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TEST PROBLEMS FOR INVIScid TRANSONIC FLOW

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SEPTEMBER 1, 1979

TEXAS ENGINEERING EXPERIMENT STATION
TEST PROBLEMS FOR
INVIScid TRANsONIC FLOW

Leland A. Carlson
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TEES REPORT NO. TAMRF-3224-7804

September 1, 1979
This report constitutes a pre-presentation publication of a paper and results to be presented at the GAMM Workshop on "Numerical Methods for the Computation of Inviscid Transonic Flow with Shock Waves". This conference will be held at the Aeronautical Research Institute (FFA), Stockholm, Sweden on September 18-19, 1979.

This report includes the conference paper, all results obtained for the various workshop test cases, and approximate iso-mach line plots for the supercritical cases.

This work was primarily supported by NASA Grant NSG1174 and partially supported by the Texas Engineering Experiment Station.
I. Introduction

This paper will briefly discuss some of the results obtained in the process of solving the test problems for the GAMM Workshop on "Numerical Methods for the Computation of Inviscid Transonic Flow with Shock Waves" with the TRANDES program. Briefly, this method,\textsuperscript{1-5} utilizes the full, inviscid, perturbation-potential flow equation in a Cartesian grid system that is stretched to infinity. This equation is represented by a non-conservative system of finite difference equations that includes at supersonic points a rotated difference scheme and is solved by column relaxation. The solution usually starts from a zero perturbation potential on a very coarse grid (typically 13 x 7) followed by several grid halvings until a final solution is obtained on a fine grid (97 x 49). Occasionally, for cases having high local Mach numbers, the solution must be started on the coarse grid (25 x 13). Since the airfoil does not coincide with the grid points, the surface boundary conditions are represented as two-term Taylor series about dummy points inside the airfoil. On the outer boundaries, the exact infinity conditions are used. This method can, if desired, include the effects of weak viscous interaction or be used in the design mode.\textsuperscript{2-6}

All of the results presented at the workshop and in this paper were obtained at the rate of 10\textsuperscript{4} pts/sec on an Amdahl 470/V6 using a FORTG compiler and single precision arithmetic (less than 7 significant digits). A typical run took 4-8 minutes, although good engineering results were obtained in one minute on the medium grid; and convergence was obtained on the medium grid to at least a maximum cyclic perturbation change of less than 5\textsuperscript{E}-5. On the fine grid, the $\Delta\phi_{\text{max}}$ was usually larger due to significant digit error in the far-field. It should be noted that the workshop cases were run 1.5-5.0 times longer than usual due to the availability of computer time from TAMU, and that the TRANDES program has never been optimized for time.

In order to maintain the goal of common discretization, the grid stretchings were setup so that the number of airfoil points, number of wake points, $\Delta x$ and $\Delta y$ at the trailing edge, and the location of the last finite vertical grid column matched the suggested grids as closely as possible. For cases involving large supersonic zones, the y-grid was extended so that the last finite horizontal grid line was subsonic. Otherwise, the rotated difference scheme might have used undefined values.

* Professor, Aerospace Engineering Department
for the test cases appear to show correct trends. (I.E. For 
\[ C_L = 0.0 \quad -M_\infty = 0.72, \quad CD = 0.0004; \quad M_\infty = 0.8, \quad CD = 0.0100; \quad M = 0.85, \quad CD = 0.0381; \quad M_\infty = 0.95, \quad CD = 0.0989) \]

b. Bump in Channel

This problem was solved by treating the bump as a symmetrical airfoil in a solid wall wind-tunnel. The solution used 41 points on the upper surface, 16 in the wake, 21 points vertically from the centerline to the wall, \( \Delta x_{te} = 0.0251, \) \( \Delta y_{te} = 0.05, \) and \( \phi = 0 \) on the channel walls. For the upstream/downstream infinity conditions two approaches were tried. The first used the asymptotic form derived by Murman\(^8\) while the second had \( \phi = 0 \) imposed and allowed the solution to float. The results on the bump and between it and the wall were identical. However, some slight differences were observed upstream and downstream, with the floating solution showing less blockage type influence. While the floating approach is easier to implement, it is more prone to significant digit errors due to the magnitude of the floating potential \( O(1) \) instead of \( O(0.1) \).

Finally, several other upstream/downstream boundary conditions were imposed, such as \( \phi = 0; \) and all yielded essentially the same results. Since the channel case was coded rapidly for the workshop, this lack of sensitivity may be due to either coding errors or a special feature of the test case.

c. RAE 2822

This problem was solved using the same grid as for the NACA 0012 cases. During the solutions, it was discovered that the subcritical results were sensitive to the location of the first and last points on the airfoil, which are normally at 0.01c and 0.99c \((X4 = 0.49)\). The initial results \((\text{solid line, Fig. 2})\) indicated a possible peak in \( C_{pu} \) near the leading edge. To resolve this peak, the first points were moved to 0.005c and 0.995c \((X4 = 0.495)\), and slightly different results were obtained as shown on Fig. 2. Interestingly, EPSS had to be increased to 1.0 for this second case due to the appearance of supersonic points on the coarser grids. Obviously, the Cartesian grid placement sometimes affects the airfoil effective \( \alpha \); and it is probably best to correlate results versus \( C_L \) rather than \( \alpha \).

d. CAST 7

Again the basic NACA 0012 grid was used. Here, in both the subcritical and supercritical cases, oscillations were observed in \( C_{pu} \) at 0.03c and in \( C_{pu} \) at 0.92c. Since the surface slopes were also oscillatory in these areas, it is believed this behavior was due to the sparsity of the given coordinates and the use of spline fits to determine the computational ordinates and slopes. Also, in both cases, the
aft \( C_{pl} \) bucket turned around at the TE and approached stagnation. This behavior is reasonable due to the large TE angle (12.5\(^\circ\)). In actuality, viscous effects would mitigate this trend.

e. KORN 1

The basic NACA 0012 grid was also used for this case. For the design point, the resultant \( C \) distribution was close to the theoretical hodograph values and was almost shockless. Again some sensitivity to the grid size and \( X_4 \) value was observed, as can be seen in Fig. 3. Also, a double precision calculation (16 digits) was made and essentially yielded the same results as with single precision (7 digits). However, for the 16 digit case the \( \Delta \)max on the fine grid was steadily decreasing instead of oscillatory and smaller (6E-5 vs. 2E-4).

III. Conclusions

Except for the NACA 0012, \( M = 0.95 \) case, all test problems were solved straightforwardly and appeared to be converged or close to convergence. The only difficulty was some sensitivity to grid placement, which is typical of the Cartesian formulation. Again, note that all results were obtained in single precision (less than 7 significant digits) on an Amdahl 470/V6.

IV. Acknowledgments

This work was primarily supported by NASA Grant NSG 1174 and partially supported by the Texas Engineering Experiment Station. The author expresses his appreciation to Robert Simmons, TAMU Aerospace Engineering Department, for his assistance in the preparation of the figures.

V. References

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Figure 1: Non-Conservative versus Conservative Results
Figure 2: The Effect of Grid Placement on $C_p$
Figure 3: Results for the KORN 1 Airfoil
Figure 1: Case A-I(i), Airfoil $C_p$ Distribution versus X/C
Figure 2: Case A-I(i), Airfoil $C_p$ Distribution versus $Y/C$
Figure 3: Case A-I(ii), Airfoil $C_p$ Distribution versus $X/C$
NACA 0012
M₀ = .63, α = 2.0°
10429 Pts/sec
222 Secs Total

Figure 4: Case A-I(ii), Airfoil Cₚ Distribution versus Y/C
Figure 5: Case A-II(i), Airfoil $C_p$ Distribution versus X/C
Figure 6: Case A-II(i), Airfoil $C_p$ Distribution versus $Y/C$
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Figure 15: Case A-II(v), Airfoil $C_p$ Distribution versus X/C
Figure 16: Case A-II(v), Airfoil $C_p$ distribution versus Y/C
4.2% Circular Arc Bump in Parallel Channel

\[ \text{M} = 0.85 \]

9781 Pts/sec
584 Sec Total

\[ x/c \]

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Iteration History

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Symbol | Location
--------|-----------
\( - \) | Lower Wall
\( O \) | Upper Wall

Figure 17: Case B, Upstream \( C_p \) Distribution
Figure 17a: Case B, Upstream $C_p$ Distribution, Floater Solution
4.2% Circular Arc Bump
in Parallel Channel

\( M_\infty = 0.85 \)

9781 Pts/sec
584 Sec Total

Figure 18: Case B, "Bump" \( C_p \) Distribution versus \( X/C_0 \)
4.2% Circular Arc Bump in Parallel Channel

M_\infty = 0.85
9701 Pts/sec
166 Sec Total

Figure 18a: Case B, "Bump" C\textsubscript{p} Distribution versus X/C,
Floater Solution
4.2% Circular Arc Bump in Parallel Channel

\( M_\infty = 0.85 \)

- 9781 Pts/sec
- 584 Sec Total

---

Figure 19: Case B, Downstream \( C_p \) Distribution
4.2% Circular Arc Bump in Parallel Channel

\( M_\infty = 0.85 \)

9701 Pts/sec
166 Sec Total

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Iteration History

Grid | Cycles
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19x11 | 1200
37x21 | 600
73x41 | 300

Symbol | Location
------|--------
\( \circ \) | Lower Wall
\( \bullet \) | Upper Wall

Figure 19a: Case B, Downstream \( C_p \) Distribution, Floater Solution
Figure 20: Case B, "Bump" $C_p$ Distribution versus Y/C
Figure 20a: Case B, "Bump" C_p Distribution versus Y/C, Floater Solution
Figure 21: Case C-I, Airfoil $C_p$ Distribution versus $X/C$
Figure 22: Case C-I, Airfoil $C_p$ Distribution versus $Y/C$
RAE 2822

\( M_\infty = 0.75, \alpha = 3.0^\circ \)

9864 Pts/sec
683 Sec Total

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Figure 23: Case C-II Airfoil \( C_p \) Distribution versus \( X/C \)
Figure 24: Case C-II, Airfoil $C_p$ Distribution versus Y/C
Figure 25: Case D-I, Airfoil $C_p$ Distribution versus $X/C$
CAST 7

$M_a = 0.70, \alpha = -1.0^\circ$

10499 Pts/sec

284 Sec Total

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Figure 26: Case D-I, Airfoil $C_p$ Distribution versus Y/C

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OF POOR QUALITY
CAST 7
$M_\infty = 0.76, \alpha = 0.5^\circ$
10245 Pts/sec
658 Sec Total

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Figure 27: Case D-II, Airfoil $C_p$ Distribution versus $X/C$
CAST 7
M_o=0.76, a=0.5°
10245 Pts/sec
658 Sec Total

Figure 28: Case D-II, Airfoil C_p Distribution versus Y/C
Figure 29: Case G-I, Airfoil $C_p$ Distribution versus $x/C$
Figure 30: Case G-I, Airfoil $C_p$ Distribution versus $Y/C$
Figure 31: Case G-II, Airfoil $C_p$ Distribution versus X/C
KORN 1
\(M_a = 0.75, \alpha = 1.0^\circ\)
10223 Pts/sec
739 Sec Total

Figure 32: Case G-II, Airfoil \(C_p\) Distribution versus \(Y/C\)
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Figure 35 Mach Chart, NACA 0012, $M_\infty = 0.95$, $\alpha = 0^\circ$. 
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Figure 39 Mach Chart, RAE 2822, $M_\infty = 0.75$, $\alpha = 3^\circ$. 
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Figure 41 Mach Chart, KORN 1, $M_\infty = 0.75$, $\alpha = 0.115^\circ$. 
Figure 42  Mach Chart, KORN 1, $M_\infty = 0.75$, $\alpha = 1^\circ$. 