ABSTRACT

Stable frequency sources and signal processing blocks must often be characterized by their noise spectra, both discrete and random, in the frequency domain. In this tutorial, conventional measures are outlined, and systems for performing the measurements are described. Broad coverage of system configurations which have been found useful is given. Their functioning and areas of application are discussed briefly. Particular attention is given to some of the potential error sources in the measurement procedures, system configurations, double-balanced-mixer-phase-detectors and application of measuring instruments. A general calibration scheme is detailed.

INTRODUCTION

This paper is a lightly edited transcription of the tutorial presentation which included audience participation. With that excuse, the author begs the reader's indulgence of the conversational style and disorder.

First, to point out the areas that make up the subject of frequency domain measurement systems, see Table 1. This is only roughly the order in which they will be covered. References will be cited for coverage of the basics.

Model numbers, ranges and accuracies of the various frequency domain instruments will be avoided since these are readily available on data sheets and in catalogs. However, there are some traps in the ways these instruments may be used which will be covered.
Measures

To establish a clear context, the symbols and definitions that are used for the measures are shown in Table 2. The reader may be familiar with one or more of these from various applications. Basically, each of these definitions implies a math model of a random process. When a measurement is made, it will most likely include processes which are both random and non-random - an important source of some of the traps. Accordingly, it is necessary to have a clear understanding of how a chosen measure (math model) responds to discrete spurious components as well as continuous random spectra, and as separate questions, how the measurement system relates to the measure for both classes of signals. These areas are covered in references 1, 2 and 3.

Systems

I would like to pose a question to start into the systems aspects of the subject. Why do we need a system to measure frequency domain stability? (see Figure 1). The usual reason is dynamic range. We might try to use a spectrum analyzer to measure the noise side bands, for example, which are 140 dB below the carrier in a 1 Hz bandwidth. Even if we use a 1 kHz bandwidth in the spectrum analyzer instrument, this noise will be 110 dB below the carrier as indicated on the screen of the analyzer, and the spurs of the one kHz filter would force us to look no closer than several kHz away from the carrier. So the dynamic range in both the frequency and amplitude sense, and their interaction, are the reasons why we have to go to more complexity than this, with higher quality oscillators. There's a large family of oscillators for which this is an appropriate way of measuring their noise side bands, but it displays AM and PM.

AM versus PM is another distinction which must be cared for. The usual name for the measure that is seen on a spectrum analyzer is the rf power spectrum. This measure is almost never used as a specification for a frequency standard or any component of a frequency distribution system. What is needed is a demodulator, a phase or frequency demodulator, implying that it is relatively insensitive to AM and it needs to be highly sensitive to small PM for FM. (See Figure 2). Otherwise, in the case of those rare systems that have AM sensitivity, it should probably be specified separately since there would be a great opportunity for confusion.
Referring to the instability sensor in the middle of Figure 2, the demodulator should have a very good ratio between the demodulated signal and its own noise contribution. We could apply some other words to that central block: an error enhancer, an error multiplier (which frequency multipliers tend to function as) or conversion to base band. In many communication systems, some element of, or some of the blocks of the system itself (which is going to use the oscillator under test) might be applied here. It is important that their transfer function be well understood and modeled. If so, they may be applied as the instability sensor to make the measurement in a very realistic fashion with respect to the oscillator's performance in the final system.

Autocorrelator System

The reason for choosing the fairly complex looking system of Figure 3 as the first to be considered in detail is that it has wide application. In many of its forms, it can take a range of input frequencies without modifying the hardware, and it can test a single source without having a reference source, without requiring a second similar source as a reference. This feature makes it attractive for development work in many programs. Figure 3 may be explained by starting at its output and working backwards. The double balanced mixer is used as a phase detector. The fundamental operation that the mixer performs can be described by assuming sine waves applied to the R and L ports. The simplest model for the mixer is that it multiplies the two sinusoids together so that what comes out are two signals as in the trigonometric identity. One signal is an average term which is proportional to the phase difference between the two input sine waves (that is the difference frequency out of the mixer). The other is a very large signal which is the sum frequency output of the mixer. Since the inputs are the same frequency, this sum output is twice the input frequency. The phase shifter, shown here in what may be called the reference channel, is adjusted to bring the two mixer input signals into quadrature so that the mixer performs a phase detection. If, on the other hand, the two input signals are brought into in-phase condition (or 180° out of phase) by the use of this phase shifter, then this becomes an amplitude detector, and the AM spectrum of the input signal can be analyzed.

The functions in the other path to the mixer cause this to
be an instability enhancer or a detector of the spectrum that we want to measure. First, the directional coupler is shown as having its 10 dB attenuating port to the L port because we expect to take less loss down this path and we want as large a signal as possible to both ports of the mixer. The result with any one of the items listed on the right of Figure 3 is that this path functions as frequency-to-phase transducer. For example, in the case of the transmission line, the signal goes down the transmission line, reflects and comes back, incurring a delay. The delay of 300 feet of coaxial cable at 1 MHz is approximately one cycle; 1/2 cycle out, and 1/2 back. If the frequency then changed by 10%, the phase shift through this path length would change by 10% of a cycle. In that way, a delay line or a narrower filter on this path serves as a frequency-to-phase transducer. Then the mixer acts as a phase-to-voltage transducer. The overall result is a sensor which transforms frequency to voltage, a low noise discriminator.

There is an interesting aspect of the narrower band resonators in this system - either a crystal filter or a cavity. Within the bandwidth of the resonator, all the foregoing is still true. However, outside the bandwidth of the resonator, the resonator can be considered to be stripping off the noise sidebands, or holding the phase of the signal through this path coherent for a longer time than the period of these larger sideband or modulation frequencies. If a phase change occurs at a high rate versus a narrow resonator, the phase out of the resonator does not change, it does not follow the rapid phase change, and the system output is now the phase modulation spectrum, outside the bandwidth of the resonator.

For areas of applicability, the crystal filter, from 100 kHz to the 100 MHz region is useful. The one port cavity is best applicable from 5 GHz up. One could use a transmission cavity similarly. The long, shorted transmission line is the broadband approach which allows the single system to operate from 100 MHz to 10 GHz. This entire system does not offer as low a system noise floor as some of the other systems, but it does have extremely broad applicability and the advantage of single input source. The references appropriate to Figure 3 are: 4, 5 and 6, the last of which dealt with this system using a crystal as a filtering element.

Preamplifiers

The modulation spectrum appearing at the mixer output in
these systems still must be measured by a frequency domain instrument such as a low frequency spectrum analyzer. The sensitivity and noise figure of these instruments usually requires a preamplifier following the mixer to give best performance. The mixer will generally be a 50 ohm component which means it was designed to operate from 50 ohm sources into a 50 load, not that it looks like 50 ohms in general, particularly when both ports are driven from a relatively high level. The low noise amplifier, which is needed to achieve good performance from the system, is trying to operate from a source impedance which is probably below 50 ohms. The optimum-noise source resistance for low frequency amplifiers is very rarely this low. The only one that is commercially available which the author has seen referenced (but has no personal experience) had the brand name Ortec mentioned in Lances paper at 77 PTTI. For that purpose, the author has built his own relatively simple amplifiers. A good reference on building amplifiers of this class (and low noise rf amplifiers also) is a book by Motchenbacher and Fitchen, reference: 7 on low noise techniques.

Mixer-Phase Detectors

Since the mixer is a critical element, let us digress from the systems configuration for a moment and give the mixer more attention. The author has seen many systems where people have attenuators preceding mixer, even following the mixer, and expecting that when they click in 20 dB, to have a 20 dB change in what is going on in the system. This is probably not true because of the way the mixer works. In Figure 4 the diagram on the left is the schematic of a good many mixers. In fact, some of the best mixers for our purposes have exactly that schematic. There are also many variations, for which this is still a good first-order model. Even though their schematics look vastly more complex than this, they look, to the input and output signals, very much as this model.

Though the results are usually misleading, it is natural to attempt to measure the drive level to the mixer by looking at the voltage at its input ports. It is very easy to put a Tee connector on the BNC going into the mixer and probe the signal at that point with a good high impedance scope probe as was done for Figure 5. But, just as one wouldn't try to determine the dissipation in a zener by measuring the voltage across it and ignoring the current, one should not draw many conclusions from this voltage. A high impedance rf voltmeter looking at the voltage into
the mixer is even less informative about what is going on in the mixer because all that is being measured is the forward drop across a diode. As is indicated by the functional model in the center of Figure 4, which assumes that the L port signal is the larger one, it simply saturates the mixer. With a large L signal, one series pair of diodes or the other, for either polarity signal, is strongly forward biased. This amounts then to connecting either one side, one polarity, or the other, of the R signal to the X port determined by the polarity of the L signal. The results of this action are shown in the right-hand part of Figure 4 for R and L in a quadrature phase relationship. Corresponding to this function, the double balanced mixer is sometimes also called a reversing switch. It is interesting that there is very little literature on the mixer used as phase detector. References: 8 and 9.

Figure 5 shows mixer waveforms which are careful tracings of scope photos. Both the L and R port waveforms appeared to be identical. Both were driven with +7 dBm incident power, in quadrature phase. That was measured with the coax disconnected from the mixer, into a good 50 ohm load, with an rf power meter. Then the coax was reconnected onto the mixer and probed with a high impedance probe for the photo. Notice that the waveform is distorted by the action of the diodes. Another important point is that this signal came from a broad band resistive 50 Ω source. This is probably not true of most of the output stages of the frequency standard devices which might be connected to a mixer. Now, given that the signal came from a 50 ohm source, and was sinusoidal into a 50 Ω load, the mixer looks like various things. It may look like more than 50 Ω as the input is crossing zero, and it clearly looks like less than 50 Ω as the diodes start to conduct more and more. The mixer, at this drive level, probably drops down to the order of 20 Ω or below, over portions of the waveform.

At the output X port the 2f output or sum frequency appears. The important thing to notice is how large it is relative to the inputs. In other words, when a mixer is used in one of these systems, one must be aware of the fact that with a 5 MHz input signal to the mixer, there will be a very large 10 MHz output signal from the mixer. That large output signal can deal a very strange blow to the spectrum measuring instrument, or to the low noise amplifier that is connected to the mixer. Although the X output does not look exactly sinusoidal, spectral analysis of that waveform at the bottom of Figure 5, shows that its third harmonic is still 30 db below the fundamental. The top of the
screen was set to equal the power incident on the mixer at 10 MHz, so the output signal is about 6 dB below the power that is incident on the inputs.

Mixer Loads and Sensitivities

Most systems will require a low pass filter following the mixer to reject the large 2f output signal. Most familiar filter designs have input impedances which are a large mismatch outside their passband. That is to say that they achieve attenuation by reflecting the unwired input. A low loss filter, made of only reactive elements, must then appear as a highly reactive mismatch at its input to signal frequencies outside the passband. This situation could result in the mixer being terminated reactively at the frequency of its largest output signal component. This condition has been seen to cause surprisingly gross distortions of the input waveforms. This can raise serious questions of the validity of the measurement because the device under test is driving a mixer input which is an extremely non-typical load, having a large reactive mismatch and violent non-linearities.

The low pass filter following the mixer can be designed to present a 50 ohm load to the 2f output component by making the input element of the filter a shunt capacitance with a 50 ohm resistance in series. Figure 6 is an example from reference 10. This arrangement terminates the mixer in a non-reactive 50 ohm for all frequencies above 1 MHz, but unloads the mixer to maximize its sensitivity below 100 kHz. Since the filter element values depend on seeing a 50 ohm source resistance in the mixer, this filter develops one to two decibels of peaking when the mixer is driven above 0 dBm at both inputs. This is a good example showing the necessity for checking system flatness, over the entire frequency range to be measured, during the calibration procedure.

The transducer coefficient of the mixer, used as a phase detector, in volts per radian, (that is, its ratio of converting radians of phase shift to volts of output signal, dc average, or low frequency average) can be quite sensitive to the terminating impedance at its output port. See Figure 7. Taking the output into a broadband 50 ohm load as a base line, the output slope can be increased up to 6 dB by raising the load resistance. However, another 6 dB, for a total of 12 dB, approximately, increase in sensitivity can be gained by careful choice of a parallel capacitive reactance well below 10 ohm (at the input
frequency) as a termination, reference: 11. Again, being a reactive termination, this can raise the above questions due to mixer input waveform distortion, and has been shown to roll off the modulation frequency response, requiring thorough calibration.

All but one of the curves in Figure 7 were taken with the same RF power level applied to both the L and R ports. Using an HP 3335A, a calibrated phase shift was inserted and the static transducer coefficient was measured using a DVM. One-tenth radian positive and one-tenth radian negative about 0 were the phase shifts used. Note that if the mixer is terminated at a high impedance, a gain from 4 dB, to much more than 6 dB is realized at lower levels, versus terminating the mixer with a broadband 50 Ω. And there is an interesting fact here that all the curves for all the mixers, at least at some input power levels, were asymptotic to the line whose equation is \( K_p = 1.8 \) times the incident power level expressed in rms voltage; and the relationship seems to be that unloading doubles the peak voltage available out of the mixer. The lower frequency average 2.2 (2/π) which is 1.8.

This completes the digression to mixer details and attention returns to other ways of configuring the measurement system.

More Measurement Systems

Figure 8 shows a two-channel version of Figure 3, somewhat amplified to fit the page. This takes two of the systems of Figure 3 and splits the power of the oscillator under test into them. This allows the output signals to be cross-correlated for noise reduction. This is now possible because there are very convenient fast Fourier transform instruments which have two inputs. The Fourier transform has both amplitude and phase and it requires two inputs to the processor in order to accomplish that. There is a button on the front panel which commands a cross correlation between those two inputs. This, with averaging, will allow reduction of the noise of amplifiers which might be inserted. The noise of one amplifier is uncorrelated versus the noise of the other amplifier, allowing this improvement in system noise figure by using one of the more recently available fast Fourier instruments as the frequency domain analyzer for the spectrum. Reference: 12. Turning now from the single oscillator to the two oscillator systems, Figure 9 shows the most common system that is used for
frequency domain measurements. It requires a pair of oscillators of similar quality, unless the reference oscillator is much better than the one to be measured. This system has been discussed fairly extensively in the literature; references: 11, 13, 14. This, as far as I am aware, has been in wide use since 1964 for specifically this purpose. As the notations in Figure 9 indicate, the modulation spectrum coming out of the mixer is a phase modulation, outside the bandwidth of the lock loop, and a frequency modulation spectrum, inside the bandwidth of the lock loop.

This system can be modified slightly, as shown in Figure 10, to make measurements of a two port device, such as an amplifier. If a synthesizer or frequency multiplier is to be tested, which changes the frequency from its input to its output, then a similar device would have to be placed in both paths so that the same frequency goes to both inputs of the mixer. The phase shifter is used to bring the phases into quadrature at the mixer. Most of the noise from the reference oscillator cancels since it appears at both inputs to the mixer. Again, reference: 11 is suggested.

The cross correlation enhancement of the measurement system can be applied to this system also, as shown in Figure 11. Here the system of Figure 9 has been duplicated, amplifiers can be inserted, and their noise can be suppressed by averaging in the cross correlation process. This was suggested with an analog multiplier as a correlator in reference 15, along with other systems considerations which are important to this kind of measurement. The dashed line shows that one of the loops can be used to lock the reference oscillator to the oscillator under test, to maintain quadrature phase at the mixers.

Up to this point the systems that have been considered either measured a single source or the combined noise of two sources (one considered to be a known reference) at the same frequency. In the systems to follow, the use of a reference source whose frequency is offset from the unit under test allows the use of period or frequency counting as the measuring instrument.

Figure 12 shows the simplest of these systems. In addition to the convenience of using a commonly available instrument, a period counter, this system has the advantage of allowing measurement of time domain stability, $\sigma_Y(\tau)$, as well as aging or other drifts. Further advantages are that the
measurement resolution (dynamic range) can be extreme (even with an inexpensive counter) and that the measured raw data is in digital form, being stable and convenient for automation via an interface bus. References: 16, 17.

Since counter input stages usually do not have low enough noise to avoid degrading the signal level available from the mixer, preamplification is necessary. Optimal characteristics for this preamp are that it should have high gain in order to hard limit on millivolt inputs, low enough noise to add no more than sub-microsecond perturbations to the zero crossing of a 1.0 Hz, 1.0 Vp-p sine wave input, and have 1.0 MHz or better bandwidth. This is already a very specialized design, and further needs to have a calibrated (even adjustable) first stage bandwidth.

Recall that the output from the mixer will have a large component at twice the input frequency riding on the low frequency waveform whose period is to be measured. This requires a low pass filter of typically greater than 100 dB rejection, at twice the lowest mixer input frequency, because these perturbations are essentially uncorrelated with the unperturbed time of the zero-crossing and therefore function as a noise source. This can typically be accomplished with as few as four poles. However, the physical construction of the mixer-filter-amplifier combination has to be very sophisticated in order to obtain the attenuation that the filter was designed to provide. One of the problems in this field is that there are few systems for sale which are specified to do the overall measurement job.

An extension of the above technique is shown in Figure 13. This dual mixer time difference system also happens to take data in the time domain, as do most Fourier transform instruments for that matter, and pass it through an integral transform with a digital processor to deliver a modulation sideband spectrum. We call the result a frequency domain measurement - and don't worry about the fact, other than to make sure we are performing it properly, that the initial data was a time record. References: 18 and 19.

The high isolation power splitter in Figure 13 is worth a moment or two to look at the kind of frequency pulling effect that the two standards at the same frequency, can have on each other. For instance, in Figure 14 consider a quartz crystal resonator inside the standard with a Q of 1 million and 60 dB of net reverse transfer isolation, which would be a typical case for a quartz oscillator.
This would be the case if the signal coming out of the oscillator sees 20 dB of gain then, with 80 dB of gross isolation between the crystal and the output, the net is 60 dB. In this case, the oscillator in question could suffer around 3 parts in $10^{10}$ of frequency pulling. Clearly this deserves consideration when two standards are close to each other in frequency.

Calibration

When calibrating any system it is desirable to keep the procedure as simple and fool-proof as possible, minimizing the number of dependencies on the calibrations of support instruments. For a phase noise measurement system, the simplest general approach would be to inject a known signal and note the response of the entire system at once. Since many of the devices in a measurement system will be operating at input or output impedance levels substantially departing from 50Ω, with this departure being level dependent, it is highly desirable to require no attenuator setting changes or signal level changes between calibration and measurement. The departures from 50Ω arise unavoidably from several causes: 1. Many high quality signal sources, though designed to drive a 50Ω load, do not present a 50Ω source impedance; 2. Many devices exhibit their best noise performance at impedance levels quite different from maximum power transfer; 3. Inherently non-linear mixers have input impedances which change radically over various portions of the input waveform and are varied further by the level and phase of the other input signal and the terminating impedance.

The simplest high accuracy calibration scheme found to date is based on a single ratio of a pair of RF power measurements at the same port and similar frequencies. This kind of measurement can be performed with 0.1 to 0.5 dB uncertainties depending on frequency, for up to a 90 dB ratio with off-the-shelf standard instruments.

As shown in Figure 15, the calibrating signal is combined with the signal under test from an oscillator or other devices. The calibrating signal may just as well be combined with the signal from the reference source, especially if this results in a more convenient set-up.

The calibration procedure consists of two parts: First the levels of the calibration signal and the main signal with which it is combined are measured, second, the overall system response to the combined signals is measured. The
computation which combines these three measurements (sometimes with other constants) to yield the scale factor for the frequency domain stability measurement may be considered a third step.

Since the details of this procedure can affect the resulting accuracy, the following sequence is suggested:

1. Connect calibration signal source to combiner. Terminate other input of combiner. Connect output of combiner to power meter. Set level of calibration signal source, as shown on power meter, to desired level, at least 40 dB below the output level of the source under test. Measure C volts rms. These levels should have been pre-determined by preliminary measurements. This should likely consist of several passes through the entire calibration and measurement sequence to establish workable and convenient signal levels and control settings.

2. Connect oscillator under test in place of termination of input of power combiner. On power meter, measure M volts rms.

3. Disconnect power meter from output of combiner and connect combiner to input of measurement system.

4. Read measurement system response at pseudo-sideband modulation frequency corresponding to the difference between frequency of C signal and M signal.

5. Compute measurement system scale factor based on reading in 4. and the fact that the input to the measurement system has a peak phase deviation of C/M radians.

6. Disconnect calibration signal source from input to combiner and terminate combiner input. Calibration is now complete. Proceed with measurements;

Other than errors in the power meter's calibration, (on a ratio basis only, absolute calibration being of no consequence) the only other error source in this calibration signal set-up can arise from non-50Ω (or in general, unmatched) impedances. This concern arises only if the input
port of the measurement system differs from the power meter in impedance. This would almost always be true if a mixer is the input element of the system.

There is still no problem unless there is also a difference in the source impedance appearing at the output of the combiner for the calibration signal versus the main signal. This would again be true very typically for a frequency standard output which is designed to be loaded with 50Ω but does not provide a 50Ω source impedance. The calibration signal source is likely to be a signal generator whose output source impedance (probably through an attenuator) is a fairly accurate 50Ω. These signals may pass through a directional coupler which would tend to normalize any mismatch of the calibration source, while presenting the main signal source impedance essentially unchanged. Should this error source be present, its effect can be calculated, and/or impedance matching can alleviate it. Figure 16 shows alternatives for the coupler. Figure 17 is a vector picture of what is going on; in sine wave terms, the calibrate signal is a vector of a slightly different frequency from the main signal, and the vector resultant is both AM and PM. The system supposedly responds only to the PM, in the cases being studied here. For a very small calibrate signal and a large main signal, the phase excursion is very well defined. There is a form following Figure 17 to help keep track of the arithmetic in this calibration and measurement scheme.

Analog Analyzers

When using an analog wave or spectrum analyzer to measure the spectrum of the output of a system, it is necessary to be very cautious of the actual noise bandwidth of the instrument. The resolution bandwidth at the switch on the front panel, when set to 10 Hz, or 3 Hz, or 1 Hz will not be equal to the noise bandwidth. This can, in most cases, account for on the order of 1 dB of measurement error. For a crude measurement, this may be of no concern. Usually, the noise bandwidth is wider than the resolution bandwidth on the front panel. The number of poles in the IF that determines that resolution bandwidth versus the slope of the noise can cause problems too. This is well covered in reference: 11.

When averaging follows a log amplifier function, about 2.5 dB of error occurs due to skewing of the mean, see reference: 10. In some of the analog instruments which utilize digital storage, there is a circuit to catch the
peak of the bright lines as they sweep past them. When
noise is measured with an analog to digital converter cir-
cuit which operates in that way, it catches the peaks of
the noise and gives an answer that is higher by as much
as 6 dB, and in most cases, 2-4 dB than the true noise
level.

Digital Analyzers

In the digital Fourier transform or fast Fourier class of
analyzers, we have had a good deal less experience. When
using these analyzers, a point to watch for is the setting
of the knob which can be switched to sine or random. The
software takes care of the problems of logging and noise
bandwidth, if that switch is in the right position. When
a sinewave calibration is being made, that switch should
be in the sine position. Then, when noise is being
measured, the switch must be in the random position.
However, for discrete spur measurement, the sine position
must be used. Some problems have arisen in trying to
utilize these analyzers because their gain is insufficient
to measure the small signals coming out of the mixers.
Measuring down below 1 Hz, for the modulation frequency,
is the vast step forward that these analyzers are offering.
Yet, in some cases, a very dc stable or low frequency
stable amplifier is needed between the mixer and the
analyzer itself. This amplifier must also have good noise
performance, usually a trade-off with long-term stability.
It seems well to be cautious about the performance of such
an amplifier.
REFERENCES


Abbreviations Used In References:


Table 1
FREQUENCY DOMAIN MEASUREMENT SYSTEMS

1. Various Measures

2. Various Devices to be Tested

3. Measurement Methods and Hook-Ups
   - Applications
   - Calibration
   - Traps

4. Frequency Domain Analyzer Instruments
   - Ranges, Accuracy
   - Traps
Table 2
FREQUENCY DOMAIN STABILITY MEASURES

<table>
<thead>
<tr>
<th>MEASURE</th>
<th>SYMBOL</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>† Spectral density of fractional frequency fluctuation</td>
<td>( S_y )</td>
<td>1/Hz</td>
</tr>
<tr>
<td>Spectral density of frequency fluctuation</td>
<td>( S_{\Delta f} )</td>
<td>Hz²/Hz</td>
</tr>
<tr>
<td>† Spectral density of phase fluctuation</td>
<td>( S_{\phi} )</td>
<td>rad²/Hz (also ( S_{8\phi}, S_{\Delta \phi} ))</td>
</tr>
<tr>
<td>* Single sideband phase noise to carrier ratio</td>
<td>( \mathcal{L} )</td>
<td>1/Hz</td>
</tr>
<tr>
<td>Residual frequency modulation, rms, in bandwidth ( f_B ), located ( f_m ) from carrier</td>
<td>( \Delta f_{\text{res}} )</td>
<td>Hz</td>
</tr>
<tr>
<td>Residual phase modulation, rms, in modulation spectrum between ( f_1 ) and ( f_2 )</td>
<td>( \Delta \phi )</td>
<td>radians</td>
</tr>
</tbody>
</table>

† Recommended by IEEE, CCIR
* Widely used on procurement specifications and data sheets.

**Independent Variable: Frequency**
- Modulation Frequency \( f_m \)
- Fourier Frequency \( f \)
- Sideband Frequency \( f \)
- Offset Frequency \( f \)
- Baseband Frequency \( f \)
Figure 1

SIMPLEST FREQUENCY DOMAIN MEASUREMENT SYSTEM

OSCILLATOR UNDER TEST

SPECTRUM ANALYZER
Figure 2
FREQUENCY DOMAIN MEASUREMENT SYSTEM

SIGNAL TO BE MEASURED

OSCILLATOR UNDER TEST

INSTABILITY SENSOR

SIGNAL WHICH REPRESENTS THE ERRORS IN THE INPUT SIGNAL

FREQUENCY DOMAIN SIGNAL ANALYZER
Figure 3

AUTOCORRELATION DEMODULATOR

OSCILLATOR UNDER TEST → DIRECTIONAL COUPLER → CIRCULATOR OR HYBRID → TUNER → RESONATOR

CRYSTAL FILTER, OR ONE-PORT CAVITY, OR LONG, SHORTED TRANSMISSION LINE

PHASE SHIFTER

-10 dB

DOUBLE BALANCED MIXER

FREQUENCY MODULATION SPECTRUM (WITHIN BW OF RESONATOR)
Figure 5
DOUBLE BALANCED MIXER PHASE DETECTOR WAVEFORMS AND SPECTRUM

0.5 V/div
10 MHz
TO L (R same)
(+7 dBm incident)

0.5 V/div
20 MHz
From X

10 dB/div
10 MHz/div
Spectrum of above output from X

Top of grat. = +7 dBm
Figure 6

SIMPLE LOW PASS MIXER TERMINATION

SPECTRUM ANALYZER

HI Z

82 μH

0.039 μF

100 kΩ

DOUBLE BALANCED MIXER

50Ω

51Ω

3900 pF

EFC

0.1 μF

DC VOLTMETER

QUADRATURE MONITOR
Figure 7

DOUBLE BALANCED MIXERS USED AS PHASE DETECTORS; COMPARISON OF TRANSDUCER COEFFICIENTS VERSUS INPUT LEVEL

INCIDENT POWER L & R PORTS 50 Ω

ALL @ 10.0 MHz
OUTPUT LOAD Z
RF: 50Ω
DC & LF: AS SHOWN

KΩ, 1.8 (V/rms)
-1.3 dBm

KΩ, 30Ω

105348 HI-Z

V/rad .40

0.32

0.25

0.20

0.16

0.13

0.10

Vrms

-20

-15

-10

-5

0

5

10

15

20

25

30

mW

dBm

INCIDENT POWER L & R PORTS 50 Ω

V/rad
Figure 8

DUAL AUTOCORRELATOR, CROSS CORRELATION ENHANCED

RESONATOR OR DELAY

DIRECTIONAL COUPLER

POWER SPLITTER

OSCILLATOR UNDER TEST

TO CROSS-CORRELATION PROCESSOR

RESONATOR OR DELAY

DIRECTIONAL COUPLER
Figure 9
TWO OSCILLATOR, LOCKED

OSCILLATOR UNDER TEST

DOUBLE BALANCED MIXER

REFERENCE OSCILLATOR

PHASE AND FREQUENCY MODULATION SPECTRUM

OUTSIDE
INSIDE

BANDWIDTH OF LOCK LOOP

LOCK
Figure 10
SINGLE OR TWO, TWO-PORT

PHASE MODULATION SPECTRUM

REFERENCE OSCILLATOR

POWER SPLITTER

AMPLIFIER UNDER TEST

PHASE SHIFTER
Figure 11
TWO OSCILLATOR, LOCKED, DUAL MIXER, CROSS CORRELATION ENHANCED

REFERENCE POWER X OSCILLATOR

POWER SPINNER

OSCILLATOR UNDER TEST

REFERENCE OSCILLATOR

TO CROSS-CORRELATION PROCESSOR

LOCK
Figure 12

TWO OSCILLATOR, OFFSET

INTEGRAL TRANSFORM

MODULATION SIDEBAND SPECTRUM

PERIOD COUNTER

DEEP SKIRT LO PASS

OSCILLATOR UNDER TEST

OFFSET OSCILLATOR
Figure 13
THREE OSCILLATOR, TWO NOT OFFSET
DUAL MIXER TIME DIFFERENCE

Oscillator under test $f_0$

Offset oscillator $f_1$

Reference oscillator $f_0$

Hi-isolation power splitter

Deep skirt LO pass

Start

Time interval counter

Stop

Time record

Integral transform

Modulation sideband spectrum
Figure 14
FREQUENCY PULLING
VERSUS
REVERSE TRANSFER ISOLATION

\[ y = \frac{\Delta \phi}{2Q} \]

\[ \Delta \phi \sim \frac{2}{V_1} \]

PEAK FREQUENCY PULLING, y

NET REVERSE TRANSFER ISOLATION, dB
DIRECT RATIO CALIBRATION METHOD

Figure 15

POWER METER

DIRECTIONAL COUPLER OR PWR. COMB.

TERMINATION

LOW LEVEL

CALIBRATION OSCILLATOR

OSCILLATOR UNDER TEST

ANY MEASUREMENT SYSTEM

373
Figure 16

ALTERNATIVES FOR CALIBRATION SIGNAL INJECTION

-50 dB

-20 dB

-0.2 dB

20 dB DIRECTIONAL COUPLER

-0.5 dB

-30 dB

3 dB POWER SPLITTER-COMBINER

-6 dB

-6 dB

$ \Gamma = \frac{Z_0}{3}$

6 dB POWER SPLITTER-COMBINER
Figure 17
CALIBRATION VECTOR DIAGRAM

\[ \beta = \arcsin \frac{C}{M} \]

For small angles, \( \beta << 1 \)

\[ \frac{C}{M} \sim \beta \equiv \text{Modulation index, radians, peak} \]
SINGLE SIDEBAND PHASE NOISE ($\mathcal{L}(f)$) MEASUREMENT WORKSHEET

Using a Double-Balanced Mixer, Directional Coupler and Spectrum Analyzer

1 port signal level should be the maximum available, up to the mixer specs, and must remain constant for both calibrate and measure.

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<thead>
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<th>FREQ. Hz</th>
<th>RAW READOUT dB</th>
<th>- SYST. RESP. dB</th>
<th>SCALE FACTOR dB</th>
<th>MEAS. DATA dB</th>
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TEST CONDITIONS AND SCALE FACTOR COMPUTATION

| LO SIG INTO L PORT ( ) dBm |
| CALIBRATE FREQ. ( ) Hz |
| - ( ) dBm MAIN SIG. INTO R |
| + ( ) dBm CAL SIG. PORT |
| - ( ) dB INPUT SENS. |
| + ( ) dB SYSTEM RESP. @ CAL FREQ. |
| - ( ) dB RAW CAL READOUT "ADJUST" LAMP OUT? |
| + ( ) dB INPUT SENS. |

-6 dB RADIANS TO SSB

[ ] dB BRIGHT LINE SCALE FACTOR "ADJUST" LAMP? ( )

- ( ) dB = 10 log ( Hz BW)
+ 1.7 dB (+ 2.5 logging (- 0.8 Gauss BPF)

[ ] dB RANDOM NOISE SCALE FACTOR SMOOTHING? OR VIDEO FILTER? ( )

UNIT UNDER TEST:

REFERENCE SIGNAL SOURCE:

MIXER:

AC AMP:

DC AMP:

NAME:

DATE:

376
QUESTIONS AND ANSWERS

DR. MICHEL TETU, Laval University:
I would like to point out first that when you use digital spectrum analyzers, we again have some difficulties in defining the equivalent bandwidth over which the measurement is done. The source seems to be the digital filtering used to compute the spectrum. Second, when you do not have a wide phase noise or a wide noise observed, it is a little dangerous to use the equivalent bandwidth principle.

MR. FISCHER:
I think we are somewhat at the mercy of the vendors of those devices, until we get more familiar with them, at least, to properly characterize their noise bandwidth in a believable way.