THERMAL CONTROL OF HIGH ENERGY NUCLEAR WASTE, SPACE OPTION

By Jerry A. Peoples
Program Development

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ACKNOWLEDGMENTS

This study resulted from the expertise of several persons who shared their knowledge in the solution of a specialized problem statement provided by Dr. Rowland E. Burns, EL24. Dr. Burns' challenge was to develop an approach for cooling high energy density nuclear waste material and provide a computerized thermal model which could give quick turnaround for configuration changes. First, a discussion with Mr. Kearns resulted in the verification of the engineering soundness of the technical approach. Second, Messrs. Carl Colley, PD23, and Dave Mercier, PD33, developed the mathematics of geodetic structures relating to the thermal model. Finally, all of these ideas were coordinated by two computer programs discussed herein. The models for data output were computerized by Patricia Sage, CSC.
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<th>Symbol</th>
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<td>A</td>
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<td>ε</td>
<td>Waste surface emittance (0, 80)</td>
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<td>Element length of subdivided icosahedron — in.</td>
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TECHNICAL MEMORANDUM

THERMAL CONTROL OF HIGH ENERGY NUCLEAR WASTE, SPACE OPTION

INTRODUCTION

The space option for nuclear waste disposal presents some thermal problems which must be solved and assessed before feasibility by space disposal can be established. Primarily, the thermal problem is one of maintaining the waste temperature below the melt conditions. This is especially true for those waste compounds having very high energy densities (>0.2 W/g). Studies thus far have resulted in development of the spherical waste configuration. Thus, the thermal problem is associated with the inner core of the waste package. The core temperature is

\[ T = T_0 + \frac{\dot{Q} R_0^2}{4k} . \] (1)

The core temperature \( T \) is, therefore, greatly affected by the surface temperature, \( T_0 \). On the ground where convection cooling can be made available, the core temperature is easier to maintain at acceptable levels. However, in space, the surface temperature is determined by the radiation properties of the waste surface. The surface temperature is established by its area and emittance. Increasing the surface area decreases the surface temperature but increases the second term, \( \dot{Q} R_0^2/4k \). The net result is always an increase in the core temperature with an increase in \( R_0 \).

Thus, in space, very little control can be exercised over the core temperature. For a homogeneous waste distribution, the temperature is best limited by limiting the energy generating characteristics and mass size. However, this is self-defeating since nuclear waste has an inherently low conductivity and some of the most desirable waste to be disposed of has high energy content. Also, it is desirable to use the Shuttle payload capability for each launch.

The purpose of this study is to suggest a technique whereby the effective conductivity of the waste can be increased. This allows core temperature control with larger masses having larger energy contents. The concept is to bring the energy within the core to the surface by means of cylindrical rods having high conductivity. This does not reduce the surface temperature but does reduce the temperature difference occurring between the surface and core.

The objective herein is to evaluate this concept and to establish the effectiveness of the cooling rods as greater waste masses having greater energy densities are considered. Also, the effects of the number of rods and their size will be assessed.

The cooling rod concept is also important because of the way it lends itself to manufacture of the nuclear waste package. This advantage is very important and is the motivation for studying the rod concept in detail. This cooling rod packaging process consists of three basic steps (in order):

1) A steel hemispherical shell with rod holes located on its surface, which will later receive the rods, is packed with waste material by a suitable packing mechanism. There are thin membranes over the rod holes to contain the material during the packing process.
2) After packing two hemispheres, they are brought together and welded. The cooling rods are then inserted through the rod holes. There may be as many as 300 to 500 rod holes. The cooling rod pierces the membrane as it is driven into the waste material.

![Diagram of spherical package with weld seam, rod holes, and cooling rod driven through hole into waste material.]

3) After each rod is driven flush, it is welded to the sphere's surface. The package is now complete for deportation into space.

![Diagram showing waste material, steel shell, cooling rod, and weld.]

There are several advantages to this technique besides the symmetrical configuration which lessens the thermal analysis problem. First, there is less chance of waste spillage and tool contamination. This is an important advantage and motivates use of this concept. Second, the extra packing of the waste material as a result of driving the rods reduces the contact resistance between the rod and waste material.
GENERAL THERMAL CHARACTERISTICS

The temperature distribution within a spherical waste package having a homogeneous mass distribution can be found in Footnote 1.

\[ T = T_0 + \frac{\dot{q} R_o^2}{4k} \left[ 1 + \left( \frac{R}{R_o} \right)^2 \right] \]  \hspace{1cm} (2)

The temperature, \( T \), occurs at Radius \( R \), with \( R_o \) being the radius of the spherical package. At the core, \( R/R_o \) equals zero and equation (2) reduces to equation (1). At the surface, \( R/R_o \) is unity, and the temperature in question if equal to the surface temperature is determined by the radiation characteristics at the surface:

\[ T_o = \left[ \frac{Q}{\sigma \epsilon A} \right]^{1/4} \]  \hspace{1cm} (3)

Equation (3) states that in the steady state the total energy rate generated must be radiated at the surface. The required surface temperature is \( T_o \).

Equations (2) and (3) are very simple to apply but are not in terms of the variables which the nuclear material is usually specified. In general a waste mass, \( M \), is given which has a given energy density, \( E \), and a mass density, \( \rho \). Noting that:

\[ \dot{q} = E \rho \]

\[ R_o = \left[ \frac{3M}{4\pi \rho} \right]^{1/3} \]

\[ Q = EM \]
Equation (2) can be manipulated to give a more useful form in terms of stated characteristics. These substitutions in equation (2) give

\[
T = \left( \frac{E_M}{4\pi\sigma} \left[ \frac{3M}{4\pi\rho} \right]^{2/3} \right)^{1/4} \cdot \frac{E\rho}{4k} \left[ \frac{3M}{4\pi\rho} \right]^{2/3} \left[ 1 - \left( \frac{R}{R_0} \right)^2 \right].
\]

A typical temperature distribution resulting from this equation is given in Figure 1. In this particular example the energy density of 0.002 W/g is sufficiently low that cooling conduction rods are not required.

Most of the time, only the core temperature is of interest since therein lies the highest temperature. Additional characteristics for just the core temperature are given in Figure 2. Waste mass is the independent variable with energy density as an argument. A density of 2.88 g/cm³ and conductivity of 1.8 W/m·°K are taken as typical waste characteristics. It is noted that density and conductivity are illusive parameters. The conductivity is particularly difficult to characterize.

Figure 2 illustrates the difficulty of keeping the core temperature below 1800°F for energy densities above 0.004 W/g. Yet, some nuclear waste has densities above 0.2 W/g. Cooling rods are necessary for these energy levels.

Figure 3 has been included to illustrate the importance of conductivity. This figure emphasizes the importance of achieving an effective conductivity of at least 4 or 5 W/m·°K. From equation (4), it is realized that as the conductivity becomes greater, the core temperature approaches the surface temperature. The objective herein is to demonstrate how the cooling rod increases the effective conductivity of the waste material.

The effective conductance can be found by solving equation (4) for \(k\) and taking the value of \(T\) as the core temperature which results from application of the cooling rods:
Figure 1. Temperature distribution for a spherical waste configuration having a homogeneous mass distribution.
Equation (5) will be utilized later to compute the effective waste conductivity resulting from cooling rods.
Equation (4) has been computerized for quick assessment of core temperature. Also, if the effective conductivity is known, the core temperature can be quickly computed without resorting to the SINF4A system numerical difference analyzer program. A typical output from this program is given in Table 1. The program iterates waste mass up to 10 000 kg and energy densities up to 0.008 W/g. The values of these selected ranges can be easily changed as is discussed in Appendix A. In some material, the temperature which can be tolerated is not known. For this reason, a tool for quick evaluation of any proposed nuclear waste package can be made available to those whose expertise are with nuclear material.
### Table 1. Data Obtained from Equation (4)

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<tr>
<th>Temperature (Kelvin)</th>
<th>Temperature (°C)</th>
<th>Current (mA)</th>
<th>Voltage (V)</th>
<th>Resistance (Ω)</th>
<th>Power (W)</th>
<th>Energy Density (Watts/cm²)</th>
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</table>
THERMAL MODEL GEOMETRY

As already mentioned, the core temperature cooling mechanism consists of conductive rods which penetrate the waste radically within a few inches of the spherical center. The rod radius, \( r_o \), results in interference between rods as they approach the center (Fig. 4). The radius of the resulting sphere at the center can be found by equating the area of the inner sphere with the sum of the cross-sectional areas of the rods plus 10.7 percent:

\[
4\pi r_o^2 = 1.107 \pi N r_c^2
\]

\[
r_c = \frac{r_o}{2} \sqrt{1.107 N}
\]

For 362 rods, the inner core diameter will be 2.5 in. for 0.25-in. radius rods. This core will probably be made of steel to provide structural integrity to the waste package. In any event, the allowable rod length must be accounted for by this method. The first step to be taken is to define a relationship for the number of rods. At first, it may appear that any arbitrary number could be used. However, this is not the case if the rods are to be equally distributed throughout the sphere.

It is necessary to establish the relationship between the number of rods and the distance between rod tips as they enter the spherical surface. Finally, the distance between all adjacent rod tips would be equal. The basis for establishing these relationships is a surface constructed of 20 equilateral triangles. The solid figure formed by a single triangle is known as an icosahedron. Representation of a spherical surface by 20 icosahedrons is shown in Figure 5.

Each equilateral triangle can be subdivided into small triangles as illustrated in Figures 6 and 7. Each intersection lies on the surface of the sphere. As the number of triangles increases each icosahedron surface approaches an equilateral spherical surface. In Figure 6 the basic icosahedron leg is divided equally into two elements. This results in formation of four triangles. The number of equal subdivisions in each icosahedron sides is known as the geodetic frequency. Figure 7 has a geodetic frequency of six.
Figure 4. Cooling rod concept which conducts heat energy from the core to radiating surface of the waste.

For purposes of the thermal control mechanism, a rod would be located at each interaction. The relationship between the geodetic frequency and the surface characteristics are:

Number of Vertices \(10 r^2 + 2\)

Number of Faces \(20 r^2\)

For example, a geodetic frequency of 6 results in 362 rods and 720 triangles formed by each rod end. Since the frequency must be an integer, the possible number of rods grows as 12, 42, 92, 162, 252, 362, 492, etc. All intermediate numbers of rods are not possible with this configuration.
Figure 5. A solid spherical surface formed by 20 icosahedrons.
Figure 6. A spherical surface generated by a geodetic frequency of 2.
Figure 7. A spherical surface generated by a geodetic frequency of 6.
For a frequency of one, the geometry reduces to the basic 20 equilateral triangles. Sides which form the surface of the icosahedron are equal. However, geodetic frequencies above one do not produce all equilateral triangles. The subdivided triangles do not result in equilateral triangles, although some sides are repeated. Consider the icosahedron surface of Figure 8 where the sides have been subdivided with a frequency of four. Letters indicate members of the same length. The length of each member can be found by multiplying the encompassing sphere radius by a correction factor, CF. Some of these correction factors are given in Table 2. For an example, the length of the "c" member for a frequency of 4 is the radius times 0.294530. It is unfortunate that this situation exists; however, for purposes of the thermal model and rod placement, the problem becomes greatly simplified by generalizing on a single relation between the sphere radius and element length. For purposes of the thermal model, the following generalization has been made:

\[
\frac{R_o}{l} = 0.8684 \quad .
\]

Equation (6) is based upon the length trends occurring within the icosahedron. It is noted that this equation cannot be used for actual design construction but does allow thermal analysis of a single rod to represent all rods.

Figure 9(a) is a plain view projection of a portion of an equilateral spherical triangle. All of the element lengths are equal as per equation (6). Each intersection is the location of a rod. Around each rod is a prescriber circle of diameter, \( l \), the element length. Each of these circles represent a cone, each having identical thermal distribution.

An element cone is shown in Figure 9(b). This cone is defined by dimensions \( l \) and \( R_o \) as related by equation (6). The number of cones is equal to the number of vertices which is, \( 10v^2 + 2 \). Thus, the mass element to be thermally modeled is defined. The complex thermal and mechanical configuration has been reduced to a simple cone with a single rod located on its radius centerline. The thermal distribution in this cone characterizes the thermal characteristics of the entire nuclear waste package. It is noted that a small area exists outside of the prescribed circles of Figure 9(a).

The energy represented by this area is distributed proportionally within each cone. Thus, the temperature at the outer surface of the cone is considered to be representative of the temperature between intersecting cones. This deviation from the actual configuration is considered to be minor and the simplicity it brings to the model is warranted.
Figure 8. Subdivision of an icosahedron to form an equilateral spherical surface.

TABLE 2. ELEMENTS LENGTHS, $l$, CORRECTION FACTORS (CF)

<table>
<thead>
<tr>
<th>FREQUENCY - $\nu$</th>
<th>CF (A)</th>
<th>CF (B)</th>
<th>CF (C)</th>
<th>CF (D)</th>
<th>CF (E)</th>
<th>CF (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0514622</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>.8180339</td>
<td>.548633</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>.348615</td>
<td>.403548</td>
<td>.412414</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>.253184</td>
<td>.295241</td>
<td>.294530</td>
<td>.312889</td>
<td>.3245196</td>
<td>.2915681</td>
</tr>
</tbody>
</table>
Figure 9. Definition of a characteristics mass element for a spherical nuclear waste package.

\[ R_o = \nu \cdot 0.8684 \]
THERMAL MODEL NODE LOCATIONS

Now that the mass element to be thermally modeled has been defined, the node location and their respective capacitance and conductance are to be determined. Since each node representing the nuclear waste is an energy generating node, it became an intuitive guideline to give each node equal mass. In this way, the energy generated would be distributed equally within the model network. This approach reinforces the fidelity of the model since a finite number of nodes represents an infinite number of energy generating points. To accomplish this guideline the sphere was divided into four concentric shells each having the same mass. This subdivision of the element cone is illustrated in Figure 10. Each shell is numbered 1 to 4 and they cut through the cone as shown. For proper location of the shells, each section of the cone will have the same mass. If \( n \) is taken as the shell number, the necessary relationship between the first shell, \( n = 1 \), radius, \( R_1 \), and the radius \( R_n \), at \( n \) is

\[
\frac{4}{3} \pi R_n^3 = n \frac{4}{3} \pi R_1^3
\]

\[
R_n = n^{1/3} R_1
\]

(7)

The spherical radius, \( R_0 \), will be taken as the independent variable with \( n = 4 \). Therefore, \( R_1 \) becomes

\[
\frac{R_{n=4}}{R_1^{1/3}} = R_1
\]

(8)

where

\[
R_{n=4} = R_0
\]
Figure 10. Spherical shells having equal mass.

For an example, if the waste radius is 2.4 ft, $R_1$ becomes 1.5119 ft. From this value equation (7) can be used to find $R_n$ at other shell locations. A scaled sketch of the waste package divided into shells as above is given in Figure 11.

To complete the thermal model, the mass within the top three shells is divided into doughnuts as shown in Figure 12. The generalized radius of the outside doughnut is $r_{n2}$. The inside doughnut radius is $r_{n1}$. For equal mass within each doughnut, the following relationship exists:

$$2r^2_{n1} = r^2_{n2} + r^2_o.$$  (9)

For similar triangles,

$$\frac{r/2}{R_0} = \frac{r_{n2}}{R_n}.$$
Figure 11. Scale drawing of four shells with radii determined by $R_n = n^{1/3} R_1$.

Figure 12. Nodal structure of characteristic mass element.
Thus

\[
r_{n2} = \frac{R_n}{2(R_o/\ell)}
\]  

(10)

which determines \( r_{n1} \) of equation (9). Equations (7) through (10) completely describe the geometry of the mass element. A summary of these dimensions are given in Figure 13.

Figure 13. General dimensions of the mass element.
The nodal system consists of six doughnuts, each having different dimensions as given in Figure 13. Also, there is node 10 which represents a cone shape. The mass in the cone is twice the mass in any single doughnut. The cooling rod common to each set of doughnuts and cone has nodes 50, 40, 30, and 20, respectively. The R-C network representing the model is illustrated in Figure 12. The nodal system consists of 13 nodes with 99 being a space node and node 17 the plate which contains the waste material.

It is noted that the model as described here has not been presented to the depth where equations are available for computing the capacitance and conductance between nodes. The intention is to describe the model in sufficient detail that the modeling approach can be understood. Since it was anticipated that the model would have to be integrated many times, the model was completely computerized and the output was made compatible with the SINDA data block forms. A typical output from this program is given in Figure 14. This program is discussed in Appendix B.

DATA RESULTS

Before the computerized thermal model was complete, it was difficult to speculate upon the effectiveness of the cooling rods. The entire purpose was to assess the capability of the rods in controlling the core temperature. Results from one of the first sample cases are shown in Figure 15. While the specific temperatures obtained from this sample problem are of an academic interest only, it indicated that the cooling rod concept can be very effective in reducing core temperature. For this sample case, a mass of 10 480 lb was chosen with 362 half inch rods. The energy density was 0.2071 W/g. Without the cooling rods the core temperature was over 80 000°F. With the cooling rods the core temperature dropped to 8500°F. This is a factor of 10, a result worth noting. The surface temperature is about 1900°F. The surface temperature is the same with and without the rods since, in the steady state, the same energy must be radiated at the surface. The cooling rods used in this example were molybdenum which has a conductivity of 72 Btu/hr-ft-°F at 3000 °F.

To establish the limit of capability of the cooling rods, the passive molybdenum rods were replaced with heat pipes. These results are indicated in Figure 16. Most of these temperatures can be tolerated. Thus, in the limit, the rod cooling concept can maintain core temperature for large masses and high energy densities. It is of interest to note that the reduction in core temperature with the number of rods is not significant. That is, the reduction in
Figure 14. Typical computer output which generates nodal data for SINDA.
Figure 15. Effectiveness of cooling rods upon the temperature distribution for high density nuclear waste material.

Figure 16. Core temperature results from heat pipe cooling rods.
core temperature by using 810 rods is not significant from that of 492 rods. This is especially true for waste masses below 10 480 lb. Thus, geodetic frequencies greater than 7 (492 rods) is not warranted in view of the additional complexity of building the waste package. The effective conductivity of the entire package was calculated for the 10 480-lb package using equation (5). The effective conductivity was computed at 122 W/m·°K.

It became of particular interest to assess the performance of the cooling rods by reducing the waste mass while keeping the same high energy density. These results are shown in Figure 17. Three cases are shown for different diameters of rods. Since most waste temperatures must be less than 3500°F, the waste mass must be limited to 2000 to 3000 lb. This implies that waste material which has high energy density must consist of multiple packages of relatively low mass.

Figure 18 corresponds to Figure 2 except that much higher energy densities are considered as the argument. To approach a single waste package of 10 000 lb, the energy density must be below 0.1 W/g to keep the core temperature below 3500°F.
Figure 17. Core temperature using molybdenum rods.
Figure 18. Core temperature with energy density as an argument.
APPENDIX A

TEMPERATURE DISTRIBUTION FOR A SPHERICAL HOMOGENEOUS MASS

The computer program presented in this appendix represents the solution to Equation (4). There are three inputs: waste density, waste conductivity, and surface emittance. These parameters are noted as RHO, XK, and EMISS, respectively per statement 17 of the main program. Density has units in g/cm³ and conductivity in W/m-°K. An emittance value of 0.80 has been selected for all calculations herein. The range of mass is controlled by state 19 of subroutine PRNT. As shown the mass is iterated in increments of 2000 kg, beginning with 1000 kg up to 10 000 kg.

For each mass the energy density is initiated from 0.001 to 0.005 W/g as controlled by statements 44 through 45 of the same subroutine. The mass and energy values can be alternated to achieve any combination of mass and energy density desired. It is noted that SIG, the Stefan Boltzmann constant, has units of W/m²-°R.

Vol — Volume — m³

RAD — Radius — m

QDOT — Energy Rate — W/m³

QT — Total Energy Rate — W/kg
JPEEPS!NEPBJPJPEP!E!1.COMDEX
1
2  COMMON PROC
3
4  COMMON VARIABLES
5
6  COMMON RHO, XX, ERMISS, E, M, T, TF, TD, RADI, QT
7  END

FURRUR 27RZAH4 E33 5L73RI 04/17/79 10:05:100
JPEEPS!NEPBJPJPEP!E!1. NEWMAN
1  INCLUDE COMDEX
2
3  INITIALIZE TEXTRONIX GRAPHICS PACKAGE
4  CALL INIT(100)
5  CALL TERM(3,1024)
6  CALL CHRSIZE(14)
7  CALL TSEND
8
9  LOOP - UNTIL ALL CASES HAVE BEEN DONE
10
11  VARIABLES ARE AS FOLLOWS
12  RHO - WASTE DENSITY
13  XX - WASTE CONDUCTIVITY
14  ERMISS - SURFACE EMISSANCE
15  READ (5,100,END=999) RHO, XX, ERMISS
16
17  PRINT AND COMPUTE DATA
18  CALL PRINT
19
20  LOOP - BACK TO NEXT CASE
21  GO TO 10
22
23  100 FORMAT ( )
24
25  999 STOP
26
27  END

ORIGINAL PAGE IS OF POOR QUALITY
SUBROUTINE PRINT

INCLUDE COMDEX

CALL NEWPAQ

PRINT OUT INPUT DATA
PRINT 101,AM
PRINT 102,HX
PRINT 103,EMISS

LC = LINE COUNT
LC=6

E = ENERGY PER UNIT MASS
E=.001
30 DO 60 M=2000.10000.1000
60 CALL HDCOPY

DO COMPUTATIONS
CALL COMP

TEST NUMBER OF LINES ON PAGE
IF (LC=63) 50,40,50

IF THERE ARE 63 LINES THEN
40 CALL HDCOPY
CALL PRINT
CALL SEND

RESET COUNTER TO COUNT NEXT PAGE
LC=0

PRINT HEADINGS
PRINT 104
PRINT 105
PRINT 106
LC=LC+5

ELSE PRINT ANOTHER LINE ON PAGE
50 PRINT 107,T,T,F,T,M,RADI,GT,E
LC=LC+1

CONTINUE
E=E+.001

TEST E TO SEE WHEN TO STOP
PRINTING DATA
30 IF (E-.005) 20,30,70

IF E IS GREATER THAN .005 THEN
70 CALL HDCOPY
CALL SEND
ELSE REBEGIN DO LOOP

100 FORMAT ( )
101 FORMAT (1X,3FH WASTE DENSITY: G/CM3 ,F10.2)
102 FORMAT (1X,3FH WASTE CONDUCTIVITY: WATTS/DEG.K M,F10.2)
57:  103 FORMAT (10X,35X SURFACE EMITTANCE ,F10.2,/) 
58:  104 FORMAT (12X,4HCORE,14X,4HCORE,12X,7MSURFACE,11X,5NWASTE,11X, 
59:  11X,SWASTE,11X,5HTOTAL,12X,7ENERGY,/) 
60:  105 FORMAT (12X,4HTEMP,14X,4HTEMP,14X,4HTEMP,13X,4HMWASS,10X,6HRADIUS, 
61:  11X,SWENERGY,12X,7HDEGREE) 
62:  106 FORMAT (10X,8HEKELVIN,10X,8HEDEG. F3,10X,8HEDEG. F3,11X,4HKG3, 
63:  11X,3HEP.,11X,7HEWATT53,10X,5HEWATT53/) 
64:  107 FORMAT (F17.1,F18.1,F17.1,F18.1,F17.1,F18.1,F17.1,F18.4) 
65:  
66:  RETURN 
67:  END 
EOF  1027 SCAN: 10
01

FUPPUR 27R34W4  E33  SL73RI  04/17/7G  10:08:14

JPEEP> BIN208#JPFILE11.COM

1 SUBROUTINE COMP
  C COMPUTES EQUATIONS FOR NEWMAN
  C INCLUDE COX
  C VARIABLES ARE AS FOLLOWS
  VOL - VOLUME
  RAD - RADIUS
  QDOT - ENERGY RATE PER VOLUME
  GT - ENERGY RATE
  SIG - STEFAN-BOLTZMANN CONSTANT
  TD - SURFACE TEMPERATURE
  TX - SURFACE TEMPERATURE - KELVIN
  T - CORE TEMPERATURE - KELVIN
  TF - CORE TEMPERATURE - FAHRENHEIT

  VOL = (3.1416/4.0)*SIG*3.333
  QDOT = GT/3.333

  SIG = 5.67E-09
  TO = QDOT/(4.0*3.1416*SIG*3.333)

  TF = TO/360.0*273.16

2 RETURN
3 END

31
APPENDIX B

PRESINDA, SINDA, DATA BLOCK DEVELOPMENT

This appendix presents the PRESINDA program. This program computes the conductance, capacitance, and energy rate of energy generating nodes. There are 10 inputs as noted in statement 24 of the main program. Units are given in SUBROUTINE PRNT statements 136 through 145.
C INCLUDE COMDEK
C INITIALIZE TEKTRONIX GRAPHICS PACKAGE
CALL INITT(120)
CALL TREAD(3,31)
CALL TSEND
C REWIND 8
C LOOP - UNTIL ALL CASES HAVE BEEN DONE
C RHU - WASTE DENSITY
C RC - RADIUS OF CONDUCTING ROD
C XNL - GEOMETRIC FREQUENCY
C R - WASTE RADILS
C CPU - HEAT CAPACITY, WASTE
C CPR - HEAT CAPACITY, ROD
C KXW - CONDUCTIVITY, WASTE
C KXR - CONDUCTIVITY, ROD
C RHO - ROD DENSITY
C ED - ENERGY DENSITY
10 READ (5,100,E=999) RHU,RC,KXU,R,CPU,CPR,KXW,
50 KXR,RHO,ED
C DO COMPUTATIONS
C CALL COMP
C PRINT OUT DATA
C CALL PRNT
C LOOP - BACK TO NEXT CASE
C GO TO 10
C 100 FORMAT ( )
999 STOP
END
SUBROUTINE PRINT
INCLUDE CMODEX
CALL NEUGAG
PRINT INPUT DATA
WRITE (8,148) 18,5(SIS)
PRINT 182,RO
WRITE (8,143) 5(S7)
PRINT 183,XRU
PRINT 184,R
PRINT 185,CPW
PRINT 186,CPR
PRINT 187,XU
PRINT 188,XR
PRINT 189,RO
PRINT 190,ED
PRINT 111,XN
PRINT 112,XF
PRINT 113,RIR
PRINT 114,RL
PRINT NODE DATA HEADINGS
WRITE (8,116)
WRITE (8,117)
WRITE (8,118)
WRITE (8,119)
PRINT 118
FD=200.
SET ND EQUAL TO NUMBER OF NODE
DO 18 N=1,7
WRITE (8,148) ND,DF,DM(N)
10 PRINT 148,ND,DF,DM(N)
WRITE NODE DATA TO A
WRITE (8,148) ND,DF,DM(N)
WRITE 8,149
DO 20 M=8,12
WRITE (8,148) ND,DF,DM(N)
WRITE (8,141)
WRITE (8,118)
PRINT SOURCE DATA HEADING
WRITE (8,120)
1151 IC(19)=40
1161 JC(19)=50
1171 IC(20)=17
1181 JC(20)=50
1191 WRITE (8,144) IC(1),JC(1),IC(1),JC(1),IC(1),JC(1) 1701 END
1191 WRITE (8,145) IC(20),JC(20),IC(20),JC(20) 1701 END
1201 WRITE (8,146) 1701 RETURN
1211 WRITE (8,147) 1711 FORMAT (""******")
1221 CASES WRITTEN ON FILE 1721 WRITE (8,148) 1731 CALL WCOPY
1231 CALL NEWPAG
1241 CALL TSSEND
1351 181 FORMAT (1X,35H WASTE DENSITY LB/FT3) 1771 FORMAT (1X,14H RADIUS OF CONDUCTING ROD; FT) 1821 FORMAT (1X,35H GEODETIC FREQUENCY) 1831 FORMAT (1X,35H RADIUS OF WASTE SHELL; FT) 1841 FORMAT (1X,35H HEAT CAPACITY RADIANT BTU/LB FT) 1851 FORMAT (1X,35H ENERGY DENSITY RADIANT BTU/LB FT) 1861 FORMAT (1X,35H CONDUCTIVITY WASTE; BTU/HR-FT) 1871 FORMAT (1X,35H CONDUCTIVITY ROD; BTU/HR-FT) 1881 FORMAT (1X,35H NUMBER OF VERTICES) 1891 FORMAT (1X,35H TOTAL MASS; LBM) 1901 FORMAT (1X,35H RADIUS OF FIRST SHELL; FT) 1911 FORMAT (1X,35H RATIO OF GEODETIC ELEMENTS) 1921 FORMAT (6X,18H BCD 3 THERMAL SPCS) 1931 FORMAT (6X,12H BCD 9 ELEMENTS) 1941 FORMAT (6X,4H END) 1951 FORMAT (6X,15H BCD 3NODE DATA) 1961 FORMAT (6X,17H BCD 3 SOURCE DATA) 1971 FORMAT (6X,20H BCD 3 CONDUCTOR DATA) 1981 FORMAT (6X,29H BCD 3 CONDUCTOR DATA) 1991 FORMAT (6X,32H BCD 3 J RC ST DATA) 2001 FORMAT (9X,12H TIMES=1890) 2011 FORMAT (9X,18H OUTPUT=6) 2021 FORMAT (6X,16H BCD 3 ARRAY DATA) 2031 FORMAT (6X,15H BCD 3 EXECUTION) 2041 FORMAT (82MF DIMENSION K(200)) 2051 FORMAT (14F NDIM=200)) 2061 FORMAT (11F MTH=8) 2071 FORMAT (9Y,7H CNRMID) 2081 FORMAT (6X,17H BCD 3 VARIABLES 1) 2091 FORMAT (6X,17H BCD 3 VARIABLES 2) 2101 FORMAT (6X,18H BCD 3 OUTPUT CALLS) 2111 FORMAT (6X,17H TMT=MT) 2121 FORMAT (6X,17H BCD 3 END OF DATA) 2131 FORMAT (11X,15H ..FF.7,F7.4) 2141 FORMAT (11X,15H ..FF.4E12.6)
INCLUDE CONDCS

COMPUTE EQUATIONS FOR SOURCE DATA EQUATION

GET VARIABLES AND CONDUCTOR DATA EQUATIONS

IN COMPUTING

SUBROUTINE COMDCS

CONDCS
\begin{verbatim}
581 C(6)+(2.*X1.4)*XORT(A2+B7)/(B1-B3)/B3\text{XXW} 6 CN 1215
591 C(7)+(2.*X1.4)*XSORT(A2+B7)/(B2-B1)/B3\text{XXW} 6 CM 1216
601 C(8)+(3.*X1.4)*(A2-A1)/B3\text{XXW} 6 CM 1611
611 C(9)+(3.*X1.4)/(A2-B7)/B3\text{XXW} 6 CM 1014
621 C(10)+(3.*X1.4)*XINT(XORT(3.XXW))
631 \{2.X30\}/(4.1)*\text{XXW} 6 CN 1022
641 C(11)=(3.*X1.4)/(B3-B1)*\text{XXW} 6 CN 1426
651 C(12)=(3.*X1.4)/(B1-B3)*\text{XXW} 6 CM 1549
661 C(13)=(3.*X1.4)/(B3-B1)*\text{XXW} 6 CM 1626
671 C(14)=(3.*X1.4)/(A7-A8-B7)/B3\text{XXW} 6 CM 1313
681 C(15)=(3.*X1.4)/(A7-A8-B7)/B3\text{XXW} 6 CM 1517
691 C(16)=(B3)/(B3-B1)*\text{XXR} 6 CN 6417
701 C(17)=(B3)/(B3-B1)*\text{XXR} 6 CN 8536
711 C(18)=(B3)/(B3-B1)*\text{XXR} 6 CN 3048
721 C(19)=(B3)/(B3-B1)*\text{XXP} 6 CN 4880
731 C(20)=(1.714*9.8.283.143)/(X1.4)/(X4.1)/(X1.4)/(X1.4) 6 CN 1799
741 \{1.X32\}/(4.1)/(X1.4)/(X1.4) 6 CN 1799
751 END
\end{verbatim}
APPROVAL

THERMAL CONTROL OF HIGH ENERGY NUCLEAR WASTE, SPACE OPTION

By Jerry A. Peoples

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

C. R. Darwin
Director, Preliminary Design Office

James T. Murfin
Director, Program Development