RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES

SCHOOL OF ENGINEERING AND
APPLIED SCIENCE

UNIVERSITY OF VIRGINIA

Charlottesville, Virginia 22901

Annual Report
on NASA Grant NSG-1509

EVALUATING AND MINIMIZING NOISE IMPACT DUE TO AIRCRAFT FLYOVER

Submitted to:
NASA Scientific and Technical Information Facility
P.O. Box 8757
Baltimore/Washington International Airport
Baltimore, MD 21240

Submitted by:
Ira D. Jacobson
Associate Professor
Gerald Cook
Professor

Report No. UVA/528166/MAE79/101
May 1979
RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES

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I. INTRODUCTION

This report presents the results of a study on the evaluation and reduction of noise impact to a community due to aircraft operation. Existing techniques have been used to assess the noise impact and to optimize the flight paths of an approaching aircraft with respect to the annoyance produced. Major achievements have been: (1) the development of a population model suitable for determining the noise impact, (2) generation of a numerical computer code which uses this population model along with the steepest descent algorithm to optimize approach/landing trajectories, (3) implementation of this optimization code in several fictitious cases as well as for the community surrounding Patrick Henry International Airport.

Previous work has centered on developing noise annoyance criteria for flyover (i.e. NEF, NNI, CNR, etc.) and ground noise signatures for aircraft. Some of these criteria are discussed in References 1-5 with a review of many of the noise effect measures being summarized in Ref. 6. Typical of the noise footprint work is Ref. 7. The annoyance criterion used in the study is the noise impact index (NII).

The details of the models used, their advantages and disadvantages and the results obtained are outlined in the following sections.
II. PROBLEM FORMULATION

A. Overview

Analysis of the problem consists of six parts: (1) aircraft noise signatures, (2) population models, (3) cost (annoyance) function, (4) aircraft flight-path model, (5) aircraft constraints, and (6) approach/landing path optimization. A modular concept has been employed so that modification of any of these segments may be effected with relative ease. The sections below describe each of these parts in detail.

B. A/C Noise Signature

The aircraft noise signature is obtained using data from Ref. 3. Here the effective perceived noise level (EPNdB) is given as a function of slant range to the closest point of approach for a variety of aircraft. A typical plot of the slant range variation is shown in Figure 1. These data were fit using standard least squares techniques to yield an expression for EPNdB given by

\[ \text{EPNdB} = 115 - 22.5 \log_{10} x (\text{Slant Range}) \]  

(1)

This equation is used for calculation of the maximum noise level at each location for a flyover. A typical footprint for a straight in approach along a 3-degree glide slope is shown in Figure 2.

C. Population Model

To model the population, a map of the community is overlaid with a grid and the population in each section of the grid determined. The population distribution within each section is assumed to be uniform. Several grid geometries were examined (see Figure 3). These geometries
Figure 1. EPNL vs. Slant Range

Flyby Noise Level
(1.93 - 1.95 EPR 727 Aircraft) FIG. A-1**
(1.94 EPR DC-9 Aircraft) FIG. D-1**

*Closest Point of Approach
**FAA-RD-71-83 (Ref. 6)
CUSSEM LANDING NOISE FOOTPRINT

Figure 2. Noise Foot Print.
Figure 3. Grid Geometries.
included: (1) rectangular sections of equal size, (2) rectangular sections whose dimensions increased with distance from the airport runway, and (3) concentric circles divided by several radial lines. The second scheme was chosen since it requires fewer rectangular sections than the first and is easier to implement than the third. Computer time required for determining the optimum trajectory varies directly with the number of grid sections. Furthermore, in light of the dependence of noise levels on distance and the fact that the aircraft has higher altitude when further from the runway, the need for high resolution of the population density diminishes with distance from the airport. While the third population scheme could offer this same advantage, it is somewhat more difficult to determine the population and noise impact for each grid section with such a geometry.

Within a grid section, the population is determined by use of the SITE II system, (Ref. 8), available on the CDC 7600 computer at the NASA-Langley facility. This system requires as input the latitude and longitude of a reference point and the coordinates of the corners of each rectangular section. Although SITE II allows for simple retrieval of 1970 census data, there is some question about its resolution capabilities for small grid sections. In addition, in rapidly growing areas the population data may lag actual population. The SITE II program is capable of producing detailed census information as shown in Figure 4. However for the present analysis only population information is used.
**SEVEN CORNERS**

**SALES TERRITORY SITE TOTAL**

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**1970 CENSUS DATA**

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<tr>
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<td>25-50</td>
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<td>1109 1.1%</td>
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<td>50+ 1109 1.1%</td>
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<tr>
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<td>17183 33.1%</td>
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<td>14380 27.7%</td>
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<tr>
<td>50+</td>
<td>6012 11.6%</td>
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<tr>
<td>15</td>
<td>71744 52.2%</td>
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<tr>
<td>147</td>
<td>44475 32.3%</td>
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<tr>
<td>61.5</td>
<td>7872 5.7%</td>
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<td>1304 0.9%</td>
<td>WASHER 71594 52.1%</td>
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<td>5510 4.0%</td>
<td>DRYER 54258 39.5%</td>
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<td>11609 6.6%</td>
<td>DISHMASH 56277 40.9%</td>
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<td>31569 23.0%</td>
<td>AIRCOND 79438 57.8%</td>
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<td>20288 14.7%</td>
<td>DEZER 26600 20.8%</td>
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<td>125 0.1%</td>
<td>MOBIL 2056 2.1%</td>
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<td>45881 11.9%</td>
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<table>
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<tr>
<th>CACI: INC</th>
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**Figure 4. Demographic Profile Report**
D. Flight Path Model

There are two ways in which the trajectory of the aircraft can be determined. In one, a discrete time integration of the equations of motion with control deflections yields point by point spatial coordinates and orientation. Although this allows the flexibility of building in control constraints as well as dynamical constraints (e.g. max roll angle) it requires a considerable number of states to be stored in the optimization routine. In a multi-aircraft, multi-runway problem, as is anticipated, these storage requirements become prohibitive.

Thus, another method was adopted which utilizes only the functional form of the trajectory to describe the flight path. First a starting path was assumed which went from the initial point to the desired runway and ended up with the proper heading, i.e., velocity vector aligned with the runway. The following equation was used to generate this starting trajectory. (See Figure 5).

\[
y_s(x) = \left[ \frac{y_f - y_p}{x_f - x_p} \right] (x - x_p) + (y_p - y_o) \exp \left[ -c(x - x_f)/(x_o - x_f) \right] + y_o
\]

(2)

For the vertical motion a simple three degree descent path was assumed.

Next the first five Fourier sine harmonics were used to introduce deviation from this starting path. One advantage in using this type representation is the fact that each of the forms contributes zero at the end points. Therefore if the starting path satisfies the boundary conditions the curve with the deviations will also. An exponential decay at the final point was used to eliminate heading deviations.
Figure 5. Rotated Coordinate System for Establishing Nominal Flight Trajectory from Initial Point to Runway Approach.
The equations with the deviations thus become

\begin{align}
    y(x) &= \left( \sum_{i=1}^{5} \alpha_i \sin[\pi i (x-x_0)/(x_f-x_0)] \right) \left\{ 1 - \exp[(x-x_f)/C_1] \right\} + y_s(x) \\
    Z(x) &= \left( \sum_{i=1}^{5} \beta_i \sin[\pi i (x-x_0)/(x_f-x_0)] \right) \left\{ 1 - \exp[(x-x_f)/C_1] \right\} + Z_s(x)
\end{align}

(3a) (3b)

A second advantage to using a Fourier Series representation for the curve is that it provides a means of representing a function with a finite number of parameters. This reduces the optimization problem from a variational one to an ordinary one.

E. Constraints

The use of a functional form of the flight path for the trajectory requires the reformulation of constraints into parameters which can be used in the optimization. This is accomplished by translating the steady state solutions of the lateral and longitudinal perturbation equations into geometric constraints. An exact derivation is given in Appendix A. The constraints are incorporated by determining maximum curvature and slope parameters as a function of aerodynamics and physical constraints. For example the constraint of a maximum roll angle, \( \phi_{\text{max}} \), yields

\[ \frac{d^2y}{dx^2} \leq \frac{C_2 + C_1 C_3}{C_4 + C_1 C_5} \frac{\phi_{\text{max}}}{V_{\text{avg}}} \]

(4)

where \( C_1 \) through \( C_5 \) depend upon aircraft stability and control derivations (see Appendix A for details) and \( V_{\text{avg}} \) is the average velocity. Similar expressions are given in Appendix A for constraints on aileron rudder
and elevator deflection, flight path angle and pitch rate limits.

F. Cost Function

A large number of criteria have been proposed by evaluating noise annoyance (e.g., EPNdB, NII, sleep interference index, speech interference index, etc.). The recent trend in noise assessment work is toward a universal measure—the noise impact index (NII). This measure is a weighted day-night model which accounts for population density. It is described in detail in Ref. 9. Briefly, the total population exposed to each incremental average day-night model sound level is multiplied by the weighting function for the level. The weighting function used in shown in Figure 6. This weighting factor \( W(L_{dn}) \) multiplied by the population exposed to that \( L_{dn} \) is summed and normalized by the total population giving the Noise Impact Index for the area.

\[
NII = \frac{\sum_{L_{dn}} P(L_{dn}) W(L_{dn})}{\sum_{L_{dn}} P(L_{dn})} \tag{5}
\]

The cost function or payoff for the optimization procedure is taken to be the NII plus penalties for violating constraints. Basically the optimization procedure is set up to "drive" the aircraft trajectory to the path which will minimize the NII and at the same time not violate any constraints. As an example of the constraint of flight path angle not exceeding a maximum descent angle, \( \gamma_d \), nor a maximum climb angle, \( \gamma_c \), is written as

\[
\tan \gamma_c < \frac{dZ}{dx} < \tan \gamma_d \tag{6}
\]
SOUND LEVEL WEIGHTING FUNCTION
FOR OVERALL IMPACT ANALYSIS

RECOMMENDED WEIGHTING FUNCTION

LINEAR WEIGHTING FUNCTION USED IN EARLY IMPACT ANALYSIS

Figure 6. Sound Level Weighting Function for Overall Impact Analysis.
Each is converted to a penalty which is added to the NII in the form

\[ \text{Cost} = \text{NII} + \left( \frac{dZ}{dx}/\tan \gamma_d \right)^2 + \left( \frac{\tan \gamma_c}{dZ/dx} \right)^2 \]  

(7)

As is seen for values of the flight path angle within the allowable range the penalty is negligible; however for values outside this range the penalty and thus the increase in cost is great. Other terms are added in a like manner.
III. OPTIMIZATION

The optimum trajectory is determined by calculating values of the $\alpha_i$'s and $\beta_i$'s (Eq. 2) which minimize the total cost (NII plus penalties). A steepest descent algorithm is employed here. Basically, this method computes the gradient of the cost function, $C$, with respect to the $\alpha_i$'s and $\beta_i$'s, then searches along the negative gradient direction for values of $\alpha_i$'s and $\beta_i$'s which reduce the cost. The change in cost is given by

$$\Delta C = \sum_{i=1}^{5} \left( \frac{\partial C}{\partial \alpha_i} \Delta \alpha_i + \frac{\partial C}{\partial \beta_i} \Delta \beta_i \right)$$

The process continues iteratively until the cost converges to within a specified tolerance. While implementation of the algorithm is fairly straightforward, convergence near the optimal set of $\alpha_i$'s and $\beta_i$'s is inherently slow. Most of the cost reduction, however, occurs in the first few iterations.

A. The Optimization Algorithm

A computer code has been developed which implements the functions described above. Figure 7 shows a flow chart for this code. Initial data (population map, aircraft constraints, initial and final aircraft positions, etc.) are required for each airport/airplane configuration to be evaluated. To facilitate calculation of the Fourier coefficients, the coordinate axes are rotated such that a line joining the initial and final aircraft positions is made to be parallel to the x axis. A nominal trajectory is generated which constrains the heading of the aircraft to asymptotically approach the runway. The steepest descent search then begins and continues until the stopping criterion is met. This criterion
READ POPULATION MAP, AIRCRAFT CONSTRAINTS
INPUT INITIAL CONDITIONS,
FINAL CONDITIONS,
FOURIER PARAMETERS = x's, β's

COORDINATE ROTATION =
ROTATE AXIS SUCH THAT LINE JOINING
INITIAL AND FINAL POINTS IS PARALLEL
TO THE x-AXIS IN ORDER TO APPLY
FOURIER SERIES

GENERATE A NOMINAL CURVE WHICH FORCES
THE HEADING OF AIRCRAFT AT FINAL POINT
TOWARD THE RUNWAY

STEEPEST DESCENT
GRADIENT SEARCH

NO

OPTIMAL REACHED?

YES

OPTIMAL LANDING TRAJECTORY

Figure 7. Flow Chart.
is met if successive improvements become negligible.

In order to provide a more accurate noise impact in each population section the impact is integrated using quadratures. This procedure can be found in Ref. 9.

The various functions such as the population model, the cost function, and the aircraft signature are incorporated as subroutines. This will allow ease of upgrading or modification if different models are desired.

Appendix B contains the Fortran code as written for a CDC Cyber 172 machine.

B. Results

Several cases have been run to test the benefits that can be obtained by this approach. First, a fictitious set of data incorporating a population valley is used. As can be seen in Figure 8 the optimization algorithm moves the aircraft (a Convair 880) towards the valley (i.e. fewer people impacted) with a corresponding improvement in the NII of 32%.

The second case models the Patrick Henry Airport in Hampton, Virginia. Here the SITE II program was used to generate the census data for each block as shown in Figure 9. Two initial trajectories were flown. One entering the area from the northwest over the Swing VOR station and the other from the southwest over the Franklin VOR station. Both of these paths are specified IFR trajectories (Figure 10). The aircraft enters the area approximately 30,000 meters from the runway. In addition several straight-in paths were evaluated. Figure 9 shows each of the trajectories. The associated
Figure 8. Optimization Results Using Fictitious Population Data.
Figure 9. Population Model and Optimization Results for Patrick Henry Airport.
Figure 10. Conventional Approach Pattern
Table I
Northwest Approach
Entry Point: Swing

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<tr>
<th>Traj. No.</th>
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<th>Cost (NII x 10^{-2})</th>
<th>% Change from Present</th>
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<td>60 deg wrt runway</td>
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<td>+3.2%</td>
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<td>2</td>
<td>30 deg wrt runway</td>
<td>2.438</td>
<td>+6.0%</td>
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<td>3</td>
<td>Initial iteration</td>
<td>2.27</td>
<td>-1.3%</td>
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<td>4</td>
<td>Optimal</td>
<td>2.213</td>
<td>-3.8%</td>
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<td>5</td>
<td>Straight in</td>
<td>2.316</td>
<td>+.1%</td>
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<tr>
<td>N1</td>
<td>Presently used</td>
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<td>0%</td>
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Table II
Southwest Approach
Entry Point: Franklin

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<th>Traj. No.</th>
<th>Description</th>
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<th>% Change From Present</th>
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<td>5</td>
<td>Straight in</td>
<td>2.316</td>
<td>-1.3%</td>
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<tr>
<td>6</td>
<td>Initial iteration</td>
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NII's are summarized in Tables I and II.

As is seen that even for this case, where the population is sparse away from the runway and congested near the end of it, an improvement of 3 to 5% is achieved using the optimization algorithm. It should also be noted that this technique allows not only the optimization of the path but also can be used to evaluate existing or proposed paths, such as the nominal and straight-in paths indicated.

Conclusion

A method has been formulated to optimize the path of an aircraft during approach or take-off from any airport. Models have been developed using available data where possible for population, aircraft signature, noise impact, constraints and flight path. An algorithm using steepest descent has been implemented and tested. This approach allows

1) The evaluation of the noise impact of existing flight paths,

2) The evaluation of the noise impact of proposed flight paths, and

3) The optimization of the flight path to minimize the noise impact under constraints.

This method has been applied to the Patrick Henry International Airport. Both nominal and other straight paths were evaluated. Also an optimal path was determined for each of two terminal area entry points. Performance ranged from a 15% degradation of the NII to 4.5% improvement compared to the presently used approaches. It is significant that as much as 3 to 5% improvement could be achieved in light of the fact that most of the population is concentrated at the end of the runway.
REFERENCES


APPENDIX A

Derivation of Parameterized Trajectory Constraints

Lateral perturbation equations

\[ Y \text{ eq'n: } - \frac{b}{2V_T} C_{y_p} \dot{\phi} - \frac{mg}{q_{\infty} S} \cos \phi \dot{\phi} + \left( \frac{mV_T}{q_{\infty} S} - \frac{b}{2V_T} C_{y_r} \right) \dot{\psi} - \frac{mg}{q_{\infty} S} \sin \theta \dot{\psi} \]
\[ + \frac{mV_T}{q_{\infty} S} \ddot{\beta} = C_{y_{\delta a}} \delta_a + C_{y_{\delta r}} \delta_r \]

\[ L \text{ eq'n: } \frac{I_{xz}}{q_{\infty} S b} \dot{\phi} - \frac{b}{2V_T} C_{x_p} \dot{\phi} - \frac{I_{xz}}{q_{\infty} S b} \ddot{\psi} - \frac{b}{2V_T} C_{x_r} \dot{\psi} - C_{x_{\delta a}} \delta_a + C_{x_{\delta r}} \delta_r \]

\[ N \text{ eq'n: } - \frac{I_{xz}}{q_{\infty} S b} \phi - \frac{b}{2V_T} C_{n_p} \dot{\phi} + \frac{I_{zz}}{q_{\infty} S \beta} - \frac{b}{2V_T} C_{n_r} \ddot{\psi} - C_{n_{\delta a}} \delta_a + C_{n_{\delta r}} \delta_r \]

If we assume all turns to be coordinated (no sideslip)

Then letting \( \frac{b}{2V_T} C_{y_p} = \overline{C}_{y_p}, \) etc.

\[ \frac{mg}{q_{\infty} S} \cos \theta = \overline{g}_1 \quad \frac{mg}{q_{\infty} S} \sin \theta = \overline{g}_2 \]

\[ \frac{I_{xx}}{q_{\infty} S b} = i_x, \text{ etc.} \quad \frac{mV_T}{q_{\infty} S} = \overline{m} \]

\[ L \text{ eq'n: } i_x \dot{\phi} - C_{x_p} \delta_a - i_x \overline{C}_{x_r} \gamma_{n_r} \dot{\gamma} = C_{x_{\delta a}} \delta_a + C_{x_{\delta r}} \delta_r \]

\[ N \text{ eq'n: } -i_x \overline{C}_{n_p} \dot{\phi} + i_{zz} \overline{C}_{n_r} \ddot{\gamma} = C_{n_{\delta a}} \delta_a + C_{n_{\delta r}} \delta_r \]

\[ Y \text{ eq'n: } -\overline{C}_{y_p} \dot{\gamma} + (\overline{m} - \overline{C}_{y_r}) \dot{\gamma} - \overline{g}_2 \dot{\psi} = C_{y_{\delta a}} \delta_a + C_{y_{\delta r}} \delta_r \]

Taking the Laplace transform (I.C.'s = 0)

\[ L \text{ eq'n: } (i_x s^2 - \overline{C}_{x_p} s) \phi (s) + (-i_x s^2 - \overline{C}_{x_r} s) \psi (s) = C_{x_{\delta a}} \delta_a (s) + C_{x_{\delta r}} \delta_r (s) \]
N eqn: \(-ixz^2-C_{np}\phi(s) + (izs^2-C_{nr}s)\psi(s) = c_n\delta_a(s) + c_n\delta_r\delta_r(s)\)

Y eqn: \((-C_{yp}s-g_1)\phi(s) + [(m-C_{yp})s-g_2]\psi(s) = c_y\delta_a(s) + c_y\delta_r\delta_r(s)\) 

To determine the required \(\delta_a\) for a given \(\delta_r\) we consider \(\delta_a\) an unknown along with \(\phi(s)\) and \(\psi(s)\) [i.e. move \(\delta_a\) to the left hand side of the equations] and solve for \(\delta_a/\delta_r\) using Cramer's rule

\[
\frac{\delta_a}{\delta_r} = \frac{\begin{vmatrix}
ixs^2-C_{lp}s & -ixz^2-C_{lr}s & +C_{\delta_r}
-iZs^2-C_{np}s & izs^2-C_{nr}s & +c_n\delta_r
-C_{yp}s-g_1 & (m-C_{yp})s-g_2 & +C_{\delta_r}
\end{vmatrix}}{\begin{vmatrix}
ixs^2-C_{lp}s & -ixz^2-C_{lr}s & -C_{\delta_a}
-iZs^2-C_{np}s & izs^2-C_{nr}s & -c_n\delta_a
-C_{yp}s-g_1 & (m-C_{yp})s-g_2 & -C_{\delta_a}
\end{vmatrix}}
\]

The denominator (characteristic eqn.) is given by:

\[
\Delta(s) = s^4(-c_y\delta_a(iZ-iZ^2)) + s^3(c_y\delta_a[iZ-C_{lp}+iZ-C_{np}+(m-C_{yp})C_{lr}+C_{np}])
\]

\[
+ c_{n}\delta_a[-ixZC_{yp}+(m-C_{yp})ix]+C_{\delta_a}[iXZC_{yp}-(m-C_{yp})ixZ]
\]

\[
+ s^2(c_y\delta_a(C_{np}C_{lr}-C_{np}C_{nr}) + c_{n}\delta_a[-ixZ\overline{g_1}-ixZ\overline{g_2}-C_{yp}\overline{g_r}-(m-C_{yp})\overline{C_{lr}}])
\]

\[
+ c_{\delta_a}[(\overline{g_1}iZ-\overline{g_2}iXZ-C_{yp}\overline{C_{lr}}+(m-C_{yp})\overline{C_{lr}})]
\]

\[
+ s(c_n\delta_a(\overline{g_2}C_{lp}\overline{g_1}C_{lr}) + C_{\delta_a}(\overline{g_2}C_{np}\overline{g_1}C_{nr}))
\]

(5)
The numerator is:

\[ N(s) = s^4\{C_y\delta \dot{r} \left(1r_\delta z - i_\delta y \right) + s^3\{C_y \delta \left[i_\delta z C_p \dot{r} + i_\delta x C_n + i_\delta x (C_r + C_p)\right]
\]

\[ -C_n \delta_r \left[-i_\delta x C_y + (m - C_{yr}) \dot{r}\right] - C_\theta \delta_r \left[i_\delta x C_y - (m - C_{yr}) i_\delta x\right]\]

\[ + s^2\{C_y \delta_r (C_p C_r - C_p C_n) + C_n \delta_a \left[-i_\delta x \dagger - i_\delta x \dagger C_{y} C_{z} (m - C_{yr}) C_p\right]\]

\[ - C_{\theta} \delta_r \left[\dagger i_\delta x - i_\delta x \dagger i_\delta y C_{y} C_{n} + (m - C_{yr}) C_{n}\right]\]

\[ + s\{-C_n \delta_r (C_{\theta} \dagger C_p - C_{\theta} \dagger C_{x}) - C_{\theta} \delta_r (C_{\theta} \dagger C_{y} - C_{\theta} \dagger C_{n})\} \quad (6) \]

Now assuming that only the steady state (st. st.) condition is of interest,

\[ \lim_{s \to 0} \frac{N(s)}{\Delta(s)} = \frac{\delta_a}{\delta_r} \text{ st. st.} \]

we get

\[ \frac{\delta_a}{\delta_r} \text{ st. st.} = \frac{-C_n \delta_r (C_{\theta} \dagger C_p - C_{\theta} \dagger C_{x}) - C_{\theta} \delta_r (C_{\theta} \dagger C_{y} - C_{\theta} \dagger C_{n})}{C_n \delta_a (C_{\theta} \dagger C_p - C_{\theta} \dagger C_{x}) + C_{\theta} \delta_a (C_{\theta} \dagger C_{y} - C_{\theta} \dagger C_{n})} \quad (7) \]

\[ \cos \theta_0 (C_n \delta_r C_{\theta} + C_{\theta} \delta_r C_n) - \sin \theta_0 (C_n \delta_r C_{\theta} + C_{\theta} \delta_r C_n) \]

\[ \frac{\delta_a}{\delta_r} \text{ st. sc.} = \frac{-\cos \theta_0 (C_n \delta_r C_{\theta} + C_{\theta} \delta_r C_n) + \sin \theta_0 (C_n \delta_r C_{\theta} + C_{\theta} \delta_r C_n)}{\cos \theta_0 (C_n \delta_r C_{\theta} + C_{\theta} \delta_r C_n) + \sin \theta_0 (C_n \delta_r C_{\theta} + C_{\theta} \delta_r C_n)} \quad (8) \]

For small initial flight path angle (i.e. \( \theta_0 \approx 0 \))

\[ \frac{\delta_a}{\delta_r} \text{ st. st.} = -\frac{C_n \delta_r C_{\theta} + C_{\theta} \delta_r C_n}{C_n \delta_r C_{\theta} + C_{\theta} \delta_r C_n} = C_1 \quad (9) \]

Assuming \( \theta_0 = 0 \) to simplify we can write the transfer functions for \( \phi \) and \( \dot{\psi} \) as (in the st. st.)
\[ \frac{\dot{\psi}}{\delta_r} = \frac{c_{2\delta_r} c_{n_\beta} - c_{n_\delta_r} c_{\beta}}{c_{\beta_\beta} c_{n_r} - c_{n_\beta} c_{\beta_r}} = c_2 \] (10)

\[ \frac{\dot{\psi}}{\delta_a} = \frac{c_{\phi_\delta_a} c_{n_\beta} - c_{n_\phi} c_{\beta}}{c_{\beta_\beta} c_{n_r} - c_{n_\beta} c_{\beta_r}} = c_3 \] (11)

\[ \frac{\phi}{\delta_r} = \frac{c_{\phi_\delta_r} (c_{\beta_r} c_{n_\beta} - c_{\beta_\beta} c_{n_r}) + c_{\phi_\delta_r} (c_{\beta_a} c_{n_\beta} + c_{n_\phi} (m - c_{\gamma_r}) + c_{n_\delta_r} (c_{\beta_\beta} (m - c_{\gamma_r}) + c_{\beta_\gamma} c_{\gamma_r})}{mg} \frac{q_{\infty}}{s} \left( c_{\beta_\beta} c_{n_r} - c_{n_\beta} c_{\beta_r} \right) = c_4 \] (12)

\[ \frac{\phi}{\delta_a} = \frac{c_{\phi_\delta_a} (c_{\beta_r} c_{n_\beta} - c_{\beta_\beta} c_{n_r}) + c_{\phi_\delta_a} (c_{\beta_a} c_{n_\beta} + c_{n_\phi} (m - c_{\gamma_r}) + c_{n_\delta_a} (c_{\beta_\beta} (m - c_{\gamma_r}) + c_{\beta_\gamma} c_{\gamma_r})}{mg} \frac{q_{\infty}}{s} \left( c_{\beta_\beta} c_{n_r} - c_{n_\beta} c_{\beta_r} \right) = c_5 \] (13)

Consider the aircraft trajectory shown
The slope at any point is \( \frac{dy}{dx} \) and the angle the slope makes with the x axis is \( \tan^{-1}\left(\frac{dy}{dx}\right) \).

The angular rate \( \psi \) is then \( \frac{d}{dt}\tan^{-1}\left(\frac{dy}{dx}\right) \)

or \( \frac{3}{dx} \left\{\tan^{-1}\frac{dy}{dx}\right\} \frac{dx}{dt} = V_{avg} \frac{3}{dx} \left\{\tan^{-1}\left(\frac{dy}{dx}\right)\right\} \)

Then \( \psi = V_{avg} \frac{\frac{d^2 y}{dx^2}}{1 + \left(\frac{dy}{dx}\right)^2} = V_{avg} \left\{\frac{f''(x)}{1 + f''[f'(x)]^2}\right\} \) \hfill (14)

If we know \( \delta_r \) we can determine \( \delta_a \) from \( \delta_a = C_1\delta_r \)

Also \( \dot{\psi} = C_2\delta_r + C_3\delta_a = (C_2+C_1+C_3)\delta_r \) \hfill (15)

We can also write

\[ \phi = C_4\delta_r + C_5\delta_a = (C_4+C_1C_5)\delta_r \] \hfill (16)

Constraining \( \delta_a \) to be \( \leq \delta_{a_{\text{max}}} \), \hfill (17)

\( \delta_r \) to be \( \leq \delta_{r_{\text{max}}} \), \hfill (18)

and \( \phi \) to be \( \leq \phi_{\text{max}} \) \( (= \text{max bank angle}) \) \hfill (19)

we get the following expressions

\[ \delta r_1 \leq \frac{\phi_{\text{max}}}{C_4+C_1C_5} \] \hfill (20)

\[ \delta r_2 \leq \delta_{r_{\text{max}}} \] \hfill (21)

\[ \delta r_3 \leq \frac{\delta_{a_{\text{max}}}}{C_1} \] \hfill (22)
The constraining value is given by

$$\delta r_{\text{max}} = \min(\delta r_1, \delta r_2, \delta r_3)$$  \hspace{1cm} (23)

which yields

$$\dot{\gamma}_{\text{max}} = (C_2+C_1C_3) \min(\delta r_1, \delta r_2, \delta r_3)$$  \hspace{1cm} (24)

This condition incorporates all three constraints ((17)-(19)) as

$$\frac{d^2\gamma}{dx^2} = \frac{f''(x)}{1+f'(x)^2} \leq \frac{(C_2+C_1C_3)}{\bar{V}_{\text{avg}}} \min(\delta r_1, \delta r_2, \delta r_3)$$

Longitudinally we wish to constrain the behavior of the trajectory so that we restrict \(\gamma\) (the flight path angle) and \(\theta\) (the pitching rate).

The trajectory is given by

\[
\begin{align*}
\begin{array}{c}
\text{x} \\
\text{z} \\
\text{Z} = g(x)
\end{array}
\end{align*}
\]

Then, assuming the aircraft center of mass follows this trajectory \(\gamma\) is given by

$$\gamma = \tan^{-1} \frac{dz}{dx}$$

or

$$\frac{dz}{dx} = \tan \gamma$$

We wish to constrain \(\gamma\) to a maximum descent angle, \(\gamma_{d_{\text{max}}}\) and a maximum angle, \(\gamma_{c_{\text{max}}}\).

Thus

$$\tan \gamma_{c_{\text{max}}} \leq \frac{dz}{dx} \leq \tan \gamma_{d_{\text{max}}}$$
PROGRAM NOISE

COMMON ALFA(5), BETA(5), POS(5,5), ARRAY(57,9), NMAP
COMMON /CURVE/ YCURVE(51), ADY(51), ADDY(51)
COMMON /LABEL/ LINE(4), LLOC(5)
COMMON /XPORT/ XPORT, YPORT, ZPORT
COMMON /SCALE/ XMIN, XINC, YMIN, YINC

INTEGER COUNT, HALF

DIMENSION ALFA0D(5), BETA0D(5), GY(5), GZ(5), DALFA(5), DBETA(5)
DIMENSION AGY(5), AGZ(5)

COMMON ICURVE, YCURVE, ADY, ADDY, ARRAY, ALFA, BETA, POS
COMMON IA11, A12, XPORT, YPORT, ZPORT
COMMON ISCALE, XMIN, XINC, YMIN, YINC

INTEGER CNT, HALF

DIMENSION ALFA0D, BETA0D, GY, GZ, DALFA, DBETA
DIMENSION AGY, AGZ

C

C COORDINATE ROTATION

THETA = ATAN2(YF-YO)/(XF-XO)
A = (XF*COS(THETA)+YF*SIN(THETA))-(XO*COS(THETA)+YO*SIN(THETA))
PHI = ATAN2((ZF-ZO),(A))
XOCAP = XO*COS(THETA)+YO*COS(PHI)+Z0*SIN(PHI)
XFACAP = XF*COS(THETA)*COS(PHI)+YF*SIN(THETA)*COS(PHI)+ZF*SIN(PHI)
YOCAP = Y0*COS(THETA)+Y0*COS(PHI)+Z0*SIN(PHI)
YFCAP = YF*COS(THETA)+YF*SIN(THETA)*COS(PHI)+ZF*SIN(PHI)
ZFCAP = ZF*COS(THETA)+ZF*SIN(THETA)*COS(PHI)+YF*SIN(PHI)
ZPORT = -XPORT*COS(THETA)-YPORT*COS(THETA)

C COORDINATE ROTATION

THETA = ATAN2(YF-YO)/(XF-XO)
A = (XF*COS(THETA)+YF*SIN(THETA))-(XO*COS(THETA)+YO*SIN(THETA))
PHI = ATAN2((ZF-ZO),(A))
XOCAP = XO*COS(THETA)+YO*COS(PHI)+Z0*SIN(PHI)
XFACAP = XF*COS(THETA)*COS(PHI)+YF*SIN(THETA)*COS(PHI)+ZF*SIN(PHI)
YOCAP = Y0*COS(THETA)+Y0*COS(PHI)+Z0*SIN(PHI)
YFCAP = YF*COS(THETA)+YF*SIN(THETA)*COS(PHI)+ZF*SIN(PHI)
ZFCAP = ZF*COS(THETA)+ZF*SIN(THETA)*COS(PHI)+YF*SIN(PHI)
ZPORT = -XPORT*COS(THETA)-YPORT*COS(THETA)

C
C  *  START OPTIMIZATION
C  *
60  C  
INDEX = 0
DLXCAP = (XFCAP-XCAP)/50.
C  
65  C  
C  FIRST FIND A CURVE WHICH FORCES THE HEADING OF THE
C  AIRCRAFT TOWARD THE RUNWAY AT THE FINAL POINT
C  
70  C  
SLOPE = (YFCAP-YPORT)/(XFCAP-XPORT)
YCURVE(1) = YOCAP
ADY(1) = 0.
ADDY(1) = 0.
XCAP = XOCAP
DO 10 I = 1,50
XCAP = XCAP+DLXCAP
EXP0 = -5.5*(XCAP-XFCAP)/(XOCAP-XFCAP)
YCURVE(I+1) = (SLOPE*(XCAP-XPORT)+YPORT-YOCAP)*EXP(EXP0)+YOCAP
ADY(I+1) = -5./(XOCAP-XFCAP)*YCURVE(I+1)-YOCAP+(SLOPE)*EXP(EXP0)
1 0
ADDY(I+1) = (-5./(XOCAP-XFCAP)**2)*(YCURVE(I+1)-YOCAP)+(-5./IX
QCAP-XFCAP))*SLOPE*EXP(EXP0)*2.0
10  CONTINUE
COUNT = 0.
C  
90  C  INITIAL COST
95  C  
XMIN = -40000
XINC = 2500
YMIN = -40000
YINC = 2500
H = 0.07
CALL COST (0.1,XCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA PHI,TOTAL,PNAL
110  C  
CALL COST (0.1,XCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA PHI,TOTAL,PNAL
111  C  
COST1 = TOTAL
A = COST1+PNALTY
WRITE (6,9150) COUNT,COST1,A,PNALTY
915  C  
WRITE (6,9220)
WRITE (6,9230) (POSIT(I,J)+J=1,3) I=1,51
WRITE (6,9250) (POSIT(I,J)+J=1,3) I=1,51
925  C  
923  C  
921  C  
920  C  
910  C  
91  C  
90  C  
85  C  
80  C  
75  C  
70  C  
65  C  
60  C  
55  C  
50  C  
45  C  
40  C  
35  C  
30  C  
25  C  
20  C  
15  C  
10  C  
5  C  
0  C  

C 50 DO 60 I = 1,5
  ALFA(I) = ALFA(I) + DALFA(I)
  CALL COST (I, XOCAP, YOCAP, ZOCAP, XFCAP, DELXCAP, THEETA, PHI, TOTAL, P
C 60   )
C 125  1  ALTY)
  COST2 = TOTAL
  GY(I) = (COST2-COST1)/ABS(ALFA(I))
  IF (INDEX.EQ.0) AGY(I) = GY(I)
  IF (INDEX.EQ.1) BGY(I) = GY(I)
  WRITE (*,9160) I, GY(I)
C 130  60 ALFA(I) = ALFA(I)-DALFA(I)
C 140  DO 70 I = 1,5
  BETA(I) = BETA(I) + DBETA(I)
  CALL COST (I, XOCAP, YOCAP, ZOCAP, XFCAP, DELXCAP, THEETA, PHI, TOTAL, P
C 70   )
C 150  1  ALTY)
  COST2 = TOTAL
  GZ(I) = (COST2-COST1)/ABS(DBETA(I))
  IF (INDEX.EQ.0) AGZ(I) = GZ(I)
  IF (INDEX.EQ.1) BGZ(I) = GZ(I)
  WRITE (*,9170) I, GZ(I)
C 160  70 BETA(I) = BETA(I)-DBETA(I)
C 170  IF INDEX.EQ.1) GO TO 190
C 180  GYMAX = ABS(GY(I))
  GZMAX = ABS(GZ(I))
  DO 80 I = 2,5
    IF (GYMAX.LT.ABS(GY(I))) GYMAX = ABS(GY(I))
    IF (GZMAX.LT.ABS(GZ(I))) GZMAX = ABS(GZ(I))
C 190  80 GYMAX = ABS(GY(I))
  GZMAX = ABS(GZ(I))
  INDEX = INDEX + 1
  IF INDEX.EQ.5) GO TO 100
  CALL COST (1, XOCAP, YOCAP, ZOCAP, XFCAP, DELXCAP, THEETA, PHI, TOTAL, P
C 200   )
C 210  90 ALFA(I) = ALFA(I) + YRATIO*GY(I)
  ALFA(I) = ALFA(I) - YRATIO*GY(I)
C 220  BETA(I) = BETA(I) + ZRATIO*GZ(I)
  BETA(I) = BETA(I) - ZRATIO*GZ(I)
C 230  CALL COST (1, XOCAP, YOCAP, ZOCAP, XFCAP, DELXCAP, THEETA, PHI, TOTAL, P
C 240   )
C 250  IF COST2.LT.COST1) GO TO 150
C 260  100 PRCT = (ABS(COST2-COST1))/COST1

C STOP CRITERION -- PERCENTAGE CHANGE IN COST INSIGNIFICANT
C
IF (PRCCNT.GE.1.E-5) GO TO 110
COUNT = COUNT+1
CALL COST (0.1,XOCAP,YOCAP,ZOCAP,XFCAP,DLCAP,THETA,PHI,TOTAL,PNAL
ITY)
WRITE (6,9180) COUNT
CALL MONIT (COUNT,COST1,PNALTY)
STOP
110 CALL COST (0.1,XOCAP,YOCAP,ZOCAP,XFCAP,DLCAP,THETA,PHI,TOTAL,PNAL
ITY)
COST1 = TOTAL
COUNT = COUNT+1
A = COST1-PNALTY
WRITE (6,9150) COUNT,COST1,A,PNALTY
WRITE (6,9220)
WRITE (6,9230) ((POS8(I,J)+J=1,3),J=1,51)
DO 120 I = 1,5
STOP CRITERION -- ALL GRADIENT COMPONENTS EQUAL TO ZERO
C
IF (GY(I).NE.0.) GO TO 130
IF (GZ(I).NE.0.) GO TO 130
CONTINUE
205 WRITE (6,9060) COUNT
CALL MONIT (COUNT,COST1,PNALTY)
STOP
130 WRITE (6,9070)
DO 140 I = 1,5
WRITE (6,9190) I,ALFA(I),BETA(I)
CONTINUE
COST2 = TOTAL
C
STOP CRITERION -- MAXIMUM NUMBER OF ITERATIONS REACHED
C
IF (COUNT.LT.MAXIT) GO TO 30
WRITE (6,9200)
CALL MONIT (COUNT,COST1,PNALTY)
STOP
150 HALF = 1
C REDUCE SIZE OF STEP CHANGE BY HALF
C  IF COST HAS NOT DECREASED

C

230  DO 170 J = 1,3

C

235  DO 160 I = 15

C

160  ALFA(I) = (ALFA(I) + ALFADD(I)) / 2.

C

HALF = J

C

170  WRITE (6,9210) HALF

C

240  CALL COST (0.0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA PHI,TOTAL,PFN)

C

170  CONTINUE

C

245  INEX = 1

C

180  DALFA(I) = -DALFA(I)

C

250  DBETA(I) = -DBETA(I)

C

255  GO TO 50

C

190  DO 200 I = 15

C

260  IF (AGY(I).LT.0.) GO TO 220

C

200  CONTINUE

C

265  ALFA(I) = ALFADD(I)

C

210  BETAD(I) = BETADD(I)

C

220  CALL COST (0.0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA PHI,TOTAL,PNAL)

C

270  CALL MONIT (COUNT,COST1,PNALTY)

C

STOP

C

225  BGYMAX = ABS(AGY(I))

C

230  BGZMAX = ABS(AGZ(I))

C

275  DO 240 I = 25

C

240  WRITE (6,9210) HALF

C

280  CHECK EACH GRADIENT COMPONENT TO DETERMINE SIZE

C

285  DO 320 I = 15
IF (HALF.EQ.7) GO TO 250
IF (GYMAX.NE.0.) AY = YALLOW/GYMAX/FLOATHALF-3*AY(I)
IF (GYMAX.NE.0.) BY = YALLOW/GYMAX/FLOATHALF-3*BY(I)
IF (GZMAX.NE.0.) AZ = ZALLOW/GZMAX/FLOATHALF-3*AZ(I)
IF (GZMAX.NE.0.) BZ = ZALLOW/GZMAX/FLOATHALF-3*BZ(I)

GO TO 260

AY = -OALFA(I)
BY = OALFA(I)
AZ = -OBETA(I)
BZ = OBETA(I)

IF (AGY(I).LE.0.) GO TO 270
IF (AGY(I).LT.0.) ALFA(I) = ALFAO(I)
IF (AGY(I).LT.0.) BETA(I) = BETAO(I)
GO TO 290

IF (AGY(I).LE.0.) GO TO 290
IF (AGY(I).LT.0.) ALFA(I) = ALFAO(I)+BY
IF (AGY(I).LT.0.) BETA(I) = BETAO(I)+BZ
GO TO 320

IF (AGZ(I).LE.0.) GO TO 310
IF (AGZ(I).LT.0.) ALFA(I) = ALFAO(I)+AZ
IF (AGZ(I).LT.0.) BETA(I) = BETAO(I)+AZ
GO TO 340

BETA(I) = BETA(I)-AZ

CALL COST (0+0,XOCAP,YOCAP,ZCAP,XFCAP,YFCAP,THETA,PHI,TOTAL,PNAL)
COST2 = TOTAL
IF (COST2.LT.COST1) GO TO 120
HALF = HALF+1
WRITE (6,9210) HALF
GYMIN = AGY(I)
J = 1
DO 310 I = 2,5
IF (GYMIN.LE.AGY(I)) GYMIN = AGY(I)
J = I
GO TO 310

K = 1
DO 340 I = 2.5
IF (GYMIN.LE.AGZ(I)) GYMIN = AGZ(I)
K = I
GO TO 340

CONTINUE

DO 360 I = 1.5
IF (GYMIN.LE.BGY(I)) GYMIN = BGY(I)
J = I

CONTINUE
350 IF (GZMIN.LE.BGZ(I)) GO TO 360
  GZMIN = BGZ(I)
  K = I+5

360 CONTINUE
  IF (NOT (GMIN.LT.0.0), OR, (GMIN.GE.0.0)) Go To 370
  CALL COST (X,Y,Z,YCAP,TCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL)

350 CONTINUE
  WRITE (6,9090) COUNT

370 GO TO 380 I = 1+5

355 IF IIGYMIN.LE.BGZ(I) GO TO 360
  GZMIN = BGZ(I)
  K = I+5

360 CONTINUE
  IF IIGYMIN.LT.O.O, OR, IGZMIN.LT.O.O) GO TO 370
  CALL COST (X,Y,Z,YCAP,TCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL)

370 IF IIGYMIN.LT.O.O, AND, IGZMIN.GE.O.O) GO TO 390
  IF IIGYMIN.LT.O.O, AND, IGZMIN.LT.O.O) GO TO 310
  IF IIGYMIN.LT.O.O, AND, IGZMIN.LE.O.O) GO TO 310

390 IF IIGYMIN.LT.O.O, AND, IGZMIN.GE.O.O) GO TO 390
  IF IIGYMIN.LT.O.O, AND, IGZMIN.LE.O.O) GO TO 390
  CALL COST (X,Y,Z,YCAP,TCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL)

370 CALL COST (X,Y,Z,YCAP,TCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL)

370 IF ICOST2.LT.COST1) GO TO 310

380 CALL I, J = 0

375 CALL COST (X,Y,Z,YCAP,TCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL)

375 GO TO 100

9010 FORMAT (5X,14MINITAL X,Y,Z: 13F12.2,3X),7H METERS: 1X,5X,13FINAL X

9020 FORMAT (5X,14MINITAL X,Y,Z: 13F12.2,3X),7H METERS: 1X,5X,13FINAL X

9030 FORMAT (1X,14HALFA,16X,HNBETA)

9040 FORMAT (10X,11PE16.9,4X,1PE16.9)

9050 FORMAT (10X,11PE16.9,4X,1PE16.9)

9060 FORMAT (10X,11PE16.9,4X,1PE16.9)

9070 FORMAT (10X,11PE16.9,4X,1PE16.9)

9080 FORMAT (10X,11PE16.9,4X,1PE16.9)

9090 FORMAT (10X,11PE16.9,4X,1PE16.9)

9100 FORMAT (10X,11PE16.9,4X,1PE16.9)

9110 FORMAT (10X,11PE16.9,4X,1PE16.9)

9120 FORMAT (10X,11PE16.9,4X,1PE16.9)

9130 FORMAT (10X,11PE16.9,4X,1PE16.9)
| 400 | 9140 FORMAT (5X,22HINITIAL ALFA AND BETAI++,/13X,4HALFA+16X,4HBETA+5/+/1) A 4000 | 400 |
| 405 | 9150 FORMAT (/*/1X,10HITRATION */1X,14HTOTAL COST IS ,1PE16.9*/ | 405 |
| 410 | 1X,22HTRUE ANNOYANCE(I) IS ,1PE16.9*/ | 410 |
| 415 | A 4040 | 415 |
| 420 | 9160 FORMAT (10X,12XH Y-GRADIENT IS ,1PE16.9) | 420 |
| 425 | 9170 FORMAT (10X,12XH Z-GRADIENT IS ,1PE16.9) | 425 |
| 430 | 9180 FORMAT (/*/1X,13HAT Iteration */12,24HPERCENTAGE CHANGE IN CO-J3 | 430 |
| 435 | A 4070 | 435 |
| 440 | 9190 FORMAT (10X,11X2X,1PE16.9,4X,1PE16.9) | 440 |
| 445 | 9200 FORMAT (10X,41HRREACH MAXIMUM ITERATION SET, PROGRAM STOP) A 4100 | 445 |
| 450 | 9210 FORMAT (10X,7HHALF = */12) A 4110 | 450 |
| 455 | 9220 FORMAT (10X,10HTRAJECTORY/*/10X,12HCOORDINATE);8X,12HCOORDINATE | 455 |
| 460 | 1X,8X,12HCOORDINATE/*/12X,7H(METER);13X,7H(METER);13X,7H(METER)) A 4130 | 460 |
| 465 | 9230 FORMAT (10X,31PE16.9,4X)) | 465 |
| 470 | END | 470 |

END
SUBROUTINE COST (ISGRAD,WRITE,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA)  
I = I+TOTAL-PNALTY)  
I = I+TOTAL-PNALTY)  
COMMON ALFA(I),BETA(I),POSIT(I),ARRAY(IS,19),NMAP  
COMMON CURVE(SET),ADD(SET),ADDY(SET)  
COMMON AIRPORT, XPORT, YPORT, ZPORT  
COMMON AC, XY, Z  
EXTERNAL FCN  
PNALTY = 0.  
XCAP = XOCAP  
PI = ATAN(I)  
C2 = PI/ABS(XFCAP-XCAP)  
C3 = ABS(XFCAP-XCAP)/PI  
DO 10 I = 1,NMAP  
ARRAY(I,1) = 0.  
10 ARRAY(I,2) = 0.  
* MULTIPY BY EXPONENTIAL TERM SUCH THAT THE FINAL  
* HEADING OF AIRCRAFT IS TOWARD THE RUNWAY  
DO 50 I = 1,N  
Y2 = 1.*EXP(-XFCAP-XCAP)/C3  
Y5 = (Y2-1.)/C3  
Y9 = 0.0  
Y6 = Y9  
Y7 = Y6  
Y3 = Y6  
* GENERATE SINE HARMONICS  
DO 20 J = 1,5  
TRIGO = FLOAT(J)*(XFCAP-XCAP)*C2  
Y3 = Y3+ALFA(J)*SIN(TRIGO)  
Y8 = Y8+BETA(J)*SIN(TRIGO)  
Y6 = Y6+FLOAT(J)*C2*ALFA(J)*COS(TRIGO)  
Y7 = Y7+FLOAT(J)*C2+BETA(J)*COS(TRIGO)  
DLYCAP = Y2*Y3  
DLZCAP = Y2*Y6  
ZCAP = ZOCAP+DLZCAP  
YCAP = DLYCAP+YCURVE(I)  
QY = Y2+Y6+Y3+Y5  
* AIRCRAFT CONSTRAINTS  
*
SUBROUTINE COST 73/172 TS

DOY = DY + ADY(I)

60

DDY = Y2*Y12 + Y5*Y6 + Y3*Y5/C3

60

DDY = DDD + ADY(I)

60

DDY = DDD/(1 + DOY**2)

DO = Y2*Y9 + Y6

60

DO = DZ + TAN(PHI)

60

DO = 0.

65

PNALTY = PNALTY + (DDY - 0.01)**(20) + (DOY - 1.14)**(20)

1

PHI

1

Y = XCAP*CSN(PHI) + YCAP*COS(PHI) - ZCAP*CSN(PHI) + SIN(PHI)

70

Z = XCAP*CSN(PHI) + ZCAP*CSN(PHI)

70

DO 40 K = 1, NMAP

40

RANGE = ((X - ARRAY(K,1))**2 + (Y - ARRAY(K,2))**2 + Z**2)**.5

40

DB = 15.22.5*ALOS10 (3.281*RANGE/500,1)

40

IF (DB LE ARRAY(K,1)) GO TO 40

40

ARRAY(K,4) = X

40

IF (ARRAY(K,4) LT 55.) GO TO 40

40

IF (ARRAY(K,3) EQ 0.) GO TO 30

30

ARRAY(K,5) = TEMP*SMALLP

30

CALL GAUSS (ARRAY(K,6), ARRAY(K,7), ARRAY(K,8), ARRAY(K,9), FC4+1E

30

MPI

1

1

ARRAY(K,5) = TEMP*SMALLP

1

GO TO 40

40

CONTINUE

40

IF (WRITE.EQ.0) GO TO 50

50

II = I

50

POSU(I1+1) = X

50

POSU(I1+2) = Y

50

POSU(I1+3) = Z

50

XCAP = XCAP + DLXCAP

50

PEOPLE = 0.

100

DO 60 K = 1, NMAP

60

IF (ARRAY(K,5) EQ 0.) GO TO 60

PEOPLE = ARRAY(K,5)*PEOPLE

60

CONTINUE

60

FX = 0.

60

DO 70 K = 1, NMAP

70

ARRAY(K,5) = ARRAY(K,5)/PEOPLE

70

FX = FX + ARRAY(K,5)

70
SUBROUTINE COST 73/172 TS

115 CONTINUE
TOTAL = FX+PNALTY
RETURN
END

41000B CM STORAGE USED .074 SECONDS
SUBROUTINE MONIT (IA, AA, BB)
COMMON ALFA(5), BETA(5), POSIT(5), ARRAY(5), NMAP
COMMON /SCALE/ XM, YM, XINC, YINC
DIMENSION PCRT(10)
DIMENSION XM(1026), YM(1026)
DIMENSION X(5), Y(5), ZP(5), NA(5), NB(3)
EQUIVALENT (XM(1), ARRAY(1), Y(1))
EQUIVALENT (X(1), POSIT(1), Y(1), POSIT(1), ZP(1), POSIT(1))

DATA NB/AUTHORY, POPULATION ANN.9HOYANCE = /

C DOCUMENTATION

Cc = AA - BB
WRITE (6, 9010) IA, AA, CC, BB
DO 10 I = 1, 51
  WRITE (6, 9030) (POSIT(I, J) + J = 1, 3)
  CONTINUE
10
WRITE (6, 9040)
DO 20 I = 1, NMAP
  WRITE (6, 9050) (ARRAY(I, J) + J = 1, 5)
  CONTINUE
20
WRITE (97, 9060) (POSIT(I, J) + J = 1, 31 + I = 1, 51)
WRITE (97, 9070) (ARRAY(I, J) + J = 1, NMAP)
RETURN

9010 FORMAT (10X, 5S, HOPTIMUM TRAJECTORY FOR LANDING AT PATRICK HENRY AIR
1PORT.
5X, 5S, MAX NOISE BELOW 55 EPNDB IS CONSIDERED NOT NOISY, ANN
5X, 5S, NANCE = 0/10X, 5S, UNIT FOR NOISE IS EPMS/10X, 5S, UNIT FOR COORDI
5X, 5S, NATES.
35
9020 FORMAT (10X, 5S, MAT ITERATION +12/15X, 1H, TOTAL COST = , 1PE16, 9/,
115X, 15S, TRUE ANNOYANCE(INI) IS , 1PE16, 9/15X, 7S, PENALTY IS , 1PE16,
5X, 15S, H, COORDINATE , 5X, 15S, H, COORDINATE , 5X, 15S, H, COORDINATE
40
9030 FORMAT (10X, 5S, H, INDEX, 5X, 15S, NOISE-ANNOYANCE CHART ,/10X, 15S, POPUL
5X, 15S, INDEX, 5X, 15S, NOISE LEVEL, 4X, 9HANN
45
9040 FORMAT (10X, 30E12.6)
9050 FORMAT (10X, 15S)
9060 FORMAT (3E12.6)
9070 FORMAT (5E12.6)

END

41000B CM STORAGE USED .284 SECONDS
SUBROUTINE GAUSS

5  COMMON /AC/ XA, YA, ZA
DIMENSION X(5), Y(5), F(5), XI(5), WI(5)
DATA XI, WI, N/-0.577350269, 0.577350269, 0, 0, 1, -0.577350269, 0.577350269, 0, 0, 1, 0, 0, 0, 2/

C
C GAUSSIAN QUADRATURE INTEGRATION WITH FOUR POINTS
C

00 10 I = 1, N
10 Y(I) = (YX-YN)/2.*XI(I)+Y(I)+YN)/2.
20 X(I) = (XX+XN)/2.*XI(I)+(XX+XN)/2.
30 FINT = 0.
40 DO 10 J = 1, N
50 F(J) = 0.
60 DO 20 I = 1, N
70 F(J) = F(J)*WI(I)*F(I)*Y(I))
80 FINT = FINT+WI(I)*F(J)
90 FINT = FINT*(YX-YN)/2
100 RETURN
110 END

*1000B CM STORAGE USED .168 SECONDS
FUNCTION FCN

FUNCTION FCN (X,Y)
COMMON /AC/ XA,YA,ZA
RANGE = SQRT((X-XA)**2+(Y-YA)**2+ZA**2)
ARG = 129.12-22.5*ALOG10(RANGE)
FCN = 13.36E-6*10**((.03*ARG)*1.43E-4*10**(-.2*10**((.03*ARG)))/(108*ARG))
RETURN
END

410008 CM STORAGE USED .100 SECONDS
### COST REPORT FOR LISTOAF

<table>
<thead>
<tr>
<th>RESOURCE</th>
<th>BILLING RATE</th>
<th>UNITS USED</th>
<th>COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>CENTRAL PROCESSOR</td>
<td>$105.00 /HOUR</td>
<td>9,314 CP SECONDS</td>
<td>$ .27</td>
</tr>
<tr>
<td>I/O</td>
<td>30.00 /HOUR</td>
<td>9,737 PP SECONDS</td>
<td>$ .05</td>
</tr>
<tr>
<td>FIELD LENGTH</td>
<td>3.00 /KILO-WRD-HOUR</td>
<td>205,576 KILO-WRD-SECS.</td>
<td>$ .17</td>
</tr>
</tbody>
</table>

*BASIC COST EXCLUDES LINES PRINTED, CARDS PUNCHED AND PLOTTER TIME CHARGES*

**JOB PRIORITY** 3  **PRIORITY COST FACTOR** 1.00  **APPROXIMATE ADJUSTED COST** $ .56

AS OF LAST ACCOUNT UPDATE, ACCOUNT EXPIRES 04/30/79, FUNDS LEFT $ 6037.31

04/27/79  UVA NOS/BE 1.2  LEVEL 454-03/11/78
11.45.47.LISTOAF FROM *GD/AB*
11.45.47.LIST=M3117A+T100.
11.45.47.ATTACH=Q+NEW/107.
11.45.47.PF CYCLE NO. = 002
11.45.47.FTN(=G)
11.45.59. 450008 CM STORAGE USED
11.45.59. 9.292 CP SECONDS COMPILATION TIME
11.45.59. STOP
11.45.00.EJ END OF JOB; AB

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End of Document