f = \text{Var} \{f(x,y)\} / \text{Var} \{N_{f'}(x,y)\} \quad (32)$$

$$(SNR)'_{f'} = \text{Var} \{f'(x,y)\} / \text{Var} \{N'_{f'}(x,y)\} \quad (33)$$

Recalling the functional dependence of  $W_s$  on the input scene spatial correlation, it follows that the ratio of the variance of a white noise process at the scanner output to the corresponding input quantity is of the order of 5% to 10%, higher or lower depending on the IFOV size.

Therefore,

$$\text{Var} \{f'(x,y)\} < \text{Var} \{f(x,y)\} \quad (34)$$

$$\text{Var} \{N'_{f'}(x,y)\} \ll \text{Var} \{N_{f'}(x,y)\} \quad (35)$$

hence

$$(SNR)'_{f'} \gg (SNR)_f \quad (36)$$

It then follows that the noise component of the output process prior to detector and quantization noise is negligible in most cases.

In order to observe the effect of noise on the scanner output class separability the test class statistics were modified to exhibit the effect of random noise. The assumed properties of the noise are additive, white and Gaussian. Let  $f''(x,y)$  be the signal to be telemetered to Earth.

$$f''(x,y) = f'(x,y) + N_f(x,y) \quad (37)$$

the statistics of  $f''(x,y)$  and  $f'(x,y)$  are related by

$$\sum_{f''} = \sum_{f'} + \sum_{N_f} \quad (38)$$

the simple addition is due to the signal and noise independence. Assuming a zero mean  $N_f$ , the mean vector are identical, i.e.,

$$E \{f''\} = E \{f'\}$$

Among the four assumptions about the noise, its Gaussian property is the weak link due to the Poisson distributed detector noise and uniformly distributed quantization noise. Relaxing the Gaussian noise assumption, however, would mean the design of an optimum classifier for non-normal classes and evaluation of its performance. A task that would complicate matters considerably. Due to the relatively insufficient documentation of the characteristics of random noise in multispectral data, the initial Gaussian assumption is adhered to.

Following the adopted SNR definition, three different noise levels are considered and the corresponding overall classification accuracies for the three previously used test classes are estimated. Figure 18 is the variation of  $\hat{P}_c$  vs. IFOV with SNR as the running parameter. For a fixed IFOV,  $\hat{P}_c$  increases with increasing SNR. For a fixed SNR,  $\hat{P}_c$  increases with increasing IFOV size. These illustrations have shown that with a proper coupling between the ACAP and the scanner characteristic function, the progress of the population statistics through the system can be studied on an analytical and entirely parametric basis. The accompanying classification accuracies can measure the designer's success in selecting the spatial and/or spectral characteristics of a Multispectral Scanner System.

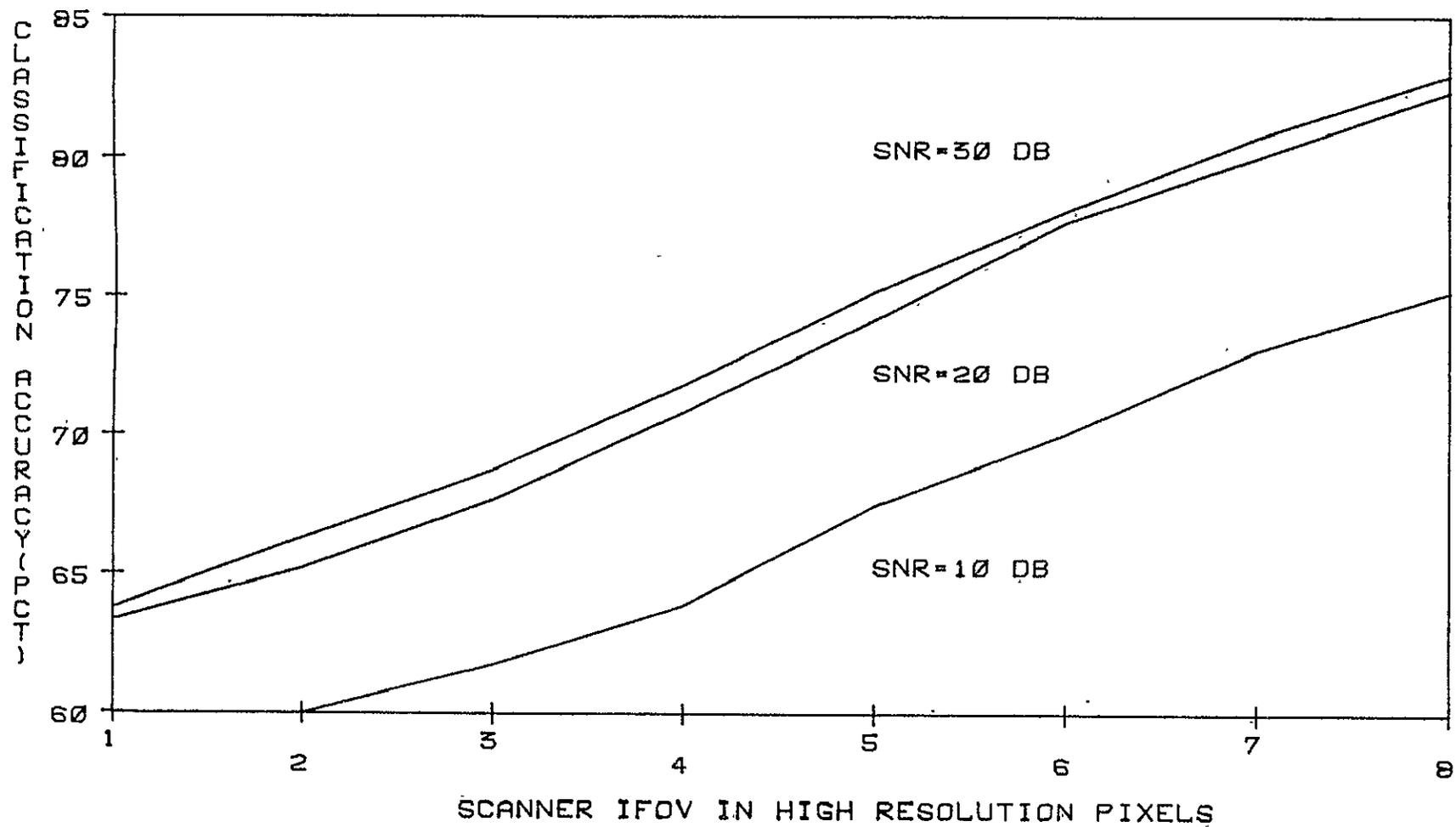


Figure 18. Overall Output Classification Accuracy Variation with Noise and IFOV.

## 2.4 Optimum Spectral Function Research

In earth observational remote sensing much work has been done with extracting information from the spectral variations in the electromagnetic energy incident on the sensor. Of primary importance for a multispectral sensor design is the specification of the spectral channels which sample the electromagnetic spectrum. An analytical technique is developed for designing a sensor which will be optimum for any well-defined remote sensing problem and against which candidate sensor systems may be compared.

Let the surface of the earth at a given time be divided in strata where each stratum is defined to be the largest region which can be classified by a single training of the classifier. Each point in the stratum is mapped into a spectral response function  $X(\lambda)$  as in Figure 19. That is if one observes a point in the stratum with the sensor, then the function  $X(\lambda)$  describes the response variations with respect to the wavelength,  $\lambda$ . The stratum together with its probabilistic description defines a random process, and the collection of all of such functions  $X(\lambda)$  which may occur in the stratum is called an ensemble.

The general concept of a pattern recognition system in this application requires that if each  $X(\lambda)$  is to be classified by a classification algorithm, this can be accomplished by first measuring a finite number of attributes of  $X(\lambda)$ , called features. This is the function of the sensor system as depicted in the upper left portion of Figure 20 where  $X_1, X_2, \dots, X_N$  are the values of  $N$  features for a given  $X(\lambda)$ . It may be viewed as a filtering operation on  $X(\lambda)$ .

For example, on the right portion of Figure 20 the function of MSS of the Landsat satellites is illustrated. In this case a number proportional to the average energy in a wavelength interval is reported out by the sensor for each of four wavelengths. Mathematically this may be expressed as

$$X_n = \int X(\lambda) \phi_n(\lambda) d\lambda \quad n = 1, 2, 3, 4$$

Next we must consider what would constitute an optimum sensor. We first note that in general the sensor may be used over any part of the earth's surface, at anytime, and for many different applications (sets of classes). Therefore the sensor must be optimized with respect to the entire set of strata represented by these cases. As a result of the large size of this set and the fact that its statistical description is not known, we will optimize the sensor with respect to its signal representation characteristics. The  $\{X(\lambda)\}$  each contain information useful to the classifier; we require of the sensor design that for a given  $N$  a maximum of this information which was in  $X(\lambda)$  still be present in  $\{X_n\}$ . Since the specific nature of this information is not known a priori, we can only assure that this will be the case for any stratum if  $X(\lambda)$  is recoverable from  $\{X_n\}$ .

Let  $\hat{X}(\lambda)$  be the result of attempting to reconstruct  $X(\lambda)$  from  $\{X_n\}$ . A fidelity criterion which is useful in this instance is

$$\epsilon = \int [X(\lambda) - \hat{X}(\lambda)]^2 d\lambda \quad (39)$$

the so called mean square error or mean square difference between  $X(\lambda)$  and  $\hat{X}(\lambda)$ .

It is known [12] that a reconstruction scheme which minimizes  $\epsilon$  for a given  $N$  is

$$\begin{aligned} X(\lambda) &= X_1\phi_1(\lambda) + X_2\phi_2(\lambda) + \dots + X_N\phi_n(\lambda) \\ &= \sum_{n=1}^N X_n\phi_n(\lambda) \end{aligned} \quad (40)$$

provided that the  $\{\phi_n(\lambda)\}$  are orthogonal over the wavelength interval of interest, i.e.,

$$\int \phi_m(\lambda) \phi_n(\lambda) d\lambda = 0 \quad m \neq n \quad (41)$$

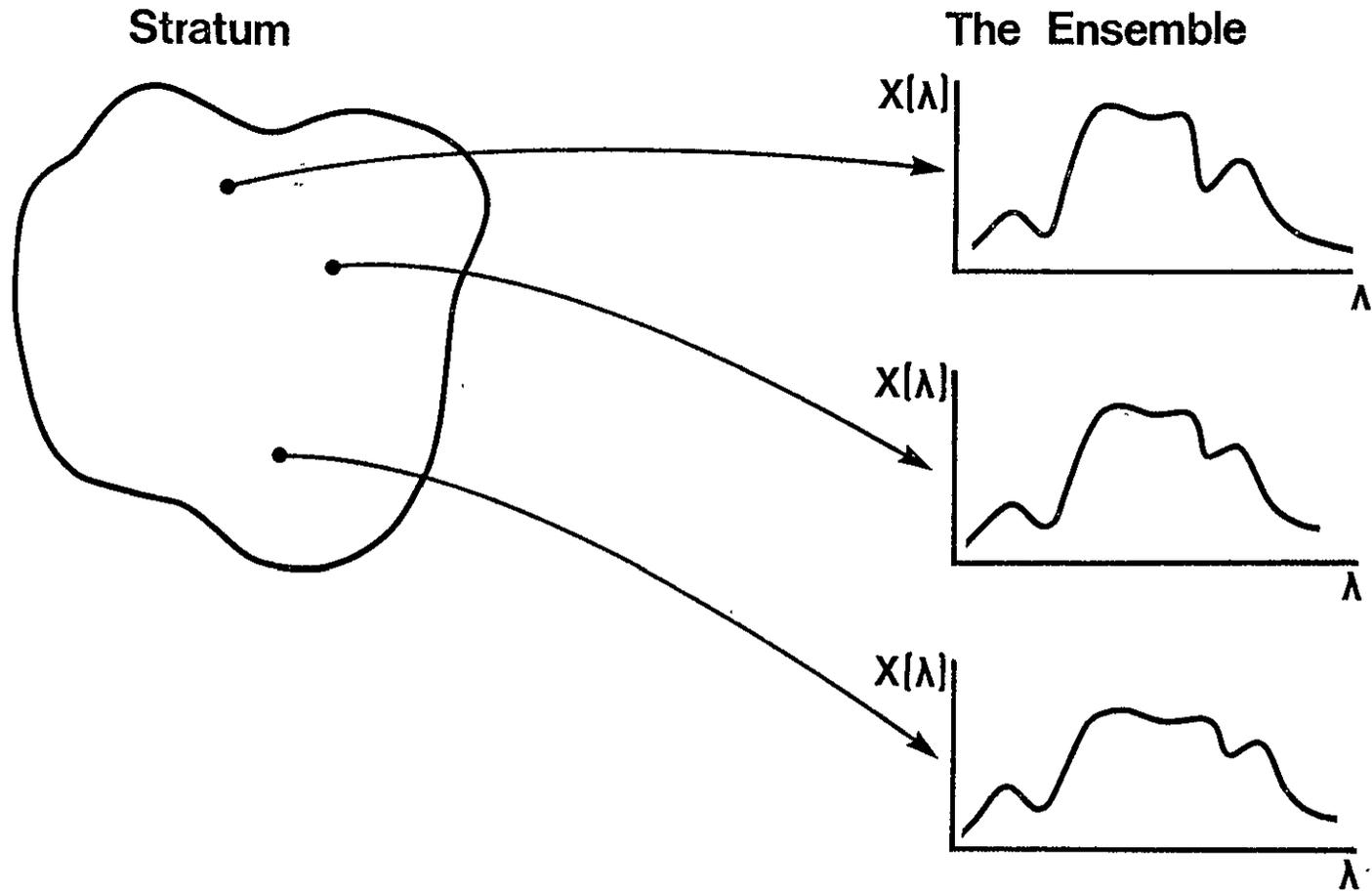
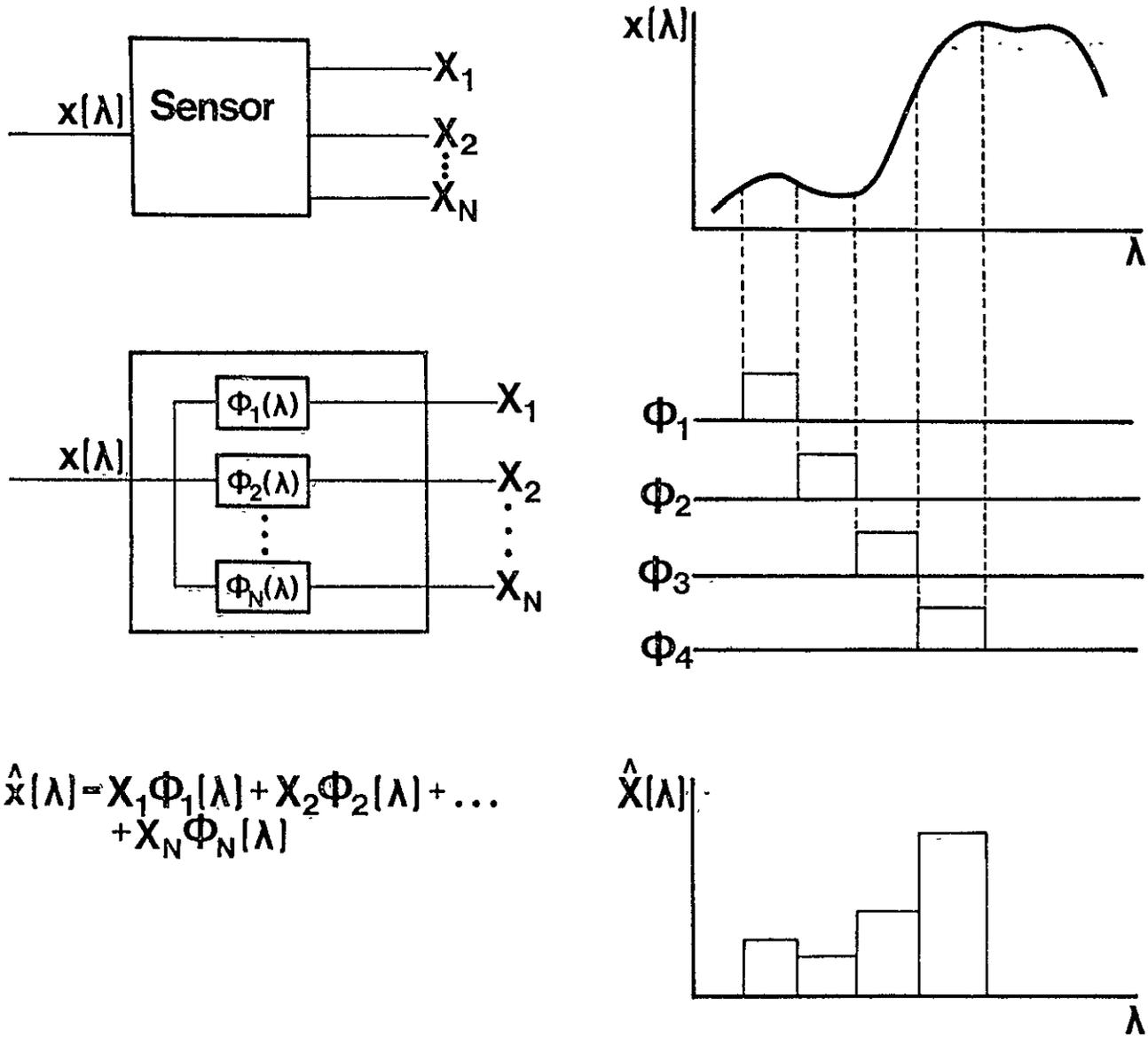


Figure 19. Realization of a Stratum as the Ensemble of Spectral Sample Functions.

Figure 20. Basis Function Expansion of a Random Process.



and the  $\{X_n\}$  are calculated by

$$X_n = \int X(\lambda) \phi_n(\lambda) d\lambda \quad (42)$$

Note for example that the Landsat example of Figure 20 satisfies these conditions. In the lower right of Figure 20 is depicted the result of such a reconstruction for the Landsat example.

While use of Eq. (42) in the case does minimize  $\epsilon$  with respect to the choice of values of  $\{X_n\}$ , a further improvement may be obtained by choosing a set of  $\{\phi_i|\lambda|\}$  which minimizes  $\epsilon$ . It can be shown [5, 12] that the set  $\{\phi_n(\lambda)\}$  which accomplishes this must satisfy the equation

$$\sigma_i \phi_i(\lambda) = \int R(\lambda, \xi) \phi_i(\xi) d\xi \quad (43)$$

where

$$R(\lambda, \xi) = E\{[X(\lambda) - m(\lambda)] [X(\xi) - m(\xi)]\} \quad (44)$$

is the correlation function of the random process and  $m(\lambda)$  is its mean value at  $\lambda$ .

Such a signal representation defined by Eqs. (40-44) is known as a Karhunen-Loève expansion [13]. It provides not only for the most rapid convergence of  $\hat{X}(\lambda)$  to  $X(\lambda)$  with respect to  $N$  but in addition the random variables  $\{X_n\}$  are uncorrelated and since the random process is Gaussian they are statistically independent. Further the only statistic required of the ensemble is  $R(\lambda, \xi)$ . This representation of  $\{X(\lambda)\}$  is therefore not only optimal, it is convenient.

A useful generalization of the Karhunen-Loève expansion can be made. Suppose a priori information concerning portions of the spectral interval are known and it is desired to incorporate this knowledge into the analysis. A weighting function  $w(\lambda)$ , is introduced which weights portions of the interval according to the a priori information. As an example, measure-

ments were taken over the spectrum and it was observed that there was considerable variation in the signal in the water absorption bands around 1.4 and 1.8 micrometers. This variation was due to measurement and calibration difficulties rather than being a result of variations in the scene. Therefore, the weighting function was set to zero in these absorption bands. This generalization is referred to as the weighted Karhunen-Loève expansion [5]. Eqs. (40), (42) and (43) become

$$\hat{X}(\lambda) = \sum_{i=1}^N x_i \phi_{w_i}(\lambda) \quad \lambda \in \Lambda \quad (45)$$

$$\sigma_{w_i} \phi_{w_i}(\lambda) = \int R(\lambda, \xi) w(\xi) \phi_{w_i}(\xi) d\xi \quad (46)$$

$$x_i = \int X(\lambda) w(\lambda) \phi_{w_i}(\lambda) d\lambda \quad (47)$$

where the eigenfunction,  $\phi_{w_i}(\lambda)$  are solutions to the integral Eq. (46) with the weight  $w(\lambda)$ . The special case where  $w(\lambda) = 1.0$  for all  $\lambda \in \Lambda$  reduces the expansion to the original form in Eq. (40), (42) (43), and (44).

The results of having utilized this means of optimal basis function scheme on spectral data are contained in reference [5]. From them one can see the significant improvement in classification accuracy which decreased spectral representation error will provide. One can also determine the spectral resolution and band placement needed to achieve such classification accuracy improvement.

## 2.5 Information Theory Approach to Band Selection

The problem of selecting a set of "optimum" windows in the electromagnetic spectrum for observing the reflected sunlight has always been of considerable interest. Depending on the definition of the optimality different methods have been developed. One such approach was shown in Section 2.4 using K-L expansion to select an optimum set of basis functions. In this section an information theoretic definition of optimality developed in [3] is explored.

Mutual Information and Stochastic Modeling.

The reflected energy from the target is detected by the scanner and corrupted by various noise sources. If  $S$  is the "noise-free" signal,  $Y$  the observation and  $N$  a random disturbance, then

$$Y = S + N \quad (48)$$

the reduction of uncertainty about  $S$  obtained from  $Y$  is called the average or mutual information between the observation and original signal. Since the reconstruction of the reflected signal from the noisy observation is the highly desirable capability, the comparison of such average information and selection of these bands with the highest information content is chosen as a means of spectral band selection. Let

$$S_n = (s_1, s_2, \dots, s_n)$$

and

$$Y_n = (y_1, y_2, \dots, y_n)$$

where  $s_i$  and  $y_i$  are the coefficients of the orthonormal (K-L) expansion of  $Y$  and  $S$ , then the mutual information between  $Y$  and  $S$  is given by [3].

$$I(Y,S) = \frac{1}{2} \log \left[ \frac{\det C_y}{\det C_n} \right] \quad (49)$$

where  $C_y$  and  $C_n$  are the covariance matrices of  $(y_i, i = 1, 2, \dots)$  and  $(n_i = N_0/2, i = 1, 2, \dots)$  and  $N_0/2$  is the two sided spectral density of the additive white noise. Equivalently  $I(Y,S)$  can be represented in terms of the Wiener-Hopf optimum filter impulse response,

$$I(Y,S) = \frac{1}{2} \int_{\lambda_1}^{\lambda_2} h(\lambda, \lambda) d\lambda \quad (50)$$

$h(\lambda, \lambda)$  provides an estimate of  $S$  from  $Y$  with a minimum mean-square error. This relationship, however, is not a practical method of evaluating  $I(Y, S)$  since the actual solution of the Wiener-Hopf integral itself is a nontrivial task. This problem can be circumvented by a discrete state variable formulation, i.e.,

$$\underline{s}(k+1) = \underline{\phi} \underline{s}(k) + \underline{\Gamma} W(k) \quad k \in [\lambda_1, \lambda_2] \quad (51)$$

where

$$\underline{s}(k+1) = \begin{bmatrix} s_1(k+1) \\ s_2(k+1) \\ \cdot \\ \cdot \\ s_n(k+1) \end{bmatrix}$$

$\underline{\phi}$  is an  $(n \times n)$  matrix

$\underline{\Gamma}$  is an  $(n \times 1)$  vector

$W(k)$  = a discrete independent Gaussian zero mean random process

The formulation of the problem in the discrete domain provides a practical way of computing  $h(\lambda, \lambda)$  through Kalman filtering techniques. The discrete version of Eq. (50) is given by

$$I(Y, S) = \frac{1}{2} \sum_{k \in \{\lambda_1, \lambda_2\}} h(k, k) \quad (52)$$

The discrete nature of this approach makes the evaluation of Eq. (52) considerably more practical than its continuous counterpart. This is due to the fact that the Wiener-Hopf equation is easily solved in only those cases for which the analytical form of  $K_s(\lambda, u)$ , the signal covariance function, is fairly simple, not likely for most random processes encountered

in remote sensing. Since  $h(k,k)$  is dependent on the parameters of Eq. (51) a concise representation of  $\underline{s}(k+1)$  is needed.

The general form of Eq. (51) is given as an autoregressive (AR) model

$$\underline{s}(k) = \sum_{j=1}^{m_1} a_j s(k-j) + \sum_{j=1}^{m_2} b_j \psi(k-j) + W(k) \quad (53)$$

$s(k)$  = The spectral response at the discrete wavelength  $k$ . It is a Gaussian random process.

$w(k)$  = zero mean independent Gaussian disturbance with variance  $\rho$ .

$\psi(k-j)$  = deterministic trend term used to account for certain characteristics of the empirical data.

$a_j, b_j$  = are unknown constant coefficients to be determined.

$m_1, m_2$  = The order of the AR model.

The identification selection and validation of general AR models for the representation of a random process is a well developed technique [14,15]. The identification of an appropriate model provides the necessary parameters required for the evaluation of  $I(Y,S)$  in Eq. (52). The model selection process for a selected number of ground covers has been carried out [3] leading to the ranking of a set of spectral bands according to the criterion outlined previously. A summary of the experimental results are given below.

#### Data Base and Model Selection.

Two different sets of empirical data are used to demonstrate the techniques developed here. The first set consists of observations of wheat scenes. The second set consists of several vegetation cover types such as oats, barley, grass, etc. For each scene the spectral responses, collected by the Exotech 20C field spectroradiometer, are averaged over the ensemble. It is thought the resultant average

spectral response provides a relatively good data set for demonstration purposes. Figures 21 and 22 show the average response for both cover types.

In order that the study be carried out in a context that is relatively realistic for multispectral scanners, the spectral response of the two data sets is divided into a number of spectral bands. The division is fairly arbitrary, but each band must contain a sufficient number of data points to ensure accurate parameter estimation for model identification. The spectrum is divided into 9 bands from 0.45  $\mu\text{m}$  to 2.38  $\mu\text{m}$  with two gaps in the 1.34-1.45  $\mu\text{m}$  and 1.82-1.96  $\mu\text{m}$  range due to atmospheric absorption, see Tables 5 and 6.

Table 5. Spectral Bands for Wheat Scene.

Band Number	Spectral Wavelength Interval ( $\mu\text{m}$ )
1	0.45-0.54
2	0.54-0.62
3	0.62-0.71
4	0.71-0.85
5	0.85-0.99
6	0.99-1.13
7	1.13-1.34
8	1.45-1.82
9	1.96-2.38

Table 6. Spectral Bands for Combined Scene.

Band Number	Spectral Wavelength Interval ( $\mu\text{m}$ )
1	0.45-0.54
2	0.54-0.62
3	0.62-0.71
4	0.71-0.85
5	0.85-0.98
6	0.98-1.12
7	1.12-1.30
8	1.45-1.82
9	1.96-2.38

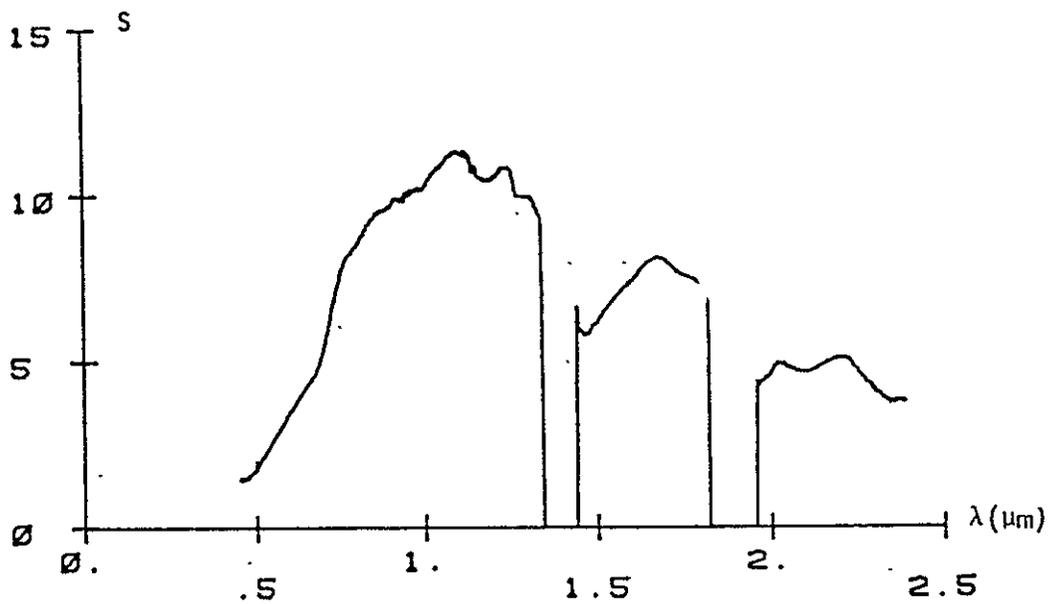


Figure 21. Average Spectral Response -- Wheat Scene.

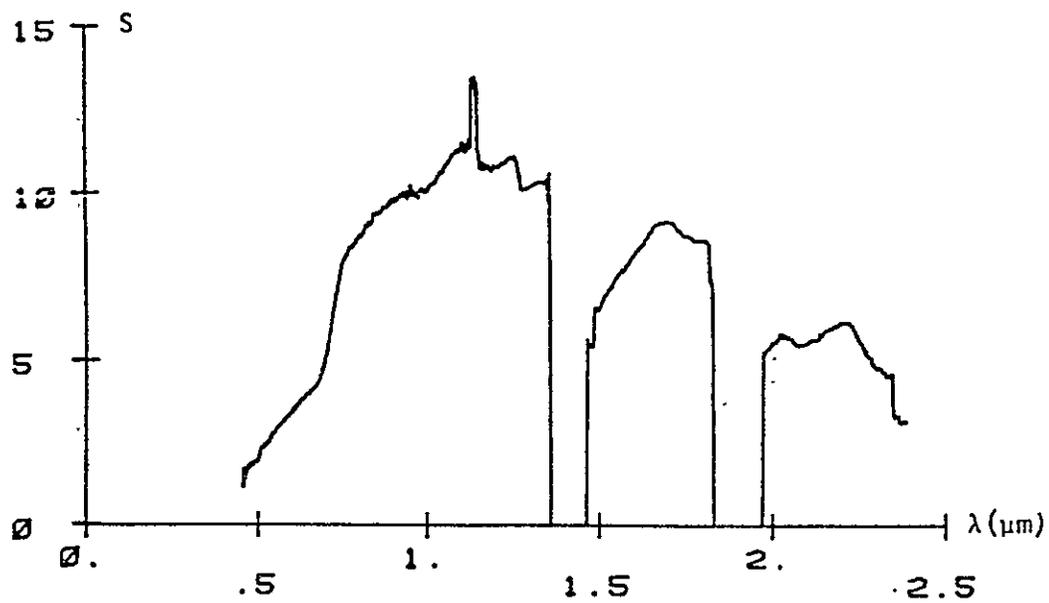


Figure 22. Average Spectral Response -- Combined Scene.

The next step is the identification and validation of models that would adequately describe the aforementioned spectral responses. Three different models were tested and compared, (a) autoregressive (AR), (b) autoregressive plus a constant trend (AR+C), (c) integrated autoregressive (IAR). Following the standard selection and validation techniques, one of the above 3 models is selected which describes the scene most satisfactorily. Tables 7 and 8 show the selected models for the wheat and combined scene respectively, IAR-2 in Table 8 in a second order IAR.

Table 7. Modeling of the Wheat Scene.

Band	Order of Model	Type of Model
1	7	AR
2	2	AR
3	11	IAR
4	1	AR+C
5	1	AR
6	2	AR+C
7	5	IAR
8	8	IAR
9	6	IAR

Table 8. Modeling of the Combined Scene.

Band	Order of Model	Type of Model
1	11	IAR-2
2	2	AR
3	11	IAR
4	1	AR+C
5	3	AR
6	1	AR
7	9	AR+C
8	8	IAR
9	1	AR

### Spectral Band Selection.

It was initially stated that the information content of a set of spectral bands can be used in the selection of an optimum subset. Here, the preceding regression analysis will be used to evaluate the mutual information between the reflected energy and the observed signal in the 9 spectral bands under study.

The first step is the computation of the average information in  $y(k)$ , the received spectral process, about  $s(k)$ . The reflected spectral scene response as a function of spectral bandwidth for each band of both scene types. The average information is computed for several values of the noise variance,  $\sigma_N^2$ . Appropriate software is developed to carry out the calculation of  $I(Y,S)$ . Figures 23 and 24 show the variation of  $I(Y,S)$  in nats for the wheat and combined scene in band 1. Similar plots are shown for the infrared band 7, Figures 25 and 26. Selecting a  $\sigma_n^2 = 10^{-3}$  for demonstration purposes, the average information for wheat and combined scenes are tabulated in Tables 9 and 10 for 9 spectral bands.

Table 9. Average Information for Wheat Scene I

Band	$I(Y,S)$ nats
1	34.50
2	10.52
3	20.35
4	30.00
5	44.96
6	37.20
7	60.31
8	34.80
9	50.10

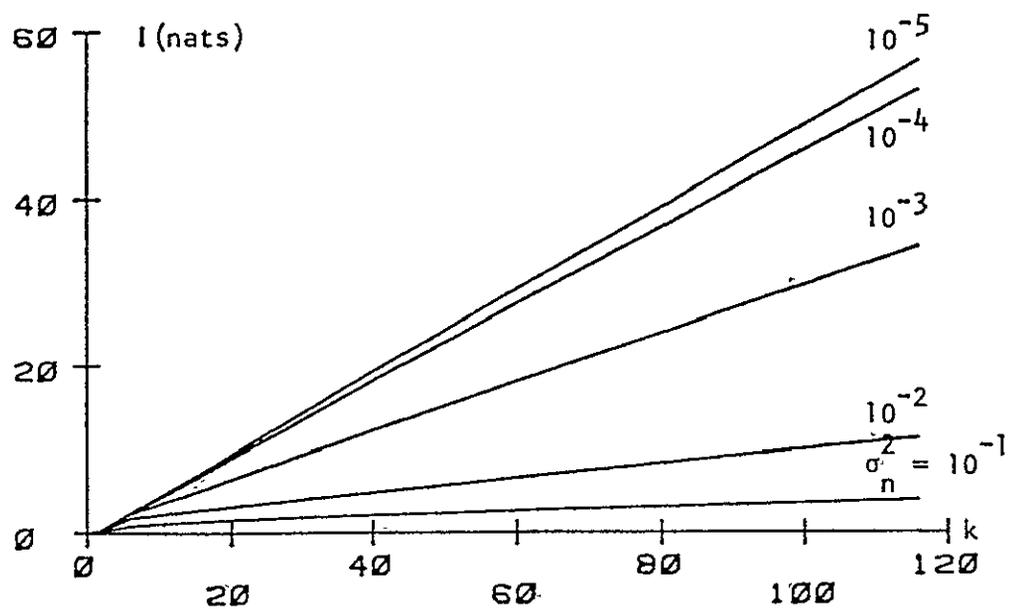


Figure 23. Average Information, Band 1, Wheat Scene.

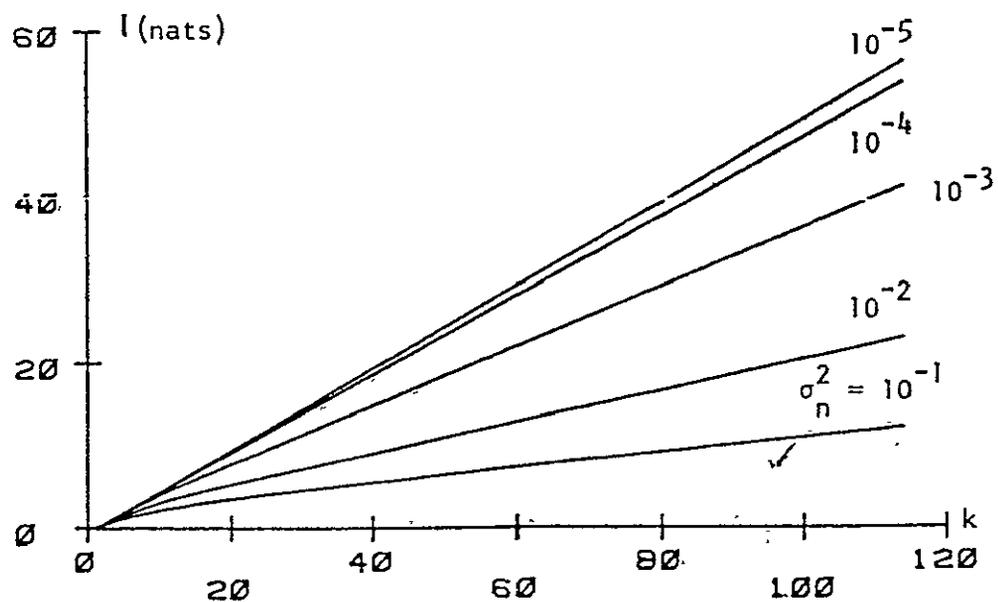


Figure 24. Average Information, Band 1, Combined Scene.

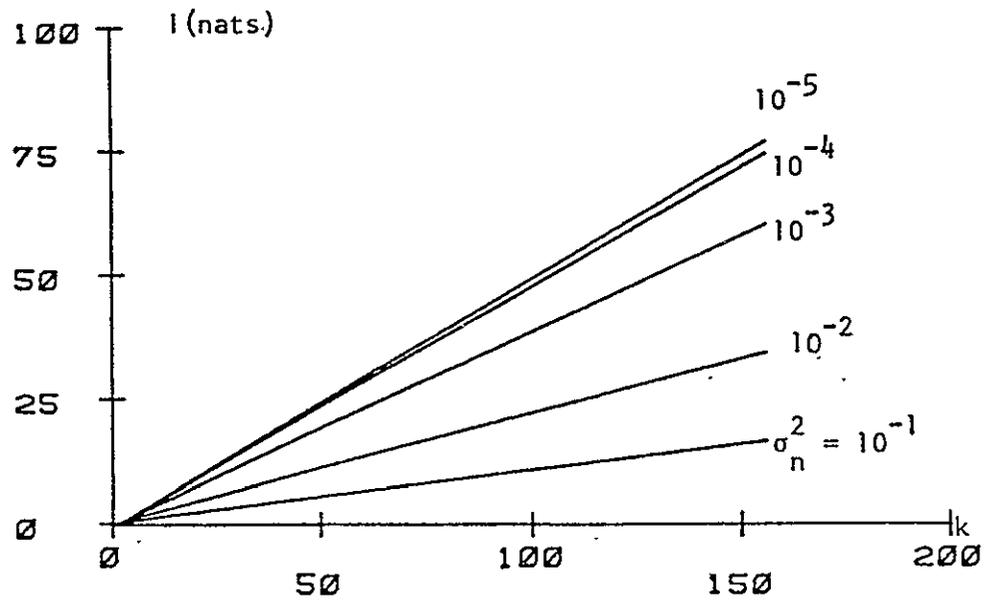


Figure 25. Average Information, Band 7, Wheat Scene.

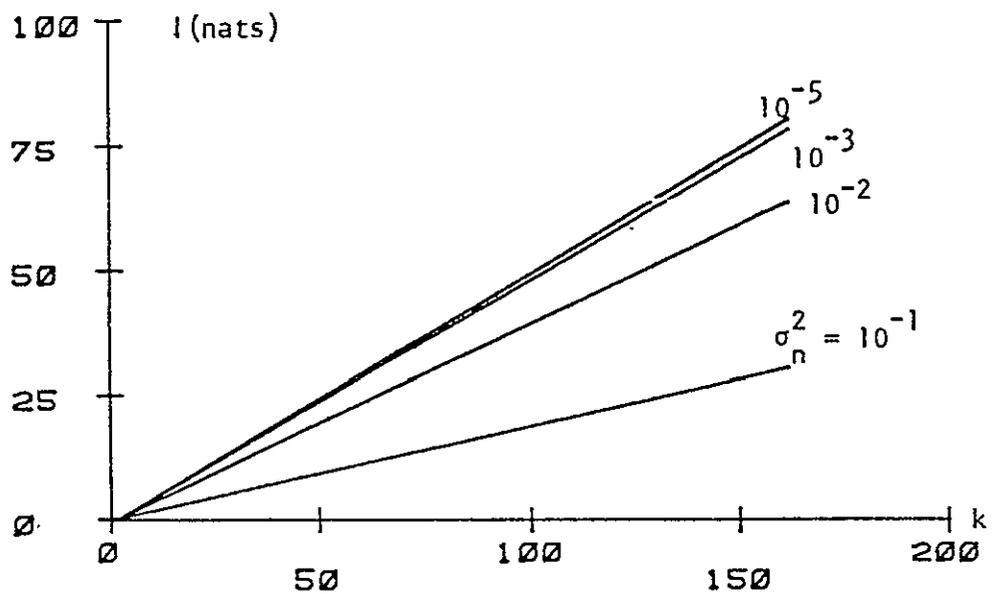


Figure 26. Average Information, Band 7, Combined Scene.

Table 10. Average Information for Combined Scene Band.

Band	I(Y,S) nats
1	41.33
2	16.17
3	22.98
4	40.08
5	45.73
6	40.96
7	78.25
8	64.15
9	74.19

Using the information content of each band as an optimality criterion, the 9 spectral bands can be ranked, see Table 11.

Table 11. Order of Preference of Spectral Bands for the Wheat and Combined Scenes.

Rank	Wheat Scene Band	Combined Scene Band
1	7	7
2	9	9
3	5	8
4	6	5
5	8	1
6	1	6
7	4	4
8	3	3
9	2	2

The top 6 highest ranked bands, although ordered differently, are the same for both cover types. Moreover, other than band 1 which is in the visible portion, the remaining 5 are all in the infrared portion of the spectrum. Thus, relative to the average information criterion, the infrared portions of the spectrum is generally preferred to the visible portion since bands 2 and 3 are ranked lowest for both the wheat scene and combined scene.

The selection of a subset of the available spectral bands using the idea of their information content is a new approach in band selection and requires further investigation to evaluate its optimality in more concrete terms. One of the most useful optimality criterion is the selection of these bands that maximize the overall classification accuracy. No documented relationship exists between the average information contents of a set of bands and the subsequent class separability. It is true however, that such information measure is directly related to the optimum Weiner filter thereby providing a basis for the optimality of this ranking technique.

### 3. THE UNIFIED SCANNER ANALYSIS PACKAGE BLOCK DIAGRAM

The identification and development of a set of individual techniques and algorithms is only the first step toward a complete system simulation package. The usefulness of this package is fully realized only when the elementary modules are interconnected in a logical and clear fashion. The objective here is the integration of the available processors such that starting with a raw data base, the question of optimum spectral bands, IFOV size and the noise model can be answered with the classification accuracy as a primary performance index.

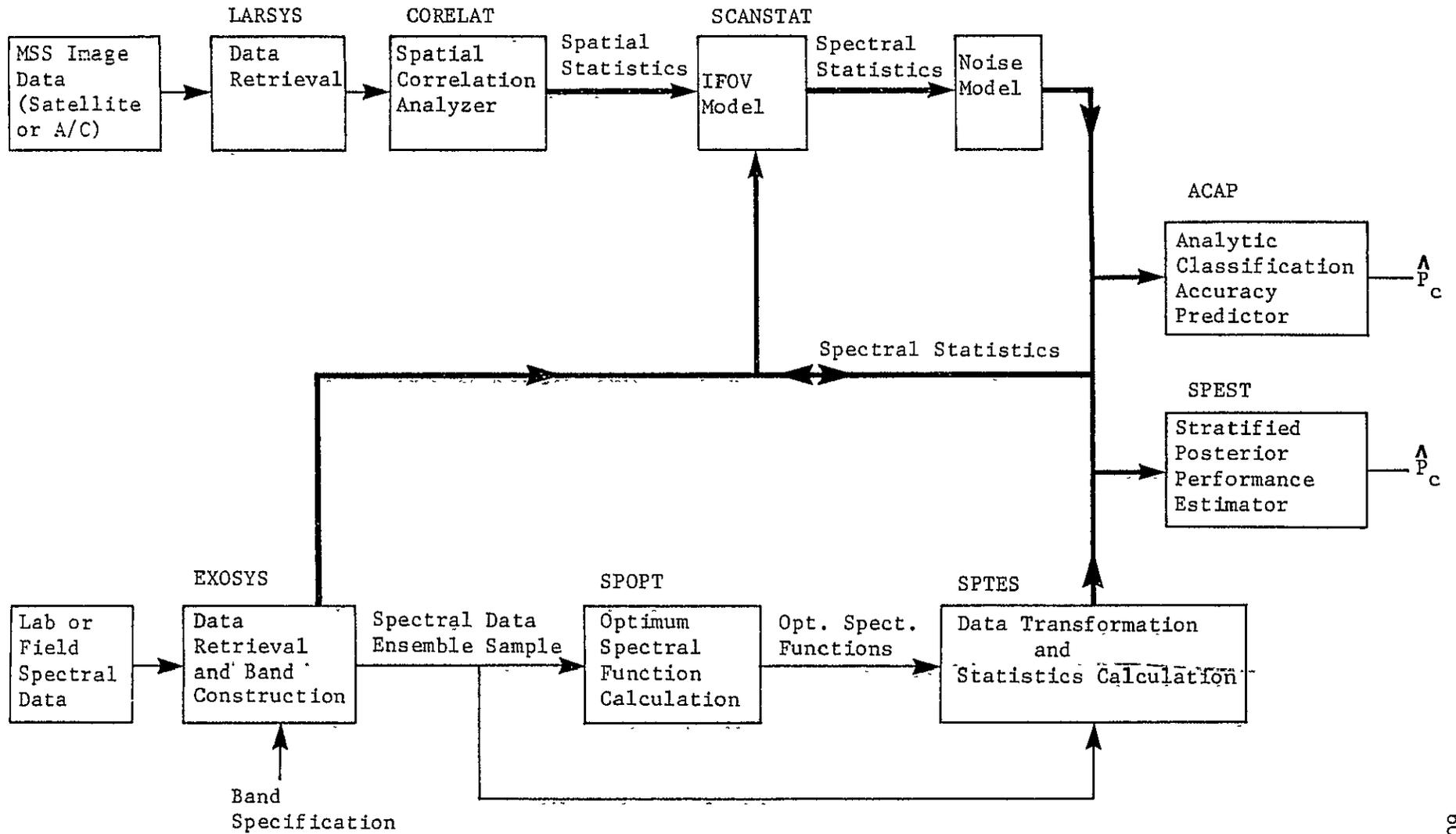
#### 3.1 System Structure

One realization of such simulation model was shown in Figure 4 and is repeated here for convenience in Figure 27. USAP is basically composed of three distinct parts (a) a spatial path, (b) a spectral path and (c) a means for classification performance estimation. In the following individual software modules are discussed.

#### Classification Accuracy Estimators.

There are two classification performance estimators available (a) the analytic classification accuracy predictors and (b) the stratified posterior performance estimator.

Figure 27. The Block Diagram of the Unified Scanner Analysis Package (USAP).



 Implies path for spectral/spatial statistics  
 Implies path for data or spectral functions

Analytic Classification Accuracy Predictor. The ACAP algorithm discussed in Section 2.1 is the primary processor in evaluating the performance of a scanner system when the probability of correct classification is defined as the primary performance index. This piece of software, as shown in its theoretical development, requires one major input in the form of the population statistics. In order to facilitate the operations, the format of the statistics deck is chosen to be identical to the one produced by LARSYS statistics processor although it contains a considerable amount of redundancy such as field coordinates. These cards are skipped. Among other user-supplied information is the desired spectral bands to be used in the analysis and the sampling and grid fineness in the form of number of elementary cells per axis. There is obviously a trade-off between the estimator's variance, a decreasing function of the grid size, and the computation time. If  $N$  is the number of spectral bands and  $n$  the number of cells per axis, the per class number of cells to be tested in a set of  $M$  quadratic discriminant functions is  $n^N$ . This exponential relationship calls for a careful selection of  $n$  particularly for a high dimensional space. On the other hand a small variance is very much required property of any estimator.

The relationship between grid structure and the estimator's variance has been covered in detail [2]. It was shown that the classification accuracy obtained using ACAP exhibits a relative independence from  $n$  for  $n \geq 12$ . This property is preceded by a fairly rapid rise to a steady state value after which the magnitude of the  $\hat{P}_c$  oscillations is within 0.5% of the true value or the Monte Carlo derived reference. The choice of  $n$  is ultimately decided by the user depending on his specific needs and after some experimentation. Initially, however, a default option of  $n = 9$  cells per axis is considered to provide quick turn-around time while keeping the quality of the estimate high. The output, in addition to the classification accuracy estimate, contains information on the transformed class statistics, feature space and sampling grid structure.

Stratified Posterior Classification Performance Estimator. This is the software implementation of the algorithm discussed in Section 2.2. The maximum conditional a posteriori class probability is the criterion for classification and error estimation purposes. The program does not provide any options and the size of the internally generated random data is fixed. ACAP and SPEST produce different, but very close results.

Spatial Path.

Data Base. The input data to the spatial scanner model is via the multispectral image storage tape containing satellite or aircraft collected data. This tape has been reformatted and is compatible with any LARSYS processor.

Data Retrieval. The individual software units can access the available data base through various system support routines or any of the LARSYS processors.

Spatial Correlation Analyzer. The determination of the scanner characteristic function requires a knowledge of the spatial properties of the input data, therefore a class conditional estimate of the spatial auto and crosscorrelation functions is needed. Let  $f_k(x,y)$  be a two dimensional image of size  $N_o \times N_o$  pixels in the  $k$ th spectral band then the spatial autocorrelation function estimate is given by [16].

$$\hat{R}_{kk}(\tau, \eta) = C \sum_{i=1}^{N_o - \tau} \sum_{j=1}^{N_o - \eta} [f_k(i, j) - \mu_k] [f_k(i + \tau, j + \eta) - \mu_k]$$

$$\tau, \eta = 0, 1, \dots, n_o - 1 \quad (54)$$

where  $\mu_k = E\{f_k(x,y)\}$ . The multiplicative factor  $C$  can be chosen to be one of the following

$$C_1 = \frac{1}{(N_o - \tau)(N_o - \eta)} \quad (55)$$

$$C_2 = \frac{1}{N_0}$$

if  $\mu_k$  is known and  $C = C_1$  then  $E\{R_{kk}(\tau, \eta)\} = R_{kk}(\tau, \eta)$ . If  $\mu_k = \hat{\mu}_k$  then neither selection of  $C_1$  or  $C_2$  will produce an unbiased estimate. The actual derivation of the mean and variance of  $\hat{R}_{kk}$  when the mean is estimated is rather complicated. The bias of the estimate in one dimension is given by [16].

$$E\{\hat{R}_k(\tau) - R(\tau)\} = \frac{-|\tau|}{N_0} R(\tau) - \frac{N_0 - |\tau|}{N_0} \text{Var}\{\hat{\mu}_k\} + o(N_0^{-2}) \quad (56)$$

From Eq. (56) it follows that  $\hat{R}_k(\tau)$  is asymptotically unbiased. This result can be extended to the two dimensional functions provided the autocorrelation function is separable along each spatial axis. In general the maximum lag,  $n_0$ , must be chosen such that  $n_0 \ll N_0$ . As a rule of thumb it is desirable to keep the maximum lag less than one-tenth the sample size  $N_0$ . This will tend to avoid certain instabilities that can occur in autocorrelation function estimates. The across-band correlation function estimate is obtained using an identical relationship to Eq. (54).

The empirically obtained spatial correlation matrix needs further processing to be used in the scanner spatial model developed in Section 2.3. Specifically a Markov correlation model is fitted to the experimentally obtained  $\hat{R}_{kk}(\tau, \eta)$ . By invoking the separability assumption for small lag values,

$$\hat{R}(\tau; \eta) \approx \hat{R}(\tau) \hat{R}(\eta) \quad (57)$$

where no subscript indicates either auto or crosscorrelation function. Table 12 shows the magnitude of the errors involved in carrying out this approximation on an aircraft data set. The error is expressed as a percentage of the experimental values. An exponentially dropping function is then fitted to the individual correlation functions along the sample and line directions. The fitting is accomplished using a

Table 12. Error Matrix for Cross Correlation Function Approximation Between Channels 2 and 8.

$$R_{28} = \begin{bmatrix} 1.00 & .92 & .81 & .69 & .59 & .50 & .44 \\ .93 & .88 & .78 & .67 & .56 & .48 & .41 \\ .73 & .71 & .64 & .54 & .44 & .36 & .3 \\ .48 & .47 & .43 & .36 & .28 & .21 & .16 \\ .30 & .31 & .29 & .24 & .18 & .12 & .08 \\ .23 & .25 & .24 & .21 & .16 & .12 & .08 \\ .22 & .24 & .24 & .22 & .19 & .15 & .12 \end{bmatrix}$$

$$\hat{R}_{28} = \begin{bmatrix} 1.00 & .92 & .81 & .69 & .59 & .50 & .44 \\ .93 & .86 & .75 & .64 & .55 & .46 & .4 \\ .73 & .67 & .6 & .5 & .43 & .36 & .32 \\ .48 & .44 & .38 & .33 & .28 & .24 & .21 \\ .30 & .27 & .24 & .2 & .17 & .15 & .13 \\ .23 & .21 & .18 & .16 & .13 & .11 & .1 \\ .22 & .2 & .18 & .15 & .13 & .11 & .1 \end{bmatrix}$$

$$E_{28} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.2 & 3.8 & 4.5 & 1.8 & 4.16 & 2.43 \\ 0 & 5.6 & 6.2 & 7.4 & 2.3 & 0 & 6.75 \\ 0 & 6.4 & 11.6 & 8.3 & 0 & 17.5 & 23.8 \\ 0 & 13.0 & 17.2 & 16.6 & 5.5 & 20 & 38.4 \\ 0 & 16 & 25 & 23.8 & 18.7 & 8.3 & 20 \\ 0 & 16.6 & 25 & 31.8 & 31.6 & 26.6 & 16.6 \end{bmatrix}$$

weighted least square approach to the logarithm of  $R(\tau)$  and  $R(\eta)$ . The slope of this linear fit determines the adjacent sample or line correlation coefficients. Specifically let

$$F(i) = \ln \hat{R}(i) \quad i = 0, 1, \dots, n_o - 1$$

then the parameters of the linear fit,  $C_1 + C_2 x$ , are given by [17]

$$\underline{C} = (\underline{H}^T \underline{W} \underline{H})^{-1} \underline{H}^T \underline{W} \underline{F} \quad (58)$$

where

$$\begin{cases} H(i) = 1 & i = 0, 1, \dots, n_o - 1 \\ H(i) = 2n_o - 1 - i & i = n_o, n_o + 1, \dots, 2n_o - 1 \end{cases} \quad (59)$$

and the diagonal weighting matrix,  $W$

$$W(i) = \alpha^{(n_o - 1 - i)} \quad i = 0, 1, \dots, n_o - 1 \quad (60)$$

with  $\alpha$  as the weighting matrix diagonal base,  $0 \leq \alpha \leq 1$ . The weighting matrix via the control parameter assigns a progressively smaller weight to  $\hat{R}(\tau, \eta)$  for succeeding lag values. This weighting is necessary since the properties of the correlation functions show an increasing deviation from the underlying assumptions of separability and Markovian structure for higher lag values.

A complete specification of the spatial properties of the available spectral classes requires determination of

$$N + \frac{N!}{2! (N-2)!}$$

auto and crosscorrelation functions per spatial axis per class where  $N$  is the total number of spectral bands used in the analysis. The implementing software contains various default provisions in case the corre-

lation properties of the input data differs considerably from the aforementioned assumptions.

The user specifies the area to be correlated by a run table entry run number followed by the field coordinates. In order to perform either the auto or crosscorrelation operations, appropriate spectral bands(s) need to be specified. The maximum lag,  $n_0$ , in computing  $\hat{R}(\tau, \eta)$  is also a variable and is entered as a percentage of the image size in pixels. The value of  $n_0$  is dependent on the size of the area to be correlated. Since the magnitude of  $\hat{R}(\tau, \eta)$  for Landsat data is generally negligible for more than 4 or 5 pixels lag,  $n_0$  as a percentage can take on small values for large fields and vice versa.

Scanner IFOV Model. The scanner IFOV software is the computer implementation of the scanner characteristic function discussed in Section 2.3. The input consists of (a) spectral covariance matrix, (b) spatial correlation matrix along the samples (c) spatial correlation matrix along the lines and (d) IFOV size in terms of the number of high resolution pixels. A standard LARSYS statistics deck produced by the statistics processor constitutes the first item. The spatial correlation information is entered through an  $N \times N$  symmetric matrix the  $(i, j)$  element of which

$$\rho_{ij} = e^{-a_{ij}}$$

is the pixel-to-pixel correlation for bands  $i$  and  $j$ .  $a_{ij}$  is estimated by the spatial correlation software. The IFOV size is expressed on a relative scale in terms of the number of high resolution pixels within 1 IFOV of the scanner PSF, e.g., 1, 2, 3, etc. There are two choices available for the functional form of the PSF, Gaussian and rectangular.

The output generated by this software module is a spectral statistics deck which is the input class statistics transformed by the scanner characteristic function. This deck is used as input to the ACAP processor.

Additive Noise Model. By virtue of the parametric approach adopted here, the incorporation of the noise effect takes on a simple form. The noise statistics is characterized by a zero mean vector and a diagonal covariance matrix with zero off diagonal elements. This matrix is then simply added to each class covariance matrix,

$$\sum_{-f} = \sum_{-f} + \begin{bmatrix} \sigma_{n_1}^2 & & & & \\ & \sigma_{n_2}^2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \sigma_{n_N}^2 \end{bmatrix} \quad (62)$$

The diagonal elements of  $\sum_{-N}$ ,  $\sigma_{n_i}^2$ , determines the SNR in each spectral band. By the appropriate selection of  $\sigma_{n_i}$ 's, different SNR can be specified for each band. Let  $\sigma_{f_k}^2$  denote the variance of the noise-free signal at the scanner output, then the SNR in the kth spectral band is defined by

$$(\text{SNR})_k = \sigma_{f_k}^2 / \sigma_{n_k}^2 \quad (63)$$

The choice of equal or unequal SNR in different bands based on experimental or theoretical results is at the analyst's discretion.

### Spectral Path.

Data Base and Retrieval. EXOSYS is a software package which provides access to field measurement data taken with a variety of field instruments. A brief overview of the EXOSYS package will be given here, with more detail available in the EXOSYS manual [18]. Data is collected and stored on magnetic tape in field measurements format. During the reformatting procedure the data is calibrated and ancillary information such as

weather readings soil conditions, and plant growth status is placed in the identification record for each run.

There are three processors in EXOSYS which are used to access the field data information - IDLIST, GSPEC, and DSEL. The IDLIST processor scans the tape and lists information from the identification record as required. One can use this information to select appropriate runs to represent informational classes.

The GSPEC processor provides a punched deck consisting of the numerical values of the spectral response function for all of the desired runs. One can select a set of run numbers as input and the output will consist of a punched deck. Options exist for plotting the spectral response functions for the desired runs.

The DSEL processor simulates rectangular spectral channels and uses data from the tape to evaluate the response in each channel for the ensemble. The inputs are the spectral band locations and the run numbers on the data tape. The output is a set of statistics for the specified channels.

Optimum Spectral Basis Function System. For the optimum spectral function calculation the output of the GSPEC processor is required. The appropriate ensemble can be selected by specifying a set of identification parameters such as date of collection, scene type, run number, etc.

The cards containing the numerical values for the spectral response functions are used and stored on disk in a format which is compact and convenient for future processing by the program SPRDCT. The files that are stored on disk may be transferred to magnetic tape for future use to avoid repeating the procedure involving the EXOSYS package. SPRDCT requires some information to be entered at the terminal to provide ID information for the ensemble. A list of all runs used by run number is printed after the data has been stored on disk.

Optimum Basis Function Calculation. The calculation of the optimum set of basis functions for an ensemble is accomplished by solving the matrix equation

$$\phi\Gamma = KW\phi \quad (64)$$

to get the eigenvalues  $\sigma_1, \sigma_2, \dots, \sigma_N$  and the eigenvectors  $\phi_1, \phi_2, \dots, \phi_N$ . The matrix  $\phi$  is the matrix of eigenvectors  $\phi = [\phi_1, \phi_2, \dots, \phi_N]$  and  $\Gamma$  is the diagonal matrix of eigenvalues

$$\Gamma = \begin{bmatrix} \sigma_1 & 0 & & \\ 0 & \sigma_2 & & \\ & & \cdot & \\ & & & \cdot \\ & & & & \sigma_N \end{bmatrix} \quad (65)$$

The matrix  $W$  is a diagonal matrix of weights

$$W = \begin{bmatrix} W_1 & 0 & & \\ 0 & W_2 & & \\ & & \cdot & \\ & & & \cdot \\ & & & & W_N \end{bmatrix} \quad (66)$$

$R$  is the covariance matrix for the ensemble. Let the mean vector for the ensemble be  $m = [m_1, m_2, \dots, m_N]^T$  then

$$R_{ij} = E \{ (x_i - m_i) (x_j - m_j)^T \} \quad (67)$$

The maximum likelihood estimate is

$$\hat{R}_{kj} = \frac{1}{N_S - 1} \sum_{k=1}^{N_S} (x_{ik} - m_i) (x_{jk} - m_j)^T \quad (68)$$

where  $N_s$  is the number of sample functions in the ensemble.

If we let  $A=KW$ , then  $A$  is a real general matrix. An algorithm for solving for the eigenvalues and eigenvectors of a real general matrix is available [19] and is used here with only slight modification. The algorithm makes use of Householder's method and the QR double-step iterative process to compute the eigenvalues. The eigenvectors are obtained by the inverse iteration technique. A sorting routine was added to order the eigenvalues and the corresponding eigenvectors.

The required inputs are the data in the appropriate format and the weight function. The output of this processor is a set of  $N$  eigenvectors or basis functions punched onto cards. Also, the eigenvalues and mean-square representation error are printed. The eigenvectors can be plotted using GCS subroutines.

Data Transformation and Statistics Calculation. The eigenvectors are used to perform a linear transformation on the original data vectors  $\{X\}$ . The transformed vectors  $\{Y\}$  have the desired properties provided by the Karhunen-Loève expansion. Each element of the transformed vector is given by

$$y_i = \phi_{i1}x_1 + \phi_{i2}x_2 + \dots + \phi_{in}x_n \quad (69)$$

where  $\phi_{ij}$  is the  $j$ th element of the  $i$ th eigenvector.

The inputs to this processor are the eigenvectors and the data set stored on the disk. The output is the set of statistics for each class. The statistics are printed and punched on a deck of cards for future processing.

#### 4. USER'S GUIDE TO USAP

The block diagram of the scanner parameter study, Figure 27, is made operational by a collection of compatible software packages. Each module is individually compatible with the LARSYS environment facilitating

incorporation of LARSYS processor in the overall system performance evaluation. This section provides a guide for the acquisition and execution of each program available on the LARS IBM 370/148 computer system.

Prior to a discussion of the individual modules some general remarks are in order. The access to this program library is simplified by the allocation of two special disk storage devices designated by DHSYS and DHDSK. The former contains the text version of the software while the latter holds the source. These devices can be accessed using the appropriate GETDISK commands.

GETDISK DHSYS

and

GETDISH DHDSK

these commands will establish the proper links in a Read-Only mode and USAP initialization is complete. In the following subsections the required input and necessary steps to run each program are discussed.

#### 4.1 The Classification Accuracy Estimators

There are two parametric classification accuracy estimators available to the USAP user, (a) the analytic classification accuracy predictor (ACAP) and (b) the stratified posterior classification accuracy estimator (SPEST). The theoretical aspects of these processors have been discussed in Section 2. Here is a guide to their software implementation.

##### Analytic Classification Accuracy Predictor

This program evaluates the performance of a Bayes classifier when the populations statistics are multivariate normal. The following control cards are required.

<u>Control Word</u>	<u>Description</u>
*ACAP	This card specifies the particular processor requested.
CHANNELS	The desired subset of the available channels is given here. Note that the numbers appearing on this card are the order of the selected channels not their actual number. For example, if the available channels are 8, 9, 12, 14 and channels 8, 9 and 14 are requested CHANNEL card should read 1, 2, 4.
CLASSES	This card specifies the name of each class. Each name must be placed in a field 7 characters long followed by a blank. The continuation card, if required, must have the word "CLASSES" at the beginning.
GRID	This quantity controls the quality of the estimate. The higher the number the closer the estimate is to the true Bayes error rate. (See 'estimated CPU time' for more details.)
END	This card signals the end of the control card. Stat deck follows immediately.

Remarks. The ACAP processor in its current form is capable of handling up to 20 classes and 8 spectral bands. The extensions of these parameters presents no conceptual difficulty. The required statistics deck is a standard LARSYS produced deck with no modifications. It must be punched in the character format, however.

How to Run the Program. Make sure the DHSYS disk has been accessed properly. One reader file consisting of the control cards followed by a statistics deck is required. Type ACAP in the CMS environment. Appropriate terminal and printer output is produced.

Example of control card set up

```
*ACAP
CHANNELS 1,3,4
CLASSES SOYBEAN ALFALFA WHEAT
END
```

Since GRID card is not included, its default value (9) is selected.

Estimated CPU Time. The execution time is quite sensitive to the GRID card specifying the number of cells per axis. For the default grid size and a 4-dimensional space it takes approximately 2 minutes of CPU time per class to provide the requested classification accuracy estimates. The CPU time is most sensitive to the dimensionality of the feature space. Hence if the number of spectral bands is limited (less than 4) considerable increase in GRID number is possible. The default number of cells per axis is considered to be the minimum while still providing acceptable performance. Increasing the parameter improves the quality of the estimate somewhat at the expense of higher CPU time. The choice is left at the user's discretion.

Stratified Posterior Error Estimator.

This program is identical in purpose but different in approach to the ACAP processor. Given a set of multivariate normal populations, SPEST provides the classification accuracies associated with each class using an internally generated random data base. The different estimation procedures between the two methods is transparent to the user.

Description of the Control Cards

<u>Control Word</u>	<u>Description</u>
*SPEST	This card specifies the particular processor requested.
CHANNELS	The desired subset of the available channels is given here. Note that the numbers appearing on this card are the order of the selected channels not their actual number.
CLASSES	This card specifies the name of each class. Each name must be placed in a field 7 characters long followed by a blank. The continuation card, if required, must have the control word "CLASSES" in the beginning followed by the rest of the names.
END	End of the control cards.

Remarks. In usage, this program is identical to \*ACAP. The standard LARSYS statistics deck follows the control and disk immediately. Printer output contains the estimated conditional classification accuracies. By virtue of their separate approaches, \*ACAP and \*SPEST provide different, but very close, estimates of the correct classification accuracies.

How to Run the Program. The reader file contains the control cards followed by the LARSYS statistics deck. A sample control card deck follows:

```
*SPEST
CHANNELS 1,2,4
CLASSES ALFALFA SOYBEAN WHEAT
END
```

## 4.2 Spatial Path

The spatial path in USAP consists of two main software units. The spatial correlation analyzer, CORELAT and the SCANNER IFOV model, SCANSTAT.

Spatial Correlation Modeling. This program is a 2-dimensional spatial correlator the primary output of which is a normalized spatial auto (cross) correlation matrix for any specified area. The user specifies the coordinates of the desired segment in the form of an initial and final line and column along with the appropriate spectral bands(s). Following the estimation of the correlation matrix, the exponential fit option, if invoked, will fit an exponentially dropping function to the experimental values of  $R_{kk}(\tau)$  or  $R_{kk}(\eta)$  using a weighted linear least squares technique.

## Description of the Control Cards

<u>Control Word</u>	<u>Control Parameter</u>	<u>Description</u>
*CORRELATE		This card specifies the particular processor requested.
INPUT	RUN(.) TAPE(.) FILE(.)	Run number of the desired area. Tape number of the desired area. File number of the desired area.
BLOCK	LINE(.,.) COLUMN(.,.)	Initial and final lines. Initial and final columns.
FUNCTION	AUTO CROSS	Autocorrelation function requested. Crosscorrelation function requested.
CHANNELS		Channels used for correlation operation.
SAMPLELAG†		Maximum cross track lag used as a percentage of the total number of samples.
LINELAG†		Same as SAMPLELAG except for along track lag.
EXPOFIT†		If included exponential fitting operation is carried out.
END		End of control cards.

Remarks. This program is capable of processing areas containing up to 2400 pixels. The maximum lag default is set at 20 percent of the total number of lines and columns. Both quantities can be altered by user supplied control cards. The exponential fit option provides a pixel-to-pixel correlation coefficient for the channel(s) specified. This number is computed from the estimated parameters of the exponential correlation model.

How to Run the Program. The only required reader file is the control card deck, an example of which follows:

---

† optional

```

*CORRELATE
INPUT RUN (74028500), TAPE (2689), FILE (3)
BLOCK LINE (1,25), COLUMN (1,25)
FUNCTION AUTO
CHANNELS 2
SAMPLELAG† 25
LINELAG† 25
EXPOFIT†
.END

```

after DHSYS disk has been properly linked to, type CORELAT to start execution. Appropriate terminal and printer output is generated.

### Scanner IFOV Model.

This program computes the spectral statistics of a population at the output of a multispectral scanner provided the data spatial correlation approximately follows a Markov model. The scanner IFOV shape is limited to either a Gaussian or rectangular shape. No assumptions are made or indeed required about the type of the population statistical distribution.

### Description of the Control Cards

<u>Control Word</u>	<u>Description</u>
*SCANSTAT	This card specifies the particular processor requested.
CHANNELS	The desired subset of the available channels is given here. Note that the numbers appearing on this card are the order of the selected channels not their actual number.
CLASSES	This card specifies the name of each class. Each name must be placed in a field 7 characters long followed by a blank. The continuation card, if required, must have the control work "CLASSES" in the beginning followed by the rest of the names.
IFOV	This card specifies the IFOV size of the scanner in terms of high resolution input pixels.

---

† optional

APERTURE	The choices here are "GAUSSIAN" or "RECTANGULAR."
SNR†	Output signal energy to noise energy in dB.
PUNCH†	The output statistics is punched out in an ACAP/SPEST format. Redundancies are added to replace field description cards.
END	End of control cards.

Remarks. This program is limited to 20 classes and 8 spectral bands. Execution time is quite short and extension of those parameters is straightforward. The input data immediately following the control cards consists of 3 parts:

1. Standard LARSYS statistics deck in character format.
2. Spatial correlation parameters (cross track) are entered via a NXN symmetric matrix where N is the number of channels. The (i,j) element of this matrix is the adjacent sample correlation between channels i and j. The lower triangular part of this matrix is punched in a 5 (E13.7,1X) format.
3. Spatial correlation parameter matrix except for along track pixels.

The above decks follow the control cards in the order listed. The signal-to-noise ratio is defined as the ratio of the output signal energy in a particular channel (diagonal element of the class covariance matrix) to the noise energy in the same bands expressed in dB and defined by

$$(\text{SNR})_k = 10 \log_{10} \frac{\sigma_{s_k}^2}{\sigma_{n_k}^2}$$

where k refers to the particular spectral band.

---

† optional

How to Run the Program. One reader file consisting of 4 consecutive decks and appropriate link to DHSYS disk is required before the program execution. An example of a control card set up follows:

```
*SCANSTAT
CHANNELS 1,2,4
CLASSES ALFALFA SOYBEAN WHEAT
IFOV 2
APERTURE GAUSSIAN
SNR† 10
PUNCH†
END†
```

The output consists of an ACAP/SPEST compatible statistics deck for the modified population. This deck can be used in the ACAP/SPEST processors to obtain the new set of classification accuracy estimates.

#### 4.3 Spectral Path

The spectral path in USAP consists of three main pieces of software (a) data retrieval through EXOSYS processor (b) optimum spectral function calculation and (c) data transformation and statistics calculation.

##### Procedure for Computing and Evaluating Optimum Basis Functions.

Data Retrieval. The data retrieval system is stored on EXOSYSDV and it is necessary to define storage as 768K. A card file containing the data points for each run will be constructed on a temporary disk (25 cylinders). It is desirable to make the temporary disk a P-disk and the permanent user disk a T-disk.

In CMS

```
RELEASE 191 P
LOGIN 191 T
GETDISK TEMP MODE P25CYL NO-UFD
```

---

† optional

In CP

```
I EXOSYSDV
CCINPUT TERMINAL
RUN EXOSYSDV
```

The control cards will be entered through the terminal. The tape on which the data is stored must be specified as well as the cover type and the collection date. A typical sequence of cards is as follows:

```
$ TAPE 4896
$ GSPEC
  GRAPH SPEC (SPRING WHEAT), DACO (770508)
  LIST NO LIST
  OPTIONS PUNCH, NOGRAPH
  END
$ END
$ EXIT
```

The runs taken over Williams Co., North Dakota for May 8, 1977 are on tape 4896. The crop species is spring wheat and the collection date is May 8, 1977. The output is a deck of cards with one hundred data values for each run punched onto the cards. Header information must be flushed at the line printer. Return the system to CMS and read the cards onto a disk file SPR100 DATA. The number of records in the file is equal to the number of runs. This number should be recorded as it will be needed later. This procedure is repeated for the second class. The cards from GSPEC are read onto the file INPUT DATA and the number of records recorded. The two files are combined by typing (in CMS)

```
COMBINE SPR100 DATA P1 SPR 100 DATA P1 INPUT DATA P1
ERASE INPUT DATA
```

This procedure is repeated until all classes have been included in the file SPR 100 DATA. The crop species SUMMER FALLOW and PASTURE are used in GRAPH SPEC(.) to complete the data set.

At this point the program SPRDCT is loaded and run. A disk file will be created using DSRN of 2 and file type FUNC. The following information is requested at the terminal by SPRDCT

Experiment Number				100
Number of Classes				3
Number of Sample Points per Run				100
(Dimensions)				
Class Name	Wheat	Fallow	Pasture	
Number of Samples per Class	664	437	164	

The information is requested and is entered between the slash marks, right justified.

At this point the data is ready to be used by the system. It is a good idea to store the file on tape for future use. Type

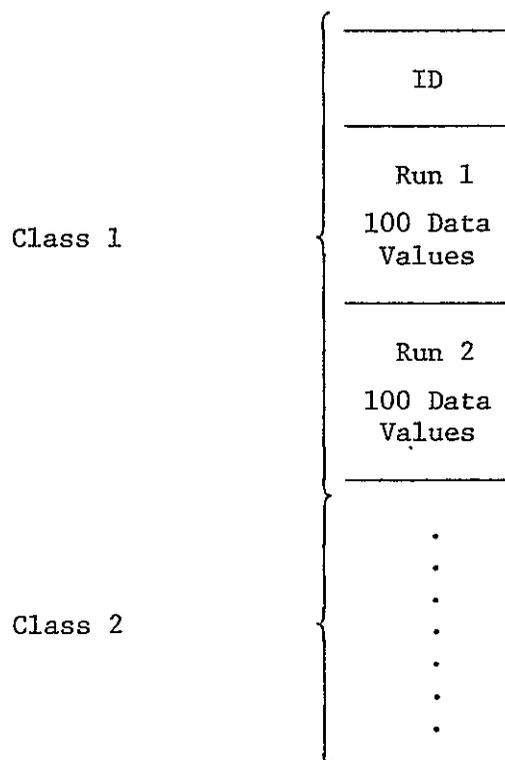
```
TAPMOUNT 156 TAP2 RI
T DUMP SPR 100 FUNC P1
```

The tape on which the data is stored is 156 in this example. To recall the data to the disk type

```
TAPMOUNT 156 TAP2 RO (If not already mounted)
T SLOAD SPR 100 FUNC
```

Note that the P-disk must be a large fairly empty disk (10 cyl)

The format for data storage is



## ID Information Record

<u>Item</u>	<u>Words</u>
Date	1-15
Exp. Number	16
Number of Classes	17
Number of Dimensions	18
Number of Samples for Class 1	21
Number of Samples for Class 2	22
Number of Samples for Class 3	23
Number of Samples for Class 4	24
Number of Samples for Class 5	25
Number of Samples for Class 6	26
Number of Samples for Class 7	27
Name of Class 1	30-39
Name of Class 2	40-49
Name of Class 3	50-59
Name of Class 4	60-69
Name of Class 5	70-79

Name of Class 6	80-89
Name of Class 7	90-99

#### Optimum Spectral Functions Calculations.

Once the data set is on disk it is necessary to issue the following CMS commands to compute the eigenvectors.

```
FILEDEF 2 DSK-P4 SPR100 FUNC RECFM VS LRECL 400 BLKSIZE
400 (PERM
CP SP PUN TO USERID
LOAD SPOPTM (XEQ
```

#### 4.4 Example Outputs

This section presents a sample output for the individual software units used in USAP. The example set consists of sample outputs from the ACAP and SPEST processors. CORELAT and SCANSTAT in the spatial path and SPOPT and SPTEs in the spectral path. Graham Co., Kansas is used as the test site.

#### Classification Accuracy Estimators.

The following control card set up is used for the ACAP processor and output is shown in Table 13.

```
*ACAP
CHANNELS 1,2,3,4
CLASSES BARES01 CORN SOYBEAN WHEAT
END
```

The required control cards for the SPEST processor are

```
*SPEST
CHANNELS 1,2,3,4
CLASSES BARES01 CORN SOYBEAN WHEAT
END
```

the output is shown in Table 14.

Table 13. \*ACAP Sample Output.

ANALYTIC CLASSIFICATION ACCURACY PREDICTION - A C A P

CLASS WHEAT

SAMPLING GRID CHARACTERISTICS

GRID SIZE= 9 CELLS PER DIMENSION

TOTAL NO OF CELLS IN THE GRID= 6561

TRANSFORMED FEATURE SPACE CHARACTERISTICS

EIGENVALUES

2.9182E 01    1.1413E 01    1.3430E 00    7.6125E-01

EIGENVECTORS

2.1045E-01	2.0478E-01	8.2319E-01	4.8593E-01
4.5844E-01	8.2659E-01	-1.5239E-01	-2.8873E-01
-6.7334E-03	1.9331E-01	-5.3271E-01	8.2390E-01
8.6342E-01	-4.8728E-01	-1.2389E-01	4.1283E-02

TRANSFORMED MEAN VECTORS

1.1654E 01	2.3928E 01	3.8200E-01	-5.4750E-02
-2.2766E 00	1.5248E 01	1.3871E-01	1.0838E-01
1.3111E 01	5.1333E 00	-5.9814E-01	1.0702E 00
0.0	0.0	0.0	0.0

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PROBABILITY OF CORRECT CLASSIFICATION = 93.486 (

\*\*\*\*\*TOTAL PROB OF CORRECT CLASSIFICATION\*\*\*\*\*= 89.287 PERCENT

Table 14. \*SPEST Sample Output.

STRATIFIED POSTERIOR ERROR ESTIMATOR

CLASS BARESOI

PROBABILITY OF CORRECT CLASSIFICATION = 79.2180

CLASS CORN

PROBABILITY OF CORRECT CLASSIFICATION = 92.7210

CLASS SOYBEAN

PROBABILITY OF CORRECT CLASSIFICATION = 96.3590

CLASS WHEAT

PROBABILITY OF CORRECT CLASSIFICATION = 92.6140

OVERALL PROBABILITY OF CORRECT RECOGNITION = 90.228

Spatial Path.

The control card set up for CORELAT is as follows:

```
*CORRELAT
INPUT RUN(74028500), TAPE(2689), FILE(3)
BLOCK LINE(50,98), COLUMN(50,98)
FUNCTION AUTO
CHANNELS 1
EXPOFIT
END
```

The sample output is shown in Table 15.

The control card set up for SCANSTAT is as follows:

```
*SCANSTAT
CHANNELS 1,2,3,4
CLASSES BARESOI CORN SOYBEAN WHEAT
IFOV 2
APERTURE GAUSSIAN
END
```

The sample output is shown in Table 16.

Spectral Path.

The optimum basis function calculation and computation of the transformed data statistics comprises the main spectral processors. The sequence of required commands has been shown in Section 4.3. The following example is a weighted basis function calculation.

The weighting function  $w(\lambda)$  is zero for the water absorption bands near 1.4 and 1.8 micrometer and zero elsewhere on the interval (.4-2.4  $\mu\text{m}$ ). The printer output is shown in Table 17, listing the first 30 eigenvalues. The first 4 eigenvectors are sent in card format to the reader. They can be stored on the disk by issuing the command.

```
0 READ EIG100 DATA T1
```

Table 15. \*CORRELAT Sample Output.

TWO DIMENSIONAL SPATIAL CORRELATION ANALYSIS

CHANNELS 1 1

2-D SPATIAL CORRELATION MATRIX

1.00	0.75	0.50	0.36	0.28	0.23	0.19	0.13	0.08
0.70	0.60	0.44	0.33	0.26	0.20	0.17	0.12	0.07
0.50	0.45	0.36	0.27	0.21	0.17	0.14	0.10	0.05
0.38	0.37	0.31	0.24	0.17	0.13	0.11	0.08	0.03
0.31	0.32	0.27	0.21	0.14	0.11	0.10	0.08	0.04
0.25	0.26	0.23	0.16	0.11	0.09	0.07	0.05	0.02
0.20	0.20	0.18	0.13	0.10	0.07	0.06	0.05	0.03
0.14	0.15	0.14	0.10	0.07	0.05	0.04	0.04	0.02
0.10	0.11	0.12	0.11	0.09	0.06	0.03	0.02	0.00

WEIGHTED LEAST SQUARES FIT INFORMATION

WEIGHTING MATRIX DIAGONAL BASE=0.40  
WEIGHTED LSF ERROR (CROSS TRACK)= 0.1037569E-02  
ADJACENT SAMPLE CORRELATION= 0.7193716E 00  
WEIGHTED LSF ERROR (ALONG TRACK)= 0.6734634E-05  
ADJACENT LINE CORRELATION= 0.7026460E 00

Table 16. \*SCANSTAT Sample Output.

SCANNER OUTPUT STATISTICS

APERTURED GAUSSIAN

IFOV SIZE 2 HIGH RESOLUTION PIXELS

CLASS CORN

INPUT COVARIANCE MATRIX

9.29			
12.26	19.79		
10.63	16.37	16.09	
4.43	7.14	6.24	3.45

OUTPUT COVARIANCE MATRIX

4.86			
4.61	10.37		
4.00	6.15	8.43	
1.67	2.68	2.34	1.81

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LABORATORY FOR APPLICATIONS OF REMOTE SENSING  
 PUDDUE UNIVERSITY  
 SAMPLE FUNCTION INFORMATION 10 JULY, 1978

EXP. NO. 1  
 NUMBER OF CLASSES.....100  
 CLASS.....WHEAT  
 NUMBER OF SAMPLE FUNCTIONS.....664  
 CLASS.....FALLOW  
 NUMBER OF SAMPLE FUNCTIONS.....437  
 CLASS.....PASTURE  
 NUMBER OF SAMPLE FUNCTIONS.....164

## WEIGHTING FUNCTION NUMBER 3

N	EIGENVALUE	VAR (GAM)	VAR (PHI)	MEAN-SQUARE ERROR
1	311.4133	307.0752	0.0086	299.156749
2	229.3520	166.5619	0.0087	64.804748
3	21.1702	1.4191	0.0104	42.634554
4	15.4660	0.7574	0.0126	33.168544
5	8.8838	0.2499	0.0106	24.284784
6	5.7765	0.1057	0.0129	18.508269
7	3.5611	0.0402	0.0237	14.947187
8	2.6887	0.0229	0.0573	12.258489
9	2.3128	0.0154	0.0635	9.945677
10	1.8363	0.0107	0.0444	8.109393
11	1.4195	0.0064	0.0663	6.589860
12	1.2304	0.0048	0.0843	5.454494
13	0.9300	0.0027	0.0336	4.524464
14	0.6806	0.0015	0.0338	3.848328
15	0.5217	0.0009	0.0351	3.327086
16	0.3517	0.0004	0.1206	2.975436
17	0.3138	0.0003	0.3414	2.601549
18	0.2945	0.0003	0.4782	2.307111
19	0.2771	0.0002	0.3430	2.090019
20	0.2336	0.0002	0.2146	1.856411
21	0.2102	0.0001	35.7262	1.646236
22	0.2092	0.0001	35.7220	1.437049
23	0.1792	0.0001	0.1469	1.257806
24	0.1514	0.0001	0.5298	1.106359
25	0.1452	0.0001	0.5206	0.961120
26	0.1137	0.0000	0.0443	0.847458
27	0.0976	0.0000	0.1201	0.749854
28	0.0851	0.0000	0.1318	0.664785
29	0.0744	0.0000	0.1153	0.540391
30	0.0583	0.0000	0.9213	0.532112

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Table 17. Eigenvalue and Mean-Square Representation Error for the Data Set.

A GCS routine was used to plot the graphs of the eigenvectors the first of which are displayed in Figures 28 through 31.

The statistics for the first 4 terms are computed by using the same FILEDEF command as above plus

```
CP SP PUN TO USERID
O PUNCH EIG100 DATA TI
LOAD SPTES (XEQ)
```

The program will ask 'NUMBER OF TERMS?', to get all 4 terms type '4.'

The output will be a statistics deck with the following format:

Card

```
1      Number of Classes, Number of Terms
2      Apriori Probabilities for each Class = 1/Number of Classes
      Mean Class 1 [Format (20A4)]
      Covariance Matrix Class 2
      Mean Class 2
      .
      .
      .
```

The covariance are in upper triangular form. This statistics deck can be used as the input to the classification error estimator algorithms. Table 18 is a sample of the statistics obtained from the data set using the first 4 optimum basis functions. Also, the statistics were used as input to the classification performance estimator \*SPEST to get an estimate of the probability of correct classification.

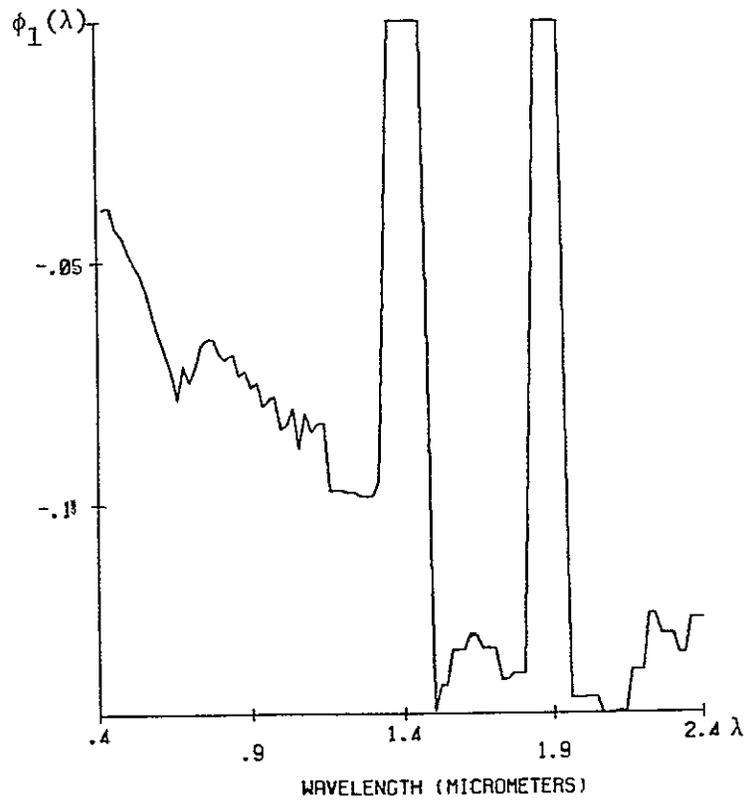


Figure 28. Eigenvector 1.

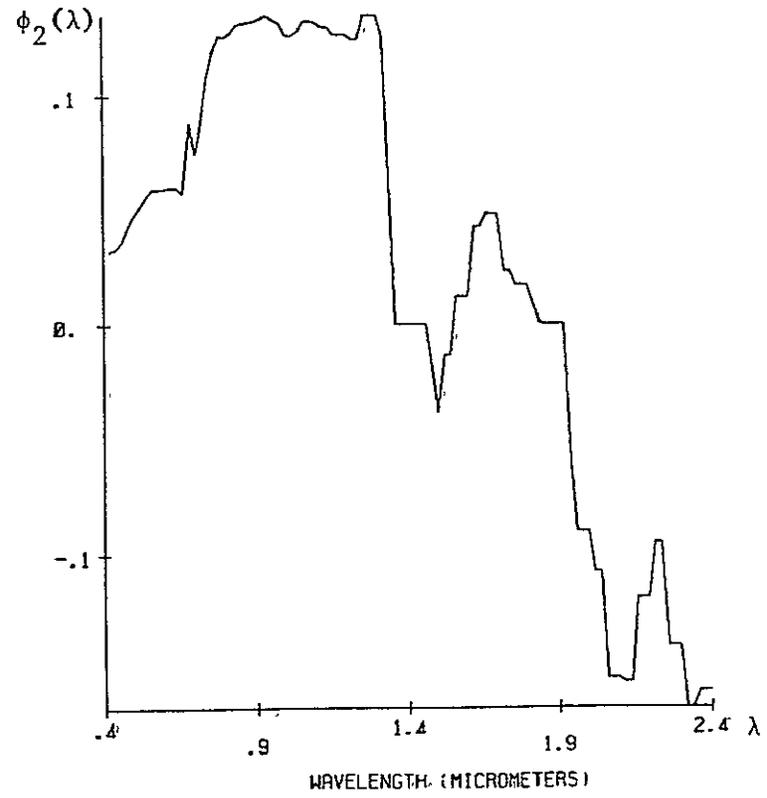


Figure 29. Eigenvector 2.

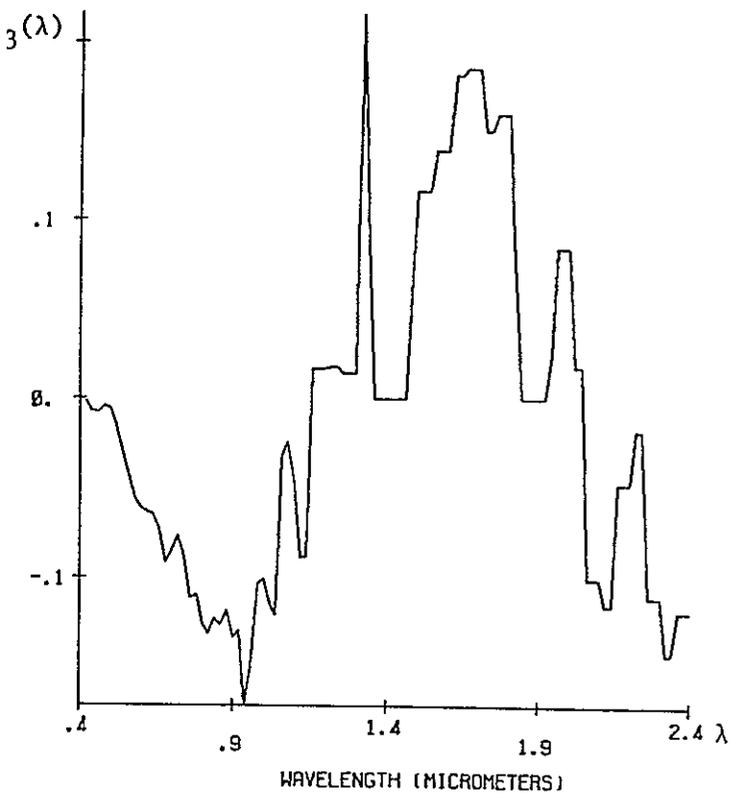


Figure 30. Eigenvector 3.

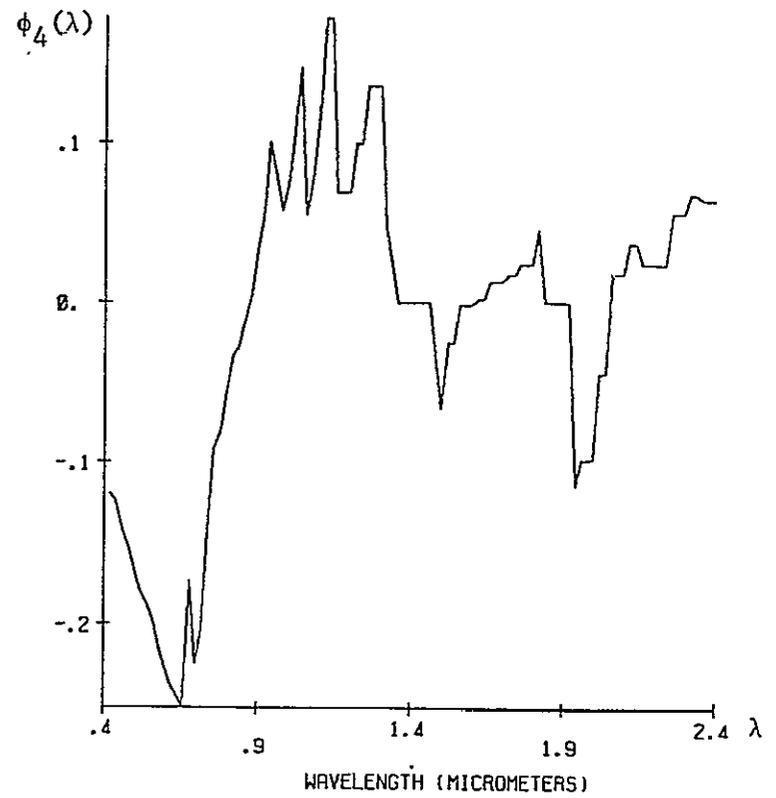


Figure 31. Eigenvector 4.

MEAN VECTOR  
 -200.3751 22.0529 4.5466 18.8553

COVARIANCE MATRIX  
 312.5391  
 -24.4102 62.8445  
 8.4233 -20.7412 14.0594  
 -5.2539 -8.6057 4.5237 12.2612

MEAN VECTOR  
 -202.1029 35.2806 5.7945 16.3977

COVARIANCE MATRIX  
 244.7227  
 -69.3594 152.5959  
 -15.8333 10.3513 24.0876  
 -2.2070 1.3667 -0.7130 12.3877

MEAN VECTOR  
 -187.5431 54.8315 8.2578 19.0705

COVARIANCE MATRIX  
 286.1719  
 -1.2813 168.4688  
 -19.2971 -47.7388 29.0520  
 10.3206 54.3401 -16.4006 26.1763

PROBABILITY OF CORRECT CLASSIFICATION FOR CLASS 1 = 0.9187

PROBABILITY OF CORRECT CLASSIFICATION FOR CLASS 2 = 0.6624

PROBABILITY OF CORRECT CLASSIFICATION FOR CLASS 3 = 0.9003

OVERALL PROBABILITY OF CORRECT RECOGNITION = 0.8270

Table 18. SPTEs and SPEST Sample Output Using the First 4 Eigenvectors and Estimates of the Classification Accuracy.

*t*

## 5. SUMMARY

The task of evaluating the performance of a multispectral scanner system while incorporating every spatial, spectral, electronic and telemetric parameter is exceedingly complicated. The primary objective of this project has been the investigation of the two important, and most relevant in remote sensing applications, of scanner parameters, namely spatial and spectral. The development of analytic techniques for system performance evaluation differentiates the approach adopted here from other experimental methods. This property provides an ease of parameter manipulation not available through some heavily data dependent algorithms. Although the development of individual components of such systems is fundamental to the overall system operation, it is the logical and proper integration of individual modules that determines its ultimate processing capabilities.

The Unified Scanner Analysis Package (USAP) is a fully integrated system with complete input-output compatibility of software units. It consists of a spatial and spectral path plus a shared unit providing the desired performance index in the form of probabilities of correct classification.

### 5.1 Classification Error Estimators

The primary performance index throughout this study is defined as the probability of correct classification of the various populations present in a data set. In keeping with the underlying requirement of a parametric approach, the available LARSYS and other classification accuracy estimators using a randomly generated data base were deemed less than satisfactory. It is well known that the exact probability of correct classification is a multiple integral over an appropriate domain. The direct evaluation of such integral in a continuous  $N$  dimensional space is a complicated and mathematically cumbersome task. This problem is circumvented by a deterministic sampling algorithm of the feature space preceded by an orthogonal transformation. This transformation when applied to the Gaussian probability density function of a particular class under consideration would conditionally decouple the feature space

and hence reduce the N dimensional error integral to a product of N one dimensional integrals each of which is a widely tabulated quantity. This algorithm requires the population statistics as the only major input and provides classification error estimates of high quality without excessively fine feature space quantization. The second classification accuracy estimator uses the maximum a posteriori principle coupled with a Monte-Carlo type integration technique. Although this algorithm is in a way dependent on a simulated data base, from a userspoint of view the difference between ACAP and SPEST are essentially transparent since both methods require the spectral statistics of the populations as their primary input. The aforementioned classification accuracy estimation techniques provide the basic tools for the scanner system performance evaluation.

## 5.2 Scanner Spatial Parameters Selection

The scanner spatial modeling algorithm and software consists of one main plus two supporting routines, i.e., IFOV modeling (SCANSTAT), spatial correlation analyzer (CORELAT) and classification error estimator (ACAP). The objectives of an analytical representation of scanner IFOV model is the establishment of a parametric relationship between the system's input and output statistics in terms of the class conditional mean vectors and covariance matrices. This relationship is established using linear system analysis techniques extended to a 2 dimensional space. In order to derive any specific results two basic characteristics need to be specified: (a) scanner PSF and (b) ground scene spatial correlation model.

The choice of a Gaussian shaped PSF has been widespread in the field of image processing as applied to the Landsat data. This model closely approximates the averaging property of the scanner aperture. An added feature of a Gaussian shaped PSF is the simplification of an otherwise intractable and cumbersome mathematics. Generally speaking the amount of information available about the spatial correlation properties of remotely sensed data is sparse. It has been frequently observed however, that the ground scene spatial correlation model approximately follows an

exponentially dropping function [2]. On the basis of previous experimental evidence and the mathematical simplicity afforded by these assumptions, a Gaussian PSF and a Markov scene spatial correlation model is adopted. Like many other instances, the choice of the problem assumptions does not necessarily rest on their strict validity but also on the tractability of the ensuing algebra. It is entirely conceivable that much more elaborate scene correlation models and PSF shapes can be envisioned. This approach, however, could and would complicate the underlying mathematics to the point where the gains initially expected from the more accurate model are balanced out. For simulation purposes the entire analysis is repeated for a rectangular shaped PSF although no currently operational Landsat is equipped with such a scanner system.

Based on the foregoing discussion the scanner characteristics function is derived in a closed form. This function relates the input and output statistics as a function of the IFOV size and pixel-to-pixel correlation. SCANSTAT is the software implementation of this linear transformation. The auxiliary program, CORELAT, estimates the class conditional correlation functions and provides the best exponential curve fit to the experimental data using a weighted least-squares fit algorithm. The resulting output statistics is modified by additional random noise the power of which is computed from the specified SNR. The ACAP or SPEST processors provide the new classification accuracy sets. The probabilities of correct classification at the scanner output provide the basic information needed to evaluate the system performance under various operating conditions. For test purposes a hypothetical set consisting of 3 populations is selected and their statistics (mean vectors and covariance matrices) specified. The scanner output statistics and associated classification accuracies are computed for various IFOV sizes and scene correlations. The results are in close agreement with the numerically oriented experiments. For any fixed scene correlation, the population separability and hence the overall classification accuracy increases monotonically with IFOV size. The rate of increase, however, is a function of the scene spatial correlation. The classification accuracy increase per IFOV step is small for a highly correlated scene compared to a scene with a small adjacent sample correlation.

This property stems from the features of the scanner characteristic function and its particular weighting process. The addition of white Gaussian noise predictably degrades the output separability. The experimental results show that for a fixed IFOV size, SNR and classification accuracy increase monotonically. Same relationship exists between the classification accuracy and IFOV size when SNR is fixed.

### 5.3 Scanner Spectral Parameters Selection

The task of information extraction from remotely sensed data here primarily deals with the development of methods and techniques to select a set of spectral bands to enhance population separability. The first criterion employed in selecting a set of optimum spectral channels is the Karhunen-Loève expansion of the ensemble of spectral responses associated with a cover type. This expansion provides a set of optimum basis functions, a linear combination of which reconstructs the original stochastic process with a minimum mean square error. These basis functions in effect define a set of optimum windows in the electromagnetic spectrum. The associated software consists of EXOSYS data retrieval package, SPOPT spectral function calculation and SPTEs data transformation and statistics calculation. The classification accuracy estimates used to check the resulting separability is obtained using either the ACAP or SPEST processors.

The second approach employs an information theoretic concept for the specification of the optimal spectral bands. The observed spectral random process is modeled as the sum of a noise free signal plus an additive random noise component. For a candidate set of channels the quantity of interest, mutual information between the reflected and observed energy, is computed. The method consists of representing each random process as an autoregressive model. This type of representation facilitates the evaluation of the mutual information when expressed in terms of the Wiener-Hopf filter PSF. Experimental results consists of selecting a wheat scene and dividing the continuum of electro magnetic spectrum into 9 distinct bands. In each band a proper autoregressive model is fitted to the particular random process. Following the

estimation of the parameters of the regression models, the average information content in each band is computed and on this basis spectral channels are ranked. Therefore on the basis of maximum mutual information optimality criterion, the top N bands represent the N "best" choice. The significant result obtained from the ranking method is that out of the 6 top ranked channels 5 lie in the infrared portion of the spectrum thus future scanner systems used in remote sensing application should contain more infrared spectral bands according to this analysis.

#### 5.4 Conclusions

This report has presented a brief description of the algorithms and results of the Scanner Parameter Selection culminating in the development of a Unified Scanner Analysis Package by proper integration of the available software modules. Although this report is the final document in this project, it actually represents the first step toward a well coordinated scanner system parameter study technique. The current structure of USAP basically represents a skeleton of the future analysis packages. There exists a considerable software and theory development potential. The software by and large can take most of the streamlining to further facilitate their usage. Specific topics include extended diagnostic handling and error recovery capability, accelerated algorithms to further reduce execution times, etc.

An overall evaluation of the methods and results presented in this report shows that the objectives initially outlined have been successfully met. The resulting analysis package, starting from a data base, produces specific guidelines on the selection of spatial and spectral parameters of a multispectral scanner system and it does so on an entirely analytic basis. In closing it should be pointed out that USAP can have a pivotal role in any follow up project providing by far the widest and most economical parameter manipulation scope, fully complimenting any numerical or experimental scanner analysis techniques.

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APPENDIX I

\*\*\*\*\*  
 SUPERVISOR FOR THE ANALYTIC CLASSIFICATION ACCURACY PREDICTION  
 WRITTEN 08/16/78 BIJAN G. MOBASSERI  
 \*\*\*\*\*

DESCRIPTION AND PURPOSE

THE ANALYTIC CLASSIFICATION ACCURACY PREDICTION - ACAP - IS A SOFTWARE PACKAGE, THE PRIMARY OUTPUT OF WHICH IS A SET OF PROBABILITIES OF CORRECT CLASSIFICATION FOR VARIOUS NUMBER OF CLASSES IDENTIFIED IN THE DATA SET.

THE ALGORITHM IMPLEMENTED HERE DIFFERS FROM THE DATA DEPENDENT TECHNIQUES AND RATIO ESTIMATORS SUCH AS \*CLASSIFYPOINT OF LARSYS. WHEREAS LARSYS RETURNS TO THE DATA BASE FOLLOWING THE ESTIMATION OF THE CLASS STATISTICS IN ORDER TO PROVIDE THE CLASSIFICATION ACCURACIES, ACAP BYPASSES THIS STEP AND PROVIDES THE SAME QUANTITY DIRECTLY FROM THE STATISTICS DECK GENERATED BY \*STAT. THE METHOD EMPLOYED IS A MATHEMATICAL ONE (AS OPPOSED TO STATISTICAL) AND INVOLVES PERFORMING A MULTIDIMENSIONAL INTEGRATION OF EACH PROBABILITY DENSITY FUNCTION OVER THE HYPERVOLUME OF CORRECT DECISION DOMAIN.

DESCRIPTION OF CONTROL CARDS

\*ACAP

THIS CARD SPECIFIES THE PARTICULAR PROCESSOR REQUESTED

CHANNELS

THE DESIRED SUBSET OF THE AVAILABLE CHANNELS IS GIVEN HERE. IT IS IMPORTANT TO REMEMBER THAT THE NUMBERS APPEARING ON THIS CARD IS THE ORDER OF THE SELECTED CHANNELS NOT THEIR ACTUAL NUMBER. FOR EXAMPLE IF THE AVAILABLE CHANNELS ARE 8,9,12,14 AND CHANNELS 8,9 AND 14 ARE REQUESTED, SUBCHANNELS CARD SHOULD READ 1,2,4.

CLASSES

THIS CARD SPECIFIES THE NAME OF EACH CLASS. EACH NAME MUST BE PLACED IN A FIELD 7 CHARACTERS LONG FOLLOWED BY A BLANK. THE CONTINUATION CARD, IF REQUIRED, MUST HAVE THE WORD \*CLASSES\* IN THE BEGINNING FOLLOWED BY THE REST OF THE NAMES.

GRID

THIS QUANTITY CONTROLS THE QUALITY OF THE ESTIMATE. THE HIGHER THE NUMBER THE BETTER THE ESTIMATE (ON A MACRO SCALE). THE CPU TIME, HOWEVER IS VERY SENSITIVE TO THIS QUANTITY AND FOR INITIAL USAGE IT IS STRONGLY RECOMMENDED TO LEAVE THIS TO ITS DEFAULT VALUE OF 9 CELLS/AXIS UNTIL THE USER HAS ACQUIRED A FEEL FOR THE PROCESS.

END

THIS CARD SIGNALS THE END OF THE CONTROL CARDS. STAT DECK FOLLOWS IMMEDIATELY.

\*\*\* STAT DECK MUST BE PUNCHED IN CHARACTER FORMAT \*\*\*

REMARKS

THE PROGRAM IS CURRENTLY CAPABLE OF HANDLING 20 CLASSES AND 8 SPECTRAL BANDS. THE EXTENSION OF THESE PARAMETERS IS ONLY LIMITED BY COMPUTATION TIME. THE USE OF MORE THAN 5 FEATURES IS NOT RECOMMENDED UNLESS THE GRID SIZE IS REDUCED. TRIAL AND ERROR IS PROBABLY THE QUICKEST WAY OF FINDING THE PROPER TRADE OFF.

HOW TO RUN THE PROGRAM

THE FOLLOWING EXEC FILE, NAMED ACAP, MUST RESIDE ON THE USER'S

PRIMARY DISK

&TYPEOUT OFF  
 GLOBAL TXLIB SYSLIB CMSLIB SSPLIB  
 GETDISK LARSYS  
 CP C C  
 C> SP RDR HOLD  
 L&AD ACAP B&OVAL (CLEAR NOMAP KEQ)  
 &EXIT

AFTER THE READER FILE IS LOADED, TYPE ACAP IN CMS ENVIRONMENT. THIS COMMAND INITIATES THE EXECUTION FOLLOWED BY APPROPRIATE TERMINAL MESSAGES AND PRINTER OUTPUT.

EXAMPLE OF THE CONTROL CARD SET UP

```
*ACAP
CHANNELS 1,3,4
CLASSES BARESOI CORN PASTURE WHEAT
GRID 9
END
```

DEFAULT VALUE OF GRID IS SELECTED IF GRID CARD IS NOT INCLUDED.

ESTIMATED CPU TIME

WITH THE DEFAULT GRID SIZE AND USING 4 FEATURES, IT TAKES APPROXIMATELY 2 MINUTES OF CPU TIME PER CLASS TO PROVIDE THE REQUESTED CLASSIFICATION ACCURACY ESTIMATES. THE CPU TIME IS MOST SENSITIVE TO THE DIMENSIONALITY OF THE FEATURE SPACE. HENCE IF THE NO OF SPECTRAL BANDS IS LIMITED (LESS THAN 4) CONSIDERABLE INCREASE IN GRID NUMBER IS FEASIBLE. EXPERIENCE HAS SHOWN HOWEVER, THAT THERE IS ONLY A MARGINAL (FRACTION OF PERCENT) IMPROVEMENT IN THE CLASSIFICATION ACCURACY ESTIMATE BY INCREASING THE GRID SIZE BEYOND ITS DEFAULT VALUE.

\*\*\*\*\*

```
REAL*4 A(720),AA(720),SIGMA(1280),MA(160),MAA(160),M(160),SD(8),
1 DELTA(8),P(160),PC(20),V(20),WV1(8),WV2(8),XPRIM(64),
2 K(64),Q(64),PR(13),EV(8),LL(8),UL(8),W(20),Z(104),DEF(20)
3 EMA(160),EA(720)
REAL*4 ICSET(90)
INTEGER*4 NADR(720),IC(20),INDX(20),HEAD(20),SDIM
INTEGER*4 LIST(5),*ACA*,*CHAN*,*CLAS*,*GRID*,*ENO */,*IVEC(1),
1 ICARD(20)
INTEGER*4 BLANK*/ */,*FSTCRD*/LARS*/
INTEGER*2 ICSEL(30),NC(30),NCC(30)
LOGICAL*1 FLAG(5)/5*.FALSE./
```

```
COMMON /CAPCOM/ NO,NDIM,NS,NCLS,NP,ISIZE,KNTR
COMMON /ACUCOM/ NTS,NTM,NSS,NSA,NSH,SD14,NSIZE,ISKP
```

```
11 FORMAT(//)
1010 F&FORMAT(1X,'ERROR IN CYLWRD. EXECUTION TERMINATED.')

```

\*\*\*\*\*

DECODE CONTROL CARDS

\*\*\*\*\*

SET THE DEFAULT GRID SIZE

```
NS=9
DO 777 I=1,30
ICSEL(I)=0
NC(I)=0
777 CONTINUE
```

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```

FILE. . . ACAP FORTRAN B1
C
778 DO 778 I=1,90
      ICSET(I)=-50000.0
C
      FLAG(4)=.TRUE.
C
100 CONTINUE
      LSZ=5
      IER=0
      INRD=5
C
      CALL CTLHRD (ICARD,ICOL,LIST,LSZ,ICODE,INRD,IER)
      IF (IER.NE.0) GO TO 1001
      GO TO (99,101,102,103,104), ICODE
99 FLAG(1)=.TRUE.
      GO TO 100
C
      CHANNELS CARD
101 CALL CHANNEL (ICARD,ICOL,NCR,ICSEL,ICSET,NCC,900)
      FLAG(2)=.TRUE.
      NDIM=NCR
      GO TO 100
C
      CLASS NAMES CARD
102 DO 10 I=1,20
10 HEAD(I)=ICARD(I)
      FLAG(3)=.TRUE.
      GO TO 100
C
      GRID SIZE CARD
103 LSZ=1
      CALL IVAL (ICARD,ICOL,IVEC,LSZ,1002)
      MS=IVEC(1)
      GO TO 100
C
      END CARD
104 LSZ=1
      CALL IVAL (ICARD,ICOL,IVEC,LSZ,1002)
      FLAG(5)=.TRUE.
      GO TO 201
C
1002 WRITE(6,1011)
      WRITE(16,1011)
      GO TO 999
C
201 CONTINUE
      CHECK IF ALL CONTROL CARDS HAVE BEEN READ
C
      DO 250 I=1,5
      IF(.NOT.FLAG(I)) GO TO 321
      GO TO 250
321 WRITE(16,1013)
      WRITE(6,1013)
      GO TO 999
250 CONTINUE
C
      WRITE(16,11)
      WRITE(6,11)
      WRITE(16,1014)
      WRITE(6,1014)
C
      WRITE(16,1015)
      WRITE(6,1015)
C
      GO TO 680
1001 WRITE(16,1010)
      WRITE(6,1010)
      GO TO 999
680 CONTINUE

```

```

FILE. . . ACAP FORTRAN B1
900 WRITE(6,1012)
      WRITE(16,1012)
      GO TO 999
720 CONTINUE
C
*****
C
      READ THE TOTAL NO OF CHANNELS AND CLASSES FROM THE STAT DECK
*****
C
502 READ(5,501) ICRD
501 FORMAT(A4)
      ICRDSQ=ICRDSQ+1
C
      IF(ICRD.EQ.BLANK) GO TO 503
      GO TO 202
503 CONTINUE
C
      REWIND 5
602 READ(5,501) ICRD
      IF(ICRD.EQ.FSTCRD) GO TO 601
      GO TO 602
601 CONTINUE
C
      NUM=ICRDSQ-2
      DO 506 I=1,NUM
507 READ(5,507)
506 FORMAT(I8A4,I8)
      CONTINUE
C
      READ(5,508) NCLS,NFLD,SDIM
508 FORMAT(I5,6X,I5,6X,I5)
C
      NUM2=SDIM+1
      DO 509 I=1,NUM2
509 READ(5,507)
      CONTINUE
C
      FIND THE CHANNEL SET THAT IS NOT REQUESTED
      DO 611 I=1,SDIM
      DO 612 J=1,NDIM
C
          IF(I.EQ.NCC(J)) GO TO 611
C
612 CONTINUE
C
          K=K+1
          NC(K)=I
C
611 CONTINUE
C
      ND=NDIM**2
      NP=NS-1
      ISIZE=NDIM*(NDIM+1)/2
      NSIZE=SDIM*(SDIM+1)/2
C
      NTS=NCLS*ISIZE
      NTM=NCLS*NDIM
      NSS=NCLS*ND
      NSA=NCLS*NSIZE
      NSM=NCLS*SDIM
      CONTINUE
20
C
      CALL ACUTST (A,EA,AA,SIGMA,EMA,NA,MAA,M,SO,DELTA,P,PC,V,WY1,WY2,
1
2
          XPRIM,R,Q,PR,EV,LL,UL,M,DET,Z,IO,INX,NADR,NC,NCC,
          HEAD)
C
999 STOP
      END
C
.....

```

FILE. . . ACAP FORTRAN B1

SUBROUTINE ACUTST

PURPOSE

PERFORM THE INITIAL TRANSFORMATIONS OF THE COORDINATES PRIOR TO CLASSIFICATION ERROR ESTIMATION

DESCRIPTION OF PARAMETERS

- A - COVARIANCE MATRIX ARRAY FOR THE SPECIFIED SUB CHANNEL
AA - DIRECT COPY OF A USED IN INITIALIZATION
DELTA - ARRAY OF CELL WIDTHS ALONG EACH FEATURE AXIS
DET - ARRAY OF DETERMINANTS OF EACH TRANSFORMED COVARIANCE MATRIX
EV - ARRAY OF EIGENVALUES OF THE CURRENT CLASS
EA - COVARIANCE MATRIX ARRAY FOR ALL THE CHANNELS
EMA - MEAN VECTOR FOR ALL THE CHANNELS
HEAD - ARRAY DISPLAYING TITLES
INDX - POINTER ARRAY
IQ - ARRAY DETERMINING THE CYCLIC STRUCTURE OF INDX
LL - ARRAY OF THE LOWER COORDINATES OF A CELL
MA - ORIGINAL MEAN VECTORS FOR THE SPECIFIED SUB CHANNEL
MAA - COPY OF MA FOR INITIALIZATION
M - TRANSFORMED MEAN VECTORS
NADR - UNWANTED COVARIANCE MATRIX ENTRIES
NC - COMPLEMENT OF NCC
NCC - DESIGNATED CHANNELS (SUBSET OF THE TOTAL)
P - ARRAY REPRESENTING ONE CELL CENTER COORDINATES
PC - ARRAY OF CLASSIFICATION ACCURACIES
PR - ONE DIMENSIONAL PROBABILITY ARRAY
Q - WORK VECTOR
R - VECTOR OF EIGENVALUES
SD - STANDARD DEVIATION ARRAY FOR THE CURRENT CLASS
UL - UPPER COORDINATE OF A SAMPLING CELL
V - ARRAY COMPRISING PART OF W
W - THE RESULTING DISCRIMINANT FUNCTION
WV1 - WORK VECTOR
WV2 - WORK VECTOR
XPRIM - WORK VECTOR
Z - ARRAY CONTAINING THE COORDINATES OF EACH SAMPLING CELL

REMARKS

NONE

SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED

GMPRD, ADRES,PMC,EIGEN,DIAG,GMTRA

METHOD

EACH CLASS IS PROCESSED SEQUENTIALLY. THE ORTHOGONAL TRANSFORMATION THAT DECOUPLES THE FEATURE SPACE IS DERIVED BY OBTAINING THE EIGENVECTOR OF THE CLASS UNDER CONSIDERATION. THE ENTIRE SET OF STATISTICS, MEAN VECTORS AND COVARIANCE MATRICES, ARE TRANSFORMED RESULTING IN A DIAGONAL COVARIANCE MATRIX FOR THE CURRENT CLASS AND GENERAL FORM FOR OTHERS. THE MEAN OF THE NEW COORDINATE SYSTEM IS AT THE CURRENT MEAN. THIS PROCESS IS REPEATED UNTIL THE ENTIRE SET OF POPULATIONS IS EXHAUSTED.

SUBROUTINE ACUTST (A,EA,AA,SIGMA,EMA,MA,MAA,M,SD,DELTA,P,PC,V,WV,WV2,XPRIM,R,Q,PR,EV,LL,UL,W,DET,Z,IQ,INDX,NADR,NC,NCC,HEAD)

LOCAL VARIABLES DEFINITION

REAL\*4 A(NTS),AA(NTS),SIGMA(NSS),MA(NTM),MAA(NTM),M(NTM),SD(NDIM),DELTA(NDIM),P(NTM),PC(NCLS),V(NCLS),WV1(NDIM),WV2(NDIM),

FILE. . . ACAP FORTRAN B1

XPRIM(ND),R(ND),Q(ND),PR(NS),EV(NDIM),LL(NDIM),UL(NDIM),N(NCLS),FA(NSA),CMAINSM,Z(NDIM,NS),DET(NCLS)

INTEGER\*4 IQ(NDIM),INDX(NDIM),NADR(NSA),HEAD(1),SDIM
INTEGER\*2 NC(SDIM),NCC(SDIM)
INTEGER\*4 FMT1(6) / 0, 1, 1, 1, 1, 1 /
INTEGER\*4 FMT2(4) / 1, 2, 1, 1 /
INTEGER\*4 FMT3(4) / 1, 2, 1, 1 /
INTEGER\*4 N1 / 0, N2 / 0, N3 / 0

COMMON /CAPCOM/ ND,NDIM,NS,NCLS,NP,ISIZE,KNTR
COMMON /ACUCOM/ NTS,NTM,NSS,NSA,NSM,SDIM,NSIZE,ISKP

COMMON BLOCK VARIABLES DESCRIPTION

- ISIZE - STORAGE SPACE REQUIRED FOR EACH COVARIANCE MATRIX
KNTR - COUNTER FOR THE CURRENT CLASS
NCLS - NO OF CLASSES
ND - ENTIRE ENTRIES OF A COVARIANCE MATRIX
NDIM - NO OF DIMENSIONS (FEATURES)
NSA - NO OF CELLS PER AXIS IN THE SAMPLING GRID
NSA - CORE SPACE ALLOCATED TO ALL COV MATRICES USING ALL CHAN
NSIZE - CORE SPACE ALLOCATED FOR ONE COV MAT USING SUBCHAN
NSM - CORE SPACE ALLOCATED TO ALL MEAN VEC USING ALL CHAN
NTM - CORE SPACE ALLOCATED TO TRAN COV MATRICES
NTM - CORE SPACE ALLOCATED TO MEAN VEC USING THE SUBCHAN
NTS - CORE SPACE ALLOCATED TO COV MAT USING THE SUBCHAN
SDIM - TOTAL NO OF CHANNELS AVAILABLE

FORMAT(1H1)
FORMAT(//)
FORMAT(///)
FORMAT(////)

FORMAT(135X,'ANALYTIC CLASSIFICATION ACCURACY PREDICTION - A C A P

1)
FORMAT(149X,'CLASS ',2A4)
FORMAT(11X,'SAMPLING GRID CHARACTERISTICS')
FORMAT(11X,'GRID SIZE=',13,' CELLS PER DIMENSION')
FORMAT(11X,'TOTAL NO OF CELLS IN THE GRID=',18)
FORMAT(11X,'TRANSFORMED FEATURE SPACE CHARACTERISTICS')
FORMAT(11X,'EIGENVALUES')
FORMAT(11X,'EIGENVECTORS')
FORMAT(11X,'EIGENVECTORS')
FORMAT(131PE,12.4,3X)
FORMAT(131PE,12.4,3X)
FORMAT(11X,'TRANSFORMED MEAN VECTORS')
FORMAT(125X,'PROBABILITY OF CORRECT CLASS

1)
FORMAT(125X,'\*\*\*\*\*TOTAL PROB OF CORRECT CLASSIFICATION\*\*\*\*\*',
F7.3,' PERCENT')
FORMAT(180A1)
FORMAT(11X,'ALL CLASS STATISTICS HAVE BEEN READ')
IS=NS\*NDIM
N1=N1+NDIM
N2=N2+SDIM
N3=N3+5
IF(NSIZE.LT.5) N3=NSIZE

READ IN THE MEAN VECTORS AND COVARIANCE MATRICES

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OF POOR QUALITY

FILE. . . ACAP FORTRAN B1

```
C
C
C READ (5,FMT2) EMA
C READ (5,FMT3) EA
C WRITE(16,550)
C WRITE( 6,550)
C
C*****
C SELECT THE REQUIRED SUBSET OF CHANNELS
C*****
C DO 803 JCLS=1,NCLS
C MDS=(JCLS-1)*SDIM
C DO 802 KDIM=1,NDIM
C DO 801 J=1,NDIM
C IF(KDIM.EQ.NCC(J)) K=K+1
C IF(KDIM.EQ.NCC(J)) MA(K)=EMA(MDS+NCC(J))
C CONTINUE
C CONTINUE
C CONTINUE
C
C DO 530 JCLS=1,NCLS
C DO 530 KDIM=1,NDIM
C MDS=(JCLS-1)*NDIM+KDIM
C MAA(MDS)=MA(MDS)
C 30 CONTINUE
C
C*****
C OBTAIN THE ADDRESS OF THE SELECTED ENTRIES INTO COVARIANCE MATRIX
C*****
C CALL ADRES (NADR,NC,SDIM,NDIM)
C
C K=0
C DO 603 JCLS=1,NCLS
C MDS=(JCLS-1)*NSIZE
C DO 602 I=1,NSIZE
C DO 601 J=1,NSIZE
C IF(I.EQ.NADR(J)) GO TO 602
C 601 CONTINUE
C K=K+1
C A(K)=EA(MDS+I)
C CONTINUE
C CONTINUE
C DO 20 KDIM=1,NTS
C AA(KDIM)=A(KDIM)
C 20 CONTINUE
C 381 CONTINUE
C DO 628 I=1,NCLS
C PC(I)=0.
C PR(I)=0.
C CONTINUE
C 628 ICNT=ICNT+1
```

FILE. . . ACAP FORTRAN B1

```
C TPCC=0.
C DO 888 KNTR =1,NCLS
C WRITE(16,70)
C WRITE(16,72)
C*****
C RESET THE MEAN VECTORS
C*****
C DO 321 JCLS=1,NCLS
C DO 321 KDIM=1,NDIM
C MDS=(JCLS-1)*NDIM
C MA(MDS+KDIM)=MAA(MDS+KDIM)
C 321 CONTINUE
C*****
C PERFORM A DECOUPLING TRANSFORMATION ON THE CURRENT CLASS
C*****
C*****
C TRANSLATE THE ORIGIN TO THE MEAN OF THE CURRENT CLASS
C*****
C
C DO 35 JCLS=1,NCLS
C IF(JCLS.EQ.KNTR) GO TO 35
C DO 38 KDIM=1,NDIM
C MDS=(JCLS-1)*NDIM
C NOS=(KNTR-1)*NDIM
C MA(MDS+KDIM)=MA(MDS+KDIM)-MA(NOS+KDIM)
C 38 CONTINUE
C CONTINUE
C DO 36 KDIM=1,NDIM
C MA(NDS+KDIM)=0
C
C*****
C FIND THE REQUIRED TRANSFORMATION MATRIX
C*****
C
C FORMAT(1X,4(1PE11.4,1X))
C NOS=(KNTR-1)*ISIZE+1
C CALL EIGEN (A(NOS),R,NDIM,MV)
C L=0
C DO 40 KDIM=1,NDIM
C L=L+KDIM
C EV(KDIM)=A(NOS+L-1)
C 40 CONTINUE
C
```

```

FILE. . . ACAP FORTRAN B1
*****
RESET THE CURRENT CLASS COVARIANCE MATRIX
*****
DO 901 KDIM=1,ISIZE
  A(NDS+KDIM-1)=AA(NDS+KDIM-1)
901 CONTINUE
*****
PERFORM THE TRANSFORMATION ON COVARIANCE MATRICES
*****
NR1=NDIM
NC1=NDIM
NC2=NDIM
MSA=0
MSB=1
ICD=0
DO 701 JCLS=1,NCLS
  KDS=(JCLS-1)*ISIZE+1
  KSS=(JCLS-1)*ND+1
  CALL DIAG (R,A(KCS),SIGMA(KSS),XPRIM,MSA,MSB,NR1,NC1,NC2,ICD,
1 NDIM)
701 CONTINUE
*****
PERFORM THE TRANSFORMATION ON MEAN VECTORS
*****
CALL GMTRA (R,Q,NDIM,NDIM)
NR1=NDIM
NC1=NDIM
NC2=1
DO 702 JCLS=1,NCLS
  MUS=(JCLS-1)*NDIM+1
  CALL GMPRD (Q,MA(MDS),M(MDS),NR1,NC1,NC2)
702 CONTINUE
*****
PREPEARE THE PRINTED OUTPUT
*****
KS=2*KNTR
WRITE(6,536)
WRITE(6,74)
WRITE(6,537) (HEAD(KS+I),I=1,2)
C
WRITE(6,74)
WRITE(6,538)
C
WRITE(6,72)
WRITE(6,539) NS
C
WRITE(6,72)
WRITE(6,540) IGS
C
WRITE(6,74)
WRITE(6,541)

```

```

FILE. . . ACAP FORTRAN B1
WRITE(6,72)
WRITE(6,542)
WRITE(6,72)
WRITE(6,FMT1) EV
C
WRITE(6,74)
WRITE(6,544)
WRITE(6,72)
WRITE(6,FMT1) R
C
WRITE(6,74)
WRITE(6,546)
C
WRITE(6,72)
WRITE(6,FMT1) M
C
WRITE(6,74)
*****
START ESTIMATION OF THE PROBABILITY OF MISCLASSIFICATION
*****
99 CONTINUE
CALL PMC (M,SIGMA,P,PC,EV,Z,SD,DELTA,WV1,WV2,V,XPRIM,W,R,
1 Q,DET,INDX,IQ,LL,UL,PR,HEAD)
C
CONVERT THE RESULTING PCC TO PERCENTAGE
*****
PC(KNTR)=100.*PC(KNTR)
C
WRITE(16,72)
WRITE(16,49) (HEAD(KS+I),I=1,2)
49 FORMAT(25X,'CLASS ',2A4)
WRITE(16,71)
43 WRITE(16,43) PC(KNTR)
FORMAT(10X,'--- PROBABILITY OF CORRECT CLASSIFICATION= ',F7.3,' -
1')
WRITE(16,72)
C
WRITE(6,547) PC(KNTR)
C
888 CONTINUE
*****
FIND THE TOTAL PROBABILITY OF CORRECT CLASSIFICATION
*****
DO 568 I=1,NCLS
  TPCC=TPCC+PC(I)
C
568 CONTINUE
TPCC=TPCC/FLOAT(NCLS)
WRITE(6,74)
WRITE(6,548) TPCC
WRITE(6,72)
C
999 CONTINUE
C
RETURN
END
*****
SUBROUTINE ADRES

```

FILE. . . ACAP FORTRAN B1

```

C      PURPOSE
C      FIND THE DESIRED SUBSET OF A COVARIANCE MATRIX
C      DESCRIPTION OF PARAMETERS
C      NADR - ADDRESS ARRAY OF THE ENTRIES TO BE DELETED
C      NC   - ARRAY OF THE CHANNELS TO BE DELETED
C      SDIM - TOTAL NO OF BANDS SUPPLIED
C      NDIM - DESIRED SUBSET OF SDIM
C      REMARKS
C      NONE
C      SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C      NONE
C      METHOD
C      USING A ONE DIMENSIONAL STORAGE MODE FOR EACH COVARIANCE
C      MATRIX IN AN UPPER TRIANGULAR FORM, THE PARAMETRIC ADDRESSES
C      FOR EACH ENTRY IS DERIVED AND USING NC ARRAY THE LOCATION OF
C      ALL THE ENTRIES THAT LIE IN THE UNWANTED LINES AND COLUMNS
C      IS COMPUTED AND STORED IN NADR

```

.....

```

C      SUBROUTINE ADRES (NADR,NC,SDIM,NDIM)
C      INTEGER*4 NADR(1),PDIM,SDIM
C      INTEGER*2 NC(1)
C      PDIM=SDIM-NDIM
C      IF(PDIM.LE.0) GO TO 100
C      DO 20 J=1,PDIM
C      N1=NC(J)
C      DO 10 I=1,N1
C      K=K+1
C      NADR(K)=(NC(J)*(NC(J)-1)/2)+I
C      CONTINUE
C      CONTINUE
C      DO 45 J=1,PDIM
C      NFIN=SDIM-NC(J)+1
C      DO 35 I=1,NFIN
C      K=K+1
C      NADR(K)={(NC(J)+I-1)*(NC(J)+I-2)/2)+NC(J)
C      WRITE(116,233) K,NADR(K)
C      FORMAT(2(15,3X))
C      CONTINUE
C      CONTINUE
C      RETURN
C      END

```

.....

```

C      SUBROUTINE PMC
C      PURPOSE
C      TO COMPUTE THE PROBABILITY OF MISCLASSIFICATION OF THE
C      CLASSES
C      DESCRIPTION OF PARAMETERS
C      DET - DETERMINANT ARRAY FOR EACH COV MATRIX
C      DELTA - ARRAY OF SAMPLING CELL DIMENSIONS
C      EV   - EIGENVALUES
C      HEAD - HEADER ARRAY
C      INDX - POINTER ARRAY

```

FILE. . . ACAP FORTRAN B1

```

C      LL - LOWER COORD. OF A CELL
C      H   - TRANSFORMED MEAN VECTORS
C      P   - GRID POINT VECTOR
C      PC  - CLASSIFICATION ACCURACY RESULTS
C      PR  - PROBABILITY ASSOCIATED WITH EACH GRID CELL
C      Q   - WORK VECTOR
C      R   - EIGENVECTORS
C      SD  - CLASS STANDARD DEVIATION VECTOR
C      V   - PART OF THE DISCRIMINANT FUNCTIONS
C      WV1 - WORK VECTOR
C      WV2 - WORK VECTOR
C      W   - DISCRIMINANT FUNCTION
C      XPRIM - WORK VECTOR
C      Z   - COORD OF CELL CENTERS

```

```

C      REMARKS
C      NONE

```

```

C      SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED

```

```

C      DIAG, GRPROB, MINV

```

```

C      METHOD

```

```

C      THE PRIMARY INPUT TO THIS SUBROUTINE IS THE TRANSFORMED
C      MEAN AND COVARIANCE MATRICES. THE STRUCTURE OF THE SAMPLING
C      GRID IS DETERMINED BY COMPUTING THE CELL WIDTH ALONG EACH
C      DIMENSION AND THE COORD OF THE CENTER OF EVERY CELL THROUGHOUT
C      THE GRID. THE ENTIRE GRID IS SCANNED AND NCLS DISCRIMINANT
C      FUNCTION IS CALCULATED FOR EACH CELL. USING MAXIMUM LIKELIHOOD
C      RULE A CELL IS ASSIGNED TO EITHER THE CURRENT CLASS OR
C      OUTSIDE (NEED NOT KNOW EXACTLY WHICH CLASS). SUBROUTINE
C      GRPROB CALCULATES THE HYPERVOLUME UNDER THE PDF AND OVER THE
C      GRID CELL. AFTER ALL THE POINTS ARE EXHAUSTIVELY TESTED THE
C      ELEMENTARY UNITS OF PROBABILITY ARE SUMMED AND STORED IN THE
C      PC ARRAY AND RETURNED TO THE CALLING PROGRAM.

```

.....

```

C      SUBROUTINE PMC (H,SIGMA,P,PC,EV,Z,SD,DELTA,WV1,WV2,V,XPRIM,W,R,
C      1 Q,DET,INDX,IQ,LL,UL,PR,HEAD)

```

.....

```

C      LOCAL VARIABLES DEFINITION

```

.....

```

C      REAL M(1),LL(1),UL(1),PR(1)
C      1 SIGMA(1),Z(NDIM,NS),SD(1),DELTA(1),
C      2 P(1),NC(1),WV(1),WV2(1),PC(1),V(1),
C      3 XPRIM(NDIM),W(1),R(1),Q(1),DET(1),EV(1)

```

```

C      INTEGER*4 GS,HEAD(1),IQ(1),INDX(1)

```

```

C      COMMON /CAPCOM/ ND,NDIM,NS,NCLS,NP,ISIZE,KNTR

```

```

C      GS=NS*NDIM
C      ICD=1

```

.....

```

C      FIND THE VARIANCES ALONG EACH FEATURE AXIS

```

.....

```

C      DO 706 KDIM=1,NDIM

```

```

C      SD(KDIM)=SQRT(EV(KDIM))

```

```

C      706 CONTINUE

```

```

C      CONTINUE

```

```

C      CONTINUE

```

```

FILE. . . ACAP FORTRAN 81
*****
      INVERT THE TRANSFORMED COVARIANCE MATRICES
*****
      DO 707 JCLS=1,NCLS
        KDS=(JCLS-1)*ND+1
        CALL MINV (SIGMA(KDS),NDIM,DET(JCLS),MV1,MV2)
707    CONTINUE
      33  FORMAT(1X,3(1PE12.4,2X))
      32  FORMAT(1X,2(1PE12.4))
*****
      SAMPLE THE FEATURE SPACE BY A BINOMIAL APPROX. TO NORMAL D.F.
*****
*****
      FIND THE WIDTH OF A SAMPLING CELL ALONG EACH DIMENSION
*****
      DO 44 KDIM=1,NDIM
        DELTA(KDIM)=2.*SD(KDIM)/SQRT(FLOAT(NP))
      CONTINUE
*****
      FIND THE COORDINATES OF EACH AND EVERY SAMPLING CELL CENTER
*****
      DO 30 KDIM=1,NDIM
      DO 30 I=1,NS
        Z(KDIM,I)=2.*SD(KDIM)*(FLOAT(I-1)-NP/2)/SQRT(FLOAT(NP))
      CONTINUE
*****
      CALCULATE THE QUADRATIC DISCRIMINANT FUNCTIONS
*****
      ICNT=0
      DO 404 KDIM=1,NDIM
        INDX(KDIM)=1
*****
      PERFORM A ONE-TO-N DIMENSIONAL MAPPING OF POSITIVE INTEGERS
*****
      DO 520 I=1,NDIM
        IQ(I)=NS*(NDIM-I)
      CONTINUE
      DO 120 J=1,GS
        IF(J.EQ.1) GO TO 30

```

```

FILE. . . ACAP FORTRAN 81
C
DO 122 KDIM=1,NDIM
  C
  IF(MOD(J-1,10(KDIM)).EQ.0) INDX(KDIM)=INDX(KDIM)+1
  IF(INDX(KDIM).GT.NS) INDX(KDIM)=1
C
122  CONTINUE
38  CONTINUE
C
*****
COMPLETE THE DISCRIMINANT FUNCTION CALCULATION
*****
DO 124 JCLS=1,NCLS
DO 126 KDIM=1,NDIM
  C
  MDS=(JCLS-1)*NDIM
  P(MDS+KDIM)=Z(KDIM,INDX(KDIM))-M(MDS+KDIM)
C
126  CONTINUE
124  CONTINUE
C
DO 128 JCLS=1,NCLS
  C
  MDS=(JCLS-1)*NDIM+1
  KDS=(JCLS-1)*ND+1
  MSA=0
  MSB=0
  CALL DIAG (P(MDS),SIGMA(KDS),V(JCLS),XPRIM,MSA,MSB,NR1,
1      NC1,NC2,ICD,NDIM)
  C
  W(JCLS)=V(JCLS)+ALOG(DET(JCLS))
C
128  CONTINUE
C
*****
CHECK THE CONDITION FOR CORRECT CLASSIFICATION
*****
TERM=1.0E 10
DO 134 JCLS=1,NCLS
  C
  IF(W(JCLS).LE.TERM) TERM=W(JCLS)
C
134  CONTINUE
*****
FIND THE ELEMENTARY UNIT OF PROBABILITY
*****
IF(TERM.NE.W(KNTR)) GO TO 120
ICNT=ICNT+1
CALL GRPROB (INDX,DELTA,SD,NS,NDIM,Z,PR,LL,UL)
C
855  CONTINUE
  PRP=1.
DO 710 KDIM=1,NDIM
  C
  PRP=PRP*PR(INDX(KDIM))
C
710  CONTINUE
  PC(KNTR)=PC(KNTR)+PRP
C
120  CONTINUE
RETURN
END
C
.....

```

FILE. . . ACAP FORTRAN B1

SUBROUTIN GRPROB

PURPOSE  
COMPUTE THE GRID PROBABILITY

DESCRIPTION OF PARAMETERS

DELTA - GRID CELL DIMENSIONS  
INDX - POINTER ARRAY  
LL - LOWER COORD OF A CELL  
NDIM - NO OF SELECTED SUBSET CHANNELS  
NS - NO OF CELL PER DIMENSION  
PR - PROBABILITY ARRAY AROUND EACH CELL  
SD - STANDARD DEVIATION VECTOR  
UL - UPPER COORD OF A CELL  
Z - CELL CENTER COORD ARRAY

REMARKS

NONE

SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED

ERF

METHOD

THE NORMALIZED CLASS DENSITY FUNCTION IS INTEGRATED OVER  
A HYPERVOLUME WHOSE SIZE IS DETERMINED BY 'DELTA'.

.....  
SUBROUTINE GRPRCB (INDX,DELTA,SD,NS,NDIM,Z,PR,LL,UL)

REAL\*4 Z(NDIM,NS),DELTA(1),PR(1),SD(1),LL(1),UL(1)  
INTEGER\*4 INDX(1)

DO 100 KDIM=1,NDIM

UL(KDIM)=Z(KDIM,INDX(KDIM))+DELTA(KDIM)/Z.  
LL(KDIM)=Z(KDIM,INDX(KDIM))-DELTA(KDIM)/Z.

UL(KDIM)=UL(KDIM)/SD(KDIM)  
LL(KDIM)=LL(KDIM)/SD(KDIM)  
PR(INDX(KDIM))=0.5\*(ERF(UL(KDIM)/SQRT(2.))-ERF(LL(KDIM)/SQRT(2.)))

1 CONTINUE

RETURN  
END

.....  
SUBROUTINE DIAG

PURPOSE  
TO PERFORM THE FOLLOWING OPERATION. (QTRANSPOSE)\*X\*(Q)

USAGE  
CALL DIAG (Q,X,SIGMA,XPRIM,MSA,MSB,NR1,NC1,NC2,ICD,N)

DESCRIPTION OF PARAMETERS

Q - THE FIRST MATRIX  
X - THE SECOND MATRIX  
SIGMA - THE OUTPUT MATRIX  
MSA - FLAG FOR THE STORAGE MODE OF FIRST MATRIX  
MSB - FLAG FOR THE STORAGE MODE OF SECOND MATRIX  
NR1 - NO OF ROWS IN THE FIRST MATRIX  
NC1 - NO OF COLUMNS IN THE FIRST MATRIX  
NC2 - NO OF COLUMNS IN THE SECOND MATRIX  
ICD - FLAG TO RESET NR AND NC PARAMETERS  
N - NO OF DIMENSIONS

FILE. . . ACAP FORTRAN B1

REMARKS  
NONE

SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED  
TPRD,GMTRD

METHOD  
THO SUBROUTINES TPRD AND GMTRD ARE USED IN SEQUENCE TO  
PERFORM THE TRANSPOSITION AND MULTIPLICATION.

.....  
SUBROUTINE DIAG (Q,X,SIGMA,XPRIM,MSA,MSB,NR1,NC1,NC2,ICD,N)  
REAL Q(1),X(1),SIGMA(1),XPRIM(1)  
WRITE(16,76) X  
FORMAT(10(F5.2,1X))  
IF(ICD.EQ.0) GO TO 55

NR1=N  
NC1=1  
NC2=N

CONTINUE  
CALL TPRD (Q,X,XPRIM,NR1,NC1,MSA,MSB,NC2)

IF(ICD.EQ.0) GO TO 121

NR1=1  
NC1=N  
NC2=1

121 CONTINUE  
CALL GMTRD (XPRIM,Q,SIGMA,NR1,NC1,NC2)  
RETURN  
END

.....  
SUBROUTINE GMTRA

PURPOSE  
TRANSPOSE A GENERAL MATRIX

USAGE  
CALL GMTRA(A,R,N,M)

DESCRIPTION OF PARAMETERS  
A - NAME OF MATRIX TO BE TRANSPOSED  
R - NAME OF RESULTANT MATRIX  
N - NUMBER OF ROWS OF A AND COLUMNS OF R  
M - NUMBER OF COLUMNS OF A AND ROWS OF R

REMARKS  
MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A  
MATRICES A AND R MUST BE STORED AS GENERAL MATRICES

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED  
NONE

METHOD  
TRANSPOSE N BY M MATRIX A TO FORM M BY N MATRIX R

.....  
SUBROUTINE GMTRA(A,R,N,M)  
DIMENSION A(1),R(1)

IR=0  
DO 10 I=1,M  
IJ=I-N  
DO 10 J=1,M  
IJ=IJ+N  
IR=IR+1  
10 R(IR)=A(IJ)

FILE. . . ACAP FORTRAN B1

RETURN  
END

.....  
SUBROUTINE TPRD

PURPOSE

TRANSPOSE A MATRIX AND POSTMULTIPLY BY ANOTHER TO FORM  
A RESULTANT MATRIX

USAGE

CALL TPRD(A,B,R,N,M,MSA,MSB,L)

DESCRIPTION OF PARAMETERS

A - NAME OF FIRST INPUT MATRIX  
B - NAME OF SECOND INPUT MATRIX  
R - NAME OF OUTPUT MATRIX  
N - NUMBER OF ROWS IN A AND B  
M - NUMBER OF COLUMNS IN A AND ROWS IN R  
MSA - ONE DIGIT NUMBER FOR STORAGE MODE OF MATRIX A  
    0 - GENERAL  
    1 - SYMMETRIC  
    2 - DIAGONAL  
MSB - SAME AS MSA EXCEPT FOR MATRIX B  
L - NUMBER OF COLUMNS IN B AND R

REMARKS

MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRICES A OR B

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED

LOC

METHOD

MATRIX TRANSPOSE OF A IS NOT ACTUALLY CALCULATED. INSTEAD,  
ELEMENTS IN MATRIX A ARE TAKEN COLUMNWISE RATHER THAN  
ROWWISE FOR MULTIPLICATION BY MATRIX B.  
THE FOLLOWING TABLE SHOWS THE STORAGE MODE OF THE OUTPUT  
MATRIX FOR ALL COMBINATIONS OF INPUT MATRICES

A	B	R
GENERAL	GENERAL	GENERAL
GENERAL	SYMMETRIC	GENERAL
GENERAL	DIAGONAL	GENERAL
SYMMETRIC	GENERAL	GENERAL
SYMMETRIC	SYMMETRIC	GENERAL
SYMMETRIC	DIAGONAL	GENERAL
DIAGONAL	GENERAL	GENERAL
DIAGONAL	SYMMETRIC	GENERAL
DIAGONAL	DIAGONAL	DIAGONAL

.....  
SUBROUTINE TPRD(A,B,R,N,M,MSA,MSB,L)  
DIMENSION A(1),B(1),R(1)

SPECIAL CASE FOR DIAGONAL BY DIAGONAL

MS=MSA\*10+MSB  
IF(MS-22) 30,10,30  
10 DO 20 I=1,N  
20 R(I)=A(I)\*B(I)  
RETURN

MULTIPLY TRANSPOSE OF A BY B -

30 IR=1  
DO 90 K=1,L  
DO 90 J=1,M  
R(IR)=0.0  
DO 80 I=1,N  
IF(MS) 40,60,40  
40 CALL LOC(I,J,A,N,M,MSA)  
CALL LOC(I,K,B,N,L,MSB)  
IF(A) 50,80,50  
50 IF(B) 70,80,70  
60 IA=N\*(J-1)+I  
IB=N\*(K-1)+I  
70 R(IR)=R(IR)+A(IA)\*B(IB)

FILE. . . ACAP FORTRAN B1

80 CONTINUE  
90 IR=IR+1  
RETURN  
END

ORIGINAL PAGE IS  
OF POOR QUALITY

FILE. . . SPEST FORTRAN B1

\*\*\*\*\*  
SUPERVISOR FOR THE STRATIFIED POSTERIOR CLASSIFICATION ACCURACY ESTIMATOR

WRITTEN 01/24/78 D. J. WIERSMA

\*\*\*\*\*

AMENDED 10/31/78 B. G. MOBASSERI

DESCRIPTION AND PURPOSE

THE STRATIFIED POSTERIOR ESTIMATOR - SPEST- IS SIMILAR TO TH ACAP PROCESSOR IN PURPOSE. IT PROVIDES A SET OF CLASSIFICATION ACCURACY ESTIMATES FOR A BAYES CLASSIFIER WHEN THE POPULATION STATISTICS ARE MULTIVARIATE NORMAL. ALTHOUGH THE ALGORITHM EMPLOYS THE LIKELIHOOD PRINCIPAL, IT DIFFERS FROM ACAP APPROC BY USING A RANDOMLY GENERATED DATA BASE. FURTHER DETAILS ON THE THEORY CAN BE FOUND IN THE LARS FINAL REPORT, NOV 30, 1978

DESCRIPTION OF CONTROL CARDS

\*SPEST

THIS CARDS SPECIFIES THE PARTICULAR PROCESSOR REQUESTED

CHANNELS

THE DESIRED SUBSET OF THE AVAILARLE CHANNELS IS GIVEN HE IT IS IMPORTANT TO REMEMBER THAT THE NUMBERS APPEARING ON THIS CARD IS THE ORDER OF THE SELECTED CHANNELS NOT THEIR ACTUAL NUMBER. FOREXAMPLE, IF THE AVAILABLE CHANNELS ARE 8,9,12,14 AND CHANNELS 8,9 AND 14 ARE REQUE THIS CARD SHOULD READ 1,2,4.

CLASSES

THIS CARD SPECIFIES THE NAME OF EACH CLASS. EACH NAME MUST BE PLACED IN A FIELD 7 CHARACTERS LONG FOLLOWED BY A BLANK. THE CONTINUATION CARD, IF REQUESTED, MUST YE THE NAME 'CLASSES' IN THE BEGINNIG FLOWED BY THE REST OF THE NAMES.

END

SIGNALS THE END OF THE CONTROL CARDS

\*\*\* STAT DECK FOLLOWS IMMEDIATELY. MUST BE IN CHARACTER FORMAT  
REMARKS

THIS PROGRAM IS CURRENTLY CAPABLE OF PROCESSING UP TO 20 CLASSES AND 8 SPECTRAL BANDS.

HOW TO RUN THE PROGRAM

TYPE 'GETDISK OHSYS' TO ESTABLISH THE PROPER LINKS. THE RE/ ER FILE CONTAINS ONE DECK CONSISTING OF THE CONTROL CARD AND LARSYS STATISTICS DECK. TYPE 'SPEST' FOR THE PROGRAM EXECUTION.

EXAMPLE OF A CONTROL CARD SET UP

\*SPEST  
CHANNELS 1,2,4  
CLASSES BARESOI CORN PASTURE WHEAT  
END

\*\*\*\*\*

REAL\*4 PR(20),PHI(8,8,20),PI(20),AM(8,20),COV(36,20)  
REAL\*4 QP(20),COVT(36),GAM(8,20),DET(20),COVIN(36,20)  
REAL\*4 Y(8),DEL(8),COVU(8,8),PX(20),SOET(20)  
REAL\*4 EXAM(8,20),EXCOV(36,20)

FILE. . . SPEST FORTRAN B1

REAL\*4 ICSET(90)  
INTEG\*4 NADR(720),HEAD(20),SDIM  
INTEG\*4 LIST(4),\*SPE,ICHAN,CLAS,END/  
INTEG\*4 IVEC(1),ICARD(20)  
BLANK//FSICRD/LARS/  
INTEG\*2 ICSEL(30),NCL(30),NCC(30)  
LOGICAL\*1 FLAG(4)/4\*.FALSE./  
COMMON /SPCOM/ NTH,NTS,ISIZE,NSIZE,NSA,SDIM,NDIM

C  
11 FORMAT(//)  
1010 FORMAT(IX,'ERROR IN CTLWRD. EXECUTION TERMINATED')  
1012 FORMAT(IX,'ERROR IN CHANEL. EXECUTION TERMINATED')  
1013 FORMAT(IX,'MISSING CONTROL WORD. EXECUTION TERMINATED')  
1014 FORMAT(IX,'SPEST PROCESSING STARTED')  
1015 FORMAT(IX,'ALL CONTROL CARDS HAVE BEEN READ')  
C\*\*\*\*\*

DECODE CONTROL CARDS

C\*\*\*\*\*

DO 777 I=1,30  
ICSEL(I)=0  
NCL(I)=0  
CONTINUE

777 C  
DO 778 I=1,90  
778 ICSET(I)=-50000.0  
C  
100 CONTINUE  
LSZ=4  
IER=0  
INRD=5

C  
CALL CTLWRD (ICARD,ICOL,LIST,LSZ,ICODE,INRD,IER)  
IF(IER.NE.0) GO TO 1001  
GO TO (99,101,102,103), ICODE  
99 FLAG(1)=.TRUE.  
GO TO 100

CHANNELS CARD

C  
C  
101 CALL CHANEL (ICARD,ICOL,NCR,ICSEL,ICSET,NCC,900)  
FLAG(2)=.TRUE.  
NDIM=NCR  
GO TO 100

CLASS NAMES CARD

C  
C  
102 DO 10 I=1,20  
10 HEAD(I)=ICARD(I)  
FLAG(3)=.TRUE.  
GO TO 100

END CARD

C  
C  
103 FLAG(4)=.TRUE.  
GO TO 201

CONTINUE

C  
C  
201 CHECK IF ALL CONTROL CARDS HAVE BEEN READ

DO 250 I=1,4  
IF(.NOT. FLAG(I)) GO TO 321  
GO TO 250  
321 WRITE(16,1013)  
WRITE(6,1013)  
GO TO 999  
250 CONTINUE

C  
WRITE(16,11)  
WRITE(6,11)  
WRITE(16,1014)  
WRITE(6,1014)

C  
WRITE(16,1015)

FILE. . . SPEST FORTRAN B1

```

C      WRITE( 6,1015)
C      GO TO 680
1001  WRITE(16,1010)
      WRITE( 6,1010)
      GO TO 999
680   CONTINUE
C      GO TO 720
900   WRITE(6,1012)
      WRITE(16,1012)
      GO TO 999
720   CONTINUE
*****
C      READ THE TOTAL NO OF CHANNELS AND CLASSES FROM THE STAT DECK
*****
502   READ(5,501) ICRD
501   FORMAT(A4)
      ICRDSQ=ICRDSQ+1
C      IF(ICRD.EQ.BLANK) GO TO 503
      GO TO 502
503   CONTINUE
C      REWIND 5
602   READ(5,501) ICRD
      IF(ICRD.EQ.FSTCRD) GO TO 601
      GO TO 602
601   CONTINUE
C      NUM=ICRDSQ-2
      DD 506 I=1,NUM
      READ(5,507)
507   FORMAT(18A4,I8)
506   CONTINUE
C      READ(5,508) NCLS,NFLD,SDIM
508   FORMAT(15,6X,15,6X,15)
      M=NCLS
      N=NDIM
      NUM2=SDIM+1
      DD 509 I=1,NUM2
      READ(5,507)
509   CONTINUE
C      FIND THE CHANNEL SET THAT IS NOT REQUESTED
      DD 611 I=1,SDIM
      DD 612 J=1,NDIM
C      IF(I.EQ.NCC(J)) GO TO 611
612   CONTINUE
      K=K+1
      NC(K)=I
C      CONTINUE
611   CONTINUE
      ISIZE=NDIM*(NDIM+1)/2
      NSIZE=SDIM*(SDIM+1)/2
C      NTS=NCLS+ISIZE
      NTH=NCLS+NDIM
      NSS=NCLS*ND
      NSA=NCLS*NSIZE
      NSM=NCLS*SDIM
C      NCT = N*(N+1)/2
      DD 12 I=1,M
      P(I)=1./FLOAT(M)
12

```

FILE. . . SPEST FORTRAN B1

```

C      CALL MCOVP(N,AM(1,1),COV(1,1))
C      CONTINUE
30   CALL SPESTM(M,N,PHI,P,AM,COV,PR,PC,OP,COVT,GAM,DET,COVIN,
      Y,TE1,DEL,COVU,PX,SDET,NAOR,NC,NCC,EXAM,EXCOV,
      HEAD)
C      PC=100.*PC
      WRITE(6,60)PC
60   FORMAT(///30X,'OVERALL PROBABILITY OF CORRECT RECOGNITION = ',F1
      1.3)
999  STOP
      END
-----
SPEST IS AN ESTIMATOR OF THE CLASSIFICATION PERFORMANCE FROM A
GIVEN SET OF STATISTICS FROM M CLASSES. THE ESTIMATOR IS A
STRATIFIED POSTERIOR ESTIMATOR (REF. WHITSITT AND LANGREBE).
THE PROBABILITY DISTRIBUTIONS ARE ASSUMED TO BE MULTIVARIATE
GAUSSIAN
19 JANUARY, 1978
-----
SUBROUTINE SPESTM(M,N,PHI,P,AM,COV,PR,PC,OP,COVT,GAM,DET,COVIN,
      Y,TE1,DEL,COVU,PX,SDET,NAOR,NC,NCC,EXAM,EXCOV,
      HEAD)
      INTEGER*4 NADR(NSA),SCIM
      INTEGER*4 FMT1(4) // (2X,' ',0,'E14.0','7')//
      INTEGER*4 FMT2(4) // (2X,' ',0,'E14.0','7')//
      INTEGER*4 N1 // 0 //, N2 // 0 //
      REAL*4 OPIM,P(M),PR(M),EXAM(SDIM,M),EXCOV(NSIZE,M),COVT(ISIZE)
      REAL*4 GAM(N,M),PHI(N,M),DET(M),COVIN(ISIZE,M)
      REAL*4 Y(N),TE1(N),DEL(N),COVIN(N)
      REAL*4 AM(N,M),HEAD(20),COV(ISIZE,M)
      REAL*8 PX(N),RIG,DE1,SDET(M),RETA,Z0,Z1,Z2,Z3
      INTEGER*2 NC(30),NCC(30)
      COMMON /SPCOM/ NTH,NTS,ISIZE,NSIZE,NSA,SDIM,NDIM
-----
LIST OF VARIABLES
M = NUMBER OF CLASSES
N = NUMBER OF DIMENSIONS
PHI = APRIORI PROBABILITIES OF CLASS I
PR(I) = CLASS CONDITIONAL PERFORMANCE
PC = OVERALL PERFORMANCE
AM(J,I) = MEAN VECTOR OF CLASS I
COV(J,I) = COVARIANCE MATRIX OF CLASS I (STORED IN UPPER TRIANGULAR
FORM)
-----
*****
      READ IN THE MEAN VECTORS AND COVARIANCE MATRICES
*****
C      N1=N1+SDIM
      N2=N2+5
      IF(NSIZE.LT.5) N2=N2+NSIZE
C      FMT1(2)=N1
      FMT2(2)=N2
C      DD 30 I=1,M
      READ(5,FMT1) (EXAM(J,I),J=1,SDIM)
      CONTINUE
30   CONTINUE
      DD 31 I=1,M
      READ(5,FMT2) (EXCOV(K,I),K=1,NSIZE)
31   CONTINUE
C

```

FILE. . . SPEST FORTRAN 81

```
C *****
C SELECT THE REQUIRED SUBSET OF THE CHANNELS
C *****
C DO 803 JCLS=1,M
C     K=0
C DO 802 KDIM=1,SDIM
C DO 801 J=1,N
C     IF(KDIM.EQ.NCC(J)) K=K+1
C     IF(KDIM.EQ.NCC(J)) AM(K,JCLS)=EXAM(NCC(J),JCLS)
C01 CONTINUE
C02 CONTINUE
C03 CONTINUE
C *****
C OBTAIN THE ADDRESS OF THE SELECTED ENTRIES INTO COVARIANCE MATRIX
C *****
C CALL ADRES (NADR,NC,SDIM,NDIM)
C     K=0
C DO 603 JCLS=1,M
C     K=0
C DO 602 I =1,NSIZE
C DO 601 J =1,NSIZE
C     IF(I.EQ.NADR(J)) GO TO 602
C01 CONTINUE
C     K=K+1
C     COV(K,JCLS)=EXCOV(I,JCLS)
C02 CONTINUE
C03 CONTINUE
C     IX = 947913
C     NCT = N*(N+1)/2
C COMPUTE EIGENVALUES AND EIGENVECTORS FOR EACH MATRIX
C     MV = 0
C     EPS = 1.0E-6
C     DO 100 IJ=1,M
C     DO 55 I=1,NCT
C     COVT(I) = COV(I,IJ)
C     CALL EIGEN(COVT,PHI(1,I,IJ),N,MV)
C     L = 0
C     DO 60 I =1,N
C     L = L + I
C     GAM(I,IJ) = COVT(L)
C60 COMPUTE DETERMINANT AND INVERSE OF EACH MATRIX
C     DO 65 I=1,NCT
C     COVT(I) = COV(I,IJ)
C     CALL SMINV(COVT,N,DET(IJ),MV,EPS,IER)
C     IF(IER)1000,70,1000
C70 CONTINUE
C     SOET(IJ) = SQRT(DET(IJ))
C     DO 75 I=1,NCT
C     COVIN(I,IJ) = COVT(I)
C75 CONTINUE
C     MV = 0
C     DO 105 I=1,M
C     QP(I) = 0.0
C105
```

FILE. . . SPEST FORTRAN 81

```
C LOOP ON CLASS ICL
C     PC = 0.0
C     DO 500 ICL=1,M
C     AVEQ = 0.0
C LOOP ON THE NUMBER OF SAMPLES
C     NS = 1000
C     DO 300 IJ=1,NS
C GENERATE Y VECTOR FROM CLASS ICL
C     DO 110 I=1,N
C     CALL RANDU(IX,IY,XP)
C     IX = IY
C     CALL NOTRI(XP,Y(I),XD,IER)
C110 CONTINUE
C COMPUTE CONDITIONAL PROBABILITIES FOR EACH CLASS
C     DO 200 JCL=1,M
C     IF(JCL.EQ.ICL) GO TO 180
C     DO 130 I=1,N
C     TE1(I) = 0.0
C     DEL(I) = AM(I,ICL) - AM(I,JCL)
C     DO 130 J=1,N
C     TE1(I) = TE1(I) + SQRT(GAM(J,ICL))*Y(J)*PHI(I,J,ICL)
C130 CONTINUE
C     JJ = 0
C     DO 140 I=1,N
C     DO 140 J=1,I
C     JJ = JJ + 1
C     COVU(I,J) = COVIN(JJ,JCL)
C     COVU(J,I) = COVIN(JJ,JCL)
C140 CONTINUE
C     Z1 = 0.0
C     Z2 = 0.0
C     Z3 = 0.0
C     DO 150 I=1,N
C     DO 150 J=1,N
C     Z1 = Z1 - 0.5*TE1(I)*COVU(I,J)*TE1(J)
C     Z2 = Z2 - TE1(I)*COVU(I,J)*DEL(J)
C     Z3 = Z3 - 0.5*DEL(I)*COVU(I,J)*DEL(J)
C150 CONTINUE
C     ZSUM = Z1 + Z2 + Z3
C     IF(ZSUM.LT.-100) GO TO 190
C     BETA = P(JCL)*1.0
C     PX(JCL) = BETA*DEXP(Z1+Z2+Z3)/SOET(JCL)
C     IF(PX(JCL).EQ.0.0) WRITE(16,919) ICL,JCL,ZSUM,SOET(JCL),PX(JCL)
C170 CONTINUE
C     GO TO 200
C180 CONTINUE
C     ZO = 0.0
C     DO 185 I=1,N
C     ZO = ZO + 0.5*Y(I)*Y(I)
C185 CONTINUE
C     IF(ZO.LT.-100) GO TO 190
C     BETA = P(JCL)*1.0
C     PX(JCL) = BETA*DEXP(ZO)/SOET(JCL)
C     IF(PX(JCL).EQ.0.0) WRITE(16,919) ICL,JCL,ZO,SOET(JCL),PX(JCL)
C     GO TO 200
C190 PX(JCL) = 0.0
C200 CONTINUE
C919 FORMAT(5X,Z19,3E12.4)
C CHOOSE THE LARGEST
C     BIG = -1000
C     DO 220 I=1,M
C     IF(PX(I).GT.BIG) LOC = I
C     IF(PX(I).GT.BIG) BIG = PX(I)
C220 CONTINUE
C     DEN = 0.0
C     DO 230 I=1,M
C     DEN = DEN + PX(I)
```

```

FILE. . . SPEST  FORTRAN  B1
230 CONTINUE
   Q = BIG/DEN
C
AVERAGE
   QP(LOC) = QP(LOC) + P(ICL)*Q/P(LOC)
300 CONTINUE
500 CONTINUE
   WRITE(6,73)
   WRITE(6,536)
536 FORMAT(35X,'STRATIFIED POSTERIOR ERROR ESTIMATOR')
73   FORMAT(1H1)
   DD 510 ICL=1,M
   PR(1CL) = QP(1CL)/FLOAT(NS)
   PC = PC + P(1CL)*PR(1CL)
   WRITE(6,72)
72   FORMAT(77)
   DSPL=2*ICL
   WRITE(6,537) (HEAD(DSPL+1),I=1,2)
537 FORMAT(49X,'CLASS ',2A4)
C
   PCPR=100.*PR(1CL)
   WRITE(6,82)
82   FORMAT(77)
   WRITE(6,482) PCPR
482 FORMAT(25X,'PROBABILITY OF CORRECT CLASS
LS I F I C A T I O N = ',F7.3,'( )')
510 CONTINUE
   RETURN
1000 WRITE(6,1100)IER
1100 FORMAT(10X,'***INVERSION ERROR(',I2,')***')
   RETURN
   END

```

```

.....
SUBROUTINE ADRES
PURPOSE
    FIND THE DESIRED SUBSET OF A COVARIANCE MATRIX
DESCRIPTION OF PARAMETERS
    NADR - ADDRESS ARRAY OF THE ENTRIES TO BE DELETED
    NC   - ARRAY OF THE CHANNELS TO BE DELETED
    SDIM - TOTAL NO OF BANDS SUPPLIED
    NDIM - DESIRED SUBSET OF SDIM
REMARKS
    NONE
SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
    NONE
METHOD
    USING A ONE DIMENSIONAL STORAGE MODE FOR EACH COVARIANCE
    MATRIX IN AN UPPER TRIANGULAR FORM, THE PARAMETRIC ADDRESSES
    FOR EACH ENTRY IS DERIVED AND USING THE NC ARRAY THE LOCATION
    OF ALL THE ENTRIES THAT LIE IN THE UNWANTED LINES AND COLUMNS
    IS COMPUTED AND STORED IN NADR.
.....

```

```

SUBROUTINE ADRES(NADR,NC,SDIM,NDIM)
C
INTEGER*4 NADR(1),PDIM,SDIM
INTEGER*2 NC(1)
C
PDIM=SDIM-NDIM
C
IF(PDIM.LE.0) GO TO 100
DO 20 J=1,PDIM
N1=NC(J)
DO 10 I=1,N1
K=K+1

```

```

FILE. . . SPEST  FORTRAN  B1
10 NADR(K1)=(NC(J)*(NC(J)-1)/2)+I
20 CONTINUE
C
DD 45 J=1,PDIM
NFIN=SDIM-NC(J)+1
DO 35 I=1,NFIN
C
K=K+1
NACR(K)={(NC(J)+I-1)*(NC(J)+I-2)/2)+NC(J)
35 CONTINUE
45 CONTINUE
C
100 RETURN
END

```

AAAAAAAAAAAAAAAAAAAA

ORIGINAL PAGE IS  
OF POOR QUALITY

FILE. . . CORELAT FORTRAN B1

SUPERVISOR FOR A TWO DIMENSIONAL SPATIAL CORRELATION  
 WRITTEN 10/04/78 BIJAN G. MOBASSERI

DESCRIPTION AND PURPOSE

THIS PROGRAM IS A TWO DIMENSIONAL SPATIAL CORRELATOR THE  
 PRIMARY OUTPUT OF WHICH IS A NORMALIZED SPATIAL CORRELATION  
 MATRIX FOR ANY SPECIFIED AREA.

THE USER SPECIFIES THE COORDINATES OF HIS DESIRED AREA IN THE  
 FORM OF INITIAL AND FINAL LINES AND COLUMNS ALONG WITH THE  
 RESPECTIVE SPECTRAL BAND(S). IF THE AUTOCORRELATION FUNCTION  
 IS DESIRED ONLY ONE CHANNEL NUMBER NEED BE SPECIFIED.

FOLLOWING THE ESTIMATION OF THE CORRELATION MATRIX, THE  
 EXPONENTIAL FIT OPTION, IF INVOKED, WILL FIT AN EXPONENTIALLY  
 DROPPING FUNCTION TO THE EXPERIMENTAL DATA USING A WEIGHTED  
 LINEAR LEAST SQUARES FIT TECHNIQUE. IN CASES WHERE THIS  
 ASSUMPTION IS NOT VALID, THIS OPERATION IS BYPASSED.

THE PURPOSE HERE IS TO DEVELOP A MARKOV MODEL FOR THE  
 SPATIAL CORRELATION FUNCTION OF THE MSS DATA.

DESCRIPTION OF CONTROL CARDS

\*CORRELATE

THIS CARD SPECIFIES THE PARTICULAR PROCESSOR REQUESTED

INPUT RUN(.),TAPE(.),FILE(.)

THE INPUT RUNTABLE FROM WHICH DATA IS READ

BLOCK LINE(.,.),COLUMN(.,.)

SEGMENT TO BE CORRELATED

FUNCTION

EITHER AUTO OR CROSS FUNCTION CAN BE SPECIFIED

CHANNELS

SPECTRAL BANDS USED IN CORRELATING

SAMPLELAG

CROSS TRACK LAG USED IN ESTIMATING THE CORRELATION FUNCTION  
 EXPRESSED AS A PERCENTAGE OF TOTAL NO OF SAMPLES IN  
 THE AREA. THIS CARD IS OPTIONAL.

LINELAG

SAME AS SAMPLELAG EXCEPT FOR LINES (ALONG TRACK)

EXPFIT

THIS CARD STARTS THE EXPONENTIAL FITTING PROCEDURE. OPTIONAL

END

END OF CONTROL CARDS

REMARKS

THIS PROGRAM IS CURRENTLY CAPABLE OF PROCESSING AN AREA  
 2401 PIXELS LARGE. SINCE ALL THE SUBROUTINES ARE DYNAMICALLY  
 DIMENSIONED, ANY ENLARGEMENTS CAN BE ACCOMPLISHED BY ALTERING  
 THE DIMENSIONS OF THE ARRAYS IN THE MAIN PROGRAM.

FILE. . . CORELAT FORTRAN B1

HOW TO RUN THE PROGRAM

USE THE COMMAND 'GETDISK DHSYS' TO ESTABLISH THE PROPER LINK  
 THEN TYPE \*CORRELATE.

EXAMPLE OF CONTROL CARD SETUP

```
*CORRELATE
INPUT RUN(74028500),TAPE(2689),FILE(3)
BLOCK LINE(1,25),COLUMN(1,25)
FUNCTION AUTO
CHANNELS 2
SAMPLELAG 25
LINELAG 25
EXPFIT
END
```

OR FOR CROSSCORRELATION,

```
*CORRELATE
INPUT RUN(74028500),TAPE(2689),FILE(3)
BLOCK LINE(1,25),COLUMN(1,25)
FUNCTION CRSS
CHANNELS 1,3
SAMPLELAG 25
LINELAG 25
EXPFIT
END
```

IF SAMPLELAG AND LINELAG ARE LEFT OUT, THE DEFAULT IS 20  
 PERCENT OF THE TOTAL NO OF LINES AND COLUMNS.

\*\*\*\*\*

```
IMPLICIT INTEGER (A-Z)
REAL*4 F(2500),G(2500),R(2500),ICSET(90),RNORM,MEAN1,MEAN2
INTEGER*4 RUNTAB(10,3),ID(200),LDATA1(2500),LDATA2(2500)
INTEGER*4 LIST1(9),COR,'INPUT','BLOC','FUNC','CHAN','SAMP',
1 'LINE','EXP','END',/
2 LIST2(3),RUN,'TAPE','FILE',/
3 LIST3(2),LINE,'COL',/
INTEGER*4 ICARD(20),ICT(25),IVEC(2)
INTEGER*2 CSEL(8),ICSEL(30),ACC(30)
LOGICAL*1 LDATA(2500),FLAG(12)/12*,FALSE,/
COMMON /CORCOM/ FSTLN,LINE,FSTCL,LCOL,NSIZEX,NSIZEY,LAGX,LAGY,
1 PCTX,PCTY

C
1011 FORMAT(1X,'ERROR IN CTLWRD. ERROR CODE=',I3,' EXECUTION TERMINAT
11)
1012 FORMAT(1X,'ERROR IN CTLPRM. EXECUTION TERMINATED')
1013 FORMAT(1X,'ERROR IN IVAL. EXECUTION TERMINATED')
1014 FORMAT(1X,'ERROR IN CHANEL. EXECUTION TERMINATED')
1015 FORMAT(1X,'MISSING CONTROL CARD OR PARAMETER. EXECUTION TERMINATI
11)

C
PCTX=20
PCTY=20
DO 777 I=1,30
NCC(I)=0
ICSEL(I)=0
777 CONTINUE

C
DO 778 I=1,90
ICSET(I)=-50000.0
778 CONTINUE

100 CONTINUE
LSZ=9
IER=0
IVRD=5
CALL CTLWRD (ICARD,ICOL,LIST1,LSZ,ICODE,INRD,IER)
IF (IER.NE.0) GO TO 1001
GO TO (99,101,102,103,104,105,106,107,108),ICODE
99 FLAG(1)=.TRUE.
GO TO 100

C
INPUT RUNTABLE
```

FILE. . . CORELAT FORTRAN B1

```

C 101  LSZ=3
      CALL CTLPRM(ICARD,ICOL,LIST2,LSZ,ICODE,1002)
      LSZ=1
      CALL IVAL(ICARD,ICOL,IVEC,LSZ,1003)
      GO TO (501,502,503), ICODE
C
C 501  RUN NUMBER
      RUNTAB(1,1)=IVEC(1)
      FLAG(2)=.TRUE.
      IF(ICOL.LT.72) GO TO 101
      GO TO 100
C
C 502  TAPE NUMBER
      RUNTAB(1,2)=IVEC(1)
      FLAG(3)=.TRUE.
      IF(ICOL.LT.72) GO TO 101
      GO TO 100
C
C 503  FILE NUMBER
      RUNTAB(1,3)=IVEC(1)
      FLAG(4)=.TRUE.
      IF(ICOL.LT.72) GO TO 101
      GO TO 100
C 102  LSZ=2
      CALL CTLPRM(ICARD,ICOL,LIST3,LSZ,ICODE,1002)
      LSZ=2
      CALL IVAL(ICARD,ICOL,IVEC,LSZ,1003)
      GO TO (601,602), ICODE
C
C 601  NO OF LINES
      FSTLN=IVEC(1)
      LLINE=IVEC(2)
      FLAG(5)=.TRUE.
      IF(ICOL.LT.72) GO TO 102
      GO TO 100
C
C 602  NO OF COLUMNS
      FSTCL=IVEC(1)
      LCOL =IVEC(2)
      FLAG(6)=.TRUE.
      IF(ICOL.LT.72) GO TO 102
      GO TO 100
C
C
C 103  AUTO OR CROSS
      DD 151 I=1,20
      FCT(I)=ICARD(I)
      CONTINUE
      FLAG(7)=.TRUE.
      GO TO 100
C
C
C 104  CHANNELS CARD
      CALL CHANEL(ICARD,ICOL,NCR,ICSEL,ICSET,NCC,1004)
      NOIM=NCR
      FLAG(8)=.TRUE.
      GO TO 100
C
C
C 105  CROSS TRACK LAG
      LSZ=1
      CALL IVAL(ICARD,ICOL,IVEC,LSZ,1003)
      PCTX=IVEC(1)
      FLAG(9)=.TRUE.
      IF(ICOL.LT.72) GO TO 101
      GO TO 100
C
C
C 106  ALONG TRACK LAG
      LSZ=1
      CALL IVAL(ICARD,ICOL,IVEC,LSZ,1003)

```

FILE. . . CORELAT FORTRAN B1

```

      PCTY=IVEC(1)
      FLAG(10)=.TRUE.
      IF(ICOL.LT.72) GO TO 101
      GO TO 100
C
C 107  EXPONENTIAL FIT
      FLAG(11)=.TRUE.
      GO TO 100
C
C 108  END CARD
      FLAG(12)=.TRUE.
C
C
C
      CHECK IF ALL CONTROL CARDS HAVE BEEN READ
      DO 651 I=1,8
      IF(.NOT.FLAG(I)) GO TO 652
      CONTINUE
      IF(.NOT.FLAG(12)) GO TO 652
C
C 651  GO TO 654
      WRITE(16,1015)
      GO TO 999
C
C 652  CONTINUE
      GO TO 654
      WRITE(16,1015)
      GO TO 999
C
C 653  CONTINUE
      GO TO 125
      WRITE(16,1011) IER
      GO TO 999
C
C 654  CONTINUE
      GO TO 125
      WRITE(16,1012)
      GO TO 999
C
C 1001  WRITE(16,1013)
      GO TO 999
C
C 1002  WRITE(16,1014)
      GO TO 999
C
C 1003  WRITE(16,1014)
      GO TO 999
C
C 1004  WRITE(16,1014)
      GO TO 999
C
C 125  NSIZEX=LCOL-FSTCL+1
      NSIZEY=LLINE-FSTLN+1
C
C
C
      LAGX=PCTX*NSIZEX/100
      LAGY=PCTY*NSIZEY/100
C
C
      CALL CRT (F,G,R,LDATA1,LDATA2,RUNTAB,NCC,FCT,FLAG)
C
C 999  STOP
      END

```

ORIGINAL PAGE IS  
OF POOR QUALITY

```

.....
SUBROUTINE CRT
PURPOSE
  TO PERFORM A TWO DIMENSIONAL CORRELATION OF AN IMAGE IN TWO
  ARBITRARY BANDS AND DETERMINE A SPATIAL CORRELATION MATRIX.
DESCRIPTION OF PARAMETERS
  F          - ARRAY TO STORE THE INPUT IMAGE (FIRST CHANNEL)
  G          - ARRAY TO STORE THE INPUT IMAGE (SECOND CHANNEL)
  R          - RESULTING SPATIAL CORRELATION MATRIX
  LDATA1    - STORAGE ARRAY
  LDATA2    - STORAGE ARRAY
  RUNTAB(1,1) - RUN NUMBER OF THE DESIRED AREA
  RUNTAB(1,2) - TAPE NUMBER OF THE DESIRED AREA
  RUNTAB(1,3) - FILE NUMBER OF THE DESIRED AREA
  VCC       - CHANNEL ARRAY
  FCT       - DESIGNATING AUTO OR CROSS FUNCTION
  FLAG      - OPTION ARRAY
REMARKS
  THIS SUBROUTINE HAS VARIABLE DIMENSION PROPERTY. ALL THE
  ARRAYS MUST BE DIMENSIONED IN THE MAIN PROGRAM ACCORDING
  TO THE SIZE OF THE PICTURE.
  THE LAG IN ESTIMATING THE CORRELATION FUNCTION IS USER SUPPLIED
  IT IS ADVISABLE HOWEVER TO KEEP THIS QUANTITY AT
  OR BELOW 20 PERCENT OF THE AREA IN ORDER TO PROVIDE AN
  ESTIMATE WITH SMALL VARIANCE.

```

FILE. . . CORELAT FORTRAN B1

ARRAYS F,G,AND R ARE EMPTY UPON ENTERING THE SUBROUTINE.  
 SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED  
 ACF, TAPOP, GETRUN  
 METHOD

THE STANDARD LAGGED-PRODUCT SUM METHOD IS IMPLEMENTED IN TWO  
 DIMENSIONS AND CARRIED OUT BY SUBROUTINE ACF.

SUBROUTINE CRLT (F,G,R,LDATA1,LDATA2,RUNTAB,NCC,FCT,FLAG)

LOCAL VARIABLES DEFINITION

```

IMPLICIT INTEGER (A-Z)
REAL*4 F(NSIZEZ,NSIZEY),G(NSIZEZ,NSIZEY),R(LAGX,LAGY),
H(5),P(6),S(7),T(8),NCT(3),WR1(2),WR2(2),ERROR(3),
EM(3),KMC(1),RN(3),FHAT(3),RNORM,MEAN1,MEAN2,
ER1,ER2,KHDX,RFOY,C12)
INTEGER*4 RUNTAB(10,3),ID(200),LDATA1(NSIZEZ),LDATA2(NSIZEZ)
INTEGER*4 ACORZ,AUT,CCOR,CRO
INTEGER*4 F1(16),F2(16),F3(16),F4(16),F5(16),F6(16),F7(16)
INTEGER*4 F11(16),F12(16),F13(16),F14(16),F15(16),F16(16)
INTEGER*4 F112(16),F122(16),F132(16),F142(16),F152(16),F162(16)
INTEGER*2 CSEL1(8),NCC(30)
LOGICAL*1 IDATA(2500),FLAG(12)
COMMON /COMCOM/ FSTLN,LLINE,FSTCL,LCOL,NSIZEZ,NSIZEY,LAGX,LAGY,
PCTX,PCTY
    
```

COMMON BLOCK VARIABLES DESCRIPTION

FSTLN - FIRST LINE  
 FSTCL - FIRST COLUMN  
 LLINE - LAST LINE  
 LCOL - LAST COLUMN  
 NSIZEZ - NO OF COLUMNS  
 NSIZEY - NO OF LINES  
 PCTX - LAG AS A PERCENTAGE OF NSIZEZ  
 PCTY - LAG AS A PERCENTAGE OF NSIZEY

```

FORMAT(1X,'OFTRN ERROR=',I2)
FORMAT(1X,'TOPFS ERROR=',I2)
FORMAT(1X,'TOPRV ERROR=',I2)
    
```

INUNIT=12  
 LAG=3

SETUP THE RUNTABLE AND FIND THE PROPER FLIGHTLINE

FSTCL=FSTCL-1

RUNSEL=RUNTAB(1,1)

```

CALL GETRUN (RUNSEL,INUNIT,LD,ERR,RUNTAB,1)
IF(ERR.NE.0) WRITE(16,60) ERR
IF(ERR.NE.0) GO TO 881
    
```

FILE. . . CORELAT FORTRAN B1

NRTRY=2

POSITION THE TAPE AT THE START OF DESIRED RECORD

```

ICOUNT=FSTLN-1
IF(ICOUNT.EQ.0) GO TO 211
CALL TOPFS (INUNIT,ICOUNT,ERR)
IF(ERR.NE.0) WRITE(16,61) ERR
IF(ERR.NE.0) GO TO 881
CONTINUE
    
```

NSC=ID(6)  
 NC=ID(5)

```

IF(FCT(3).EQ.ACOR) DSPL=FSTCL
IF(FCT(3).EQ.CCOR) DSPL=NSC+FSTCL
DO 11 I=1,8
CSEL(I)=0
    
```

```

IF(FCT(3).EQ.ACOR) GO TO 14
CSEL(NCC(1))=1
CSEL(NCC(2))=1
CSEL(NCC(3))=1
    
```

READ IN THE DESIRED AREA

```

DO 30 J=1,NSIZEZ
CALL TOPRV (INUNIT,NSC,ERR,LDATA,NRTRY,NC,CSEL,LDID)
IF(ERR.NE.0) WRITE(16,62) ERR
IF(ERR.NE.0) GO TO 881
    
```

```

CALL MOVBYT (LDATA,FSTCL,1,LDATA1,3,4,NSIZEZ)
CALL MOVBYT (LDATA,DSPL,1,LDATA2,3,4,NSIZEZ)
    
```

```

DO 20 I=1,NSIZEZ
F1(J)=LDATA1(I)
G1(J)=LDATA2(I)
CONTINUE
CONTINUE
    
```

FIND THE MEAN OF THE PICTURE

```

DO 200 J=1,NSIZEZ
DO 200 I=1,NSIZEZ
MEAN1=MEAN1+F1(I,J)
MEAN2=MEAN2+G1(I,J)
CONTINUE
MEAN1=MEAN1/FLOAT(NSIZEZ*NSIZEY)
MEAN2=MEAN2/FLOAT(NSIZEZ*NSIZEY)
    
```

SUBTRACT THE MEAN OUT

```

DO 255 J=1,NSIZEZ
DO 255 I=1,NSIZEZ
F1(I,J)=F1(I,J)-MEAN1
G1(I,J)=G1(I,J)-MEAN2
FORMAT(1X,I2,1X,I2,1X,2(1PE11.4,1X))
CONTINUE
CONTINUE
    
```

COMPUTE THE AUTO(CROSS) CORRELATION FUNCTION

FILE. . . CORELAT FORTRAN B1

```

C      CALL ACF (F,G,R,NSIZEX,NSIZEY,LAGX,LAGY)
C
C*****
C      NORMALIZE THE RESULTING CORRELATION FUNCTION
C*****
      RNORM=R(1,1)
      DO 68 J=1,LAGY
      DO 68 I=1,LAGX
      R(I,J)=R(I,J)/(RNORM)
68      CONTINUE
      IF(.NOT.FLAG(1)) GO TO 902
C*****
      EXPONENTIAL CORRELATION MODEL DEVELOPMENT
C*****
      USE A LINEAR LEAST SQUARE FIT TO THE LOGARITHM OF THE FUNCTION
      IF(R(1,1).LT.R(2,1)) GO TO 901
      IF(R(1,1).LT.R(1,2)) GO TO 901
      DO 831 I=1,LAG
      IF(R(1,I).LE.0) GO TO 901
      FN(I)=-ALOG(R(LAG-(I-1),1))
831      CONTINUE
      CALL LSF (H,WGT,P,S,FN,FHAT,C,IER,ERROR,WR1,WR2,LAG,RMSE,RMU)
      ER1=RMSE(1)
      RHDX=EXP(-C(2))
      DO 832 I=1,LAG
      IF(R(1,I).LE.0) GO TO 901
      FN(I)=-ALOG(R(1,LAG-(I-1)))
832      CONTINUE
      CALL LSF (H,WGT,P,S,FN,FHAT,C,IER,ERROR,WR1,WR2,LAG,RMSE,RMU)
      ER2=RMSE(1)
      RHOY=EXP(-C(2))
      GO TO 902
901      WRITE(16,1013)
1013      WRITE(16,1013)
      FORMAT(1X,'EXPONENTIAL FUNCTION DOES NOT DESCRIBE THE ACF OR CCF O
      IF THIS AREA. REQUEST IGNORED')
902      CONTINUE
C*****
      GENERATE THE OUTPUT
C*****
      IF(FCY(3).EQ.ACOR) NCC(2)=NCC(1)
      FMT1(3)=LAGX*N1
      IF(LAGX.GE.10) FMT1(3)=N1+9
      FMT2(2)=FMT1(3)
      WRITE(6,1000)
1000      FORMAT(1H1)
      WRITE(6,1001)
1001      FORMAT(40X,'TWO DIMENSIONAL SPATIAL CORRELATION ANALYSIS')
      WRITE(6,1002)
1002      FORMAT(//)
      WRITE(6,1003) NCC(1),NCC(2)
1003      FORMAT(50X,'CHANNELSO ',2(I2,1X))
      WRITE(6,1002)
      WRITE(6,1004) (CLASS(I+2),I=1,2)
C
1004      FORMAT(40X,'CLASS',2A4)

```

FILE. . . CORELAT FORTRAN B1

```

      WRITE(6,1005)
      FORMAT(//)
      WRITE(6,1006)
1006      FORMAT(30X,'2-D SPATIAL CORRELATION MATRIX')
      WRITE(6,1002)
      DO 320 J=1,LAGY
      WRITE(6,FMT1) (R(I,J),I=1,LAGX)
320      CONTINUE
C
      IF(R(1,1).LT.R(2,1)) GO TO 671
      IF(R(1,1).LT.R(1,2)) GO TO 671
      IF(.NOT.FLAG(1)) GO TO 671
C
      WRITE(6,1005)
      WRITE(6,1007)
1007      FORMAT(40X,'WEIGHTED LEAST SQUARES FIT INFORMATION')
      WRITE(6,1002)
      WRITE(6,1008) RMU
1008      FORMAT(30X,'WEIGHTING MATRIX DIAGONAL BASE=',F4.2)
      WRITE(6,1002)
      WRITE(6,1009) ER1
1009      FORMAT(30X,'WEIGHTED LSF ERROR (CROSS TRACK)='E14.7)
      WRITE(6,1002)
      WRITE(6,1010) KFOX
1010      FORMAT(30X,'ADJACENT SAMPLE CORRELATION='E14.7)
      WRITE(6,1002)
      WRITE(6,1011) ER2
1011      FORMAT(30X,'WEIGHTED LSF ERROR (ALONG TRACK)='E14.7)
      WRITE(6,1002)
      WRITE(6,1012) RHOY
1012      FORMAT(30X,'ADJACENT LINE CORRELATION   ='E14.7)
C
671      CONTINUE
      WRITE(16,1002)
C
2001      FORMAT(//)
      WRITE(16,2002)
2002      FORMAT(15X,'TWO DIMENSIONAL SPATIAL CORRELATION ANALYSIS')
      WRITE(16,2001)
      WRITE(16,2003) NCC(1),NCC(2)
2003      FORMAT(20X,'CHANNELSO ',2(I2,1X))
      WRITE(16,2001)
      WRITE(16,2004)
2004      FORMAT(17X,'2-D SPATIAL CORRELATION MATRIX')
      WRITE(16,2001)
      DO 330 J=1,LAGY
      WRITE(16,FMT2) (R(I,J),I=1,LAGX)
330      CONTINUE
      WRITE(16,1005)
881      RETURN
      END
C*****
      USAGE
      CALL ACF (F,G,R,NSIZEX,NSIZEY,LAGX,LAGY)
C*****
      DESCRIPTION OF PARAMETERS
      F - ARRAY CONTAINING THE AREA TO BE CORRELATED(1ST COLUMN)
      G - ARRAY CONTAINING THE AREA TO BE CORRELATED(2ND COLUMN)
      R - RESULTING SPATIAL CORRELATION MATRIX
      NSIZEX - NUMBER OF COLUMNS IN THE PICTURE
      NSIZEY - NUMBER OF LINES IN THE PICTURE
      LAGX - LAG IN PIXELS ALONG THE COLUMNS
      LAGY - LAG IN PIXELS ALONG THE LINES
C*****
      REMARKS
      NONE
C*****
      SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
      NONE
C*****
      METHOD
      THE LAGGED PRODUCT SUM METHOD IS CARRIED OUT

```

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FILE. . . CORELAT FORTRAN B1

.....  
SUBROUTINE ACF (F,G,R,NSIZE,NSIZEY,LAGX,LAGY)

.....  
LOCAL VARIABLES DEFINITION

.....  
IMPLICIT INTEGER (A-Z)  
REAL\*4 F(NSIZE,NSIZEY),G(NSIZE,NSIZEY),R(LAGX,LAGY),  
1 MEAN1,MEAN2,RDIM  
RDIM=NSIZE\*NSIZEY

.....  
SUM THE LAGGED PRODUCTS

.....  
DO 10 I=1,LAGX  
DO 10 J=1,LAGY  
DO 30 K=1,NSIZEY  
DO 30 L=1,NSIZE

IF((I+TAU-1).GT.NSIZE).OR.(J+ETA-1).GT.NSIZEY) GO TO 25

R(I,ETA)=R(I,ETA)+G(I+TAU-1,J+ETA-1)\*F(I,J)  
CONTINUE  
R(I,ETA)=R(I,ETA)/RDIM  
CONTINUE

RETURN  
END

.....  
SUBROUTINE LSF

PURPOSE

TO FIT A LINEAR FUNCTION THROUGH A DATA SET,USING LEAST SQUARES

DESCRIPTION OF PARAMETERS

H - MATRIX OF VALUES OF LINEAR FUNCTIONS  
WGT - WEIGHTING MATRIX  
P - TRANSPOSE\*WGT  
R - P\*H  
S - R-INVERSE\*P  
C - LEAST SQUARES COEFF.  
F - MATRIX OF DATA VALUES  
FHT - ESTIMATE OF F  
ERR - WEIGHTED F-FHAT  
LAG - NO OF DATA VALUES  
WR1 - WORK VECTOR  
WR2 - WORK VECTOR

REMARKS

THE EQUATION OF THE LINEAR FIT IS OF THE FORM,C1+C2\*X  
THE FOLLOWING MATRICES ARE DIMENSIONED 2\*LAG

H,P,S

THE FOLLOWING MATRICES ARE DIMENSIONED LAC IN THE MAIN PROG

WGT,WR1,WR2,ERROR,ERR,FN,FHAT

FILE. . . CORELAT FORTRAN B1

THE FOLLOWING MATRICES HAVE CONSTANT DIMENSIONS  
R(4),RMSE(1),C(2) ,

SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED  
TPRO,GMPRD,MINV

METHOD  
GENERALIZED LEAST SQUARES TECHNIQUE IS IMPLEMENTED.FORMULA USE  
(HTR\*HGI\*H)\*\*(-1)\*HTR\*WGT\*F  
FOR FURTHER DETAILS SEE 'DICRETE PARAMETER ESTIMATION ',BY  
J.M.MENDEL,1973

.....  
SUBROUTINE LSF (H,WGT,P,S,F,FHAT,C,ERR,ERROR,WR1,WR2,LAG,RMSE,RMU)

.....  
LOCAL VARIABLES DEFINITION

.....  
IMPLICIT INTEGER (A-Z)  
REAL\*4 H(1),WGT(1),P(1),S(1),F(1),FHAT(1),C(1),ERR(1),ERROR(1)  
1 WR1(1),WR2(1),RMSE(1),R(4),RMU,DET

80 FORMAT(//)  
81 FORMAT(//)  
NPNTS=LAG

DO 30 I=1,NPNTS  
H(I)=1  
CONTINUE

K1=NPNTS+1  
K2=2\*NPNTS  
DO 40 I=K1,K2  
H(I)=K2-I  
CONTINUE

.....  
\* INITIALIZE THE WEIGHTING MATRIX

RMU=0.4

DO 28 I=1,NPNTS  
K=NPNTS-I  
WGT(I)=RMU\*\*K  
CONTINUE

.....  
DETERMINE THE LEAST SQUARES FIT COEFFICIENTS

.....  
N=NPNTS  
M=2  
MSA=0  
MSB=2  
L=NPNTS

CALL TPRD (H,WGT,P,N,M,MSA,MSB,L)

N=2  
H=NPNTS  
L=2



SUPERVISOR FOR THE COMPUTATION OF THE SCANNER OUTPUT STATISTICS  
 WRITTEN 09/25/78 BIJAN G. MOBASSERI

DESCRIPTION AND PURPOSE

THE SCANNER OUTPUT STATISTICS PROGRAM IMPLEMENTS A LINEAR TRANSFORMATION TO OBTAIN A SET OF STATISTICS AT THE OUTPUT OF A MULTIBAND SCANNING RADIOMETER IN TERMS OF THE CORRESPONDING INPUT QUANTITIES. THIS TRANSFORMATION IS ACCOMPLISHED BY THE COMPUTATION OF THE 'SCANNER CHARACTERISTIC FUNCTION', AN ANALYTICAL EXPRESSION, THE MAIN PARAMETERS OF WHICH ARE THE IFOV SIZE AND INFORMATION ON DATA SPATIAL CORRELATION.

DESCRIPTION OF CONTROL CARDS

\*SCANSTAT

THIS CARD SPECIFIES THE PARTICULAR PROCESSOR REQUESTED

CHANNELS

THE DESIRED SUBSET OF THE AVAILABLE CHANNELS IS GIVEN HERE. IT IS IMPORTANT TO REMEMBER THAT THE NUMBERS APPEARING ON THIS CARD ARE THE ORDER OF THE SELECTED CHANNELS NOT THEIR ACTUAL NUMBER. FOR EXAMPLE IF THE AVAILABLE CHANNELS ARE 8,9,12,14 AND CHANNELS 8,9 AND 14 ARE SELECTED, CHANNELS CARD SHOULD READ 1,2,4

CLASSES

THIS CARD SPECIFIES THE NAME OF EACH CLASS. EACH NAME MUST BE PLACED IN A FIELD 7 CHARACTER LONG FOLLOWED BY A BLANK. THE CONTINUATION CARD, IF REQUIRED, MUST HAVE THE WORD 'CLASSES' IN THE BEGINNING FOLLOWED BY THE REST OF THE NAMES.

IFOV

THIS CARD SPECIFIES THE SPATIAL RESOLUTION OF THE OUTPUT DATA IN TERMS OF THE INPUT. BASICALLY IT IS THE NUMBER OF HIGH RESOLUTION PIXELS WITHIN ONE IFOV OF THE SCANNER. E. G. FOR MSS OPERATING ON A 6 METER DATA AND IFOV=5 ACCORDING TO THE ABOVE CONVENTION THE ACTUAL SPATIAL RESOLUTION IS 30 METERS

APERTURE

THE CHOICES HERE ARE 'GAUSSIAN' OR 'RECTANGULAR'

SNR (OPTIONAL)

THIS CARD SIMULATES THE EFFECT OF RANDOM ADDITIVE NOISE ON THE POPULATIONS STATISTICS AT THE SCANNER OUTPUT. THE NOISE COVARIANCE MATRIX IS DIAGONAL WITH OFF DIAGONAL ELEMENTS EQUAL TO ZERO. THE SNR IS DEFINED AS THE RATIO OF SIGNAL ENERGY (DIAGONAL ELEMENTS) TO NOISE ENERGY. SNR MUST BE GIVEN IN DECIBELS DEFINED  $SNR = 10 \cdot \log_{10}(\text{SIGNAL ENERGY/NOISE ENERGY})$ . THIS SNR WILL BE THE SAME IN ALL CHANNELS.

PUNCH (OPTIONAL)

IF PRESENT THE OUTPUT STATISTICS IS PUNCHED OUT. NOTE THAT SOME BLANK CARDS ARE INCLUDED IN THE DECK FOR COMPATIBILITY REASONS

END

THIS CARD SIGNALS THE END OF CONTROL CARDS. DATA FOLLOWS IMMEDIATELY

INPUT DATA STRUCTURE

INPUT TO \*SUS CONSISTS OF 3 SEPARATE DECKS  
 1- LARSYS STATISTICS DECK WITH NO CHANGES IT SHOULD HOWEVER BE IN \*\* CHARACTER FORMAT \*\*  
 2- SPATIAL CORRELATION PARAMETERS ARE ENTERED VIA A NDIM X NDIM

MATRIX. THE (I,J) ELEMENT OF IT IS THE PIXEL-TO PIXEL CORRELATION IN CHANNELS I AND J. THIS DECK CORRESPONDS TO THE CROSS TRACK CORRELATION.

3- SAME AS 2 EXCEPT FOR ALONG TRACK DIRECTION.

REMARKS

THIS PROGRAM IS CURRENTLY CAPABLE OF PROCESSING UP TO 20 CLASSES AND 8 SPECTRAL BANDS. THE EXECUTION TIME IS QUITE SHORT AND EXTENSION TO A HIGHER NO OF CLASSES AND DIMENSIONS PRESENTS NO PARTICULAR PROBLEM. THE STATISTICS DECK PRODUCED HERE DOES NOT CARRY A SEQUENCE NUMBER IN THE 72-80 COLUMNS.

HOW TO RUN THE PROGRAM

THE SOURCE AND TEXT FILES ARE LOCATED ON THE DHDISK AND DHSYS DISKS RESPECTIVELY. AUTHORIZED TD'S ARE AUTOMATICALLY LINKED TO BOTH DISKS AT LOGIN TIME. OTHERWISE ANYONE CAN ACCESS THE DISKS THRU THE COMMANDS 'GETDISK DHSYS' FOR TEXT AND 'GETDISK DHDISK' FOR THE SOURCE. AFTER THE PROPER LINKS ARE ESTABLISHED, TYPE IN \*SOS.

EXAMPLE OF THE CONTROL CARD SET UP

```
*SCANSTAT
CHANNELS 1,2,4
CLASSES BARESO1 CORN SOYBEAN WHEAT
IFOV 2
APERTURE GAUSSIAN
PUNCH
END
```

```
*****
INTEGER*4 LIST(8)/*SCA*,*CHAN*,*CLAS*,*IFOV*,*APER*,*SNR*,*PUNC*,
1*END*,*
INTEGER*4 IVEC(1),ICARD(20),NADR(720)
INTEGER*4 BLANK/* /FSTCRD/LARS*/
INTEGER*4 SDIM,SIGMAX,SIGMAY,HEAD(20),APERT(20)
INTEGER*4 FHT1(5)/*1,7,11,15,19,23,27,31,35,39,43,47,51,55,59,63,67,71,75,79,83,87,91,95,99,103,107,111,115,119,123,127,131,135,139,143,147,151,155,159,163,167,171,175,179,183,187,191,195,199,203,207,211,215,219,223,227,231,235,239,243,247,251,255,259,263,267,271,275,279,283,287,291,295,299,303,307,311,315,319,323,327,331,335,339,343,347,351,355,359,363,367,371,375,379,383,387,391,395,399,403,407,411,415,419,423,427,431,435,439,443,447,451,455,459,463,467,471,475,479,483,487,491,495,499,503,507,511,515,519,523,527,531,535,539,543,547,551,555,559,563,567,571,575,579,583,587,591,595,599,603,607,611,615,619,623,627,631,635,639,643,647,651,655,659,663,667,671,675,679,683,687,691,695,699,703,707,711,715,719,723,727,731,735,739,743,747,751,755,759,763,767,771,775,779,783,787,791,795,799,803,807,811,815,819,823,827,831,835,839,843,847,851,855,859,863,867,871,875,879,883,887,891,895,899,903,907,911,915,919,923,927,931,935,939,943,947,951,955,959,963,967,971,975,979,983,987,991,995,999,1003,1007,1011,1015,1019,1023,1027,1031,1035,1039,1043,1047,1051,1055,1059,1063,1067,1071,1075,1079,1083,1087,1091,1095,1099,1103,1107,1111,1115,1119,1123,1127,1131,1135,1139,1143,1147,1151,1155,1159,1163,1167,1171,1175,1179,1183,1187,1191,1195,1199,1203,1207,1211,1215,1219,1223,1227,1231,1235,1239,1243,1247,1251,1255,1259,1263,1267,1271,1275,1279,1283,1287,1291,1295,1299,1303,1307,1311,1315,1319,1323,1327,1331,1335,1339,1343,1347,1351,1355,1359,1363,1367,1371,1375,1379,1383,1387,1391,1395,1399,1403,1407,1411,1415,1419,1423,1427,1431,1435,1439,1443,1447,1451,1455,1459,1463,1467,1471,1475,1479,1483,1487,1491,1495,1499,1503,1507,1511,1515,1519,1523,1527,1531,1535,1539,1543,1547,1551,1555,1559,1563,1567,1571,1575,1579,1583,1587,1591,1595,1599,1603,1607,1611,1615,1619,1623,1627,1631,1635,1639,1643,1647,1651,1655,1659,1663,1667,1671,1675,1679,1683,1687,1691,1695,1699,1703,1707,1711,1715,1719,1723,1727,1731,1735,1739,1743,1747,1751,1755,1759,1763,1767,1771,1775,1779,1783,1787,1791,1795,1799,1803,1807,1811,1815,1819,1823,1827,1831,1835,1839,1843,1847,1851,1855,1859,1863,1867,1871,1875,1879,1883,1887,1891,1895,1899,1903,1907,1911,1915,1919,1923,1927,1931,1935,1939,1943,1947,1951,1955,1959,1963,1967,1971,1975,1979,1983,1987,1991,1995,1999,2003,2007,2011,2015,2019,2023,2027,2031,2035,2039,2043,2047,2051,2055,2059,2063,2067,2071,2075,2079,2083,2087,2091,2095,2099,2103,2107,2111,2115,2119,2123,2127,2131,2135,2139,2143,2147,2151,2155,2159,2163,2167,2171,2175,2179,2183,2187,2191,2195,2199,2203,2207,2211,2215,2219,2223,2227,2231,2235,2239,2243,2247,2251,2255,2259,2263,2267,2271,2275,2279,2283,2287,2291,2295,2299,2303,2307,2311,2315,2319,2323,2327,2331,2335,2339,2343,2347,2351,2355,2359,2363,2367,2371,2375,2379,2383,2387,2391,2395,2399,2403,2407,2411,2415,2419,2423,2427,2431,2435,2439,2443,2447,2451,2455,2459,2463,2467,2471,2475,2479,2483,2487,2491,2495,2499,2503,2507,2511,2515,2519,2523,2527,2531,2535,2539,2543,2547,2551,2555,2559,2563,2567,2571,2575,2579,2583,2587,2591,2595,2599,2603,2607,2611,2615,2619,2623,2627,2631,2635,2639,2643,2647,2651,2655,2659,2663,2667,2671,2675,2679,2683,2687,2691,2695,2699,2703,2707,2711,2715,2719,2723,2727,2731,2735,2739,2743,2747,2751,2755,2759,2763,2767,2771,2775,2779,2783,2787,2791,2795,2799,2803,2807,2811,2815,2819,2823,2827,2831,2835,2839,2843,2847,2851,2855,2859,2863,2867,2871,2875,2879,2883,2887,2891,2895,2899,2903,2907,2911,2915,2919,2923,2927,2931,2935,2939,2943,2947,2951,2955,2959,2963,2967,2971,2975,2979,2983,2987,2991,2995,2999,3003,3007,3011,3015,3019,3023,3027,3031,3035,3039,3043,3047,3051,3055,3059,3063,3067,3071,3075,3079,3083,3087,3091,3095,3099,3103,3107,3111,3115,3119,3123,3127,3131,3135,3139,3143,3147,3151,3155,3159,3163,3167,3171,3175,3179,3183,3187,3191,3195,3199,3203,3207,3211,3215,3219,3223,3227,3231,3235,3239,3243,3247,3251,3255,3259,3263,3267,3271,3275,3279,3283,3287,3291,3295,3299,3303,3307,3311,3315,3319,3323,3327,3331,3335,3339,3343,3347,3351,3355,3359,3363,3367,3371,3375,3379,3383,3387,3391,3395,3399,3403,3407,3411,3415,3419,3423,3427,3431,3435,3439,3443,3447,3451,3455,3459,3463,3467,3471,3475,3479,3483,3487,3491,3495,3499,3503,3507,3511,3515,3519,3523,3527,3531,3535,3539,3543,3547,3551,3555,3559,3563,3567,3571,3575,3579,3583,3587,3591,3595,3599,3603,3607,3611,3615,3619,3623,3627,3631,3635,3639,3643,3647,3651,3655,3659,3663,3667,3671,3675,3679,3683,3687,3691,3695,3699,3703,3707,3711,3715,3719,3723,3727,3731,3735,3739,3743,3747,3751,3755,3759,3763,3767,3771,3775,3779,3783,3787,3791,3795,3799,3803,3807,3811,3815,3819,3823,3827,3831,3835,3839,3843,3847,3851,3855,3859,3863,3867,3871,3875,3879,3883,3887,3891,3895,3899,3903,3907,3911,3915,3919,3923,3927,3931,3935,3939,3943,3947,3951,3955,3959,3963,3967,3971,3975,3979,3983,3987,3991,3995,3999,4003,4007,4011,4015,4019,4023,4027,4031,4035,4039,4043,4047,4051,4055,4059,4063,4067,4071,4075,4079,4083,4087,4091,4095,4099,4103,4107,4111,4115,4119,4123,4127,4131,4135,4139,4143,4147,4151,4155,4159,4163,4167,4171,4175,4179,4183,4187,4191,4195,4199,4203,4207,4211,4215,4219,4223,4227,4231,4235,4239,4243,4247,4251,4255,4259,4263,4267,4271,4275,4279,4283,4287,4291,4295,4299,4303,4307,4311,4315,4319,4323,4327,4331,4335,4339,4343,4347,4351,4355,4359,4363,4367,4371,4375,4379,4383,4387,4391,4395,4399,4403,4407,4411,4415,4419,4423,4427,4431,4435,4439,4443,4447,4451,4455,4459,4463,4467,4471,4475,4479,4483,4487,4491,4495,4499,4503,4507,4511,4515,4519,4523,4527,4531,4535,4539,4543,4547,4551,4555,4559,4563,4567,4571,4575,4579,4583,4587,4591,4595,4599,4603,4607,4611,4615,4619,4623,4627,4631,4635,4639,4643,4647,4651,4655,4659,4663,4667,4671,4675,4679,4683,4687,4691,4695,4699,4703,4707,4711,4715,4719,4723,4727,4731,4735,4739,4743,4747,4751,4755,4759,4763,4767,4771,4775,4779,4783,4787,4791,4795,4799,4803,4807,4811,4815,4819,4823,4827,4831,4835,4839,4843,4847,4851,4855,4859,4863,4867,4871,4875,4879,4883,4887,4891,4895,4899,4903,4907,4911,4915,4919,4923,4927,4931,4935,4939,4943,4947,4951,4955,4959,4963,4967,4971,4975,4979,4983,4987,4991,4995,4999,5003,5007,5011,5015,5019,5023,5027,5031,5035,5039,5043,5047,5051,5055,5059,5063,5067,5071,5075,5079,5083,5087,5091,5095,5099,5103,5107,5111,5115,5119,5123,5127,5131,5135,5139,5143,5147,5151,5155,5159,5163,5167,5171,5175,5179,5183,5187,5191,5195,5199,5203,5207,5211,5215,5219,5223,5227,5231,5235,5239,5243,5247,5251,5255,5259,5263,5267,5271,5275,5279,5283,5287,5291,5295,5299,5303,5307,5311,5315,5319,5323,5327,5331,5335,5339,5343,5347,5351,5355,5359,5363,5367,5371,5375,5379,5383,5387,5391,5395,5399,5403,5407,5411,5415,5419,5423,5427,5431,5435,5439,5443,5447,5451,5455,5459,5463,5467,5471,5475,5479,5483,5487,5491,5495,5499,5503,5507,5511,5515,5519,5523,5527,5531,5535,5539,5543,5547,5551,5555,5559,5563,5567,5571,5575,5579,5583,5587,5591,5595,5599,5603,5607,5611,5615,5619,5623,5627,5631,5635,5639,5643,5647,5651,5655,5659,5663,5667,5671,5675,5679,5683,5687,5691,5695,5699,5703,5707,5711,5715,5719,5723,5727,5731,5735,5739,5743,5747,5751,5755,5759,5763,5767,5771,5775,5779,5783,5787,5791,5795,5799,5803,5807,5811,5815,5819,5823,5827,5831,5835,5839,5843,5847,5851,5855,5859,5863,5867,5871,5875,5879,5883,5887,5891,5895,5899,5903,5907,5911,5915,5919,5923,5927,5931,5935,5939,5943,5947,5951,5955,5959,5963,5967,5971,5975,5979,5983,5987,5991,5995,5999,6003,6007,6011,6015,6019,6023,6027,6031,6035,6039,6043,6047,6051,6055,6059,6063,6067,6071,6075,6079,6083,6087,6091,6095,6099,6103,6107,6111,6115,6119,6123,6127,6131,6135,6139,6143,6147,6151,6155,6159,6163,6167,6171,6175,6179,6183,6187,6191,6195,6199,6203,6207,6211,6215,6219,6223,6227,6231,6235,6239,6243,6247,6251,6255,6259,6263,6267,6271,6275,6279,6283,6287,6291,6295,6299,6303,6307,6311,6315,6319,6323,6327,6331,6335,6339,6343,6347,6351,6355,6359,6363,6367,6371,6375,6379,6383,6387,6391,6395,6399,6403,6407,6411,6415,6419,6423,6427,6431,6435,6439,6443,6447,6451,6455,6459,6463,6467,6471,6475,6479,6483,6487,6491,6495,6499,6503,6507,6511,6515,6519,6523,6527,6531,6535,6539,6543,6547,6551,6555,6559,6563,6567,6571,6575,6579,6583,6587,6591,6595,6599,6603,6607,6611,6615,6619,6623,6627,6631,6635,6639,6643,6647,6651,6655,6659,6663,6667,6671,6675,6679,6683,6687,6691,6695,6699,6703,6707,6711,6715,6719,6723,6727,6731,6735,6739,6743,6747,6751,6755,6759,6763,6767,6771,6775,6779,6783,6787,6791,6795,6799,6803,6807,6811,6815,6819,6823,6827,6831,6835,6839,6843,6847,6851,6855,6859,6863,6867,6871,6875,6879,6883,6887,6891,6895,6899,6903,6907,6911,6915,6919,6923,6927,6931,6935,6939,6943,6947,6951,6955,6959,6963,6967,6971,6975,6979,6983,6987,6991,6995,6999,7003,7007,7011,7015,7019,7023,7027,7031,7035,7039,7043,7047,7051,7055,7059,7063,7067,7071,7075,7079,7083,7087,7091,7095,7099,7103,7107,7111,7115,7119,7123,7127,7131,7135,7139,7143,7147,7151,7155,7159,7163,7167,7171,7175,7179,7183,7187,7191,7195,7199,7203,7207,7211,7215,7219,7223,7227,7231,7235,7239,7243,7247,7251,7255,7259,7263,7267,7271,7275,7279,7283,7287,7291,7295,7299,7303,7307,7311,7315,7319,7323,7327,7331,7335,7339,7343,7347,7351,7355,7359,7363,7367,7371,7375,7379,7383,7387,7391,7395,7399,7403,7407,7411,7415,7419,7423,7427,7431,7435,7439,7443,7447,7451,7455,7459,7463,7467,7471,7475,7479,7483,7487,7491,7495,7499,7503,7507,7511,7515,7519,7523,7527,7531,7535,7539,7543,7547,7551,7555,7559,7563,7567,7571,7575,7579,7583,7587,7591,7595,7599,7603,7607,7611,7615,7619,7623,7627,7631,7635,7639,7643,7647,7651,7655,7659,7663,7667,7671,7675,7679,7683,7687,7691,7695,7699,7703,7707,7711,7715,7719,7723,7727,7731,7735,7739,7743,7747,7751,7755,7759,7763,7767,7771,7775,7779,7783,7787,7791,7795,7799,7803,7807,7811,7815,7819,7823,7827,7831,7835,7839,7843,7847,7851,7855,7859,7863,7867,7871,7875,7879,7883,7887,7891,7895,7899,7903,7907,7911,7915,7919,7923,7927,7931,7935,7939,7943,7947,7951,7955,7959,7963,7967,7971,7975,7979,7983,7987,7991,7995,7999,8003,8007,8011,8015,8019,8023,8027,8031,8035,8039,8043,8047,8051,8055,8059,8063,8067,8071,8075,8079,8083,8087,8091,8095,8099,8103,8107,8111,8115,8119,8123,8127,8131,8135,8139,8143,8147,8151,8155,8159,8163,8167,8171,8175,8179,8183,8187,8191,8195,8199,8203,8207,8211,8215,8219,8223,8227,8231,8235,8239,8243,82
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FILE. . . SCANSTAT FORTRAN B1

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C*****
C
C DO 777 I=1,30
C ICSEL(I)=0
C CONTINUE
777
C DO 778 I=1,90
C ICSET(I)=-50000.0
778
C CONTINUE
C LSZ=8
C IER=0
C INIO=5
C
C CALL CTLWRD (ICARD,ICOL,LIST,LSZ,ICODE,INRD,IER)
C IF (IER.NE.0) GO TO 1001
C GO TO (99,101,102,103,104,105,106,107),ICODE
99 FLAG(1)=.TRUE.
C GO TO 100
C
C CHANNELS CARD
101 CALL CHANEL (ICARD,ICOL,NCR,ICSEL,ICSET,NCC,900)
C FLAG(2)=.TRUE.
C NDIR=NCR
C GO TO 100
C
C CLASS NAMES CARD
102 DO 10 I=1,20
C HEAD(I)=ICARD(I)
C FLAG(3)=.TRUE.
C GO TO 100
10
C
C IFOV SIZE SPECIFICATION
103 LSZ=1
C CALL IVAL (ICARD,ICOL,IVEC,LSZ,1002)
C FLAG(4)=.TRUE.
C SIGMAX=IVEC(1)
C SIGMAY=SIGMAX
C GO TO 100
C
C IFOV SHAPE SPECIFICATION
104 DO 30 I=1,20
C APER(I)=ICARD(I)
C FLAG(5)=.TRUE.
C GO TO 100
30
C
C SIGNAL TO NOISE RATIO
105 LSZ=1
C CALL IVAL (ICARD,ICOL,IVEC,LSZ,1002)
C SYN=IVEC(1)
C FLAG(6)=.TRUE.
C GO TO 100
C
C TO PUNCH OR NOT TO PUNCH
106 FLAG(7)=.TRUE.
C GO TO 100
C
C END CARD
107 LSZ=1
C FLAG(8)=.TRUE.
C GO TO 201
C
C 1002 WRITE (6,1011)
C WRITE (6,1011)
C GO TO 999
C
C 900 WRITE (6,1012)
C WRITE (6,1012)

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FILE. . . SCANSTAT FORTRAN B1

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201 GO TO 999
C CONTINUE
C
C CHECK IF ALL CONTROL CARDS HAVE BEEN READ
C
C DO 250 I=1,5
C IF (.NOT.FLAG(I)) GO TO 321
C GO TO 250
321 WRITE (6,1013)
C WRITE (6,1013)
C GO TO 999
C
C 250 CONTINUE
C
C IF (.NOT.FLAG(8)) GO TO 321
C
C GO TO 680
1001 WRITE (6,1010)
C WRITE (6,1010)
C GO TO 999
680 CONTINUE
C
C*****
C READ THE TOTAL NO OF CHANNELS AND CLASSES FROM THE STAT DECK
C*****
502 READ (5,501) ICRD
501 FORMAT (A4)
C ICRDSQ=ICRSQ+1
C
C IF (ICRD.EQ.BLANK) GO TO 503
C GO TO 502
503 CONTINUE
C
C REWIND 5
602 READ (5,501) ICRD
C IF (ICRD.EQ.FSTCRD) GO TO 601
C GO TO 602
601 CONTINUE
C
C NUM=ICRSQ-2
C DO 506 I=1,NUM
C READ (5,507)
507 FORMAT (18A4,18)
C CONTINUE
C
C 1014 READ (5,1014) NCLS,NFLD,SDIM
C FORMAT (15,6X,15,6X,15)
C
C NUM2=SDIM+1
C DO 509 I=1,NUM2
C READ (5,507)
509 CONTINUE
C
C NSIZE=SDIM*(SDIM+1)/2
C ISIZE=NDIM*(NDIM+1)/2
C
C NTS=NCLS*ISIZE
C NTH=NCLS*NDIM
C NSM=NCLS*SDIM
C NSA=NCLS*NSIZE
C
C FIND THE CHANNEL SET THAT IS NOT REQUESTED
C
C K=0
C DO 611 I=1,SDIM
C DO 612 J=1,NDIM
C
C IF (I.EQ.NCC(J)) GO TO 611
C
C 612 CONTINUE

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ORIGINAL PAGE IS  
OF POOR QUALITY

FILE. . . SCANSTAT FORTRAN B1

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      K=K+1
      NC(K)=1
011  CONTINUE
*****
      START COMPUTATION OF THE STATISTICS AT THE MSS OUTPUT
*****
      CALL SCANNER (COVIN,COVOUT,SUBCOV,SLPX,SLPY,SUBSLX,SUBSLY,PX,PY,
      1  C1,C2,C,NC,NCC,RHO,NADR,FLAG,MU,SUBMU,APERT)
*****
      GENERATE THE OUTPUT
*****
      DO 842 JCLS=1,NCLS
          FMT1(2)=N1
          KS=2+JCLS
          NS=(JCLS-1)*ISIZE
          WRITE(6,1015)
          WRITE(6,1016)
          WRITE(6,1016)
          WRITE(6,1017)
          WRITE(6,1018)
          WRITE(6,1016)
          WRITE(6,1016)
          WRITE(6,1019) (APERT(1+2),I=1,3)
          WRITE(6,1020) (APERT(1+2),I=1,3)
          WRITE(6,1016)
          WRITE(6,1016)
          WRITE(6,1021) SIGMAX
          WRITE(6,1022) SIGMAX
          WRITE(6,1027)
          WRITE(6,1016)
          WRITE(6,1023) (HEAD(KS+1),I=1,2)
          WRITE(6,1024) (HEAD(KS+1),I=1,2)
          WRITE(6,1016)
          WRITE(6,1027)
          WRITE(6,1025)
          WRITE(6,1026)
          WRITE(6,1016)
          WRITE(6,1016)
      DO 841 KDIM=1,NDIM
          ISTART=KDIM*(KDIM-1)/2+1
          ISTOP=KDIM*(KDIM+1)/2
          FMT1(2)=FMT1(2)+1
          FMT2(2)=FMT1(2)
          FMT3(2)=FMT1(2)
          FMT4(2)=FMT1(2)
          WRITE(6,FMT1) (SUBCOV(I+MS),I=ISTRT,ISTOP)
          WRITE(6,FMT2) (COVOUT(J+MS),J=ISTRT,ISTOP)
          WRITE(6,FMT3) (SUBCOV(I+MS),I=ISTRT,ISTOP)
          WRITE(6,FMT4) (COVOUT(J+MS),J=ISTRT,ISTOP)
          WRITE(6,1016)
041  CONTINUE
042  CONTINUE
999  STOP
      END

```

FILE. . . SCANSTAT FORTRAN B1

SUBROUTINE SCANNER

PURPOSE

TO COMPUTE THE SCANNER CHARACTERISTICS FUNCTION AND GENERATE THE TRANSFORMED STATISTICS

DESCRIPTION OF PARAMETERS

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APERT  = ARRAY CONTAINING THE IFDV SHAPE
C1     = ONE DIMENSIONAL SCANNER CHARACTERISTIC FUNCTION
C2     = ONE DIMENSIONAL SCANNER CHARACTERISTIC FUNCTION
C      = C1+C2
COVIN  = INPUT COVARIANCE MATRICES
COVOUT = OUTPUT COVARIANCE MATRICES
FLAG   = FLAG IN CONTRL CARDS
MU     = INPUT MEAN VECTORS
NADR   = ADDRESS OF THE MATRIX ELEMENTS TO BE DELETED
NC     = THE CHANNEL SET THAT IS NOT REQUESTED
NCC    = COMPLEMENT OF NC
RHO    = SPECTRAL CORRELATION MATRIX
PX     = AUX ARRAY
PY     = AUX ARRAY
SLPX  = SPATIAL CORRELATION PARAMETERS, CROSS TRACK
SLPY  = SPATIAL CORRELATION PARAMETERS, ALONG TRACK
SUBSLX = SUBSET OF SLPX
SUBSLY = SUBSET OF SLPY
SUBCOV = SUBSET OF THE INPUT SPECTRAL COV MATRICES
SUBMU  = SUBSET OF THE INPUT MEAN VECTORS

```

REMARKS

NONE

SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED

METHOD

ADRES

THE REQUIRED SUBSET OF THE INPUT SPECTRAL AND SPATIAL CORRELATION MATRICES IS COMPUTED. THE SCANNER CHARACTERISTIC FUNCTION IS CALCULATED BASED ON EITHER A GAUSSIAN OR RECTANGULAR IFDV AND SPECIFIED SIZE. THIS WEIGHTING FACTOR IS THEN APPLIED TO THE INPUT STATISTICS AND THE RESULTING OUTPUT IS PRINTED AND PUNCHED (IF REQUESTED). THE PUNCHED STAT DECK IS COMPATIBLE WITH SACAP AND VARIOUS LARSYS PROCESSORS. UNITY FILTER GAIN MAINTAINS THE EQUALITY OF INPUT AND OUTPUT MEAN VECTORS

SUBROUTINE SCANNER (COVIN,COVOUT,SUBCOV,SLPX,SLPY,SUBSLX,SUBSLY,PX,PY,C1,C2,C,NC,NCC,RHO,NADR,FLAG,MU,SUBMU,APERT)

LOCAL VARIABLES DEFINITION

```

REAL*4 COVIN(NSA),MU(NSH),COVOUT(NTS),SUBCOV(NTS),SUBMU(NTS),SLPX(
1  NSA),SLPY(NSA),SUBSLX(NTS),SUBSLY(NTS),PX(NTS),PY(NTS),
2  C1(NTS),C2(NTS),C(NTS),RHO(NTS)
INTEGER*4 FMT1(4),FMT2(4),FMT3(4),FMT4(4),N1(1),N2(1)
INTEGER*4 SIGMAX,SIGMAX,SIGM,NADR(NSA),APERT(20)
INTEGER*4 SHAPE(1),REC(1)
INTEGER*2 NC(30),NCC(30)
LOGICAL*1 FLAG(0)

```

FILE. . . SCANSTAT FORTRAN B1

```

601 CONTINUE
K=K+1
SUBCDV(K)=CDV(MDS+1)
SUBSLX(K)=SLPX(MDS+1)
SUBSLY(K)=SLPY(MDS+1)
602 CONTINUE
603 COMMON BLOCK VARIABLES DESCRIPTION
K=0
DO 803 JCLS=1,NCLS
MDS=(JCLS-1)*SDIM
DO 802 KOIM=1,SDIM
DO 801 J=1,NDIM
[[FKDIM.EQ.NCC(J)] SUBMU(K)=MU(MDS+NCC(J))
CONTINUE
CONTINUE
802 CONTINUE
803 CONTINUE
DETERMINE THE SCANNER CHARACTERISTIC FUNCTION
*****
GAUSSIAN SCANNER POINT SPREAD FUNCTION
DO 100 I=1,NTS
[[FAPERT(3).EQ.SHAPE] GO TO 641
PX(I)=SIGMAX**2+(SUBSLX(I)**2)
PY(I)=SIGMAX**2+(SUBSLY(I)**2)
SPX=SQRT(PX(I))
SPY=SQRT(PY(I))
CZ(I)=Z*(0.5*ERFC(SPX/SORT(Z.)))*EXP(PY(I)/Z.
C1(I)=Z*(0.5*ERFC(SPY/SORT(Z.)))*EXP(PX(I)/Z.
RECTANGULAR SCANNER POINT SPREAD FUNCTION
CONTINUE
PX(I)=SIGMAX*SUBSLX(I)
PY(I)=SIGMAX*SUBSLY(I)
C1(I)=Z/PX(I)*[1-EXP(-PX(I)/PY(I))]
C2(I)=Z/PY(I)*[1-EXP(-PY(I)/PX(I))]
CONTINUE
C(I)=C1(I)*C2(I)
C(I)=C(I)*C2(I)
FIND THE OUTPUT COVARIANCE MATRICES
*****
CDVOUT(I)=SUBCOV(I)*C(I)
CONTINUE
IF (.NOT.FLAG(6)) GO TO 456
*****
ADD NOISE TO THE SCANNER OUTPUT SIGNAL

```

FILE. . . SCANSTAT FORTRAN B1

```

*****
COMMON /SOSCOM/ NCLS,SDIM,NDIM,NSIZE,ISIZE,NTS,NSA,SIGMAX,SIGMAY,
NIM,NSH,SNR
*****
SIZE - CORE SPACE FOR ONE SUBSET OF COVARIANCE MATRIX
NCLS - NO OF CHANNELS
NIM - NO OF REQUESTED CHANNELS
NSA - CORE SPACE FOR A SUBSET OF MEAN VECTORS
NSH - CORE SPACE FOR ONE COV MATRIX
NIM - CORE SPACE FOR THE ENTIRE INPUT MEAN VECTORS
NIM - NO OF AVAILABLE CHANNELS
SDIM - NO OF SPREAD ALONG TRACK
SIGMAY - IGV SPREAD, CROSS TRACK
SIGMAX - IGV SPREAD, ALONG TRACK
*****
FORMAT(20A)
FORMAT(10F4.2,1X)
FORMAT(15F4.2,15X)
FORMAT(15F4.2,15X)
FORMAT(10F4.2,1X)
*****
READ IN THE INPUT SPECTRAL AND SPATIAL STATISTICS
*****
N1=NI*SDIM
N2=N2*5
IF(NSIZE.LT.5) N2=N3+NSIZE
FMT(2)=N1
FMT(2)=N2
READ(5,FMT) MU
READ(5,FMT) CDVIM
READ(5,507)
READ(5,511) SLPX
READ(5,511) SLPY
DD 521 I=1,NSA
SLPX(I)=ALOG(SLPX(I))
SLPY(I)=ALOG(SLPY(I))
CONTINUE
*****
SELECT THE REQUESTED SUBSET OF THE INPUT MATRICES
*****
CALL ADRES (NADR,NC,SDIM,NDIM)
*****
DO 603 JCLS=1,NCLS
MDS=(JCLS-1)*NSIZE
DO 602 J=1,NSIZE
DO 601 I=1,NSIZE
IF ((.EQ.NADR(I)) GO TO 602

```

FILE. . . SCANSTAT FORTRAN B1

```

C *****
C DD 456 JCLS=1,NCLS
C MDS=(JCLS-1)*ISIZE
C DD 455 KDIM=1,NDIM
C NAD=KDIM*(KDIM+1)/2
C VARNSE=((1/C)**(-SNR/10.))*COVOUT(MDS+NAD)
C COVOUT(MDS+NAD)=COVOUT(MDS+NAD)+VARNSE
C 455 CONTINUE
C 456 CONTINUE
C IF(.NOT.FLAG(7)) GO TO 544
C *****
C PUNCH OUT THE SCANNER OUTPUT STATISTICS
C *****
C WRITE(7,508)
C WRITE(7,509)
C WRITE(7,523)
C 523 FORMAT(1X,' ')
C WRITE(7,510) NCLS,NFLO,NDIM
C
C NUM2=SDIM+1
C DD 533 I=1,NUM2
C WRITE(7,523)
C 533 CONTINUE
C FMT1(2)=N3+NDIM
C FMT2(2)=N2
C
C WRITE(7,FMT1) SUBNO
C WRITE(7,FMT2) COVOUT
C 544 CONTINUE
C RETURN
C END

```

```

C *****
C SUBROUTINE ADRES
C PURPOSE
C FIND THE DESIRED SUBSET OF A COVARIANCE MATRIX
C DESCRIPTION OF PARAMETERS
C NADR - ADDRESS ARRAY OF THE ENTRIES TO BE DELETED
C NC - ARRAY OF THE CHANNELS TO BE DELETED
C SDIM - TOTAL NO OF BANDS AVAILABLE
C NDIM - DESIRED SUBSET OF SDIM
C REMARKS
C NONE
C SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED
C NONE
C METHOD
C USING A ONE DIMENSIONAL STORAGE MODE FOR EACH COVARIANCE
C MATRIX IN AN UPPER TRIANGULAR FORM, THE PARAMETRIC ADDRESSES
C FOR EACH ENTRY IS DERIVED. USING THE NC ARRAY, THE LOCATION
C OF ALL THE ENTRIES THAT LIE IN THE UNWANTED LINES AND COLU
C IS COMPUTED AND STORED IN NADR.

```

FILE. . . SCANSTAT FORTRAN B1

```

C .....
C SUBROUTINE ADRES (NADR,NC,SDIM,NDIM)
C INTEGER*4 NADR(1),PDIM,SDIM
C INTEGER*2 NC(1)
C PDIM=SDIM-NDIM
C IF(PDIM.LE.0) GO TO 100
C DD 20 J=1,PDIM
C N1=NC(J)
C DD 10 I=1,N1
C K=K+1
C NADR(K)=(NC(J)+NC(J)-1)/2+I
C 10 CONTINUE
C 20 CONTINUE
C DD 45 J=1,PDIM
C NFIN=SDIM-NC(J)+1
C DD 35 I=1,NFIN
C K=K+1
C NADR(K)=((NC(J)+I-1)*(NC(J)+I-2)/2)+NC(J)
C 35 CONTINUE
C 45 CONTINUE
C 100 RETURN
C END

```

FILE. . . SPROCT FORTRAN P1

SPROCT

PURPOSE  
EXOSYS DATA IN PUNCHED FORMAT IS READ AND STORED ON TAPE

REVISED  
3 JULY, 1978

```
COMMON ID(100)
INTEGER*4 IN(5),DAY(3),TIME(3),NOGRPS,GROUPS(5)
INTEGER*4 ISAM,SINT,DATE(15),INFO(10,7)
REAL*4 HBCDEF(2,5),DATA(2500),X(100)
EQUIVALENCE(DATE(1),ID(1)),(INFO(1,1),ID(30)),(INFO(1,2),ID(40)),
*(INFO(1,3),ID(50)),(INFO(1,4),ID(60)),(INFO(1,5),ID(70)),(INFO(1,
*),ID(80)),(INFO(1,7),ID(90))
REWIND 11
NT = 100
```

ID INFORMATION

```
WRITE(16,10)
FORMAT(5X,'TYPE IN DATE',/1X,15('/'))
READ(15,15)DATE
FORMAT(15A1)
```

EXP. NO., NUMBER OF CLASSES, AND NUMBER OF DIMENSIONS

```
WRITE(16,20)
FORMAT(9X,'TYPL EXP.NO.,CLASSES,AND DIMENSIONS',/' /// // ///'
READ(15,25)ID(16),ID(17),ID(18)
FORMAT(13,2X,12,3X,13)
```

EXPERIMENT INFORMATION

```
NCLS = ID(17)
DO 35 I=1,NCLS
WRITE(16,30)I
FORMAT(5X,'TYPE CLASS INFO AND NO. SAMPLES FOR CLASS ',I1,/1X,10
*/',1,4X,17('/))
READ(15,40)INFO(I,1),L=1,10, ID(20+I)
FORMAT(10A1,4X,13)
WRITE(11) ID
CALL SPLBL
```

READ SAMPLE FUNCTIONS FROM EACH CLASS

```
DO 500 K=1,NCLS
WRITE(16,80)
FORMAT(15(7),10X,'SAMPLE FUNCTIONS')
NF = ID(20+K)
DO 200 JJ=1,NF
READ(5,1000)DAY,TIME,IN,NOGRPS,NOSAMS
1000 FORMAT(3A4,2X,3(12,1X),7,45X,5A4,7,20X,11,8X,13,/)
1100 FORMAT(20A4)
WRITE(16,150)IN
FORMAT(10X,20A4)
DO 160 I=1,NT
160 X(I) = DATA(I+1)
WRITE(11) X
CONTINUE
CONTINUE
END FILE 11
STOP
END
```

FILE. . . SPOPTH FORTRAN P1

SPOPTH

PURPOSE  
TO DESIGN THE OPTIMUM SENSOR FOR A GIVEN DATA SET.

USAGE  
CALLED FROM EXEC ROUTINE

DESCRIPTION OF PARAMETERS  
AM - MEAN VECTOR OF DATA  
COV - COVARIANCE MATRIX OF DATA  
PHI - MATRIX OF EIGENVECTORS.  
GAM - EIGENVALUES  
N - DIMENSIONALITY OF DATA SET  
NCLS - NUMBER OF CLASSES

SUBROUTINE AND FUNCTION SUBPROGRAMS CALLED  
EIGENP,EISORT,SPWGT

METHOD  
THE KARHUNEN-LOEVE EXPANSION WITH THE MAXIMUM LIKELIHOOD ESTIMATE  
OF THE COVARIANCE MATRIX AS THE KERNEL IS USED TO REPRESENT THE  
RANDOM PROCESS.

REVISED  
14 AUG, 1978

```
COMMON ID(100)
REAL*4 AM(100),Y(100),COV(5050),PHIP(100,100)
REAL*8 VEC(100,100),EVI(100),INDIC(100),ACOV(100,100),GAM(100)
REAL*8 PHI(100,100),SUM
REAL*4 X(100),M(100)
REWIND 2
WRITE(16,5)
FORMAT(15X,'OPTIMUM SENSOR DESIGN')
```

READ ID INFORMATION

```
READ(2) ID
WRITE(16,8)
FORMAT(11H,5(/))
CALL SPLBL
N = ID(18)
NCT = N*(N+1)/2
```

COMPUTE COVARIANCE

```
WRITE(16,10)
FORMAT(15X,'COVARIANCE BEING ESTIMATED (SPOPTH)')
NFT = ID(17)
NFT = 0
DO 20 I=1,NCLS
NFT = NFT + ID(20+I)
CON = DFLOAT(NFT)/DFLOAT(NFT-1)
DO 30 I=1,N
AM(I) = 0.0
DO 35 J=1,NCT
COV(I) = 0.0
DO 65 IJ=1,NFT
READ(2) X
IN = 0
DO 50 I=1,N
AM(I) = AM(I) + X(I)/DFLOAT(NFT)
DO 50 J=1,I
IN = IN + 1
COV(IN) = COV(IN) + X(I)*X(J)/DFLOAT(NFT-1)
50 CONTINUE
CONTINUE
IN = 0
DO 60 I=1,N
DO 60 J=1,I
IN = IN + 1
COV(IN) = COV(IN) - CON*AM(I)*AM(J)
```

FILE. . . SPOPTM FORTRAN P1

```
60 CONTINUE
C C WEIGHTING FUNCTION
C IN = 0
DO 210 I=1,N
DO 210 J=1,I
IV = IN + I
ACOV(I,J) = COV(IN)
ACOV(J,I) = COV(IN)
210 CONTINUE
C CALL SPWGT3(W)
C DO 250 I=1,N
DO 250 J=1,N
250 ACOV(I,J) = ACOV(I,J)*W(J)
C C COMPUTE TRACE OF COVARIANCE
C SUM = 0.0
DO 80 I=1,N
80 SUM = SUM + ACOV(I,I)
C C COMPUTE EIGENVALUES AND EIGENVECTORS
C WRITE(16,75)
75 FORMAT(5X,'EIGENVALUES AND EIGENVECTORS (EIGENP)')
NM = N
I = 56.
CALL EIGENP(N,ACOV,T,GAM,EVI,PHI,VEC),INDIC,W)
CALL EISORT(N,GAM,PHI)
C C PRINT EIGENVALUES AND MEAN-SQUARE ERROR
C CO = FLOAT(NFT)/(FLOAT(NFT-1)*FLOAT(NFT-1))
C1 = FLOAT(4*NFT-1)/(FLOAT(NFT-1)*FLOAT(NFT-1))
WRITE(6,110)
110 FORMAT(5,17),5X,'N',5X,'EIGENVALUE',5X,'VAR(GAM)',5X,'VAR(PHI)',5X
*,'MEAN-SQUARE ERROR')
DO 150 I=1,30
VARP = 0.0
DO 120 J=1,100
IF(I.EQ.J) GO TO 115
VARP = VARP + CO*GAM(I)*GAM(J)/(GAM(I) - GAM(J))**2
115 CONTINUE
120 CONTINUE
VARG = C1*GAM(I)*GAM(I)
SUM = SUM - GAM(I)
WRITE(6,145)I,GAM(I),VARG,VARP,SUM
145 FORMAT(4X,12,4X,F10.4,4X,F10.4,2X,F10.4,2X,F14.6)
150 CONTINUE
DO 155 J=1,N
DO 155 I=1,N
155 PHIP(I,J) = PHI(I,J)
DO 180 J=1,20
WRITE(7,160)(PHIP(I,J),I=1,N)
160 FORMAT(20A4)
180 CONTINUE
STOP
END
```

FILE. . . SPWGT3 FORTRAN P1

```
C C C WEIGHTING FUNCTION NUMBER 3
C SUBROUTINE SPWGT3(W)
REAL*4 W(100)
C WRITE(6,15)
15 FORMAT(75X,'WEIGHTING FUNCTION NUMBER 3')
C DO 20 I=1,100
W(I) = 1.0
20 CONTINUE
DO 30 I=48,53
30 W(I) = 0.0
DO 40 I=72,76
40 W(I) = 0.0
W(47) = 0.5
W(54) = 0.5
W(71) = 0.5
W(77) = 0.5
RETURN
END
```

FILE. . . SPTES FORTRAN P1

SPTES TRANSFORMS THE DATA USING THE OPTIMUM SET OF BASIS VECTORS, COMPUTES THE MEAN-SQUARE ERROR, AND COMPUTES THE STATISTICS FOR EACH CLASS.

6 FEBRUARY, 1978

```
COMMON ID(100)
REAL*4 P(10),PHI(100,10),X(100),Y(100),Z(100)
REAL*4 AM(100),AVE(20,10),COV(210,10)
```

SELECT NUMBER OF TERMS

```
WRITE(16,10)
FORMAT(5X,'NUMBER OF TERMS *')
READ(15,15)INTERM
FORMAT(12)
REWIND 2
READ(2) ID
NCLS = ID(17)
N = ID(18)
NCT = NTERM*(INTERM + 1)/2
NFT = 0
DO 20 I=1,NCLS
NFT = NFT + ID(20+I)
DO 25 I=1,NCLS
P(I) = 1./FLOAT(NCLS)
CONTINUE
WRITE(17,28)NCLS,NTERM
FORMAT(12,3X,12)
WRITE(17,30)(P(I),I=1,NCLS)
FORMAT(10F6.4)
```

COMPUTE MEAN FUNCTION

```
DO 300 I=1,N
AM(I) = 0.0
DO 320 K=1,NFT
READ(2) X
DO 320 I=1,N
AM(I) = AM(I) + X(I)/FLOAT(NFT)
CONTINUE
REWIND 2
READ(2) ID
```

READ EIGENVECTORS

```
DO 40 J=1,NTERM
READ(5,35)(PHI(I,J),I=1,N)
FORMAT(20A4)
CONTINUE
```

LOOP ON THE SAMPLE FUNCTIONS IN THE DATA SET

```
AVESQ = 0.0
DO 200 ICLS=1,NCLS
DO 50 I=1,NTERM
AVE(I,ICLS) = 0.0
DO 55 I=1,NCT
COV(I,ICLS) = 0.0
```

```
NF = ID(20+ICLS)
CON = FLOAT(NF)/FLOAT(NF-1)
DO 150 ISAM=1,NF
```

READ SAMPLE POINTS FROM FUNCTION

```
READ(2) X
```

TRANSFORM DATA USING BASIS FUNCTIONS

```
DO 70 J=1,NTERM
Y(J) = 0.0
DO 70 I=1,N
Y(J) = Y(J) + PHI(I,J)*(X(I) - AM(I))
```

FILE. . . SPTES FORTRAN P1

70 CONTINUE

COMPUTE SQUARED ERROR

```
DO 80 I=1,N
Z(I) = 0.0
DO 85 J=1,NTERM
DO 85 I=1,N
Z(I) = Z(I) + PHI(I,J)*Y(J)
CONTINUE
DO 88 I=1,N
Z(I) = Z(I) + AM(I)
XSQ = 0.0
ZSQ = 0.0
KZ = 0.0
TSQ = 0.0
DO 90 I=1,N
XSQ = XSQ + X(I)*X(I)
ZSQ = ZSQ + Z(I)*Z(I)
KZ = KZ + 2.0*X(I)*Z(I)
CONTINUE
ESQ = (XSQ - KZ + ZSQ)/FLOAT(N)
AVESQ = AVESQ + ESQ
```

COMPUTE STATISTICS

```
DO 100 I=1,NTERM
AVE(I,ICLS) = AVE(I,ICLS) + Y(I)/FLOAT(NF)
CONTINUE
IN = 0
DO 110 J=1,NTERM
DO 110 I=1,J
IN = IN + 1
COV(IN,ICLS) = COV(IN,ICLS) + Y(I)*Y(J)/FLOAT(IN-1)
CONTINUE
CONTINUE
```

PRINT STATISTICS

```
IN = 0
DO 160 J=1,NTERM
DO 160 I=1,J
IN = IN + 1
COV(IN,ICLS) = COV(IN,ICLS) - CON*AVE(I,ICLS)*AVE(J,ICLS)
CONTINUE
WRITE(16,165)ICLS
FORMAT(5I7,10X,'STATISTICS FOR CLASS',I4)
CALL MCOVP(INTERM,AVE(I,ICLS),COV(I,ICLS))
WRITE(17,170)(AVE(I,ICLS),I=1,NTERM)
FORMAT(20A4)
WRITE(17,175)(COV(I,ICLS),I=1,NCT)
FORMAT(20A4)
```

```
CONTINUE
AVESQ = AVESQ/FLOAT(NFT)
WRITE(16,210)AVESQ
FORMAT(7//10X,'MEAN-SQUARE ERROR = ',E10.4)
STOP
END
```

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