STRESS-INTENSITY FACTORS FOR INTERNAL SURFACE CRACKS IN CYLINDRICAL PRESSURE VESSELS

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SUMMARY

Failures of many pressure vessels have been traced to surface cracks. Accurate stress analyses of these surface-cracked components are needed for reliable prediction of their crack-growth rates and fracture strengths. Because of the complexities of such problems, all investigators have used engineering estimates or approximate analytical methods to obtain the stress-intensity factors.

A few three-dimensional stress analyses of semi-elliptical surface cracks in pressurized cylinders have been reported recently. However, these investigators considered only an internal surface crack with a crack-depth-to-crack-length ratio of 1/3 and a wall-thickness-to-vessel-radius ratio of 0.1.

The purpose of this paper is to present stress-intensity factors for a wide range of semi-elliptical surface cracks on the inside of pressurized cylinders. The ratio of crack depth to crack length ranged from 0.2 to 1; the ratio of crack depth to wall thickness ranged from 0.2 to 0.8; and the ratio of wall thickness to vessel radius was 0.1 or 0.25. The stress-intensity factors were calculated by a three-dimensional finite-element method. The finite-element models employed singularity elements along the crack front and linear-strain elements elsewhere. The models had about 6500 degrees of freedom. The stress-intensity factors were evaluated from a nodal-force method. In this method, the nodal forces normal to the crack plane and ahead of the crack front were used to obtain the stress-intensity factors.

An empirical equation for the stress-intensity factors was fitted to the results of the present analysis as a function of crack depth, crack length, wall thickness, and vessel radius. The equation applies over a wide range of configuration parameters and was within about 5 percent of the present results.

The present results were compared to other analyses of internal surface cracks in cylinders. The surface-crack configuration had a crack-depth-to-crack-length ratio of 1/3 and a wall-thickness-to-vessel-radius ratio of 0.1. Results from the literature using a boundary-integral equation method were in good agreement (±2 percent) and those from a finite-element method were in fair agreement (±8 percent) with the present results.

The stress-intensity factors and equations presented herein should be useful in correlating fatigue-crack-growth rates and in calculating fracture toughness for the surface crack in a pressurized cylinder.
1. Introduction

Failures of many pressure vessels have been traced to surface cracks. Accurate stress analyses of these surface-cracked components are needed for reliable prediction of their crack-growth rates and fracture strengths. Because of the complexities of such problems, all investigators have used engineering estimates or approximate analytical methods to obtain the stress-intensity factors.

Some engineering estimates for the stress-intensity factors for surface cracks in pressurized cylinders have been made by Underwood [1] and Kobayashi [2]. Their estimates did not include the effects of wall thickness. Recently, however, Kobayashi, Emery, Polvanich, and Love [3] have estimated stress-intensity factors for internal surface cracks that did include the effects of wall thickness.

A few three-dimensional stress analyses of semi-elliptical surface cracks in pressurized cylinders have been reported recently. Atluri and Kathiresan [4] and McGowan and Raymund [5] used three-dimensional finite-element methods, while Heliot, Labbens, and Pellissier-Tanon [6] used the boundary-integral equation method, to obtain stress-intensity factor variations along the crack front for a limited range of configuration parameters. References [5] and [6] considered only an internal surface crack with a crack-depth-to-crack-length ratio of \( \frac{a}{c} \) and a wall-thickness-to-vessel-radius ratio of 0.1.

The purpose of this paper is to present Mode I stress-intensity factors, calculated by a three-dimensional finite-element method [7-9], for a wide range of semi-elliptical surface cracks in pressurized cylinders. The cracks were located on the inside of the cylinders. The ratio of crack depth to wall thickness ranged from 0.2 to 0.8; the ratio of crack depth to crack length ranged from 0.2 to 1; and the ratio of wall thickness to vessel radius was 0.1 or 0.25. The stress-intensity factors were calculated by using a nodal-force method [7-9]. An equation for the stress-intensity factors was also developed for a wide range of configuration parameters. The stress-intensity factor variations along the crack front are presented and, where possible, compared with other solutions from the literature.

2. Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( a )</td>
<td>depth of surface crack</td>
</tr>
<tr>
<td>( b )</td>
<td>half-length of pressurized cylinder</td>
</tr>
<tr>
<td>( c )</td>
<td>half-length of surface crack</td>
</tr>
<tr>
<td>( f_c )</td>
<td>ratio of boundary-correction factors (cylinder to flat plate)</td>
</tr>
<tr>
<td>( F )</td>
<td>boundary-correction factor for an internal surface crack</td>
</tr>
<tr>
<td>( G_{ij} )</td>
<td>boundary-correction factors for the ( j )-th stress distribution on crack</td>
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<tr>
<td>( K_I )</td>
<td>stress-intensity factor (Mode I)</td>
</tr>
<tr>
<td>( p )</td>
<td>internal pressure on the cylinder</td>
</tr>
<tr>
<td>( Q )</td>
<td>shape factor for an elliptical crack</td>
</tr>
<tr>
<td>( R_i, R_o )</td>
<td>inner and outer radii of cylinder</td>
</tr>
<tr>
<td>( t )</td>
<td>cylinder wall thickness</td>
</tr>
<tr>
<td>( x, y, z )</td>
<td>local Cartesian coordinates centered at crack mouth</td>
</tr>
</tbody>
</table>
X,Y,Z  global Cartesian coordinates
θ  angular measurement on cylinder
σ_j  applied stress on crack surfaces
ϕ  parametric angle of elliptical crack

3. Three-Dimensional Analysis
A surface crack in an internally pressurized cylinder is shown in figure 1. The elastic cylinder of wall thickness t, internal radius R, and length 2b, contains a semi-elliptical surface crack of length 2c and depth a on the inner surface of the cylinder. The stress-intensity factors for the surface-crack configurations were obtained by using a three-dimensional finite-element analysis.

3.1 Finite-Element Idealization
Two types of elements (isoparametric singular [7]) were used in combination to model the cylindrical vessels. Figure 2(a) shows a typical finite-element model for an internal surface crack. The model, which employed nearly 6500 degrees of freedom, idealized one-eighth of the vessel (0 ≤ θ ≤ 90 degrees and 0 ≤ X ≤ b). The isoparametric (linear-strain, eight-noded hexahedron) elements were used everywhere except near the crack front, where eight singularity elements, each in the shape of a pentahedron, were used [8]. The singularity elements produced a square-root singularity in stress and strain at the crack front.

A typical finite-element pattern along the crack plane is shown in figure 2(b). The finite-element models in the neighborhood of the crack were identical to those used in references [8] and [9] for surface cracks in flat plates, except that the models were curved to conform to the desired cylindrical shape. The vessel half-length-to-radius ratio, b/R, ranged from 1 to 5, the b/c ratio ranged from 10 to 50, and the t/R ratio was 0.1 or 0.25. Further details on modeling and formulation of the two types of elements used are given in references [7-9] and are not repeated here.

3.2 Boundary Conditions and Applied Loading
Symmetry boundary conditions were applied on the X = 0 plane; Y = 0 plane, and Z = 0 plane, and the model simulated a vessel with two symmetric surface cracks (180 degrees apart). The X = b plane was free.

The stress-intensity factor solutions were obtained by solving the complementary problem of applied stresses on the crack surfaces. Four applied stress distributions on the crack surfaces were analyzed: constant, linear, quadratic, and cubic. These stresses, which were applied to the crack surfaces as shown in figure 2(c), were symmetric about the y = 0 plane and were given by

$$σ_j = \left( \frac{z}{a} \right)^j \quad \text{for}\; j = 0 \; \text{to} \; 3 \quad (1)$$

where z is measured from the crack mouth toward the crack front. Solutions for these four stress distributions were superimposed to obtain stress-intensity factors for the pressurized cylinder. (These four solutions can be superimposed to obtain stress-intensity factors for other stress distributions, such as those caused by thermal shock.)
4. **Stress-Intensity Factor**

The Mode I stress-intensity factor, $K_I$, at any point along the surface crack was taken to be

$$K_I = \sqrt{\frac{\pi \alpha}{Q}} G_j \left( \frac{a}{R} \right)$$

for $j = 0$ to 3. $G_j$ is the boundary-correction factor corresponding to the $j$th stress distribution from eq. (1). $Q$, the shape factor for an elliptical crack, is given by the square of the complete elliptic integral of the second kind. The vessel length ($2b$) was always chosen large enough that the length would have a negligible effect on stress intensity ($b/c > 10$).

The stress-intensity factors for a surface crack in an internally pressurized cylinder were obtained by appropriate superposition of the results given by eq. (2). For convenience, the stress-intensity factor was written as

$$K_I = \frac{\sigma R}{t} \sqrt{\frac{\pi \alpha}{Q}} F \left( \frac{a}{R} \right)$$

where $\sigma R/t$ is the hoop stress and $F$ is the boundary-correction factor for a surface crack on the inside of an internally pressurized cylinder. The expression for $F$, in terms of $G_j$, was obtained from the first four terms of a power-series expansion of Lame's solution [10] for the hoop stress in an internally pressurized cylinder. The result is

$$F = \left( \frac{R_0}{R} \right)^2 \left[ 2G_1 - \frac{2}{R_0} \left( \frac{a}{R} \right) G_2 + \frac{4}{R_0} \left( \frac{a}{R} \right)^2 G_3 \right]$$

where each $G_j$ was obtained from the appropriate finite-element solution. The correction factor $F$ includes also the influence of the internal pressure, $p$, acting on the crack surfaces. Values for $F$ were calculated as a function of $a/c$, $a/t$, and $\phi$ for $t/R$ values of 0.1 and 0.25. The $a/c$ ratios were 0.2, 0.4, and 1; and the $a/t$ ratios were 0.2, 0.5, and 0.8.

The stress-intensity factors from the finite-element models were obtained by using a nodal-force method, details of which are given in references [7] and [9]. In this method, the nodal forces normal to the crack plane and ahead of the crack front are used to evaluate the stress-intensity factors.

5. **Results and Discussion**

In the following sections, results are presented for two symmetric semi-elliptical surface cracks on the inside of pressurized cylinders. The stress-intensity factor variations along the crack front for various surface cracks ($a/c = 0.2$ and 1) are presented as a function of $a/t$. An empirical equation for the stress-intensity factor is also developed for a wide range of configuration parameters. An estimate for the stress-intensity factor for a single surface crack is also presented. The stress-intensity factors are compared with other solutions from the literature.

5.1 **Semi-Circular Surface Cracks**

Figure 3 shows the boundary-correction factors for two symmetric semi-circular surface cracks ($a/c = 1$) as a function of the parametric angle, $\phi$, and $a/t$ for $t/R = 0$, 0.1, and 0.25. For $t/R = 0$ (flat plate [8,9]), the $\sigma R/t$ stress in eq. (3) is replaced by $S_t$, a
remote uniform applied stress. For a fixed t/R ratio, the correction factors are higher for larger a/t ratios. Also, for a fixed a/t ratio, the correction factors are higher for smaller t/R ratios. The maximum correction factor (or stress-intensity factor) occurred at the intersection of the crack front with the inner surface (φ = 0).

5.2 Semi-Elliptical Surface Cracks

Figure 4 shows the boundary-correction factors for two symmetric semi-elliptical surface cracks (a/c = 0.2) as a function of φ and a/t, for t/R = 0, 0.1, and 0.25. Again, for a fixed t/R ratio, the correction factors are higher for larger a/t ratios. For a given a/t ratio, smaller t/R ratios gave higher correction factors. In contrast to results for a semi-circular surface crack, the maximum correction factor (or stress-intensity factor) occurred at the maximum depth point (φ = π/2).

5.3 Stress-Intensity Factor Equation

The results shown in figures 3 and 4 suggest that the ratio of the correction factors for a given t/R and those for a flat plate (t/R = 0) are nearly independent of the parametric angle, so that the curve for t/R = 0 can be scaled to approximate the results for t/R = 0.1 and 0.25. Figure 5 shows f_c (the ratio of F for a given t/R to F_0) as a function of t/R. F_0 is the correction factor for a flat plate [8,9]. The labels (and bars) give the average (and range) of f_c for a given value of a/t with a/c = 0.2, 0.4, or 1 and any value of φ. (For clarity, results for a/t = 0.5 are not shown.) These results were found to be closely approximated by

\[ f_c = \left[ \frac{R^2 + R'^2}{R^2 - R'^2 + 1 - 0.5 \sqrt{\frac{a}{t}}} \right] (5) \]

In figure 5, the upper curve shows the exact limiting solution for a/c = 0 and a/t = 0. The upper curve was obtained from Lame's stresses [10] on the inside of the cylinder and the solution for an edge-crack in a semi-infinite plate, and is given by eq. (5) with a/t = 0. The other curves show results from eq. (5) for various a/t ratios.

The stress-intensity factor for two symmetric surface cracks on the inside of a pressurized cylinder is given by eq. (3) where the following approximate expression for F has been fitted to the present results and those of references [8] and [9]:

\[ F = 0.97 \left[ M_1 + M_2 \left( \frac{a}{t} \right)^2 + M_3 \left( \frac{a}{t} \right)^3 \right] F_0 f_c \] (6)

\[ M_1 = 1.13 - 0.09 \frac{a}{c} \] (7)

\[ M_2 = 0.54 + \frac{0.89}{0.2 + \frac{a}{c}} \] (8)

\[ M_3 = 0.5 - \frac{1}{0.65 + \frac{a}{c}} + 14 \left( 1 - \frac{a}{c} \right)^{24} \] (9)

\[ g = 1 + \left[ 0.1 + 0.35 \left( \frac{a}{t} \right)^2 \right] (1 - \sin \phi)^2 \] (10)
\[
\phi = \left( \sin^2 \phi + \left( \frac{E}{\sigma} \right)^2 \cos^2 \phi \right)^{1/4}
\]

for \(0 \leq \frac{a}{\epsilon} \leq 0.8, 0 < \frac{t}{\epsilon} \leq 1, t/R \leq 0.25,\) and any \(\phi\). Equation (6) is within about ±5 percent of the finite-element results.

An estimate for the stress-intensity factors for a single crack in terms of two symmetric cracks was obtained from an equation (eq. (7)) given in reference [11]. For \(a/c = 0\) and \(t/R = 0.1\), the stress-intensity factor for a single crack was about 2 percent lower than that for two symmetric cracks, whereas for \(t/R = 0.25\) the stress-intensity factor for a single crack was about 4 percent lower than that for two cracks. For larger \(a/c\) ratios the differences are smaller. Thus, eq. (6) can also be used for a single surface crack without appreciable error.

5.4 Comparisons with Other Solutions

Figure 6 shows the stress-intensity boundary-correction factors obtained by several investigators for a semi-elliptical surface crack \((a/c = 1/3\) and \(a/t = 0.8)\) on the inside of a cylindrical vessel with \(t/R = 0.1\). The crack surfaces were subjected to stress distributions given by eq. (1). The correction factors, \(H_j\), are those used in references [5] and [6], and are related to \(G_j\) by

\[
H_j = \frac{B_j G_j}{f_{\phi}} \quad \text{for} \quad j = 0 \text{ to } 3
\]

where \(B_0 = 1, B_1 = \pi/2, B_2 = 2, B_3 = 3\pi/4,\) and \(f_{\phi}\) is given by eq. (11). The present results are shown as symbols. The results from reference [6], obtained from the boundary-integral equation method, are shown as solid curves and are in good agreement (+2 percent) with the present results for \(\phi > \pi/4\). The dashed curves show the results from reference [5]. Their results were obtained from the finite-element method and were within ±3 percent of the present results. (For clarity, the results for \(H_2\) were not shown.)

6. Concluding Remarks

A three-dimensional finite-element elastic stress analysis was used to calculate stress-intensity factors for a wide range of semi-elliptical surface cracks on the inside of cylindrical pressure vessels. The ratio of crack depth to crack length ranged from 0.2 to 1; the ratio of crack depth to wall thickness ranged from 0.2 to 0.8; and the ratio of wall thickness to vessel radius was 0.1 and 0.25. Singularity elements were used along the crack front and linear-strain elements were used elsewhere. The models of these configurations had about 6500 degrees of freedom. A nodal-force method was used to evaluate the stress-intensity factors.

The stress-intensity factors for surface cracks in pressurized cylinders were similar to those calculated for surface cracks in flat plates as a function of the parametric angle. For semi-circular cracks, the stress-intensity factors were maximum at the intersection of the crack with the inside surface of the cylinder, but for semi-elliptical cracks the values were largest at the maximum depth point. Larger crack-depth-to-wall-thickness ratios and larger wall-thickness-to-vessel-radius ratios gave higher stress-intensity factors for all surface-crack configurations considered.
The present results were compared to other analyses of internal surface cracks in cylinders. The surface-crack configuration had a crack-depth-to-wall-thickness ratio of 0.8, a crack-depth-to-crack-length ratio of 1/3, and a wall-thickness-to-vessel-radius ratio of 0.1. The cracks were subjected to constant, linear, quadratic, and cubic stress distribution on the crack surfaces. The results from a boundary-integral equation method were in good agreement (generally ±2 percent) and those from a finite-element method were in fair agreement (±8 percent) with the present results.

An empirical equation for the stress-intensity factors for an internal surface crack in a pressurized cylinder was developed to estimate the present results. The equation applies over a wide range of configuration parameters and was within about ±5 percent of the present results. The results obtained herein should be useful in correlating fatigue-crack-growth rates and in calculating fracture toughness for the surface crack in a pressurized cylinder.

References


Fig. 1.- Surface crack in an internally pressurized cylinder.

(a) Finite-element model.  (c) Loading on crack surfaces.

(b) Element pattern on \( Y = 0 \) plane.

Fig. 2.- Finite-element model and loading on a semi-elliptical surface crack in a cylinder.
Fig. 3. - Stress-intensity boundary-correction factors along crack front for a semi-circular surface crack (a/c = 1.0) in a pressurized cylinder.

Fig. 4. - Stress-intensity boundary-correction factors along crack front for a semi-elliptical surface crack (a/c = 0.2) in a pressurized cylinder.
Fig. 5.- Boundary-correction factor for a surface crack in a pressurized cylinder normalized by the correction factor for a surface crack in a flat plate.

Fig. 6.- Stress-intensity correction factors computed by several methods for a semi-elliptical surface crack in a cylinder.