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MAGNETOPAUSE SURFACE FLUCTUATIONS  
OBSERVED BY VOYAGER 1

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## ABSTRACT

As Voyager 1 moved out of the dawnside of the Earth's magnetosphere at 1634 UT on September 5, 1977 [at position  $(-2.6, -16.5, 1.1)$  earth radii in GSE], it crossed the magnetopause apparently seven times, despite the high spacecraft speed of 11 km/sec. Normals to the magnetopause and their associated error cones were estimated for each of the crossings using a minimum variance analysis of the internal magnetic field. The oscillating nature of the ecliptic plane component of these normals indicates that most of the multiple crossings were due to a wave-like surface disturbance moving tailward along the magnetopause. We modeled the wave, which was aperiodic, as a sequence of sine waves with amplitude  $A_i$ , wavelength  $\lambda_i$ , and speed  $V_i$ . These quantities were determined for two pairs of intervals from the measured slopes, occurrence times, and relative positions of six magnetopause crossings. The average amplitude was  $A = 2100^{+1800}_{-500}$  km, and the wavelengths were on the order of  $47,000^{+30,000}_{-12,000}$  km. The wave speed was approximately  $340^{+210}_{-95}$  km/s, and typical periods were in the neighborhood of  $170 \pm 60$  sec. The magnetopause thickness was estimated to lie in the range 300 to 700 km with higher values possible.

The estimated amplitude of these waves was obviously small compared to their wavelengths; this conclusion is independent of any bulk normal motion of the magnetopause that might have been present.

## 1. INTRODUCTION

Earth orbiting spacecraft moving from the magnetosphere to the magnetosheath (or vice versa) have often observed multiple, "discontinuous", transitions in the magnetic field, each with the characteristics of magnetosheath fields on one side and magnetosphere fields on the other side. There are three classes of interpretations of such multiple transitions. One is that the multiple transitions are due to the motion of a single discontinuity (magnetopause) back and forth across the spacecraft (Willis, 1971; Fairfield, 1978). This could be due to bulk displacements of the magnetopause caused by changes in upstream conditions, to tailward propagating waves (e.g., generated by a Kelvin-Helmholtz instability (Southwood, 1968), or to a disturbance convected tailward by irregularities in the magnetosheath flow. Another class of interpretations of multiple magnetopause transitions is that they are due to a complex, quasi-stationary structure resulting from the penetration of sharply bounded filaments of magnetosheath plasma into the outer magnetosphere (Lemaire and Roth, 1978). Generally, one can always construct such a configuration which will describe observations from just one spacecraft, but such constructions may be complex, arbitrary, and non-unique. Based on data from a dual spacecraft mission (ISEE-1 and 2) the third class is one in which the transition zone is viewed, during periods of southward  $B_z$  magnetosheath magnetic fields, as being composed of ripped-off magnetospheric flux tubes, possibly implying sporadic field reconnection (Russell and Elphic, 1978).

In some multiple magnetopause transitions, an oscillation of the normal for each successive discontinuity is observed (e.g. alternately inclined tailward or sunward from the mean direction) with a particular phase corresponding to entry or exit from the magnetosphere (sunward-inclined when the spacecraft moves from the magnetosphere to the magnetosheath and tailward for the opposite situation.)

This type of multiple transition is most simply explained as multiple crossings of a discontinuous magnetopause due to a tailward moving wave-like disturbance on the magnetopause (Kaufman and Konradi, 1969; Aubry et al., 1971). Aubry et al. (1971) observed such a disturbance on OGO - 5. Assuming that the disturbance was a wave moving tailward relative to the earth at a speed  $V_w = 500$  km/s, they determined that the wavelength was  $\sim 3600$  km ( $\sim 0.52 R_E$ ) and that its thickness was  $\sim 140$  km ( $\sim 2$  Larmor radii). They could not determine the amplitude of the wave. Ledly (1971) and Fairfield (1976) carried out similar analyses for other magnetopause crossings. Again, their results depend on an assumed wave speed, and they were unable to determine the wave amplitude.

Helzer et al. (1966) interpreted multiple magnetopause crossings observed by OGO as the result of large-scale "normal" motions of a discontinuous magnetopause, toward and away from the average magnetopause position. Taking the oscillation amplitude to be one half of the radial distance between the first and last crossing and using the observed times between the crossings, they estimated that the speed of the magnetopause normal to the average magnetopause direction was  $V_{Nmp} \approx 10$  km/s.

Howe and Siscoe (1972) estimated that at the lunar orbit the magnetopause moves with a speed  $V_{Nmp}$  in the range 10 to 20 km/s with an amplitude of 1 to  $2 R_E$  on the dawn side at periods of  $\approx 17$  min. Their model does not consider the possibility of a wave moving tailward, and such a model cannot explain an oscillation of the magnetopause normals.

In general, both small-scale tailward moving disturbances on the magnetopause and large-scale normal motions of the magnetopause can occur.

Earth-orbiting spacecraft move at a speed of a few km/s relative to the average magnetopause position, which is significantly less than  $V_{Nmp}$ ; such spacecraft cannot separate normal magnetopause motions from tangential (tailward) wave motions. Voyager 1 moved through the magnetopause at a high normal speed, 9.2 km/s, which is probably greater than or comparable to  $V_{Nmp}$ . Thus, the effect of normal motions is much smaller at Voyager 1 than at earth orbiting spacecraft, and Voyager 1's observations of multiple crossings provide a better opportunity to study the small scale tailward moving disturbance. It will be shown that although the normal motions are not negligible, one can obtain a good estimate of the wave amplitude, wavelength and wave speed from the Voyager 1 data.

## 2. CHARACTERISTICS OF THE VOYAGER 1 MAGNETOPAUSE CROSSINGS

Figure 1 shows the magnetic field observations made by Voyager 1 as it passed through the magnetopause between 1626:27 and 1645:26 on launch day, September 5, 1977. The position of the spacecraft was  $(-2.6, -16.5, 1.1) R_E$  (earth radii) in GSE coordinates, and the spacecraft speed was 10.6 km/s. (Also see Figures 1 and 2 of Lanzerotti et al., 1979, for the trajectory and an alternate view of the field observations.) The plasma science instrument was not yet turned on. Seven nearly discontinuous transitions in the magnetic field were observed, with the characteristics of magnetosheath fields on one side ( $\sim 10\gamma$  intensity  $\pm 2\gamma$  variability, based on  $2\sigma$  of pre-or post-48 s averages for all seven crossings) and the characteristics of magnetosphere fields on the other side ( $\sim 27\gamma$  intensity,  $\pm 8\gamma$  variability). Notice that both  $B_R$  and  $B_T$  are distinctly negative in the magnetosphere, positive in the magnetosheath, and usually of mixed sign in the transition zones. The center times of each transition ( $T_c$ ), durations of the transition ( $\Delta T$ ), and the change in magnetic field direction across each transition ( $\omega$ ) are given in Table 1.

A minimum variance (MV) analysis (Sonnerup and Cahill, 1967; and see Burlaga et al., 1977) was applied to the magnetic field measurements in each transition, which were made every 60ms. The minimum variance analysis yields the following parameters for each transition: a measure of the extent to which the magnetic vectors lie near a plane ( $\lambda_2/\lambda_3$ , which is the ratio of the intermediate eigenvalue to the minimum eigenvalue); the direction of the normal to the minimum variance plane (longitude,  $\lambda_{mv}$ ; and latitude,  $\delta_{mv}$ ); and the RMS of the component of  $\underline{B}$  normal to the MV plane ( $\text{RMS} \{ B_H \}$ ). The results of the minimum variance analysis of the Voyager magnetopause transitions are given in Tables 1 and 2. These are the basis of the discussion that follows.

The nature of the magnetopause transitions was that of a tangential discontinuity (TD). This is revealed in three ways. First, the angle  $\beta$  between

the minimum variance plane normal and the average field direction is close to ninety degrees (see Table 1), i.e., the magnetic field vectors are all nearly parallel to the minimum variance plane. The  $1\sigma$  uncertainties in  $\beta$ ,  $\Delta\beta$ , were computed as a function of  $\omega$ ,  $\beta$ , and  $\lambda_2/\lambda_3$  using the results of the error study by Lepping and Behannon (1979). Table 1 shows that within the uncertainties given by  $\Delta\beta$ ,  $\beta$  is consistent with  $90^\circ$  in all cases except possibly crossings 5 and 7. Also the ratio of RMS  $\{B_n\}$  to the average field intensity  $\langle B \rangle$  is unusually small (between 0.05 and 0.15 except for crossing 7) and consistent with zero within the experimental errors as it should be for well-determined discontinuity normals. Crossing 7 had the most poorly determined normal ( $\Delta\beta = 10^\circ$ ) and the largest RMS  $\{B_n\}$ , because the magnetic fields in the transition varied irregularly in three dimensions. Second, the minimum variance normals ( $\lambda_{mv}$ ,  $\delta_{mv}$ ) were found to be comparable with those computed from the magnetic fields before and after each transition, using the formula for a TD, viz.  $\hat{n} = (\lambda_{cp}, \delta_{cp}) = \frac{\hat{B}_1 \times \hat{B}_2}{|\hat{B}_1 \times \hat{B}_2|}$ . This is shown in Table 2. Finally, the average magnetopause transition normal ( $\langle \lambda_{mv} \rangle = 115^\circ$ ,  $\langle \delta_{mv} \rangle = 10^\circ$  and  $\langle \lambda_{cp} \rangle = 118^\circ$ ,  $\langle \delta \rangle = -2^\circ$ ) was found to be essentially the same as the model magnetopause normal ( $\lambda_{MOD} = 117^\circ$ ,  $\delta_{MOD} = 0$ ) computed by fitting a hyperbola to the positions of hundreds of magnetopause crossings observed by earth-orbiting spacecraft (Fairfield, 1971). The quantities  $\theta_i$  in Table 2 are defined by  $\theta_i \equiv \lambda_{MV_i} - \lambda_0$ , where  $\lambda_0 = 116^\circ$  is the average of  $\langle \lambda_{MV} \rangle$  and  $\lambda_{MOD}$ . We conclude that the magnetopause transitions observed by Voyager 1 can be regarded as tangential discontinuities

whose well determined normals ( $\lambda_{MV}, \delta_{MV}$ ) are inclined with respect to the unperturbed magnetopause normal by  $\theta_1, 1=1, \dots, 7$ , in the ecliptic plane.

The simplest interpretation of the multiple magnetopause transitions observed by Voyager 1 is that they were due to a wave-like disturbance moving tailward along the unperturbed magnetopause direction. In other words, the magnetopause at this time may be regarded as a single surface which had an irregular profile that moved "tailward" without much distortion (See Figure 1). The evidence supporting this view is the oscillation of the magnetopause normal direction on successive crossings: that  $\lambda_{MV}$  is greater than average for odd numbered crossings and smaller than average for even numbered crossings (Table 2). The larger than average direction of  $\lambda_{MV}$  ("sunward" directed normals) on the odd numbered crossings is consistent with the fact that the spacecraft moved from the magnetosphere to the magnetosheath on odd numbered crossings (See Figure 2). We cannot exclude the possibility that there were also bulk motions of the magnetopause toward and/or away from the earth. In fact, the relatively long intervals between crossings 3 and 4 and between crossings 4 and 5 may be the result of bulk motions. However, the presence of such motions does not exclude the presence of a tailward moving disturbance as well, and bulk motions alone would not explain the oscillations in  $\lambda_{MV}$ .

### 3. A MODEL FOR DESCRIBING QUASI - PERIODIC FLUCTUATIONS

We assume that the multiple magnetopause crossings observed by Voyager 1 were due to a tailward moving, quasi-periodic wave train, and we seek to estimate the "amplitude", "wavelength", "speed", and "period" of the fluctuations. There is no unique way to fit the observations. Our approach is to formulate a model which gives a closed set of relationships between the characteristics of the fluctuations (amplitude, wavelength, and speed) and the measured quantities (time intervals between successive **intercepts** of the wave, the slope of the magnetopause surface at each crossing ( $\tan \theta_1$ ), and the observer's (spacecraft) velocity relative to the undisturbed magnetopause). The model is general, and its application is not restricted to magnetopause observations.

The model is based on seven defining characteristics:

1. It is 2-dimensional, **i.e.**  $\delta = 0^\circ$  for all normals and all latitudinal changes are zero.
2. Between two successive crossings of the wave, the surface has the form:

$$y = A \cos (k x + \epsilon),$$

where A, k, and  $\epsilon$  can be different for successive pairs of crossings;  $\hat{y}$  is normal to the unperturbed magnetopause surface and  $\hat{x}$  is parallel to that surface and to the ecliptic plane.

3. The speed of the wave between two successive crossings a and b is the same as that for crossings b and c, where

c follows b.

4. There is no bulk motion of the surface in the  $\hat{y}$  direction.
5. An "observer" moves at a constant velocity relative to Earth in the (x,y) plane over the period between a and c.
6. The magnetopause thickness is smaller than the amplitude of the wave.
7. Local curvature of unperturbed magnetopause is negligible.

Thus, for each set of three successive crossings the data are fitted to two sinusoids using the measured times and slopes of each of the crossings and the known (constant) velocity of the observer.

The specific equations of the model are the following.

Between points a and b (see Figure 2)

$$y_i = A' \cos (k' \Delta V t_i + \epsilon') \quad (1)$$

$$\tan \theta_i = \frac{dy}{dx} = -A' k' \sin (k' \Delta V t_i + \epsilon'), \quad i = a, b., \quad (2)$$

$$\text{where we set } x_i = (V_w - V_T) t_i \equiv \Delta V t_i (\Delta V > 0), \quad (3)$$

$V_w$  being the wave's speed and  $V_T$  the x component (tangential to magnetopause) of the observer's speed. Likewise, between points b and c,

$$y_i = A \cos (k \Delta V t_i + \epsilon) \quad (4)$$

$$\tan \theta_i = -A k \sin (k \Delta V t_i + \epsilon), \quad i = b, c \quad (5)$$

where we have used the assumption that the wave speed between a and b is the same as that between b and c. The positions  $y_i$  are not measured directly, but they are given by the equations

$$y_i = -V_N t_i + y_0 \quad i = a, b, c, \quad (6)$$

where  $V_N > 0$  is the speed of the observer in the  $-\hat{y}$  direction (normal to magnetopause). One may take  $t_i = 0$  at point a, in which case  $y_a = y_0$ ,  $y_b = -V_N t_b + y_0$ , and  $y_c = -V_N t_c + y_0$ , where  $V_N$ ,  $t_b$ , and  $t_c$  are known (7a,b,c) quantities.

Evaluating (1) and (2) at points a and b with  $y_i$  given by (6) gives,

respectively,

$$y_o = A' \cos \epsilon' \quad (8)$$

$$\tan \theta_a = -A'k' \sin \epsilon' \quad (9)$$

$$y_o - V_N t_b = A' \cos (\beta + \epsilon') \quad (10)$$

$$\tan \theta_b = -A'k' \sin (\beta + \epsilon'), \quad (11)$$

where  $\beta \equiv k' \Delta V t_b \quad (12)$

Similarly, evaluating (4) and (5) at points b and c with  $y_1$  given by (6) gives

$$y_o - V_N t_b = A \cos (\alpha + \epsilon) \quad (13)$$

$$\tan \theta_b = -Ak \sin (\alpha + \epsilon) \quad (14)$$

$$y_o - V_N t_c = A \cos (p\alpha + \epsilon) \quad (15)$$

$$\tan \theta_c = -Ak \sin (p\alpha + \epsilon), \quad (16)$$

where  $\alpha \equiv k \Delta V t_b \quad (17)$

and  $p \equiv t_c / t_b \quad (18)$

Equations (8)-(12) and (13)-(18) are eight independent equations

in terms of the 8 unknowns  $k, k', A, A', \epsilon, \epsilon', \Delta V$ , and  $y_o$ . [Equations (12).

(17) and (18) are defining equations used for simplification only.] The

problem of describing the wave trains has thus been reduced to the problem

of solving these equations in terms of the measured values of  $\theta_a, \theta_b, \theta_c, t_b,$

$t_c$ , and  $V_N$ .

Equations 8-18 may be combined to give a transcendental equation of the form  $T(\beta) = 0$ , viz  $T(\beta) \equiv \frac{\psi E \sin \psi}{p-1} + \beta \sin \beta (1 - Q_o \cos \psi) = 0 \quad (19)$

where:

$$\psi \equiv \cos^{-1} \left[ \frac{EQ_o - G_o + \cos \beta}{E - Q_o G_o + Q_o \cos \beta} \right] \quad (20)$$

$$E = [p (G_o + 1) - G_o] \cos \beta + 1 - p (G_o + 1) \quad (21)$$

$$Q_o = \frac{\tan \theta_c}{\tan \theta_b} \quad (22)$$

$$\text{and } G_o = \frac{\tan \theta_a}{\tan \theta_b} \quad (23)$$

The solution of (19) provides a unique root  $\beta = \beta_o$  for  $0 \leq \beta \leq 360^\circ$ , and  $\alpha$  is given by (20), viz.

$$\alpha = \Psi(\beta_o) \quad (24)$$

Given  $\alpha$  and  $\beta_o$ , one can solve for the eight basic unknowns using the following eight equations, which were derived from equations (8) through (18):

$$y_o = \left( \frac{1 - G_o \cos \beta_o}{1 - \cos \beta_o} \right) \left( \frac{V_N t_b}{G_o + 1} \right) \quad (25)$$

$$\epsilon' = \tan^{-1} \left( \frac{G_o \sin \beta_o}{1 - G_o \cos \beta_o} \right) \quad (26)$$

$$\epsilon = \tan^{-1} \left( \frac{\sin p \alpha - Q_o \sin \alpha}{Q_o \cos \alpha - \cos p \alpha} \right) \quad (27)$$

$$A' = y_o / \cos \epsilon' \quad (28)$$

$$A = \frac{V_N t_b (\cos \beta_o - G_o)}{(G_o + 1) (1 - \cos \beta_o) \cos (\alpha + \epsilon)} \quad (29)$$

$$\lambda' = 2 \pi / k' = -2 \pi A' \sin \epsilon' / \tan \theta_a \quad (30)$$

$$\lambda = 2 \pi / k = \beta_o \lambda' / \alpha \quad (31)$$

$$\Delta V = V_w - V_T = \frac{\alpha \lambda}{2\pi t_b} \quad (32)$$

The wavelength and amplitude are better determined when there is a larger phase separation between the successive wave intercepts. Thus, it is meaningful to define a weighted average wavelength and amplitude for each Set, viz.,

$$\lambda_e = \frac{\phi \lambda + \phi' \lambda'}{\phi_T} \quad (33)$$

$$\text{and } A_e = \frac{\phi A + \phi' A'}{\phi_T} \quad (34)$$

$$\text{where } \phi = |(p\alpha + \epsilon) - (\alpha + \epsilon)| = |(p-1)\alpha| \quad (35)$$

$$\phi' = |(\beta + \epsilon') - \epsilon'| = |\beta| \quad (36)$$

$$\text{and } \phi_T = \phi + \phi' \quad (37)$$

#### 4. APPLICATION OF THE MODEL TO THE VOYAGER 1 MAGNETOPAUSE OBSERVATIONS

The x-y components of the Voyager 1 velocity in earth-centered solar ecliptic coordinates were (-0.734, -10.6) km/s at the mid-point time of the magnetopause crossings (1634 UT on September 5, 1977). The spacecraft velocity in the (x,y) coordinate system is  $(V_T, V_N)$  where  $V_N = 9.2$  km/s (outward) and  $V_T = 5.3$  km/s ("tailward"). The times of the magnetopause crossings are given in Table 1. The slope ( $\tan \theta$ ) of the magnetopause at each crossing is given in Table 2. This set of numbers provides the necessary inputs for the model described in the previous section. In fitting the first three crossings (1, 2, 3 : Set I) we chose the origin such that  $x = 0$  at crossing 1 (point a), and we chose  $\hat{y}$  pointing "toward the earth" and  $\hat{x}$  anti-"tailward" as shown in Figures 2 and 3. For the last three crossings (5, 6, 7; Set II) we chose the origin such that  $x = 0$  at crossing 7 (point a), and we chose  $\hat{y}$  pointing "away from earth" and  $\hat{x}$  pointing "tailward". The inversion of the signs of  $\hat{x}$  and  $\hat{y}$  is equivalent to rotating the coordinate system by  $180^\circ$  with respect to the wave. In this "rotated" coordinate system the slopes and time intervals between crossings for Set II resemble those for Set I. This choice of coordinates was convenient for the numerical computations.

Solving (19) and (24)-(32) for Sets I and II gave the wave characteristics listed in Table 3. Note that for both Sets I and II  $\lambda > \lambda'$ ,  $A > A'$ , and  $\epsilon > \epsilon'$ ; this is a consequence of our choice of coordinate systems. The wave profiles (i.e., the shape of the disturbed magnetopause in the wave frame) were computed for Sets I and II using (1) and (4) with the values of  $A$ ,  $A'$ ,  $\lambda$ ,  $\lambda'$ ,  $\epsilon$ ,  $\epsilon'$ ,  $\Delta V$ , and  $y_0$

given in Table 3; they are plotted in Figure 3. Set III will be discussed below (error section). Qualitatively, three basic results are apparent in Figure 3 and in Table 3: 1) The amplitudes of the perturbations are nearly all the same  $\sim 2,000$  km; 2) The wavelengths are variable, i. e., the perturbations are **not strictly periodic**; and 3) the **amplitudes are much smaller than the wavelengths**. It is significant that the results for Set I are similar to those for the independent Set II, for it indicates that the numbers are in some sense representative of waves on the magnetopause. The only marked difference between the results for **Sets I and II is** in  $V_w$ . Probably the value 507 km/s for Set I is more accurate than the value 170 km/s for Set II, since the uncertainty in the slope of crossing 7 in Set II is relatively large.

For Sets I and II respectively, the average amplitudes,  $A_e$ , are  $2,100^{+3800}_{-540}$  km and  $2,000^{+3800}_{-540}$  km; the average wavelengths,  $\lambda_e$ , are  $57,000^{+30,000}_{-12,000}$  km and  $37,000^{+30,000}_{-12,000}$  km; and the ratios  $\lambda_e/A_e$  are 19 and 26. The wave speeds for Sets I and II are  $510^{+210}_{-95}$  km/s and  $170^{+210}_{-95}$  km/s, respectively. The error estimates will be derived in the next section.

The periods  $\tau_e = \lambda_e/V_w$  are 113 s and 221 s for Sets I and II, respectively. These are typical of the longer period micropulsations observed at the earth's surface (pc 4,5). Disturbances on the magnetopause have been suggested as a possible source of hydromagnetic waves responsible for these micropulsations (Nishida, 1978). Notice that for Set I  $t_c/\tau_e = 0.973$  is approximately the same as  $\phi_T/360^\circ = 0.964$ ; this is because  $t_c/\tau_e$  is that fraction of the composite wave which is between a and c. Similarly for Set II  $t_c/\tau_e = 0.941$  is approximately equal to  $\phi/360^\circ = 0.911$ .

Since  $R_{MP}/2 A_e$  and  $R_{MP}/\lambda_e$  are significantly greater than unity for both sets, our assumption that the nominal magnetopause in the model was locally curvature free is found to be justified. The wave is clearly a small or moderate amplitude wave, since  $\lambda_e/(4A_e)$  is approximately 6. Nevertheless, such a wave may play a role in the transmission of stress from the solar wind to the earth's geomagnetic tail.

## 5. ERROR ESTIMATES

The estimated values quoted in the previous section have two distinct types of errors:

1. Errors due to uncertainties in the measured times and slopes of the magnetopause crossings, and
2. systematic errors due to normal motions of the magnetopause.

To estimate the effects of measurement errors on  $V_w$ ,  $A$ ,  $\lambda$ , and  $\tau$ , we applied the model for various values of the parameters  $\theta_a$ ,  $\theta_b$ ,  $\theta_c$ , and  $p \equiv t_c/t_b$  consistent with the "observed" values within the estimated errors. Specifically,  $\theta_a$ ,  $\theta_b$ ,  $\theta_c$ , and  $p$  were varied with respect to the average values of Sets I and II:  $\langle \theta_i \rangle = (\theta_{iI} + \theta_{iII})/2$ ,  $i = a, b, c$ , and  $\langle p \rangle = \langle t_c \rangle / \langle t_b \rangle$ , where  $\langle t_i \rangle = (t_{iI} + t_{iII})/2$ ,  $i = a, b, c$ . These averages are  $\langle \theta_a \rangle = 18^\circ$ ,  $\langle \theta_b \rangle = -5$ ,  $\langle \theta_c \rangle = 1^\circ$ ,  $\langle p \rangle = 1.66$ , and  $\langle t_b \rangle = 96$  s.  $\theta_i$  were varied by  $\pm 1^\circ$ ,  $\pm 2^\circ$ , and  $\pm 3^\circ$  with respect to the average values for  $i = a, b, c$ , respectively. The results are shown in Table 4.

The uncertainties in  $V_w$ ,  $\lambda$ , and  $A$  (i.e., the resulting variations) should be associated with the deviations from the mean of  $V_e$ ,  $\lambda_e$ , and  $A_e$  in Table 3. We distinguish mean deviations of values greater than the average and of values less than the average. Excluding case 5, which is discussed below, we find characteristic uncertainties to be:

$$\Delta \lambda = \begin{array}{l} +30,000\text{km} \\ -12,000\text{km} \end{array}$$

$$\Delta A = \begin{array}{l} +3,800\text{km} \\ -540\text{km} \end{array}$$

$$\Delta V_w = \begin{matrix} +210\text{km/s} \\ -100\text{km/s} \end{matrix}$$

The values of  $A_e$ ,  $\lambda_e$ , and  $V_w$  for case 5 are clearly out of line with the values for the other cases in Table 4. The value of  $A_e$  is comparable to the distance from the magnetopause to earth, and the value of  $\lambda_e$  is comparable to the size of the magnetosphere. Obviously the model breaks down under these circumstances and consequently the error estimate is meaningless in this case. It does show, however, that the results of our model can be very sensitive to the slopes that are being fitted.

Motions of the magnetopause normal to its average position clearly did take place during the Voyager 1 passage. This is implied by Figure 3 where the spacecraft was near the maximum of the perturbation as it entered the magnetosheath on crossing 3 but near the minimum of the perturbation on crossing 5, i.e. the minimum of the perturbations between crossings 5, 6, 7 was higher than the maximum of the perturbations between crossings 1, 2, 3. Large normal motions are also suggested by the relatively long times between crossings 3 and 4 and between 4 and 5 (5 min to 10 min, see Table 1) which lead to peculiar values of wave characteristics when the model is applied to crossings 3, 4, 5, (Set III, Table 3). This time scale is comparable to the eigen period of the geomagnetic tail, and it is close to the "short" period oscillation observed by Howe and Siscoe (1972).

We believe our model is not applicable to Set III due to the normal magnetopause motion, both positive and negative, between crossings 3 and 5. It is likely that points 3, 4, and 5 are not part of the same cycle of the "wave" because of the normal motion. It is also interesting that Sets I and II differ in the latitudes of their normals,  $\delta$  (Table 2), but are internally consistent with crossing 4 again appearing out of place. This change in average  $\delta$  from Set I to Set II might indicate also that a third motion, transverse to the tailward moving disturbance, is taking place, at least during this interval; notice that  $\delta_{II} - \delta_I$  is  $\approx 20^\circ$  for both  $\delta_{mp}$  and  $\delta_{cp}$ . Our model, of course, is concerned only with the ecliptic plane component of the disturbance, but in actuality the disturbance could have been a complicated 3-dimensional one.

It is possible to estimate the uncertainties due to normal motions (or latitudinal motions masquerading as normal motions) of the magnetopause, which were neglected in our model. A uniform motion from the position of  $y = 0$  for Set I to the position of  $y = 0$  for Set II implies a normal speed of  $\approx 15$  km/s with respect to earth, or  $\approx 6$  km/s with respect to Voyager 1. A large earthward motion of the magnetopause between crossings 3 and 4 and a large excursion in the opposite direction between crossings 4 and 5 would imply normal speeds  $> 15$  km/s with respect to earth at some time between points 3 and 5. One could explain the intervals between crossings 1-2, 2-3, 5-6, 6-7 as entirely the result of a normal motion rather than a wave motion, if the normal speed of the magnetopause were  $\sim 10$  km/s. Similar normal speeds were derived by Holzer et al. (1966), Williams, (1978),

Russell and Elphic (1978), and Howe and Siscoe (1972), but the normal motion would not account for the observed oscillations of the normals. Purely normal motions would imply no change in the normal directions from one crossing to the next, which is not observed, even after normal error cone angles  $\Delta\theta$ , are considered. Thus, the question is, What effect would a normal motion have on our determination of the wave characteristics?

Since our equations were formulated in the wave frame with respect to an unperturbed magnetopause, the effect of motions of the unperturbed magnetopause can be represented by replacing  $V_N$  (the component of the spacecraft speed along the average magnetopause normal, relative to earth) by  $V_N - V_{Nmp}$ , where  $V_{Nmp}$  is the speed of the magnetopause in the direction normal to the average magnetopause. The equations for  $\alpha$ ,  $\beta$ ,  $\epsilon'$  and  $\epsilon$  ( (19), (24), (26) and (27) ) are independent of  $V_N$ , so those quantities are not affected by normal motions of the magnetopause. The other quantities,  $y_0$ ,  $A'$ ,  $A$ ,  $\lambda'$ ,  $\lambda$ , and  $\Delta V \approx V_w$ , (i.e. all the quantities which contain a dimension of a length) are directly proportioned to  $V_N$  (see (25), (28), (29), (30), (31), and (32) ). Thus, if the magnetopause were moving away from the earth at one half the spacecraft speed (i.e.  $\approx 5$  km/s), then  $y_0$ ,  $A'$ ,  $A$ ,  $\lambda'$ ,  $\lambda$ , and  $\Delta V$  would be one half the values given in Table 4; the shape of the disturbances shown in Figure 3 would not change, but the x, y scales would be reduced by a factor of 2. The ratios  $A'/\lambda'$  and  $A/\lambda$  are independent of the magnetopause motion, so our conclusion that the perturbations are of small amplitudes is rather general.

### Magnetopause Thickness

If our assumption that normal bulk motion of the magnetopause is negligible over the interval through crossings 1 to 3, inclusive, and likewise through crossings 5 to 7, inclusive, then the results of applying our model to Sets I and II provide us with sufficient knowledge to estimate the magnetopause thickness for each of the six crossings quasi-independently. From straight-forward geometrical considerations the thickness,  $D$ , is given by

$$D = | \Delta T [(V_w - V_T) \sin \theta + V_N \cos \theta] | \quad (38)$$

for each crossing. The quantity  $\Delta T$  is given in Table 1. Computation of  $D$  for the six crossings using Equation (39) yields results shown in Table 5; also given are  $\Delta T$  and  $\Delta \theta$ . The latter quantities are useful in assessing the quality of the estimation. Notice for instance that crossings 1 and 7 yield thicknesses far in excess of the others. It is not too surprising that this is the case for crossing 7, since  $\Delta T_7$  and  $\theta_7$  (because of large  $\Delta \theta_7$ ) were poorly known. This fact obviously weakens our confidence in all of the parameters associated with Set II, as stated earlier, but apparently estimating the thickness is crucially sensitive to errors in the magnetopause normal. Also the scatter in  $D$  is due in part to some violation of the assumption that there was no normal bulk motion of the magnetopause over Sets I and II; we know that such motion must have occurred between these sets and also clearly was responsible for the beginning and end times of Set I, for example. It is further not unreasonable to expect that the magnetopause current layer is in actuality temporally variable in thickness, especially in light

of surface wave and bulk motion. The average thickness for crossings 2, 3, 5, and 6 is 450 km (with  $\sigma = 260$  km), which is in agreement with the lower estimations of this parameter by Russell and Elphic (1978) derived from ISEE-1 and -2 spacecraft measurements when the spacecraft were nearer the nose of the magnetosphere (local times 1000 to 1200):  $D \sim 500$  to 1000 km. Their thickness estimations showed a correlation with the sense of the magnetosheath  $B_z$  such that  $B_z$  -northward was associated with the thicker boundaries and  $B_z$  -southward with thinner ones, but with great variability. (Also see Elphic and Russell, 1978.) Recall (Figure 1) that  $B_N$  (or  $B_z$ ) was clearly "northward" for both Sets I and II. However, our thickness estimates for most cases indicate the thinner end of the Russel-Elphic range.

## 6. SUMMARY AND CONCLUSION

Voyager 1 magnetic field observations of the earth's magnetopause on launch day revealed an oscillatory nature in the attitude of its local normal in the ecliptic plane for seven crossings over an interval of  $\approx 20$  min.

The surface normals were determined by use of a minimum variance method applied to the transition magnetic field sampled every 60 ms. The method is that described by Sonnerup and Cahill (1967). Error cones were estimated for the normals according to a technique developed by Lepping and Behannon (1979). Magnetopause surface waves were assumed present and modeled in terms of sequential sinusoids. The model depends only on estimates of wave slopes and temporal separations of crossings.

Due to the relatively large errors in the estimated wave slopes compared to the angles between the longitudes of adjacent normals, the resulting wave properties have rather large uncertainties. The results may realistically be considered order-of-magnitude estimates, which are:  $A_e \approx 2000\text{km}$ ,  $\lambda_e \approx 50,000\text{km}$ , and  $V_w \approx 340\text{km/sec}$ , with  $\lambda_e$  being best determined and  $A_e$  being worst determined; also  $\tau_e \approx 170$  s. The characteristic period,  $\tau_e$ , is typical of those of pc 4,5 micropulsations which have been suggested as due to magnetopause surface waves. The phase wave speed,  $V_w$ , is not inconsistent with what one expects if the mechanism producing the waves is the Kelvin-Helmholtz instability. In this case (for  $V_{sw} \approx 440\text{km/sec}$ ,  $N_{sw} \approx 3.3 \text{ cm}^{-3}$  and  $B_{sh} \approx 10\gamma$ ) the wave speed, which is characteristically the sum of the local magnetosheath and Alfvén speeds, would be  $\sim 470\text{km/s}$ , close to  $V_w \approx 510\text{km/s}$  for Set I (the first three crossings), where  $V_{sw}$  and  $N_{sw}$  were adjusted to

probable magnetosheath values according to Spreiter et al., 1968. There are indications that the magnetopause thickness, D, is very variable under the conditions observed, but in any case D was for most crossings poorly estimated: 300 km  $\leq$  D  $\leq$  700 km with higher values possible. The relatively high speed of the Voyager 1 spacecraft ( $\approx$ 11km/sec) at the time of the magnetopause crossings made possible the estimations performed here in that the normal speed of the boundary was apparently less than the normal spacecraft velocity for most of the seven crossings (i.e., except between points 3 and 5), and possibly much less. For a much lower spacecraft speed (say 1-2km/sec, as for IMP's 6,7,8, for example) deconvolution of wave and bulk speeds for a single spacecraft study would be exceedingly difficult or impossible. If Voyager had been much faster, possibly only one crossing would have been encountered and no realistic modeling possible. Our conclusion that the amplitude of the wave is small compared to its wavelength is independent of any bulk normal motions of the magnetopause that might have been present.

### ACKNOWLEDGEMENTS

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TABLE 1  
 Voyager 1 Magnetopause Crossings, September 5, 1978, hour 16

Crossing	Center Time ( $T_c$ ) m:s	Duration $\Delta T$ (sec)	$\psi$	$\theta$	$\Delta B$	$\langle \theta \rangle (\gamma)$	$\frac{\text{RMS }  B_n }{\langle B \rangle}$	$\lambda_2/\lambda_3$
1	26:45	56	56°	87°	3°	21.8	0.07	4.1
2	27:55	38	80°	83°	4°	13.0	0.15	2.8
3	28:35	10	73°	90°	1°	19.0	0.05	6.6
4	33:58	20	139°	88°	1°	19.7	0.07	13.9
5	42:44	58	38°	84°	1°	22.5	0.05	14.4
6	44:10	28	124°	84°	3°	15.2	0.06	4.6
7	46:12?	90?	103°	66°	10°	20.2	0.20	2.6

TABLE 2

## Magnetopause Normals\*

Crossing	$\lambda_{MV,i}$	$\lambda_{cp,i}$	$\delta_{MV,i}$	$\delta_{cp,i}$	$\theta_i$	$\tan\theta_i$
1	129°	123°	-5°	-18°	13°	0.231
2	113°	120°	-3°	-17°	-3°	-0.0524
3	117°	121°	-6°	-17°	1°	0.0175
-----						
4	80°	95°	42°	32°	-36°	-0.727
-----						
5	117°	118°	13°	3°	1°	0.0175
6	109°	119°	15°	3°	-7°	-0.123
7	139°	130°	17°	3°	23°	0.424
-----						
Average	115°	118°	10°	-20		
-----						
Model:		$\lambda=117^\circ$	$\delta=0^\circ$			

\* The longitude ( $\lambda$ ) and latitude ( $\delta$ ) angles are given in spacecraft centered heliographic coordinates.  $\lambda$  ( $0^\circ$  to  $360^\circ$ ) is measured counterclockwise in a plane parallel to the sun's equator plane and is zero antisunward;  $\delta$  ( $-90^\circ$  to  $+90^\circ$ ) is the inclination with respect to this plane, position "northward". To a very good approximation  $\lambda = \phi_{GSE} - 180^\circ$  for small  $\delta$  for the time interval of this data set.

TABLE 3  
Disturbance Characteristics

	Set I	Set II	Ave I and II	Set III
Crossing Number	1,2,3	5,6,7		3,4,5
$\alpha$ (deg)	77	74		150
$\beta$ (deg)	303	276		91
A ( $10^3$ km)	2.5	2.6		31.1
A' ( $10^3$ km)	2.1	1.9		4.8
$\lambda$ ( $10^3$ km)	160	97		25.5
$\lambda'$ ( $10^3$ km)	42	26		42
$\epsilon$ (deg)	69	60		-60
$\epsilon'$ (deg)	-133	-111		-1
$y_0$ ( $10^3$ km)	-1.4	-0.7		4.8
$V_w$ (km/s)	510	170	340	26
$\phi_T$ (deg)	347	328		183
$A_e$ ( $10^3$ km)	2.1	2.0	2.1	18
$\lambda_e$ ( $10^3$ km)	57	37	47	34
$\tau_e$ (sec)	110	220	170	1300
$\lambda_e/(4A_e)$	6.7	4.7	5.7	0.5
$R_{mp}^*/(2A_e)$	25	27	26	3.0
$2R_{mp}^*/\lambda_e$	3.7	5.7	4.7	6.3

\* $R_{mp}$  (=16.7  $R_E$ ) is the distance from the earth's center to the magnetopause at mid-point crossing (1634 UT)

TABLE 4  
SIMULATED PARAMETER VARIATIONS: ERROR STUDY  
 $V_T = 5.3 \text{ km/s}$ ;  $V_N = 9.2 \text{ km/s}$ ;  $t_b = 96 \text{ s}$

Case No	Ave. I + II	1	2	3	4	5	3'	4''	
P		←————— 1.66 —————→						1.5	1.8
$\theta_a$	18°	21°	15°	21°	15°	21°	21°	15°	
$\theta_b$	5°	3°	7°	3°	7°	3°	3°	7°	
$\theta_c$	1°	0°	0°	1°	1°	2°	1°	1°	
$A'$ ( $10^3 \text{ km}$ )	1.8	3.4	0.9	6.6	1.0	23	6.7	1.0	
$A$ ( $10^3 \text{ km}$ )	23	4.0	1.4	7.2	1.5	24	7.2	1.6	
$\lambda'$ ( $10^3 \text{ km}$ )	30	38	20	54	23	100	55	22	
$\lambda$ ( $10^3 \text{ km}$ )	110	250	57	360	60	840	310	67	
$\phi_T$	334°	344°	291°	358°	302°	370°	355°	305°	
$V_w$ (km/s)	250	350	140	520	160	1000	520	160	
$\lambda_e$ ( $10^3 \text{ km}$ )	42	57	27	82	30	160	76	32	
$A_e$ ( $10^3 \text{ km}$ )	1.8	3.5	1.0	6.7	1.1	23	67	1.1	
$\tau_e$ ( s )	170	165	190	160	180	150	145	200	

Table 5  
Magnetopause Thickness, D

Crossing no.	$\Delta T$ (sec)	Error cone angle, $\Delta\beta$	D (km)
1	56	$3^\circ$	6890
2	38	$4^\circ$	658
3	10	$1^\circ$	181
5	58	$1^\circ$	692
6	28	$3^\circ$	277
7	90 ?	$10^\circ$	6250 ?

## FIGURE CAPTIONS

Figure 1 The magnitude (B), RMS deviation (Pythagorean mean RMS of component RMS's based on 60 ms samples), and R, T, N components of the magnetic field in 1.92 s average form. The spacecraft centered heliographic R, T, N coordinate system is defined by (Also see the footnote of Table 2):  $\hat{R}$  is radially away from the sun,  $\hat{T}$  is perpendicular to  $\hat{R}$  and parallel to the sun's equator plane, and positive in sense of the sun's rotation, and  $\hat{N} = \hat{R} \times \hat{T}$ .

The shaded regions are the magnetopause transition zones, denoted 1 to 7 at top.

Figure 2 A schematic view of the magnetopause wave-like disturbance in a magnetopause reference frame; the spacecraft motion is shown with respect to a fixed wave.

Figure 3 Pictorial results of the model calculations for Sets I and II.

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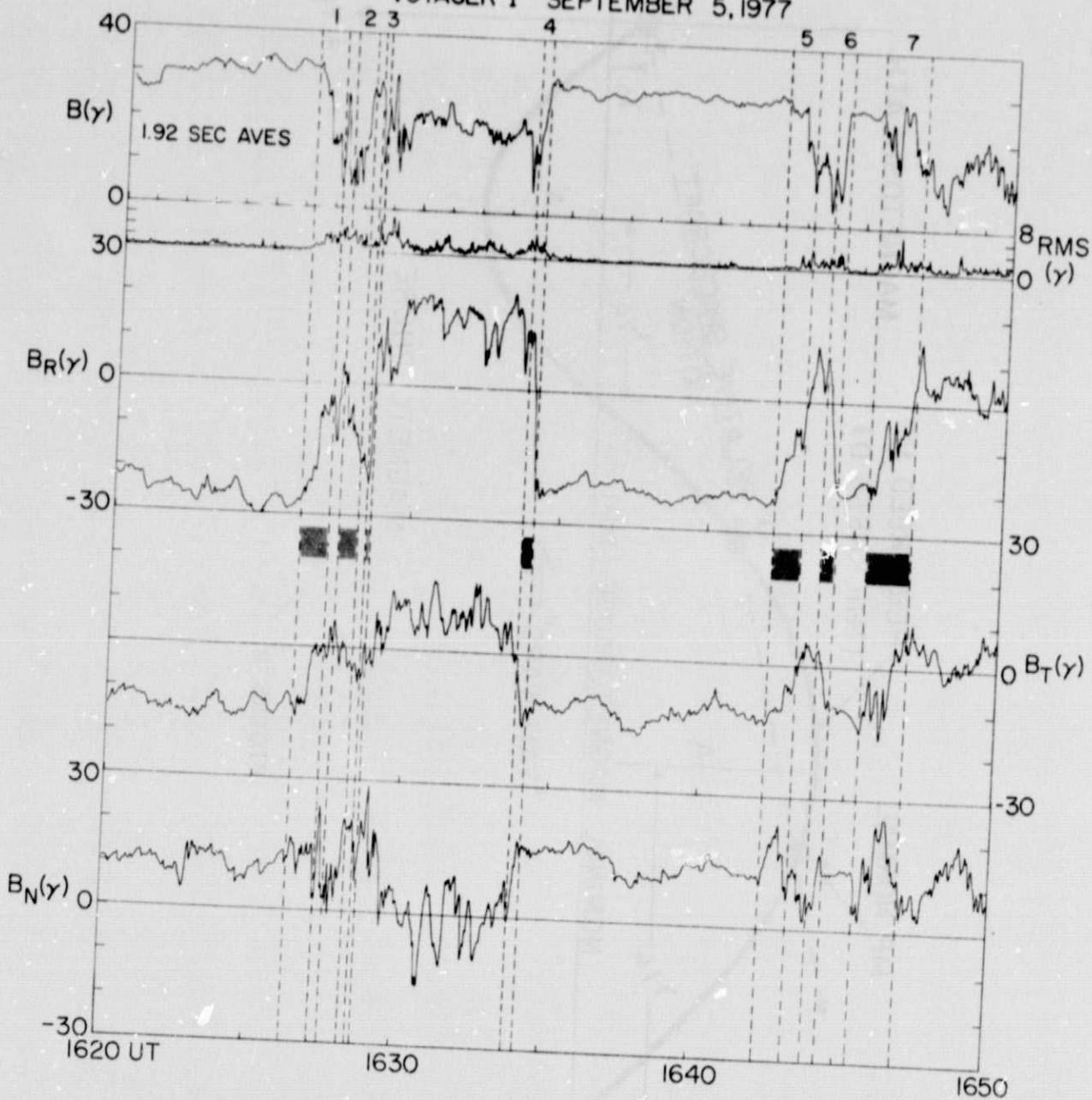


FIGURE 1

ORIGINAL PAGE IS  
OF POOR QUALITY

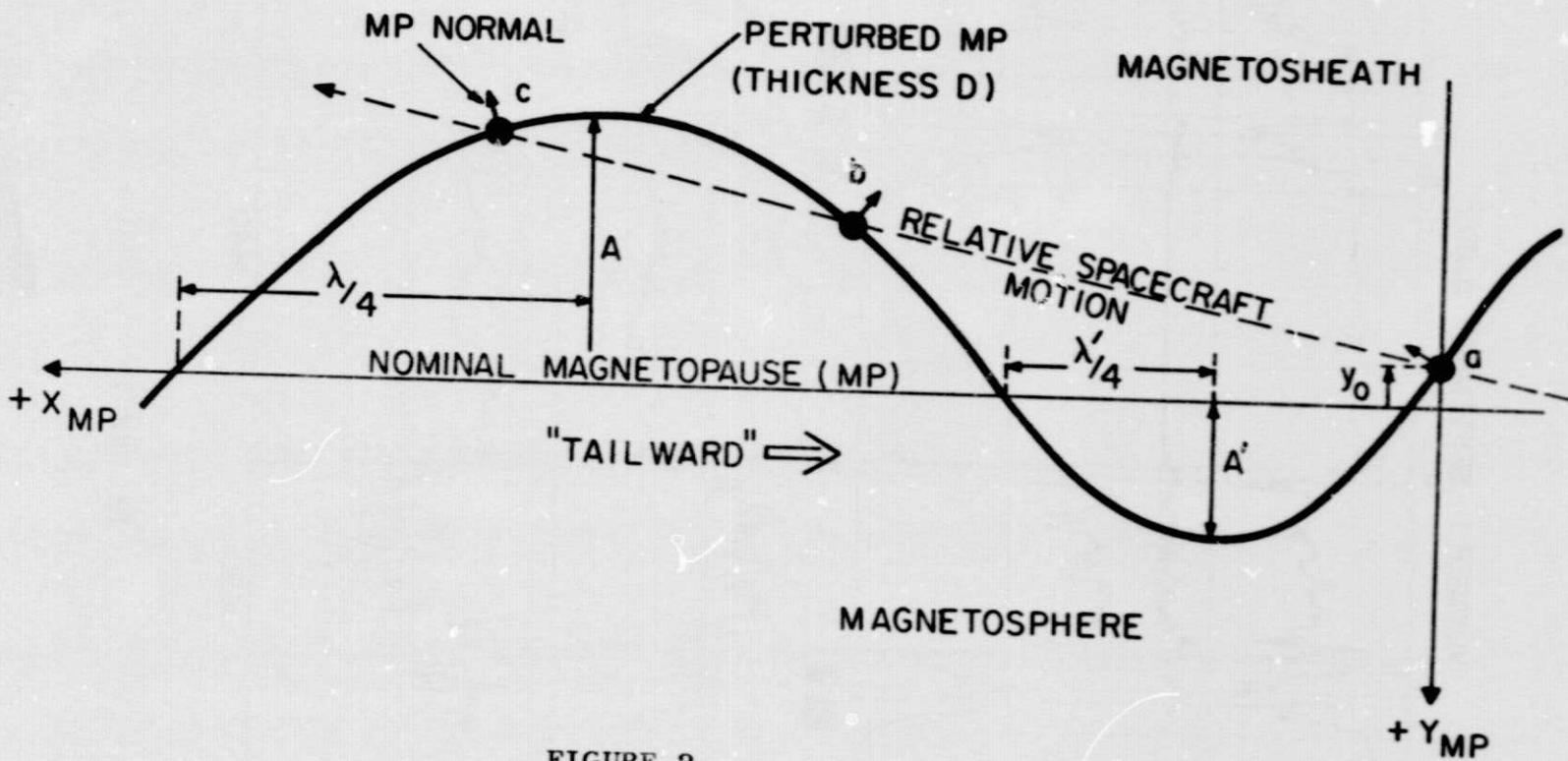


FIGURE 2

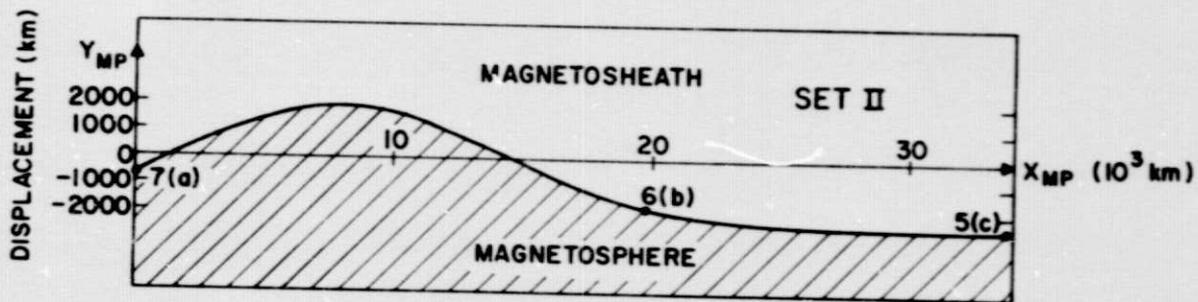
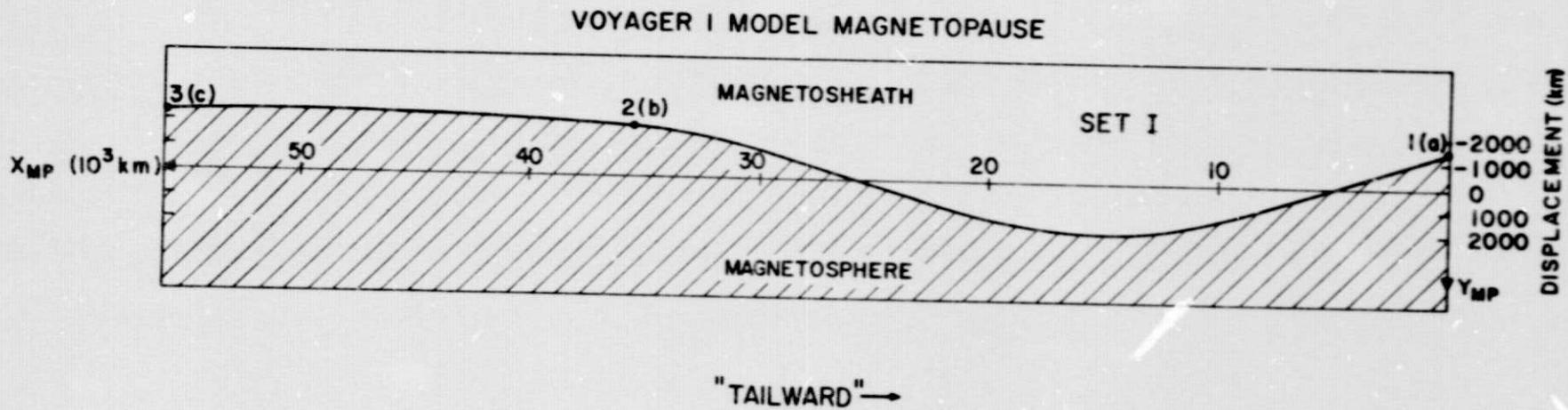


FIGURE 3

## ABSTRACT

As Voyager 1 moved out of the dawnside of the Earth's magnetosphere at 1634 UT on September 5, 1977 [at position  $(-2.6, -16.5, 1.1)$  earth radii in GSE], it crossed the magnetopause apparently seven times, despite the high spacecraft speed of 11 km/sec. Normals to the magnetopause and their associated error cones were estimated for each of the crossings using a minimum variance analysis of the internal magnetic field. The oscillating nature of the ecliptic plane component of these normals indicates that most of the multiple crossings were due to a wave-like surface disturbance moving tailward along the magnetopause. We modeled the wave, which was aperiodic, as a sequence of sine waves with amplitude  $A_i$ , wavelength  $\lambda_i$ , and speed  $V_i$ . These quantities were determined for two pairs of intervals from the measured slopes, occurrence times, and relative positions of six magnetopause crossings. The average amplitude was  $A = 2100 \begin{smallmatrix} +1800 \\ -500 \end{smallmatrix}$  km, and the wavelengths were on the order of  $47,000 \begin{smallmatrix} +30,000 \\ -12,000 \end{smallmatrix}$  km. The wave speed was approximately  $340 \begin{smallmatrix} +210 \\ -95 \end{smallmatrix}$  km/s, and typical periods were in the neighborhood of  $170 \pm 60$  sec. The magnetopause thickness was estimated to lie in the range 300 to 700 km with higher values possible.

The estimated amplitude of these waves was obviously small compared to their wavelengths; this conclusion is independent of any bulk normal motion of the magnetopause that might have been present.