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RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES

SCHOOL OF ENGINEERING AND
APPLIED SCIENCE

UNIVERSITY OF VIRGINIA

Charlottesville, Virginia 22901

Annual Report
on NASA Grant NSG-1509

EVALUATING AND MINIMIZING NOISE IMPACT DUE TO AIRCRAFT FLYOVER

Submitted to:
NASA Scientific and Technical Information Facility
P.O. Box 8757
Baltimore/Washington International Airport
Baltimore, MD 21240

Submitted by:
Ira D. Jacobson
Associate Professor

Gerald Cook
Professor

Report No. UVA/528166/MAE79/101
May 1979
RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES

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I. INTRODUCTION

This report presents the results of a study on the evaluation and reduction of noise impact to a community due to aircraft operation. Existing techniques have been used to assess the noise impact and to optimize the flight paths of an approaching aircraft with respect to the annoyance produced. Major achievements have been: (1) the development of a population model suitable for determining the noise impact, (2) generation of a numerical computer code which uses this population model along with the steepest descent algorithm to optimize approach/landing trajectories, (3) implementation of this optimization code in several fictitious cases as well as for the community surrounding Patrick Henry International Airport.

Previous work has centered on developing noise annoyance criteria for flyover (i.e. NEF, NNI, CNR, etc.) and ground noise signatures for aircraft. Some of these criteria are discussed in References 1-5 with a review of many of the noise effect measures being summarized in Ref. 6. Typical of the noise footprint work is Ref. 7. The annoyance criterion used in the study is the noise impact index (NII).

The details of the models used, their advantages and disadvantages and the results obtained are outlined in the following sections.
II. PROBLEM FORMULATION

A. Overview

Analysis of the problem consists of six parts: (1) aircraft noise signatures, (2) population models, (3) cost (annoyance) function, (4) aircraft flight-path model, (5) aircraft constraints, and (6) approach/landing path optimization. A modular concept has been employed so that modification of any of these segments may be effected with relative ease. The sections below describe each of these parts in detail.

B. A/C Noise Signature

The aircraft noise signature is obtained using data from Ref. 3. Here the effective perceived noise level (EPNdB) is given as a function of slant range to the closest point of approach for a variety of aircraft. A typical plot of the slant range variation is shown in Figure 1. These data were fit using standard least squares techniques to yield an expression for EPNdB given by

\[
\text{EPNdB} = 115 - 22.5 \log_{10} x (\text{Slant Range}).
\]

This equation is used for calculation of the maximum noise level at each location for a flyover. A typical footprint for a straight in approach along a 3-degree glide slope is shown in Figure 2.

C. Population Model

To model the population, a map of the community is overlaid with a grid and the population in each section of the grid determined. The population distribution within each section is assumed to be uniform. Several grid geometries were examined (see Figure 3). These geometries...
727 & DC-9

727-200 (Ref. 8)
JT8D-9

DC-9 727 (JT8D-1)

4000 Ft. 1000 Ft. 500 Ft. 250 Ft.

Effective Perceived Noise Level (EPNdB)

Slant Range at CPA* (FT)

FLYBY NOISE LEVEL
(1.93 - 1.95 EPR 727 Aircraft) FIG. A-1**
(1.94 EPR DC-9 Aircraft) FIG. D-1**

*Closest Point of Approach
**FAA-RD-71-83 (Ref. 6)

Figure 1. EPNL vs. Slant Range
CUSSEM LANDING NOISE FOOTPRINT

110 EPNDS
100 EPNDS
90 EPNDS
80 EPNDS

x,y Coordinates Unit: Mile
A.C. Original Altitude: 20000 Feet
A.C. Speed: 279 ft./sec
3 Degree Landing

Figure 2. Noise Foot Print.
1. Equal Size Blocks

2. Variable Size Blocks

3. Concentric Circles

Figure 3. Grid Geometries.
included: (1) rectangular sections of equal size, (2) rectangular sections whose dimensions increased with distance from the airport runway, and (3) concentric circles divided by several radial lines. The second scheme was chosen since it requires fewer rectangular sections than the first and is easier to implement than the third. Computer time required for determining the optimum trajectory varies directly with the number of grid sections. Furthermore, in light of the dependence of noise levels on distance and the fact that the aircraft has higher altitude when further from the runway, the need for high resolution of the population density diminishes with distance from the airport. While the third population scheme could offer this same advantage, it is somewhat more difficult to determine the population and noise impact for each grid section with such a geometry.

Within a grid section, the population is determined by use of the SITE II system, (Ref. 8), available on the CDC 7600 computer at the NASA-Langley facility. This system requires as input the latitude and longitude of a reference point and the coordinates of the corners of each rectangular section. Although SITE II allows for simple retrieval of 1970 census data, there is some question about its resolution capabilities for small grid sections. In addition, in rapidly growing areas the population data may lag actual population. The SITE II program is capable of producing detailed census information as shown in Figure 4. However for the present analysis only population information is used.
### Demographic Profile Report

**1970 Census Data**

<table>
<thead>
<tr>
<th>Population Total</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>387009</td>
<td>18646</td>
<td>9.2%</td>
<td>52.2%</td>
</tr>
<tr>
<td>White</td>
<td>367224</td>
<td>94.9%</td>
<td>52.3%</td>
</tr>
<tr>
<td>Negro</td>
<td>15414</td>
<td>4.0%</td>
<td>52.3%</td>
</tr>
<tr>
<td>Other</td>
<td>4371</td>
<td>1.1%</td>
<td>52.2%</td>
</tr>
<tr>
<td>Span</td>
<td>13839</td>
<td>3.6%</td>
<td>52.2%</td>
</tr>
<tr>
<td>Family Income (000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0-5</td>
<td>7945</td>
<td>7.8%</td>
<td>52.2%</td>
</tr>
<tr>
<td>$5-7</td>
<td>6942</td>
<td>6.8%</td>
<td>52.2%</td>
</tr>
<tr>
<td>$7-10</td>
<td>14752</td>
<td>14.4%</td>
<td>52.2%</td>
</tr>
<tr>
<td>$10-15</td>
<td>25949</td>
<td>25.4%</td>
<td>52.2%</td>
</tr>
<tr>
<td>$15-25</td>
<td>32623</td>
<td>31.9%</td>
<td>52.2%</td>
</tr>
<tr>
<td>$25-50</td>
<td>12867</td>
<td>12.6%</td>
<td>52.2%</td>
</tr>
<tr>
<td>$50+</td>
<td>1109</td>
<td>1.1%</td>
<td>52.2%</td>
</tr>
<tr>
<td>Total</td>
<td>102187</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>$15763</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>$14134</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rent</td>
<td>$0-100</td>
<td>10.5%</td>
<td>52.2%</td>
</tr>
<tr>
<td>$100-150</td>
<td>35292</td>
<td>42.5%</td>
<td>52.2%</td>
</tr>
<tr>
<td>$150-200</td>
<td>28662</td>
<td>34.5%</td>
<td>52.2%</td>
</tr>
<tr>
<td>$200-250</td>
<td>6645</td>
<td>8.0%</td>
<td>52.2%</td>
</tr>
<tr>
<td>$250+</td>
<td>3792</td>
<td>4.6%</td>
<td>52.2%</td>
</tr>
<tr>
<td>Total</td>
<td>87128</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>$150</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>$147</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Renter</td>
<td>61.5%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Occupation**

<table>
<thead>
<tr>
<th>Occupation</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGR/PROF</td>
<td>68537</td>
</tr>
<tr>
<td>SALES</td>
<td>12291</td>
</tr>
<tr>
<td>CLERICAL</td>
<td>48735</td>
</tr>
<tr>
<td>CRAFT</td>
<td>12810</td>
</tr>
<tr>
<td>LABORER</td>
<td>2144</td>
</tr>
<tr>
<td>FARM</td>
<td>11469</td>
</tr>
<tr>
<td>SERVICE</td>
<td>11663</td>
</tr>
</tbody>
</table>

**Units in Structure**

<table>
<thead>
<tr>
<th>Structure</th>
<th>Households With:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>126239 91.8% TV</td>
</tr>
<tr>
<td>2</td>
<td>71594 52.1% WASH</td>
</tr>
<tr>
<td>3</td>
<td>54258 39.5% DRYER</td>
</tr>
<tr>
<td>5</td>
<td>56277 40.9% DISHWSH</td>
</tr>
<tr>
<td>10</td>
<td>79438 57.8% AIRCND</td>
</tr>
<tr>
<td>50+</td>
<td>26600 20.8% FREEZER</td>
</tr>
<tr>
<td>MOBILE</td>
<td>2056 2.1% 2 HOMES</td>
</tr>
</tbody>
</table>

**Household Parameters**

- Fam Pop: 335153
- Indivs: 45881
- GRP WGRS: 5975
- Tot Pop: 387009
- NO of HH'S: 13747
- NO of FAM'S: 101961

**Average**

- TV: 126239
- WASH: 71594
- DRYER: 54258
- DISHWSH: 56277
- AIRCND: 79438
- FREEZER: 26600
- 2 HOMES: 2056

**Figure 4. Demographic Profile Report**

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D. Flight Path Model

There are two ways in which the trajectory of the aircraft can be determined. In one, a discrete time integration of the equations of motion with control deflections yields point by point spatial coordinates and orientation. Although this allows the flexibility of building in control constraints as well as dynamical constraints (e.g. max roll angle) it requires a considerable number of states to be stored in the optimization routine. In a multi-aircraft, multi-runway problem, as is anticipated, these storage requirements become prohibitive.

Thus, another method was adopted which utilizes only the functional form of the trajectory to describe the flight path. First a starting path was assumed which went from the initial point to the desired runway and ended up with the proper heading, i.e., velocity vector aligned with the runway. The following equation was used to generate this starting trajectory. (See Figure 5).

\[ y_s(x) = \left[ \frac{y_f - y_p}{x_f - x_p} \right] (x-x_p) + (y_p-y_o) \exp \left[ -C(x-x_f)/(x_o-x_f) \right] + y_o \] (2)

For the vertical motion a simple three degree descent path was assumed.

Next the first five Fourier sine harmonics were used to introduce deviation from this starting path. One advantage in using this type representation is the fact that each of the forms contributes zero at the end points. Therefore if the starting path satisfies the boundary conditions the curve with the deviations will also. An exponential decay at the final point was used to eliminate heading deviations.
Figure 5. Rotated Coordinate System for Establishing Nominal Flight Trajectory from Initial Point to Runway Approach.
The equations with the deviations thus become

\[ y(x) = \left( \sum_{i=1}^{5} \alpha_i \sin\left( \frac{\pi i (x-x_0)}{(x_f-x_0)} \right) \right) \left( 1 - \exp\left[ \frac{(x-x_f)}{C_1} \right] \right) + y_s(x) \]  

\[ Z(x) = \left( \sum_{i=1}^{5} \beta_i \sin\left( \frac{\pi i (x-x_0)}{(x_f-x_0)} \right) \right) \left( 1 - \exp\left[ \frac{(x-x_f)}{C_1} \right] \right) + Z_s(x) \]  

A second advantage to using a Fourier Series representation for the curve is that it provides a means of representing a function with a finite number of parameters. This reduces the optimization problem from a variational one to an ordinary one.

E. Constraints

The use of a functional form of the flight path for the trajectory requires the reformulation of constraints into parameters which can be used in the optimization. This is accomplished by translating the steady state solutions of the lateral and longitudinal perturbation equations into geometric constraints. An exact derivation is given in Appendix A. The constraints are incorporated by determining maximum curvature and slope parameters as a function of aerodynamics and physical constraints. For example the constraint of a maximum roll angle, \( \phi_{\text{max}} \), yields

\[ \frac{d^2 y}{dx^2} \leq \frac{C_2 + C_1 C_3}{C_4 + C_1 C_5} \frac{\phi_{\text{max}}}{V_{\text{avg}}} \]  

(4)

where \( C_1 \) through \( C_5 \) depend upon aircraft stability and control derivations (see Appendix A for details) and \( V_{\text{avg}} \) is the average velocity. Similar expressions are given in Appendix A for constraints on aileron rudder.
and elevator deflection, flight path angle and pitch rate limits.

F. Cost Function

A large number of criteria have been proposed by evaluating noise annoyance (e.g., EPNdB, NII, sleep interference index, speech interference index, etc.). The recent trend in noise assessment work is toward a universal measure—the noise impact index (NII). This measure is a weighted day-night model which accounts for population density. It is described in detail in Ref. 9. Briefly, the total population exposed to each incremental average day-night model sound level is multiplied by the weighting function for the level. The weighting function used is shown in Figure 6. This weighting factor \( W(L_{dn}) \) multiplied by the population exposed to that \( L_{dn} \) is summed and normalized by the total population giving the Noise Impact Index for the area.

\[
NII = \frac{\sum_{L_{dn}} P(L_{dn}) W(L_{dn})}{\sum_{L_{dn}} P(L_{dn})} \tag{5}
\]

The cost function or payoff for the optimization procedure is taken to be the NII plus penalties for violating constraints. Basically the optimization procedure is set up to "drive" the aircraft trajectory to the path which will minimize the NII and at the same time not violate any constraints. As an example of the constraint of flight path angle not exceeding a maximum descent angle, \( \gamma_d \), nor a maximum climb angle, \( \gamma_c \), is written as

\[
\tan \gamma_c \left< \frac{dZ}{dx} \right< \tan \gamma_d \tag{6}
\]
SOUND LEVEL WEIGHTING FUNCTION
FOR OVERALL IMPACT ANALYSIS

Figure 6. Sound Level Weighting Function for Overall Impact Analysis.
Each is converted to a penalty which is added to the NII in the form

\[ \text{Cost} = \text{NII} +\left( \frac{dZ}{dx}/\tan\gamma_d \right)^2 + \left( \tan\gamma_c/\frac{dZ}{dx} \right)^2 \]  

(7)

As is seen for values of the flight path angle within the allowable range the penalty is negligible; however for values outside this range the penalty and thus the increase in cost is great. Other terms are added in a like manner.
III. OPTIMIZATION

The optimum trajectory is determined by calculating values of the $\alpha_i$'s and $\beta_i$'s (Eq. 2) which minimize the total cost (NII plus penalties). A steepest descent algorithm is employed here. Basically, this method computes the gradient of the cost function, $C$, with respect to the $\alpha_i$'s and $\beta_i$'s, then searches along the negative gradient direction for values of $\alpha_i$'s and $\beta_i$'s which reduce the cost. The change in cost is given by

$$\Delta C = \sum_{i=1}^{n} \left( \frac{3C}{\partial \alpha_i} \Delta \alpha_i + \frac{3C}{\partial \beta_i} \Delta \beta_i \right)$$

(8)

The process continues iteratively until the cost converges to within a specified tolerance. While implementation of the algorithm is fairly straightforward, convergence near the optimal set of $\alpha_i$'s and $\beta_i$'s is inherently slow. Most of the cost reduction, however, occurs in the first few iterations.

A. The Optimization Algorithm

A computer code has been developed which implements the functions described above. Figure 7 shows a flow chart for this code. Initial data (population map, aircraft constraints, initial and final aircraft positions, etc.) are required for each airport/airplane configuration to be evaluated. To facilitate calculation of the Fourier coefficients, the coordinate axes are rotated such that a line joining the initial and final aircraft positions is made to be parallel to the x axis. A nominal trajectory is generated which constrains the heading of the aircraft to asymptotically approach the runway. The steepest descent search then begins and continues until the stopping criterion is met. This criterion
READ POPULATION MAP, AIRCRAFT CONSTRAINTS, 
INPUT INITIAL CONDITIONS, 
FINAL CONDITIONS, 
FOURIER PARAMETERS = x's, β's

COORDINATE ROTATION = 
ROTATE AXIS SUCH THAT LINE JOINING 
INITIAL AND FINAL POINTS IS PARALLEL 
TO THE x-AXIS IN ORDER TO APPLY 
FOURIER SERIES

GENERATE A NOMINAL CURVE WHICH FORCES 
THE HEADING OF AIRCRAFT AT FINAL POINT 
TOWARD THE RUNWAY

STEEPEST DESCENT 
GRADIENT SEARCH

NO

OPTIMAL REACHED?

YES

OPTIMAL LANDING TRAJECTORY

Figure 7. Flow Chart.
is met if successive improvements become negligible.

In order to provide a more accurate noise impact in each population section the impact is integrated using quadratures. This procedure can be found in Ref. 9.

The various functions such as the population model, the cost function, and the aircraft signature are incorporated as subroutines. This will allow ease of upgrading or modification if different models are desired.

Appendix B contains the Fortran code as written for a CDC Cyber 172 machine.

B. Results

Several cases have been run to test the benefits that can be obtained by this approach. First, a fictitious set of data incorporating a population valley is used. As can be seen in Figure 8 the optimization algorithm moves the aircraft (a Convair 880) towards the valley (i.e. fewer people impacted) with a corresponding improvement in the NII of 32%.

The second case models the Patrick Henry Airport in Hampton, Virginia. Here the SITE II program was used to generate the census data for each block as shown in Figure 9. Two initial trajectories were flown. One entering the area from the northwest over the Swing VOR station and the other from the southwest over the Franklin VOR station. Both of these paths are specified IFR trajectories (Figure 10). The aircraft enters the area approximately 30,000 meters from the runway. In addition several straight-in paths were evaluated. Figure 9 shows each of the trajectories. The associated
Figure 3. Optimization Results Using Fictitious Population Data.
Figure 9. Population Model and Optimization Results for Patrick Henry Airport.
Figure 10. Conventional Approach Pattern

- Harcum
- Cape Charles 112.2 CCV Chan 59
- Franklin
- Norfolk 116.9 ORF Chan 116
- 2000 to Swing Int 255° (42.4)
- 1600
- 2100
- 530
- 365
- 428
- 531
- 216
- R-250
- 50
### Table I
Northwest Approach

**Entry Point: Swing**

<table>
<thead>
<tr>
<th>Traj. No.</th>
<th>Description</th>
<th>Cost (NII x 10^2)</th>
<th>% Change from Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60 deg wrt runway</td>
<td>2.373</td>
<td>+3.2%</td>
</tr>
<tr>
<td>2</td>
<td>30 deg wrt runway</td>
<td>2.438</td>
<td>+6.0%</td>
</tr>
<tr>
<td>3</td>
<td>Initial iteration</td>
<td>2.27</td>
<td>-1.3%</td>
</tr>
<tr>
<td>4</td>
<td>Optimal</td>
<td>2.213</td>
<td>-3.8%</td>
</tr>
<tr>
<td>5</td>
<td>Straight in</td>
<td>2.316</td>
<td>+1.1%</td>
</tr>
<tr>
<td>N1</td>
<td>Presently used</td>
<td>2.300</td>
<td>0%</td>
</tr>
</tbody>
</table>

### Table II
Southwest Approach

**Entry Point: Franklin**

<table>
<thead>
<tr>
<th>Traj. No.</th>
<th>Description</th>
<th>Cost (NII x 10^2)</th>
<th>% Change From Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Straight in</td>
<td>2.316</td>
<td>-1.3%</td>
</tr>
<tr>
<td>6</td>
<td>Initial iteration</td>
<td>2.408</td>
<td>+2.6%</td>
</tr>
<tr>
<td>7</td>
<td>Optimal</td>
<td>2.241</td>
<td>-4.5%</td>
</tr>
<tr>
<td>8</td>
<td>30 deg wrt runway</td>
<td>2.598</td>
<td>+10.7%</td>
</tr>
<tr>
<td>9</td>
<td>60 deg wrt runway</td>
<td>2.687</td>
<td>+14.5%</td>
</tr>
<tr>
<td>N2</td>
<td>Presently used</td>
<td>2.346</td>
<td>0%</td>
</tr>
</tbody>
</table>
NII's are summarized in Tables I and II.

As is seen that even for this case, where the population is sparse away from the runway and congested near the end of it, an improvement of 3 to 5% is achieved using the optimization algorithm. It should also be noted that this technique allows not only the optimization of the path but also can be used to evaluate existing or proposed paths, such as the nominal and straight-in paths indicated.

Conclusion

A method has been formulated to optimize the path of an aircraft during approach or take-off from any airport. Models have been developed using available data where possible for population, aircraft signature, noise impact, constraints and flight path. An algorithm using steepest descent has been implemented and tested. This approach allows

1) The evaluation of the noise impact of existing flight paths,
2) The evaluation of the noise impact of proposed flight paths, and
3) The optimization of the flight path to minimize the noise impact under constraints.

This method has been applied to the Patrick Henry International Airport. Both nominal and other straight paths were evaluated. Also an optimal path was determined for each of two terminal area entry points. Performance ranged from a 15% degradation of the NII to 4.5% improvement compared to the presently used approaches. It is significant that as much as 3 to 5% improvement could be achieved in light of the fact that most of the population is concentrated at the end of the runway.
REFERENCES


APPENDIX A

Derivation of Parameterized Trajectory Constraints

Lateral perturbation equations

\[ Y \text{ eq'n: } - \frac{b}{2V_T} C_y \dot{\phi} + \frac{mg}{q_{\infty} S} \cos \phi \dot{\phi} + \left( \frac{mV_T}{q_{\infty} S} - \frac{b}{2V_T} C_y \right) \ddot{\psi} - \frac{mg}{q_{\infty} S} \sin \theta_o \dot{\psi} \]

\[ + \frac{mV_T}{q_{\infty} S} \dddot{\beta} - C_y \beta = C_y \delta_a + C_y \delta_r \]

\[ L \text{ eq'n: } \frac{I_{xx}}{q_{\infty} S b} \dddot{\phi} - \frac{b}{2V_T} C_{\beta} \phi - \frac{I_{xz}}{q_{\infty} S b} \dddot{\psi} - \frac{b}{2V_T} C_{\beta} \dddot{\psi} - C_{\beta} \beta = C_{\beta} \delta_a + C_{\beta} \delta_r \]

\[ N \text{ eq'n: } - \frac{I_{xz}}{q_{\infty} S b} \dddot{\phi} - \frac{b}{2V_T} C_{n_p} \dddot{\phi} + \frac{I_{zz}}{q_{S b}} \dddot{\psi} - \frac{b}{2V_T} C_{n_r} \dddot{\psi} - C_{n_r} \beta = C_{n_r} \delta_a + C_{n_r} \delta_r \] (1)

If we assume all turns to be coordinated (no sideslip)

Then letting \[- \frac{b}{2V_T} C_y = \overline{C}_y, \text{ etc.} \]

\[ \frac{mg}{q_{\infty} S} \cos \theta_o = \overline{g}_1 \frac{mg}{q_{\infty} S} \sin \theta_o = \overline{g}_2 \]

\[ \frac{I_{xx}}{q_{\infty} S b} = i_x, \text{ etc. } \frac{mV_T}{q_{\infty} S} = \overline{m} \]

\[ L \text{ eq'n: } i_x \dddot{\phi} - C_{\beta} \dddot{i_x Z \overline{C}_r \psi} = C_{\beta} \delta_a + C_{\beta} \delta_r \]

\[ N \text{ eq'n: } -i_x Z \dddot{\phi} - C_{n_p} \dddot{\phi} + i_x Z \dddot{\psi} = C_{n_r} \delta_a + C_{n_r} \delta_r \]

\[ Y \text{ eq'n: } -C_{y_p} \dddot{\phi} - \overline{g}_1 + (m - C_{\overline{C}_{yr}}) \dddot{\psi} - \overline{g}_2 \psi = C_y \delta_a + C_y \delta_r \] (2)

Taking the Laplace transform (I.C.'s = 0)

\[ L \text{ eq'n: } (i_x Z^2 - C_{\beta} s) \phi (s) + (-i_x Z s^2 - C_{\beta} s) \psi (s) = C_{\beta} \delta_a (s) + C_{\beta} \delta_r (s) \]
N eq'n: 
\[-i_{xz}s^2-C_{np} \phi(s) + (i_{zs}^2-C_{nr}s) \psi(s) = C_n \delta_a(s) + C_{n \delta_r}s\]  

Y eq'n: 
\[-C_{yp}s^{-g_1} \phi(s) + [(m-C_{yp})s^{-g_2}]\psi(s) = C_y \delta_a(s) + C_{y \delta_r}s\]  

To determine the required \( \delta_a \) for a given \( \delta_r \) we consider \( \delta_a \) an unknown along with \( \phi(s) \) and \( \psi(s) \) [i.e. move \( \delta_a \) to the left hand side of the equations] and solve for \( \delta_a/\delta_r \) using Cramer's rule.

\[
\frac{\delta_a}{\delta_r} = \frac{\begin{vmatrix} i_{xs}^2-C_{\ell_p}s & -i_{xz}s^2-C_{\ell_r}s & +C_{\ell_r}
-i_{xzs}^2-C_{np}s & i_{zs}^2-C_{nr}s & +C_{n}\delta_r
-C_{yp}s^{-g_1} & (m-C_{yp})s^{-g_2} & +C_{y\delta_r}
i_{xs}^2-C_{\ell_p}s & -i_{xz}s^2-C_{\ell_r}s & -C_{\ell\delta_a}
-i_{xzs}^2-C_{np}s & +i_{zs}^2-C_{nr}s & -C_{n\delta_a}
-C_{yp}s^{-g_1} & (m-C_{yp})s^{-g_2} & -C_{y\delta_a}\end{vmatrix}}{N(s)}
\]  

The denominator (characteristic eqn.) is given by:

\[
\Delta(s) = s^4(-C_{y\delta_a}(i_x^2-Z_{iZ})) + s^3(C_{y\delta_{a}}[i_{xZ}^C_{\ell_p} + i_{xZ}C_{n_r} + i_{xZ}(C_{\ell_r} + C_{np})] 
+ c_{n\delta_a}[-i_{xZ}C_{y\ell} + (m-C_{yr})i_x] + C_{\ell\delta_{a}}[i_{xZ}C_{y\ell} - (m-C_{yr})i_x^Z]) 
+ s^2(C_{y\delta_{a}}(C_{np}C_{\ell_p}C_{n_r}) + c_{n\delta_a}[-i_{xZ}^g_{g_1} - i_{xZ}^g_{g_2} - C_{yp}C_{\ell_r} - (m-C_{yr})C_{\ell_p}] 
+ C_{\ell\delta_{a}}[g_{1Z}^g_{g_1}g_{1Z}^g_{g_2}C_{np}C_{n_r} + (m-C_{yr})C_{nr}]) 
+ s(c_{n\delta_a}(g_{2Z}C_{np}g_{1Z}C_{n_r}) + C_{\ell\delta_{a}}(g_{2Z}C_{np}g_{1Z}C_{n_r}))
\]  

(5)
The numerator is:

\[ N(s) = s^3 \{C_y \delta r (1 + i \frac{z}{x} - i \frac{z^2}{x^2}) \} + s^2 \{-C_{n} \left[ -i_x Z \overline{C}_p + i_x \overline{C}_n - \overline{C}_r \right] \} + s \left[ \frac{-C \left( C_{n} \overline{C}_{n_{r}} - \overline{C}_{z_{p}} \overline{C}_{z_{r}} \right) - C_{\delta} \left[ -i_x \overline{C}_p + \overline{C}_n \right] \} \}

\]

Now assuming that only the steady state (st. st.) condition is of interest,

\[ \lim_{s \to 0} N(s) = \left( \frac{\delta_a}{\delta_r} \right) \text{ st. st.} \]

we get

\[ \left( \frac{\delta_a}{\delta_r} \right) \text{ st. st.} = \frac{-C_{n}\left( g_2 \overline{C}_{p} - g_1 \overline{C}_{n_{r}} \right) - C_{\delta_6} \left( g_2 \overline{C}_{n_{p}} - g_1 \overline{C}_{n_{r}} \right)}{C_{n_{6}} \left( g_2 \overline{C}_{p} - g_1 \overline{C}_{n_{r}} \right) + C_{\delta_{6}} \left( g_2 \overline{C}_{n_{p}} - g_1 \overline{C}_{n_{r}} \right)} \]  

\[ \left( \frac{\delta_a}{\delta_r} \right) \text{ st. sc.} = \frac{\cos \theta (C_{n_{r}} C_{n_{r}} + C_{\delta_{r}} C_{n_{r}}) - \sin \theta (C_{n_{r}} C_{p} + C_{\delta_{r}} C_{p})}{\cos \theta (C_{n_{r}} C_{n_{r}} + C_{\delta_{r}} C_{n_{r}}) + \sin \theta (C_{n_{r}} C_{p} + C_{\delta_{r}} C_{p})} \]  

For small initial flight path angle (i.e. \( \theta_0 \approx 0 \))

\[ \left( \frac{\delta_a}{\delta_r} \right) \text{ st. st.} = - \frac{C_{n_{r}} C_{p} + C_{\delta_{r}} C_{n_{r}}}{C_{n_{r}} C_{n_{r}} + C_{\delta_{r}} C_{n_{r}}} = C_1 \]  

Assuming \( \theta_0 = 0 \) to simplify we can write the transfer functions for \( \phi \) and \( \hat{\psi} \) as (in the st. st.)
\[
\frac{\dot{\psi}}{\delta_r} = \frac{C_{2,\delta_r}C_{n_\beta} - C_{n_\delta_r}C_{2,\beta}}{C_{2,\beta}C_{n_r} - C_{n_\beta}C_{\delta_r}} = c_2
\]  

(10)

\[
\frac{\dot{\psi}}{\delta_a} = \frac{C_{2,\delta_a}C_{n_\beta} - C_{n_\delta_a}C_{2,\beta}}{C_{2,\beta}C_{n_r} - C_{n_\beta}C_{\delta_r}} = c_3
\]  

(11)

\[
\frac{\phi}{\delta_r} = \frac{C_{y,\delta_r}(C_{2,\beta}C_{n_\beta} - C_{2,\beta}C_{n_r}) + C_{2,\delta_r}(C_{y_p}C_{n_r} + C_{n_\beta}(m - C_{y_r})) + C_{n_\delta_r}(C_{2,\beta}(m - C_{y_r}) + C_{y_\beta}C_{\delta_r})}{\frac{mg}{q_\omega s}(C_{2,\beta}C_{n_r} - C_{n_\beta}C_{\delta_r})} = c_4
\]  

(12)

\[
\frac{\phi}{\delta_a} = \frac{C_{y,\delta_a}(C_{2,\beta}C_{n_\beta} - C_{2,\beta}C_{n_r}) + C_{2,\delta_a}(C_{y_p}C_{n_r} + C_{n_\beta}(m - C_{y_r})) + C_{n_\delta_a}(C_{2,\beta}(m - C_{y_r}) + C_{y_\beta}C_{\delta_r})}{\frac{mg}{q_\omega s}(C_{2,\beta}C_{n_r} - C_{n_\beta}C_{\delta_r})} = c_5
\]  

(13)

Consider the aircraft trajectory shown
The slope at any point is $\frac{dy}{dx}$ and the angle the slope makes with the x axis is $\tan^{-1}\left(\frac{dy}{dx}\right)$.

The angular rate $\psi$ is then $\frac{d}{dt} \tan^{-1}\left(\frac{dy}{dx}\right)$

or $\frac{\partial}{\partial x} \{\tan^{-1}\frac{dy}{dx}\} \frac{dx}{dt} = \psi \frac{\partial}{\partial x} \{\tan^{-1}\left(\frac{dy}{dx}\right)\}$

Then $\psi = V_{avg} \frac{d^2y}{dx^2} = V_{avg} \left(\frac{f''(x)}{1+f''[x]}\right)^2$  \hspace{1cm} (14)

If we know $\delta_r$, we can determine $\delta_a$ from $\delta_a = C_1 \delta_r$

Also $\dot{\psi} = C_2 \delta_r + C_3 \delta_a = (C_2+C_1+C_3) \delta_r$ \hspace{1cm} (15)

We can also write

$\phi = C_4 \delta_r + C_5 \delta_a = (C_4+C_1C_5) \delta_r$  \hspace{1cm} (16)

Constraining $\delta_a$ to be $\leq \delta_a_{\text{max}}$, \hspace{1cm} (17)

$\delta_r$ to be $\leq \delta_r_{\text{max}}$, \hspace{1cm} (18)

and $\phi$ to be $\leq \phi_{\text{max}}$ (max bank angle) \hspace{1cm} (19)

we get the following expressions

$\delta_{r1} \leq \frac{\phi_{\text{max}}}{C_4+C_1C_5}$  \hspace{1cm} (20)

$\delta_{r2} \leq \delta_r_{\text{max}}$  \hspace{1cm} (21)

$\delta_{r3} \leq \frac{\delta_{a_{\text{max}}}}{C_1}$  \hspace{1cm} (22)
The constraining value is given by
\[
\delta r_{\text{max}} = \min(\delta r_1, \delta r_2, \delta r_3)
\]  
which yields
\[
\dot{\psi}_{\text{max}} = (C_2 + C_1 C_3) \min(\delta r_1, \delta r_2, \delta r_3)
\]  
This condition incorporates all three constraints \((17)-(19)) as
\[
\frac{d^2\psi}{dx^2} = \frac{f''(x)}{1 + f'(x)^2} \leq \frac{(C_2 + C_1 C_3)}{\bar{v}_{\text{avg}}} \min(\delta r_1, \delta r_2, \delta r_3)
\]
Longitudinally we wish to constrain the behavior of the trajectory so that we restrict \(\gamma\) (the flight path angle) and \(\theta\) (the pitching rate).

The trajectory is given by

\[
Z = g(x)
\]

Then, assuming the aircraft center of mass follows this trajectory \(\gamma\) is given by
\[
\gamma = \tan^{-1} \frac{dz}{dx}
\]
or
\[
\frac{dz}{dx} = \tan \gamma
\]
We wish to constrain \(\gamma\) to a maximum descent angle, \(\gamma_{d_{\text{max}}}\) and a maximum angle, \(\gamma_{c_{\text{max}}}\).

Thus
\[
\tan \gamma_{c_{\text{max}}} \leq \frac{dz}{dx} \leq \tan \gamma_{d_{\text{max}}}
\]
PROGRAM NOISE

COMMON ALFA(5), BETA(5), POSIT(5,5), ARRAY(578,9), NMAP
COMMON CURVE/ YCURVE(51), ADY(51), ADDY(51)
COMMON /LABEL/ LINE(4), LLOC(5)
COMMON /AIRPORT/ XPORT, YPORT, ZPORT
COMMON /SCALE/ XMIN, XINC, YMIN, YINC

INTEGER CJNT, HALF

DIMENSION ALFADD(5), BETAADD(5), GY(5), GZ(5), DALFA(5), DBETA(5)

COMMON ICURVE, YCURVE, ADY, ADDY, ARRAY, NMAP

COMMON ICURV, YCURV, ADY, ADDY, ARRAY, NMAP

READ (5,*) ICURV, YCURV, ADY, ADDY, ARRAY, NMAP

READ (5,*) A11, A12
READ (5,*) NMAP, XPORT, YPORT
READ (5,*) (ARRAY(I,J), J=1,9), I=1,NMAP

WRITE (16,9110) ICURV, YCURV, ADY, ADDY
WRITE (16,9120) NMAP
WRITE (16,9130) XPORT, YPORT
WRITE (16,9140) ICURV, YCURV, ADY, ADDY

WRITE (16,9150) ICURV, YCURV, ADY, ADDY

THETA = ATAN2(YF-YO, XF-XO)
A = (XF*COS(TTHETA) + YF*SIN(TTHETA))*(X0*COS(TTHETA) + Y0*SIN(TTHETA))

XOCP = X0*COS(TTHETA)*COS(PHI) + Y0*COS(PHI)*SIN(TTHETA) + Z0*SIN(PHI)
XFCAP = XF*COS(TTHETA)*COS(PHI) + YF*SIN(TTHETA)*COS(PHI) + ZF*SIN(PHI)

XPORT = XPORT*COS(TTHETA)*COS(PHI) + YPORT*SIN(TTHETA)*COS(PHI)

YFCAP = YF*COS(TTHETA)*COS(PHI) + YF*SIN(TTHETA)*COS(PHI) + ZF*SIN(PHI)

ZFCAP = ZF*COS(TTHETA)*COS(PHI) + ZF*SIN(TTHETA)*COS(PHI) + ZF*SIN(PHI)

ZPORT = XPORT*COS(TTHETA)*SIN(PHI) + YPORT*SIN(TTHETA)*SIN(PHI) + ZF*SIN(PHI)
60 C * START OPTIMIZATION

65 C * INDEX = 0

OLXCAP = (XFCAP-XOCAP)/50.

70 C * FIRST FIND A CURVE WHICH FORCES THE HEADING OF THE

C * AIRCRAFT TOWARD THE RUNWAY AT THE FINAL POINT

C * INDEX = 0

SLOPE = (YFCAP-YPORT)/(XFCAP-XPORT)

75 YCURVE(1) = YOCAP

ADY(1) = 0.

ADDY(1) = 0.

XCAP = XOCAP

DO 10 I = 1,50

XCAP = XCAP+DLXCAP

80 EXP = -5.*(XCAP-XFCAP)/(XOCAP-XFCAP)

YCURVE(I+1) = (SLOPE*(XCAP-XPORT)+YPORT-YOCAP)+EXP(EXP)+YOCAP

ADY(I+1) = -5./(XOCAP-XFCAP)*(YCURVE(I+1)-YOCAP)+(SLOPE)*EXP(EXP)

10 CONTINUE

COUNT = 0.

90 C * INITIAL COST

95 XMIN = -40000

XINC = 2500

YMIN = -40000

YINC = 2500

H = 0.07

CALL COST (0.1*XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA PHI,TOTAL,PNAL

110 1)

COST1 = TOTAL

A = COST1-PNALTY

WRITE (6,9150) COUNT,COST1,A,PNALTY

105 WRITE (6,9220) 

WRITE (6,9230) ((POSI(I,J)+I=1+3,J=1+51)

WRITE (6,9030) 

00 20 I = 1+5

110 WRITE (6,9040) I,ALFA(I),BETA(I)

20 CONTINUE

WRITE (6,9050)

30 DO 40 I = 1+5

30 GALFA(I) = A11

40 DBETA(I) = A12
115 C
120 C
50 DO 60 I = 1,5
   ALFA(I) = ALFA(I)+DALFA(I)
   CALL COST (I,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THEMA,PHI,TOTAL,P)
   GY(I) = (COST2-COST1)/ABS(ALFA(I))
   IF (INDEX,EQ,0) GY(I) = GY(I)
   IF (INDEX,NE,1) GY(I) = GT(I)
60 ALFA(I) = ALFA(I)-DALFA(I)
   DO 70 I = 1,5
   BETAI = BETAI+DBETAI
   CALL COST (I,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THEMA,PHI,TOTAL,P)
   GZ(I) = (COST2-COST1)/ABS(BETAI)
   IF (INDEX,NE,0) GZ(I) = GZ(I)
   IF (INDEX,NE,1) GZ(I) = GT(I)
70 BETAI = BETAI-DBETAI
   IF (INDEX,EQ,1) GO TO 190
135 C
   1 ALTY
   COST2 = TOTAL
   GZ(I) = (COST2-COST1)/ABS(BETAI)
   IF (INDEX,EQ,0) GZ(I) = GZ(I)
   IF (INDEX,NE,1) GZ(I) = GZ(I)
   WRITE (6,9170) I,GZ(I)
90 BETAI = BETAI-ZRATIO*GZ(I)
C 50 DO 60 I = 1,5
   ALFAOD = ALFA(I)
   ALFA(I) = ALFA(I)-YRATIO*GT(I)
   BETAO = BETAI
   BETAI = BETAI-ZRATIO*GZ(I)
   CALL COST (I,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THEMA,PHI,TOTAL,P)
   COST2 = TOTAL
   IF (COST2,GE,COST1) GO TO 150
100 PRCENT = ABS(COST2-COST1)/COST1
STOP CRITERION -- PERCENTAGE CHANGE IN COST INSIGNIFICANT

IF (PRCCNT.GE.1.E-51) GO TO 110
COUNT = COUNT+1
CALL COST (O+1,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA PHI,TOTAL,PNAL,
IT)
WRITE (6+9180) COUNT
CALL MONIT (COUNT,COST2,PNALTY)
STOP

STOP CRITERION -- ALL GRADIENT COMPONENTS EQUAL TO ZERO

IF (GYII.NE.0.) GO TO 130
IF (GZII.NE.0.) GO TO 130
CONTINUE
WRITE (6+9060) COUNT
CALL MONIT (COUNT,COST2,PNALTY)
STOP

WRITE (6+9070) I
DO 140 I = 1,5
WRITE (6+9190) I,ALFA(I),BETA(I)
CONTINUE
COST2 = TOTAL

STOP CRITERION -- MAXIMUM NUMBER OF ITERATIONS REACHED

IF (COUNT.LT.MAXIT) GO TO 30
WRITE (6+9200) I
CALL MONIT (COUNT,COST2,PNALTY)
STOP
HALF = 1
REDUCE SIZE OF STEP CHANGE BY HALF
C. IF COST HAS NOT DECREASED

C

230 DO 170 J = 1,3
       DO 170 I = 1,5

       ALFA(I) = (ALFA(I)+ALFAADD(I))/2.
       BETA(I) = (BETA(I)+BETAADD(I))/2.

160 IF (HALF .EQ. J) WRITE (6,9210,HALF)

170 CALL COST (I,D02CAP,Y0CAP,Z0CAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,N)

C

240 IF (COST2.LT.COST1) GO TO 100

C

170 CONTINUE

HALF = 4

245 INDEX = 1
       DO 180 I = 1,5

       DALFA(I) = -DALFA(I)
       DBETA(I) = -DBETA(I)

250 CALL COST

C

C. PERTURB CURVE IN THE OPPOSITE DIRECTION

C

255 GO TO 50

190 DO 200 I = 1,5

       IF (AGY(I).LT.0.) GO TO 220

260 IF (AGZ(I).LT.0.) GO TO 220

200 CONTINUE

WRITE (6,9060)

DO 210 I = 1,5

ALFA(I) = ALFAADD(I)

210 BETA(I) = BETAADD(I)

CALL COST (I,D02CAP,Y0CAP,Z0CAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,N)

CALL MONIT (COUNT,COST1,PNALTY)

STOP

220 BGYMAX = ABS(AGY(I))
       BGZMAX = ABS(AGZ(I))

       DO 230 I = 2,5

          IF (BGYMAX.LT.ABS(AGY(I))) BGYMAX = ABS(AGY(I))

235 IF (BGZMAX.LT.ABS(AGZ(I))) BGZMAX = ABS(AGZ(I))

240 WRITE (6,9210,HALF)

270 C

280 CHECK EACH GRADIENT COMPONENT TO DETERMINE SIZE

C

C

285 DO 320 I = 1,5
IF (HALF.EQ.7) GO TO 250
IF (GYMAX.NE.0.) AY = YALLOW/GYMAX/FLOAT(HALF-3)*AGY(I)
IF (GYMAX.NE.0.) BY = YALLOW/GYMAX/FLOAT(HALF-3)*BGY(I)
IF (GYMAX.NE.0.) AZ = 0.
IF (GYMAX.NE.0.) BZ = 0.
IF (GYMAX.NE.0.) AY = YALLOW/GYMAX/FLOAT(HALF-3)*AGY(I)
IF (GYMAX.NE.0.) BZ = YALLOW/BGYMAX/FLOAT(HALF-3)*BGY(I)
290 GO TO 260

250 AY = -OMEGA(I)
BY = -OMEGA(I)
AZ = -OMEGA(I)
BZ = -OMEGA(I)

300 IF (AGY(I).LE.0.) GO TO 270
IF (BGY(I).LE.0.) ALFA(I) = ALFADD(I)
IF (BGY(I).LE.0.) ALFA(I) = ALFADD(I)+BY
GO TO 290

270 IF (AGY(I).LE.0.) GO TO 260

305 IF (AGY(I).LE.0.) GO TO 290
IF (AGY(I).LE.0.) ALFA(I) = ALFADD(I)+BY
GO TO 290

310 IF (AGY(I).LE.0.) GO TO 310
IF (AGY(I).LE.0.) ALFA(I) = ALFADD(I)+BY

315 IF (BGY(I).LE.0.) BETA(I) = BETA(I)+BZ
GO TO 320

310 BETA(I) = BETA(I)-AZ

320 CONTINUE
CALL COST (0*0, YOCAP, ZOCAP, YFCAP, ZFCAP, THETA, PHI, TOTAL, PNAL)

325 COST2 = TOTAL
IF (COST2 .LT. COST1) GO TO 420
HALF = HALF+1

350 CONTINUE
DO 340 I = 2,5
GZMIN = AGZ(I)
K = 1
DO 340 J = 1,5
IF (GZMIN .LE. AGY(I)) GO TO 330
GMIN = AGY(I)
J = I
335 CONTINUE
330 IF (GZMIN .LE. AGZ(I)) GO TO 340
GZMIN = AGZ(I)
K = I
340 CONTINUE
DO 360 I = 1,5
IF (GMIN .LE. BGY(I)) GO TO 350
GMIN = BGY(I)
J = I+5
340 CONTINUE
350 IF (GZMIN.LE.BGZ(I)) GO TO 360

GZMIN = BGZ(I)

K = I+5

360 CONTINUE

IF ((GMIN.LT.0.0).OR.(GZMIN.LT.0.0)) GO TO 370

CALL COST (0.0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL,11)

COUNT = COUNT+1

WRITE (6,9090) COUNT

CALL MONIT (COUNT,COST1,PNALTY)

STOP

370 DO 380 I = 1,5

380 ALFA(I) = ALFADD(I)

BETA(I) = BETAADD(I)

IF (GMIN.LT.0.0).AND.(GZMIN.GE.0.0)) GO TO 390

IF (GMIN.LT.0.0).AND.(GZMIN.LT.0.0)) GO TO 400

IF (K.LT.5) BETA(K) = BETA(K)+DBETA(K)

IF (K.GT.5) BETA(K-5) = BETA(K-5)+DBETA(K-5)

CALL COST (0.0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL,11)

STOP

390 IF (I.J.LE.5) ALFA(J) = ALFA(J-5) 

IF (I.J.GT.5) ALFA(J-5) = ALFA(J-5) 

IF (I.J.LE.5) ALFA(J) = ALFA(J) 

IF (I.J.GT.5) ALFA(J-5) = ALFA(J-5)

CALL COST (0.0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL,11)

STOP

400 IF (I.J.LE.5) BETA(J) = BETA(J-5) 

IF (I.J.GT.5) BETA(J-5) = BETA(J-5) 

IF (I.J.LE.5) BETA(J) = BETA(J) 

IF (I.J.GT.5) BETA(J-5) = BETA(J-5)

CALL COST (0.0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL,11)

STOP

1100 GO TO 110

110 BETA(I) = BETAADD(I)

INDEX = 0

CALL COST (0.0,XOCAP,YOCAP,ZOCAP,XFCAP,DLXCAP,THETA,PHI,TOTAL,PNAL,11)

GO TO 100

9010 FORMAT (5X,14MINITAL X,Y,Z: /3IF12.2,3X),7H METERS, /5X,13FINAL X /3IF12.2,3X),7H METERS/ /5X,13AIRPORT LOCATION, X,Y, /21F12.2,7H METERS)

9020 FORMAT (5X,43PERTURB TRAJECTORY IN Y AND Z DIRECTIONS BY .F6.2,.5H AND .F6.2,.5H METERS, RESPECTIVELY FOR CALCULATING GRADIENTS)

9030 FORMAT (1X,WHALFA,16X,HBETA)

9040 FORMAT (10X+11P16.9,4X+1P16.9)

9050 FORMAT (///)

9060 FORMAT (///,1X,15HAT ITERATION +I2.49H ALL GRADIENTS EQUAL TO 0)

9070 FORMAT (///,1X,ZERO PROGRAM STOPS)

9080 FORMAT (5X,43HALL GRADIENTS PERTURBED BOTH DIRECTIONS > 0)

9090 FORMAT (1X,13HAT ITERATION +I2.16H optimum reached)

9100 FORMAT (3A10:'/4A1.1)

9110 FORMAT (1H1+20X+3A10:'/4A10://)

9120 FORMAT (1X,19HINFORMATION INPUT: /15X,21HMAXIMUM ITERATION SET+1

9130 FORMAT (5X,47HMAXIMUM ALLOWED CHANGES PER ITERATION IN Y AND Z,27M

1DIRECTIONS RESPECTIVELY),1PE10.5,5H AND 1PE10.5,7H METERS)
Program Noise 73/172 TS

9140 FORMAT (5x,22hInitial Alfa and Beta: 13x+4hAlfa+16x+4hBeta+5f+1) A 4000
10x+11+1x+1PE16.9+4x+1PE16.9)
9150 FORMAT (//,1x,10hIteration +13+1x,14hTotal Cost Is +1PE16.9+1
1+5+22hTrue Annoyance(n) Is +1PE16.9+5x+2hPenalty Due to AI
2hCraft Constraints Is +1PE16.9//)
A 4020
A 4030
9160 FORMAT (10x,12+17+1hY Gradient Is +1PE16.9) A 4040
9170 FORMAT (10x,12+17+1hZ Gradient Is +1PE16.9) A 4050
9180 FORMAT (1//,1x,13+1hPercent Change in Co. But
1hLess Than .001%, Program Stops) A 4060
9190 FORMAT (10x,11+12+1x+1PE16.9+4x+1PE16.9) A 4070
9200 FORMAT (10x,41hReach Maximum Iteration Set, Program Stop) A 4080
9210 FORMAT (10x,7+1x+1hHalf = +12) A 4090
9220 FORMAT (10x,10+12+1hTrajectory / / 10x+12+1hCoordinate, 8x+12+1hCoordinate
1x+5+12+1hCoordinate / / 12x+7+1hMeter) A 4100
9230 FORMAT (10x,3+1PE16.9+4x)) A 4110
END

450008 CM Storage Used 7.828 Seconds
SUBROUTINE COST (IGRAD, IWRITE, XOCAP, YOCAP, ZOCAP, XFCAP, YLCAP, ZLCAP, THETA)

COMMON ALFA(5), BETA(5), POSIT(5), 3, ARRAY(578, 9), NMAP
COMMON /CURVE_ YCURVE(51), ADY(51), ADDY(51)
COMMON /AIRPORT/ XPORT, YPORT, ZPORT
COMMON /AC/ XYF, Z
EXTERNAL FCN

PNALTY = 0.
XCAP = XOCAP
PI = ATAN(I) * 4.
C2 = PI/ABS(XFCAP - XCAP)
C3 = ABS(XFCAP - XCAP)/C2
DO 10 I = 1, NMAP
ARRAY(I, 5) = 0.
10 ARRAY(I, 5) = 0.

* MULTIPLY BY EXPONENTIAL TERM SUCH THAT THE FINAL
* HEADING OF AIRCRAFT IS TOWARD THE RUNWAY

DO 20 J = 1:5
TRIG0 = FLOAT(J) * (XFCAP - XCAP) * C2
Y3 = Y3 + ALFA(J) * SIN(TRIG0)
Y8 = Y8 + BETA(J) * SIN(TRIG0)
Y6 = Y6 + FLOAT(J) * C2 * ALFA(J) * COS(TRIG0)
Y7 = Y7 + FLOAT(J) * C2 * BETA(J) * COS(TRIG0)
DLCAP = Y2 * Y3
ZLCAP = ZOCAP + DLCAP
YCAP = DLCAP + YCURVE(I)

* AIRCRAFT CONSTRAINTS

C = Y2 * Y6 + Y3 * Y5

PAGE 1
SUBROUTINE COST

DY = DY + ADY(I)
DY = Y2*Y7+2.*Y5+Y6+Y3*Y5/C3
DY = DDD + ADDY(I)
DY = DDD/(1+DY**2)
DZ = Y2*Y9+Y5+Y6
DZ = DZ*TAN(PHI)
DZ = 0.

PNALTY = PNALTY + (DY/100.)**2*(20.)*3**(10.)*2**(20.)
X = XCAP*COS(THETAI)+YCAP*SIN(THETAI)-ZCAP*COS(THETAI)*SIN

Y = XCAP*SIN(THETAI)+ZCAP*COS(THETAI)*SIN

Z = XCAP*COS(PHI)+ZCAP*COS(PHI)

DO 40 K = 1,NMAP
RANGE = ((X-ARRAY(K,1))**2+(Y-ARRAY(K,2))**2)**2.5
DB = 115.-22.5+ALOG10(3.261*RANGE/500.)
IF(DB.LE.ARRAY(K,1)) GO TO 40

ARRAY(K,4) = n
IF (ARRAY(K,3),LT,55.) GO TO 40
IF (ARRAY(K,3),EQ,0.) GO TO 30

Z = XCAP*SIN(PHI)+ZCAP*COS(PHI)

DO 40 K = 1,NMAP
RANGE = ((X-ARRAY(K,1))**2+(Y-ARRAY(K,2))**2)**2.5
DB = 115.-22.5+ALOG10(3.261*RANGE/500.)
IF(DB.LE.ARRAY(K,1)) GO TO 40

ARRAY(K,4) = n
IF (ARRAY(K,3),LT,55.) GO TO 40
IF (ARRAY(K,3),EQ,0.) GO TO 30

CONTINUE

CALL GAUSS

ARRAY(K,5) = TEMP*SMALLP
GO TO 40

CONTINUE

IF (WRITE,EQ.0) GO TO 50
II = I
POS(I1+1) = X
POS(I1+2) = Y
POS(I1+3) = Z
XCAP = XCAP+DLXCAP

PEOPLE = 0.
DO 60 K = 1,NMAP
IF (ARRAY(K,5),EQ,0.6) GO TO 60
PEOPLE = ARRAY(K,5)+PEOPLE

CONTINUE

FX = 0.
DO 70 K = 1,NMAP
ARRAY(K,5) = ARRAY(K,5)/PEOPLE
FX = FX+ARRAY(K,5)

CONTINUE
SUBROUTINE COST 73/172 TS

115 70 CONTINUE
TOTAL = FX*PNALTY
RETURN
END

41000B CM STORAGE USED .874 SECONDS
SUBROUTINE MONIT

COMMON ALFA(5), BETA(5), POSIT(13, 3), ARRAY(578, 9), NMAP
COMMON /SCALE/ XMIN, XINC, YMIN, YINC
COMMON /CRTIT/ (10)
DIMENSION X(1026), Y(1026)
DIMENSION XP(53), YP(53), ZP(53), NA(5), NB(3)
EQUIVALENT (XP(1), ARRAY(1, 1)), (Y(1), ARRAY(1, 1))
DATA NB/10HTOTAL POPULATION ANN. HOYANCE = /
SUBROUTINE GAUSS

SUBROUTINE GAUSS (XN,XX,YN,YX,FCN,FINT)
COMMON /AC/ XA,YA,ZA
DIMENSION X(5), Y(I), F(5), XI(I), W(I)
DATA XI,W,-0.577350269,0.577350269,0.0,0.1,0.0,0.2/1
C
C GAUSSIAN QUADRATURE INTEGRATION WITH FOUR POINTS
C
DO 10 I = 1,N
   Y(I) = (YX-YN)/2.*XI(I)+(YX+YN)/2.
10   X(I) = (XX+XN)/2.*XI(I)+(XX+XN)/2.,
FINT = 0.
DO 30 J = 1,N
   F(J) = 0.
20   DO 30 I = 1,N
      F(J) = F(J)*X(I)*W(I)*FCN(X(I),Y(J))
   30 FINT = FINT+W(I)*F(J)
FINT = FINT*(YX-YN)/2.
RETURN
END

100008 CM STORAGE USED .186 SECONDS
FUNCTION FCN (X,Y)  
COMMON /AC/ XA,YA,ZA  
RANGE = SQRT((X-XA)**2+(Y-YA)**2+ZA**2)  
ARG = 129.12-22.5*ALOG10(RANGE)  
FCN = 13.36E-6*(1.2*10**((0.03*ARG)+1.43E-4)*10**((108*ARG))))  
RETURN 
END  

410008 CM STORAGE USED .100 SECONDS
COST REPORT FOR LISTOAF

04/27/79

RESOURCES

BILLING RATE

UNITED USED

COST

CENTRAL PROCESSOR
$105.00 /HOUR
9.314 CP SECONDS

$27

PERIPHERAL PROCESSOR
20.00 /HOUR
9.737 PP SECONDS

$25

I/O
80.00 /HOUR
2.926 10 SECONDS

$7

FIELD LENGTH
3.00 /KILO-WRD-HOUR
205.576 KILO-WRD-SECS.

$17

(BASIC COST EXCLUDES LINES PRINTED, CARDS PUNCHED
AND PLOTTER TIME CHARGES)

JOB PRIORITY 3
PRIORITY COST FACTOR 1.00
APPROXIMATE ADJUSTED COST 

AS OF LAST ACCOUNT UPDATE: ACCOUNT EXPIRES 04/30/79, FUNDS LEFT $ 6037.31

04/27/79 UVA NOS/BE 1.2 LEVEL 454-03/11/78
11.45.47.LISTOAF FROM *GD/AB
11.45.47.LIST-M3117A-T100.
11.45.47.ATTACH=0=NEW=0=10.
11.45.47.PF CYCLE NO. = 002
11.45.47.FTN(=G)
11.45.59. 450008 CM STORAGE USED
11.45.59. 9.292 CP SECONDS COMPILATION TIME
11.45.59. STOP
11.46.00,EJ END OF JOB, AB

PRINT COST $000.88 LISTOAF /// END OF LIST /// 0000863 LINES
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UNIVERSITY OF VIRGINIA
School of Engineering and Applied Science

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