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Material Science and Solid State Physics
Studies With Positive Muon Spin Precession

Virginia State University
Petersburg, Virginia 23803
U.S.A.

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Progress Report


Carey E. Stronach
Program Director.
MUON HYPERFINE FIELDS IN Fe(Al) ALLOYS

C.E. Stronach∗
Virginia State University, Petersburg VA 23803

W.J. Kossler, J. Lindemuth, and K.G. Petzinger
The College of William and Mary, Williamsburg VA 23186

A.T. Fiory and R.P. Minnich
Bell Laboratories, Murray Hill NJ 07974

W.F. Lankford
George Mason University, Fairfax VA 22030

J.J. Singh
NASA Langley Research Center, Hampton VA 23665

and

K.G. Lynn
Brookhaven National Laboratory, Upton NY 11973

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∗ Present address: Nuclear Research Centre, University of Alberta, Edmonton, Alberta, Canada T6G 2N5.

† Present address: Siemens Corporation, Cherry Hill NJ 08003
Abstract

The hyperfine field on the muon, $B_{hf}$, at interstitial sites in dilute Fe(Al) alloys has been measured for four different concentrations of Al and as a function of temperature by the muon spin rotation method. The magnitude of $B_{hf}$, which is negative, decreases at rates ranging from $0.09 \pm 0.03\%$ per at.\% Al at 200 K to an asymptotic limit of $0.35 \pm 0.03$ above 440 K. This behavior shows that sites near the Al impurity are weakly repulsive to the muon, with an interaction potential of $13 \pm 3$ meV. In order to fit the temperature dependence of the hyperfine field, it is necessary to hypothesize the existence of a small concentration of unidentified defects, possibly dislocations, which are attractive to the muon. Although the Al impurity acts as a non-magnetic hole in the Fe lattice, the observed decrease in $B_{hf}$ is only 35\% of the decrease in the bulk magnetization. We conclude that $B_{hf}$ is determined mainly by the enhanced screening of conduction electrons in Fe and Fe(Al). Since the influence of the Al impurity on the neighboring Fe moments is very small, most of the change in $B_{hf}$ is therefore attributed to the increase in conduction electron polarization at the Al impurity. [PACS 75.50.Bb].
1. Introduction

Knowledge of the local electronic structure around the isotopes of hydrogen, when present as impurities in metals and alloys, is fundamental to a basic understanding of the diffusion of these particles and their poorly-understood interaction with defects and impurities, particularly in Fe. Measurements of the Fermi contact or hyperfine field on positive muons can help provide this information [1-3]. We have studied Fe(Al) alloys with muon spin rotation methods (µSR) to help formulate a physical picture of the local magnetic field and how the host electrons participate in screening the muon’s positive charge. Information is also obtained on the mutual interaction between the positive muon and the Al impurities in the Fe host.

The muon probes the magnetic fields at interstitial sites, so that it is specifically sensitive to the extent to which electron states in metals are delocalized, or band-like. As an example of how this is accomplished, we refer to the recent work of Hayano et al. who have compared µSR and host NMR measurements in a study of the itinerant magnetism of the helimagnet MnSi [4]. General applicability of this aspect of such measurements is of course contingent on being able to correctly account for the perturbation which the muon creates in the metal. The problem appears to be tractable and several theoretical approaches have been advanced [1-3,5]. The pure elemental ferromagnets Ni [6-10], Fe [6,7,9-12], Co [7,9,13,14], Gd [15,16], and Dy [17] have been among the metals studied with muons.

The basic properties of the muon in metals may be derived from the assumption that the s-like conduction band electrons are mainly responsible for screening the muon’s charge. This interpretation has been proposed for the case of Ni [1,2] assuming d-like states contribute very weakly to the screening at the muon site, even though there is a high density of d states at the Fermi level. However, the s-like bands of Ni have little or no polarization, and the muon therefore experiences a negative hyperfine field which is dominated by the tails of the minority-spin d-like wavefunctions in the
interstitial region. The theoretical basis for this model was discussed by Petzinger and Munjal [1] and by Petzinger [2]. This model takes the opposite view to what had been previously supposed for hydrogen in transition metals [18]. Patterson and Keller [5] recently carried out a finite cluster calculation which lends additional support to this model, in that the d-electrons remain on the neighboring Ni atoms in the cluster.

The above model is thought to be applicable to Fe in spite of the more complicated band structure, where more significant s-d hybridization is found [19]. Jena has discussed the non-linear screening of the muon in Fe in terms of a free-electron gas [3], where the ambient spin density is increased by a factor of 9.8. The measured hyperfine field on the muon is $-11 \text{ kG}$, so that on applying this model one finds that the ambient polarization density is effectively $-0.014 \mu_B \AA^{-3}$. By comparison, neutron diffraction measurements [20] give a value of $-0.014 \pm 0.004 \mu_B \AA^{-3}$, averaged over a 0.5 Å cube centered at the tetrahedral interstitial site where the muon is presumed to reside [21]. The neutron data also show that there is a delocalized background polarization of $-0.21 \mu_B$ per atom, which is equivalent to a homogeneous polarization density of $-0.018 \mu_B \AA^{-3}$ [18]. This indicates that the muon hyperfine field is obtained following two quite different assumptions, that the muon experiences either the average conduction electron polarization or the local polarization at a particular site. Band structure calculations have found that the $3s$ electron contribution to the polarization is between $-0.04$ and $-0.07 \mu_B$ per atom [19]. This does not include the contribution from itinerant d states, which is apparently larger. Thus, our view is that previous work indicates that itinerant states in Fe might be treated as a free electron gas in screening the muon.

Alloys of various non-transition elements in Fe have been studied by bulk magnetization measurements [22,23], neutron diffraction [24,25], NMR [26-29] and the Mössbauer effect [29-31]. These measurements found that the Al impurity produces simple magnetic dilution. Neutron diffraction studies have confirmed that the Al is
non-magnetic and that the surrounding Fe neighbors exhibit very little perturbation on their moments. The NMR and Mössbauer satellite lines have been associated with Al in various neighbor shells around the Fe. Grüner et al. [29] have shown that these results can be explained by the spin-polarized conduction electron cloud around the Al. More recently, Terakura has proposed a theoretical explanation based on an ab initio calculation of the electronic densities and polarizations at the Al site [32]. He found a net increase in the s-p polarization at the Al site. His predictions for Si impurities are quite similar to those for Al.

This paper is an elaboration and an extension of our previous studies of Fe(Al) with muons [33]. Additional studies of ferromagnetic alloys were recently reported by Kossler et al. [34] for Ni(Co) and Ni(Cu) and by Nishida et al. [35] for Ni(Cr), Fe(Si) and Fe(Ti).

2. Experiment

µSR is a perturbed angular distribution technique and it has been discussed extensively in the literature [6,36,37]. Spin-polarized positive muons are implanted into the sample under investigation and the time intervals for individual positron decays are recorded. The anisotropy of the positron decay and the precession of the muon spin in the local magnetic field give rise to an oscillatory component in the time dependence of the measured positron emission rate in a given direction. The present studies were done at zero external magnetic field on the samples and used positron detectors at 0° and 180° with respect to the µ+ beam. The time distribution of the positrons is given by the following formula:

\[ N_\tau(t) = N_0 \exp(-t/\tau_\mu) \left[ 1 + a + b \exp(-X t) \cos(\omega t + \phi) \right] + N_{\text{sec}}, \]  

(1)

\( N_0 \) is an overall normalization, \( \tau_\mu \) the muon lifetime of 2.2 \( \mu \)sec, \( \lambda \) the depolarization
rate, $\omega$ the spin precession angular frequency and $\phi$ a geometrical phase, essentially the angular coordinate of the positron detector. The values of $a$ and $b$ depend upon the distribution of the orientations of the local fields in the sample and upon the angle subtended by the detector [37]. Taking the case of an unmagnetized ferromagnetic sample, where the local fields are isotropically distributed, and longitudinal placement of the detector, we have $a = P/3$ and $b = 2P/3$, where $P$ is a function of the polarization of the muon beam and the energy dependence of the positron detection efficiency [37]. For our experiment $P \approx 0.1$. The term $N_{\text{back}}$ is included to account for accidental background counts in the data.

The experiments were performed at temperatures ranging from 80 to 433 K. The data were fitted with Eq. (1) using a multi-parameter least-squares fitting routine. We did not attempt to find the term proportional to $a$, but rather include it into the definition of $N_0$ and $b$ in fitting the data. The field on the muon is given by $B_\mu = \omega \gamma_\mu^{-1}$, where $\gamma_\mu = 8.51 \times 10^4 \text{sec}^{-1} \text{G}^{-1}$.

Four spherical samples were fabricated, one of 99.99\%-pure Fe, the others containing 1.60, 4.29 and 5.81 at.\% Al. The sample materials were melted in a MgO crucible in a He atmosphere by rf induction. The Al was added by including Fe(Al) alloy in the melt. The melt was poured into an Al$_2$O$_3$-coated Fe mold and allowed to cool in the furnace. The castings were 6 cm in diameter and 20 cm in length. Each casting was reheated to 1273 K and hot pressed, reducing the length by a factor of 2. This is a standard procedure for removing most of the casting structure in Fe and its alloys. The castings were then machined to 5.715 ± 0.001-cm-diameter spheres and annealed in H$_2$ at 1088 K for one hour. At the concentrations used here, the Al is in a random solid solution [38]. The results of a chemical analysis on the samples is given in Table 1. After the $\mu$SR runs, the spheres were sectioned for analysis. The sizes of the macroscopic crystallites vary between 0.1 and several mm. The 4.29\%-Al sample contains, in addition, dispersed Al$_2$O$_3$ precipitates on the order of 1 \mu m across, which
accounts for the high oxygen concentration in the analysis (There is a thermite reaction between aluminum and oxygen). From transmission electron microscopy, the dislocation density is on the order of $10^8 \text{cm}^{-2}$ and the subgrain cells are about $1 \mu\text{m}$ across.

3. Data Analysis

3.1 Extraction of the Hyperfine Field

The results for the magnetic fields on the muon and the depolarization rates are listed in Table II. Values of $B_\mu$ given in brackets were obtained by linearly interpolating the temperature dependence of the results on pure Fe given by Nishida et al. [11]. The hyperfine fields were extracted in the following manner.

The field experienced by the muon may be decomposed [39] into

$$B_\mu = B_{\text{ext}} - B_{DM} + B_L + B_{\text{dp}} + B_{hf},$$

where $B_{\text{ext}}$ is the applied external field, $B_{DM}$ is the demagnetizing field which depends upon sample geometry, $B_L$ is the Lorentz cavity field $4\pi M/3$ where $M$ is the domain magnetization, $B_{\text{dp}}$ is the sum of the dipole fields inside the Lorentz sphere, and $B_{hf}$ is the Fermi contact or hyperfine field. Because we performed the experiment with zero external field, and since the $\mu^+$ particles hop rapidly from site to site, averaging $B_{\text{dp}}$ to zero, the expression for $B_{hf}$ reduces to

$$B_{hf} = B_\mu - B_L.$$

The temperature dependence of the Lorentz cavity field was computed following the parameterization of magnetization data for pure Fe given by Redi [40]:

$$M = M_{hf}(T) = M_0 \frac{1}{1 - A(T/T_c)^{3/2} - B(T/T_c)^{7/2}}.$$
where \( A = 0.102 \pm 0.005 \), \( B = 0.33 \pm 0.07 \), and \( M_o = 1749 \text{G} \). We have assumed that
the temperature dependence scales with the Curie temperature of the alloy, given
as [41]

\[
T_c = T_{eo}(1 - 0.1 c),
\] (5)

where \( T_{eo} = 1044 \text{K} \) is the Curie temperature of pure Fe and \( c \) is the Al
cconcentration.

The magnetization is also corrected for the presence of Al in the sample by summing
the effects of simple dilution, i.e., replacing Fe atoms with Al atoms and taking into
account the known change in lattice spacing [42]. The change in lattice spacing \( \Delta a_o \)
gives a density factor \( 3\Delta a_o/a_o \), which becomes 0.157 \( c \). Combining these factors yields

\[
B_L = \frac{4\pi M}{3} (1 - 1.157 c),
\] (6)

with \( M \) given by Eq.(4).

The resulting values of \( B_{hf} \) are listed in Table II. The data for sphere No.1 are
consistent with pure Fe data of Nishida et al. [11]. The data and fit
for \( B_{hf} \) at 301 K as a function of concentration are plotted in Fig. 1. The change in \( B_{hf} \)
is \( -0.234 \pm 0.004 \) per at.% impurity at 301K.

The reduced quantity \( \Delta B_{hf}/c B_{hf} \) for \( c = 0.0429 \) is plotted as a function of temperature
in Fig.2. Muon precession oscillations were observed only at 200 K and above,
preumably because the muons are trapped at defects below about 200 K.

It is seen that the fractional change in hyperfine field is temperature dependent.

3.2 Temperature Dependence of \( B_{hf} \)

The temperature dependence of the change in \( B_{hf} \) with Al concentration (Fig.2)
can be explained if the diffusing muons do not randomly sample the interstitial sites in
the alloy. This does not invalidate the assumption that \( B_{dip} \) averages
to zero because the preference in sampling relates to the presence of
an Al nearest neighbor rather than to the two magnetically inequivalent
tetrahedral interstitial sites. In general, the average hyperfine field is obtained
by summing local contributions from all available sites, weighted according to
Boltzmann population factors:

\[ B_{hf} = \sum_i B_{hf}^i \exp(-\beta E^i) / Z, \]  

(7)

where \( \beta = (kT)^{-1} \), \( E^i \) the state energies of the muon at sites \( i \), \( B_{hf} \) the local hyperfine fields at sites \( i \), and \( Z \) the thermodynamic partition function.

In order to give this effect a quantitative treatment, we consider the following model. Firstly, we assume that the \( E^i \) vary significantly only at sites immediately adjacent to impurities and defects. Secondly, we assume that the muon energy is changed by an amount \( E' \) for a fraction \( c' \) of sites which are near the Al impurity. Defects are taken into account by including a fraction \( c'' \) of sites with energy \( E'' \) relative to the unperturbed sites. Since there are 24 tetrahedral interstitial sites around each Al atom, as opposed to the 6 tetrahedral sites per Fe atom, we take \( c' = 4c \) for consistency. If we write the field near the Al sites as \( B_{hf}' \), the field at the defect sites \( B_{hf}'' \), and the field elsewhere as \( B_{hf}^0 \), then the average hyperfine field is given by

\[ \Delta B_{hf} / B_{hf}^0 = [f' c' \exp(-\beta E') + f'' c'' \exp(-\beta E'')] / [1 - c' - c'' + c' \exp(-\beta E') + c'' \exp(-\beta E'')], \]  

(8)

where we have used the reduced parameters

\[ f' = 1 - B_{hf}' / B_{hf}^0 \]

and

\[ f'' = 1 - B_{hf}'' / B_{hf}^0. \]

The data of Fig.2 were fitted with Eq.(8) by treating \( f' \), \( f'' \), \( c' \), \( E' \), and \( E'' \) as adjustable parameters. The results are listed in Table III.

We find that the rapid dependence of \( \Delta B_{hf} / c B_{hf} \) near 250 K. followed by a comparatively gentle dependence at higher temperatures, can therefore easily be explained in terms of a small concentration of strongly attractive defect sites and weakly...
repulsive Al sites. The reduced $\chi^2$ for 3 degrees of freedom is 0.54 for the fit given in Table III. For comparison, we also tested an alternative fit to the data by omitting the defect term, i.e., with $c'' = 0$. The asymptotic value of $\Delta B_{hf}/c B_{hf}$ turns out to be 40% larger and $c'$ a factor of 2 larger. However, the reduced $\chi^2$, for 6 degrees of freedom, increases to 10. The statistical probability that the defect term gives a better fit is therefore 0.99.

The asymptotic value of $B_{hf}/cB_{hf}$ approaches $-0.35 \pm 0.03$ at very high temperatures (it is $-0.31$ at the Curie point, 1043K), and we take this to be the average hyperfine field shift for random sampling by the muon. The hyperfine field on the muon at sites near Al is weaker by $8 \pm 1\%$ with respect to pure Fe. The three parameters characterizing the defect sites might be determined better if we had more data in the low-temperature region. We therefore regard these values as approximate. From the small value of the defect concentration found in the fit, $c'' = 10^{-9}$, it seems that dislocations and subgrain boundaries would be the likely explanation. Our result does, however, indicate that the shift in the hyperfine field with impurity doping can be developed as a technique for studying the muon-impurity interaction. It could be complementary to depolarization measurements which study the motional narrowing effect of muon motion [9-12]. These points are discussed further in the following sections.

3.3 Depolarization Rates

The $\mu$SR signal is observable in Fe and Fe(Al) when the rapid motion of the muon nearly averages out the dipolar field. From the depolarization rate the correlation time of the local dipolar field on the muon can be calculated, and this is nearly the same as the mean time of stay at an interstitial site. The second moment of the dipolar field distribution is about $(\Delta B_{dip})^2 = (2.6 \text{kG})^2$, owing to the existence of two magnetically inequivalent tetrahedral sites in pure Fe [10]. The value is nearly the same for the Fe(Al) alloy, although the spatial distribution differs. The depolarization
rate measured at 301 K for our samples is about a factor of 10 larger than those measured in high-purity Fe [12] and does not appear to be sensitive to the Al concentration. Since samples of Fe of nominally lesser purity typically show larger depolarization rates, this effect has been attributed to longer mean times of stay for muons at sites near or at defects. We could assume, therefore, that the diffusing muons spend part of their lifetime sampling some kind of defect sites throughout the temperature range covered. Clues as to the nature of these defects are obtained by comparing the hyperfine field shift and depolarization data. The fact that the depolarization in the alloys is weaker than for our pure Fe may be due to the scavenging of impurities such as oxygen and nitrogen by the aluminum.

The depolarization measurements for the 4.29% alloy given in Table II show a monotonic decrease with temperature, with a tendency towards saturation at high temperatures. Thus it appears that the mean time of stay decreases steadily with temperature and that there is a background contribution to the depolarization rate. We consider expressing these data as follows:

\[ \lambda = \gamma \mu (\Delta B_{	ext{dep}})^2 f^2(T) \tau_e + \Delta B f(T). \]  

The first term represents the motionally-narrowed local dipolar inhomogeneity and the second term the macroscopic magnetization inhomogeneities. \( \tau_e \) is a correlation time, which we assume has an Arrhenius temperature dependence:

\[ \tau_e = \tau_0 \exp\left(\frac{U}{kT}\right). \]  

The function \( f(T) \) is the same as that given in Eq.(4) and it is included in Eq.(9) in order to correct for the temperature dependence of the dipolar fields. The data do not show any evidence for diffusion limited capture by deep traps (no de-trapping), which would have a \( \tau_e^{-1} \) dependence [43,12]. A trapping term had been considered by Kossler et al.[43] in their interpretation of the non-Arrhenius temperature dependence of the depolarization rate in Cr and by Nishida et al.[12] in a study of Fe.
The results of a non-linear least-squares fit of Eqs. (9) and (10) to the depolarization data yields the Arrhenius plot of the correlation time given in Fig. 3. The parameters of the fit are $\Delta B = 8.8 \pm 0.2 \text{G}$, $\tau_p = 0.3 \text{ps}$, and $U = 0.11 \pm 0.02 \text{eV}$. The statistical uncertainty in $\tau_p$ is about a factor of 10.

We note that the activation energy $U$ for the muon jump processes is about a factor of 4 smaller than the magnitude of the defect interaction energy $E''$ (see Table III) found in the fit of the hyperfine field shift. This difference needs explanation. One consistency check concerns whether the muon jump rate is high enough for the muons to reach the defects. A fair test can be made at $T = 250 \text{K}$, where the change in the hyperfine field with temperature is most rapid. At this temperature $c'' \exp(-\beta E'') = 1$, with muons spending about one-half their time at the defect sites, according to our previous analysis. It can be readily estimated that over the mean duration of the measurement, which is $\lambda^{-1} \approx 0.3 \mu\text{sec}$ at 250 K, the muon executes $N = \lambda^{-1} \tau^{-1}$ jumps, where $\tau$ is the jump time. A possible value for $\tau$ is 5 ps, obtained by Nishida et al. [12] for high-purity Fe at 250 K, with the result $N \approx 6 \times 10^4$. Equilibrium sampling of defects would be approached when $N c'' = 1$. On the contrary, we find $N c'' < 10^{-4}$. To resolve the apparent discrepancy we propose the following: (a) The defects are dislocations and subgrain boundaries, whose effective site concentration is very small; (b) $E''$ is some average muon-dislocation binding energy; (c) the muons are mobile along the dislocations, with an activation energy of migration $U$; and (d) the muon mean time of stay at unperturbed interstitial sites in Fe is probably much smaller than 5 ps at 250 K.
4. Discussion

A simple view of the effect of dilute Al impurities is first that the Al is substitu-
tional in Fe and the contribution from the d-states of the removed atom can be simply
subtracted. Invoking the observation that the moments of the Fe atoms are nearly
 unperturbed, we then assume that the d-electron contribution to the average
hyperfine field on the muon is simply decreased by 1% per atomic per cent Al.

Further support for this assumption comes from the fact that the magnetic disturbance
has only a weak dependence on the valence of the impurity atom [24,25]. There is a
similarly small perturbation on the neighboring Fe moments for Si impurities, which
have a valence difference of 3 with respect to Fe [25]. The valence difference is 2 for
Al. Terakura [32] has discussed the d-band filling and emptying effects for non-
transition element impurities in Fe and concludes that these nearly cancel each other.

Simple dilution of the d-electron contribution to the hyperfine field on the muon, if
that were the main contribution, would lead to \( \Delta B_{hf}/c B_{hf} \) of \(-1.0\) which is three times
larger than the data indicate. The apparent disagreement can be resolved with an
explanation in terms of the contribution from conduction electrons.

NMR and Mössbauer measurements have found that the hyperfine field on Fe
nuclei at sites nearest neighbor to Al impurities is less negative by 7% and this has
been attributed to a small reduction in the local, negative conduction electron polariza-
tion [29]. At the Al site, the conduction electron polarization is negative due to the
negative hyperfine field on the Al [40]. From a different point of view, Stearns has
treated the pick-up of conduction electron polarization at impurities from the neigh-
boring Fe atoms in terms of the volume misfit of the impurity atom [44,45]. This
effect is small for the Al impurity and can be neglected. Referring to the theoretical
results calculated by Terakura [32], the fractional change in the s-electron polarization
within the Wigner-Seitz sphere at the Al is \(+0.08\) and the p-polarization change is
\(-0.93\), for a total of \(+0.05\). These findings can be used to make a prediction for the
change in the average $B_{hf}$ for muons per unit concentration of Al impurity, taking into account the free electron spin density enhancement factor of 9.8:

$$\Delta B_{hf}/c = 9.8 \frac{8\pi}{3} \left(0.05 \times 2\mu_B/a_0^3\right) = 3.3 \text{ kG.}$$  \hspace{1cm} (11)

This is equivalent to $\Delta B_{hf}/c B_{hf} = -0.30$ and is close to the measured asymptotic value of $-0.35 \pm 0.03$.

The average change in the hyperfine field at the Fe sites in Fe(Al) has been given by several authors [46,47]. The fractional change is in the range $-0.4$ to $-0.5$.

Nishida et al. [35] have recently measured $B_{hf}$ for Fe doped with 5\% Si at 300K. They find $B_{hf}/c B_{hf} = -0.24$. This is the same as the value we find for our 301 K measurements on Fe(Al). Although the asymptotic limit was not checked in the Fe(Si) measurement, this result suggests an insensitivity to the valence difference between Al and Si [30]. The magnetic properties of Fe(Al) and Fe(Si) are also similar [27].

Our explanation for the change in the hyperfine field on muons assumes that distortions in the localized d-wavefunction amplitudes at interstitial sites near the Al impurity can be neglected. This is based on the assumption that local changes in electron spin density associated with these states are not enhanced when the muon is present, as is the case for itinerant states. Thus we are assuming that the $\mu^+$ impurity acts in a manner similar to Al and Si impurities by creating minimal magnetic disturbance in their vicinity. Nevertheless, we cannot completely rule out the possibility that the weak dependence of $B_{hf}$ on Al concentration may arise in part from a large increase in the minority-spin d-like wavefunctions at interstitial sites near the Al. It would be interesting, therefore, to investigate the systematic behavior of $B_{hf}$ for other non-magnetic impurities in Fe as well.

Owing to the apparently extreme sensitivity of $B_{hf}$ to attractive defect potentials, it would be interesting to investigate the systematic effects of dislocations and
impurities such as C, O, and N in Fe alloy samples. Our interpretation predicts a logarithmic dependence of the temperature of inflection in Fig. 2 on the defect concentration. This could be tested with deformed alloys.

Our basic conclusion is that one obtains consistency by treating both the local magnetic perturbation around Al impurities [29] and the hyperfine field on the muon [3] as problems of conduction electron screening and spin density perturbation. It is hoped that these measurements will motivate more fundamental calculations for determining the origins of the muon hyperfine field and the local muon potential, such as finite cluster calculations treating the ternary system $\mu^-$-Al in Fe.

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TABLE I. Properties of the Fe(Al) spheres used in this study. The impurity concentrations measured by atomic absorption analysis for Al, by vacuum fusion analysis for C, O, and N, and by mass spectrographic analysis for the other elements, are given in atomic per cent under the element headings. He and H were below the limits of detectability, 0.0001% and 0.0005%, respectively. An additional estimated 0.2 at.% Al is present in sphere No. 3 in the form of \( \text{Al}_{2}O_{3} \) precipitates.

<table>
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<th>Mass (g)</th>
<th>Al</th>
<th>C</th>
<th>O</th>
<th>N</th>
<th>Si</th>
<th>Ni</th>
<th>Zr</th>
<th>Ti</th>
<th>Mg</th>
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<td>0.005</td>
<td>0.003</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0002</td>
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TABLE II. Observed magnetic fields on the muon $B_\mu$, muon-spin depolarization rates $\lambda$, calculated Lorentz cavity fields $B_L$, and resultant Fermi-contact hyperfine fields $B_{hf}$. Errors of measurement are enclosed in parenthesis. Values of $B_\mu$ enclosed in brackets were interpolated from the data of Nishida et al. [11]. $c$ is the Al impurity concentration in atomic per cent.

<table>
<thead>
<tr>
<th>$c$ (\text{at. pt})</th>
<th>$T$ (K)</th>
<th>$B_\mu$ (G)</th>
<th>$\lambda$ (\text{\mu sec}^{-1})</th>
<th>$B_L$ (G)</th>
<th>$B_{hf}$ (G)</th>
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<tr>
<td>0</td>
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<td>7256</td>
<td>-10.928</td>
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<tr>
<td>240</td>
<td></td>
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<td>7230</td>
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<tr>
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<tr>
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<td>7198</td>
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<tr>
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</tr>
<tr>
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<td>3.0(3)</td>
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<td>-10.771</td>
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<tr>
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<td>-3930(2)</td>
<td>1.3(2)</td>
<td>6695</td>
<td>-10.625</td>
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</tbody>
</table>
TABLE III. Parameters fitting Eq. (8) to the temperature dependence of the hyperfine field in the 4.29% Fe(Al) alloy. $f$ and $f''$ are the fractional changes in the hyperfine field, $c'$ and $c''$ the site concentrations, $E'$ and $E''$ the site energies, for Al and defect sites, respectively. The parameter $c' = 4c$ was held constant.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'$</td>
<td>$-0.087(7)$</td>
</tr>
<tr>
<td>$f''$</td>
<td>$-0.0032(8)$</td>
</tr>
<tr>
<td>$c'$</td>
<td>$0.172$</td>
</tr>
<tr>
<td>$c''$</td>
<td>$10^{-11.7+3.6}$</td>
</tr>
<tr>
<td>$E'$</td>
<td>$0.013(3)$ eV</td>
</tr>
<tr>
<td>$E''$</td>
<td>$-0.6(2)$ eV</td>
</tr>
</tbody>
</table>
Figure Captions

Figure 1. The Fermi-contact hyperfine field on the positive muon in Fe(Al) as a function of impurity concentration at a temperature of 301 K. The line is a least-squares fit.

Figure 2. Temperature dependence of the fractional change in the hyperfine field with aluminum concentration. Data were taken for $c = 0.0429$. The curve is the fitted function Eq.(8) divided by $c$, using the parameters of Table III.

Figure 3. Dependence of the local field correlation time upon inverse temperature, obtained from depolarization rates according to Eqs.(9) and (10). The curve is the fitted function.
FIGURE 1

T = 301 K
Figure 2
$\tau_c$ (ps) vs. $T^{-1}(10^{-3} \text{K}^{-1})$

FIGURE 3
Measurements of local magnetic fields and depolarization rates were made as a function of temperature in zero external field in samples of Ni doped with either 0.76 at. % Cu or 0.76 at. % Co. At temperatures below 50K the magnetic field on the \( \mu^+ \) in the Ni-Cu alloy is 1462±2G and for the Ni-Co alloy the field is 1524±7G.

I. Introduction

The purpose of the present study is to determine the effect of either Cu or Co impurities on the magnetic field experienced by the positive muon probe in ferromagnetic Ni. Previous work [1-3] has established that the Fermi-contact or hyperfine field on the positive muon in pure Ni is negative, i.e., it is directed oppositely to the bulk magnetization, and equals -66G at low temperatures. The hyperfine field is also only slightly temperature dependent up to room temperature, decreasing by about 1% as the magnetization of Ni decreases by 7% [3]. The muon occupies the octahedral interstitial site, a region that lies between the Ni second nearest neighbors and where the ambient magnetic field is particularly uniform at about -66G [4].

The magnitude of the contact field has been explained theoretically [5-9] as arising from the negative polarization of the 3d orbitals in the interstitial region. The 4s electrons are mainly responsible for screening the muon's charge and seem to make a negligible contribution to the contact field [10].

In pure Ni the majority d bands are fully occupied and there is a 0.6-electron vacancy in the minority band. The addition of Cu decreases the bulk magnetization by 1.9% per atomic per cent Cu, with a critical concentration of 53 at. % Cu, while the Cu atom is non-magnetic [11]. Locally, the Cu atom is non-magnetic [11], while the moments of the neighboring Ni atoms are decreased. The addition of Co to Ni has the opposite effect: the magnetization increases and the Co retains its magnetic moment.

II. Experimental Procedure

Polycrystalline samples for uSR experiments were prepared from 99.99%-pure starting materials. Two alloys, Ni with 0.76% Cu and Ni with 0.76% Co were cast in a mould, hot pressed to a thickness of 1.6cm, annealed, machined to a final size of 10.2cm x 1.3cm, annealed again at 550°C for one hour in a hydrogen atmosphere, and cooled overnight in the furnace.

The uSR data were collected at zero external magnetic field for positron decays in the forward and backward directions with respect to the muon polarization. The time histograms were fitted with an exponentially-damped oscillatory function in order to extract the field on the muon, \( B_{\mu} \), and the depolarization rate \( \lambda \). For comparison, a 99.99%-pure Ni sample was also studied.

* Supported in part by the NSF and the Commonwealth of Virginia
† Supported by the Department of Energy
‡ Supported in part by NASA
III. Experimental Results and Discussion

The temperature dependence of $B_4$ and $\lambda$ are given in Figs. 1 and 2. The initial asymmetry in the $\mu$SR signal is about 0.12 on the average and does not depend upon temperature, to within the statistical accuracy of 10%. The larger measured depolarization in the alloys, ranging from 6 to 10 usec$^{-1}$, permitted more precise determination of $B_u$.

The static field broadening associated with the impurities makes a contribution to the depolarization that may depend upon where the muon stops. It tends to be averaged out if the muon is diffusing rapidly. The depolarization rate calculated for the case where the muon stops randomly in the lattice is given by [12]:

$$\lambda = \gamma_\mu \frac{B_\mu^2 N_c u^2}{9\sqrt{3}},$$

where $N$ is the density of lattice atoms, $\mu$ approximately equals one Bohr magneton, and $c$ is the impurity concentration. For $c = 0.007b$, we calculate $\lambda = 2.8$ usec$^{-1}$. If the muon were to selectively stop at a site adjacent to one of the Cu or Co impurities, the depolarization rate would be about 80 usec$^{-1}$. We conclude that the observed depolarization rate is due mainly to macroscopic field inhomogeneities of 70 to 120G. The muons do not appear to be rapidly depolarized by trapping at the Cu or Co impurities. This is ruled out because we do not find any temperatures with an appreciable loss of signal amplitude, as measured by the initial asymmetry.

The sign of $B_u$, which has been deduced from previous work [1], is positive and is given by the following formula:

$$B_u = 4\pi M_s / 3 + B_{hf},$$

where $M_s$ is the saturation magnetization and $B_{hf}$ the hyperfine field. We have assumed that the average magnetic field within a magnetic domain is correctly given by the Lorentz local field, although there may be random contributions.

---

Figure 1. Temperature dependence of the local field on the muon.

Figure 2. Temperature dependence of the depolarization rate.
from magnetization inhomogeneities and impurities. Any random fields would contribute to the depolarization rate.

At low temperatures, the values of $B_0$ for Ni-Cu are smaller than they are for pure Ni, while for Ni-Co they are larger. The differences become less pronounced as the temperature is raised above 100K. The difference in $B_0$ between Ni-Cu and pure Ni increases again as the temperature is further raised to 330K. There are fluctuations in the temperature dependence of both $\lambda$ and $B_0$, which may possibly be significant. The changes evident at 100K occur in the temperature region where the onset of thermal diffusion of the $\nu^+$ is observed in pure Cu [13].

At low temperatures, the $4\pi M_s/3$ field is 30.6G smaller in Ni-Cu than it is in pure Ni. Thus most of the observed decrease of $34.3\pm4.0$G that we find for temperatures less than 50K can be attributed to the decreased macroscopic magnetization, assuming that the muons stop at randomly selected interstitial sites in the alloy. The remainder is due to a change in $B_{hf}$. Values of the reduced fractional change, $\Delta B_{hf}/cB_{hf}$, are given for the low-temperature and 330K data in the Table.

<table>
<thead>
<tr>
<th>TABLE. Experimental values for the fractional change in the hyperfine field per unit concentration, $\Delta B_{hf}/cB_{hf}$.</th>
<th>5-60K</th>
<th>330K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni-Cu</td>
<td>0.8 (0.8)</td>
<td>2.8 (1.1)</td>
</tr>
<tr>
<td>Ni-Co</td>
<td>0.7 (1.5)</td>
<td>3.7 (1.3)</td>
</tr>
</tbody>
</table>

The results at 330K are interpreted with the assumption that the muons diffuse rapidly through the lattice. It is at first sight unusual that the hyperfine field shift at 330K has the same sign for both alloys. To explain this, one could assume the Cu impurity to be repulsive to the muon, so that sites near the Cu would be selectively sampled less frequently by the diffusing muon. The sites near the Cu impurity, on the other hand, need to be sampled more frequently. With this interpretation, the result: given in the second column of the Table show that the hyperfine field at sites distant from a Cu impurity is larger than it is in pure Ni, while for the Cu impurity, sites near the Co have the larger field. A repulsive local potential has recently been observed by Stronach et al. [14], who studied the Al impurity in Fe.

IV. Conclusions

Both our results and the results for pure Ni [1-3] can be interpreted with the model that attributes the hyperfine field on the positive muon in Ni to the local d-band polarization. The field is negative because it is dominated by minority spin bands, whose wavefunctions are more spread out and thus overlap the muon at the octahedral interstitial site. Petzinger has presented arguments showing how the temperature-dependent shift of the hyperfine field in pure Ni is weak because minority-band filling compensates the decrease in the magnetization with temperature [9]. One could suppose that similar arguments apply in the case of alloying with small amounts of Cu.

The data also show that the shift in the hyperfine field at sites near the impurities tends to be opposite to the shift elsewhere, with the result that the average shift is very small. We have presumed that the hyperfine field is weaker near Cu impurities because the local d-band splitting is smaller near Cu impurities. Conversely, the sites adjacent to the Co have a larger hyperfine field, which one might have anticipated since the hyperfine field in pure Co, -6.2 kG, is larger than it is in pure Ni [15]. Our results are also consistent with previous conclusions that the local spin-density enhancement factor is effectively unity for the 3d-like orbitals.
The authors thank K. G. Petzinger for stimulating conversations.

References:

14. C.E. Stronach et al., to be published.
DISCUSSION

D.G. FLEMING:

Since the T dependence of $B_{hf}$ does not follow $M(T)$ depending on impurity concentration, we should be a little cautious in interpreting $B_{hf}(T) - M(T)$ as due to the host metal rather than the presence of (unknown) impurities.

A.T. FIORY:

I agree. The possibility that diffusing muons do not randomly sample the sites in the lattice has not been extensively checked. There is some information on this: in Fe doped with Al, the muon-Al repulsion has been observed by Stronach et al. at SREL. A change in the observed Fermi-contact field occurs when $C_{imp} \exp (E/kT)$ becomes comparable to unity, where $C_{imp}$ is the impurity concentration and $E$ the muon-impurity binding enthalpy. For trapped muons, self-trapped or otherwise, thermal equilibrium treatment does not apply. For fcc lattice like Ni, the low-temperature data may contain selective trapping anomalies: we do not know at present if that is the case.


**μ⁺ DEPOLARIZATION MEASUREMENTS OF Al ALLOYED WITH 0.1 at.% Ag, Cu, Mg, Si, and Zn**


a) College of William and Mary, Williamsburg VA 23186 USA
b) Bell Laboratories, Murray Hill NJ 07974 USA
c) George Mason University, Fairfax VA 22030 USA
d) Brookhaven National Laboratory, Upton NY 11973 USA
e) Virginia State College, Petersburg VA 23803 USA

To try to understand the nature of μ⁺ trapping by impurities in Al, as has recently been observed for copper impurities, Al alloyed with 0.1 at.% Ag, Cu, Mg, Si, and Zn has been studied as a function of temperature and heat treatment.

### I. Introduction

In high purity, undamaged aluminum muons are observed to not depolarize over the temperature range 2.8 to 300K [1], implying rapid diffusion. On the other hand a small admixture of Cu produces trapping as inferred from temperature-dependent depolarization [2]. The temperature dependence of the depolarization rate has been observed to change with impurity concentration [3], and neutron induced vacancies created and maintained at low temperature have been observed to trap muons [4]. These vacancies disappear somewhere above 220K [5].

It is reasonable to expect that the problem of the muon in aluminum under various circumstances is amenable to calculation. Theoretical predictions for the heat of solution for hydrogen have already been obtained [6]. The relaxation of the lattice due to the muon has been treated by Shaw using pseudopotentials [7]. The extension of pseudopotential theory to include binding to impurities is approachable. The tractability of aluminum is associated with its having only s and p shells filled and having a very small core. The heavier metals do not share this advantage.

The complete temperature structure in the depolarization rate, which has been observed for Al with dilute Cu, would seem to indicate a variety of trapping sites. These sites could be clusters of impurities or impurity-vacancy complexes.

Aluminum can thus be viewed as a medium in which trapping sites can form and which by itself does not trap, suggesting that different impurities be added to it to find muon-impurity binding energies and by the use of differing heat treatments purposely creating different impurity-impurity and impurity-vacancy complexes for which concentrations and detrapping energies might be obtained.

### II. Experiment

Appropriate quantities of 99.99% Al and alloying element were melted in an MgO crucible by rf induction heating. The molten alloy was then cast into 2Kg ingots, which subsequently were hot rolled to a thickness of 3mm. The molten alloy was handled in a dry helium atmosphere. Rectangular pieces 7.6 × 10.2 cm were then cut out. These samples were heat treated for 8 hours at 800-825K and quenched to 273K in a brine bath with cooling times measured to be between 0.05 and 0.2 second. The samples were stored at temperatures below 200K to avoid thermal aging.

After precession experiments to 330K, the samples were annealed at 800K for 8 hours, then cooled by standing in air so that a time constant of typically 250
seconds was obtained, and again used for precession measurements. These data were designated: air cooled.

Following these measurements, the samples were again heat treated for 8 hours again at 800K, then cooled in the oven with the power off, typically with a time constant of 4 hours. Precession measurements on these samples were designated: oven cooled.

Analysis by mass spectrograph showed the samples to be: Al (0.1 at.% Ag), Al (0.09 at.% Si), Al (0.042 and 0.13 at.% Cu), Al (0.1 at.% Mg), and Al (0.06 at.% Zn). These measurements are expected to be approximately 5% accurate.

Residual Cu impurities of 15-30 ppm were seen, as well as possible traces of Si.

Seven or eight 3mm-thick sample plates were stacked inside a cryostat. Temperatures were measured with carbon and platinum resistors and controlled to typically better than 0.1K.

An iron core magnet with shims and trim coils provided a field of 500 G. The measured inhomogeneity was about 25μG over the sample volume and during the μSR measurement the field was controlled by a feedback system to better than 1μG. A continuous record was maintained of any deviation.

Positrons were detected with plastic scintillators at 0° and 180° with respect to the incident beam direction. Both were 20 × 20 cm, the 180° detector having a 6.4-cm diameter hole for the beam to enter. Typical stopping muon rates were 7000 sec⁻¹ with event rates of approximately 2000 sec⁻¹. Data were taken for about 45 minutes per point, corresponding to 4 million events. These data were fitted on-line using the usual expression with Gaussian damping of the polarization, exp(-a²(t)). We have found that the dimensionless parameter:

\[ \alpha = \int_{0}^{\infty} e^{-\gamma(t)} dt \]

with \[ \gamma(t) = \sum_{i} c_{i} \int_{0}^{t} dt_{1} F_{i}(t_{1}) G_{i}(t-t_{1}) \]

is a natural representation of the depolarization in terms of the occupation probability \( F_{i}(t) \) and autocorrelation function \( G_{i}(t) \) for site \( i \). As a matter of practice the Gaussian fit to the data is used and then \( \alpha = 2e^{-\tau_{0}} \).

III. Results and Discussion

For comparison to the data it is useful to know that \( \alpha \) is 0.44 for the muon localized at an octahedral site and 0.12 for a substitutional site neglecting lattice relaxation and distortion of the muon wavefunction.

In Al, deformed at 77K, we had previously found [3] depolarization rates consistent with trapping in open volume defects and not at interstitial sites due to the strain fields around the defects. These data are presented in Fig. 1a for reference.

On the other hand, with Cu impurities in Al we had found a concentration dependence to the structure, see Fig. 1b, which indicates stronger trapping for increased impurity density.

The general features seen for Al(Cu) are seen in Al(Si) with peaks occurring in different places. A maximum in \( \alpha \) occurs at 2K of 0.2 then \( \alpha \) falls linearly to 0.15 at 5K, a peak of 0.2 arises at 10K followed by a plateau of 0.12 between 15
Fig. 1 Temperature Dependence of the Depolarization Rate Parameter.

and 30K, a peak of 0.13 occurs at 35K, followed by a decrease to 0.04 at 120K. A final peak is seen at 180K of 0.07 and by 250K α has been reduced to 0.015.

Complex structure is also observed for Al(Mg) but with a different character to the pattern. This is shown in Fig. 1c.

For the Ag alloy α decreases from 0.07 at 2K to 0.16 at about 15K, dropping off starting a little above 20K to a minimum of 0.12 at 35K followed by a peak of 0.15 at 60K and a further minimum of about 0.06 at 100K and a final slight rise at 180K.

The Al(Zn) data presented in Fig. 1d have the least pronounced structure of all the samples. This may indicate a smaller lattice strain associated with the more readily soluble Zn. The fall off in α at only 4K, compare Al(Mg) Fig. 1c, would indicate a very small binding, possibly with a trap rate comparable to the detrapping rate and both sufficiently large to allow motional narrowing from the expected interstitial value for α of 0.44.

A common feature to all our data seems to be a small peak in α of between 0.05 and 0.08 at about 180K.
There is a remarkable insensitivity to heat treatment for all our data. The number and rough position of peaks and valleys remain throughout. At most there may be a narrowing of some of the peaks with annealing. This could indicate that even the rapid quench is too slow to freeze sufficient vacancies to be observed, or that the strain fields induced by the impurities do not allow motion to the vacancies and that the bonding of vacancies and impurities in clusters is sufficiently strong so as to be insensitive to heat treatment.

In the future, several directions could be explored. Very thin samples so as to be able to quench the sample in milliseconds or less would clarify the question of whether or not the cooling rates were fast enough. The use of single crystals and varying the magnetic field would better determine the trapping site. Following an alloy system through known annealing stages including the formation of precipitates would be useful in interpreting the features we are seeing. Certainly the structure is complex and is a challenge to be understood.

IV. Acknowledgements

We would like to thank H. K. Birnbaum, K. G. Petzinger and W. B. Gauster for many helpful discussions. The artistry of S. Hummel with light pipes is gratefully acknowledged. The staff of SREL, even in the cyclotron's last hours, provided truly marvelous support. The work at the College of William and Mary and at George Mason University was supported in part by the NSF and the Commonwealth of Virginia; work at Brookhaven National Laboratory by the Dept. of Energy; work at Virginia State College by NASA.

References

Diffusion and Trapping of Positive Muons in Al:Cu Alloys and in Deformed Al


College of William and Mary, Williamsburg, Virginia 23187, and Bell Laboratories, Murray Hill, New Jersey 07974, and George Mason University, Fairfax, Virginia 22030, and Brookhaven National Laboratory, Upton, New York, 11973, and Virginia State College, Petersburg, Virginia 23803

(Received 8 June 1976)

Depolarization of $\mu^+$ particles implanted into quenched alloys of 0.042, 0.13, and 0.42 at.% Cu in Al shows peaks in the temperature dependence which are attributed to trapping of the muons by various metastable Cu impurity complexes, and provides evidence that positive muons can be used to study impurity correlations in metals.

Previous work has shown complex muon diffusion phenomena in metals. This work is a study of one of the simplest systems of dilute impurities in metals which shows interesting muon depolarization dependence on temperature and therefore should be useful in understanding muon trapping and diffusion. For low concentrations of Cu in Al, though precipitation is not observed by transmission electron microscopy (TEM), there is indirect evidence from resistivity and NMR measurements as well as theoretical grounds to suspect that Cu is not uniformly distributed.

To study this low-concentration region, we have measured the depolarization of implanted spin-polarized $\mu^+$ particles (muon spin rotation) in the temperature range 1.7 to 300 K. These experiments with muons provide a unique way of studying the thermal diffusion and transitory trapping processes for implanted particles, as well as the nature of impurities and other defects that act as traps. Over the temperature range studied, the weak depolarization that has been previously reported for high-purity Al is explained by rapid diffusion of the muon. Depolarization has previously been observed in Al:Cu. The present data, taken with a finer temperature mesh on quenched Al:Cu alloys reveal previously unobserved peaks in the temperature dependence, which can be explained on the basis of a model involving muon trapping at sites near Cu impurity complexes. So far trapping by impurities has not been treated theoretically. We presume that it arises from the local lattice distortion and electronic perturbation at sites near the impurity.

Alloys containing 0.042, 0.13, and 0.42 at.% Cu were prepared from 99.999% pure Al and Cu. Eight 0.3-cm×7.5-cm×10-cm plates fabricated from each ingot were combined to make muon-spin-rotation samples. Each plate was given a homogenizing treatment by annealing in air at 820 K followed by a quench (0.1 sec time constant) into agitated brine at 290 K. Prior to measurement, the 0.42% alloy was aged for 1.5 days at 390 K, the 0.13% alloy about 0.5 h at 300 K, and the 0.042% alloy 3 days at 300 K. Stereoscopic TEM measurements find no evidence of precipitation down to 200 nm and find an average dislocation density of $10^6$ cm$^{-2}$.

In order to test the role of point defects and dislocations on muon trapping, we also studied a 99.999% pure Al sample, deformed 50% in compression at approximately 77 K to a final thickness of 2 cm. Prior to the muon-spin-rotation measurements, the temperature of the sample was kept below 16°C to avoid the loss of vacancies created during deformation would not be lost. Subsequent TEM analysis at room temperature found dislocation cells about 1 µm across, with a dislocation density of $10^6$ to $10^{10}$ cm$^{-2}$.

The results of the muon-spin-rotation measurements are shown in Fig. 1 and were analyzed in the following manner. The decay of the asymmetry in the muon-spin-rotation signal can be represented as $\exp[\gamma(t)]$. The muon spin decay rate, which we define as $\gamma$, has an average value which is statistically weighted according to the radioactive decay law. Thus we can define a dimensionless depolarization parameter:

$$\sigma = \int_0^\infty \exp(-st)(dy/dt)dt,$$  \hspace{1cm} (1)
The calculated value of $\alpha$ is 0.44 for the muon localized at an octahedral interstitial site and 0.12 for a substitutional site, neglecting lattice relaxation and the finite extent of the muon wave function,8,11.

Nonlinear screening calculations using the jellium model predict trapping of protons as well as muons at vacancies in Al.13,14 We had assumed a priori that dislocations would also be capable of trapping the muons.

Our results for the deformed Al in the vicinity of 100 K are consistent with the trapping at open volume defects as opposed to trapping at interstitial sites by the strain fields surrounding defects.15 A smaller fraction of the muons diffuse to traps at lower temperatures, indicating that the diffusion time increases with decreasing temperature.

The depolarization in the alloys at $T \leq 50$ K is sufficiently large to be caused by the trapping of muons at interstitial sites. For the two more dilute alloys $\alpha$ does not reach a constant maximum at low temperatures nor does it drop off smoothly with temperature, as it does for Cu (Fig. 1). In Cu the depolarization at low temperatures is the result of self-trapping at an octahedral interstitial site,16 and the decrease at high temperatures is attributed to thermal diffusion.12 The peaks observed in the alloy data at low temperatures increase in magnitude with increasing Cu concentration. This is expected for thermal diffusion of the $\mu^+$ to traps whose population of Cu increases with impurity concentration.17 What is surprising here is that complex structure is found above 10 K.

A motional-narrowing theory for $\gamma(t)$, related to that used for nuclear magnetic resonance, has been proposed18 to explain muon diffusion and trapping in terms of correlation functions:

$$\gamma(t) = \sum_i C_i^2 \int_0^t dt' \int_0^t dt'' F_i(t'') G_i(t' - t''). \quad (2)$$

The sum extends over all interstitial sites in the sample, each having a local frequency distribution of second moment $\sigma_i^2$ from nuclear dipolar fields. $F_i(t')$ is the probability that the muon occupies site $i$ at time $t'$ and $G_i(t')$ is the site autocorrelation function. Equation (2) can be rewritten to represent equivalent unperturbed sites in the host and $n$ types of trapping sites. The result is similar to Eq. (2), where we now define $f_0$ as the probability that any host site is occupied, and $f_m$ for $m=1, \ldots, n$ as the probability that a trap of type $m$ of concentration $c_m$ is occupied.

where the muon lifetime $s^{-1} = 2.2 \mu$sec. We evaluate $\alpha$ by fitting the experimental time histogram with a Gaussian function, $\gamma(t) = \frac{1}{2} \alpha^2 t^2$, which we find gives a better fit than an exponential. We have chosen to represent the data in terms of the $\alpha$ parameter because $\alpha$ is also conveniently calculated from the theory, discussed below. For comparison, values of $\alpha$ calculated from previous measurements on annealed polycrystalline samples of Al14 and Cu12 are also shown in Fig. 1.
good approximation to the autocorrelation function for any site of type \( m \) is \( \alpha_m = \exp(-p_m s) \).

When the traps are assumed to be dilute, the \( f_m \) are then solutions to the following rate equations:

\[
\frac{df_m}{dt} = -p_m f_m + \sum_{m=1}^{\infty} p_m f_m \left[ f_m(0) = 1 - c \right],
\]

\[
\frac{df_m}{dt} = p_m f_m - p_m f_m \left[ f_m(0) = c_m; m \neq 0 \right],
\]

where \( c = \sum_m c_m \). The renormalized jump rate between host sites is \( p_\alpha \). The \( p_m \) for \( m > 0 \) are detrapping rates given as \( p_m = p_\alpha \exp(-H_m/kT) \), with \( H_m \) the binding enthalpy to the trapping site. Actual lattice structure is not taken into account explicitly in this model.

Since our definition of \( \alpha \) is identical to the Laplace transform, the solution for \( \alpha \) can be expressed as

\[
\alpha = \sum_{m=0}^{\infty} c_m f_m(s)/(s + p_m),
\]

where \( f_m(s) \) is the transform of \( f_m(t) \). Thus one can solve the theoretical Eq. (2) quite easily as one does not need to solve the integral convolution of this equation in the time domain. Since the depolarization in pure Al is small, \( p \gg s \) and the \( m = 0 \) term is negligible. The model predicts a peak in the temperature dependence of \( \alpha \) when \( p_\alpha \) increases monotonically with temperature and the trapping times and detrapping lifetimes are comparable to the muon lifetime, i.e., when the conditions \( p_\alpha s = s \) are satisfied. Thus, the peak appearing at the lowest temperature is associated with traps present in the highest concentration, assumed to be a single Cu impurity. The additional peak structure observed at higher temperatures is due to a lower concentration of traps with higher binding enthalpies, assumed to be di- and tri-Cu clusters.

Equation (5) was used to fit the data on the alloys for \( T \leq 100 \) K using a nonlinear least-squares procedure with \( n = 5 \). Because of a lack of a quantitative theoretical treatment, we have used an empirical formula for the temperature dependence of \( p_\alpha \):

\[
p_\alpha = a \exp(-H_0/kT) + b + dT^2,
\]

where the three terms are to represent the following. At high temperatures the Flynn-Stoneham model predicts an Arrhenius dependence and at low temperatures a constant that is proportional to the coherent tunneling rate plus a one-phonon-assisted tunneling term that is linear in temperature and dependent upon strain broadening. We find that the \( dT^2 \) term fits the data better than a linear term. The fitted parameters are given in Table I. For the highest-concentration alloy (0.42 at %) the total number of traps is sufficiently high that the contribution of the individual traps is more difficult to observe and the data therefore were not fitted.

The significance of these results is that qualitatively our model describes the data at low temperatures. Quantitatively, the binding enthalpies giving rise to the trapping at low temperatures are quite small. The concentration of traps producing the peak at 10 K is consistent with individual Cu impurity traps. The structure seen at \( T > 10 \) K is attributed to appreciable concentrations of microclusters of two or more Cu. We note that if the distribution of impurities were random, the fraction of interstitial sites with more than one Cu nearest neighbor would be about two to three orders of magnitude smaller than the fraction with one neighbor. Our larger fractions indicate that clustering is energetically more favorable even at these low Cu concentrations.

The depth of the \( m = 5 \) trap requires a different interpretation. On the basis of our findings for the deformed Al and our TEM results, we suspect that dislocations are responsible for the structure observed at temperatures near 100 K. This is also consistent with our small \( c_m \) parameter. Diffusion along these dislocations explains the smaller depolarization at 300 K.

The results of this Letter are the first evidence demonstrating that positronium muons provide a unique way of studying impurity correlations in...
metals. Extension of this work to other impurities in Al is currently in progress.

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11The Zeeman energy is expected to exceed electric quadrupole splittings for $^{27}$Al near the $\mu^+$ and Cu (Ref. 5) impurities. Electric field gradients and the location of the muon can be determined from the magnetic field dependence for single crystals. [See P. Jena et al., Phys. Rev. Lett. 40, 264 (1978); and O. Hartmann, Phys. Rev. Lett. 58, 852 (1977).] In weak fields $\alpha$ may be a factor 5/3 larger.
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Dynamic field on 40-MeV \( \mu^+ \) traversing magnetized iron

J. M. Brennan, N. Bencze-Koller, and M. Hass
Rutgers University, New Brunswick, New Jersey 08903

W. J. Kossler and J. Lindemuth
The College of William and Mary, Williamsburg, Virginia 23186

A. T. Fiory, D. E. Murnick, and R. P. Minnich
Bell Laboratories, Murray Hill, New Jersey 07974

W. F. Lankford
George Mason University, Fairfax, Virginia 22030

C. E. Stronach
Virginia State College, Petersburg, Virginia 23803

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Previous experiments have demonstrated the existence of large dynamic magnetic fields that act on positive heavy ions traversing magnetized iron. These studies have been extended to the limit of ionization and to \( v/c = 0.68 \) by using spin-polarized muons. The angular precession of the spin of \( \mu^+ \) particles which traversed magnetized-iron plates and stopped in aluminum was measured. Decay positrons were detected at \( 0^\circ, 90^\circ, \) and \( 270^\circ \). Relative phases of the corresponding muon-spin-rotation spectra were obtained as a function of the magnetization direction in the iron. After subtracting the phase shift resulting from the saturation magnetization of the iron and the fringing fields, the experimental result for the dynamic field \( \Delta B = -0.5 \pm 2.6 \) kG is obtained, in agreement with theoretical expectations for very fast totally stripped ions.

INTRODUCTION

The existence of an intense transient magnetic field acting on fast ions slowing down in magnetic media was discovered in 1968 by Borchers et al.\(^1\) The transient field was theoretically described by Lindhard and Winther\(^2\) in 1971 in terms of scattering of polarized electrons in the ferromagnetic medium by the moving ions. Hubler et al.\(^3\) in 1974 parametrized the Lindhard and Winther model using existing data for medium-weight ions of low initial velocity, \( v/c \ll 1 \). In this model, the field varies inversely with the ion's velocity for \( v/c \gtrsim 0.004 \), is constant at lower velocity (0.0005 \( \lesssim v/c \lesssim 0.004 \)) and vanishes at low velocity.

In recent measurements\(^4-10\) on faster ions, a dynamic field much larger than that predicted by the adjusted Lindhard-Winther theory\(^2\) was observed and, within the velocity range studied, the field increases with the ion's velocity through the medium. This field was qualitatively described in terms of polarized-electron capture into electron vacancies produced in the moving ion.\(^8\) A recent review of the subject has been given by van Middelkoop.\(^11\)

Experiments have been carried out at the Rutgers-Bell tandem accelerator on \(^{56}\)Fe, \(^{82}\)Se, \(^{106}\)Pd, and \(^{112}\)Cd ions in iron and on \(^{66}\)Se ions in gadolinium in the velocity range 0.01 \( \lesssim v/c \lesssim 0.05 \). In these experiments, the ions were excited to their \( 2^+ \) state by a beam of 72-MeV \(^{56}\)Fe\(^{39+}\) ions, recoiled through the ferromagnetic foil and stopped in a copper backing where they decayed in an environment free of perturbations. The precession of the angular correlation of the deexcitation \( \gamma \) rays was measured as a function of the direction of polarization of the ferromagnetic material. As the \( g \) factor of these states was known, the velocity and \( Z \) dependence of the magnetic field was determined. The results obtained with the heavier ions show unambiguously that the dynamic field which acts on the ions studied increases with the velocity of the ion. The observed dynamic field has been phenomenologically described by a function of the velocity and atomic number of the ion: \(^5,9,10\)

\[
B(Z, v) = aZ^{2/3}(v/v_0)\mu_BN_p, \tag{1}
\]

where \( v_0 = e^2/h \), \( N_p \) is the polarized-electron density and \( \mu_B \) is the Bohr magneton. For iron \( \mu_BN_p = 1752 \) Oe and \( a = 12 \pm 0.5 \).

An alternate parametrization in terms of the effective charge of the ion passing through the magnetic material fits the data equally well:

\[
B(Z, v) = a'Z_{eff}^{2/3}\mu_BN_p, \tag{2}
\]

where

\[
Z_{eff} = Z[1 - \exp(-v/v_0Z^{0.54})] \tag{3}
\]
is the best fit to the observed effective charge of ions leaving a solid foil,\textsuperscript{12} and $a' = 140 \pm 8$.

In both parameterizations, the field depends on the type of ferromagnet only through the number of polarized electrons per unit volume, suggesting that atomic shell or band-structure effects may not play a significant role for heavier ions.

As the nature of the dynamic field—a hyperfine interaction between the partially stripped ion and the magnetized medium—is not yet fully understood, it is not at all evident that the parameterization of the field which seems to fit the data in the velocity range $0.01 \leq v/c \leq 0.05$, should extend to higher velocity. At velocities greater than $Zv_p$, where the ions are completely stripped, the solid can probably be treated as a gas of polarized electrons and a dynamic magnetic field inversely proportional to the ion velocity should result.\textsuperscript{2,13}

In order to test the validity of the charge and velocity dependence of the alternate parameterizations, high velocity, totally stripped ions should be used. For this purpose, the effect of the dynamic field on polarized positive muons traversing a thin magnetized iron plate was investigated.

Previous studies using the spin precession of muons stopped in unmagnetized iron have found that the field is $3.6 \text{ kG}$ at room temperature, for a sample in zero external field.\textsuperscript{14,15} This field is explained in terms of a dipolar contribution from the iron ions and a contact interaction with the polarized screening electrons.\textsuperscript{16} These results imply that the dynamic field on the muon is not very large. Otherwise the residual polarization of the muon's spin would have been lost on traversing the unmagnetized iron, washing out the spin precession signal of the stopped muon.\textsuperscript{15}

In the present experiment, the effects on muons which traverse a magnetized iron plate and subsequently stop in an aluminum target located in a fixed externally applied magnetic field were observed. The angle through which the spin has precessed in the iron is detected as a phase shift in the oscillatory $\mu$SR (muon spin rotation) spectra of the stopped $\mu^+$.\textsuperscript{17}

**EXPERIMENTAL PROCEDURE**

The experiment was carried out with the apparatus shown schematically in Fig. 1. Positive pions are produced at an internal target in the SREL (Space Radiation Effects Laboratory) synchrocyclotron\textsuperscript{18} and focused into an external meson channel. The momentum analyzing magnet $M$ is adjusted to select $\mu^+$ particles which originate from $\pi^+$ decays in the backwards direction in the $\pi^+$ frame of reference. The $\pi^+$ decays produce a beam of spin polarized muons, a well-known consequence of parity nonconservation in weak interactions. The arrangement of charged particle scintillation detectors is also shown in Fig. 1.

Figure 2 shows the measured particle flux $\frac{dI}{dE}$ detector $A$ as a function of momentum, as determined by varying the current through the deflecting magnet $M$. The $\mu^+$ beam passes through a magnetized Fe plate in the reversible magnet $C_1$, and stops in an Al target located in a constant magnetic field produced by magnet $C_2$.

The logic coincidence $ABF$ establishes that a $\mu^+$ stopped in the target. The mean momentum of the incident $\mu^+$ beam, 103 MeV/c, was deduced from the range measurements shown in Fig. 3, using the known momentum dependence of the muon's range in various materials.\textsuperscript{19} The mean kinetic energy of the incident beam is 42 MeV and the

![FIG. 1. Layout of the meson channel at SREL and the $\mu$SR apparatus in the target area. $Q_1$ and $Q_2$ are input and output quadrupole magnets, $Q_0$ the "drift tube" where the pions decay into muons, and $M$ the momentum selection magnet. The gap of magnet $C_2$ is 30 cm. The Al stopping target is located 55 cm downstream from the Fe plate.]
mean velocity is 0.68c. Two range curves are shown, for the aluminum stopping target used for the phase-shift measurement, and the other for a thinner Cu target. The width of the momentum distribution is determined more accurately with the Cu target. The momentum spread is about ±8 MeV/c, corresponding to a velocity spread of ±0.03c. The experiment was carried out separately for different Fe plates of thicknesses varying from 0.1 to 0.8 cm inserted in the C1 magnet. The mean velocity of the muons in the Fe was decreased by up to 6%. The energy of the muon beam hitting the target was kept approximately constant by adding appropriate amounts of CH₄ degrader to compensate for the thinner Fe plates.

The iron plates were fabricated from 99.99% Fe, machined into sheets 12.7-cm square and 0.05- to 2.5 cm thick and subsequently annealed. The electromagnet C1 has a movable iron yoke that mates evenly against the edges of the plates. The direction of the magnetization of the Fe plate is vertical. The magnetic induction in the plate was measured by integrating the voltage produced in a sense coil wound around the plate as the C1 magnet is energized. For 1600 ampere-turns a magnetic induction of 16 kG is produced in the plate. The profile of the fringing field outside the plate was mapped along the beam axis, and is shown in Fig. 4. The fringing field was insensitive to the Fe plate thickness in the range 0.1-0.8 cm.

The muon beam was collimated to an approximately 12-cm square aperture by the C1 magnet yoke and additional 5-cm thick Pb shields placed at each side of the Fe plate. The profile of the μ⁺ beam at the Al target position (Fig. 5) was scanned horizontally in the direction transverse to the beam, for the two directions of magnetization of the Fe plate with a 2.5-cm square scintillation detector. These measurements show a bending of the beam, due mainly to the deflection in the magnetization field in the Fe plates and to a lesser extent to the fringing field. For an Fe plate of thickness dₑ, a bending angle ψₑ may be calculated from the measured magnetization and fringing fields:

$$\psi_e = (0.067 \text{ cm}^{-1}) d_e + 0.006. \quad (4)$$

The aluminum target was 5-cm wide by 10-cm high and 2.5-cm thick. The local field at the tar-
The true zero of time was periodically recorded by detecting prompt signals associated with scattered muons. The timing calibration of the analyzer was also checked before and after each run. A low magnetic field in C2 is desirable since spurious phase shifts originating from electronic timing drifts increase in proportion to the spin precession frequency.

Several runs were carried out on each of a variety of Fe plates 0.1-, 0.4-, 0.6-, and 0.8-em thick, respectively. The spectra corresponding to either of the three detectors at 0°, 90°, or 270° were fitted using a nonlinear least-squares method with the following model μSR function.

$$N(t) = A_1 e^{-t/\tau} \left[ 1 + a_1 e^{-\lambda t} \cos (\omega \pm \Delta \omega) t + \phi \pm \Delta \phi \right] + B_1,$$

where (+) or (−) applies to the spectra for the two directions of Fe plate magnetization. The fitting procedure treats $A_1$, $A_2$, $a_1$, $a_2$, $\lambda$, $\omega$, $\Delta \omega$, $\phi$, $\Delta \phi$, $B_1$, and $B_2$ as 11 adjustable parameters. The muon lifetime $\tau = 2.1594 \mu$sec is used.

The parameters $a_1$ give the magnitude of the asymmetry, which depends upon geometrical details such as the polarization of the muon beam and the selectivity in the $e^-$ detection. These effects are illustrated in Fig. 6, where the asymmetry measured for a thin Cu target is shown as a function of the incident $\mu^-$ momentum as determined by the thickness of the degrader interposed in the beam. The results display the dependence of the asymmetry upon the average momentum of the stopped $\mu^-$ and the target thickness, which affect the energy of the emitted $e^-$. The angle $\phi$ is an average geometric angle determined by the average initial spin orientation of the $\mu^-$ beam and the location of the positron detector. The angular precession frequency of the stopped muons is $\omega = \gamma \mu \times 105 \text{ G} = 9.0 \times 10^8 \sec^{-1}$. The factor $\exp(-\lambda t)$ takes into account the inhomogeneity in the local field in the target.

The phase-shift parameter $\Delta \phi$ gives the change monitored.
in the spin orientation of the muon beam produced by the magnetization and dynamic fields in the Fe plate and the fringing field. Both the magnitude and the statistical uncertainty in $\Delta \varphi$ are insensitive to the sampling time interval of the multichannel analyzer and to the magnitude of the field on the Al target. This result was deduced from the analysis of model simulations of the experiment. The parameter $\Delta \omega$ was included in the model function in order to account for a possible small change in local field when $C1$ is reversed. The parameter $\Delta \varphi$ is strongly correlated with $\Delta \omega$, the correlation coefficient being 0.8. The correlation coefficients between $\Delta \omega$ and the other parameters have magnitudes less than 0.1. Typical values obtained for the 0.4-cm Fe plate are given in Table I, while the phase shifts obtained for the other thicknesses of Fe are displayed in Table II.

An alternate procedure was also tested, whereby $\Delta \omega$ was set to 0 and 10 adjustable parameters were varied. This analysis did not produce statistically significant changes in $\Delta \varphi$, although the statistical errors in $\Delta \varphi$ were reduced by 40%. In the 11-parameter fits, the $\Delta \omega$ values were smaller than the statistical errors in $\Delta \omega$.

DISCUSSION OF THE RESULTS

As the muon velocity in the iron is approximately constant, the phase shift is proportional to the time spent in the iron plate and hence varies line-
Here, the field would be proportional to the velocity of the muon, and the resulting phase shift would in turn be proportional to the Fe plate thickness \( d_{Fe} \):

\[
\Delta \phi = \gamma_{\mu} B \int \frac{v}{v_0} \frac{dx}{v} \mu_{\mu} N_e \int \gamma_{\mu} B dt = \gamma_{\mu} a Z \mu_{\mu} N_e \int \frac{v}{v_0} \frac{dx}{v} \mu_{\mu} N_e d_{Fe} / v_0.
\]

This phase shift turns out to be \((8.2 \text{ cm}^{-1})d_{Fe}\), radians—a rather dramatic prediction. The data easily rule out such an extrapolation of the field to high velocity.

Although we do not have a precise measure of the dynamic field, our result is in agreement with the Sak and Bruno prediction\(^1\) for \( v \gg v_0 Z \):

\[
\Delta B = 4 \pi \gamma_{\mu} a Z \mu_{\mu} N_e / v_0.
\]

Lindhard and Winther did not actually evaluate the enhancement of the magnetic field at high velocity. Their theory was designed to explain the large magnetic fields acting on relatively slow \((v \ll Z v_0)\) ions. For this reason, they assume the field is approximately spherically symmetric and neglect the asymmetric corrections represented in their notation by the parameter \( \lambda \). These corrections are, however, important at high velocity. Hence the Lindhard-Winther theory is not directly applicable to fast ions or muons. Furthermore, even though in principle their theory could apply to slow ions, they also neglect atomic shell effects and assume that the ions are essentially stripped. Shell effects can be neglected only for swift ions with \( v \gg v_0 Z \), hence in just the region where the asymmetry corrections are important.

It would be very interesting to fill in the large gap in velocity between the present muon experiment and the heavy-ion experiments. The two experiments taken together indicate that there is a maximum in the velocity dependence of the dynamic field. However, the possibility exists that the large dynamic field observed with heavy ions may partly be due to atomic effects such as equilibrium between hole production and capture of polarized electrons from the medium. Experiments on very slow muons \((v \ll v_0)\) and on very fast totally stripped heavy ions \((v \gg Z v_0)\) may be necessary for a better understanding of the dynamic field.

In conclusion we mention two examples of related work with positive muons. It is known that the \( \mu^+ \) picks up a bound electron in semiconductors and insulators, forming a muonium atom. Recently, a muonium stage for \( \mu^+ \) stopping in a Ge

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**TABLE IV.** Calculation of the dynamic field and ensuing total phase shifts \( \Delta \phi \) for muons at \( r/e = 0.68 \), or \( r/z_9 = 93 \), based on three proposed parametrizations of the dynamic field. The saturation field of the iron was taken as 16 kG, the measured value for the iron plates used in the experiment.

<table>
<thead>
<tr>
<th>( \Delta B ) (kG)</th>
<th>( \Delta \phi / d_{Fe} ) (cm(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic field Eq. (1)</td>
<td>1530</td>
</tr>
<tr>
<td>Dynamic field Eq. (2)</td>
<td>178</td>
</tr>
<tr>
<td>Sak and Bruno Eq. (7)</td>
<td>0.17</td>
</tr>
<tr>
<td>Experiment</td>
<td>&lt;2.6</td>
</tr>
</tbody>
</table>

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crystal has been observed through a measurement of the phase shift in the \( \mu \)SR signal associated with muonium precession in a transverse external field. Also, our results show that magnetized solid Fe deflectors and lenses may indeed be used for the transport and focussing of high-energy muon beams without significant loss of polarization.

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*Present address: Weizmann Institute, Rehovot, Israel.
13J. Sak and J. Bruno (private communication).