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A SEMI-SPECTRAL NUMERICAL MODEL FOR THE LARGE SCALE STRATOSPHERIC CIRCULATION

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1. INTRODUCTION

A numerical model for simulation of the global circulation of the stratosphere and mesosphere is currently under development at the University of Washington. The complete model is a semi-spectral model in which the longitudinal dependence is represented by expansion in zonal harmonics while the latitude and height dependencies are represented by a finite difference grid. Since many of the dynamical processes which occur in the stratosphere and mesosphere are the result of interactions between the zonal mean flow and planetary waves, it is useful to formulate a model in which the zonal mean and wave portions are explicitly separated, as is done here.

The model is based on the primitive equations in the log pressure coordinate system as given by Holton (1975). In order to avoid the problems inherent in simulating tropospheric meteorological processes, the lower boundary of the model domain is set at the 100 mb level (i.e., near the tropopause) and the effects of tropospheric forcing are included in the lower boundary condition. The upper boundary is at approximately 96 km, and the latitudinal extent is either global or hemispheric.

In this report we first outline the basic differential equations and boundary conditions. We next describe the finite difference equations. We then discuss the initial conditions and present a sample calculation. Finally, the Fortran code is given in the appendix.
2. BASIC EQUATIONS

In setting down the basic equations we will make use of the following symbols:

\[ \lambda \] longitude

\[ \theta \] latitude

\[ z \] a measure of "height" \( = -H \ln \left( \frac{p}{p_s} \right) \)

\[ H \] scale height \( = \frac{RT_s}{g} \)

\[ R \] gas constant for dry air

\[ T_S \] a constant stratospheric mean temperature

\[ g \] gravitational acceleration

\[ p \] pressure

\[ p_s \] a constant reference pressure

\[ u \] eastward velocity

\[ v \] northward velocity

\[ w \] a measure of "vertical velocity" \( = \frac{dz}{dt} \)

\[ T_0 \] a basic state temperature \( = T_0(z) \)

\[ \phi \] a basic state geopotential \( = \phi_0(z) \)

\[ T \] departure of local temperature from \( T_0(z) \)

\[ \phi \] departure of local geopotential from \( \phi_0(z) \)

\[ \Omega \] angular velocity of earth

\[ a \] radius of earth

\[ J \] diabatic heating rate per unit mass

\[ c_p \] specific heat at constant pressure

\[ \kappa \] ratio of gas constant to specific heat at constant pressure \( = \frac{R}{c_p} \)

\[ dx \] eastward distance increment \( = a \cos \theta \, d\lambda \)

\[ dy \] northward distance increment \( = a \, d\theta \).
The horizontal momentum equations can then be written in flux form as

\[
\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{1}{\cos^2 \theta} \frac{\partial}{\partial y} (uv \cos^2 \theta) + \frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 w) - 2\Omega v \sin \theta = -\frac{\partial \Phi}{\partial x} + D_1(u) \tag{2.1}\]

\[
\frac{\partial v}{\partial t} + \frac{\partial (uv)}{\partial x} + \frac{1}{\cos \theta} \frac{\partial}{\partial y} (v^2 \cos \theta) + \frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 v) + \frac{u^2 \tan \theta}{a} + 2\Omega u \sin \theta = -\frac{\partial \Phi}{\partial y} + D_2(v) \tag{2.2}\]

Here, \( \rho_0 \equiv \rho_s \exp(-z/H) \), where \( \rho_s \) is the mean density at \( z = 0 \). \( D_1(u) \) and \( D_2(v) \) represent subgrid scale momentum diffusion. Explicit forms for these terms will be given in Section 4.

Using the above notation the hydrostatic approximation and continuity equation become

\[
\frac{d \Phi_0}{dz} = \frac{RT_0}{H}, \quad \frac{\partial \Phi}{\partial z} = \frac{RT}{H}, \tag{2.3}\]

and

\[
\frac{\partial u}{\partial x} + \frac{1}{\cos \theta} \frac{\partial}{\partial y} (v \cos \theta) + \frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 w) = 0 \tag{2.4}\]

The variables \( T_0 \) and \( \Phi_0 \) define a hydrostatically balanced basic state which is specified to be the U.S. standard atmosphere. Using (2.3) we can write the thermodynamic energy equation for the departure from the basic
state as follows

\[ \frac{\partial \Phi_z}{\partial t} + \frac{\partial}{\partial x} (u \Phi_z) + \frac{1}{\cos \theta} \frac{\partial}{\partial y} (v \Phi_z \cos \theta) \]

\[ + \frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 \Phi_z w) + wN^2 = kJ/H + D_2(\Phi_z) \quad (2.5) \]

where

\[ N^2 = \frac{R}{H} \left( \frac{dT_o}{dz} + \frac{\kappa T_o}{H} \right) \]

is the buoyancy frequency squared, and we have let \( D_2(\Phi_z) \) denote the subgrid scale diffusion.

The basic state temperature profile is assumed to be in radiative equilibrium (see Section 9), so that the horizontal average of the diabatic heating will vanish provided that the horizontally averaged temperature equals the basic state temperature \( T_o(z) \). Because of the nonlinearity of (2.5) the horizontally averaged temperature need not remain equal to \( T_o(z) \) as the flow evolves in time. However, in practice we find that departures of the horizontally averaged total temperature from \( T_o(z) \) are at most a few degrees so that for practical purposes the horizontally averaged diabatic heating remains very small, and \( J \) can be regarded as the differential heating.

\[ ^1 \text{Following Holton (1975) we here neglect the small term } w_k T/H \text{ compared to } w_k T_o/H. \text{ This approximation is necessary if we wish to define available potential energy in terms of the temperature variance.} \]
3. ZONAL HARMONIC EXPANSION

The basic equations of the model were given as (2.1), (2.2), (2.4), and (2.5). In order to develop the semi-spectral model we expand the basic equations in zonal harmonic series by letting

\[ f(\lambda, y, z, t) = e^{z/2H} \sum_{n=-\infty}^{+\infty} F_n(y, z, t)e^{in\lambda} \]  \hspace{1cm} (3.1)

where \( f(\lambda, y, z, t) \) stands for any field variable and \( F_n \) is the Fourier transform of \( f \) defined by

\[ F_n = \frac{e^{-z/2H}}{2\pi} \int_{-\pi}^{+\pi} f(\lambda, y, z, t)e^{-in\lambda} d\lambda \]  \hspace{1cm} (3.2)

so that \( F_{-n} = F_n^* \), where the asterisk denotes the complex conjugate.

To transform the nonlinear terms in the basic equations we need the convolution theorem:

\[ \frac{1}{2\pi} \int_{-\pi}^{+\pi} [f(\lambda)g(\lambda)]e^{-in\lambda} d\lambda = e^{z/H} \sum_{m=-\infty}^{+\infty} G_m F_{n-m} \]

from which we find

\[ \frac{1}{2\pi} \int_{-\pi}^{+\pi} \left[ \frac{\partial f}{\partial \lambda} g \right] e^{-in\lambda} d\lambda = e^{z/H} \sum_{m=-\infty}^{+\infty} \text{im} G_{n-m} F_m \]
To Fourier transform (2.1), (2.2), (2.4), and (2.5) we define the following transform pairs:

\[ f(\lambda): u \quad v \quad w \quad \phi \quad kJ/H \]

\[ F_n : U_n \quad V_n \quad W_n \quad \psi_n \quad Q_n \]

The transformed equations are as follows:

\[
\frac{\partial U_n}{\partial t} - fV_n = -\frac{in}{a \cos \theta} \psi_n + D_2 (U_n)
\]

\[
- e^{-z/2H} \sum_{m=-\infty}^{+\infty} \left[ -\frac{2im}{a \cos \theta} U_m U_{n-m} \right. \\
\left. + \frac{1}{\cos^2 \theta} \frac{\partial}{\partial y} (U_m V_{n-m} \cos^2 \theta) + \frac{\partial}{\partial z} (U_m W_{n-m}) \right]
\]

where \( \lambda = 1 \) for \( n = 0 \), \( \lambda = 2 \) for \( n \neq 0 \).

\[
\frac{\partial V_n}{\partial t} + fU_n = -\frac{\partial \psi_n}{\partial y} + D_2 (V_n)
\]

\[
- e^{-z/2H} \sum_{m=-\infty}^{+\infty} \left[ \frac{im}{a \cos \theta} (U_m V_{n-m} + V_m U_{n-m}) \right. \\
\left. + \frac{1}{\cos \theta} \frac{\partial}{\partial y} (V_m V_{n-m} \cos \theta) \right. \\
\left. + \frac{\partial}{\partial z} (V_m W_{n-m}) + \tan \theta \frac{\partial}{a} (V_m U_{n-m}) \right]
\]
\[
\frac{\partial}{\partial t} \left[ \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \psi_n \right] + N^2 w_n =
\]

\[
Q_n + D_2 \left[ \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \psi_n \right] - e^{z/2H} \sum_{m=-\infty}^{+\infty} \left\{ \frac{im}{a \cos \theta} \left[ U_m \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \psi_{n-m} \right] + \frac{1}{\cos \theta} \frac{\partial}{\partial y} \left[ \cos \theta V_{n-m} \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \psi_{m} \right] \right\} (3.5)
\]

\[
\frac{\text{in} U_n}{a \cos \theta} + \frac{1}{\cos \theta} \frac{\partial}{\partial y} (\cos \theta V_n) + \left( \frac{\partial}{\partial z} - \frac{1}{2H} \right) w_n = 0 (3.6)
\]

We now severely truncate the wave spectrum by assuming that the flow consists of a single wave of wavenumber \( n = s \), and the zonal mean \( n = 0 \). To exclude all other wave modes we must replace the summations in (3.3) - (3.5) by a summation over the two values \( m = 0 \) and \( m = s \).

3.1 The zonal mean equations

If we set \( n = 0 \) in (3.3) - (3.6) and replace \( () \) by \( (\overline{\cdot}) \) for all field variables we obtain the zonal mean equations:

\[
\overline{\frac{\partial U}{\partial t} - fV} = -e^{z/2H} \left[ \frac{1}{\cos^2 \theta} \frac{\partial}{\partial y} \left( \overline{U V \cos^2 \theta} \right) + \frac{\partial}{\partial z} \left( \overline{U W} \right) \right]
\]

\[
+ F_M + D_1 (\overline{U}) (3.7)
\]
\[
\frac{\partial \bar{V}}{\partial t} + f \bar{U} = - \frac{\partial \bar{V}}{\partial y} - \frac{e^2/2H}{U^2} \tan \frac{\theta}{a} + D_2(\bar{V}) 
\]  (3.8)

\[
\frac{1}{\cos \theta} \frac{\partial}{\partial y} (\bar{V} \cos \theta) + \left[ \frac{\partial}{\partial z} - \frac{1}{2H} \right] \bar{W} = 0 
\]  (3.9)

\[
\frac{\partial}{\partial t} \left( \frac{\partial \bar{V}}{\partial z} + \frac{\bar{V}}{H} \right) + N^2 \bar{W} = + F_T - e^2/2H \left\{ \frac{1}{\cos \theta} \frac{\partial}{\partial y} \left[ \bar{V} \cos \theta \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \bar{\psi} \right] \right. 
\]

\[
\left. + \frac{\partial}{\partial z} \left[ \bar{W} \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \bar{\psi} \right] \right\} + D_2 \left( \frac{\partial \bar{V}}{\partial z} + \frac{\bar{V}}{2H} \right) + \bar{Q} 
\]  (3.10)

where \( f \equiv 2\Omega \sin \theta \) is the Coriolis parameter. Here \( F_M \) denotes the convergence of the momentum flux due to zonally asymmetric motions (e.g., planetary waves) while \( F_T \) denotes the convergence of the eddy heat flux.

We have neglected the advection by the mean meridional circulation and the eddy momentum flux terms in (3.8) since the mean zonal wind is nearly in gradient wind balance. The terms \( \partial \bar{V}/\partial t \) and \( D_2(\bar{V}) \) are also very small but must be retained for our method of numerical solution.

With the aid of (3.9) we can define a mean meridional streamfunction, \( \bar{X} \), by letting

\[
\bar{W} \cos \theta = \frac{\partial \bar{X}}{\partial y}, \quad \bar{V} \cos \theta = - \left[ \frac{\partial}{\partial z} - \frac{1}{2H} \right] \bar{X} 
\]  (3.11)

The \( \bar{X} \) field proves useful in specifying boundary conditions and solving the zonal mean component equations.

The eddy flux convergence terms in (3.7) and (3.10) have the following forms:

\[
F_M = - \frac{e^2/2H}{\cos^2 \theta} \frac{\partial}{\partial y} \left[ \left( \frac{1}{U_s} V_s^* + \frac{U_s^* V_s}{s_s} \right) \cos^2 \theta \right] 
\]

\[
+ \frac{\partial}{\partial z} \left( \frac{U_s^* W_s^* + U_s W_s}{s_s} \right) \right\} 
\]  (3.12a)
3.2 The eddy equations

When we set \( n = s \) in (3.3) - (3.6) and again designate zonal means by an overbar rather than the \( m = 0 \) subscript, we obtain the eddy equations,

\[
\begin{align*}
\frac{\partial U_s}{\partial t} - fV_s &= -\frac{is}{a \cos \theta} \psi_s - e^{z/2H} \left\{ \frac{isU}{a \cos \theta} + \frac{V_s}{\cos \theta} \frac{\partial}{\partial y} (\bar{U} \cos \theta) \right. \\
&\left. + W_s \left\{ \frac{\partial}{\partial z} + \frac{1}{2H} \right\} \bar{U} \right\} + D_2 (U_s) \\
\frac{\partial V_s}{\partial t} + fU_s &= -\frac{\partial}{\partial y} \psi_s - e^{z/2H} \left\{ \frac{2U_s \tan \theta}{a} + \frac{isV}{a \cos \theta} \right\} + D_2 (V_s) \\
\frac{\partial}{\partial t} \left[ \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \psi_s \right] + N^2 W_s &= Q_s - e^{z/2H} \left\{ \frac{isU}{a \cos \theta} \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \psi_s \right. \\
&\left. + W_s \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right)^2 \bar{\psi} + V_s \frac{\partial}{\partial y} \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \bar{\psi} \right\} \\
&\left. + D_2 \left[ \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \psi_s \right] \right. \\
&\left. \frac{isU_s}{a \cos \theta} + \frac{1}{\cos \theta} \frac{\partial}{\partial y} (\cos \theta \psi_s) + \left( \frac{\partial}{\partial z} - \frac{1}{2H} \right) W_s = 0 \right)
\end{align*}
\]

Here all terms involving advection by the mean meridional circulation, \( \bar{V}, \bar{W} \) have been neglected.
Conditions for the zonal mean:

The model can be integrated on either a hemispheric or global domain. For global integrations the horizontal boundary conditions are as follows:

\[ \bar{X} = \bar{U} = \bar{V} = \partial \bar{\Psi} / \partial y = 0 \text{ at } \theta = \pm \pi/2 \]  

For hemispheric integrations the same boundary conditions are used at \( \theta = 0, \pi/2 \) except that a value of \( \bar{U} \) not equal to zero may be specified at \( \theta = 0 \).

Vertical boundary conditions are specified as follows:

\[ \bar{U} \equiv \bar{U}_B(y,t) \text{ at } z = 0 \]  

where \( z = 0 \) designates the lower boundary (i.e., the tropopause level) and \( \bar{U}_B \) is an externally specified mean zonal wind. The boundary mean zonal flow is assumed to be in gradient wind balance. Thus from (2.10) we see that at \( z = 0 \)

\[ \bar{V} = 0 \]  

and

\[ - \frac{\partial \bar{\Psi}}{\partial y} = f \bar{U} + \bar{U}^2 \frac{\tan \theta}{a} e^{z/2H} \]  

Using the conditions (4.2a) we can integrate (4.2c) to obtain \( \bar{\Psi}(y,t) \) at \( z = 0 \), provided that we let the horizontal average of \( \bar{\Psi}(y,0,t) \) vanish.

At the upper boundary (\( z = z_T \)) we assume that the vertical shear of the mean zonal wind, the mean meridional wind, and the mean geopotential
all vanish. Thus,

\[
\frac{\partial}{\partial z} \left( -ue^{z/2H} \right) = \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \vec{U} = 0 \quad (4.3a)
\]

\[
\left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \vec{V} = 0 \quad (4.3b)
\]

and

\[
\left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \vec{\Psi} = 0 \quad (4.3c)
\]

Condition (4.3c) of course implies that the zonal mean temperature must equal the basic state \( T_0(z_T) \) at \( z = z_T \).

In addition to these conditions it is clear from (3.7) and (3.10) that boundary conditions are also required for the vertical momentum and heat fluxes associated with the mean meridional circulation. We wish to avoid specifying \( \bar{W} \) or the fluxes themselves at \( z = 0 \). Instead we assume that the flux divergences vanish at the lower boundary:

\[
\frac{\partial}{\partial z} (\bar{U} \bar{W}) = \frac{\partial}{\partial z} \left[ \bar{W} \left( \frac{\partial}{\partial z} + \frac{1}{2H} \right) \bar{\Psi} \right] = 0 \quad (4.4)
\]

However, for simplicity we assume that the fluxes themselves vanish at the upper boundary. If in addition we let \( \bar{Q} = \bar{F}_T = 0 \) at the upper boundary, then from (3.10) we have

\[
\bar{W} = 0 \text{ at } z = z_T \quad (4.5)
\]
Boundary conditions for the eddy equations:

The boundary conditions for the eddy motions are analogous to the conditions for the zonal mean. However, the case $s = 1$ must be treated separately because $V_s$ does not vanish at the poles for $s = 1$. Thus, for integrations on the global domain we have at $\theta = \pm \pi/2$:

$$\psi_s = 0$$

$$U_s = V_s = 0 \text{ for } s > 1 \quad (4.6)$$

$$\partial U_s / \partial \theta = \partial V_s / \partial \theta = 0 \text{ for } s = 1$$

For a hemispheric domain the conditions at $\theta = 0$ depend on the symmetry conditions assumed. If geopotential is symmetric we have

$$\partial \psi_s / \partial \theta = \partial U_s / \partial \theta = V_s = 0 \text{ at } \theta = 0 \quad (4.7)$$

If geopotential is antisymmetric we have

$$\psi_s = U_s = \partial V_s / \partial \theta = 0 \text{ at } \theta = 0 \quad (4.8)$$

Conditions at the horizontal boundaries are specified as follows:

At the lower boundary a geopotential height perturbation is specified so that

$$\psi_s (y,t) = gh_s (y,t) \text{ at } z = 0 \quad (4.9)$$

while at the upper boundary the wave perturbations are assumed to vanish

$$\psi_s = 0 \text{ at } z = z_T \quad (4.10)$$
The latter condition requires that we impose strong damping in the layers near $z_T$ to prevent spurious reflection of wave energy from the upper boundary. Finally, in order to compute $F_M$ and $F_T$ at the upper and lower boundaries we assume that the vertical momentum and heat flux divergences vanish at the boundaries.
5. ENERGETICS

It can be shown that the eddy equations (3.13) - (3.16) are energetically consistent with the mean flow equations (3.7) - (3.10). In fact the system is governed by a Lorenz type energy cycle which (neglecting the diffusion terms) may be written as follows:

\[
\frac{dK}{dt} = \langle K_s \cdot \bar{K} \rangle + \langle \bar{A} \cdot \bar{K} \rangle + B(K) 
\]

(5.1)

\[
\frac{dA}{dt} = - \langle \bar{A} \cdot \bar{K} \rangle - \langle \bar{A} \cdot \bar{A}_s \rangle + \bar{G} + B(A)
\]

(5.2)

\[
\frac{dK_s}{dt} = - \langle K_s \cdot \bar{K} \rangle + \langle A_s \cdot K_s \rangle + B(K_s)
\]

(5.3)

\[
\frac{dA_s}{dt} = \langle \bar{A} \cdot A_s \rangle - \langle A_s \cdot K_s \rangle + G_s
\]

(5.4)

where

\[
\bar{K} \equiv \int_0^{\pi/2} \int_0^\infty \left( \frac{U^2 + V^2}{2} \right) \cos \theta \, d\theta \, dz
\]

\[
\bar{A} \equiv \int_0^{\pi/2} \int_0^\infty \frac{1}{2N^2} \left( \frac{\partial \bar{W}}{\partial z} + \frac{\bar{W}}{2H} \right)^2 \cos \theta \, d\theta \, dz
\]

\[
\langle K_s \cdot \bar{K} \rangle \equiv - \int_0^{\pi/2} \int_0^\infty e^{-z/2H} \left\{ \frac{1}{\cos \theta} \frac{\partial}{\partial y} \left[ (U V_s^* + U^* V_s) \cos^2 \theta \right] + \cos \theta \frac{\partial}{\partial z} (U V_s^* + U^* V_s) \right\} d\theta \, dz
\]
\[ \langle A \cdot K \rangle \equiv \int_0^\pi \int_0^\pi \frac{1}{N^2} \left( \frac{\partial \psi}{\partial z} + \frac{1}{2H} \right) \psi \cos \theta \, d\theta \, dz \]

\[ B(K) \equiv \int_0^{\pi/2} \left\{ [\bar{\psi} (\psi + \frac{\bar{U}}{2}) + \bar{U} (U_{1s} W_{1s}^* + U_{2s} W_{2s}^*)] \cos \theta \, d\theta \right\}_z = 0 \]

\[ \langle A \cdot A_s \rangle \equiv \int_0^\infty \int_0^{\pi/2} \frac{e^{-z/2H}}{N^2} \left( \frac{\partial \psi}{\partial z} + \frac{1}{2H} \right) \left\{ \frac{\partial \psi}{\partial z} \cos \theta \left[ V_{1s} \left( \frac{\partial \psi}{\partial z} + \frac{1}{2H} \right) \right] + \psi_s \left( \frac{\partial \psi}{\partial z} + \frac{1}{2H} \right) \psi_s^* 
+ \psi_s \left( \frac{\partial \psi}{\partial z} + \frac{1}{2H} \right) \psi_s \right\} + \psi_s \left( \frac{\partial \psi}{\partial z} + \frac{1}{2H} \right) \psi_s^* \right\} \, d\theta \, dz \]

\[ \bar{G} \equiv \int_0^\infty \int_0^{\pi/2} \frac{Q}{N^2} \left( \frac{\partial \psi}{\partial z} + \frac{\psi}{2H} \right) \cos \theta \, d\theta \, dz \]

\[ B(A) \equiv \int_0^{\pi/2} \left\{ \left( \frac{1}{2N^2} \left( \frac{\partial \psi}{\partial z} + \frac{\psi}{2H} \right)^2 \bar{W} + \frac{1}{N^2} \left( \frac{\partial \psi}{\partial z} + \frac{\psi}{2H} \right) \left( W_{1s} \left( \frac{\partial \psi}{\partial z} + \frac{1}{2H} \right) \psi_s \right. 
+ W_{2s} \left( \frac{\partial \psi}{\partial z} + \frac{1}{2H} \right) \psi_s \right) \right\} \cos \theta \, d\theta \right\}_z = 0 \]

\[ K_s \equiv \int_0^\infty \int_0^{\pi/2} \left( |U_s|^2 + |V_s|^2 \right) \cos \theta \, d\theta \, dz \]
Thus, in the above equations the terms enclosed by angle brackets represent transfers of energy among the components \( \overline{K}, \overline{A}, K_s, \) and \( A_s \) while the terms \( \overline{G} \) and \( G_s \) represent diabatic heat sources and the terms \( \overline{B}(K), \overline{B}(A), \) and \( B(K_s) \) represent energy fluxes across the lower boundary. Summing (5.1) – (5.4) we see that the total energy \( \overline{K} + \overline{A} + K_s + A_s \) is conserved in the absence of diabatic heating and boundary fluxes.
6. FINITE DIFFERENCE EQUATIONS

6.1 The grid mesh

All field variables are represented on a staggered grid in the meridional plane with grid points identified by the indices \((j,k)\). Here, \(j\) increases southwards and \(k\) increases upwards. To minimize truncation errors the grid points are staggered as shown in Fig. 1. The grid staggering is arranged in the horizontal so that \(\bar{U}, \bar{V}, F_M, \bar{X}, \Psi'\) and \(W'\) are defined at the meridional points given by \(y = (\pi a/2) - (j - 1)\Delta y\), \(j = 1,2,\ldots,J_m\) where

\[
\Delta y = \begin{cases} 
\pi a/(J_m - 1), \text{global domain} \\
\pi a \\
2(J_m - 1), \text{hemispheric domain}
\end{cases}
\]

The variables \(\bar{V}, U', V', \bar{W}\) and \(F_T\) are defined at the meridional points

\[y = \frac{\pi a}{2} - (j - 1/2)\Delta y, \quad j = 1,2,\ldots,J_m - 1\]

Thus, \(\bar{U}, \bar{V}, \bar{X}, \Psi'\) and \(W'\) are defined at the horizontal boundary points while \(\bar{V}, U', V', \bar{W}\) are defined a distance \(\Delta y/2\) inside the boundaries. This form of staggering is natural for use with the horizontal boundary conditions (4.1).

The vertical staggering is arranged so that \(\bar{U}, \bar{V}, \bar{W}, U', V', \Psi'\) and \(F_M\) are defined at the levels

\[z = (k - 1)\Delta z, \quad k = 1,2,\ldots,K_N\]

where \(\Delta z\) is the vertical grid increment; while the variables \(\bar{W}, \bar{X}, W'\) and \(F_T\) are defined at the levels

\[z = (k - 1/2)\Delta z, \quad k = 1,2,\ldots,(K_N - 1)\].
6.2 The difference equations for the zonal mean

For time differencing we choose a semi-implicit method in which the inertia-gravity terms (i.e., the Coriolis, pressure gradient, and adiabatic heating terms) are treated implicitly while the nonlinear advection terms and forcing terms are represented by centered differences. However, in order to prevent a weak time splitting of the solutions, which occurs due to the "leapfrog" scheme for the advection terms, a forward time step is used once every 48 steps.

The time differencing scheme can be expressed efficiently if we define a time average as follows

\[
\hat{F} = \beta_1 F^{n+1} + \beta_2 F^n + \beta_3 F^{n-1}
\]  

(6.1)

Here \( F \) stands for any dependent variable, \( n \) is the time index given by

\[ t = n\Delta t, \ n = 0,1,2,\ldots \]

where \( \Delta t \) is the time step, and \( \beta_1, \beta_2, \beta_3 \) are defined as follows:

(a) leapfrog step, \( \beta_1 = 1/4, \beta_2 = 1/2, \beta_3 = 1/4 \)

(b) forward step, \( \beta_1 = 1/2, \beta_2 = 1/2, \beta_3 = 0 \)

For leapfrog steps the time difference can then be written as

\[
\left( \frac{\partial F}{\partial t} \right)^n = \frac{F^{n+1} - F^{n-1}}{2\Delta t} = \frac{\hat{F} - \frac{1}{2}(F^n + F^{n-1})}{(\Delta t/2)}
\]  

(6.2a)

While for forward steps we have

\[
\left( \frac{\partial F}{\partial t} \right)^n = \frac{F^{n+1} - F^n}{\Delta t} = \frac{\hat{F} - F^n}{(\Delta t/2)}
\]  

(6.2b)
In writing out the space differences it is convenient to use the following differencing and averaging operators:

\[ \delta_{j+1/2} = \frac{[\psi_j - \psi_{j+1}]}{\Delta y} \]  \hspace{1cm} (6.3a)

\[ \langle \psi_j + \frac{1}{2} \rangle = \frac{[\psi_j + \psi_{j+1}]}{2} \]  \hspace{1cm} (6.3b)

Furthermore, to write the required vertical differences we let

\[ \left( \frac{\partial F}{\partial z} + \frac{F}{2H} \right) = e^{-z/2H} \frac{\partial}{\partial z} (F e^{z/2H}) \approx (F_{k+1}^+ - F_k^-)/\Delta z \]  \hspace{1cm} (6.4a)

where \( e^+ = e^{\Delta z/4H} \) and \( e^- = e^{-\Delta z/4H} \). Similarly, we have

\[ \left( \frac{\partial F}{\partial z} - \frac{F}{2H} \right) \approx (F_{k+1}^- - F_k^+)/\Delta z \]  \hspace{1cm} (6.4b)

where in each case the difference is centered at the \( k + 1/2 \) level.

Using the operators defined in (6.1) - (6.4) we can write finite difference approximations to (3.7) - (3.10) as follows:

\[ \hat{\psi} - (f\Delta t/2) \hat{\psi} = \hat{A} \]  \hspace{1cm} (6.5)

\[ \hat{\psi} + (f\Delta t/2) \hat{U} + (\Delta t/2) \delta_{j-1/2} \hat{\psi} = \hat{B} \]  \hspace{1cm} (6.6)

\[ (\hat{\psi}_{k+1}^+ - \hat{\psi}_{k-1}^+) + \frac{\Delta z}{\cos \theta} \delta_{j+1/2} (\hat{\psi}_k \cos \theta^\star) = 0 \]  \hspace{1cm} (6.7)

\[ (\hat{\psi}_{k+1}^- - \hat{\psi}_k^+) + \frac{\Delta t \Delta z}{2} \hat{\psi}_k = R \]  \hspace{1cm} (6.8)

Here the terms involving the unknown variables have been collected on the left hand sides, and the source terms involving known quantities at time
levels \( n \) and \( n - 1 \) appear on the right hand sides. In writing out these equations the subscripts \( j,k \) have been omitted wherever no ambiguity would result. In the continuity equation \( \cos \theta \) is required at both the \( \bar{V} \) and \( \bar{W} \) grid points. Thus, we define

\[
\theta_j = \pi/2 - (j - 1/2)\Delta y/a
\]

\[
\theta^*_j = \pi/2 - (j - 1)\Delta y/a
\]

(6.9a)

(6.9b)

The source terms \( \bar{A} \) and \( \bar{B} \) are defined as follows:

\[
\bar{A} = \mu_1 \bar{u}^n + \mu_2 \bar{u}^{n-1} - \frac{\Delta t}{2} e^{z/2H} \left\{ \frac{1}{\cos^2 \theta^*_j} \delta_j \left( \bar{u}^n \cos \theta^*_j \right) \left( \bar{v}^n \cos \theta^*_j \right)_j \right\} \\
+ \frac{1}{2\Delta z \cos \theta^*_j} \left\{ \left( \bar{u}^n_{j,k} \epsilon^+ + \bar{u}^n_{j,k+1} \epsilon^- \right) \left( \bar{w}^n \cos \theta \right)_{j-1/2,k-1/2} \right\}
\]

\[
\bar{B} = \mu_1 \bar{v}^n + \mu_2 \bar{v}^{n-1} - \frac{\Delta t}{2} e^{z/2H} \left\{ \bar{v}^n_{j-1} \left( \cos \theta^*_j \right) \left( \cos \theta^*_j \right)_{j-1} - \cos \theta^*_j \left( \cos \theta^*_j \right)^{-1}_{j-1} \right\} \\
+ \frac{\Delta t}{2} \left\{ \frac{\cos \theta^*_j}{\cos \theta^*_j+1} - \frac{\cos \theta^*_j}{\cos \theta^*_j+1} \right\} + \frac{\Delta t}{2} D_2 \left( \bar{v}^{n-1} \right)
\]

(6.10a)

(6.10b)

Here the coefficients \( \mu_1 \) and \( \mu_2 \) are defined as follows:

(a) leapfrog step, \( \mu_1 = \mu_2 = 1/2 \)

(b) forward step, \( \mu_1 = 1, \mu_2 = 0 \)
In formulating $\bar{B}$ we have used a special form for the so-called "metric" term, $\bar{U}^2 \tan \theta/a$, which is required to keep the difference equations energy conserving in adiabatic, frictionless flow. To derive this form we note that

$$ - \frac{\bar{U}^2 \tan \theta}{a} = \frac{\bar{U}^2}{2 \cos^2 \theta} \frac{d}{dy} (\cos^2 \theta) $$

This may be approximated as

$$ \approx \frac{\bar{U}}{4\Delta y} \left[ \frac{\bar{U}}{j-1} \left( \frac{\cos^2 \theta^*_{j-1} - \cos^2 \theta^*_{j}}{\cos \theta^*_{j} \cos \theta^*_{j-1}} \right) + \frac{\bar{U}}{j+1} \left( \frac{\cos^2 \theta^*_{j+1} - \cos^2 \theta^*_{j}}{\cos \theta^*_{j} \cos \theta^*_{j+1}} \right) \right] $$

which easily reduces to the form given in (6.10b).

In order to write the thermodynamic source term, $\bar{R}$, in a compact form we define a density weighted "thickness" by letting

$$ \bar{S}^n_{j,k} = \left( \bar{\nu}^n_{j,k+1} e^+ - \bar{\nu}^n_{j,k} e^- \right) $$

(6.11)

We then have:

$$ \bar{R} = \mu_1 \bar{S}^n + \mu_2 \bar{S}^{n-1} + \frac{\Delta t}{2} \left[ \Delta z (\varepsilon_T + Q) + D_2 (\bar{S}^{n-1}) \right] $$

$$ - \frac{\Delta t}{2} e^{z/2H} \left\{ \frac{1}{2 \cos \theta} \delta_{j+1/2} [\cos \theta^* (\bar{\nu}^n_{j,k+1} + \bar{\nu}^n_{j,k}) (\bar{S}^n)_{j-1/2,k}] $$

$$ + \frac{1}{4\Delta z} \left[ (\bar{\nu}^n_{j,k+1} + \bar{\nu}^n_{j,k}) (\bar{S}^n_{j,k+1} e^+ + \bar{S}^n_{j,k} e^-) \right] $$

$$ - (\bar{\nu}^n_{j,k} + \bar{\nu}^n_{j,k-1})(\bar{S}^n_{j,k} e^+ + \bar{S}^n_{j,k-1} e^-) \right\} $$

(6.12)
6.3 The difference equations for the eddies

Using the above notation the equations may be written as follows:

\[
\hat{U}_s - (f\Delta t/2) \hat{V}_s = -im_s \frac{\Delta t}{2} \langle \psi_s \rangle_{j+1/2} + A_s \tag{6.13}
\]

\[
\hat{V}_s + (f\Delta t/2) \hat{U}_s = \frac{\Delta t}{2} \delta_{j+1/2} \langle \psi_s \rangle + B_s \tag{6.14}
\]

\[
\langle \psi_{s,k+1}^+ - \psi_{s,k}^- \rangle + \frac{N^2 \Delta t \Delta z}{2} \hat{W}_s = R_s \tag{6.15}
\]

\[
\langle \hat{W}_{s,k}^- - \hat{W}_{s,k-1}^+ \rangle + \frac{\Delta z}{\cos \theta^*} \left[ i \langle m_s \hat{U}_s \cos \theta \rangle_{j-1/2} + \delta_{j-1/2} \langle \hat{V}_s \cos \theta \rangle \right] = 0 \tag{6.16}
\]

Where here \( m_s = s/(a \cos \theta) \)

\[
A_s = \mu_1 U^n_s + \mu_2 U^{n-1}_s - \frac{\Delta t}{2} e^{z/2H} \left\{ i \langle m_s \hat{U}_s \rangle_{j+1/2} U^n_s + \frac{\hat{V}^n_s}{\cos \theta} \delta_{j+1/2} \langle \hat{U} \cos \theta^* \rangle \right\}
\]

\[
+ \frac{1}{2\Delta z} \left[ \langle W_{s,j,k} \hat{U}_{j,k+1}^+ - \hat{U}_{j,k}^- \rangle \right]_{j+1/2}
\]

\[
+ \langle W_{s,j,k-1} \hat{U}_{j,k}^+ - \hat{U}_{j,k-1}^- \rangle_{j+1/2} \right\} + \frac{\Delta t}{2} D_{2}(U^{n-1}_s) \tag{6.17}
\]

\[
B_s = \mu_1 V^n_s + \mu_2 V^{n-1}_s - \frac{\Delta t}{2} e^{z/2H} \left\{ i \langle m_s \hat{V}_s \rangle_{j+1/2} V^n_s \right\}
\]

\[
- \frac{U^n_s}{\Delta y} \left[ \hat{U}_j \left( \frac{\cos \theta^*}{\cos \theta_j} - \frac{\cos \theta_{j+1}}{\cos \theta^*_{j+1}} \right) \right]
\]

\[
+ \hat{U}_{j+1} \left[ \frac{\cos \theta_{j+1}^*}{\cos \theta_{j+1}} - \frac{\cos \theta_j}{\cos \theta^*_{j+1}} \right] \right\} + \frac{\Delta t}{2} D_{2}(V^{n-1}_s) \tag{6.18}
\]
(Note that if \( \overline{U} \) is symmetric about the equator then \( A_s \) and \( B_s \) have the same symmetry as \( U_s \) and \( V_s \), respectively.)

In formulating \( B_s \) we have used a special form for the metric term, 
\[-2\overline{U}U_s \tan \theta/a,\] which is required to keep the difference equations energy conserving in adiabatic frictionless flow. To derive this form note that

\[-2\overline{U}U_s \tan \theta/a = \frac{\overline{U} U_s}{\cos^2 \theta} \frac{d}{dy} (\cos^2 \theta)\]

This may be approximated as

\[\frac{U_s}{\Delta y} \left( \frac{\cos^2 \theta^* - \cos^2 \theta}{\cos \theta^* \cos \theta} \right) + \frac{U_{s+1}}{\Delta y} \left( \frac{\cos^2 \theta - \cos^2 \theta^*}{\cos \theta \cos \theta^*} \right)\]

which easily reduces to the form given in (6.18).

The term \( R_s \) in (6.15) can be written in a fairly compact form if we first define a density weighted "thickness"

\[S_{s,j,k} \equiv (\psi_{s,j,k+1} e^+ - \psi_{s,j,k} e^-)\]

We then can write

\[R_s \equiv \mu_1 S_{s,j,k}^n + \mu_2 S_{s,j,k}^{n-1} + \frac{\Delta t}{2} Q_s + \frac{\Delta t}{2} D_s (S_{s,j,k}^{n-1})\]

\[-\frac{\Delta t}{2} e^{\Delta z/2H} \left( \frac{\imath m k}{2} (\overline{U}^n_{j,k+1} + \overline{U}^n_{j,k}) S_{s,j,k}^n\right)\]

\[+ \frac{\overline{W}_{s,j,k}^n}{2\Delta z} \left( \langle \overline{S}^n \rangle_{j-\frac{1}{2},k+1} e^{\Delta z/2H} - \langle \overline{S}^n \rangle_{j-\frac{1}{2},k-1} e^{-\Delta z/2H} \right)\]

\[+ \left( \frac{\langle V_s \rangle_{j-\frac{1}{2},k+1} + \langle V_s \rangle_{j-\frac{1}{2},k}}{2} \right) \delta_{j-\frac{1}{2}} \overline{S}_{j,k}^n\]

(6.19)
7. SOLUTION METHOD

7.1 The zonal mean equations

The system (6.5) - (6.8) is a set of simultaneous equations for the unknowns \( \hat{U}, \hat{V}, \hat{\Psi}, \) and \( \hat{W} \). To solve this set we first eliminate \( \hat{U} \) between (6.5) and (6.6) to obtain

\[
\hat{V} = \gamma_j \left[ -\frac{\Delta t}{2} \delta_{j-1/2} \hat{\Psi} + \mathcal{B} - \frac{f \Delta t}{2} \right]
\] 

(7.1)

where \( \gamma_j = (1 + f^2 \Delta t^2 / 4)^{-1} \).

We next substitute from (7.1) and (6.8) into (6.7) to eliminate \( \hat{W} \) and \( \hat{V} \). The result is an elliptic difference equation in \( \hat{\Psi} \):

\[
\begin{align*}
\Gamma_k \hat{\Psi}_{j,k+1} - (\Gamma_{k-1} e^{\Delta z/2H} + \Gamma_k e^{-\Delta z/2H}) \hat{\Psi}_{j,k} \\
+ \Gamma_{k-1} \hat{\Psi}_{j,k-1} + A_j \hat{\Psi}_{j-1,k} + B_j \hat{\Psi}_{j,k} + C_j \hat{\Psi}_{j+1,k} = D_{j,k}
\end{align*}
\]

(7.2)

Here we have let \( N^2(z) = N_0^2 / \Gamma(z) \) where \( N_0^2 = \) constant, and then expressed \( \Gamma(z) \) (the vertical variation of static stability) at \( z = (k - 1/2) \Delta z \) as \( \Gamma_k \).

The coefficients, \( A_j, B_j, C_j, D_j \) are then defined by

\[
A_j \equiv \frac{N_0^2 \Delta z^2 \Delta t^2}{4 \Delta y^2 \cos \theta_j} (\gamma_j \cos \theta_j)
\]

\[
C_j \equiv \frac{N_0^2 \Delta z^2 \Delta t^2}{4 \Delta y^2 \cos \theta_j} (\gamma_{j+1} \cos \theta_{j+1})
\]

\[
B_j \equiv -A_j - C_j
\]
The elliptic system (7.2) may be solved for \( \Psi_{j,k} \) using the NCAR subroutine BLKTRI (Swarztrauber and Sweet, 1975). In this case the solution is carried out for the grid points in the range

\[
1 \leq j \leq J_m - 1, \quad 2 \leq k \leq K_N \leq 1
\]

The lateral boundary condition (4.1) is incorporated by setting

\[ A_j = 0 \text{ and } C_{J_m - 1} = 0 \]

The lower boundary condition is incorporated by setting \( \Psi_{j,1} \) equal to the value obtained from integrating (4.2c). The upper boundary condition (2.16c) in finite difference form requires

\[
\hat{\Psi}_{j,K_N} e^+ = \hat{\Psi}_{j,K_N - 1} e^-
\]  

(7.3)

Once \( \hat{\Psi} \) has been obtained by inversion of (7.2) it is a simple matter to compute the remaining fields. If, however, one attempts to compute \( \hat{V} \) from (7.1) the results are rather unsatisfactory due to large truncation errors. This problem arises due to the fact that the first and third terms on the right side are generally two orders of magnitude greater than \( \hat{V} \) so that \( \hat{V} \) is obtained as a small residual of two large but opposite terms. To avoid this problem it is useful to utilize the meridional streamfunction
defined by (3.11). In finite difference form we have

$$\hat{W}_{j,k} = \frac{1}{\cos \theta} \frac{1}{\delta_{j+1/2} X} \hat{X}$$  \hspace{1cm} (7.4a)$$

$$\hat{V}_{m,k} = \frac{1}{\Delta z \cos \theta X} \left( \hat{X}_{j,k} e^+ - \hat{X}_{j,k-1} e^- \right)$$  \hspace{1cm} (7.4b)$$

which identically satisfies the finite difference form of the continuity equation (6.7).

Substituting from (7.4a) into (6.8) and noting that $\hat{X}_{1,k} = 0$ we can solve for $\hat{X}_{j,k}$:

$$\hat{X}_{j+1,k} = \hat{X}_{j,k} \frac{-2Ay \cos \theta}{N^2 \Delta z \Delta t} \left[ \hat{U} - \left( \hat{\Psi}_{k+1} e^+ - \hat{\Psi}_k e^- \right) \right]$$  \hspace{1cm} (7.5)$$

We next use (7.4b) to solve for $\hat{V}$ and finally use (6.5) to obtain $\hat{U}$. The final step of the solution is then to use the definition (6.1) to obtain all fields at time $n+1$. For example,

$$\bar{U}^{n+1} = \left( \hat{U} - \beta_2 \bar{U}^n - \beta_3 \bar{U}^{n-1} \right) / \beta_1$$  \hspace{1cm} (7.6)$$

and similarly for $\bar{V}^{n+1}$, $\bar{\Psi}^{n+1}$, and $\bar{W}^{n+1}$.

7.2 The eddy equations

The system (6.13) - (6.16) is a set of simultaneous equations for the unknowns $\hat{U}_s$, $\hat{V}$, $\hat{\Psi}_s$, $\hat{W}_s$ which is exactly analogous to the mean flow set discussed above. To solve this set we first solve for $\hat{U}_s$ and $\hat{V}_s$ in terms of $\hat{\Psi}_s$.
using (6.13) and (6.14):

\[
\hat{U}_s = \gamma_j \left[ \frac{-\text{im} \Delta t}{2} \left( \hat{\psi}_{s,j+1/2} + \frac{\Delta t}{2} \delta_{j+1/2} \right) \psi_s + A_s + \frac{\Delta t}{2} B_s \right] \tag{7.7}
\]

\[
\hat{V}_s = \gamma_j \left[ \frac{\text{im} f \Delta t^2}{4} \left( \hat{\psi}_{s,j+1/2} + \frac{\Delta t}{2} \delta_{j+1/2} \right) \psi_s + B_s - \frac{\Delta t}{2} A_s \right] \tag{7.8}
\]

where \( \gamma_j \equiv (1 + f^2 \Delta t^2/4)^{-1} \).

Combining (6.15), (6.16), (7.7), and (7.8) we get a single equation for \( \hat{\psi}_s \):

\[
\Gamma_{k,s,j,k+1} = (\Gamma_{k-1} e^{\Delta z/2H} + \Gamma_k e^{-\Delta z/2H}) \hat{\psi}_{s,j,k} + \Gamma_{k-1} \hat{\psi}_{s,j,k-1} + D_{s,j} \hat{\psi}_{s,j-1,k} + E_{s,j} \hat{\psi}_{s,j,k} + F_{s,j} \hat{\psi}_{s,j,k+1,k} = T_{s,j,k} \tag{7.9}
\]

where \( \Gamma_k \) is as defined below (7.2), and the coefficients \( D_s, E_s, F_s \) are

\[
D_{s,j} = \frac{N^2 \Delta z^2 \Delta t^2}{4 \cos \theta_j^\infty} \left[ \frac{\gamma_{l-1} \cos \theta_{l-1}}{\Delta y^2} - \frac{m^2_{s,j-1} \gamma_{l-1} \cos \theta_{l-1}}{4} \right]
\]

\[
E_{s,j} = \frac{N^2 \Delta z^2 \Delta t^2}{4 \cos \theta_j^\infty} \left[ \frac{\gamma_{l-1} \cos \theta_{l-1}}{\Delta y^2} + \frac{m^2 \gamma_{l-1} \cos \theta_{l-1}}{4} \right]
\]

\[
F_{s,j} = -\frac{N^2 \Delta z^2 \Delta t^2}{4 \cos \theta_j^\infty} \left[ \frac{\gamma_{l-1} \cos \theta_{l-1}}{\Delta y^2} + \frac{m^2_{s,j-1} \cos \theta_{l-1} \gamma_{l-1} + m^2 \gamma_{l-1} \cos \theta_{l-1} \gamma_{l-1}}{4} \right]
\]

\[
+ \frac{i \Delta t}{\Delta y} \left( \gamma_{l-1} m_{s,j-1} \cos \theta_{l-1} f_{l-1} - \gamma_j m_{s,j} \cos \theta_j f_j \right)
\]
and the source term is

\[ T_{s,j,k} = (\Gamma_k R_{s,j,k} e^{-} - \Gamma_{k-1} R_{s,j,k} e^{+}) \]

\[ + \frac{N^2 \Delta t \Delta z^2}{2 \cos \theta_j^k} [(\gamma_{j-1} \cos \theta_{j-1} q_{s,j-1,j} - \gamma_j \cos \theta_j q_{s,j,j})/\Delta y \]

\[ + \frac{i}{2} (m_{s,j-1,j} \gamma_{j-1} \cos \theta_{j-1} p_{s,j-1,j} + m_{s,j,j} \gamma_j \cos \theta_j p_{s,j,j})] \]

where

\[ p_{s,j} = A_s + \frac{f\Delta t}{2} B_s, \quad q_{s,j} = B_s - \frac{f\Delta t}{2} A_s,j \]

The elliptic system (7.9) may be solved for \( \Psi_{s,j,k} \) using the NCAR subroutine CBLKTRI.

For global or hemispheric antisymmetric modes the solution is carried out for grid points

\[ 2 \leq j \leq J_m - 1; \quad 2 \leq k \leq K_N - 1 \]

However, for symmetric hemispheric calculations the solution includes the point \( j = J_m \).

In the global or antisymmetric hemispheric case we thus require

\[ \Psi_{s,1,k} = \Psi_{s,J_m,k} = 0 \] (7.10)

while for the symmetric hemispheric case we must have \( \Psi_{s,1,k} = 0 \) and

\[ \Psi_{s,J_m-1,k} = \Psi_{s,J_m+1,k} \] (7.11)
Condition (7.10) is incorporated by letting $D_s, 2 = 0$ and $F_{s, J_{m-l}} = 0$ while the condition (7.11) requires replacing $D_s, J_m$ as defined above by $D_s, J_m + F_s, J_m$.

In all cases the upper boundary condition is $\psi_{s, j, K_N} = 0$ and the lower boundary condition is a specified forcing

$$\psi_{s, j, l} = gh(y, t) \quad (7.12)$$

Once $\psi_{s, j, k}$ is obtained from (7.9) we compute $\hat{W}_{s, j, k}$ from (6.15)

$$\hat{W}_{s, j, k} = \frac{2\Gamma}{\Delta t N_s^2} R_{s, j, k} - (\psi_{s, j, k+1} e^+ - \psi_{s, j, k} e^-) \quad (7.13)$$

We then use (7.7) and (7.8) to solve for $\hat{U}_s$ and $\hat{V}_s$. Finally, these results are used to obtain $U_{s}^{n+1}, V_{s}^{n+1}, \psi_{s}^{n+1}$ by a formula analogous to (7.6). A similar treatment of $\hat{W}_s$ proved unstable. Therefore in computing fluxes and vertical advection terms $\hat{W}_s$ is used in place of $W_s^{n+1}$.

7.3 The eddy flux terms

The eddy momentum flux convergence (3.11) and the eddy heat flux convergence (3.12) must be written in finite differences so that the energy integrals of the flow remain satisfied. It turns out that energetically consistent forms are:
\[ F_m = -e^{z/2H} \left\{ \frac{1}{\cos^2 \theta_j} \delta_{j-\frac{1}{2}} \left[ \langle \Psi_{s}^* \Psi_{s} + \Psi_{s}^* \Psi_{s} \rangle \cos^2 \theta \right] \right. \]

\[ + \frac{1}{2\Delta z} \left\{ \langle U_{s,j,k} e^- + U_{s,j,k+1} e^+ \rangle_{j-\frac{1}{2}} \Psi_{s,j,k} \right. \]

\[ - \langle U_{s,j,k} e^+ + U_{s,j,k-1} e^- \rangle_{j-\frac{1}{2}} \Psi_{s,j,k-1} \]

\[ + \langle U_{s,j,k} \Psi_{s,j,k+1} e^+ + U_{s,j,k+1} \Psi_{s,j,k} e^- \rangle_{j-\frac{1}{2}} \Psi_{s,j,k} \]

\[ - \langle U_{s,j,k} \Psi_{s,j,k+1} e^- + U_{s,j,k-1} \Psi_{s,j,k} e^+ \rangle_{j-\frac{1}{2}} \Psi_{s,j,k-1} \} \]  

(7.14)

and,

\[ \Delta z F_T = -\frac{e^{(z+\Delta z/2)/H}}{2 \cos \theta_j} \delta_{j+\frac{1}{2}} \left\{ \cos \theta_j [\langle \nabla_{s,j-\frac{1}{2},k+1} + \langle \nabla_{s,j-\frac{1}{2},k} \rangle S_{s,j,k} \right. \]

\[ + (\langle \nabla_{s,j-\frac{1}{2},k+1} + \langle \nabla_{s,j-\frac{1}{2},k} \rangle S_{s,j,k} \left. \right) S_{s,j,k} \right\} \]

\[ - \frac{e^{z/2H}}{2\Delta z} [\langle W_{s,s,j+\frac{1}{2},k+1}^* S_{s,s,j+\frac{1}{2},k+1} - \langle W_{s,s,j+\frac{1}{2},k-1} \rangle S_{s,s,j+\frac{1}{2},k-1} \]

\[ + \langle W_{s,s,j+\frac{1}{2},k+1}^* S_{s,s,j+\frac{1}{2},k+1} - \langle W_{s,s,j+\frac{1}{2},k-1} \rangle S_{s,s,j+\frac{1}{2},k-1} \} \]  

(7.15)
8. INTEGRAL CONSTRAINTS AND SUBGRID SCALE DIFFUSION

8.1 Integral constraints for the zonal mean equations

The basic equations of the model (3.7) - (3.10) satisfy certain integral constraints which also must be satisfied by the finite difference equations if satisfactory long term integrations are to be obtained. It is easily verified that when the forcing terms $F_M$, $F_T$, and $Q$ are omitted, and subgrid scale diffusion is neglected, the rate of change of zonal mean kinetic plus available potential energy is equal to the energy flux through the lower boundary:

$$
\frac{d}{dt} (\overline{P} + \overline{K}) = \int_A \left\{ \left[ \frac{U^2 + (\partial \overline{\Psi}/\partial z + \overline{\Psi}/2H)^2}{2N^2} + \overline{\Psi} \right] \right\}_{z=0} dA \tag{8.1}
$$

where

$$
\overline{P} = \int_T \frac{1}{2} \left( \frac{\partial \overline{\Psi}}{\partial z} + \frac{\overline{\Psi}}{2H} \right) / N^2 d\tau \tag{8.2a}
$$

$$
\overline{K} = \int_T \frac{1}{2} (U^2 + \overline{V^2}) d\tau \tag{8.2b}
$$

and

$$
dA = a^2 \cos \theta \, d\theta \, d\lambda \, dz
$$

$$
d\tau = a^2 \cos \theta \, d\theta \, d\lambda \, d\tau
$$

Another important constraint is the conservation of relative angular momentum. If we multiply (3.7) by $\exp(-z/2H)\cos \theta$ and integrate the result over the entire domain we find that relative angular momentum is conserved except for the flux of angular momentum through the lower boundary:
\[ \frac{d}{d\tau} \int e^{-z/2H} \overline{U \cos \theta} \, d\tau = \int [\overline{U \overline{W}} \cos \theta]_{z=0} \, dA + \int \overline{fx(z=0)} \, dA \]  

(8.3)

In deriving (8.3) we have neglected eddy momentum fluxes through the lower boundary. It is important to note in connection with (8.3) that horizontal diffusion can not change relative angular momentum so that we require

\[ \int D_1(\overline{U}) \cos \theta \, dA = 0 \]  

(8.4)

which constrains the possible forms for the operator \( D_1(\cdot) \).

Also, horizontal diffusion can not change the horizontally averaged temperature (thickness) on a horizontal surface. Thus from (3.10):

\[ \int D_2 \left( \frac{\overline{\psi}}{2H} + \frac{\overline{\psi}}{2H} \right) \, dA = 0 \]  

(8.5)

The constraints (8.1), (8.3), and (8.5) must also be satisfied by our system of finite difference equations\(^2\) if satisfactory results are to be obtained. In finite difference form the integrals are replaced by sums over the grid points:

\[ \int (\cdot) \, d\tau \approx \sum_{j,k} (\cdot)2\pi a \cos \theta \Delta y \Delta z \]  

(8.6a)

\[ \int (\cdot) \, dA \approx \sum_j (\cdot)2\pi a \cos \theta \Delta y \]  

(8.6b)

\(^2\)Except for the effects of time truncation errors.
The value of $\theta_j$ used in (8.6a) or (8.6b) is given by either (6.9a) or (6.9b) depending on the location of the dependent variable; e.g., in (8.5) we use (6.9a) and in (8.3) we used (6.9b).

Next, multiplying (6.5) by $e^{-z/2H} \cos^2 \theta^* (2\pi a \Delta y \Delta z)$ we find after summing over all grid points ($2 \leq j \leq J^{'}, 2 \leq k \leq K^{'}, 1$), and using 6.2a):

$$\frac{d}{dt} \sum_{j,k} e^{-z/2H} \frac{\bar{U} \cos^2 \theta^*}{2} = \frac{1}{\Delta z} \sum_j [f \bar{X}_{j,1} \cos \theta^*]$$

$$+ \frac{1}{\Delta z} \sum_j [(\bar{U}_{j,2} e^+ + \bar{U}_{j,1} e^-) \langle \bar{w} \cos \theta \rangle_{j-\frac{1}{2},1} \cos \theta^*]$$

(8.7)

which is consistent with the differential form for angular momentum conservation, (8.3). (We have here assumed that (8.4) holds for the finite difference form of subgrid scale diffusion.)

The finite difference analogy to the energy integral (8.1) may be obtained by multiplying (6.5) by $\bar{U} \cos \theta^*$, (6.6) by $\bar{V} \cos \theta^*$, and (6.8) by $$(\cos \theta)/N^2 \Delta z \left( \bar{\Psi}_{k+1}^+ e^+ - \bar{\Psi}_k^- e^- \right)$$ then adding the three resulting equations together and summing over all gridpoints. Using (6.2) to express the time derivatives in differential form we can then write

$$\frac{d}{dt} \left[ \sum_{j=2, J^{'}, 1 \atop k=2, K^{'}, 1} \frac{(\bar{U}^2_j + \bar{V}^2_j)}{2} \cos \theta^* \right] + \sum_{j=1, J^{'}, 1 \atop k=1, K} \frac{(\bar{\Psi}_{k+1}^+ - \bar{\Psi}_k^-)^2 \cos \theta^*}{2 \Delta z^2 N^2}$$

$$= \frac{1}{\Delta z} \sum_{j=1, J^{'}, 1} \left[ \bar{W}_{j,1} \bar{\Psi}_{j,1} e^- \cos \theta^* j + \frac{\bar{U}_{j,2} + \bar{U}_{j,1} \langle \bar{w} \cos \theta \rangle_{j-\frac{1}{2},1} \cos \theta^*}{2} \right.$$

$$+ \frac{(\bar{W}_{j,2} + \bar{W}_{j,1}) \langle \bar{\Psi}_{j,3}^+ - \bar{\Psi}_{j,2}^- \rangle (\bar{\Psi}_{j,2}^+ - \bar{\Psi}_{j,1}^- \rangle \langle \bar{\Psi}_{j,2} \rangle + \bar{\Psi}_{j,1} \rangle ^+ (8.8)$$
where \( F_M = F_T = Q = 0 \) and friction terms have all been neglected. Clearly, (8.9) is a reasonable analogue to the differential relationship (8.1).

8.2 Subgrid scale diffusion for the mean flow equations

In order to suppress nonlinear instability it is necessary to smooth all fields in the meridional direction. In order to prevent this smoothing from damping the large scale motions we have chosen to use a fourth order linear diffusion operator. In applying the diffusion in the zonal momentum and thermodynamic energy equations we must recall that both relative angular momentum and horizontal average temperature must be conserved [see (8.4) and (8.5)]. In addition the diffusion terms should make negative definite contributions to the energy equation.

In order to satisfy both these requirements it turns out that in the zonal momentum equation relative angular velocity should be diffused. Thus,

\[
D_1(\bar{U}) = - \frac{K}{\cos^2 \theta} \frac{\partial^2}{\partial y^2} \left( \frac{\bar{U}}{\cos \theta} \right)
\]

(8.9)

This automatically satisfies (8.4) provided that \( (\partial^3 / \partial y^3)(\bar{U}/\cos \theta) = 0 \) at the meridional boundaries. If we multiply (8.9) by \( \bar{U} \cos \theta \) and integrate the result in \( y \) we obtain

\[
\int_A \bar{U} \cos \theta \, D_1(\bar{U}) \, dA = - K \int \left[ \frac{\partial^2}{\partial y^2} \left( \frac{\bar{U}}{\cos \theta} \right) \right]^2 \, dy \, d\lambda
\]

which is negative definite. Thus, the diffusion term (8.9) acts as an
energy sink. In finite difference form we write

\[
D_1(\overline{U}) = \frac{- K}{\cos^2 \theta \Delta y^*} \left[ \left( \frac{\overline{U}}{\cos \theta \Delta y^*} \right)_{j-2} - 4 \left( \frac{\overline{U}}{\cos \theta \Delta y^*} \right)_{j-1} + 6 \left( \frac{\overline{U}}{\cos \theta \Delta y^*} \right)_{j} + \left( \frac{\overline{U}}{\cos \theta \Delta y^*} \right)_{j+1} + \left( \frac{\overline{U}}{\cos \theta \Delta y^*} \right)_{j+2} \right]
\] (8.10)

In order that this finite difference form satisfy the difference analogue of (8.4) i.e., \( \sum_j \cos^2 \theta \Delta y^* D_1(\overline{U}) = 0 \) the formula must be modified at the points adjacent to the boundaries. Thus

\[
[D_1(\overline{U})]_{j=2} = \frac{- K}{\cos^2 \theta \Delta y^*} \left[ \left( \frac{2\overline{U}}{\cos \theta \Delta y^*} \right)_{j=2} - 3 \left( \frac{\overline{U}}{\cos \theta \Delta y^*} \right)_{j=3} + \left( \frac{\overline{U}}{\cos \theta \Delta y^*} \right)_{j=4} \right] (8.11a)
\]

\[
[D_1(\overline{U})]_{j=3} = \frac{- K}{\cos^2 \theta \Delta y^*} \left[ -3 \left( \frac{\overline{U}}{\cos \theta \Delta y^*} \right)_{j=2} + 6 \left( \frac{\overline{U}}{\cos \theta \Delta y^*} \right)_{j=3} - 4 \left( \frac{\overline{U}}{\cos \theta \Delta y^*} \right)_{j=4} + \left( \frac{\overline{U}}{\cos \theta \Delta y^*} \right)_{j=5} \right] (8.11b)
\]

with analogous expressions for \( j = J_m - 1 \) and \( j = J_m - 2 \). Again using the notation of (6.11) we can write the finite difference diffusion term in the thermodynamic energy equation as follows:

\[
D_2(\overline{S}) = \frac{- K}{\cos \theta \Delta y^*} \left[ \overline{S}_{j-2} - 4 \overline{S}_{j-1} + 6 \overline{S}_j - 4 \overline{S}_{j+1} + \overline{S}_{j+2} \right] (8.12)
\]

Again the points adjacent to the boundaries require special treatment:

\[
[D_2(\overline{S})]_{j=1} = \frac{- K}{\cos \theta \Delta y^*} \left[ 2 \overline{S}_{j-1} - 3 \overline{S}_{j=2} + \overline{S}_{j=3} \right] (8.13)
\]
\[ [D_2(S)]_{j=2} = \frac{-K}{\cos \theta \Delta y} \left[ -3S_{j=1} + 6S_{j=2} - 4S_{j=3} + S_{j=4} \right] \] \hfill (8.14)

\[ [D_2(S)]_{j=J_m-1} = \frac{-K}{\cos \theta \Delta y} \left[ S_{j=J_m-3} - 3S_{j=J_m-2} + 2S_{j=J_m-1} \right] \] \hfill (8.15)

Finally we write

\[ D_2(\overline{V}) = \frac{-K}{\cos \theta \Delta y} \left[ \overline{V}_{j-2} - 4\overline{V}_{j-1} + 6\overline{V}_j - 4\overline{V}_{j+1} + \overline{V}_{j+2} \right] \] \hfill (8.16)

with the special cases

\[ [D_2(\overline{V})]_{j=2} = \frac{-K}{\cos \theta \Delta y} \left[ 3\overline{V}_{j=2} - 3\overline{V}_{j=3} + \overline{V}_{j=4} \right] \] \hfill (8.17)

and an analogous form for \( j = J_m - 1 \).

8.3 Subgrid scale diffusion for the eddy equations

To filter out small scale noise so as to suppress nonlinear instability the eddy equations include fourth order linear diffusion terms similar to those discussed in Section 8.2 for the zonal mean flow. \( D_2(U_s) \) and \( D_2(V_s) \) have the same form as \( D_2(S) \) given in (8.12), while \( D_2(S_s) \) has the form of \( D_2(\overline{V}) \) given in (8.16). These forms must, however, be modified next to the boundaries to insure that diffusion does not change the meridional average of any field. For a global domain the modification to \( D_2(S_s) \) is identical to that given in (8.17) for \( D_2(\overline{V}) \).

For the \( U_s \) and \( V_s \) field the situation is more complicated since the boundary conditions are different in the \( s = 1 \) and \( s > 1 \) cases.
For the case \( s = 1 \), \( D_2(U_s) \) and \( D_2(V_s) \) are computed using formulas analogous to (8.12), (8.13), and (8.14). For global integrations formulas similar to (8.13) and (8.14) are applied at \( j = J_m - 1 \) and \( j = J_m - 2 \).

For the case \( s > 1 \) the polar boundary condition requires that

\[
D_2(U_s)_{j=1} = \frac{K}{\cos \theta \Delta y^4} \left[ 4U_{s,j=1} - 3U_{s,j=2} + U_{s,j=3} \right]
\]

(8.18)

\[
D_2(U_s)_{j=2} = \frac{K}{\cos \theta \Delta y^4} \left[ -5U_{s,j=1} + 6U_{s,j=2} - 4U_{s,j=3} + U_{s,j=4} \right]
\]

(8.19)

with similar expressions for \( V_s \).

In the case of hemispheric integrations the diffusion operators at \( j = J_m - 1 \) and \( j = J_m - 2 \) are modified as follows:

For antisymmetry conditions on \( \hat{\Psi} \) the form of \( D_2(S_s) \) is the same as in the global case; however, since \( U_s, j_m = -U_s, j_m - 1 \) and \( V_s, j_m = +V_s, j_m - 1 \) we use a diffusion form analogous to (8.18) and (8.19) for \( D_2(U_s)_{j_m-1} \) and \( D_2(U_s)_{j_m-2} \) while for \( D_2(V_s)_{j_m-1} \) and \( D_2(V_s)_{j_m-2} \) we use forms analogous to (8.14) and (8.15).

For symmetric conditions on \( \hat{\Psi} \) the form of \( D_2(S_s) \) must be modified as follows:

\[
D_2(S_s)_{j_m-1} = \frac{-K}{\cos \theta \Delta y^4} \left[ -4S_{s,j_m} + 7S_{s,j_m-1} - 4S_{j_m-2} + S_{j_m-3} \right]
\]

(8.20)

\[
D_2(S_s)_{j_m} = \frac{-K}{\cos \theta \Delta y^4} \left[ 3S_{j_m} - 4S_{j_m-1} + S_{j_m-2} \right]
\]

(8.21)

and since \( U_s, j_m = U_s, j_m - 1 \) and \( V_s, j_m = -V_s, j_m - 1 \) we use forms similar to (8.14) and (8.15) for \( D_2(U_s)_{j_m-1} \) and \( D_2(U_s)_{j_m-2} \) and forms analogous to (8.18) and (8.19) for \( D_2(V_s)_{j_m-1} \) and \( D_2(V_s)_{j_m-2} \).
9. DIABATIC HEATING COMPUTATION

9.1 Infrared heating

This study has utilized Dickinson's (1973) parameterization of infrared cooling consisting of the sum of the cooling for a reference temperature \( T_0 \) and a Newtonian cooling approximation for the departures from that profile. Thus the net heating terms take the following forms: For the eddies,

\[
Q_s = - \alpha T_s
\]

while for the mean flow

\[
\overline{Q} = \overline{Q}_e - (\overline{Q}_r + \alpha \overline{T})
\]

where \( \overline{Q}_e \) is the diabatic heating due to the absorption of solar radiation by ozone. \( Q_r \) is the net infrared cooling at each level for the reference temperature profile, and \( \alpha \) is the Newtonian cooling coefficient. \( \overline{Q}_e \) and \( \overline{T} \) are functions of altitude and latitude while \( \overline{Q}_r, \alpha, \) and \( T_0 \) depend on altitude alone.

The values of the Newtonian cooling coefficients have been calculated for levels between 30 and 80 km by Dickinson (1973). Below 30 km Trenberth's (1973) values are adopted. Although the accuracy of the Newtonian cooling representation breaks down above about 70 km, it shall be retained at this time for lack of a better representation. Following Schoeberl and Strobel (1978), the value of \( \alpha \) between 80 and 96 km was taken to be the \( \text{CO}_2 \) cooling rate in the fundamental band at 15\( \mu \) (see Fig. 2).

Dickinson's (1973) careful computations of \( \alpha \) and \( \overline{Q}_r \) were made for atmospheric temperature profiles that differ little from the reference temperature profile. Because the actual temperatures may vary considerably
from this reference profile, especially in the winter polar region, alternative values of $Q_r$ are here computed in the following manner.

At a given level the globally averaged diabatic heating $\tilde{Q}$ is given by

$$\tilde{Q} = \tilde{Q}_e - (\tilde{Q}_r - \tilde{dT})$$

where ($\tilde{}$) designates a horizontal average on the sphere. Since the observed globally averaged temperature profile $T_o$ is fairly well known, we choose $\tilde{Q}_r$ so that global radiative equilibrium ($\tilde{Q} = 0$) is achieved when the globally averaged temperature profile $\tilde{T}$ is equal to $T_o$. Therefore

$$\tilde{Q}_r = \tilde{Q}_e.$$

9.2 Solar Heating

Below 96 km ozone is the only significant absorber of solar radiation. The parameterizations of Lacis and Hansen (1974) are used to compute the solar heating term $Q_s$. The diurnally averaged solar heating is calculated by fixing the sun angle at its average value between sunrise and sunset (approximation 1 of Cogley and Borucki, 1976). The sun angle may remain fixed for the duration of a given run, or may be varied according to the seasonal cycle depending on the objectives of the particular run.
10. A TEST APPLICATION OF THE MODEL

In order to demonstrate the capabilities of the model we have computed the zonal mean annual cycle for the stratosphere and mesosphere for conditions of zonal mean forcing only. In this experiment the eddy forcing was set to zero at the lower boundary. The mean zonal winds at the lower boundary (16 km) were specified to vary over the annual cycle according to the observations of Labitzke (1972) for the northern hemisphere and Taljaard et al. (1969) for the southern hemisphere. The diabatic heating was also specified to vary on the annual cycle by including seasonal variations in the solar zenith angle and sun-earth distance.

10.1 Rayleigh friction parameterization

In order to produce a realistic mean wind profile it proved necessary to specify strong damping in the mean momentum equations above 70 km. In the atmosphere the mechanical damping of the mean wind near the mesopause is probably due to the breaking of gravity waves and tides. For the present model this effect is parameterized in the simplest possible form by using a height dependent Rayleigh friction coefficient

\[ \kappa_R = \kappa_0 + \kappa_1 \left[ 1 + \tanh \left( \frac{z - 71}{10} \right) \right] \]

where \( \kappa_0 = 1/80 \) days, \( \kappa_1 = 1/4 \) days and \( z \) is in kilometers. This profile is shown in Fig. 2.

The biharmonic horizontal diffusion coefficient is given the value \( K/\Delta y^4 = 10^{-8} \) s\(^{-1} \) which is the minimum necessary to suppress nonlinear computational instability when \( \Delta t = 1 \) hr.
10.2 The zonal mean annual cycle

Figs. 3-8 show the zonal mean wind, mean meridional wind, and vertical velocity profiles for southern hemisphere winter solstice and spring equinox conditions computed using the above described parameters and a grid resolution of 10° latitude and 5 km height. During the solstice season there is a thermally direct mean meridional circulation with rising motion in the summer hemisphere and sinking in the winter hemisphere. At the equinox, on the other hand there is a two cell meridional circulation with rising in the equatorial zone and sinking near both poles. Zonal mean winds computed in both seasons are quite realistic. This example shows that a zonal mean model is capable of simulating many important features of the general circulation in the middle atmosphere. Further details of this annual cycle simulation are reported in Holton and Wehrbein (1979).
References


Figure 1: A portion of the grid mesh in the meridional plane showing the arrangement of variables on the staggered grid.
Figure 2: Vertical profiles of the Newtonian cooling coefficient (solid line) and Rayleigh friction coefficient (dashed line) in units of $d^{-1}$. Time in units of $d^{-1}$. 
Figure 3: Computed mean zonal winds (m s$^{-1}$) for the Southern Hemisphere winter solstice.

Figure 4: Computed mean zonal winds (m s$^{-1}$) for the Southern Hemisphere vernal equinox.
Figure 5: Computed mean meridional wind (m s\(^{-1}\)) for the Southern Hemisphere winter solstice.

Figure 6: Computed mean meridional wind (m s\(^{-1}\)) for the Southern Hemisphere vernal equinox.
Figure 7: Computed mean vertical velocity (mm s\(^{-1}\)) for the Southern Hemisphere winter solstice.

Figure 8: Computed mean vertical velocity (mm s\(^{-1}\)) for the Southern Hemisphere vernal equinox.
APPENDIX

FORTRAN CODE FOR THE SEMI-SPECTRAL MODEL

The present version of the model computes the interaction of a single wave mode with the zonal mean flow. The program consists of the main program, PROGRAM WAVE2, in which the fields are initialized and the calling sequence for the various subroutines is established. The main dynamical computations are carried out in SUBROUTINE ASTREAM (mean flow equations) and SUBROUTINE EDDY (wave equations). The radiative heating calculations are carried out in SUBROUTINE HEAT, SUBROUTINE RADEQU, FUNCTION DELT, and FUNCTION OZUV. The output fields are created in SUBROUTINE AOUT. All of the above routines are given in the following program listing. In addition the program requires the NCAR library routines SUBROUTINE BLKTRI and SUBROUTINE CBLKTRI.

In order to facilitate reading of the code a dictionary of the principal FORTRAN symbols is provided below. In addition to defining the symbols, we have indicated the location in the program where each symbol first appears.
### Dictionary of FORTRAN Symbols

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<th>Location</th>
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<td>$\alpha$, Newtonian cooling coefficient</td>
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</tr>
<tr>
<td>AIRDEN</td>
<td>air density</td>
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<tr>
<td>ALA(J)</td>
<td>$\Delta t/\Delta y (2\Omega \sin \theta)$</td>
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<td>ALB(J)</td>
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<td>CM(J)</td>
<td>$C_j$ coefficient in (7.2)</td>
<td>A 105</td>
</tr>
<tr>
<td>CN(K-1)</td>
<td>Coefficient of $\hat{\psi}_{k+1}$ in (7.2)</td>
<td>A 176</td>
</tr>
<tr>
<td>CNA(J)</td>
<td>$N^2\Delta z^2\Delta t/(2\Delta y \cos \theta)$</td>
<td>A 77</td>
</tr>
<tr>
<td>CNB(J)</td>
<td>$\Delta t/(2\Delta y \cos \theta)$</td>
<td>A 68</td>
</tr>
<tr>
<td>COR(J)</td>
<td>$2\Omega \sin \theta$</td>
<td>A 65</td>
</tr>
<tr>
<td>CORIOL(J)</td>
<td>$2\Omega \sin \theta^*$</td>
<td>A 66</td>
</tr>
<tr>
<td>COZONE</td>
<td>(unused)</td>
<td>C 8</td>
</tr>
<tr>
<td>CR(K)</td>
<td>Coefficient of 3rd term in (7.9)</td>
<td>A 180</td>
</tr>
<tr>
<td>CS(J)</td>
<td>$\gamma \cos \theta^*$</td>
<td>A 63</td>
</tr>
<tr>
<td>CSA(J)</td>
<td>$\gamma \cos \theta^*$</td>
<td>A 64</td>
</tr>
<tr>
<td>CTS(J,K)</td>
<td>(unused)</td>
<td>A 42</td>
</tr>
<tr>
<td>CUB(J)</td>
<td>$\gamma \cos \theta^*$</td>
<td>A 78</td>
</tr>
<tr>
<td>CURTIS(J,K)</td>
<td>(unused)</td>
<td>A 37</td>
</tr>
<tr>
<td>C1(J)</td>
<td>$F_{s,j}$ coefficient in (7.9)</td>
<td>A 187</td>
</tr>
<tr>
<td>DCA(K)</td>
<td>$a_k\Delta t/2$</td>
<td>A 227</td>
</tr>
<tr>
<td>DCOS(J)</td>
<td>$1./(a \cos \theta^*)$</td>
<td>A 87</td>
</tr>
<tr>
<td>DELAY</td>
<td>forcing switch-on time delay</td>
<td>A 229</td>
</tr>
<tr>
<td>DEL(J)</td>
<td>$N^2\Delta t\Delta z^2/\cos \theta^*$</td>
<td>A 85</td>
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<tr>
<td>DELT</td>
<td>$\Delta t$</td>
<td>A 30</td>
</tr>
<tr>
<td>DELTA</td>
<td>solar declination</td>
<td>B 24</td>
</tr>
<tr>
<td>DENS(K)</td>
<td>$\exp(z/2H)$</td>
<td>A 94</td>
</tr>
<tr>
<td>DKY</td>
<td>$K/\Delta y^k$</td>
<td>A 228</td>
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<tr>
<td>DR(J)</td>
<td>$\overline{B} - \overline{A}\Delta t/2$</td>
<td>F 130</td>
</tr>
<tr>
<td>DT</td>
<td>$\Delta t/2$</td>
<td>A 31</td>
</tr>
<tr>
<td>FORTRAN Symbol</td>
<td>Definition</td>
<td>Location</td>
</tr>
<tr>
<td>----------------</td>
<td>-----------------------------------------------------------</td>
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<tr>
<td>DTODY</td>
<td>$\Delta t/(4\Delta y)$</td>
<td>A 57</td>
</tr>
<tr>
<td>DUM(J)</td>
<td>dummy</td>
<td>I 18</td>
</tr>
<tr>
<td>DY</td>
<td>$\Delta y$</td>
<td>A 51</td>
</tr>
<tr>
<td>DY2</td>
<td>$(\Delta y)^2$</td>
<td>A 54</td>
</tr>
<tr>
<td>DZ</td>
<td>$\Delta z$</td>
<td>A 29</td>
</tr>
<tr>
<td>DZ2</td>
<td>$\Delta z^2$</td>
<td>A 53</td>
</tr>
<tr>
<td>DZN</td>
<td>$N^2\Delta z^2\Delta t^2/(4\Delta y^2 \cos \theta)$</td>
<td>A 103</td>
</tr>
<tr>
<td>EC</td>
<td>Orbital eccentricity of the earth</td>
<td>B 21</td>
</tr>
<tr>
<td>EMI</td>
<td>$\exp(-\Delta z/4H)$</td>
<td>A 59</td>
</tr>
<tr>
<td>EPL</td>
<td>$\exp(\pm z/4H)$</td>
<td>A 58</td>
</tr>
<tr>
<td>FDAY</td>
<td>fraction of day that sun shines</td>
<td>B 49</td>
</tr>
<tr>
<td>FM(J,K)</td>
<td>$F_M (7.14)$</td>
<td>H 38</td>
</tr>
<tr>
<td>FREQ</td>
<td>$\Omega$</td>
<td>A 55</td>
</tr>
<tr>
<td>FT(J,K)</td>
<td>$F_T (7.15)$</td>
<td>H 22</td>
</tr>
<tr>
<td>GAM(J)</td>
<td>$\gamma_j + \frac{1}{2}$</td>
<td>A 67</td>
</tr>
<tr>
<td>GAMB(J)</td>
<td>$\gamma$</td>
<td>A 76</td>
</tr>
<tr>
<td>GBV(K)</td>
<td>$\Gamma_k$</td>
<td>A 95</td>
</tr>
<tr>
<td>GMEP(J)</td>
<td>vertical advection of $\overline{T}$</td>
<td>F 76</td>
</tr>
<tr>
<td>GM1(J)</td>
<td>$\frac{.5i}{s} \Delta t/(a \cos \theta)$</td>
<td>A 89</td>
</tr>
<tr>
<td>GTIME</td>
<td>growth time for forcing</td>
<td>A 231</td>
</tr>
<tr>
<td>ICLOUD</td>
<td>altitude layer of clouds</td>
<td>C 43</td>
</tr>
<tr>
<td>ICT</td>
<td>Index for forward difference</td>
<td>A 33</td>
</tr>
<tr>
<td>IEND</td>
<td>Total time steps for run</td>
<td>A 45</td>
</tr>
<tr>
<td>IFD</td>
<td>Frequency of forward steps</td>
<td>A 32</td>
</tr>
<tr>
<td>IFLG</td>
<td>IFLG = 0 initializes BLKTRI</td>
<td>A 29</td>
</tr>
<tr>
<td><strong>FORTRAN Symbol</strong></td>
<td><strong>Definition</strong></td>
<td><strong>Location</strong></td>
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<tr>
<td>IMAT</td>
<td>Index for output</td>
<td>A 111</td>
</tr>
<tr>
<td>INIT</td>
<td>Input Flag ≠ 0 for TAPE 1 input on continuation runs</td>
<td>A 45</td>
</tr>
<tr>
<td>IPHAS(J)</td>
<td>Dummy</td>
<td>I 89</td>
</tr>
<tr>
<td>IPRINT</td>
<td>Frequency of output</td>
<td>A 45</td>
</tr>
<tr>
<td>TRAD</td>
<td>Index for calls to HEAT</td>
<td>A 99</td>
</tr>
<tr>
<td>IRECT</td>
<td>Frequency of calls to HEAT</td>
<td>A 100</td>
</tr>
<tr>
<td>ITIME</td>
<td>Index for time step n</td>
<td>A 110</td>
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<tr>
<td>JM</td>
<td>$J_m$</td>
<td>A 29</td>
</tr>
<tr>
<td>JML</td>
<td>$J_m - 1$</td>
<td>A 47</td>
</tr>
<tr>
<td>KAP(K)</td>
<td>$\alpha_k$, Newtonian cooling</td>
<td>A 97</td>
</tr>
<tr>
<td>KN</td>
<td>$K_N$</td>
<td>A 29</td>
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<tr>
<td>KNL</td>
<td>$K_N - 1$</td>
<td>A 48</td>
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<td>KZ</td>
<td>Diffusion Coefficient</td>
<td>A 34</td>
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<tr>
<td>M</td>
<td>$J_m - 2$</td>
<td>A 49</td>
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<tr>
<td>N</td>
<td>$K_N - 2$</td>
<td>A 50</td>
</tr>
<tr>
<td>NGC(J)</td>
<td>$\gamma_j^{+1/2} \cos \theta$</td>
<td>A 81</td>
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<tr>
<td>NU1(J)</td>
<td>$.5i s(\gamma \cos \theta)/(a \cos \theta)$</td>
<td>A 83</td>
</tr>
<tr>
<td>OZUV</td>
<td>Solar energy absorbed by ozone</td>
<td>D 1</td>
</tr>
<tr>
<td>P(J)</td>
<td>$p_s,j$</td>
<td>G 140</td>
</tr>
<tr>
<td>PB(J,K)</td>
<td>$\omega^{n+1}$</td>
<td>A 159</td>
</tr>
<tr>
<td>PBA(J,K)</td>
<td>Dummy array</td>
<td>A 138</td>
</tr>
<tr>
<td>PBO(J,K)</td>
<td>$\omega^n$</td>
<td>A 160</td>
</tr>
<tr>
<td>PERH</td>
<td>Date of herhelion after Vernal Equinox</td>
<td>B 18</td>
</tr>
<tr>
<td>PHBL1(J)</td>
<td>Amplitude of boundary forcing for wave</td>
<td>A .69</td>
</tr>
<tr>
<td>FORTRAN Symbol</td>
<td>Definition</td>
<td>Location</td>
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<tr>
<td>PHI</td>
<td>Latitude</td>
<td>B 30</td>
</tr>
<tr>
<td>PI</td>
<td>(\pi)</td>
<td>A 46</td>
</tr>
<tr>
<td>PL1(J)</td>
<td>(\hat{\psi}(k=1))</td>
<td>A 234</td>
</tr>
<tr>
<td>PRS(J,K)</td>
<td>Contains Fourier Coefficients of (\overline{U}_B(y,t)) on input</td>
<td>A 39</td>
</tr>
<tr>
<td>PI(J,K)</td>
<td>(\psi^{n+1}_s)</td>
<td>A 243</td>
</tr>
<tr>
<td>PI1A(J,K)</td>
<td>Dummy array</td>
<td>A 239</td>
</tr>
<tr>
<td>PI0(J,K)</td>
<td>(\psi^n_s)</td>
<td>A 243</td>
</tr>
<tr>
<td>Q(J)</td>
<td>(q_{s,j})</td>
<td>G 141</td>
</tr>
<tr>
<td>QA(K)</td>
<td>Globally averaged net radiative heating</td>
<td>A 259</td>
</tr>
<tr>
<td>QB(J,K)</td>
<td>(\overline{Q}), diabatic heating</td>
<td>B 93</td>
</tr>
<tr>
<td>QDOT( )</td>
<td>Net radiative heating function</td>
<td>B 89</td>
</tr>
<tr>
<td>QO(J,K), QOG(J)</td>
<td>Ozone density/Loeschmidt's number</td>
<td>A 40</td>
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<tr>
<td>QOZS(J,K), QOSZG(J)</td>
<td>Ozone column abundance</td>
<td>A 41</td>
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<tr>
<td>QR</td>
<td>Radiative cooling of reference atmosphere</td>
<td>C 23</td>
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<tr>
<td>QRS(K)</td>
<td>Globally averaged solar heating</td>
<td>A 115</td>
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<tr>
<td>QS(J,K)</td>
<td>(Q_s), diabatic heating in wave</td>
<td>A 195</td>
</tr>
<tr>
<td>R(J,K)</td>
<td>(T_{s,j,k}) (on input)</td>
<td>G 143</td>
</tr>
<tr>
<td>RAB</td>
<td>Effective albedo of lower atmosphere</td>
<td>C 49</td>
</tr>
<tr>
<td>RAD</td>
<td>(a) (radius of earth)</td>
<td>A 29</td>
</tr>
<tr>
<td>RAYF(K)</td>
<td>(\kappa_R), Rayleigh friction</td>
<td>A 225</td>
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<tr>
<td>RR1</td>
<td>Albedo of reflecting region</td>
<td>C 51</td>
</tr>
<tr>
<td>RDB</td>
<td>Spherical albedo of reflecting region</td>
<td>C 50</td>
</tr>
<tr>
<td>RED(6,K)</td>
<td>Newtonian cooling parameters</td>
<td>A 43</td>
</tr>
<tr>
<td>RG</td>
<td>Ground reflectivity</td>
<td>C 42</td>
</tr>
<tr>
<td>FORTRAN Symbol</td>
<td>Definition</td>
<td>Location</td>
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<tr>
<td>RHO</td>
<td>Distance to sun in A.U.</td>
<td>B 22</td>
</tr>
<tr>
<td>RHZ</td>
<td>H/287.1</td>
<td>A 56</td>
</tr>
<tr>
<td>RR(J,K)</td>
<td>D_{j,k} (on input)</td>
<td>F 135</td>
</tr>
<tr>
<td>S</td>
<td>Planetary Wavenumber</td>
<td>A 26</td>
</tr>
<tr>
<td>SA</td>
<td>$\theta^*$</td>
<td>A 62</td>
</tr>
<tr>
<td>SB</td>
<td>\theta</td>
<td>A 61</td>
</tr>
<tr>
<td>SH</td>
<td>H</td>
<td>A 29</td>
</tr>
<tr>
<td>SO3</td>
<td>Ozone UV heating</td>
<td>C 63</td>
</tr>
<tr>
<td>STAB(J)</td>
<td>$N^2 \Delta t^2 \Delta z^2 / \cos \theta^*$</td>
<td>A 86</td>
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<tr>
<td>STRDAY</td>
<td>Starting day (vernal equinox = 80)</td>
<td>A 36</td>
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<tr>
<td>SU(J)</td>
<td>$2\Omega \sin \theta \Delta t \gamma / \Delta y$</td>
<td>A 80</td>
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<tr>
<td>SV(J)</td>
<td>$\gamma \Delta t / \Delta y$</td>
<td>A 79</td>
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<tr>
<td>SZT(J,K)</td>
<td>Ozone UV heating field</td>
<td>B 67</td>
</tr>
<tr>
<td>T(J,K), TC(J)</td>
<td>Standard Atmosphere temperatures in each zone</td>
<td>A 38</td>
</tr>
<tr>
<td>TAU</td>
<td>Optical depth of clouds in visible</td>
<td>C 40</td>
</tr>
<tr>
<td>TB(J,K)</td>
<td>$\bar{T}$, deviation of zonally averaged temperature from global mean</td>
<td>B 76</td>
</tr>
<tr>
<td>TEMP</td>
<td>Dummy</td>
<td>A 147</td>
</tr>
<tr>
<td>TIME</td>
<td>$t = n\Delta t$</td>
<td>A 113</td>
</tr>
<tr>
<td>THETA</td>
<td>Polar angle</td>
<td>B 59</td>
</tr>
<tr>
<td>TN(J)</td>
<td>$(\Delta \gamma)^{-1}(\cos \theta^<em>_{j-1}/\cos \theta^</em><em>{j} - \cos \theta^*</em>{j}/\cos \theta^*_{j-1})$</td>
<td>A 91</td>
</tr>
<tr>
<td>TNA(J)</td>
<td>$(\cos \theta^<em>_{j}/\cos \theta^</em><em>{j} - \cos \theta^*</em>{j}/\cos \theta^*_{j})/\Delta y$</td>
<td>A 82</td>
</tr>
<tr>
<td>TNB(J)</td>
<td>$(\cos \theta^<em>_{j}/\cos \theta^</em><em>{j+1} - \cos \theta^*</em>{j+1}/\cos \theta^*_{j})/\Delta y$</td>
<td>A 90</td>
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<td>TR</td>
<td>Reference temperature profile</td>
<td>C 22</td>
</tr>
<tr>
<td>TSTAR</td>
<td>Local time of sunset</td>
<td>B 40</td>
</tr>
<tr>
<td>FORTRAN Symbol</td>
<td>Definition</td>
<td>Location</td>
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<tr>
<td>----------------</td>
<td>---------------------------------------------------------------------------</td>
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<tr>
<td>TUL(J)</td>
<td>( i \gamma s \Delta t/(2a \cos \theta) )</td>
<td>A 74</td>
</tr>
<tr>
<td>TVL(J)</td>
<td>( i \gamma s \Delta t^2(2\Omega \sin \theta)/2a \cos \theta )</td>
<td>A 75</td>
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<tr>
<td>TZO(K)</td>
<td>Global radiative equilibrium temperature</td>
<td>B 61</td>
</tr>
<tr>
<td>T1</td>
<td>( \mu_1 )</td>
<td>F 25</td>
</tr>
<tr>
<td>T2</td>
<td>( \mu_2 )</td>
<td>F 26</td>
</tr>
<tr>
<td>T3</td>
<td>( \beta_2/\beta_1 )</td>
<td>F 27</td>
</tr>
<tr>
<td>T4</td>
<td>( \beta_3/\beta_1 )</td>
<td>F 28</td>
</tr>
<tr>
<td>T5</td>
<td>( 1/\beta_1 )</td>
<td>F 29</td>
</tr>
<tr>
<td>UB(J,K)</td>
<td>( \overline{U}^{n+1} )</td>
<td>A 127</td>
</tr>
<tr>
<td>UBO(J,K)</td>
<td>( \overline{U}^n )</td>
<td>A 128</td>
</tr>
<tr>
<td>UT</td>
<td>ozone path</td>
<td>C 57</td>
</tr>
<tr>
<td>U1(J,K)</td>
<td>( \overline{U}^{n+1}_s )</td>
<td>A 243</td>
</tr>
<tr>
<td>U10(J,K)</td>
<td>( \overline{U}^n_s )</td>
<td>A 244</td>
</tr>
<tr>
<td>VB(J,K)</td>
<td>( \overline{V}^{n+1} )</td>
<td>A 250</td>
</tr>
<tr>
<td>VBO(J,K)</td>
<td>( \overline{V}^n )</td>
<td>A 250</td>
</tr>
<tr>
<td>V1(J,K)</td>
<td>( \overline{V}^{n+1}_s )</td>
<td>A 244</td>
</tr>
<tr>
<td>V10(J,K)</td>
<td>( \overline{V}^n_s )</td>
<td>A 244</td>
</tr>
<tr>
<td>WB(J,K)</td>
<td>( \overline{W}^{n+1} )</td>
<td>A 250</td>
</tr>
<tr>
<td>WBO(J,K)</td>
<td>( \overline{W}^n )</td>
<td>A 250</td>
</tr>
<tr>
<td>WRA(I)</td>
<td>Work Array</td>
<td>A 249</td>
</tr>
<tr>
<td>WR0(I)</td>
<td>Work Array in BLKTRI</td>
<td>A 244</td>
</tr>
<tr>
<td>WL(J,K)</td>
<td>( \overline{W}^{n+1}_s )</td>
<td>A 244</td>
</tr>
<tr>
<td>XBA(J,K)</td>
<td>( \overline{X} )</td>
<td>A 250</td>
</tr>
<tr>
<td>XMA1(J)</td>
<td>( s/(a \cos \theta^x) )</td>
<td>A 88</td>
</tr>
<tr>
<td>FORTRAN Symbol</td>
<td>Definition</td>
<td>Location</td>
</tr>
<tr>
<td>---------------</td>
<td>-----------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>XM1(J)</td>
<td>( s / (a \cos \theta) )</td>
<td>A 73</td>
</tr>
<tr>
<td>Z(K)</td>
<td>( z )</td>
<td>A 93</td>
</tr>
<tr>
<td>ZEN</td>
<td>Average value of cos(solar zenith angle)</td>
<td>B 37</td>
</tr>
<tr>
<td>ZT</td>
<td>( z + \Delta z / 2 )</td>
<td>I 31</td>
</tr>
</tbody>
</table>
PROGRAM WAVE2 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1=TAPE2)

COMPLEX U1(19,17),U10(19,17),V1(19,17),V10(19,17),P1(19,17),P10(19,
S17),W1(19,16),W10(19,16),P1A(19,17),B1(19,17),P1(19,15),P10(19),
Q1(19),SN1U(19),CI1G1M(19),TU1(19),TV1(19),CDU1M(19,16),P1(19),O5(19,16)

COMPLEX Y2,PHBK(19)

REAL UB(19,7),UB0(19,17),VB(19,17),VBO(19,17),PB(19,17),PPB(19,17),
$),WS(19,7),PPB(19,17),WRO(400),AN(15),BN(15),CN(15),CS(19),KAP(17),
$),WS(19,7),PPB(19,17),WRO(400),AN(15),BN(15),CN(15),CS(19),KAP(17),

REAL PHBI,1Q1A(MI(19),OEL(19),STAB(19),ALA(19),ALB(19),TNA(19),

SYM = 0 FOR GLOBAL DOMAIN, SYM = 1 FOR ANTISYMMETRIC HEMISPHERE

DATA JIKN,IFL(19,17),O/.RADDZBVFPSH/16.746,5000,4,E7.O0EB/

DELT*1800.*2.

DT = DELT/2.

KZ = 0.

SETUP CONSTANTS AND INITIALIZE

PI = 2.*AS1N(I.)

JML - JM-1

N - KN-2

DY = PI*RAD/JML

CI = (0.,1.)

DZ2 = DZ**2

DYZ = DYZ**2

FREQ = 7.29*E7.

RHZ = SH/287.1

DTODY = DT/(4.*DY)

EPL = EXP(DZ/4.*SH))

EMI = EXPDZ/(4.*SH))

DO 10 J=JM-1

SA = PI*(JM+1-2.*J)/(2.*JML)

CS(J) = COS(SB)

CSA(J) = COS(SA)

COR(J) = 2.*FREQ*SIN(SB)

COR1(J) = 2.*FREQ*SIN(SA)

GAM(J) = 1./*(1.+COR(J)**2+DT**2)

CNB(J) = 1./*DT*CS(J)

PHB(J) = 150.*SIN((3.*PI/6.)*(SA-PI/6.))*2.*9.*

IF (SA,ST-P=PI/6.) PHB(J) = 0.

ALB(J) = DT/DY*COR(J)

ALB(J) = COR(J)*DT*GAM(J)

XM1(J) = S/(SA)*CS(J)

TU1(J) = GAM(J)*CI1XM1(J)*DT*Z.

TV1(J) = TU1(J)*COR(J)*DT.

GAM(J) = 1./1.*(COR1(J)**2+DT**2)
CNA(J) = BVF*D22*DT/(DY*CS(J))  A 77
CUB(J) = GAM8(J)*CSA(J)  A 78
SV(J) = GAM(J)*DT/DY  A 79
SU(J) = COR(J)*DT*SV(J)  A 80
NGC(J) = GAM(J)*CSA(J)  A 81
THA(J) = (CSA(J)/CS(J)-CS(J)/CSA(J))/DY  A 82
10 NUL(J) = CI*DI*HM(J)*GAM(J)*CSA(J)  A 83
DO 15 J=2,JM  A 84
DEL(J) = BVF*DT*DZ2/CSA(J)  A 85
STAB(J) = DEL(J)*DT  A 86
DCOS(J) = 1./(R4*CSA(J))  A 87
XNA1(J) = S/(R4*CSA(J))  A 88
GRI(J) = .5*(C*XM(A1(J)+DT)  A 89
TNB(J-1) = (CSA(J-1)/CSA(J)-CSA(J-1))/DY  A 90
15 TN(J) = 1./(4.*DY)*CSA(J)-CSA(J)/DY  A 91
DO 20 K=1,KN  A 92
Z(K) = (K-1)*DZ  A 93
DENS(K) = EXP(Z(K)/(Z.*SH))  A 94
GBV(K) = 1.  A 95
IF (K.GT.KNL) GO TO 20  A 96
KAP(K) = (RED(5,K)/86400.)*DT  A 97
20 CONTINUE  A 98
IRAD = 0  A 99
IRCT = 1  A 100
WRITE (6,195) (KAP(K),K=1,KN)  A 101
DO 25 J=1,JM  A 102
DIN = BVF*D22/(CS(J)+DY)*DT**2  A 103
AM(J) = D2N*GAM8(J)*CSA(J)  A 104
CM(J) = D2N*GAM8(J+1)*CSA(J+1)  A 105
25 BM(J) = AM(J)-CM(J)  A 106
BM(JML) = BM(JML)+CM(JML)  A 107
AM(J) = CM(JML) = (O.,O.)  A 108
ITIME = 0  A 109
IMAT = 0  A 110
MT = 1.  A 111
TIME = 0  A 112
IF (INIT.NE.0) GO TO 90  A 113
QRS(L) = -101.  A 114
C READ INITIAL ZONAL FLOW  A 115
DO 30 J=1,JM  A 116
30 CONTINUE  A 117
DO 45 J=1,JM  A 118
Y = 2*PI*STRDAY/360.  A 119
RZ = PRS1(J)  A 120
R1 = PRS2(J)  A 121
S1 = PRS3(J)  A 122
R2 = PRS4(J)  A 123
35 CONTINUE  A 124
DO 70 J=1,JM  A 125
70 CONTINUE  A 126
C COMPUTE INITIAL GEOPOTENTIAL  A 127
DO 55 K=1,KN  A 128
DO 45 J=1,JM  A 129
UBO(J,K) = UBO(J,1)  A 130
UB(J,K) = UBO(J,1)  A 131
45 CONTINUE  A 132
DO 60 J=1,JM  A 133
DO 55 K=1,KN  A 134
UB(J,K) = UBO(J,K)/DENS(K)  A 135
55 CONTINUE  A 136
DO 65 J=1,JM  A 137
65 CONTINUE  A 138
C READ INITIAL FLOW  A 139
DO 60 J=1,JM  A 140
60 CONTINUE  A 141
C COMPUTE INITIAL GEOPOTENTIAL  A 142
DO 55 K=1,KN  A 143
55 CONTINUE  A 144
DO 65 J=1,JM  A 145
DO 55 K=1,KN  A 146
SUM = 0.  A 147
TEMP = 0.  A 148
DO 65 J=1,JM  A 149
65 CONTINUE  A 150
DO 70 J=1,JM  A 151
70 CONTINUE  A 152
PBA(J,K) = PBA(J,K)-SUM1/TEMP
CONTINUE
PBA(JM,K) = PBA(JML,K)
CONTINUE
DO 85 J=1,JM
DO 80 K=1,KN
PBA(J,K) = PBA(J,K)
CONTINUE
DO 85 CONTINUE
GO TO 95
CONTINUE C
FOR CONTINUATION RUNS READ INPUT DATA
READ (1) TIME,PL,PB,P10,UB,UL,VL,V10,W1,KBO,UBO,VB,VB0,VBW,VBW0,
GBP,TZ0,NHZ,ANZ,FDY,T0,QRs,SZT
CONTINUE C
GLOBAL MEAN STABILITY PROFILE AND INITIAL HEATING
QA(1) = 101.
CALL HEAT (TIME,GBP,GBV,HN,RH,DEHo,DZ,JM,KN,TZ0,BVF,SH,PI,OA,
$STDay)
DO 100 K=1,KN
AN(K) = GBV(K)
CN(K) = GBV(K+1)
BN(K) = -GBV(K)*EPL**2-GBV(K)*EMI**2
DO 100 CONTINUE
DO 105 K=1,N
AR(K) = AN(K)
CR(K) = CN(K)
BR(K) = BN(K)
BR(N) = BR(N)+EMI/EPL*GBV(KNL)
AR(1) = CR(N) = 0.
DO 110 J=2,JML
AL(J-1) = STAB(J)*(NGC(J-1)/DYZ-XMiCJ-1)**2*NGC(J-1)/4.)
CI(J-1) = STAB(J)+NGC(J)/DYZ-XMiCJ-1)**2*NGC(J)/4.
BI(J-1) = -STAB(J)*(XMiCJ-1)**2*NGC(J-1)+XMiCJ-1)**2*NGC(J))/4+
$NGC(J-1)+NGC(J))/DYZ-C1*(NGC(J-1)+XMiCJ-1)*AL(AJ-1)-NGC(J)*XMiCJ
$J*AL(AJ))
CONTINUE C
COMPUTE EDDY NONNEWTONIAN HEATING
DO 115 J=1,JM
DO 120 K=1,KNL
DAY = TIME/(24.*60.*60.)
DO 120 J=1,JM
Y = 2.*PI*(STDay+DAY DELT1864G.)/360. A
RZ = PRS(1,J)
RI = PRS(2,J)
SI = PRS(3,J)
R2 = PRS(4,J)
PR5(5J) = RZ/2.*R1*COS(Y)+SI*SIN(Y)+RZ*C03(2.*Y)
DO 120 CONTINUE
PR5(6,J) = 0.
DO 125 J=I*JML
TEMP = TEMP+CS(J)
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DO 125 CONTINUE
PR5(6,J) = PRS(6,J-1)+DY*CORIOL(J)*PR5(5,J)+PR5(5,J-1)*PR5(6,J-1)
DELAY = 0.
DELAY = TIME - DELAY
GTIME = 4.32E+05
IF (DELAY.LT.0.) GO TO 160
DO 155 J = 2, JM
   P1J(J) = (PHBI(J)*((1.-EXP(-((DELAY+DEL)/(GTIME))))+2.*P1(J-1)+PI0 $/4.
155 CONTINUE
160 CONTINUE
   DO 165 K = 1, KNL
      P1A(JK) = (0., 0.)
165 CONTINUE
   CALL EDDY (A1, B1, C1, ALB, BVF, ANG, CI, CS, CSA, COR, DCA, DEL, DENS, DKY, $ DTDY, DT, DZ, EM1, EPL, GM1, GMJ, ICT, IFD, JH, KNL, M, S, NCS, NUL, $ RAYF, PL1, P, S, SV, TUI, TVJ, TNA, TNB, XH1, XMA1, PB, PlA, P10, R, UB, U1, $ U10, V10, W10, WR, CDUM, ANm, BN, CN, GBV, SYM, Q, KZ)
   CALL FLUX (JH, KN, DZ, CS, CS, DENS, EPL, EM1, U1, V1, W1, PlA, FM, FT, $ KZ, UBO, CDUM, CNB)
   IFLG = IFLG-2
   ICT = ICT-1
   CALL ASTREAM (AMB, CM, AR, BR, CR, IFL, IFD, ICT, DT, DZ, DY, CHI, Q, WRA, $ RR, XBA, UB, UBD, VB, VBD, WR, WBD, PB, PBO, PBA, CS, CSA, FM, FT, EPL, EM1, RAYF, $ DCA, DKY, TN, DZ, DENS, CNB, GM, COR, C1, CNA, CUB, BVF, JM, JML, KN, NS, GBV)
   IMAT = IMAT+1
   ITIME = ITIME+1
   TIME = TIME+DEL
   IF (ICT.EQ.00) ICT = 0
   IRAD = IRAD+1
   IF (IRAD.EQ.0) GO TO 170
   QA(1) = 0.
   CALL HEAT (TIME, G8, PB, TB, Q, RHZ, DENS, DZ, JH, KN, TIO, BVF, SH, PI, OA, $ STRDAY)
   IRAD = 0
170 CONTINUE
   IF (IMAT.EQ.IPRINT) GO TO 175
   GO TO 115
C 175 CONTINUE
   CALL AGUT (TIME, DT, KN, JM, DUM, DENS, IPHAS, JH, DZ, RHZ, COR, DRY, CI, XM1 $, IEND, ITIME, MT, IMAT, Pl, PB, U1, U8, V1, VB, W1, W8, FM, FT, S, QS, CS, QB, CZT, $ S2T, COT, T0)
   REWIND 1
   WRITE (1) (1, TIME, Pl, PB, P10, U1, U8, V1, VB, W1, W8, FM, FT, S, QS, CS, QB, CZT, $ S2T, COT, T0)
   END FILE 1
   END FILE 3
   IF (ITIME.EQ.IEND) GO TO 115
   STOP
C C 180 FORMAT (4F15.4) A 229
185 FORMAT (5X, 6F14.7) A 230
190 FORMAT (6F10.4, /5FI4.4) A 231
195 FORMAT (9E10.3) A 232
200 FORMAT (9F10.3) A 233
205 FORMAT (6F10.0) A 234
END
SUBROUTINE HEAT(TIME, GBV, PB, TB, QHZ, DENS, TZO, GVF, SH, PI, 
$GA, STDAY)
COMMON /BUMLEG/ R(13, 19), T(18), T(18), PRS, (18), PRS(18, 19), QDG, 
$ (16), QHZ(18, 19), QDZ(18, 19), QDZ(18, 19), CS(18, 19), CO(18, 19), ZH(18), 
$A(16), FDY(18), TQ(16, 16), CTD(17), CTD(17), CTD(17), CTD(17), CTD(17), RED(6), 
$16), QRS(16)
DIMENSION GBV(KN), PB(JHPKN), TB(JMKN), QBCJMKN), DENS(KN), TZO 
$(KN), QA(KN)
JML = JM-1
KNL = KN-1
LBOT = 4
LTOP = 19
C COMPUTE SOLAR GEOMETRY FACTORS
DAY = TIME/(24.*60.*60.)
IQAY = DAY
RES = DAY - IQAY
IF (RES.GTO,05) GO TO 60
PERH = (101.21972+180.)*360./360.8
VD = STRDAY+DAY-RES
V = VD*0.7
EC = 0.015722
DELTA = 0.4093198*SIN(2.*PI*(STRDAY+DAY-80.)/360.)
DO 30 LLI1B
LL = 19-L
RL = LL-9
PHD = RL*10.-5.
PHI = PHD*PI/180.
TSTAR = TIME OF SUNSET (OR NEGATIVE TIME OF SUNRISE)
ZEN = AVERAGE VALUE OF COS(SZA) BETWEEN -TSTAR AND TSTAR
SUN = TAN(DELTA)*TAN(PHI)
ISUN = SUN
IF (ISUN) 10,15,20
ZEN = COS(DELTA)*COS(PHI)+(12./(PI*TSTAR))*COS(DELTA)*COS(PHI)*SIN(PI*TSTAR/1Z.)
GO TO 25
AZEN = 35./SORT(1224.*ZEN*ZEN+1.)
FDAY = TSTAR/12.
IN(L) = ZEN
AZN(L) = AZEN
FDY(L) = FDAY
CONTINUE
AZN = 12.
ZEN = SIN(DELTA)*SIN(PHI)+12./(PI*TSTAR))*COS(DELTA)*COS(PHI)*SIN(PITSTAR/12.)
GO TO 25
C COMPUTE GLOBAL RADIATIVE EQUILIBRIUM TEMPERATURE
IF (TIME.GT.0.) GO TO 45
DO 40 K1,KNL
10 TZO(K) = 0.
DO 35 J1,JML
THETA = 175-(J-1)*10
THETA = THETA*PI/180.
TZO(K) = TZO(K)+T(J,K+3)*SIN(THETA)*PI/36.
CONTINUE
35 CONTINUE
DO 55 J1,JML
SIT(J,K) = SQRT(TZO(J,K+3)*RHD)
CONTINUE
55 CONTINUE
CALL RADEQU (TZO, TB, QHZ, RHD)
CONTINUE
DO 65 J1,JML
DO 65 K1,KNL
56 CONTINUE
CONTINUE
DO 65 J1,JML
CONTINUE
C COMPUTE ZOAL MEAN TEMPERATURE (DEVIATION FROM GLOBAL AVERAGE)
DO 65 J1,JML
TB(J1,K) = RHZ*(PB(J,K+3)*DENS(K+1)-P(B(J,K)*DENS(K))/DZ
CONTINUE
65 CONTINUE
IF (TIME.GT.0.) GO TO 75
C COMPUTE STATIC STABILITY FROM TZO
N = KNL-1

DO 70 K=2,N
    GBV(K) = BVF*RHZ/(I2./7.*TZO(K)/SH+(TZO(K+1)-TZO(K-1))/(2.*DZ))
    GBV(1) = GBV(2)
    GBV(KNL) = GBV(N)

70 CONTINUE
IF (QA(1).LT.100.) GO TO 60
WRITE (6,105) (GBV(I),I=1,KNL)

60 CONTINUE
C COMPUTE HEATING RATE
CALL ODOT (TZO,TB,QB,RHO)

6TIME = 1.73E+06

DO 90 K=1,KNL
    DO 85 L=1,JML
        QB(L,K) = QB(L,K)/(86400.*RHZ*DENS(K))*(1.-EXP(-TIME/GTIME))
        DZ
75 CONTINUE
80 CONTINUE

C O{d) = 0.
C QA(15) = 0.
C QA(16) = 0.
DO 100 H=LBOT,LTOP
    I = M-3
    QA(I) = 0.
DO 95 L=1,JML
    THETA = 175-(L-1)*10
    QA(I) = QA(I)+QB(L,I)*SIN(THETA)*PI/36.
95 CONTINUE
100 CONTINUE
RETURN
C
C 105 FORMAT (5X,8E15.3)
END
FUNCTION DELT(TPL, L, IM, RHO)

COMMON /BULLEG/ R(13, 19), TG(18), T(18), 19), PRSRG(18), PRS(18, 19), QOG(18), QGZ(18, 19), QOZS(18, 19), CS(18, 19), CG(18, 19), ZN(18), SZN(18), FDY(18), TQ(18, 19), CZT(19, 17), SZT(19, 17), CDT(19, 17), RED(6, 16), ORS(18)

DIMENSION TP(17), TD(19)

I = IM-3
COZONE = 0.
CCO2 = 0.
SO3 = 0.
IF (I.EQ.16) GO TO 20
KRL = 16
DO 10 J = 1, KRL
TD(J+3) = TP(4)
10 CONTINUE
C FOR Z * LT 18.5 KM U. S. STANDARD ATMOSPHERE ASSUMED
C 15 CONTINUE
SO3 = S2ZT(L, I)
C DICKINSON INFRARED COOLING
TR = RED(9, I)
OR = ORS(I)
A = RED(5, I)
CCO2 = -OR-A*(TD(IM)-TR)
C FOR Z * LT 18.5 KM U. S. STANDARD ATMOSPHERE ASSUMED
C 20 CONTINUE
CDT(L, I) = CCO2
CZT(L, I) = COZONE
DELT = SO3+CC02+COZONE
RETURN
ENTRY SOL
I = IM-3
DZ = 5000.
SH = 7000.
Z = (I-1)*DZ
AIRDEN = 1.5991E-04*EXP(-Z/SH)
ZEN = ZN(L)
AZEN = AZN(L)
FDAY = FYD(I)
TAU = 10.
CLOUD = 0.446
RG = 0.5
ICLOUD = 2
C CLEAR SKY
RAB = 0.219/(1.+0.816*ZEN)
RDB = 0.144
RBI = RAB+(1.-RAB)*RD/(1.-RD*RDB)
RB = RDI
C CLOUDY SKY
RAB = 0.13*TAU/(1.+0.13*TAU)
PDB = RAB
RBI = RAB+(1.-RAB)*RD/(1.-RD*RDB)
UT = AZEN*QOZS(L, I)CLOUD)+1.9*(GOZS(L, I)CLOUD)-QOZS(L, I)
A1 = OZUV(UT)
UT = AZEN*GOZS(L, I)+1.9*(GOZS(L, I)-QOZS(L, I))
A2 = OZUV(UT)
U = AZEN*GOZS(L, I)
A3 = OZUV(U)
SO3 = 75.374*QGZ(18, 19)+4*QGZS(18, 19)+4*QOZS(18, 19)+4*QOZS(18, 19)+4*ZEN*FDAY*(AZEN*A3+(1.-$CLOUD)*RB)+1.9*AZN*2+2+CLOUD*RB1+1.9*A1
25 CONTINUE
DELT = SO3
RETURN
END
FUNCTION O2UV(U)

C SUBROUTINE TO CALCULATE SOLAR ENERGY ABSORBED BY OZONE

F1 = 1.0 + (0.042*U) + (3.23*U**2)

F2 = (0.0212/F1) + (1.0 + ((U/F1) + (0.042 + (6.46*U**2))) )

F1 = 1.0 + (138.6*U)

F2 = F2 + (1.082/*F1**0.805) + (1.0 + (138.6*0.805*U)/F1)

F1 = 1.0 + (103.6*U)**3

OZUV = F2 + ((0.0658/F1) + (1.0 - (3.0*(F1 - 1.0)/F1)))

RETURN

END
SUBROUTINE Radequ(TZD, TB, QB, RHO)

DIMENSION TZO(17), TP(17), TB(19,17), QO(19,17), A3(2)

COMMON /BUHLEG/ R(13,19), TG(18), T(18,19), PRS(18,19), O00

S(18), QO(19,19), QOZI(18,19), QOZS(18,19), CTS(18,19), CG(18,19), ZN(18),

ZAZN(18), FOFY(18), TO(18,16), CZT(19,17), SZT(19,17), CDT(19,17), RED(6),

$16), QRS(16)

DATA LBOT, LTOP, ITOT, EPS, 4, 19, 20, 0.1, 0.1/

DATA JML, KNL, 18, j/

PI = ACOS(-1.)

IF (QRS(1).GT.-100.) GO TO 20

DO 15 K-LBOT, LTOP

I = K-3

QRS(I) = 0.

DO 10 J-1, JML

THETA = 175-(J-1)*10

THETA = THETA*PI/180.

W = SIN(THETA)*PI/36.

QRS(I) = QRS(I)+SDL(TZO, J, K, 1.)*W

10 CONTINUE
</nohighlight>

15 CONTINUE

20 CONTINUE

DO 40 IT=3, ITOT

SUM = 0.

LMP = LTOP-1

DO 35 M-LBOT, LMP

I = M-3

TZO(I) = TZO(I)+2.*DT

DO 30 J=1, 2

TZO(I) = TZO(I)-DT

A3(J) = A3(J)+DELT(TZO, M, RHO)*D

30 CONTINUE

DO - (A3(1)-A3(2))/DT

DIFF = A3(2)/DQ

TN = TZO(I)-DIFF

DTP = DIFF

IF (OTP.GT.20.) DTP = 20.

IF (OTP.LT.-20.) DTP = -20.

TN = TZO(I)-DTP

DIFF = ABS(DIFF)

IF (DIFF.LT.EPS) GO TO 45

40 CONTINUE

PRINT 65, ITOT

STOP 1

45 PRINT 70, IT

WRITE (6,75) TZO

RETURN

ENTRY QDOD

C COMPUTE RADIATIVE HEATING IN KELVIN PER DAY

DO 60 L=1, JML

DO 50 J=1, KNL

TP(J) = TB(L,J)+TZO(J)

50 CONTINUE

QB(L,1) = 0.

DO 55 M=LBOT, LTOP

I = M-3

QB(L,1) = DELT(TP, L, M, RHO)

55 CONTINUE

60 CONTINUE

RETURN

C

65 FORMAT (5X, *TEMPERATURE PROFILE FAILED TO CONVERGE AFTER*, IT, 

* ITERATIONS*)

70 FORMAT (5X, *TEMPERATURE CONVERGED AFTER *, IT, * ITERATIONS*)

75 FORMAT (1X, 16F7.2)

END
SUBROUTINE ASTPEAM(AMBMpCM,ANPBN.CN,IFLG,IFDICTDTDZ,DY,CHIrQB, F
F$WRORRXBAUBUBOiVBtVBOsWBWBOPSPBUPPBACSCSAFMFTEPLEMIj- F
FZSRAYF,DCAOKY,TNDRiDENS,CNBGMEPCURIOLCNAiCUBBVF, JMPJML,KNN. F 3
F4
DIMENSION AM(JML), BM(JML), CMCJML)t PBA(JliKN), CORIOL(JM), F
F5QB(JM,KN), DCA(KN), UB(JM,KN), UBO(JM,KN), VB(JM,KN), VBO(JM,KN), PB(JM F
F6+KN), PBO(JM,KN), TN(JM), DR(JM), WB(JM,KN), CHI(JM,KN), WO(400), F
F7$ RR(JM,15), GMEP(JM), XBA(JM,KN), DENS(KN), CS(JM), CSA(JM), FM F
F8$JM(JM, KN), FT(JM,KN), CHA(JM), CNB(JM), CUBJMJ, VB(JM,KN), RAYF(KNN), WBO(JM,KN F
F9S1, AN(15), BN(15), CN(15), GBV(KN) F
F10
COMMON /UHLEG/ CURTIS(13,19),TG(18),T(18,19),PRSG(18),PRS(18,19) F
F11
N = JML-1, KNL = JML-1, KN = N+2
F12
$CHOOSE LEAPFROG OR FORWARD DIFFERENCE F 18
F19
DO 10 J=1,JM
DO 10 K=2,KN
10 UB(J,K) = UB(J,K)/DENS(K)
ICT = ICT+1
IF (ICT.LT*IFD) GO TO 15
T1 = 1.
T2 = 0.
T3 = 1.
T4 = 0.
T5 = 2.
GO TO 20
15 T1 = T2 = .5
T3 = 2.*
T4 = 1.
T5 = 4.
GO TO 20
20 CONTINUE
C
UB SMOOTHING
DO 30 K=2,KNL
25 FM(JK) = FM(JK)-DKY*(UBO(J-2,K)/CSA(J-2)-4.*UBO(J-1,K)/CSA(J-1 F
$)+6.*UBO(J,K)/CSA(J)+4.*UBO(J+1,K)/CSA(J+1)) F 43
$CSA(J)-4.*UBO(J-K)/CSA(J+K)**2 F 44
FK(JK) = FM(JK)-DKY*(UB(J-1,K)/CSA(J-1-K))/CSA(J-K)**2 F 45
FK(2*K) = FM(2*K)-DKY*(UBO(J-2,K)/CSA(J-2)-3.*UBO(J-1,K))/CSA(J+2)+UBO(J F
$$+1,K)/CSA(J+2)**2 F 46
FK(JM,K) = FM(JM,K)-DKY*(UB(J-1,K)/CSA(J-1-K))/CSA(J-1-K)**2 F 47
$CSA(J+1-K)+2.*UBO(JM,K))/CSA(J+1-K)**2 F 48
FM(JML,K) = FM(JML,K)-DKY*(UBO(J-1,K)/CSA(J-1-K))/CSA(J-1-K)**2 F 49
$CSA(J+1-K)+2.*UBO(JM,K))/CSA(J+1-K)**2 F 50
FM(JML-K) = FM(JML-K)-DKY*(UBO(JM+1,K)/CSA(J-1-K))/CSA(J-1-K)**2 F 51
30 CONTINUE
C
THICKNESS TENDENCY
DO 40 J=1,JM
DO 35 K=1,KNL
35 PBA(J,K) = PB(J,K+1)*EPL-PB(J,K)*EMI F 57
40 PBA(J,KN) = 0.
DO 50 J=1,JML
50 CHI(J,K) = T1*PBA(J,K)+T2*(PBO(J,K)+PL-PB0(J,K)*EMI)+DT*(QF F
$$J,J)K)+FT(J,K)) F 62
$IF (J,EQ.1) GO TO 45
CHI(J,K) = CHI(J,K)-DT*DENS(K)*EPL/(4.*CS(J)*DY)*(VB(J,K+1)+ F
$$V8(J,K)+CSA(J)*(PBA(J-1,K)+PBA(J,K))-(VB(J+1,K+1)+VB(J+1,K))* F 65
$CSA(J+1)*PBA(J,K)+PBA(J+1,K)) F 66
GO TO 50
C
45 CHI(J,K) = CHI(J,K)-DT*DENS(K)*EPL/(4.*CS(J)*DY)*(VB(J,K+1)+ F
$$V8(J,K)+CSA(J)*(PBA(J-1,K)+PBA(J,K))-(VB(J+1,K+1)+VB(J+1,K))*CSA(J+1 F
$$+1)*PBA(J,K)+PBA(J+1,K)) F 69
50 CONTINUE
IF (K.EQ.1) GO TO 65
IF (K.EQ.KNL) GO TO 65
DO 65 J=1,JML
65 GMEP(J) = -DT/(4.*DZ)*EPL*DENS(K)*(DB(J,K+1)+WB(J,K)) (PBA(J F
F1
2
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\[ K+1)\text{EPL+PBA(JK)*EMI)=(WB(JK)+WB(JK+1))*(PBA(JK)*EPL+PBA(JK)*EMI) \]

55 CONTINUE F 77

DO 60 J=1,KML F 78

60 CHI(JK) = CHI(JK)+GMEP(J) F 79

65 CONTINUE F 80

C THICKNESS SMOOTHING F 81

DO 70 J=1,JK F 82

70 PBA(JK) = PBO(JK+1)*EPL-PBO(JK)*EMI F 83

74 CONTINUE F 84

C CONTINUE F 85

DO 80 J=1,KML F 86

80 CHI(JK) = CHI(JK)+GMEP(J) F 87

85 CONTINUE F 88

C CONTINUE F 89

C DO 70 J=1,JK F 90

C CONTINUE F 91

C CONTINUE F 92

C CONTINUE F 93

C CONTINUE F 94

C CONTINUE F 95

C CONTINUE F 96

C CONTINUE F 97

C CONTINUE F 98

C CONTINUE F 99

C CONTINUE F 100

C CONTINUE F 101

C CONTINUE F 102

C CONTINUE F 103

C CONTINUE F 104

C CONTINUE F 105

C CONTINUE F 106

C CONTINUE F 107

C CONTINUE F 108

C CONTINUE F 109

C CONTINUE F 110

C CONTINUE F 111

C CONTINUE F 112

C CONTINUE F 113

C CONTINUE F 114

C CONTINUE F 115

C CONTINUE F 116

C CONTINUE F 117

C CONTINUE F 118

C CONTINUE F 119

C CONTINUE F 120

C CONTINUE F 121

C CONTINUE F 122

C CONTINUE F 123

C CONTINUE F 124

C CONTINUE F 125

C CONTINUE F 126

C CONTINUE F 127

C CONTINUE F 128

C CONTINUE F 129

C CONTINUE F 130

C CONTINUE F 131

C CONTINUE F 132

C CONTINUE F 133

C CONTINUE F 134

C CONTINUE F 135

C CONTINUE F 136

C CONTINUE F 137

C CONTINUE F 138

C CONTINUE F 139

C CONTINUE F 140

C CONTINUE F 141

C CONTINUE F 142

C CONTINUE F 143

C CONTINUE F 144

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C CONTINUE F 146

C CONTINUE F 147

C CONTINUE F 148

C CONTINUE F 149

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C CONTINUE F 198

C CONTINUE F 199

C CONTINUE F 200

C CONTINUE F 201

C CONTINUE F 202

C CONTINUE F 203
**CONTINUE**

C

COMPUTE NEW UB, VB, WB, PB

C

DO 130 K=2,KNL

DO 130 J=2,JML

AS = T1*UB(J,K)+T2*UBD(J,K)

AS = AS+DT*F(J,K)+DENS(K)*(-DT/CZA(J))*2*DY*4.*((UBJ-1,K)*

CZA(J-1)+UB(J,K)+CSA(J))*VB(J-1,K)+VB(J,K)+CSA(J) =

UB(J+1,K)*CSA(J)+UB(J,K)*CSA(J)

VB(J-1,K)+VB(J,K)*CSA(J)+VB(J,K)*CSA(J)

(DT/(4.)*DY*CSA(J))*(UBJ-1,K)*CSA(J)+UB(J,K)*CSA(J)

UB(J+1,K)+UB(J,K)*CSA(J)+UB(J,K)*CSA(J)

VB(J-1,K)+VB(J,K)*CSA(J)+VB(J,K)*CSA(J)

UB(J+1,K)*CSA(J)+UB(J,K)*CSA(J)

UB(J,K) = UB(J,K)*T3-UB(J,K)*T3+(AS+CQIRDO(J)*DT*(XBA(J,K-1)

UBO(J,K) = UBO(J,K)*T3+AS*(XBA(J,K)-XBA(J,K-1))

VBO(J,K) + VBO(J,K)*T5-4*(XBA(J,K-1))

PD(J,K) = PRS(J,K)

GO TO 150

DO 140 K=2,KNL

UBO(J,K) = UBO(J,K)*EMI*EPL

VBO(J,K) = VBO(J,K)*EMI*EPL

DO 145 K=1,KNL

PBA(J,K) = PBA(J,KL)

DDO 150 J=1,JML

PBA(J,K) = PBA(J,KL)*EMI/EPL

VBO(J,K) = VBO(J,KL)*EMI/EPL

DO 150 J=1,JML

UBO(J,K) = UBO(J,KL)*EMI/EPL

DO 155 J=1,JML

DO 155 K=1,KNL

CHI(J,K) = T5*PBA(J,K)-T4*PB0(J,K)-T3*PB(J,K)

PB0(J,K) = PB(J,K)

PB(J,K) = CHI(J,K)

CHI(J,K) = WBO(J,K)

WBO(J,K) = CHI(J,K)

IF (J.EQ.1) GO TO 155

CHI(J,K) = UBO(J,K)

UBO(J,K) = UB(J,K)

UB(J,K) = CHI(J,K)

CHI(J,K) = VBO(J,K)

VBO(J,K) = CHI(J,K)

VBO(J,K) = CHI(J,K)

RETURN

END
$ J_K)*(UJ+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 77$

$ (J+K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 79$

$ IF (J+K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 81$

$ UJ*(UJ+1,K)*TNA(J+1,K)*UB(J+1,K)*TNA(J+1,K) \quad 6 \quad 82$

$ MOMENTUM SMOOTHING \quad 6 \quad 83$

$ IF (J+K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 85$

$ IF (J+K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 87$

$ BV = T*V(J+1,K)*(12-RAY(K))*V(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 89$

$ IF (J+K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 91$

$ IF (J+K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 93$

$ BV = BV*DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 95$

$ GO TO 75 \quad 6 \quad 96$

$ AV = AV-DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 97$

$ IF (J+K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 99$

$ AV - AV-DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 101$

$ IF (J+K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 103$

$ BV = BV*DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 105$

$ GO TO 100 \quad 6 \quad 107$

$ C \quad 6 \quad 108$

$ IF (S+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 109$

$ IF (S+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 111$

$ BV = BV*DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 113$

$ GO TO 100 \quad 6 \quad 115$

$ C \quad 6 \quad 116$

$ IF (SYM+EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 117$

$ BV = BV*DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 119$

$ GO TO 100 \quad 6 \quad 121$

$ C \quad 6 \quad 122$

$ IF (SYM+EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 123$

$ BV = BV*DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 125$

$ GO TO 100 \quad 6 \quad 127$

$ C \quad 6 \quad 128$

$ IF (SYM+EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 129$

$ BV = BV*DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 131$

$ GO TO 100 \quad 6 \quad 133$

$ C \quad 6 \quad 134$

$ IF (SYM+EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 135$

$ BV = BV*DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 137$

$ GO TO 100 \quad 6 \quad 139$

$ C \quad 6 \quad 140$

$ IF (SYM+EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 141$

$ BV = BV*DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 143$

$ GO TO 100 \quad 6 \quad 145$

$ C \quad 6 \quad 146$

$ IF (SYM+EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 147$

$ BV = BV*DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 149$

$ GO TO 100 \quad 6 \quad 151$

$ C \quad 6 \quad 152$

$ IF (SYM+EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 153$

$ BV = BV*DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 155$

$ GO TO 100 \quad 6 \quad 157$

$ C \quad 6 \quad 158$

$ IF (SYM+EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 159$

$ BV = BV*DT*K*Z*(UJ+1,K+1)*V(J+1,K+1)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K)*EPL-UB(J+1,K) \quad 6 \quad 161$
CONTINUE
IF (ISW.EQ.2) GO TO 145
DO 120 J=1,N
   PHI(J+1,KNL+1) = (O*O)*
120 R(J+1) = R(J+1)-PL1(J+1)*GBV(1)
C INVERT ELLIPTIC EQUATION FOR PHI
125 CALL CBKTRI (IFLG, N, AN, BN, CN, H, AA, BB, CC, MR, IER, WRL)
   IFLG = IFLG+1
   IF (IFLG-1) LE 125,125,130
130 CONTINUE
   DO 135 J=2,NML
      PHIA(J+1) = PL1(J)
      DO 135 K=1,KNL
         PHIA(J+1,K) = R(J+1,K-1)
      135 CONTINUE
   DO 140 K=1,KNL
      PHIA(K) = (O*O)*
140 PHIA(JH,K) = (O*O)*
      ISW = 2
      GO TO 60
C
145 CONTINUE
   DO 150 J=2,NML
      DO 150 K=1,KNL
         W(JK) = (CDUM(J,K)-PHIA(J,K+1)*EPL-PHIA(J,K)*EMI)/(BVF*DT*DZ)*
            $GBV(K)
   150 CONTINUE
C NEW VALUES FOR DEPENDENT VARIABLES
   DO 155 J=1,NML
      DO 155 K=1,KNL
         PHIA(J,K) = T5*PHIA(J+1,K)-T4*PHIA(J,K)-T3*PHI(J,K)
         PHID(J,K) = PHI(J,K)
         PHIA(J,K) = PHI(J,K)
         PHIA(J,K) = UO(J,K)
         UO(J,K) = U(J,K)
         U(J,K) = PHIA(J,K)
         PHIA(J,K) = VJ(K)
         VJ(K) = V(J,K)
   165 CONTINUE
155 CONTINUE
   DO 160 K=1,KNL
      U(JM,K) = -U(JML,K)
      V(JM,K) = V(JML,K)
      DO 160 J=2,NML
         UB(J,K) = UB(J,K)*DENS(K)
   160 CONTINUE
RETURN
END
SUBROUTINE FLUX(JM, KN, DY, CZA, CSA, CS, DENS, EPL, EM, U, V, W, T, FM, FT, KZ)

DIMENSION CSA(1), CS(1), DENS(1), U(JM+1), V(JM+1), W(JM+1), P(JM+1), S(1), T(JM+1), FM(JM+1), FT(JM+1), UBO(JM+1)

REAL KZ

DIMENSION VT(JM+1), CNB(1)

COMPLEX U, V, W, T

KNL = KN-1

N = KNL+1

DO 10 J=1, JM

DO 10 K=1, KNL

10 T(J,K) = P(J,K+1)*EPL-P(J,K)*EM

DO 15 J=1, JM

DO 15 K=1, KNL

VT(J,K) = VT(J+K) = 0.

DO 15 J=1, JM

DO 15 K=1, KNL

VT(J,K) = REAL(W(J+1,K))*EPL+REAL(U(J,K))*EM

S(J,K) = REAL(T(J,K))+AIMAG(V(J-1,K+1)+V(J,K)+V(J-1,K)+V(J,K))

SAIMAG(T(J,K))/2.

DO 20 J=1, JM

DO 20 K=1, N

20 FT(J,K) = -(CNB(J)*VT(J,K)-VT(J+1,K))

DO 25 J=1, JM

VT(J+K) = 0.

DO 25 J=1, JM

DO 25 K=1, N

25 VT(J,K) = REAL(W(J+1,K))*REAL(T(J,K))+REAL(W(J+1,K))*REAL(T(J,K))+

SAIMAG(W(J+1,K))*AIMAG(T(J,K))+AIMAG(W(J+1,K))*AIMAG(T(J,K))

DO 30 J=1, JM

DO 30 K=1, N

30 FT(J,K) = FT(J,K)+DENS(K)*EPL*(VT(J+K)-VT(J,K))/2.

DO 40 J=1, JM

DO 40 K=1, KNL

FM(J,K) = -(DENS(K)*T(J-1,K))*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.1) GO TO 45

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.2) GO TO 35

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.3) GO TO 25

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.4) GO TO 15

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.5) GO TO 10

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.6) GO TO 9

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.7) GO TO 8

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.8) GO TO 7

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.9) GO TO 6

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.10) GO TO 5

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.11) GO TO 4

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.12) GO TO 3

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.13) GO TO 2

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

IF (K.EQ.14) GO TO 1

FM(J,K) = FM(J,K)+DENS(K)*REAL(U(J,K-1)+U(J,K))*EPL+REAL(U(J,K-1)+U(J,K))*EPL

S(J,K) = AIMAG(U(J,K-1)+AIMAG(V(J,K-1))*CS(J)*2)/CSA(J)*2*

END
SUBROUTINE OUT (TIME, DT, KN, JM, DUM, DENS, IPHASE, Z, JM, DZ, RHZ, COR, DY, 
* CI, XMI, IEND, TIME, MT, IHAT, PI, PB, U, V, VB, W1, WB, FM, FT, SS, Q, CS, QB 
* SZ, CTZ, SZT, CTZ, TQ) 
DIMENSION DUM(JM), DENS(KN), Z(9), PB(JM, KN), UB(JM, KN), VB(JM, KN) 
* W(V), DB(JM, KN), CS(JM), QB(JM, KN), CTZ(JM, KN), SZT(JM, KN), CTZ(JM, KN) 
* TQ(JM, 16), TDQ(16, 16), LAT(19) 
* DENS(JM), COR(JM), XM(m), F(JM, KN), FT(JM, KN) 
* COMPLEX CI, PI, J, KN, U1, VH, W, FT, S, Q, CS, QB 
* INTEGER S 
* PI = 2.* ASIN(1.) 
* DAY = TIME / (3600.* 24.) 
* KNL = KN - 1 
* WRITE (6, 105) 
* WRITE (6, 130) DAY 
* DO 15 KK = 1, KN 
* K = KN - KK + 1 
* DO 10 J = 1, JM 
* DUM(J) = 0.0 
* DUM(J) = VB(J, K) * DENS(K) 
* CONTINUE 
10 ZZ = Z(K) +16000. 
* WRITE (6, 190) 
* WRITE (6, 110) ZZ, (DUM(J), J = 1, JM) 
* CONTINUE 
15 ZZ = Z(K) +16000. 
* WRITE (6, 105) 
* WRITE (6, 135) 
* DO 30 KK = 1, KNL 
* K = KN - KK 
* DO 20 J = 1, JM 
* DUM(J) = RHZ*(PB(J, K) * DENS(K + 1) - PB(J, K) * DENS(K)) / DZ 
* ZZ = Z(K) + DZ/Z, +16000. 
* WRITE (6, 156) 
* WRITE (6, 125) ZZ, (U(J), J = 1, JM) 
* CONTINUE 
20 DUM(J) = RHZ*(PB(J, K + 1) * DENS(K) - PB(J, K) * DENS(K)) / DZ 
* ZZ = Z(K) + DZ/Z, +16000. 
* WRITE (6, 125) ZZ, (U(J), J = 1, JM) 
* CONTINUE 
25 SUN = SUN + DUM(J) * CS(J) 
* SUN = SUN * PI/36. 
* WRITE (6, 190) 
* WRITE (6, 140) ZZ, (DUM(J), J = 1, JM), SUN 
* CONTINUE 
30 ZZ = Z(K) +16000. 
* WRITE (6, 105) 
* WRITE (6, 145) 
* WRITE (6, 135) 
* DO 40 KK = 1, KN 
* K = KN - KK + 1 
* DO 30 J = 1, JM 
* DUM(J) = VB(J, K) * DENS(K) * 1.E3 
* ZZ = Z(K) + DZ/Z, +16000. 
* WRITE (6, 125) ZZ, (U(J), J = 1, JM) 
* CONTINUE 
35 ZZ = Z(K) +16000. 
* WRITE (6, 105) 
* WRITE (6, 145) 
* WRITE (6, 135) 
* DO 40 LL = 1, 19 
* LAT(L) = LL 
* WRITE (6, 150) LAT 
* WRITE (6, 105) 
* WRITE (6, 155) 
* DO 50 KK = 1, KN 
* K = KN - KK + 1 
* DO 45 J = 1, JM 
* DUM(J) = VB(J, K) * DENS(K) * 1.E3 
* ZZ = Z(K) + DZ/Z, +16000. 
* WRITE (6, 190) 
* WRITE (6, 120) ZZ, (DUM(J), J = 1, JM) 
* CONTINUE 
40 LAT(L) = LL 
* WRITE (6, 150) LAT 
* WRITE (6, 105) 
* WRITE (6, 155) 
* DO 50 KK = 1, KN 
* K = KN - KK + 1 
* DO 45 J = 1, JM 
* DUM(J) = VB(J, K) * DENS(K) * 1.E3 
* ZZ = Z(K) + DZ/Z, +16000. 
* WRITE (6, 190) 
* WRITE (6, 120) ZZ, (DUM(J), J = 1, JM) 
* CONTINUE 
45 ZZ = Z(K) +16000. 
* WRITE (6, 110) ZZ, (DUM(J), J = 1, JM) 
* CONTINUE 
50 ZZ = Z(K) +16000. 
* WRITE (6, 105) 
* WRITE (6, 140) 
* WRITE (6, 135) 
* DO 60 KK = 1, KN 
* K = KN - KK + 1 
* DO 55 J = 1, JM 
* DUM(J) = CB(J, K) * DENS(K) * 86400. * RHZ/DZ 
* ZZ = Z(K) + DZ/Z, +16000. 
* WRITE (6, 190) 
* WRITE (6, 115) ZZ, (DUM(J), J = 1, JM) 
* CONTINUE 
55 ZZ = Z(K) +16000. 
* WRITE (6, 110) ZZ, (DUM(J), J = 1, JM) 
* CONTINUE 
60 WRITE (6, 105) 
* WRITE (6, 110) 
* WRITE (6, 100) 
* DO 65 KK = 1, KN 
* K = KN - KK + 1
ZD = Z(K)+16000.
WRITE (6,190)
WRITE (6,115) ZD, (SZT(J,K), J=1, JML)

CONTINUE
WRITE (6,105)
WRITE (6,165) DAY,S
DO 75 KK=1,KN
  K = KN-KK+1
  DO 70 J=1, JM
    DUM(J) = CABS(PI(J,K))
    IF (DUM(J).EQ.0.) GO TO 70
    DUM(J) = DUM(J)*DENS(K)/9.8*Z,
    IPHAS(J) = ATAN2(AIMAG(PI(J,K)),REAL(PI(J,K)))*180./PI
  70 CONTINUE
ZD = Z(K)+16000.
WRITE (6,190) ZD, (IPHAS(J), J=1, JML)
WRITE (6,190) 
75 CONTINUE
WRITE (6,105)
WRITE (6,170)
DO 85 KK=1,KNL
  K = KN-KK
  DO 80 J=1, JM
    DUM(J) = FM(1,J,K)*DENS(K)*1.66
    DUM(19) = 0.
    ZD = Z(K)+16000.
    WRITE (6,190)
  80 CONTINUE
WRITE (6,125) ZD, (DUM(J), J=1, JM)
WRITE (6,105)
WRITE (6,170)
DO 95 KK=1,KNL
  K = KN-KK
  DO 90 J=1, JM
    DUM(J) = FT(J,K)*DENS(K)*RHZ/DZ*1.66
    ZD = Z(K)+16000.
    WRITE (6,190)
  90 CONTINUE
WRITE (6,125) ZD, (DUM(J), J=1, JM)
KT = 1
IMAT = 0
RETURN

C
C
100 FORMAT (* SOLAR HEATING BY OZONE *)
105 FORMAT (1H1)
110 FORMAT (-3PF6.1,19(OPF6.2))
115 FORMAT (-3PF6.1,16(OPF6.2))
120 FORMAT (-3PF6.1,14(OPF6.1))
125 FORMAT (-3PF6.1,19(OPF6.1))
130 FORMAT (29H MEAN MERIDIONAL WIND DAY= ,F6.2)
135 FORMAT (29H MEAN TEMPERATURE )
140 FORMAT (-3PF6.2,10(OPF6.1),6X,F10.4)
145 FORMAT (18H MEAN ZUMAL WIND )
150 FORMAT (/,'AX,1916)
155 FORMAT (26H VERTICAL VELOCITY, MM/S )
160 FORMAT (30H RADIATIVE HEATING K/D )
165 FORMAT (23H GEOPOTENTIAL, DAY = ,F6.2,12H WAVENUMBER ,14)
170 FORMAT (30H EDDY MOMENTUM FLUX DIVERGENCE)
175 FORMAT (26H EDDY HEAT FLUX DIVERGENCE)
180 FORMAT (-3PF6.2,19(OPF6.1))
185 FORMAT (6X,1916)
190 FORMAT (3H )
END