SCHOOL OF ENGINEERING AND APPLIED SCIENCE

UNIVERSITY OF VIRGINIA

Charlottesville, Virginia 22901

Annual Report
on NASA Grant NSG-1509

EVALUATING AND MINIMIZING NOISE IMPACT DUE TO AIRCRAFT FLYOVER

Submitted to:
NASA Scientific and Technical Information Facility
P.O. Box 8757
Baltimore/Washington International Airport
Baltimore, MD 21240

Submitted by:
Ira D. Jacobson
Associate Professor

Gerald Cook
Professor

Report No. UVA/528166/MAE79/101
May 1979
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I. INTRODUCTION

This report presents the results of a study on the evaluation and reduction of noise impact to a community due to aircraft operation. Existing techniques have been used to assess the noise impact and to optimize the flight paths of an approaching aircraft with respect to the annoyance produced. Major achievements have been: (1) the development of a population model suitable for determining the noise impact, (2) generation of a numerical computer code which uses this population model along with the steepest descent algorithm to optimize approach/landing trajectories, (3) implementation of this optimization code in several fictitious cases as well as for the community surrounding Patrick Henry International Airport.

Previous work has centered on developing noise annoyance criteria for flyover (i.e. NEF, NNI, CNR, etc.) and ground noise signatures for aircraft. Some of these criteria are discussed in References 1-5 with a review of many of the noise effect measures being summarized in Ref. 6. Typical of the noise footprint work is Ref. 7. The annoyance criterion used in the study is the noise impact index (NII).

The details of the models used, their advantages and disadvantages and the results obtained are outlined in the following sections.
II. PROBLEM FORMULATION

A. Overview

Analysis of the problem consists of six parts: (1) aircraft noise signatures, (2) population models, (3) cost (annoyance) function, (4) aircraft flight-path model, (5) aircraft constraints, and (6) approach/landing path optimization. A modular concept has been employed so that modification of any of these segments may be effected with relative ease. The sections below describe each of these parts in detail.

B. A/C Noise Signature

The aircraft noise signature is obtained using data from Ref. 3. Here the effective perceived noise level (EPNdB) is given as a function of slant range to the closest point of approach for a variety of aircraft. A typical plot of the slant range variation is shown in Figure 1. These data were fit using standard least squares techniques to yield an expression for EPNdB given by

\[ \text{EPNdB} = 115 - 22.5 \log_{10} x \text{ (Slant Range)} \]  

This equation is used for calculation of the maximum noise level at each location for a flyover. A typical footprint for a straight in approach along a 3-degree glide slope is shown in Figure 2.

C. Population Model

To model the population, a map of the community is overlaid with a grid and the population in each section of the grid determined. The population distribution within each section is assumed to be uniform. Several grid geometries were examined (see Figure 3). These geometries
Figure 1. EPNL vs. Slant Range

FLYBY NOISE LEVEL
(1.93 - 1.95 EPR 727 Aircraft) FIG. A-1**
(1.94 EPR DC-9 Aircraft) FIG. D-1**

*Closest Point of Approach
**FAA-RD-71-83 (Ref. 6)
CUSSEM LANDING NOISE FOOTPRINT

110 EPNDS
100 EPNDS
90 EPNDS
80 EPNDS

x, y Coordinates Unit: Mile
A.C. Original Altitude: 20000 Feet
A.C. Speed: 279 ft./sec
3 Degree Landing

Figure 2. Noise Foot Print.
1. Equal Size Blocks

2. Variable Size Blocks

3. Concentric Circles

Figure 3. Grid Geometries.
included: (1) rectangular sections of equal size, (2) rectangular sections whose dimensions increased with distance from the airport runway, and (3) concentric circles divided by several radial lines. The second scheme was chosen since it requires fewer rectangular sections than the first and is easier to implement than the third. Computer time required for determining the optimum trajectory varies directly with the number of grid sections. Furthermore, in light of the dependence of noise levels on distance and the fact that the aircraft has higher altitude when further from the runway, the need for high resolution of the population density diminishes with distance from the airport. While the third population scheme could offer this same advantage, it is somewhat more difficult to determine the population and noise impact for each grid section with such a geometry.

Within a grid section, the population is determined by use of the SITE II system, (Ref. 8), available on the CDC 7600 computer at the NASA-Langley facility. This system requires as input the latitude and longitude of a reference point and the coordinates of the corners of each rectangular section. Although SITE II allows for simple retrieval of 1970 census data, there is some question about its resolution capabilities for small grid sections. In addition, in rapidly growing areas the population data may lag actual population. The SITE II program is capable of producing detailed census information as shown in Figure 4. However for the present analysis only population information is used.
# Demographic Profile Report

## 1970 Census Data

### Population

<table>
<thead>
<tr>
<th>Total</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>374709</td>
<td>186466</td>
<td>188243</td>
<td>374709</td>
</tr>
</tbody>
</table>
- White: 367224 (94.9%)
- Negro: 15414 (4.0%)
- Other: 4371 (1.1%)

### Family Income (000)

<table>
<thead>
<tr>
<th>Range</th>
<th>Total</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0-5</td>
<td>7945</td>
<td>65</td>
<td>7859</td>
<td>14113</td>
</tr>
<tr>
<td>$5-7</td>
<td>5694</td>
<td>42</td>
<td>5578</td>
<td>10272</td>
</tr>
<tr>
<td>$7-10</td>
<td>4984</td>
<td>39</td>
<td>4888</td>
<td>9872</td>
</tr>
<tr>
<td>$10-15</td>
<td>4794</td>
<td>38</td>
<td>4516</td>
<td>9310</td>
</tr>
<tr>
<td>$15-25</td>
<td>5265</td>
<td>444</td>
<td>5176</td>
<td>10441</td>
</tr>
<tr>
<td>$25-50</td>
<td>12867</td>
<td>1109</td>
<td>11758</td>
<td>24625</td>
</tr>
<tr>
<td>$50+</td>
<td>3045</td>
<td>27112</td>
<td>30717</td>
<td>60829</td>
</tr>
</tbody>
</table>

### Median (Age)

- 27.4

### Average

- 185052
- 201950

### Median

- 15763
- 14134

### Median (Age)

- 27.4

### Rent

<table>
<thead>
<tr>
<th>Range</th>
<th>Average</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0-100</td>
<td>8377</td>
<td>7312</td>
</tr>
<tr>
<td>$100-150</td>
<td>35292</td>
<td>31754</td>
</tr>
<tr>
<td>$150-200</td>
<td>48626</td>
<td>43576</td>
</tr>
<tr>
<td>$200-250</td>
<td>6945</td>
<td>12761</td>
</tr>
<tr>
<td>$250+</td>
<td>3792</td>
<td>2920</td>
</tr>
<tr>
<td>Total</td>
<td>43128</td>
<td>37704</td>
</tr>
</tbody>
</table>

### Average

- 150
- 147

### Median

- 150
- 147

### Percentage

- 61.5

### Household Parameters

- Fam Pop: 335153 (86.6%)
- Individs: 45881 (11.9%)
- GGP: 5975 (1.5%)
- Tot Pop: 387009

### Units in Households

<table>
<thead>
<tr>
<th>Structure</th>
<th>Households with:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-bed</td>
<td>TV: 126239 (91.8%)</td>
</tr>
<tr>
<td>2-bed</td>
<td>Washer: 71594 (52.1%)</td>
</tr>
<tr>
<td>3+4</td>
<td>Dryer: 54258 (39.5%)</td>
</tr>
<tr>
<td>5-9</td>
<td>Dish Wash: 56277 (40.9%)</td>
</tr>
<tr>
<td>10-49</td>
<td>Air Con: 79438 (57.8%)</td>
</tr>
<tr>
<td>50+</td>
<td>Fridge: 26600 (20.8%)</td>
</tr>
<tr>
<td>Mobile</td>
<td>Hones: 2856 (2.1%)</td>
</tr>
</tbody>
</table>

### Location

- Latitude: 38 52 10
- Longitude: 77 9 20

### Figure 4. Demographic Profile Report
D. Flight Path Model

There are two ways in which the trajectory of the aircraft can be determined. In one, a discrete time integration of the equations of motion with control deflections yields point by point spatial coordinates and orientation. Although this allows the flexibility of building in control constraints as well as dynamical constraints (e.g. max roll angle) it requires a considerable number of states to be stored in the optimization routine. In a multi-aircraft, multi-runway problem, as is anticipated, these storage requirements become prohibitive.

Thus, another method was adopted which utilizes only the functional form of the trajectory to describe the flight path. First a starting path was assumed which went from the initial point to the desired runway and ended up with the proper heading, i.e., velocity vector aligned with the runway. The following equation was used to generate this starting trajectory. (See Figure 5).

\[
y_s(x) = \left[ \frac{y_f - y_p}{x_f - x_p} \right] (x - x_p) + (y_p - y_o) \exp \left[ -C(x - x_f)/(x_o - x_f) \right] + y_o
\]

(2)

For the vertical motion a simple three degree descent path was assumed.

Next the first five Fourier sine harmonics were used to introduce deviation from this starting path. One advantage in using this type representation is the fact that each of the forms contributes zero at the end points. Therefore if the starting path satisfies the boundary conditions the curve with the deviations will also. An exponential decay at the final point was used to eliminate heading deviations.
Figure 5. Rotated Coordinate System for Establishing Nominal Flight Trajectory from Initial Point to Runway Approach.
The equations with the deviations thus become

\[
y(x) = \left( \sum_{i=1}^{5} \alpha_i \sin \left[ \pi \left( x - x_0 \right) / (x_f - x_0) \right] \right) \left[ 1 - \exp \left( (x - x_f) / C_1 \right) \right] + y_s(x)
\]

\[
Z(x) = \left( \sum_{i=1}^{5} \beta_i \sin \left[ \pi \left( x - x_0 \right) / (x_f - x_0) \right] \right) \left[ 1 - \exp \left( (x - x_f) / C_1 \right) \right] + Z_s(x)
\]

(3a)

(3b)

A second advantage to using a Fourier Series representation for the curve is that it provides a means of representing a function with a finite number of parameters. This reduces the optimization problem from a variational one to an ordinary one.

E. Constraints

The use of a functional form of the flight path for the trajectory requires the reformulation of constraints into parameters which can be used in the optimization. This is accomplished by translating the steady state solutions of the lateral and longitudinal perturbation equations into geometric constraints. An exact derivation is given in Appendix A. The constraints are incorporated by determining maximum curvature and slope parameters as a function of aerodynamics and physical constraints. For example the constraint of a maximum roll angle, \( \phi_{\text{max}} \), yields

\[
\frac{d^2y}{dx^2} \leq \frac{C_2 + C_1 C_3}{C_4 + C_1 C_5} \frac{\phi_{\text{max}}}{V_{\text{avg}}}
\]

(4)

where \( C_1 \) through \( C_5 \) depend upon aircraft stability and control derivations (see Appendix A for details) and \( V_{\text{avg}} \) is the average velocity. Similar expressions are given in Appendix A for constraints on aileron rudder.
and elevator deflection, flight path angle and pitch rate limits.

F. Cost Function

A large number of criteria have been proposed by evaluating noise annoyance (e.g., EPNdB, NII, sleep interference index, speech interference index, etc.). The recent trend in noise assessment work is toward a universal measure—the noise impact index (NII). This measure is a weighted day-night model which accounts for population density. It is described in detail in Ref. 9. Briefly, the total population exposed to each incremental average day-night model sound level is multiplied by the weighting function for the level. The weighting function used is shown in Figure 6. This weighting factor \( W(L_{dn}) \) multiplied by the population exposed to that \( L_{dn} \) is summed and normalized by the total population giving the Noise Impact Index for the area.

\[
\text{NII} = \frac{\sum_{L_{dn}} P(L_{dn})W(L_{dn})}{\sum_{L_{dn}} P(L_{dn})} \tag{5}
\]

The cost function or payoff for the optimization procedure is taken to be the NII plus penalties for violating constraints. Basically the optimization procedure is set up to "drive" the aircraft trajectory to the path which will minimize the NII and at the same time not violate any constraints. As an example of the constraint of flight path angle not exceeding a maximum descent angle, \( \gamma_d \), nor a maximum climb angle, \( \gamma_c \), is written as

\[
\tan\gamma_c < \frac{dZ}{dx} < \tan\gamma_d \tag{6}
\]
Figure 6. Sound Level Weighting Function for Overall Impact Analysis.
Each is converted to a penalty which is added to the NII in the form

$$\text{Cost} = \text{NII} + \left(\frac{dZ}{dx}/\tan\gamma_d\right)^2 + \left(\tan\gamma_c/\frac{dZ}{dx}\right)^2$$  \hspace{1cm} (7)

As is seen for values of the flight path angle within the allowable range the penalty is negligible; however for values outside this range the penalty and thus the increase in cost is great. Other terms are added in a like manner.
III. OPTIMIZATION

The optimum trajectory is determined by calculating values of the \( a_i \)'s and \( \beta_i \)'s (Eq. 2) which minimize the total cost (NII plus penalties). A steepest descent algorithm is employed here. Basically, this method computes the gradient of the cost function, \( C \), with respect to the \( a_i \)'s and \( \beta_i \)'s, then searches along the negative gradient direction for values of \( a_i \)'s and \( \beta_i \)'s which reduce the cost. The change in cost is given by

\[
\Delta C = \sum_{i=1}^{5} \left( \frac{3C}{3a_i} \Delta a_i + \frac{3C}{3\beta_i} \Delta \beta_i \right)
\]  

(8)

The process continues iteratively until the cost converges to within a specified tolerance. While implementation of the algorithm is fairly straightforward, convergence near the optimal set of \( a_i \)'s and \( \beta_i \)'s is inherently slow. Most of the cost reduction, however, occurs in the first few iterations.

A. The Optimization Algorithm

A computer code has been developed which implements the functions described above. Figure 7 shows a flow chart for this code. Initial data (population map, aircraft constraints, initial and final aircraft positions, etc.) are required for each airport/airplane configuration to be evaluated. To facilitate calculation of the Fourier coefficients, the coordinate axes are rotated such that a line joining the initial and final aircraft positions is made to be parallel to the x axis. A nominal trajectory is generated which constrains the heading of the aircraft to asymptotically approach the runway. The steepest descent search then begins and continues until the stopping criterion is met. This criterion
READ POPULATION MAP, AIRCRAFT CONSTRAINTS
INPUT INITIAL CONDITIONS,
FINAL CONDITIONS,
FOURIER PARAMETERS = x's, β's

COORDINATE ROTATION =
ROTATE AXIS SUCH THAT LINE JOINING
INITIAL AND FINAL POINTS IS PARALLEL
TO THE x-AXIS IN ORDER TO APPLY
FOURIER SERIES

GENERATE A NOMINAL CURVE WHICH FORCES
THE HEADING OF AIRCRAFT AT FINAL POINT
TOWARD THE RUNWAY

STEEPEST DESCENT
GRADIENT SEARCH

NO

OPTIMAL REACHED?

YES

OPTIMAL LANDING TRAJECTORY

Figure 7. Flow Chart.
is met if successive improvements become negligible.

In order to provide a more accurate noise impact in each population section the impact is integrated using quadratures. This procedure can be found in Ref. 9.

The various functions such as the population model, the cost function, and the aircraft signature are incorporated as subroutines. This will allow ease of upgrading or modification if different models are desired.

Appendix B contains the Fortran code as written for a CDC Cyber 172 machine.

B. Results

Several cases have been run to test the benefits that can be obtained by this approach. First, a fictitious set of data incorporating a population valley is used. As can be seen in Figure 8 the optimization algorithm moves the aircraft (a Convair 880) towards the valley (i.e. fewer people impacted) with a corresponding improvement in the NII of 32%.

The second case models the Patrick Henry Airport in Hampton, Virginia. Here the SITE II program was used to generate the census data for each block as shown in Figure 9. Two initial trajectories were flown. One entering the area from the northwest over the Swing VOR station and the other from the southwest over the Franklin VOR station. Both of these paths are specified IFR trajectories (Figure 10). The aircraft enters the area approximately 30,000 meters from the runway. In addition several straight-in paths were evaluated. Figure 9 shows each of the trajectories. The associated
Figure 8. Optimization Results Using Fictitious Population Data.
Figure 9. Population Model and Optimization Results for Patrick Henry Airport.
### Table I

Northwest Approach

**Entry Point: Swing**

<table>
<thead>
<tr>
<th>Traj. No.</th>
<th>Description</th>
<th>Cost (NII x 10^{-2})</th>
<th>% Change from Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60 deg wrt runway</td>
<td>2.373</td>
<td>+3.2%</td>
</tr>
<tr>
<td>2</td>
<td>30 deg wrt runway</td>
<td>2.438</td>
<td>+6.0%</td>
</tr>
<tr>
<td>3</td>
<td>Initial iteration</td>
<td>2.27</td>
<td>-1.3%</td>
</tr>
<tr>
<td>4</td>
<td>Optimal</td>
<td>2.213</td>
<td>-3.8%</td>
</tr>
<tr>
<td>5</td>
<td>Straight in</td>
<td>2.316</td>
<td>+1.1%</td>
</tr>
<tr>
<td>N1</td>
<td>Presently used</td>
<td>2.300</td>
<td>0%</td>
</tr>
</tbody>
</table>

### Table II

Southwest Approach

**Entry Point: Franklin**

<table>
<thead>
<tr>
<th>Traj. No.</th>
<th>Description</th>
<th>Cost (NII x 10^{-2})</th>
<th>% Change From Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Straight in</td>
<td>2.316</td>
<td>-1.3%</td>
</tr>
<tr>
<td>6</td>
<td>Initial iteration</td>
<td>2.408</td>
<td>+2.6%</td>
</tr>
<tr>
<td>7</td>
<td>Optimal</td>
<td>2.241</td>
<td>-4.5%</td>
</tr>
<tr>
<td>8</td>
<td>30 deg wrt runway</td>
<td>2.598</td>
<td>+10.7%</td>
</tr>
<tr>
<td>9</td>
<td>60 deg wrt runway</td>
<td>2.687</td>
<td>+14.5%</td>
</tr>
<tr>
<td>N2</td>
<td>Presently used</td>
<td>2.346</td>
<td>0%</td>
</tr>
</tbody>
</table>
NII's are summarized in Tables I and II.

As is seen that even for this case, where the population is sparse away from the runway and congested near the end of it, an improvement of 3 to 5% is achieved using the optimization algorithm. It should also be noted that this technique allows not only the optimization of the path but also can be used to evaluate existing or proposed paths, such as the nominal and straight-in paths indicated.

Conclusion

A method has been formulated to optimize the path of an aircraft during approach or take-off from any airport. Models have been developed using available data where possible for population, aircraft signature, noise impact, constraints and flight path. An algorithm using steepest descent has been implemented and tested. This approach allows

1) The evaluation of the noise impact of existing flight paths,
2) The evaluation of the noise impact of proposed flight paths, and
3) The optimization of the flight path to minimize the noise impact under constraints.

This method has been applied to the Patrick Henry International Airport. Both nominal and other straight paths were evaluated. Also an optimal path was determined for each of two terminal area entry points. Performance ranged from a 15% degradation of the NII to 4.5% improvement compared to the presently used approaches. It is significant that as much as 3 to 5% improvement could be achieved in light of the fact that most of the population is concentrated at the end of the runway.
REFERENCES


APPENDIX A

Derivation of Parameterized Trajectory Constraints

Lateral perturbation equations

\[ \begin{align*}
\dot{Y} & = -\frac{b}{2V} C_{y_p} \phi - \frac{mg}{q_{\infty} S} \cos \phi \dot{\phi} + \frac{mV_T}{q_{\infty} S} - \frac{b}{2V} C_{y_r} \psi - \frac{mg}{q_{\infty} S} \sin \theta_o \psi \\
& + \frac{mV_T}{q_{\infty} S} \beta - C_y \beta = C_y \delta_a + C_y \delta_r \\
L & = \frac{I_{xx} \cdot \phi - b}{2V} C_{x_p} \phi - \frac{I_{xz}}{q_{\infty} S_b} \dot{\psi} - \frac{b}{2V} C_{x_r} \psi - C_s \beta = C_s \delta_a + C_s \delta_r \\
N & = \frac{I_{xz} \cdot \phi - b}{2V} C_{n_p} \phi + \frac{I_{zz}}{q_{\infty} S} \dot{\psi} - \frac{b}{2V} C_{n_r} \psi - C_n \beta = C_n \delta_a + C_n \delta_r \quad (1)
\end{align*} \]

If we assume all turns to be coordinated (no sideslip)

Then letting \(- \frac{b}{2V} C_{y_p} = \overline{C}_{y_p}, \text{etc.} \)

\[ \begin{align*}
\frac{mg}{q_{\infty} S} \cos \theta_o = \overline{g}_1 \\
\frac{mg}{q_{\infty} S} \sin \theta_o = \overline{g}_2 \\
\frac{I_{xx}}{q_{\infty} S_b} = \overline{i}_x, \text{etc.} \quad \frac{mV_T}{q_{\infty} S} = \overline{m}
\end{align*} \]

\[ \begin{align*}
L & = \frac{i_x \overline{\phi} - C_{x_p} \cdot \delta_{i-x} \overline{Z} \cdot \psi - \overline{C}_{x_r} \cdot \psi = C_s \delta_a + C_s \delta_r \\
N & = -i_x \overline{Z} \cdot \phi + i_x \overline{Z} \cdot \psi - \overline{C}_{n_r} \cdot \psi = C_n \delta_a + C_n \delta_r \\
Y & = -\overline{C}_{y_p} \cdot \overline{g}_1 + (\overline{m} - \overline{C}_{y_r}) \cdot \overline{g}_2 \cdot \psi = C_y \delta_a + C_y \delta_r \quad (2)
\end{align*} \]

Taking the Laplace transform (I.C.'s = 0)

\[ \begin{align*}
L & = (i_x s^2 - \overline{C}_{x_p}) \phi (s) + (-i_x s^2 - \overline{C}_{x_r}) \psi (s) = C_s \delta_a (s) + C_s \delta_r (s)
\end{align*} \]
\[ \text{N eq'n: } (-i_x z^2 - C_{n_p}) \phi(s) + (i_x z^2 - C_{n_r}) \psi(s) = C_n \delta_a(s) + C_n \delta_r(s) \]

\[ \text{Y eq'n: } (-C_{y_p} s - \bar{g}_1) \phi(s) + [(m-C_{y_p}) s - \bar{g}_2] \psi(s) = C_y \delta_a(s) + C_y \delta_r(s) \]

(3)

To determine the required \( \delta_a \) for a given \( \delta_r \) we consider \( \delta_a \) an unknown along with \( \phi(s) \) and \( \psi(s) \) [i.e. move \( \delta_a \) to the left hand side of the equations] and solve for \( \delta_a/\delta_r \) using Cramer's rule

\[
\frac{\delta_a}{\delta_r} = \frac{\begin{vmatrix}
i_x s^2 - C_{y_p} s & -i_x z^2 - C_{y_p} s & +C_{\delta_r} \\
-i_x z^2 - C_{y_p} s & i_x s^2 - C_{y_p} s & +C_n \delta_r \\
-C_{y_p} s - \bar{g}_1 & (m-C_{y_p}) s - \bar{g}_2 & +C_{y_r} \\
i_x s^2 - C_{y_p} s & -i_x z^2 - C_{y_p} s & -C_{\delta_a} \\
-i_x z^2 - C_{y_p} s & +i_x s^2 - C_{y_p} s & -C_n \delta_a \\
-C_{y_p} s - \bar{g}_1 & (m-C_{y_p}) s - \bar{g}_2 & -C_{y_r} \\
\end{vmatrix}}{N(s)} \quad (4)
\]

The denominator (characteristic eqn.) is given by:

\[
\Delta(s) = s^4(-C_{y_a} (i_x i_z - i_x i_z)) + s^3(C_y + (m-C_{y_r}) + i_x C_{y_r} + i_x C_{y_p} + C_{\delta_a})
\]

\[
+ s^2(C_y (C_{n_p} C_{y_p} - C_{n_r} C_{y_r}) + C_{n_r} (-i_x z g_1 - i_x z g_2 - C_{y_p} C_{y_r} - (m-C_{y_r} C_{y_p}))
\]

\[
+ C_{\delta_a} (g_2 i_z g_2 i_x z - C_{y_p} C_{y_r} + (m-C_{y_r}) C_{n_r})
\]

\[
+ s(C_{n_r} (g_2 C_{y_p} - g_1 C_{y_r}) + C_{\delta_a} (g_2 C_{n_p} - g_1 C_{n_r}))
\]

(5)
The numerator is:

\[ N(s) = s^4\{C_y \delta_r (1, iZ-iZ^2) + s^3\{-C_y \delta_r [iZC_{\delta_r} + iZC_{\delta_r} + iZ(C_{\delta_r} + C_{np})] \}
\]

\[ -C_n \delta_r [-iZC_{\delta_r} + (m-C_{\delta_r})iZ] - C_{\delta_r} iZC_{\delta_r} - (mC_{\delta_r})iZ \}
\]

\[ + s^2\{-C_y \delta_r (C_{np}C_{\delta_r} - C_{np}C_{nr}) \} - C_{\delta_r} [iZC_{\delta_r} + (m-C_{\delta_r})C_{\delta_r}] \]

\[ + s\{-C_{\delta_r} (C_{np}C_{\delta_r} - C_{nr}) \} - C_{\delta_r} (C_{np}C_{nr} - C_{nr}) \}

(6)

Now assuming that only the steady state (st. st.) condition is of interest,

\[ \lim_{s \to 0} \frac{N(s)}{\Delta(s)} = \frac{\delta_a}{\delta_r} \text{ st. st.} \]

we get

\[ \frac{\delta_a}{\delta_r} \text{ st. st.} = \frac{-C_{n \delta_r} (C_{np}C_{\delta_r} - C_{nr}) - C_{\delta_r} (C_{np}C_{\delta_r} - C_{nr})}{C_{n \delta_a} (C_{np}C_{\delta_r} - C_{nr}) + C_{\delta_a} (C_{np}C_{\delta_r} - C_{nr})} \]

(7)

\[ \cos \theta_o (C_{n \delta_r} C_{\delta_r} + C_{\delta_a} C_{\delta_r}) - \sin \theta_o (C_{n \delta_r} C_{\delta_r} - C_{\delta_a} C_{\delta_r}) \]

\[ \frac{\delta_a}{\delta_r} \text{ st. st.} = \frac{-C_{n \delta_r} (C_{np}C_{\delta_r} + C_{\delta_r} C_{\delta_r}) C_{\delta_r} + \sin \theta_o (C_{n \delta_r} C_{\delta_r} + C_{\delta_a} C_{\delta_r})}{C_{n \delta_a} C_{\delta_r} + C_{\delta_r} C_{\delta_r} + C_{\delta_a} C_{\delta_r} + C_{\delta_r} C_{\delta_r}} \]

(8)

For small initial flight path angle (i.e. \( \theta_o \approx 0 \))

\[ \frac{\delta_a}{\delta_r} \text{ st. st.} = - \frac{C_{n \delta_r} C_{\delta_r} C_{\delta_r} C_{\delta_r}}{C_{n \delta_r} C_{\delta_r} C_{\delta_r} C_{\delta_r}} = C_1 \]

(9)

Assuming \( \theta_o = 0 \) to simplify we can write the transfer functions for \( \phi \) and \( \dot{\psi} \) as (in the st. st.)
\[
\frac{\dot{\psi}}{\delta_r} = \frac{c_2 \delta_r (c_{n\beta} - c_{n\delta_r} c_{n\beta})}{c_{\delta_r} c_{n\beta} - c_{n\beta} c_{\delta_r}} = c_2
\]  
(10)

\[
\frac{\dot{\psi}}{\delta_a} = \frac{c_3 \delta_a (c_{n\beta} - c_{n\delta_a} c_{n\beta})}{c_{\delta_a} c_{n\beta} - c_{n\beta} c_{\delta_a}} = c_3
\]  
(11)

\[
\frac{\phi}{\delta_r} = \frac{c_{y_r} (c_{r\beta} c_{n\beta} - c_{r\beta} c_{n\delta_r} c_{n\beta}) + c_{y_a} (c_{y_p} c_{n\beta} + c_{n\beta} (c_{y_r} + c_{y_r} c_{y_p} c_{n\beta} c_{n\delta_r} c_{n\beta}) + c_{n\delta_r} (c_{r\beta} (c_{y_r} + c_{y_r} c_{y_p} c_{n\beta} c_{n\delta_r} c_{n\beta}))}{m g q_o s (c_{\delta_r} c_{n\beta} - c_{n\beta} c_{\delta_r})}
\]  
= c_4

\[
\frac{\phi}{\delta_a} = \frac{c_{y_a} (c_{r\beta} c_{n\beta} - c_{r\beta} c_{n\delta_a} c_{n\beta}) + c_{y_a} (c_{y_p} c_{n\beta} + c_{n\beta} (c_{y_r} + c_{y_r} c_{y_p} c_{n\beta} c_{n\delta_a} c_{n\beta}) + c_{n\delta_a} (c_{r\beta} (c_{y_r} + c_{y_r} c_{y_p} c_{n\beta} c_{n\delta_a} c_{n\beta}))}{m g q_o s (c_{\delta_a} c_{n\beta} - c_{n\beta} c_{\delta_a})}
\]  
= c_5

(12)

(13)

Consider the aircraft trajectory shown

\[
\begin{aligned}
y &= f(x) \\
\theta &= \psi \\
E &= (x_E, y_E)
\end{aligned}
\]
The slope at any point is $\frac{dy}{dx}$ and the angle the slope makes with the $x$ axis is $\tan^{-1}\left(\frac{dy}{dx}\right)$.

The angular rate $\Psi$ is then $\frac{d}{dt} \tan^{-1}\left(\frac{dy}{dx}\right)$

or $\frac{\partial}{\partial x} \{\tan^{-1}\frac{dy}{dx}\} \frac{dx}{dt} = V_{avg} \frac{\partial}{\partial x} \{\tan^{-1}\frac{dy}{dx}\}$

Then $\Psi = V_{avg} \frac{d^2y}{dx^2} = V_{avg} \frac{\left(f''(x)\right)^2}{1 + \left(f''(x)\right)^2}$

If we know $\delta_r$, we can determine $\delta_a$ from $\delta_a = C_1 \delta_r$

Also $\dot{\Psi} = C_2 \delta_r + C_3 \delta_a = (C_2 + C_1 + C_3) \delta_r$

We can also write

$\phi = C_4 \delta_r + C_5 \delta_a = (C_4 + C_1 C_5) \delta_r$

Constraining $\delta_a$ to be $\leq \delta_{a_{max}}$,

$\delta_r$ to be $\leq \delta_{r_{max}}$

and $\phi$ to be $\leq \phi_{max}$ (max bank angle)

we get the following expressions

$\delta_{r_1} \leq \frac{\phi_{max}}{C_4 + C_1 C_5}$

$\delta_{r_2} \leq \delta_{r_{max}}$

$\delta_{r_3} \leq \frac{\delta_{a_{max}}}{C_1}$
The constraining value is given by

$$\delta r_{\text{max}} = \min(\delta r_1, \delta r_2, \delta r_3)$$

(23)

which yields

$$\dot{\gamma}_{\text{max}} = (C_2+C_1C_3) \min(\delta r_1, \delta r_2, \delta r_3)$$

(24)

This condition incorporates all three constraints ((17)-(19)) as

$$\frac{d^2\gamma}{dx^2} = \frac{f''(x)}{1 + f'(x)^2} \leq \frac{(C_2+C_1C_3)}{\nu_{\text{avg}}} \min(\delta r_1, \delta r_2, \delta r_3)$$

Longitudinally we wish to constrain the behavior of the trajectory so that we restrict \(\gamma\) (the flight path angle) and \(\theta\) (the pitching rate). The trajectory is given by

Then, assuming the aircraft center of mass follows this trajectory \(\gamma\) is given by

$$\gamma = \tan^{-1} \frac{dz}{dx}$$

or

$$\frac{dz}{dx} = \tan \gamma$$

We wish to constrain \(\gamma\) to a maximum descent angle, \(\gamma_{d_{\text{max}}}\) and a maximum angle, \(\gamma_{c_{\text{max}}}\).

Thus

$$\tan \gamma_{c_{\text{max}}} \leq \frac{dz}{dx} \leq \tan \gamma_{d_{\text{max}}}$$
APPENDIX B
PROGRAM NOISE

COMMON ALFA(5), BETA(5), POSIT(5), ARRAY(5, 9), NMAP
COMMON /CURVE/ YCURVE(5, 1), ADY(5, 1), ADDY(5, 1)
COMMON /LABEL/ LINF(4), LLOC(5)
COMMON /AIRPORT/ XPORT, YPORT, ZPORT
COMMON /SCALE/ XMIN, XINC, YMIN, YINC

INTEGER COUNT, HALF

DIMENSION ALFA(5), BETA(5), GY(5), GZ(5), DALFA(5), DBETA(5)
DIMENSION AGY(5), BGY(5), AGZ(5), BGZ(5)

C

C ••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••• A

C, READ MAP FROM DISC INTO MEMORY A
C ••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••• A

READ (5, •) A11, A12
READ (5, •) NMAP, XPORT, YPORT
READ (5, •) (ARRAY, I, J), I = 1, NMAP

C

C ••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••• A

C, INPUT INITIAL CONDITIONS A
C • A

READ (5, •) MAXIT, VALLOW, ZALLOW, IALFA(I), I = 1, 5, I,
READ (5, •) VO, ZO, XF, YF, ZF
READ (5, •) II, ALFA(I), ETA(I), I = 1, 5

C

C ••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••• A

C, COORDINATE ROTATION A
C • A

THETA = ATAN2(YF - YO) / (XF - XO)
A = (XF * COS(THETA) + YF * SIN(THETA)) / (XO * COS(THETA) + YO * SIN(THETA))
PHI = ATAN2((ZF - ZO) / A)
XDAP = XO * COS(THETA) * COS(PHI) + YO * COS(THETA) + ZO * SIN(PHI)
YDFAP = XF * COS(THETA) * COS(PHI) + YF * SIN(THETA) * COS(PHI) + ZF * SIN(PHI)
ZDFAP = -XF * COS(THETA) * SIN(PHI) + YF * SIN(THETA) * SIN(PHI) + ZF * COS(PHI)
XPORT = XPORT * COS(THETA) * COS(PHI) + YPORT * SIN(THETA) * COS(PHI)
YPORT = XPORT * SIN(THETA) * COS(PHI) + YPORT * COS(THETA) * COS(PHI)
C ** START OPTIMIZATION **

INDEX = 0
DLXCAP = (XFCAP-XOCAP)/50.

C ** FIRST FIND A CURVE WHICH FORCES THE HEADING OF THE AIRCRAFT TOWARD THE RUNWAY AT THE FINAL POINT **

SLOPE = (YFCAP-YPORT)/(XFCAP-XPORT)
YCURVE(1) = YOCAP
ADT(1) = 0.
ADDY(1) = 0.
XCAP = XOCAP
DO 10 I = 1,50
XCAP = XCAP+DLXCAP
EXP = -5.*(XCAP-XFCAP)/(XOCAP-XFCAP)
YCURVE(I+1) = (SLOPE*(XCAP-XPORT)+(YPORT-YOCAP))*EXP*EXP+YOCAP
ADY(I+1) = -5.*(XOCAP-XFCAP)*(YCURVE(I+1)-YOCAP)+SLOPE*EXP*EXP
1 CONTINUE

COUNT = 0.

C ** INITIAL COST **

XMIN = -40000
XINC = 2500
YMIN = -40000
YINC = 2500
H = 0.07
CALL COST (0.1*XOCAP,YOCAP,0.05*XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL
1
1) 

C ** CONTINUE **

WRITE (6,9050) I,ALFA(I),BETA(I)
PROGRAM NOISE  73/172  TS

C  CALCULATE GRADIENT

C

50 DO 60 I = 1,5
  ALFA(I) = ALFA(I) + DALFA(I)
  CALL COST (1.0*XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL)
  CY1(I) = (COST2-COST1)/ABS(DALFA(I))
  IF (INDEX,EQ,0) CY1(I) = GY(I)
  IF (INDEX,NE,1) CY1(I) = GT(I)
  WRITE (6,9160) I,CY1(I)
  ALFA(I) = ALFA(I) - CY1(I)
60 DO 70 I = 1,5
  BETA(I) = BETA(I) + DBETA(I)
  CALL COST (1.0*XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL)
  CIZ1(I) = (COST2-COST1)/ABS(DBETA(I))
  IF (INDEX,EQ,0) CIZ1(I) = GZ(I)
  IF (INDEX,NE,1) CIZ1(I) = GT(I)
  WRITE (6,9170) I,CIZ1(I)
  BETA(I) = BETA(I) - CIZ1(I)
70 IF (INDEX,NE,1) GO TO 190
100 PRTENT = ABS(COST2-COST1)/COST1

C  DETERMINE SIZE OF STEP CHANGE

C

YALLOW = (YALLOW-A11)*0.95+A11
ZALLOW = (ZALLOW-A12)*0.95+A12
IF (GYMAX,GE,0.) YRATIO = 0.
IF (GYMAX,LT,0.) YRATIO = YALLOW/GYMAX
IF (GZMAX,GE,0.) ZRATIO = 0.
IF (GZMAX,LT,0.) ZRATIO = ZALLOW/GZMAX
DO 80 I = 1,5
  ALFAOD(I) = ALFA(I)
  ALFA(I) = ALFA(I) - YRATIO*GY(I)
  BETAOD(I) = BETA(I)
  BETA(I) = BETA(I) - ZRATIO*GZ(I)
  CALL COST (1.0*XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL)
  CY1(I) = COST2 - TOTAL
  IF (CY1,GE,COST1) GO TO 150
  100 PRTENT = ABS(CY1,COST1)/COST1
C STOP CRITERION -- PERCENTAGE CHANGE IN COST INSIGNIFICANT

IF (PRCGR.EQ.1.E-5) GO TO 110
COUNT = COUNT+1
CALL COST (O.1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL1)
WRITE (6,9180) COUNT
CALL MONIT (COUNT,COST1,PNALTV)
STOP

110 CALL COST (O.1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL1)
COUNT = COUNT+1
A = COST1-PNALTV
WRITE (6,9190) I,ALFA,I,BETA(I)
DO 120 I = 1,5
120 CONTINUE
WRITE (6,9200) I,ALFA(I),BETA(I)

C STOP CRITERION -- ALL GRADIENT COMPONENTS EQUAL TO ZERO

IF (GY(I).NE.0.) GO TO 130
IF (GZ(I).NE.0.) GO TO 130
CONTINUE
WRITE (6,9210) I,ALFA(I),BETA(I)
DO 140 I = 1,5
140 CONTINUE

C STOP CRITERION -- MAXIMUM NUMBER OF ITERATIONS REACHED

IF (COUNT.LT.MAXIT) GO TO 30
WRITE (6,9220)
CALL MONIT (COUNT,COST1,PNALTV)
STOP
HALF = 1

C REDUCE SIZE OF STEP CHANGE BY HALF
C IF COST HAS NOT DECREASED
C
DO 170 J = 1,3
DO 160 I = 1,5
ALPHA(I) = (ALPHA(I) + ALPHAADD(I))/2.
BETA(I) = (BETA(I) + BETAADD(I))/2.
HALF = J
WRITE (6,9210) HALF
CALL COST (I, I, XCAP, YOCAP, ZOCAP, XFCAP, YFCAP, DLXCAP, DLYCAP, THETA, PHI, TOTAL, PNL)

1 CONTINUE
IF (COST2.LT.COST1) GO TO 100
DO 170 CONTINUE
HALF = 4
INDEX = 1
DO 180 I = 1,5
DALPHA(I) = -DALPHA(I)
180 DBETA(I) = -DBETA(I)

C PERTURB CURVE IN THE OPPOSITE DIRECTION
C
GO TO 50

190 DO 200 I = 1,5
IF (AGY(I).LT.0.) GO TO 220
IF (BGY(I).LT.0.) GO TO 220
IF (AGZ(I).LT.0.) GO TO 220
IF (BGZ(I).LT.0.) GO TO 220

200 CONTINUE
WRITE (6,9080)
DO 210 I = 1,5
ALPHA(I) = ALPHADD(I)
BETA(I) = BETADD(I)
CALL COST (I, I, XCAP, YOCAP, ZOCAP, XFCAP, YFCAP, DLXCAP, DLYCAP, THETA, PHI, TOTAL, PNL)
CALL MONIT (COUNT, COST1, PNLTY)
STOP

220 TERMINAL = ABS(AGY(I))
BGZMAX = ABS(BGZ(I))
DO 230 I = 2,5
IF (BGYMAX.LT.ABS(BGY(I))) BGYMAX = ABS(BGY(I))
230 IF (BGZMAX.LT.ABS(BGZ(I))) BGZMAX = ABS(BGZ(I))
240 WRITE (6,9210) HALF

C CHECK EACH GRADIENT COMPONENT TO DETERMINE SIZE
C
DO 320 I = 1,5

IF (HALF.EQ.71) GO TO 250
IF (GYMAX.NE.0.) Z = YALLOW/GYMAX/FLOATHALF*AGY(I)
IF (GZMAX.NE.0.) Z = ZALLOW/GZMAX/FLOATHALF*AGZ(I)
IF (ZGMAX.NE.0.) Z = ZALLOW/GZMAX/FLOATHALF*AZ(I)
GO TO 290

AY = -ALFA(I)
AZ = -ALFA(I)
BZ = -ALFA(I)

AY = -DALFA(I)
BY = -DALFA(I)
AZ = -DBETA(I)
BZ = -DBETA(I)

IF (AY(I),LE.,0.) GO TO 270
IF (BY(I),LE.,0.) ALFA(I) = ALFADD(I)
IF (AZ(I),LE.,0.) ALFA(I) = ALFADD(I)
GO TO 290

IF (AY(I),LT.,0.) GO TO 280
IF (BY(I),LT.,0.) ALFA(I) = ALFADD(I)+BY
IF (AZ(I),LT.,0.) ALFA(I) = ALFADD(I)+BY
GO TO 290

IF (AY(I),LT.,BY(I)) ALFA(I) = ALFADD(I)*AY
IF (BY(I),LT.,AY(I)) ALFA(I) = ALFADD(I)*BY
IF (AZ(I),LT.,AZ(I)) GO TO 300

IF (AZ(I),LT.,AZ(I)) GO TO 310
IF (AZ(I),LT.,AZ(I)) BETAI = BETADD(I)
IF (AZ(I),LT.,AZ(I)) BETAI = BETADD(I)*AZ
GO TO 320

IF (AZ(I),LT.,AZ(I)) BETAI = BETAI-AZ

320 CONTINUE
CALL COST (0+0,XOCAP,YOCAP,ZOCAP,XFCAP,OLXCAP,THETA,PHI,TOTAL,PINAL

325 DO 340 I = 1,5
325 IF (GYMIN.LE.AGY(I)) GO TO 330
    GYMIN = AGY(I)
325 J = I
325 K = 1
330 DO 340 I = 2,5
330 IF (GYMIN.LE.AGY(I)) GO TO 330
    GYMIN = AGY(I)
330 J = I
335 DO 340 I = 1,5
335 IF (GYMIN.LE.BGY(I)) GO TO 335
    GYMIN = BGY(I)
335 J = I
340 CONTINUE
340
350 IF (GZMIN <= 0 & GZ(I)) GO TO 360
GZMIN = GZ(I)

345 K = I + 5

360 CONTINUE
IF (I) (GMIN <= 0 & O, OR. (GMIN <= 0 & I)) GO TO 370
CALL COST (10, XOCAP, YOCAP, ZOCAP, XFCAP, DTCAP, THETA, PHI, TOTAL, PNAL)
11Y
COUNT = COUNT + 1
WRITE (6, 9990) COUNT
CALL MONIT (COUNT, COST1, PNALTY)
STOP

370 DO 380 I = 1, 5

355 ALFA(I) = ALFA0(I)

380 BETA(I) = BETA0(I)
IF (I) (GMIN <= 0 & AND. (GMIN <= 0 & I) GO TO 390
IF (I) (GMIN <= 0 & AND. (GMIN <= 0 & I) GO TO 400
IF (K, L, 5) BETA(K) = BETA(K) + DBETA(K)
IF (K, L, 5) BETA(K) = BETA(K) + DBETA(K)
GO TO 420

390 IF (I, L, 5) ALFA(J) = ALFA(J) - DALFA(J)
IF (I, L, 5) ALFA(J) = ALFA(J) - DALFA(J)
GO TO 420

400 IF (I, L, 5) ALFA(J) = ALFA(J) - DALFA(J)
IF (I, L, 5) ALFA(J) = ALFA(J) - DALFA(J)
GO TO 420

410 IF (I) (BETA(I) = BETA0(I))

420 INDEX = 0
CALL COST (10, XOCAP, YOCAP, ZOCAP, XFCAP, DTCAP, THETA, PHI, TOTAL, PNAL)
1 1Y
GO TO 100

C

9010 FORMAT (5X, 14MINITAL X, Y, Z, 13FIT2, 2, 3X, 10, 7METERS, /, 5X, 13FINAL X
1 + 1X, Z) + F2, 3X, 10, 7METERS/

9020 FORMAT (5X, 43PLOTERBY TRAJECTORY IN Y AND Z DIRECTIONS BY 2, 0, 2, 5H
1 AND 2, 0, 2, 5H METERS, RESPECTIVELY FOR CALCULATING GRADIENTS)

9030 FORMAT (1X, 4HALFA, 16X, 4HBETA)

9040 FORMAT (10X, 11, 4FIT2, 9, 4X, 10E16, 9)

9050 FORMAT (///)

9060 FORMAT (///, 1X, 15HAT ITERATION, +I2, 49H ALL GRADIENTS EQUAL TO
1 0)

1 1ZERO, PROGRAM STOPS)

9070 FORMAT (10X, 2X, 4HALFA, 16X, 4HBETA)

9080 FORMAT (5X, 4HALL GRADIENTS PERTURBED BOTH DIRECTIONS > 0
1 0)

9090 FORMAT (1X, 15HAT ITERATION, 12, 16H OPTIMUM REACHED)

9100 FORMAT (3A10, 4A11)

9110 FORMAT (1X, 20X, 3A10, 4A10, ///)

9120 FORMAT (1X, 19HINPUT INFORMATION INPUT: /, 5X, 21HMAXIMUM ITERATION SET = 1
1H1 + 13)

9130 FORMAT (5X, 47HMUTERION ALLOWED CHANGES PER ITERATION IN Y AND Z, 27M
1 DIRECTIONS RESPECTIVELY, 1, PER 0, 3, 5H AND 1, PE10, 3, 7H METERS)

9190
9140 FORMAT (5X,22HINITIAL ALFA AND BETA/,13X,4HALFA,16X,4HBEATA,5(/,1 A 4000
10X,11X,1PE16,9,4X,1PE16,9))
9150 FORMAT (10X,1X,10I4TERATION,13X,15X,14HTOTAL COST IS 1PE16,9/,
15X,22HTRUE ANNOYANCE(NII) IS 1PE16,9/5X,2HPRIMAL CONSTRAINTS IS 1PE16,9/)
9160 FORMAT (10X,12X,17HTH Y-GRADIENT IS 1PE16,9)
9170 FORMAT (10X,12X,17HTH Z-GRADIENT IS 1PE16,9)
9180 FORMAT (10X,13HTH iterative percentage change in costs)
9190 FORMAT (10X,12X,17HTH less than .001, program stops)
9200 FORMAT (10X,41HREACH MAXIMUM ITERATION SET, program stops)
9210 FORMAT (10X,10HHALF = 12)
9220 FORMAT (10X,10HTRAJECTORY/,10X,12HX COORDINATE,8X,12HY COORDINATE
1X,12HZ COORDINATE,13X,7H(METER),13X,7H(METER),13X,7H(METER))
9230 FORMAT (10X,1IPE16,9,4X))
END

450000 CM STORAGE USED 7.828 SECONDS
SUBROUTINE COST

SUBROUTINE COST (IWRIT, XOCAP, YOCAP, ZOCAP, XFCAP, DLYCAP, THETA)
COMMON ALFA(I), BETA(I), POSIT(I), ARRAY(I, J), NMAP
COMMON CURVE(I), YCURVE(I), ADY(I), ADDY(I)
COMMON /AIRPORT/, XPORT, YPORT, ZPORT
EXTERNAL FCN

PNALTY = 0.
XCAP = XOCAP

PI = ATAN(1., 0).
C2 = PI/ABS(XFCAP - XCAP)
C3 = ABS(XFCAP - XCAP)/4.
DO 10 I = 1, NMAP
  ARRAY(I, 1) = 0.
  ARRAY(I, 5) = 0.
10

C ----------------------------------------------------------
  * MULTIPLY BY EXPONENTIAL TERM SUCH THAT THE FINAL
  * HEADING OF AIRCRAFT IS TOWARD THE RUNWAY
  * ----------------------------------------------------------
  DO 50 I = 1, 5
    Y2 = 1. * EXP(- (XFCAP - XCAP)/C3)
    Y5 = (Y2 - 1) / C3
    Y9 = 0.0
    Y6 = Y9
    Y7 = Y6
    Y6 = Y7
    Y3 = Y6
    Y3 = Y6
 50

C ----------------------------------------------------------
  * GENERATE SINE HARMONICS
  * ----------------------------------------------------------
  DO 20 J = 1, 5
    TRIGOX = FLOAT(J) * (XFCAP - XCAP) * C2
    Y3 = Y3 + ALFA(J) * SIN(TRIGOX)
    Y8 = Y8 + BETA(J) * SIN(TRIGOX)
    Y6 = Y6 + FLOAT(J) * C2 * ALFA(J) * COS(TRIGOX)
    Y7 = Y7 + FLOAT(J) * C2 * ALFA(J) * SIN(TRIGOX)
 20

DLYCAP = Y2 * Y3
DLYCAP = Y2 * Y6
ZCAP = ZOCAP + DLYCAP
YCAP = DLYCAP + YCURVE(I)

C ----------------------------------------------------------
  * AIRCRAFT CONSTRAINTS
  * ----------------------------------------------------------
  QY = Y2 * Y6 * Y3 * Y5
SUBROUTINE COST 73/172 TS

C

C = 0.1 + AO'1'CII

C = DOY/Cl + Dy.*21

DZ = Y2*Y9tY5*Y6

DZ = DZ+TANCPHII

PNALTY = PNALTYtCDDY/.0011**C20ItIDZ/.1~1**1201

X = XCAP.COSCTHETAI*COSIPHII-YCAP*SI~CTHETAI-ZCAP*COSCTHETAI*SIN

Z = XCAP*SI~CPHII+ZCAP*COSCPHII

ANNOYANCE INTEGRATION OVER A SINGE BLOCK

SMALLP = ARRAY(K,3)/ARRAY(K,7)-ARRAY(K,6)/ARRAY(K,9)-ARRAY(K,8)

CALL GAUSS (ARRAY(K,6);ARRAY(K,7);ARRAY(K,8);ARRAY(K,9);FCV+1E

ARRAY(K,5) = TEMP*SMALLP

GO TO 40

ARRA'I'K,SI = TEMP*SHALLP

GO TO 80

ARRA'I'K,SI = 0,

CONTINUE

IF (IWRITE.EQ.0) GO TO 50

II = 1

POSIT(II+1) = X

POSIT(II+2) = Y

POSIT(II+3) = Z

XCAP = XCAP+DLXCAP

TOTAL POPULATION EXPOSED TO NOISE ABOVE 55 EPnOB

PEOPLE = 0,

DO 50 K = 1,NMAP

IF (ARRAY(K,5).EQ.0.0) GO TO 50

PEOPLE = ARRAY(K,3)/PEOPLE

CONTINUE

FX = 0.

DO 70 K = 1,NMAP

ARRAY(K,5) = ARRAY(K,5)/PEOPLE

FX = FX+ARRAY(K,5)

50 CONTINUE

100 CONTINUE

105 CONTINUE
SUBROUTINE COST 73/172 T5

115 70 CONTINUE
TOTAL = FX*PNALTY
RETURN
END

41000B CM STORAGE USED .874 SECONDS
SUBROUTINE MONIT

SUBROUTINE MONIT (IA, AA, BB)
COMMON ALFA (5), BETA (5), POSIT (5, 3), ARRAY (578, 9), NMAP
COMMON /SCALE/ XMIN, XINC, YMIN, YINC
COMMON /PCRT/ (10)
DIMENSION XM (1026), YM (1026)
DIMENSION XP (53), YP (53), NA (5), NB (3)
EQUIVALENC XP, (ARRAY (1, 1)), YM, (ARRAY (1, 1))
EQUIVALENC (XP (1), POSIT (1, 1)), (YP (1), POSIT (1, 2)), (ZP (1), POSIT (1, 3))
DATA NB, NOH, TOTAL POPULATION, ANNOYANCE = /10, 9, 0, 10/
C
C DOCUMENTATION
C
C CC = AA-BB
WRITE (6, 90100) IA, AA, CC, BB
DO 10 I = 1, S1
WRITE (6, 90200) I, IPOSIT (I, J) J = 1, S1
10 CONTINUE
DO 20 I = 1, NMAP
WRITE (6, 90500) I, IARRAY (I, J) J = 1, S1
20 CONTINUE
WRITE (97, 90700) (I, IPOSIT (I, J) J = 1, S1), (I, IARRAY (I, J) J = 1, S1), NMAP
RETURN
C
90100 FORMAT (10X, 'SHOPTIMUM TRAJECTORY FOR LANDING AT PATRICK HENRY AIRPORT', /)
10X, 'TOTAL COST IS ', 1PE16.9, 10X, 'TRUE ANNOYANCE IS ', 1PE16.9, 10X, 'POPULATION-NOISE-ANNOYANCE
90200 FORMAT (10X, 'SHOPTIMUM TRAJECTORY FOR LANDING AT PATRICK HENRY AIRPORT', /)
10X, 'TOTAL COST IS ', 1PE16.9, 10X, 'TRUE ANNOYANCE IS ', 1PE16.9, 10X, 'POPULATION-NOISE-ANNOYANCE
90400 FORMAT (10X, 'SHOPTIMUM TRAJECTORY FOR LANDING AT PATRICK HENRY AIRPORT', /)
10X, 'TOTAL COST IS ', 1PE16.9, 10X, 'TRUE ANNOYANCE IS ', 1PE16.9, 10X, 'POPULATION-NOISE-ANNOYANCE
90500 FORMAT (10X, 'SHOPTIMUM TRAJECTORY FOR LANDING AT PATRICK HENRY AIRPORT', /)
10X, 'TOTAL COST IS ', 1PE16.9, 10X, 'TRUE ANNOYANCE IS ', 1PE16.9, 10X, 'POPULATION-NOISE-ANNOYANCE
END

41000B CM STORAGE USED 284 SECONDS
SUBROUTINE GAUSS

SUBROUTINE GAUSS (XN, XX, YN, YX, FCN, FINT)
COMMON /AC/ XA, YA, ZA
DIMENSION X(5), Y(5), F(5), XI(5), W(5)
DATA XI,W,N/-0.577350269,0.577350269,0.0,0.1,1.,0,0.0,2./

C
C GAUSSIAN QUADRATURE INTEGRATION WITH FOUR POINTS
C
00 10 I = 1,N
VIII = IYX-YNI/2.*XIII)+IYX+YN)/2.
10 XII) = IXX-XNI/2.*XIIII+IXX+XNI/2.
FINT = O.
00 30 J = 1,N
FIJI = O.
00 20 I = 1,N
20 FIJI = FIJI+WIII.FCNIXIII,YIJ)
FINT = FINT+WIJI*FIJI
FINT = FINT*IYX-YN,
RETURN
END

*1000B CM STORAGE USED .186 SECONDS
FUNCTION FCN

FUNCTION FCN (X,Y)
COMMON /AC/ XA,YA,ZA
RANGE = SQRT((X-XA)**2+(Y-YA)**2+ZA**2)
ARG = 129.12-22.5*ALOG10(RANGE)
FCN = 13.36E-6*10**((.2*10**(.03*ARG)+1.43E-4*10**(-.03*ARG)))/100
RETURN
END

41000B CM STORAGE USED .100 SECONDS
## COST REPORT FOR LISTOAF

**04/27/79**

### RESOURCE BILLING RATE UNITS USED COST

<table>
<thead>
<tr>
<th>Resource</th>
<th>Rate</th>
<th>Time</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Processor</td>
<td>$105.00/HOUR</td>
<td>9,314 CP SECONDS</td>
<td>$2.27</td>
</tr>
<tr>
<td>Peripheral Processor</td>
<td>20.00/HOUR</td>
<td>9,737 PP SECONDS</td>
<td>0.05</td>
</tr>
<tr>
<td>I/O</td>
<td>80.00/HOUR</td>
<td>2,926 10 SECONDS</td>
<td>0.07</td>
</tr>
<tr>
<td>Field Length</td>
<td>3.00/KILO-WRD-HOUR</td>
<td>205,576 KILO-WRD-SECS.</td>
<td>1.17</td>
</tr>
</tbody>
</table>

(BASIC COST EXCLUDES LINES PRINTED, CARDS PUNCHED AND PLOTTER TIME CHARGES)

**JOB PRIORITY 3**  **PRIORITY COST FACTOR 1.00**  **APPROXIMATE ADJUSTED COST**  **\$56**

AS OF LAST ACCOUNT UPDATE, ACCOUNT EXPIRES 04/30/79, FUNDS LEFT \$ 6037.31

04/27/79  UVA NOS/BE 1.2  LEVEL 454-03/11/78
11.45.47.LISTOAF FROM #GD/AB
11.45.47.LIST=M3117A+T00.
11.45.47.ATTACH=Q+NEWFIOT.
11.45.47.PF CYCLE NO. = 002
11.45.47.FTN(=G)
11.45.59. 450008 CM STORAGE USED
11.45.59. 9.292 CP SECONDS COMPILATION TIME
11.45.59.  STOP
11.45.00,EJ  END OF JOB, AB

PRINT COST \$000.88 LISTOAF ///// END OF LIST ///// 0000803 LINES
<table>
<thead>
<tr>
<th>Copy No.</th>
<th>Name and Address</th>
</tr>
</thead>
</table>
| 1 - 2   | NASA Scientific and Technical Information Facility  
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9967:hfc
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Research is an integral part of the educational program and interests parallel academic specialties. These range from the classical engineering departments of Chemical, Civil, Electrical, and Mechanical to departments of Biomedical Engineering, Engineering Science and Systems, Materials Science, Nuclear Engineering, and Applied Mathematics and Computer Science. In addition to these departments, there are interdepartmental groups in the areas of Automatic Controls and Applied Mechanics. All departments offer the doctorate, the Biomedical and Materials Science Departments grant only graduate degrees.

The School of Engineering and Applied Science is an integral part of the University (approximately 1,400 full-time faculty with a total enrollment of about 14,000 full-time students), which also has professional schools of Architecture, Law, Medicine, Commerce, and Business Administration. In addition, the College of Arts and Sciences houses departments of Mathematics, Physics, Chemistry and others relevant to the engineering research program. This University community provides opportunities for interdisciplinary work in pursuit of the basic goals of education, research, and public service.
End of Document