ACCELERATION SENSITIVITY COMPENSATION IN HIGH PERFORMANCE CRYSTAL OSCILLATORS

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ABSTRACT

High stability crystal oscillators are an essential component in a wide variety of systems, from satellite based navigation and communications, to Doppler radar, time keeping and so on. In a hostile shock and vibration environment, the full potential of such systems is severely limited by the intrinsic acceleration sensitivity of the widely used resonator types. Thus, it becomes increasingly important to investigate the possibility of increased frequency stability under adverse conditions.

Two approaches to achieving reduced sensitivity will be discussed. The first involves electronic compensation within the frequency control loop. The second utilizes two resonators of comparable acceleration sensitivity to compensate each other.

Problems encountered in matching and tuning the resonators will be discussed, as well as orientation symmetry of the frequency deviation patterns. Results on frequency stability which reflect an improved static sensitivity of less than $5 \times 10^{-11}$ per g are presented.
INTRODUCTION

High performance crystal oscillators are critical components in navigation, communications and time-keeping systems which require the best attainable frequency stability and spectral purity. This high stability performance is usually realized only under quiet ambient conditions, yet systems are often required to operate, without degraded performance, in a shock and vibration environment.

Although new designs (Ref. 1) and improved resonator types will ultimately be available, the most widely used type at present is the AT-cut, which shows a typical acceleration sensitivity of $10^{-9}$ per g or greater. Some recent progress in compensating AT-cut overtone crystals for reduced sensitivity to low frequency mechanical inputs will be described. This is a continuation of the work briefly mentioned previously at these meetings (Ref. 2). The basic test bed has been the rugged high performance Model 1000 oscillator using an oven stabilized, 5th overtone 5 MHz resonator.

First, the observable effects associated with shock and vibration will be discussed and correlated with the static g sensitivity of the resonators. Particular attention is given to the spatial dependence of frequency change with relative acceleration direction, and the symmetry of this pattern.

Two basic approaches (neither of them new) have been explored and each has some limitations. The first scheme encompasses various methods of correcting the VCXO tuning via an accelerometer derived signal. Results from a straightforward systems approach using an external sensor have been reported recently by Przyjemski (Ref. 3). We have achieved similar improvement using different resonator types. The disadvantage of using a VCXO whose tuning is non-linear will be pointed out.
In the present work we have also incorporated sensors within the crystal oven for high stability, compactness, and potentially better high frequency compensation response (Ref. 4).

The alternate approach has been to use two resonators in series and arranged so that acceleration induced frequency shifts cancel as suggested by Gagnepain and Walls (Ref. 5). Here the individual g sensitivity pattern symmetries are extremely important. Several interesting problems have come to light. Among the advantages on the other hand are that resonator and sensor have nearly identical construction, and they can be placed physically close to one another. Furthermore, a very interesting experimental effect linking temperature induced stress and g-sensitivity has been found, which has to do with thermal shock and warm-up behavior in AT-cut quartz.

MOTIVATION

Continued effort toward improving the performance of existing resonator types is justified by the immediacy of demands on existing technology, and partly by the fact that it can be done.

Although considerable data have been gathered on a variety of resonator types, we concentrate here on 5th overtone oscillators having particularly good signal-to-noise ratio close in to the carrier, and short-term stability performance of better than $10^{-12}$ in the 1 - 100 second region. (See Figure 2)

Low noise performance becomes quickly degraded in a hostile physical environment. At a vibration frequency $f_m$, modulation sidebands appear with a magnitude $(\gamma a f_0/2f_m)^2$ where $\gamma$ is the intrinsic g-sensitivity coefficient, $a$, the peak acceleration, and $f_0$ the carrier frequency. (See Figure 3)

For systems involving frequency multiplication to high order, $n$, noise appearing close to the carrier and multiplied up by $n^2$ is of concern. Since the
sideband level goes as \( f_m^{-2} \), perturbations down to zero frequency dominate, while with increasing frequency both electronic and mechanical filtering are relatively easy and the sideband level is usually decreasing.

Acceleration inputs may span a wide range of amplitude, and be randomly oriented. The spectrum extends to d.c. where attitude in the earth's gravitational field may be a slowly varying parameter. In a closed loop system, accumulated phase error may be of interest (Ref. 3).

In frequency standards, such as the cesium beam instrument, frequency slewing of the flywheel oscillator may occur at a rate such as to give unacceptable frequency offset, where the time rate of oscillator drift multiplied by the control loop time constant gives the resultant error. This is particularly severe when \( T \) has been made long to exhibit short-term stability approaching the performance of the open loop oscillator. Notice, however that phase slip resulting from frequency shift in the oscillator can be reversible, averaging to zero if the mean value of g-induced shifts is zero.

Another effect can occur under sustained vibration. At the onset of 20 Hz 1 g vibration, we have observed a transient shift of the order 5 \( \times 10^{-10} \) recovering in 10 minutes to within about 1 \( \times 10^{-10} \). At the cessation of vibration, the opposite transient occurs, and the frequency settles back to nearly the original. This effect is traceable to perturbed oven control, which can be minimized by packaging techniques.

The effect of mechanical shock on high precision crystals is generally seen as a frequency jump either positive or negative. For 2300 g pyrotechnic shocks, jumps are typically severe parts in \( 10^{10} \) per pulse, with many successive shocks leaving the final value within 5 \( \times 10^{-10} \) of the original frequency.
Shock mounting of the crystal enclosure itself is possible. However, for effective compensation it is necessary to have the resonator unit rigidly coupled to the compensating element, so as to avoid phase differences between the two responses. It is thus, more useful to shock mount the oscillator as a whole to attenuate high frequency inputs, even in cases where only low frequency vibration is expected.

RESONATOR BEHAVIOR

The response of AT-cut quartz to acceleration stresses has been discussed by many authors. In relation to the resonator geometry it is found that the axis of maximum acceleration induced frequency change is dependent on, among other factors, the angle of cut, and position of resonator supports with respect to the crystal axes. Valdois and Besson (Ref. 6), Lee (Ref. 7) and others have discussed the expected stresses and it has been shown that \( \Delta f \) along a given axis is linear with \( g \) to at least 50 g. In general an approximately \( \cos \theta \) dependence for the force-frequency effect is found, implying linear response to the vector component along a principal axis. Our results with high \( Q \) resonators bear this out.

Given that linearity and regular angle dependence exist, one can then make detailed measurements of spatial symmetry at a level of 1 g. Figure 4 shows the pattern for \( 2\pi \) rotation in azimuth. The plano-convex quartz disk is supported in a three point ribbon mount. The acceleration of 1 g is fixed, downward, and the resonator rotates clockwise giving the associated values of \( \Delta f/f \) shown. A coefficient \( \gamma' \) for maximum \( \Delta f \) in this plane is \( 4 \times 10^{-10} \) per g. The heavy circle marks the reference line of zero deviation. It should be emphasized that these are simply one set of data, for a particular resonator, in one special plane. Experience shows that the correlation of symmetry pattern to the position of the mounting supports is of secondary interest. More important is the high degree of
symmetry, indicating that in this plane a single axis accelerometer could give exact cancellation.

The appearance of the plot is a function of purely arbitrary scale factors. The expression is \( A + \gamma' \cos \theta \) and for \( \gamma' \) small compared to \( A \), the data lie on a displaced circle.

The same resonator is shown in Figure 5, for rotations in polar angle \( \theta \), about an axis through the odd support point and the plane of the disk. Note that \( \gamma \) is \( 1.3 \times 10^{-9} \) per g and that the symmetry axis is tipped slightly from the vertical. (Data for 3rd overtone resonators show a similar shape with coefficients as much as 2 times greater.) The data points shown lying nearly on the zero circle illustrate the compensation which was achieved with a simple accelerometer incorporated within the oscillator, to be described in the next section. The exact degree of cancellation depends on fine tuning of axis angles and compensation magnitudes.

This is similar to what Przyjemski has already shown for an oscillator compensated by an external accelerometer.

To introduce the discussion of compensation schemes, Figure 6 shows some points which need to be considered for ideal performance, recognizing that non-ideal performance will also be useful.

1. Perfect symmetry is required so that no residual sensitivity (to the desired low level) is observed. This can also be taken to mean that the coordinate systems of resonator and compensator must be exactly coincident in space. Otherwise, angular rotation about an axis other than the line of centers produces centripetal acceleration which is distinctly not compensated, but shows a coefficient \( 2\gamma \).
2. Linear frequency tuning may be important for a system which uses a compensation signal to control the VCYO. At any given tuning voltage the frequency deviations are small enough that the linearity is sufficient even for typical varicap tuning; however, as the oscillator ages, the desired correction coefficient may vary drastically.

3. Vibration induced sidebands fall off as $f^{-2}$ (typically -120dB for 1 g at 10 KHz). Thus, large compensation bandwidths may not be necessary. An upper limit of 1 KHz might be typical, at which point mechanical isolation is easy. In any case, above several KHz one expects trouble from phase shift due to the resonator ribbon mount resonances.

It is interesting to note, however, that oscillators can exhibit an fm bandwidth of up to 20 KHz when the usual low-pass VCO filter is removed.

4. All spurious sources of phase modulation must be suppressed before the intrinsic quartz effect can be linearly compensated.

5. Vibration induced frequency shifts can be complicated by the exaggerated shifts from "thermal shock" accompanying very small crystal oven temperature perturbations.

6. Hysteresis includes effects which cannot be compensated, such as shock induced permanent offsets.

7. Finally, one wishes to compensate in such a way as to introduce no additional noise into the oscillator loop, and no additional sources of apparent aging.
Relative to point 2 above, an FTS oscillator whose linear tuning range was extended to accommodate a particular g-compensation sensitivity, is shown in Figure 7 to have integral non-linearity of less than 5%.

COMPENSATION SCHEMES

We now turn to some considerations of the series resonant circuit containing a high Q thickness shear mode crystal. The frequency determining network is such that a series (capacitive) reactance of 1 Kohm typically pulls the crystal to the operating frequency. The tuning rate $dX/df$ is about 100 ohms per Hz. Thus, given a g-sensitivity of $10^{-9}$ per g, the compensation required is $df \approx 5$ milliHz per g, or $dX \approx 0.5$ ohm per g. Figure 8 shows the circuit. The parallel combination of reactances (usually capacitive) determines the resonant frequency. This approximate analysis will ignore the resonator electrode capacitance and lead strays, lumped together as $X'$. $X_1$ is a calibration capacitor on the order of 1 Kohm at 5 MHz. $X_2$ is a variable reactance which provides electronic tuning. $X_3$ indicates various possible stray capacities adding to $X'$. Supposing $X_3$ to be quite large (stray C small) we have that the effective series X is just $(X_1 + X_2)$, controlling the frequency at 100 ohms per Hz.

The intrinsic resonant frequency shifts of interest are of order 5 mHz per g. If $X_1$ or $X_2$ were to be mechanically or electrically modulated with a coefficient 0.5 ohm per g, then frequency compensation would be achieved. With X equal to 1 K (32 pF) we need a capacitive change of 15 milli-pF per g; or, if 1 K inductive, then about 15 nH. This, with variations, encompasses one class of compensators. One either modulates the reactance $X_1$ gravitationally, or modifies $X_2$ indirectly from an external transducer.
Now consider the case where $X_3$ is not negligible. Because stray capacitance to ground is often several pF, it is easy to imagine mechanically modulated changes of the order of milli-pFs (femto-Farads). Suppose a 1 pF stray were due to leads or components spaced 0.1 cm from ground. Then for spurious modulation to be less than 1 mpF the spacing must stay constant to 10⁻⁴ cm. This means that leads and components must be firmly fixed. In fact, the conditions are less stringent since $X_3$ is shunted by much larger capacitances.

However, this leads directly to the idea of mechanically modulating $X_3$ in the same manner as for $X_1$.

A generalized capacitive transducer which responds linearly to acceleration (deflection proportional to force) will have a fractional sensitivity $\Delta X_3/X_3$ per g which is sufficiently linear for small deflections. Frequency response down to d.c. is of course assumed. As an example, changes of 10 mpF per g or so can produce the desired half-ohm compensation signals. (We have, with apologies to Michael Faraday, a 'milli-puffer').

The overall sensitivity in the network goes as $\Delta X_3/X_3)^2$ so that $X_3$ must be kept small if we wish to avoid diluting the effect. The device reactance will be proportional to some spacing, s, and fractional change in X will be proportional to fractional change in s. Acceleration induced deflections are fixed by material stiffness and mass distribution. Fundamental considerations of the dynamic response show that a natural resonance frequency of 500 Hz is reasonable. Experimental devices tend to show lower frequencies than this, but the technology of capacitance microphones has pushed response well up above 1 KHz for such devices.

Returning to the expression $\Delta X = (X_1 + X_2)^2 \Delta X_3/(X_3)^2$ we note that the shunt effect of $X_1$ and $X_2$ dilutes the transducer coefficient unless $X_3$ is kept small. However, $X_3$ cannot be too small because that dilutes the oscillator tunability. Some compromise must be chosen.
It is also clear that the compensator coefficient is a function of $X_2$ and so depends on oscillator tuning; the effective compensation may change by 20% over a tuning range of 2 Hz. This is unfortunate for the milli-puffer. However, an oscillator intended for fixed frequency operation can be exactly compensated. Quartz aging would then require eventual readjustment.

A question which should be raised is whether the acceleration induced frequency shift intrinsic to the quartz gives an observed $\Delta f$ which depends on external circuit parameters? Empirically the answer is no. Over a 2 Hz range the observed sensitivity $\Delta f/f$ per g is constant to within $\pm 2 \times 10^{-11}$. The required compensation is apparently not a function of operating point on the piezoelectric reactance curve.

Figure 9 shows results of reduced g-sensitivity using external control of the parameter $X_2$. This oscillator has been subjected to vibration testing in the range 8 to 33 Hz, with maximum g level up to 1.5 g, and the measured phase noise was -98 dB (15Hz) relative to the carrier, which checks quite well with the measured static sensitivity. The results that were shown in Figure 5 were obtained using the $X_3$ 'milli-puffer' method.

The second type of g-compensation uses two resonators in series (Ref. 5). The method is indicated in Figure 10. Initial success with this method is indicated by the low phase noise. The dual crystal within the oscillator loop exhibits the same loaded Q as for a single resonator, at the same time showing reduced g-sensitivity. Results were disappointing however, in that the nearly exact cancellation expected did not occur. What happens is that in pairing the two resonators, the relative stray capacitance for each is changed, and the individual sensitivities no longer cancel.

The feature around 2.5 Hz is associated with an incipient frequency instability which grows worse (and shifts in frequency) as the relative strays are
adjusted to achieve exact compensation. In the analysis of the dual resonator equivalent circuit, two allowed frequencies appear (Ref. 5). These may be widely separated and yet permit some degree of g-compensation. What is surprising is that the compensation can be 'tuned' at the expense of overall stability. At exact compensation a slow beat appears, correlated with the phase noise anomaly. This behavior is still under investigation.

The selection process for matching resonators is indicated in Figure 11. Here the two sensitivities are nearly identical aside from a small relative tilt. Following a parity reversal of one set of geometric axes, the two resonators are put in series and installed in the same oscillator.

Figure 12 shows results for a pair of 3rd overtone crystals. The sensitivity for one alone is $19 \times 10^{-10}$ per g and for the combination, is reduced to $3.5 \times 10^{-10}$ per g.

An interesting consequence of the partial cancellation of stress induced frequency shifts is found in the thermal shock behavior of the dual crystal units. The warm-up behavior is shown in Figure 13, along with the warm-up curve when one of the resonators is bypassed. The latter shows the typical 0.5 Hz to 1 Hz overshoot due to 'thermal shock' non-equilibrium. It is not surprising to find that frequency shifts induced by thermal gradients are intimately related to those induced by mechanical stress; what may be surprising is that in a particular oven geometry these stresses can be compensated to any extent during the warm-up time. This is reminiscent of early attempts (Ref. 8) to reduce the equilibrium temperature coefficient in A element resonators by operating two in series. This new phenomenon may shed some light on the non-time-equilibrium conditions.
SUMMARY

Within the frequency determining network for a high precision crystal we have a variety of possibilities for making use of acceleration derived compensating signals. For a capacitive device, sensitivity of the order of tens of femtoFarads per g is needed, and frequency response down to zero is desired. The analysis is straightforward in terms of series reactances. Inductive g-sensitive transducers may also be employed.

Furthermore the reactance of interest may also be controlled electronically by an externally derived correction signal. The bandwidth of the oscillator sustaining loop is more than adequate for handling the desired compensation.

There seems to be no clear-cut answer as to which scheme is preferable in all cases. All of the single crystal methods have had comparable success, and the dynamic tests have been consistent with measured reductions in static g-sensitivity.

The dual crystal measurements have produced evidence of a fundamental frequency instability, but compromises can be made for less than complete compensation, with overall improvement. Of particular interest is the fact that thermal shock frequency deviations, as noted during warmup, are reduced from the usual single crystal case.

In conclusion, Figure 14 shows some comparisons of compensated vs. uncompensated performance. The example of $5 \times 10^{-11}$ per g residual sensitivity is typical for reproducibly attainable results. Worst axis residual sensitivity can be made smaller with a considerable amount of care. Irregularities of symmetry probably set a practical lower limit around $10^{-11}$ per g for these resonators.
Perturbation of short term stability is also reduced. The quiescent oscillator shows a two sample, Allan variance, of better than $6 \times 10^{-13}$ at 1 second averaging time. When the oscillator is perturbed by a sinusoidal 0.5 g input, the stability is 1000 times worse. For the compensated oscillator an improvement by a factor of more than 20 is seen, and is consistent with the compensated static sensitivity.

In the same way, for phase noise sampled at low frequency, a typical improvement is from -70 dB to -98 dB at 15 Hz. Approximate agreement with expected $f_m^{-2}$ is also found.

The results have clear implications for navigation, communications and other systems whose performance depends on low noise high stability frequency sources operating in a non-laboratory environment.

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References


I, HIGH PERFORMANCE CRYSTAL OSCILLATORS IN VIBRATION ENVIRONMENT

A. DEGRADED STABILITY OF AT CUT RESONATORS

B. SYMMETRY OF FREQUENCY SHIFT VS. G

II. METHODS OF REDUCING SENSITIVITY

A. FREQUENCY DETERMINING NETWORK

B. REQUIREMENTS FOR COMPENSATION

C. DUAL RESONATOR APPROACH

III. SUMMARY OF RESULTS
1% G DISTURBANCE AT \( \tau = 1 \) SEC

- O Perturbed
- X Quiescent

Figure 2. Short Term Frequency Stability

\( \tau \) SECONDS

10^{-10} 10^{-11} 10^{-12}
Figure 3. Side Band Phase Noise, 1 G Sine Sweep Vibration
X. Unmodified MODEL 1000 oscillator, $\gamma = 1.2 \times 10^{-9}/G$
O. Modified for $\gamma = 3 \times 10^{-10}/G$

\[ \Phi(f) = \left( \frac{\gamma f_0}{2f_m} \right)^2 \]
Figure 4. Gravitational Frequency Shift VS. Azimuth for Fifth Overtone 5 MHz Crystal
Figure 5. Gravitational Frequency Shift VS. Polar Angle; Compensated and Uncompensated Comparison
REQUIREMENTS FOR IDEAL SYSTEM

1. PERFECT SYMMETRY FOR EXACT CANCELLATION OF VECTOR COMPONENTS
2. LINEARITY WITH G AND WITH FREQUENCY TUNING
3. MODULATION RESPONSE TO LIMIT OF FM BANDWIDTH
4. NO SPURIOUS MECHANICAL PHASE MODULATION
5. NO SPURIOUS CRYSTAL OVEN EFFECTS
6. NO ΔF HYSTERESIS
7. QUIESCENT BEHAVIOR NOT DEGRADED

FIGURE 6: SOME REQUIREMENTS FOR IDEAL COMPENSATION
\[ x_{MT} = \frac{y_2 (x_1 + y_2)}{y_2 + x_1 + x_2} = 10 \text{ Ohm} \]

\[ \delta x_{MT} = \frac{(y_1 + y_2)^2}{x_3^2} \delta y_3 \]

\[ \delta x_3 \propto \text{Hz} \]

\[ \frac{\delta x_3}{x_3} = \frac{\delta y_3}{y_3} \]

\[ s = \frac{j}{\omega} \]

Figure 8. Frequency Determining Network
Figure 9. Reduction of G Sensitivity
Figure 10. Dual Resonator Approach. Phase Noise of Quiescent Oscillator

Dual outputs for the dual resonator oscillator
X output A
O output B
Figure 11. Selection of Matched Resonators
Figure 12. Dual Resonator, 3rd Overtone, Worst Axis Sensitivities
- Single Crystal Response
- Dual Crystal Response
Figure 13. Experimental Oscillator Warmup (Thermal Shock) Characteristic

- DUAL RESONATOR
- SINGLE RESONATOR

RELATIVE FREQUENCY IN Hz

10 MINUTES 20 30
<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>UNCOMPENSATED</th>
<th>COMPENSATED</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔF/F VERSUS ACCELERATION</td>
<td>~ 1.2 × 10^{-9}/G</td>
<td>&lt; 5 × 10^{-11}/G</td>
</tr>
<tr>
<td>SHORT STABILITY σ (1 SEC) FOR 1/2G INPUT AT C</td>
<td>70 × 10^{-11}</td>
<td>3 × 10^{-11}</td>
</tr>
<tr>
<td>PHASE NOISE Λ Ø (15 Hz) WITH 1G SINE INPUT AT 15 Hz</td>
<td>-70 DB</td>
<td>-98 DB</td>
</tr>
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**FIGURE 14:** SUMMARY. AT CUT 5TH OVERTONE RESONATOR G-SENSITIVITY COMPENSATION
QUESTIONS AND ANSWERS

MR. D. A. EMMONS, Frequency and Time Systems, Inc.:

Could I just make a correction--an error of omission? I think I failed to mention that Presjinski of Draper Labs has very convincingly shown the use of an external accelerometer for g sensitivity reduction. I didn't mean to imply that that was a new technique.

I have also brought with me a desensitized oscillator which is in back beside our portable clock, which fits under the seat of a DC-9.