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THE EFFECT OF ERRORS IN THE ASSIGNMENT
OF THE TRANSMISSION FUNCTIONS ON THE ACCURACY
OF THE THERMAL SOUNDING OF THE ATMOSPHERE

Yu. M. Timofeyev

Translation of "Vliyaniye oshibok v zadaniii funktsiy propuskaniya na tochnost' termicheskogo zondirovaniya atmosfery," USSR Academy of Sciences, Moscow, Report, 1979, pp. 1-13
This is Number XVII of a series of works performed in the USSR in accordance with the program of joint Soviet-American research in the improvement of the methods of temperature sounding by satellites. It is Appendix III to the Protocol of the third conference of the Soviet-American Working Group on Space Meteorology, held in Moscow, 10-22 November, 1976.
THE EFFECT OF ERRORS IN THE ASSIGNMENT
OF THE TRANSMISSION FUNCTIONS ON THE ACCURACY
OF THE THERMAL SOUNDING OF THE ATMOSPHERE

Yu. M. Timofeyev,
USSR Academy of Sciences

In accordance with a program of Soviet-American cooperation \(^1\) in the improvement of the methods of thermal sounding of the atmosphere, the section of atmospheric physics of the NIIF of the LGU performed an investigation of the effect of errors in the assignment of transmission functions on the accuracy of the indirect method. These studies included calculations of the "effective background noise" of measurement and numerical experiments on solving the problem in reverse by replacement. The corresponding calculations and numerical experiments were performed for conditions which functionally modeled the specialized satellite-borne radiometers of the USSR and USA in space used for thermal sounding of the atmosphere. The spectral distributions of the measurement channels of thermal radiation are presented below.

USSR Radiometer

<table>
<thead>
<tr>
<th>Channel Number N</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Frequency (cm(^{-1}))</td>
<td>666.5</td>
<td>676.0</td>
<td>692.5</td>
<td>699.0</td>
<td>736.0</td>
<td>746.0</td>
</tr>
</tbody>
</table>

* Numbers in the margin refer to the pagination in the foreign text.
In order to obtain a model of the errors in the transmission function of the atmosphere (and consequently in the root of the corresponding integral equation [1,2]), the transmission functions were obtained by means of a program of linear calculations for the following model atmospheres:

1. The standard model of the atmosphere with a carbon dioxide content \( c = 330 \) ppm ("basic" variant).
2. A standard model of the atmosphere with \( c = 363 \) ppm (the "content" variant).
3. A standard model of the atmosphere \( (c = 330 \) ppm) with a displacement of the function of the measurement channel of the apparatus by \( 1 \) cm\(^{-1}\) in the axis of frequency (the "displacement" variant).
4. A standard model \( (c = 330 \) ppm) with an increase in the half-width of the function of the channel of the apparatus by \( 10\% \) (the "stretched" variant).
5. A standard model \( (c = 330 \) ppm) including "exponential" type aerosol absorption and an integral optic depth \( \tau_a(0,\infty) = 0.1 \) (the "aerosol 1" variant).
6. A standard model \( (c = 330 \) ppm) taking into account "layered" aerosol absorption and an integral optic depth \( \tau_a(0,\infty) = 0.1 \) (the "aerosol 2" variant).
7. The same as variant 5, but with \( \tau_a(0,\infty) = 0.2 \).
8. The same as variant 6, but with \( \tau_a(0,\infty) = 0.2 \).
The calculations thus performed made it possible to obtain the variation in the transmission function $P_{AV}$ and in the roots of the integral equation, relative to the "basic" variant, for the USSR and USA radiometers, caused by variations in the carbon dioxide abundance, errors in assigning the spectral characteristics of the satellite apparatus, and the presence of additional atmospheric absorption (in the aerosol frequencies).

Table 1 presents the maximum values of the variation of the function $P_{AV}$ for various variants of calculation with respect to the "basic" variant. The value of maximum variation, as the data presented here imply, does not exceed 0.1; that is, 10% absolute transmission. The maximum variation, $\Delta P_{AV}$, is observed in the first channel of the USA radiometer (at the center of the band of absorption) with a displacement of the apparatus function ($\Delta P = 9.9\%$) and in the "transparent" channels of the USSR and USA radiometers at maximum aerosol optical depth $\tau_a(0,\infty) = 0.2$ ($\Delta P = 6.3$ and $8.5\%$, respectively).

Figure 1 illustrates the weighting functions [1] for six spectral intervals of the USSR radiometer as well as their variations caused by changes in the CO$_2$ concentration (fig. 1b), "stretching" (fig. 1c), and "displacement" (fig. 1d) of the apparatus function. It follows from figure 1 that although the vertical changes in the variation of the weighting functions are similar for all the mechanisms considered "disturbing" the transmission function of the atmosphere, the amplitudes of the variation vary considerably.

Table 2 presents the values of the emitted radiation for the channels of the USSR and USA radiometers in the "basic" variant and the "effective noise" of measurement, $\Delta J$, representing the variation in emitted radiation caused by the
### TABLE 1
MAXIMUM VARIATION OF THE TRANSMISSION FUNCTION

#### a) USSR Radiometer

<table>
<thead>
<tr>
<th>Channel N</th>
<th>Disturbance</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>Variation</td>
<td>0.036</td>
<td>0.040</td>
<td>0.041</td>
<td>0.035</td>
<td>0.041</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td>Displacement</td>
<td>0.026</td>
<td>0.007</td>
<td>0.028</td>
<td>0.026</td>
<td>0.012</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>Stretch</td>
<td>0.008</td>
<td>0.004</td>
<td>0.003</td>
<td>0.007</td>
<td>0.006</td>
<td>0.005</td>
</tr>
<tr>
<td>Aerosol 1</td>
<td>Ta = 0.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.001</td>
<td>0.017</td>
<td>0.033</td>
</tr>
<tr>
<td>Aerosol 1</td>
<td>Ta = 0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.001</td>
<td>0.033</td>
<td>0.063</td>
</tr>
</tbody>
</table>

#### b) USA Radiometer

<table>
<thead>
<tr>
<th>Channel N</th>
<th>Disturbance</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>Variation</td>
<td>0.027</td>
<td>0.041</td>
<td>0.040</td>
<td>0.039</td>
<td>0.038</td>
<td>0.045</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td>Displacement</td>
<td>0.099</td>
<td>0.012</td>
<td>0.026</td>
<td>0.029</td>
<td>0.022</td>
<td>0.006</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>Stretch</td>
<td>0.018</td>
<td>0.003</td>
<td>0.005</td>
<td>0.006</td>
<td>0.003</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>Aerosol 1</td>
<td>Ta = 0.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.001</td>
<td>0.007</td>
<td>0.017</td>
<td>0.045</td>
</tr>
<tr>
<td>Aerosol 1</td>
<td>Ta = 0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.001</td>
<td>0.014</td>
<td>0.033</td>
<td>0.085</td>
</tr>
</tbody>
</table>
TABLE 2. Emitted Radiation and "Effective Noise"

a) USSR Radiometer (erg/cm² at average cm⁻¹)

<table>
<thead>
<tr>
<th>Channel Number N</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disturbance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic Variant</td>
<td>52,14</td>
<td>47,96</td>
<td>44,60</td>
<td>50,77</td>
<td>75,87</td>
<td>88,22</td>
</tr>
<tr>
<td>Content</td>
<td>0.70</td>
<td>0.62</td>
<td>0.15</td>
<td>1.22</td>
<td>2.83</td>
<td>2.12</td>
</tr>
<tr>
<td>&quot;Stretch&quot;</td>
<td>0.039</td>
<td>0.0041</td>
<td>0.12</td>
<td>0.35</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>Displacement</td>
<td>0.73</td>
<td>0.23</td>
<td>0.24</td>
<td>0.88</td>
<td>0.71</td>
<td>1.14</td>
</tr>
<tr>
<td>Aerosol 1</td>
<td>0.0020</td>
<td>0.0002</td>
<td>0.0045</td>
<td>0.095</td>
<td>1.01</td>
<td>1.72</td>
</tr>
<tr>
<td>Ta = 0.1</td>
<td>0.0041</td>
<td>0.0005</td>
<td>0.0090</td>
<td>0.18</td>
<td>1.96</td>
<td>3.34</td>
</tr>
<tr>
<td>Aerosol 2</td>
<td>0.0095</td>
<td>0.0037</td>
<td>0.022</td>
<td>0.19</td>
<td>1.35</td>
<td>2.13</td>
</tr>
<tr>
<td>Ta = 0.2</td>
<td>0.0023</td>
<td>0.0094</td>
<td>0.068</td>
<td>0.43</td>
<td>2.75</td>
<td>4.27</td>
</tr>
<tr>
<td>Aerosol 1</td>
<td>0.00003</td>
<td>0.00001</td>
<td>0.00002</td>
<td>0.00001</td>
<td>0.00001</td>
<td>0.00001</td>
</tr>
<tr>
<td>Ta = 0.1</td>
<td>0.00017</td>
<td>0.00069</td>
<td>0.00081</td>
<td>0.017</td>
<td>1.02</td>
<td>2.04</td>
</tr>
<tr>
<td>Aerosol 2</td>
<td>0.0022</td>
<td>0.0041</td>
<td>0.19</td>
<td>0.20</td>
<td>0.78</td>
<td>1.41</td>
</tr>
<tr>
<td>Ta = 0.2</td>
<td>0.0048</td>
<td>0.011</td>
<td>0.060</td>
<td>0.47</td>
<td>1.64</td>
<td>2.66</td>
</tr>
</tbody>
</table>

b) USA Radiometer (erg/cm² at average cm⁻¹)

<table>
<thead>
<tr>
<th>Channel Number N</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disturbance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic Variant</td>
<td>56.04</td>
<td>46.37</td>
<td>44.79</td>
<td>51.57</td>
<td>66.16</td>
<td>77.73</td>
<td>91.72</td>
</tr>
<tr>
<td>Content</td>
<td>0.98</td>
<td>0.54</td>
<td>0.059</td>
<td>1.45</td>
<td>2.59</td>
<td>2.93</td>
<td>1.93</td>
</tr>
<tr>
<td>&quot;Stretch&quot;</td>
<td>0.47</td>
<td>0.17</td>
<td>0.29</td>
<td>0.15</td>
<td>0.27</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>Displacement</td>
<td>3.21</td>
<td>0.18</td>
<td>0.20</td>
<td>1.20</td>
<td>1.22</td>
<td>0.41</td>
<td>1.56</td>
</tr>
<tr>
<td>Aerosol 1</td>
<td>0.00008</td>
<td>0.00002</td>
<td>0.00010</td>
<td>0.0001</td>
<td>0.0001</td>
<td>1.05</td>
<td>1.02</td>
</tr>
<tr>
<td>Ta = 0.1</td>
<td>0.00017</td>
<td>0.00069</td>
<td>0.00081</td>
<td>0.017</td>
<td>1.02</td>
<td>2.04</td>
<td>3.71</td>
</tr>
<tr>
<td>Aerosol 2</td>
<td>0.0022</td>
<td>0.0041</td>
<td>0.19</td>
<td>0.20</td>
<td>0.78</td>
<td>1.41</td>
<td>2.35</td>
</tr>
<tr>
<td>Ta = 0.2</td>
<td>0.0048</td>
<td>0.011</td>
<td>0.060</td>
<td>0.47</td>
<td>1.64</td>
<td>2.66</td>
<td>4.70</td>
</tr>
</tbody>
</table>
various variations in the transmission function (the variation of the radiation was calculated according to an average temperature profile proposed by the American side together with a corresponding covariant temperature matrix $K_T$). It follows from the data of table 2 that in the majority of cases the "effective noise" of measurement exceeds the level of average square accidental error $\sigma_J$ of contemporary satellite apparatus and in particular exceeds the value taken for numerical experiments on the solution of the reverse problem (see below) $\sigma_J = 0.15$ erg/cm at avg. cm$^{-1}$. The exceptions are the values of the "effective noise" in the optically dense channels of measurement with aerosol disturbances (this is caused by the linear nature of the vertical changes in optical density $\tau_a(0,2)$), and in the "stretched" apparatus functions $\phi(\nu-\nu')$ for channels $N = 1-3$ in the USSR radiometer and $N = 2$ in the USA radiometer. We should draw attention to the fact that the "stretched" $\phi(\nu-\nu')$ value of $\Delta J$ for the first two channels is substantially larger for the USA radiometer than for the USSR radiometer. This circumstance is a result of the spectral character of the first two channels of the USA radiometer than for the USSR radiometer. This circumstance is a result of the spectral character of the first two channels of the USA radiometer, in particular the noticeably smaller half-width $\phi(\nu-\nu')$ compared to the USSR radiometer. From this, it also follows that a high accuracy of the assigned spectral character is necessary to a radiometer, especially in the area of rapidly changing spectral and optical properties of the terrestrial atmosphere. We should note, finally, that in the remaining cases the values of the "effective noise" are similar for the two radiometer types.

Thus, the calculations of the "effective noise" of measurement in the analyzed variants of the transmission function show that these variations in the majority of cases (with the
possible exception of the "stretch" variant) appear to have a noticeable effect on the accuracy of the indirect determination of the temperature profile.

During the next stage of the investigation, numerical experiments were performed on generating the vertical temperature profiles by a replacement arrangement for 30 temperature profiles specifically chosen by the American side. For each temperature profile the emitted radiation $J_{\Delta v_i}$ was calculated for the transmission function corresponding to the "basic" variant, the accidental errors $e_i$ (distributed normally with a mean square deviation $\sigma_J = 0.15$ erg/cm$^2$ at avg. cm$^{-1}$) were added to the value $J_{\Delta v_i}$, and then the quantity $J_{\Delta v_i} + e_i$ was used to generate a vertical temperature profile. In this solution of the reverse problem, the transmission function corresponding to both the "basic" variants and to the other variants was used: the "content," "displacement," "stretch," and so on.

The generation of the temperature profile took place by means of a method of statistical regularization [1,2] using the correlation matrix $K_T$ calculated by the American team from a global ensemble of 210 profiles. It is important to emphasize that the generation of each temperature profile was performed for a large set of accidental errors (50 examples), and that the average characteristics of the accuracy of the indirect method were subsequently analyzed.

Let us enumerate the basic results and conclusions obtained in this stage:

1. The errors in generating temperature varied widely for different profiles and different levels of the atmosphere. As an example, figure 2 presents the dependence of the mean
square error in the generated temperature, $\sigma_T$, in the 1000-10 mb levels, on the distance $\sigma_K = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (T(p_i) - \overline{T}(p_i))^2}$
of the temperature from the average temperature profile. The figure illustrates the strong variability of $\sigma_T$ (from 1 to 4°K) and the weak correlation between $\sigma_T$ and $\sigma_K$.

2. A comparison of the accuracy of generating temperatures by means of the USSR and USA radiometers revealed a small advantage in the USA radiometer, possibly caused by an insufficient number of channels in the Soviet device for error calculations ($\sigma_J = 0.15$ erg/cm$^2$ at avg. cm$^{-1}$). The mean square error $\sigma_T$ (for 30 profiles, 28 levels in the range from 1000 to 10 mb pressure and 50 instances of accidental error) of the Soviet (six-channel) radiometer consists of 2.20°K, and for the USA (seven-channel) radiometer, 2.05°K, at exactly assigned roots of the integral equation. This relatively low level of accuracy in the indirect method is explained by two basic causes:

a) The ensemble of 30 temperature profiles used in the numerical experiments was chosen from an initial global ensemble of temperature profiles. Accordingly, the covariant matrix $K_T$ constructed from the initial global ensemble (and used in the method of statistical regularization) describes the global structure of the thermal state of the atmosphere. In this connection, the a priori mean square temperature deviations at many atmospheric levels exceeds 10°K. Such a large a priori indeterminacy in the knowledge of the thermal structure at an essential point determines the accuracy of the indirect method.

b) It is possible that six-channel and even seven-channel systems of measurement are not optimal for the level of accidental error ($\sigma_J = 0.15$ erg/cm$^2$ at avg. cm$^{-1}$) used in the numerical experiments and for global observations.
3. The increase of the errors of generation created by errors in the assignment of roots to the integral equation is extremely changeable. These increases vary from profile to profile, are different at different levels of the atmosphere, and so on. With an error in the CO₂ concentration of 30% the mean square error increases in the USA radiometer by 0.68°K and in the USSR radiometer by 0.52°K. It is interesting to note at this point that the effectiveness of both sounding systems is practically the same, which is easily explained by the fact that the total noise (the accidental error plus the "effective noise") has already reached the point where six- and seven-channel devices are practically equivalent. The maximum increase in error of generation ΔTₘ is observed close to the earth's surface (ΔTₘ ≈ 3°K) and the region of the tropopause (ΔTₘ ≈ 1-2°K).

"Stretching" of the apparatus function of the radiometer (increasing the half-width by 10%) does not introduce any essential change in the error of generation. Thus when the mean square errors increase by 0.03-0.05°K, the maximum increase in errors reaches ≈0.3°K.

A change in the central frequency of the channel (the systematic "displacement" of the apparatus function of the radiometer by 1 cm⁻¹) introduces an increase in the mean square error of generation $\bar{\Delta}_T$ of 0.21°K (the USSR radiometer) and of 0.27°K (the USA radiometer). The maximum increase for the USSR radiometer consists of 1.63°K and for the USA radiometer, of 5.18°K, and it is observed in the upper levels of the atmosphere. The substantial increase in the error of generation T(p) in the upper atmospheric layers for the USA radiometer is caused by the peculiarity of the spectral characteristics of its first channel (see below).
The effect of the aerosol absorption is determined above all by the value of the integral optical depth \( \tau_a(0,\infty) \). When the value \( \tau_a(0,\infty) = 0.1 \), the increase in the mean square error is 0.16-0.18\(^\circ\)K, and the maximum error (in the lower layers of the atmosphere) is \( \sim 3\)\(^\circ\)K. The corresponding value at \( \tau_a(0,\infty) = 0.2 \) are equal to 0.56-0.63\(^\circ\)K and \( \sim 6\)\(^\circ\)K. Table 3 presents the values of the increase in the mean square error (\( \Delta \bar{\sigma}_T \)) and the maximum error (\( \Delta T_m \)) in the generated temperatures for all the considered variants of "disturbed" transmission function.

**TABLE 3. INCREASE OF MEAN-SQUARE ERROR (\( \Delta \bar{\sigma}_T \)) AND MAXIMUM ERROR (\( \Delta T_m \)) OF GENERATED TEMPERATURE (IN \( \circ\)K)**

<table>
<thead>
<tr>
<th>#</th>
<th>Model Variant</th>
<th>USSR Radiometer</th>
<th>USA Radiometer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \Delta \bar{\sigma}_T )</td>
<td>( \Delta T_m )</td>
</tr>
<tr>
<td>1</td>
<td>&quot;Content&quot;</td>
<td>0.52</td>
<td>2.53</td>
</tr>
<tr>
<td>2</td>
<td>&quot;Displacement&quot;</td>
<td>0.21</td>
<td>1.63</td>
</tr>
<tr>
<td>3</td>
<td>&quot;Stretched&quot;</td>
<td>0.03</td>
<td>0.33</td>
</tr>
<tr>
<td>4</td>
<td>Aerosol 1</td>
<td>0.18</td>
<td>2.64</td>
</tr>
<tr>
<td>5</td>
<td>Aerosol 2</td>
<td>0.16</td>
<td>2.46</td>
</tr>
<tr>
<td>6</td>
<td>Aerosol 3</td>
<td>0.61</td>
<td>6.53</td>
</tr>
<tr>
<td>7</td>
<td>Aerosol 4</td>
<td>0.56</td>
<td>6.11</td>
</tr>
</tbody>
</table>

Thus, all of the "disturbances" to the transmission function (except the "stretched") considered above substantially influence that accuracy of the thermal sounding of the atmosphere.
On the basis of the results obtained, let us formulate the accuracy requirements of the assigned transmission function. In case we analyze the maximum increase of the error in generated temperature $\Delta T_m$, we find that in order to ensure $\Delta T_m$ does not exceed 1°K, it is necessary that the maximum variation in the transmission function does not exceed $\approx 0.01$. An analysis of the relationship between the increase in the mean square error of the generated temperature $\Delta \sigma_T$ and the mean square errors of the assigned transmission function $\sigma_\rho$ makes it possible to conclude that, for $\sigma_\rho < 0.02$, $\Delta \sigma_T$ usually does not exceed 0.5°K, at which point the maximum errors $\Delta P_m$ can be 0.03-0.04. As a qualitative average number, it appears advisable to infer an accuracy of assigning $P\Delta \nu$ to 2% absolute transmission. Given this level of error, the increase in the mean square, generated errors do not exceed an average of 0.5°K (or 25% of the initial value of $\sigma_T$).
Figure 1: The weight functions (upper left, measured in °K) corresponding to the measurement channels of a Soviet radiometer, plotted against pressure P in millibars; the deviations (measured in Δ°K) caused by the variants in CO₂ content (upper right), increasing half-width (lower left), and displacement (lower right), plotted against pressure.

Figure 2: Plot of the mean square errors of generation $\bar{\sigma}_T$ (y-axis, in °K) versus the values of initial deviation $\sigma_K$ (x-axis, in °K) for 30 temperature profiles.
FIGURE 1

OF POOR QUALITY
REFERENCES
