IMPACT CRATERING CALCULATIONS,
PART I:
Early Time Results

PIFR - 1220 Interim Final Report August 11, 1979

NASA Contract NASW 3168

Physics International Company
A Subsidiary of ROCKCOR/2700 Merced St., San Leandro, CA 94577, (415) 357-4610
IMPACT CRATERING CALCULATIONS, PART I:

EARLY TIME RESULTS

INTERIM FINAL REPORT

PIFR - 1220

NASA CONTRACT NASW 3168

August 11, 1979


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APPENDIX  PAPER TO BE PUBLISHED IN THE PROCEEDINGS OF THE TENTH LUNAR AND PLANETARY SCIENCE CONFERENCE.
ABSTRACT

Early time two-dimensional finite difference calculations of laboratory-scale hypervelocity (6 km/sec) impact of 0.3 g spherical 2024 aluminum projectiles into homogeneous plasticene clay targets were performed, and the resulting material motions analyzed. Results show that energy and momentum are coupled very quickly from the aluminum projectile to the target material, and in the process of this coupling, some of the plasticene clay target is vaporized while the projectile becomes severely deformed. The velocity flow field developed within the target is shown to have features quite similar to those found in calculations of near-surface explosion cratering. Specific application of Maxwell's analytic Z-Model (developed to interpret the flow fields of near-surface explosion cratering calculations), shows that this model can be used to describe the early time flow fields resulting from the impact cratering calculations as well, provided the flow field centers are located beneath the target surface, and that application of the model is made late enough in time that most of the projectile momentum has been dissipated.
The authors wish to thank Dr. William L. Quaide, the NASA Technical Monitor on this program for his constructive help in the performance of this program. We also wish to thank all those at the Lunar and Planetary Institute (LPI) in Houston, Texas, who helped us with the publication of our paper in the Proceedings of the Tenth Lunar and Planetary Science Conference.

The Principal Investigator would like to thank Dr. Jon B. Bryan of Lawrence Livermore Laboratory for many helpful technical conversations covering the wide range of physical processes involved in impact cratering, and Mrs. Vera L. Terry for her patience and perseverance in preparing the manuscript.
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SECTION I
INTRODUCTION

1.1 BACKGROUND

Impact cratering is one of the principal mechanisms of evolution and alteration of the surface of the moon as well as the planets Mercury and Mars. An improved understanding of the phenomenology and dynamics of impact cratering may be expected to improve, in general, our interpretation of the physical features observed on the lunar surface, and thus our understanding of the evolution of the Moon. More specifically, an improved understanding of the dynamics of impact cratering may be expected to contribute significantly to answering such fundamental questions as:

a) Why does lunar crater morphology generally progress from bowl-shaped craters to flat-floored craters with central uplifts, and then to multiring-type structures as crater size increases? Is this transition related to post-cratering adjustment, or related in some way to cratering dynamics and the ejection process?

b) What is the depth and shape of the transient crater as a function of time? At what depth do specific portions of ejecta deposits originate? (See, for example, Reference 1).

c) What is the thickness and particle velocity gradient in ejecta sheet, and what does this imply regarding secondary cratering and such features as herringbone pattern and dunelike structures observed surrounding some large impact craters? (See, for example, Reference 2).
The research reported here was not intended to directly answer all these very fundamental questions. Rather, the research was based on a "walk before you run" philosophy. Namely, that in order to answer such fundamental questions, it was necessary first to understand the fundamental dynamics and phenomenology associated with impact cratering, and the influence of such parameters as target material shear strength and layering on crater growth.

Current knowledge of impact cratering dynamics is primarily based on 1) observation and analysis of characteristics of planetary and lunar craters \(\{3,4\}\), b) small-scale laboratory experiments \(\{5,6,7,8\}\), and c) a very few theoretical calculations \(\{9,10,11,12\}\). Analyses of characteristics of lunar and planetary craters provide constraints on models of impact cratering dynamics. However, such observations and analysis alone provide only limited insight into the basic mechanics of the cratering process. Small-scale laboratory experiments have resulted in a greatly improved understanding of small-scale impact cratering phenomenology, and provide experimental restraints on theoretical cratering models. A difficulty, however, is the lack of understanding of how to scale or extrapolate small scale impact cratering data to the large scale cratering events of primary interest. The theoretical calculations performed by O'Keefe and Ahrens \(\{13\}\) emphasized the effects of the high pressure equation-of-state and high pressure phase transition on early-time impact cratering phenomena. These above calculations addressed important questions, and the calculations reported here are viewed as complimentary to O'Keefe and Ahren's work.

1.2 OBJECTIVES AND TECHNICAL APPROACH

Somewhat in parallel with the impact cratering studies, extensive experimental and theoretical research of the dynamics of explosion cratering had been performed. Theoretical research
employed extensive calculations using advanced finite difference continuum mechanics codes. The calculational results had been reasonably successful in modeling specific explosion cratering experiments (14). More importantly, the calculations illuminated much of the fundamental phenomenology and dynamics that govern the explosion cratering process (15,16). Analyses of a number of explosion cratering calculations revealed the following features associated with near-surface explosion cratering.

- Except at very early times, the mean stress in the material near the transient crater wall is very low (on the order of tens of bars).

- Prior to the time the crater achieves its maximum transient depth, crater growth is nearly hemispherical.

- Crater growth, subsequent to time of maximum transient depth, is achieved principally by "shearing" material from the crater wall.

- The cratering flow field, defined loosely as the material flow field in the cratering region, but behind the outgoing shock wave, closely approximates steady-state incompressible flow.

- The radial velocity characterizing the cratering flow field, \( R \), is approximately independent of angle, and may be approximated by the equation \( \dot{R} = \alpha(t)R^Z \), where to a good first approximation \( \alpha \) and \( Z \) are constants.
Ejecta is lofted from near the transient crater lip, resulting in a thin sheet of ejecta in the shape of an inverted cone. Except at very early and very late times, the angle at which ejecta leaves the ground surface is nearly constant and is usually 40° to 60° from the horizontal.

Overturned flaps are observed and are computed to occur at late times.

The observed cratering phenomenology exists at very early times, and persists until late times in explosion cratering calculations. By characterizing the cratering flow field by \( \dot{R} = \alpha R^{-2} \) and assuming incompressible flow and energy conservation, a simple but very successful model of explosion cratering was developed \( \{16\} \).

It is also known that explosion craters are influenced by the shear strength and layering of the cratering media, although the details of the influence of shear strength and layering on the cratering flow field are not presently known.

Within recent years, several investigators have shown that surface and shallow-buried explosions result in cratering phenomenology very similar to that observed in impact cratering events \( \{6,17\} \). Several pertinent questions then arise concerning the relationship between explosion and impact cratering dynamics.

1) Is the impact cratering associated with a relatively simple flow field such as observed in explosion cratering?
2) What are the characteristics of this flow field?

3) How is the flow field affected by target shear strength and layering?

4) How is the flow field altered as a function of time due to the action of gravitational forces? What does this imply regarding the change of impact crater morphology as a function of crater size (and thus time of crater formation)?

The overall objective of the research reported here is to initiate an investigation of these questions. The research is viewed as complimentary to both proposed and on-going experimental studies being performed on the NASA-AMES Vertical Gun Range. This is a two-year research effort, and this report summarizes the first year efforts.

Three impact cratering calculations will be performed using the PISCES 2DELK finite difference continuum mechanics computer code. Initial conditions for the calculations were chosen such that corresponding experiments could be performed at NASA-AMES. Those chosen (Figure 1.1) were as follows:

- Projectile material: aluminum
- Projectile mass: 0.3 g
- Projectile geometry: spherical
- Impact velocity: 6 km/s
- Atmospheric condition: vacuum
- Target material: plasticene (modeling) clay
Figure 1.1 Basic geometry for the impact cratering calculations.
Table 1.1 Summary of calculations to be performed during two year program.

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<th>Calculation</th>
<th>Shear Strength</th>
<th>Target Geometry</th>
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<tr>
<td>1</td>
<td>$Y_1 = 50 \text{ kPa}$</td>
<td>No layer, infinite homogeneous target</td>
</tr>
<tr>
<td>2</td>
<td>$Y_2 = 150 \text{ kPa}$</td>
<td>No layer, infinite homogeneous target</td>
</tr>
<tr>
<td>3</td>
<td>$Y_1 / Y_2$</td>
<td>Layer with shear strength $Y_1$ overlying half space with shear strength $Y_2$.</td>
</tr>
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Plasticene clay was chosen as the target material because a suitable high pressure equation of state was readily available and because it behaves as a nearly ideally elastic-plastic material, with a von Mises yield strength which is temperature dependent. The temperature dependence of the strength will allow experiments with different shear strengths to be performed. The (two year) calculational program consists of three calculations with initial conditions as stated above. Only the shear strengths and layering are varied as shown in Table 1.1.

1.3 **SUMMARY OF PROGRESS**

Much progress was made during this first year of effort. Initial conditions for the calculations were chosen so that a comparable experimental program would be feasible (Section IV). All equations of state were implemented in the computer code and thoroughly checked. A special post-processing routine designed to aid in the analysis of the calculated cratering flow fields was programmed and checked (Section III). Calculations 1 and 2 were performed to early times (Section V) and their results accepted for publication in the Proceedings of the Tenth Lunar and Planetary Science Conference, Houston, Texas (a copy of the paper to be published is included in the Appendix).

The third (layered) calculation has not yet been started. Instead, an experimental impact cratering program using the Vertical Gun Range at NASA-AMES has been mapped out, proposed and approved. In the two test series (August 16-21, and September 13-19, 1979) impact experiments into plasticene clay targets (analogous to the calculations reported here) will be conducted. Then an investigation of the feasibility of conducting a layered experiment will be performed, and if possible, the layered experiment will then be conducted. The corresponding layered calculation (Calculation 3) will then be performed.
Since this work will be performed very late during the first year of the research effort, results are not reported.

1.4 PRELIMINARY CONCLUSIONS

Performance of two impact calculations of aluminum projectiles impacting plasticene clay targets to 18 μsec (early times), and analysis of the results have shown that:

1) Energy and momentum coupling from the projectile to the target is essentially complete before 18 μsec. About one half of the initial impact kinetic energy is transformed into kinetic energy in the target, and therefore into energy useful for cratering.

2) A hemispherical shock wave is formed very early in the target; some of the plasticene is vaporized, but the aluminum projectile is not shock melted, although it is severely deformed by the impact.

3) The velocity flow field in the shocked target material is strikingly similar to the flow fields developed in explosion cratering calculations; thus with slight modification, Maxwell's Z-Model of explosion cratering can be applied to impact cratering calculations, at least for unlayered targets*. The flow field center was found to lie beneath the target surface, a result which is different than that found for surface-burst explosion cratering calculations. Application of the Z-Model cannot be made until almost all of the projectile momentum has been coupled to the target material.

*Note that the Z-model has not been applied to explosion cratering in layered geologies.
4) To the times calculated, the results of the low and high strength unlayered target calculations (Calculations 1 and 2) are essentially identical. The final crater sizes will be different, however, due to the different shear strengths.

We feel this calculational research will be useful in assessing the effectiveness of laboratory scale impact experiments in interpreting the observations of impact cratering structures. The laboratory scale impact experiments involve slower velocities, and different projectile materials than are suspected to have occurred in nature. Impact momentum plays a more important role in these experiments during the early development of the cratering flow field. The aluminum projectiles are not vaporized in the laboratory scale experiments at 6 km/sec; however, this may not be the case at higher impact velocities as the calculations of O'Keefe and Ahrens (Reference 13) and Bryan et al. (Reference 12) have indicated. A suggestion for future research would be to use different projectile materials such as those which would vaporize at lower shock levels or more dense ones which would increase momentum at constant total energy. The differences in energy and momentum coupling efficiency could then be investigated.

We have found that, even though energy and momentum coupling are relatively fast, the cratering process still requires a long time to complete. We hope to investigate in detail the cratering processes which have been set in motion in the first 18 μsec during the second year of this program. In particular we will compare the motions of mass elements within the calculated cratering flow fields with that predicted by the Z-Model. This will be done for both the low and high strength unlayered calculations and for a layered calculation. The specific layering for the third calculation will be determined after the impact experiments have been performed at NASA/AMES.
SECTION II

A BRIEF DESCRIPTION OF THE PISCES SYSTEM OF COMPUTER CODES

The PISCES system of codes is a set of static and dynamic finite-difference computer codes based on computational methods developed by the Atomic Energy Commission in connection with the calculation of non-linear, large-amplitude responses of structures, fluid bodies and solid media. The codes solve the fundamental partial differential equations of continuum mechanics expressed in the explicit finite difference forms developed by Wilkins for Lagrangian codes and Noh for Eulerian codes [18]. Complex geometries, time-dependent boundary conditions, space-dependent initial conditions, and standard nonlinear material models are entered into the codes with no degradation in code efficiency or accuracy. More importantly, no changes in code structure need be made to accommodate non-standard models. The codes are designed for non-linear phenomena and should not be confused with implicit techniques which are not well suited for non-linear phenomena.

The acronym "PISCES" stands for Physics International multi-Spacial Code for Engineering and Science. A list of available codes is given in Table 2.1. All of the codes can be used to calculate three-dimensional real-space geometry. However, in the one- and two-dimensional codes, there are only one and two independent space variables, respectively. The remaining dependent space variables are calculated from symmetry considerations. In a one-dimensional code there are three natural symmetries that can exist: plane, cylindrical and spherical. In a two-dimensional code there are two possible symmetries - axial and translational. The three-dimensional code is, of course, an asymmetric code (all spatial coordinates are independent variables).
Table 2.1  A list of the codes in the PISCES computer code system.

<table>
<thead>
<tr>
<th>Code Name</th>
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<td>PISCES 1DL</td>
<td>One-dimensional Lagrangian</td>
</tr>
<tr>
<td>PISCES 2DL</td>
<td>Two-dimensional Lagrangian</td>
</tr>
<tr>
<td>PISCES 2DE</td>
<td>Two-dimensional Eulerian</td>
</tr>
<tr>
<td>PISCES 3DE</td>
<td>Three-dimensional Eulerian</td>
</tr>
<tr>
<td>PISCES 2DELK</td>
<td>Two-dimensional Coupled Eulerian-Lagrangian</td>
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For all the codes, the space continuum is subdivided into zones or cells. For the dynamic codes, the time continuum is divided into time steps, while the static codes use the time steps as an iteration variable. In the Lagrangian formulations, each zone contains a constant mass element of material that moves and distorts in space and time (i.e., the zone volume may change). The motion for all the zones approximates the continuum motion. In the Eulerian formulations, each zone is a constant volume element with mass transported across zone boundaries. In either case, the position of a particular zone is defined at the zone corners, (grid points that define the zonal boundary), while other variables such as pressure, stress and energy are defined as averages over the zonal interior. In the Lagrangian codes, material velocities are defined at zone corners while in the Eulerian codes, velocities are defined at zone centers.

Except for the difference in cell definition, the Eulerian and Lagrangian codes are quite similar. Equations of state, stress tensors, yield models, boundary conditions, etc., available in one class of codes are available in the other. However, Lagrangian and Eulerian codes each have computational considerations that restrict their economical use. For certain problems dealing with phenomena in solids, Lagrangian codes are generally better suited, while Eulerian codes are usually restricted to problems dealing with fluids and/or gasses.

The PISCES 2DELK code was used to perform the impact calculations. It is unique in that Eulerian and Lagrangian computational grids may be used simultaneously in a single computation. In such a computation, Eulerian and Lagrangian grids are fulling coupled (19). The PISCES 2DELK code has been used extensively for cratering calculations where, near the explosion source or impact point, the target behavior is hydrodynamic and may best be
computed in an Eulerian grid; while at distances of a few source dimensions the elastic-plastic behavior of the target material dominates and may best be computed in the Lagrangian frame.
3.1 MAXWELL'S Z-MODEL OF EXPLOSION CRATERING

The cratering flow field has been defined previously for explosion cratering [15] as the particle velocity flow field existing in the region of the transient crater but behind the initial out-going shock front (Figure 3.1). Maxwell [16] and Orphal [15] described an analytic model called Maxwell's Z-Model that was developed to describe the cratering motions observed in near-surface explosion cratering calculations. After the initial near-surface explosion, a shock wave propagates through the target material approximately spherically from the detonation point and decays rapidly. After the shock front passes, the shocked material retains a residual velocity which, subsequently modified by surface rarefactions and by the effects of material strength and gravity, produces the final crater at much later times. Maxwell found for near-surface explosion calculations that an early-time crater growth was nearly hemispherical. In spherical polar coordinates taken about the on-axis detonation point, the flow field could be described by

\[ \dot{R} = \alpha(t)R^{-Z}, \]

where \( \dot{R} \) is the radial velocity of the flow field, \( \alpha \) is a time-dependent coupling term describing the flow field strength, and \( Z \) defines the rate of velocity decay with range, \( R \). Maxwell [16] also observed that the density in the cratering flow-field region was approximately constant, yielding incompressible flow:

\[ \nabla \cdot U(R, \theta) = 0 \]
Figure 3.1 This schematic representation shows the cratering flow field region as defined for near-surface explosion cratering calculations.
where \( U(R,\theta) \) is the vector velocity of the flow field. Combining Equations 3.1 and 3.2 permits derivation of the full equation of motion of a mass element within the cratering flow field:

\[
U(R,\theta) = \dot{R} \hat{R} + \ddot{R} (Z-2)\tan (\theta/2) \hat{\theta},
\]

(3.3)

where \( R \) and \( \theta \) are unit vectors in spherical polar coordinates. This can be integrated to give the mass element motion as a function of time. Gravity and material strength effects can also be included. It is clear from Equation 3.3 that for \( Z = 2 \), \( U \) is radial at all points, yielding an irrotational flow field. The corresponding physical case is the purely radial velocity field resulting from the explosion of a spherical source in an homogeneous medium of infinite extent in all direction. Maxwell \( \{16\} \) found for near-surface explosion cratering calculations that \( Z \approx 2 \) for \( \theta = 0^\circ \), that \( Z \approx 2.7 \) for \( 30^\circ \leq \theta \leq 60^\circ \); and that \( Z \approx 4 \) for \( \theta \geq 75^\circ \). An average value of \( Z \approx 3 \) was found to be representative of the entire cratering flow field produced by a near-surface explosion. Values of \( Z > 2 \) lead to flow fields which are rotational in the direction of the surface (e.g. Reference 15). The rotation leads to the eventual ejection of material below the ground plane in a very orderly manner, beginning with material closest to the wall of the transient cavity. Total energy, material strength and gravity limit the total mass ejected, and also control the crater depth (e.g., Reference 20).

The primary implication of Maxwell's \( Z \)-Model is that the near-surface explosion cratering process is an extremely orderly one \( \{16\} \). This has been validated for explosive cratering experiments in plasticene clay \( \{21\} \) and dry, noncohesive sand \( \{5\} \). If the \( Z \)-Model, perhaps in some modified form, can be applied to the results of impact cratering calculations, then much can be learned about the dynamics of the impact cratering process by analogy with the experience derived in explosion cratering calculations.
3.2 PISCES 2DELK SPECIAL Z-MODEL EDITING SUBROUTINE

To facilitate analysis of the impact cratering flow fields, a special subroutine was built. The subroutine manipulates the computed cell variables at any time during the impact calculation, and prints information in a form suitable for immediate comparison with Maxwell's Z-Model equations. We wanted to know in particular how well the calculated velocity field at early times during the impact process fit with Equation 3.1 Also needed was the location of the flow field center, and any variation of \( \alpha \) and \( Z \), or of the position of the center with time. Although flow fields generated in explosion cratering calculations fit the Z-Model well with a center of the flow field located at the original ground surface (Figure 3.1), there was no reason to a priori expect that this would also be the case for impact cratering. Finally, we wanted to measure the extent to which Maxwell's Z-Model assumption of incompressible flow (Equation 3.2) after shock wave passage through the target material, held for impact cratering calculations.

The subroutine prints out processed data at any selected time during the calculation. The radial velocity is calculated along rays extending out every 5 degrees from possible centers of the flow field (which are any of the zones lying on the axis of symmetry). With reference to Figure 3.2, if \((X_i, Y_i)\), etc. are the coordinates of the four corner points through which a ray passes, then the subroutine computes \( R \), \( \dot{R} \) and \( \theta \) for either Eulerian or Lagrangian zones. At each chosen edit time, Table 3.1 shows the type of information which is printed out for each ray and each selected origin. For each ray, \( \alpha \) and \( Z \) are also calculated assuming that Equation 3.1 holds. Then, for each origin, average values of \( \alpha \) and \( Z \) are calculated, together with the appropriate standard deviations.
Figure 3.2 Schematic showing an Eulerian or Lagrangian zone, the vector velocity of that zone, and the velocity components to be calculated along a ray centered at a possible flow center.
Table 3.1 Basic output of the special subroutine comparing the results of the impact cratering calculations with the equations of Maxwell's Z-Model.

<table>
<thead>
<tr>
<th>DOB</th>
<th>ZONE (I,J)</th>
<th>ORIGIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>JJJ</td>
<td>xxx 000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>θ₁</th>
<th>Col.</th>
<th>Row</th>
<th>X</th>
<th>Y</th>
<th>X</th>
<th>Y</th>
<th>R</th>
<th>R</th>
<th>θ</th>
<th>ρ</th>
<th>α</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>I₁</td>
<td>J₁</td>
<td>X₁</td>
<td>Y₁</td>
<td>X₁</td>
<td>Y₁</td>
<td>R₁</td>
<td>R₁</td>
<td>θ₁</td>
<td>ρ₁</td>
<td>α₁</td>
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<td>Iₙ</td>
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<td>Xₙ</td>
<td>Yₙ</td>
<td>Rₙ</td>
<td>Rₙ</td>
<td>θₙ</td>
<td>ρₙ</td>
<td>αₙ</td>
<td>Zₙ</td>
<td></td>
</tr>
</tbody>
</table>
Typically, a log plot of $R$ vs $R$ along any ray should resemble that shown in Figure 3.3 which shows $\dot{R}$ attenuating regularly with $R$ until the shock front is reached. An activity test eliminates the shock front region from the calculation of $\alpha$ and $Z$ for that ray. This subroutine was a major analysis tool for our investigation of the cratering flow fields formed by the impact of aluminum projectiles into plasticene clay targets. Results of this analysis will be presented later in this report.
Figure 3.3 Typical plot of $\dot{R}$ vs $R$ expected from special PISCES 2DELK Z-model editing subroutine.
SECTION IV
INITIAL CONDITIONS, MATERIAL MODELS, AND CALCULATION ZONING

Figure 4.1 summarizes the initial conditions for the calculations. The calculations simulate the normal impact of 6 mm diameter 2024 aluminum projectiles into plasticene (oil base) clay targets at an impact velocity of 6 km/sec. Axial symmetry and terrestrial gravity (1 g = 9.80 m/sec\(^2\)) are assumed. It is also assumed that the impacts occur in a vacuum. Two early time calculations were performed in which only the clay strength was varied.

The overall calculational effort is directed toward a better understanding of cratering dynamics. The above initial conditions were chosen to make it highly likely that the corresponding experiments could be performed at the NASA Ames Vertical Gun Range. Plasticene has properties which make it rather unique as a target material for this type of study. It has a simple equation of state description, primarily because it contains no air voids. Its shear strength can also be simply characterized by a von Mises \cite{22} or Mohr-Coulomb \cite{23} criterion, and the strength magnitude can be changed by varying the ambient temperature. Finally, it is readily available, and we felt this would allow close integration of the calculational effort with future experimental studies.

Static material properties determined for plasticene clay are: an initial density of 1.69 Mg/m\(^3\), a compressional wave velocity of 1.4 m/msec, a shear wave velocity of 0.475 m/msec, and a Poisson's ratio of 0.435. Its equation of state (EOS)\cite{22} is based on Hugoniot data from Christensen et al \cite{24} presented in Table 4.1. From these data, the relative volumes and internal energies were calculated using Hugoniot relationships (see, for example, Reference 27). The EOS form which matches these states
Figure 4.1 Summary of calculation initial conditions.
Table 4.1 Plasticene Hugoniot data from Christensen et al [24].

<table>
<thead>
<tr>
<th>Measured Shock Velocity $U_s$ (km/sec)</th>
<th>Calculated Particle Velocity $U_p$ (km/sec)</th>
<th>Measured Pressure $P$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.7</td>
<td>0.91</td>
<td>5.75</td>
</tr>
<tr>
<td>5.6</td>
<td>2.26</td>
<td>21.4*</td>
</tr>
</tbody>
</table>

*Represents the average of two measurements.
gives the pressure $P$ (GPa) as a function of compression, $V$
where $V = \rho_0 + \rho$; $\rho_0$ (Mg/m$^3$) is the initial density and $\rho$ (Mg/m$^3$) is
the current density; compressibility $\mu$ where $\mu = (1-V)^{-1}V$; and energy density, $E$(GJ/m$^3$):

$$P = 2.8\mu + 40.7\mu^2 - 36.0\mu^3 + 1.7\frac{E}{V} \quad \mu \geq 0$$
$$P = 1.7\frac{E}{V} \quad \mu < 0$$

This form incorporates the initial (zero pressure) bulk modulus of 2.8 GPa, provides the required fit to the Hugoniot data, and allows for reasonable extrapolation to pressures greater than those covered by experimental data ($0 \leq P \leq 21.4$ GPa). A plot of the Hugoniot is given in Figure 4.2, the data points are plotted in Figure 4.3.

A von Mises failure envelope for plasticene clay was used in the calculations reported here; the actual von Mises strength was found to be dependent on the ambient clay temperature, $T$, so two calculations with differing strengths were performed. One calculation used the estimated low strength (50 kPa, $T = 32.2^\circ C$) and the other used a higher strength (150 kPa). We did not expect the results of the two calculations to differ greatly at early times, when the impact-induced shock pressures greatly exceeded the clay strengths. Explosive cratering experiments in plasticene clay revealed, however, that the final crater volume increased by a factor of three as the ambient clay temperature increased from $17.5^\circ C$ to $32.2^\circ C$. Thus, when we examine (in the future) the calculational results at late times, we expect significant differences to appear. The difference in crater size can be estimated from some equations derived using Maxwell's Z-Model; this is done in Section 5.4.
Figure 4.2 Plasticene clay Hugoniot EOS.
Figure 4.3 Intersection of curves defining the one-dimensional theoretical pressure state occurring behind the shock front for an aluminum projectile with a velocity of 6 km/sec impacting into a plasticene clay target. (The two clay experimental data points of Christensen et al. [24] are also shown.)
The equation of state for 2024 aluminum was a Hugoniot fit reported by van Thiel {26} based on the Hugoniot measurements of McQueen et al. {27}:

\[ P(\text{GPa}) = \frac{\rho_o C_o^2 \mu (1+\mu)}{(1+\mu(1-s))^2} \quad 8.4 \leq P \leq 109 \text{ GPa}, \]  

where \( \rho_o = 2.783 \text{ Mg/m}^3 \); \( C_o \) (the bulk sound speed) = 5.343 m/msec and \( s = 1.325 \). The maximum strength of the aluminum Mohr-Coulomb failure envelope used (Figure 4.4) 320 MPa {25} is much greater than the clay strengths.

The one-dimensional pressure state generated behind the shock during the impact of the aluminum sphere into the plasticene clay target at 6 km/sec was determined from Equations 4.1 and 4.2 using the method of Gault and Heitowit {28}. The calculated pressure of 50 GPa (Figure 4.3) is not sufficient to shock melt aluminum, as a shock pressure of 60 GPa is required to cause incipient melting {29}. Maxwell and Reaugh {22} estimated that incipient vaporization of the volatile constituents of the plasticene clay begins at a pressure of about 6 GPa. This latter pressure is substantially lower than 50 GPa, indicating that vaporization of the clay target will occur during the compression stage.

The two-dimensional computer code used, PISCES 2DELK (see Section II), permitted initial modeling of the projectile using a Lagrangian finite difference approximation and the target using an Eulerian approximatation. As the projectile moves into the target, it interacts continuously along its outer boundary with the target. In the computational model, the Lagrangian grid literally "moves through" the Eulerian grid. Material cannot occupy simultaneously the same point in space in both grids, however, and the Lagrangian material predominates. Thus the Eulerian target material
Figure 4.4 Mohr-Coulomb failure envelope for aluminum 2024.
gets pushed out of the way by the impacting Lagrangian projectile; momentum and energy are transferred continuously, and a shock wave is formed in the target. The Lagrangian projectile zone size is shown explicitly in Figure 4.5. The zone size in the Eulerian target region was initially 0.3 mm square, directly beneath the projectile, and thus the initial projectile radius was spanned by ten radial zones.

The above initial grid was used until 2.45 µsec in calculation 1. At that time the Lagrangian grid describing the projectile had become badly distorted. A rezone was performed in which this Lagrangian grid was mapped onto Eulerian coordinates, and the aluminum projectile material was designated to be a second material in the Eulerian grid. The calculation was then continued (using the initial Eulerian zoning) in Eulerian coordinates entirely. The Eulerian grid was expanded at approximately 4 µsec and again at 10 µsec to accommodate the developing shock wave in the target. A similar procedure was performed for calculation 2. Table 4.2 summarizes the early time rezones for both calculations 1 and 2. Care was taken in all of the above steps to ensure that adequate grid resolution was maintained in all areas of interest. At early times, these areas include all of the target subjected to impact shock, the projectile itself, and the void areas above the target where crater lip development occurs and high-speed jetting of vaporized target material was found to take place.

The calculations were zoned and the material models and initial conditions generated for PISCES 2DELK as discussed in this section. A CDC CYBER 176 computer located at the Air Force Weapons Laboratory (AFWL) at Kirtland Air Force Base, New Mexico was used to perform the calculations.
Figure 4.5 Initial zoning for the impact calculations (Dimensions given in cm, |x, y|).
Table 4.2 Summary of the rezone history for the early time impact calculations.

<table>
<thead>
<tr>
<th>Calculation 1 (low strength clay)</th>
<th>Cycle</th>
<th>Time</th>
<th>Approximate Eulerian zone size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projectile mapped from Lagrangian to Eulerian coordinates</td>
<td>350</td>
<td>2.45 µsec</td>
<td>Same as initial grid</td>
</tr>
<tr>
<td>Eulerian grid mapped onto larger Eulerian grid</td>
<td>540</td>
<td>4.01 µsec</td>
<td>((0.07 \text{ cm} \times 0.07 \text{ cm}))</td>
</tr>
<tr>
<td>Eulerian grid mapped onto larger Eulerian grid</td>
<td>710</td>
<td>10.00 µsec</td>
<td>((0.14 \text{ cm} \times 0.14 \text{ cm}))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculation 2 (high strength clay)</th>
<th>Cycle</th>
<th>Time</th>
<th>Approximate Eulerian zone size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projectile mapped from Lagrangian to Eulerian Coordinates</td>
<td>290</td>
<td>2.30 µsec</td>
<td>Same as initial grid</td>
</tr>
<tr>
<td>Eulerian grid mapped onto large Eulerian grid</td>
<td>427</td>
<td>4.02 µsec</td>
<td>((0.07 \text{ cm} \times 0.07 \text{ cm}))</td>
</tr>
<tr>
<td>Eulerian grid mapped onto larger Eulerian grid</td>
<td>585</td>
<td>9.87 µsec</td>
<td>((0.14 \text{ cm} \times 0.14 \text{ cm}))</td>
</tr>
</tbody>
</table>
SECTION V

EARLY TIME RESULTS OF THE IMPACT CALCULATIONS

During this first year of the calculational effort, the calculations were performed to a time of 18 μsec. Analysis of the calculated results centered on the early time cratering flow fields. The computer code used, computational techniques, and flow field analysis tools have been discussed in previous sections of this final report. In this section, results from calculation 1 (low strength) and calculation 2 (high strength) are discussed over the above time period. This time period encompasses the time domain of the initial impact, vaporization of the target material, transfer of energy and momentum from the projectile to the target, and establishment of the cratering flow field in the target material. Subsequent motion of the target material in the cratering flow field leads, at a much later time, to the formation of the final crater. This (second stage) of the calculation will be performed during the second year of the research program. A third (layered) target calculations, calculation 3, will also be performed during the second year.

The results discussed in this section include first, the early-time general impact phenomenology; secondly, the energy and momentum transferred to the target; thirdly, the analysis of the resulting cratering flow field, and fourthly, a section discussing the implications of the adapted Maxwell Z-Model to estimate the final crater size, followed by some of the later time results.

5.1 EARLY TIME IMPACT PHENOMENOLOGY

Upon projectile impact a strong shock is formed in both the clay target and the projectile. The shocks cause the projectile to deform as it moves into the target, a strong shock is also
developed as the projectile compresses the target material directly beneath it. A maximum pressure of 42.1 GPa was observed in both calculations at the earliest time, when the Eulerian cell variables were printed out after the initial impact (0.115 μsec). It occurred close to the symmetry axis, directly below the projectile. This observed pressure is only slightly lower than the 50 GPa maximum pressure estimated in the previous section.

At the point of contact between the projectile and the original target surface, a state of stress cannot be maintained, and vaporized target material jets into the void area above the surface. Jetting of this target material was analyzed in detail and the results presented in the journal article to be published (included in the Appendix).

Figure 5.1 shows the very early-time impact phenomenology in pictorial form from calculation 2. It includes a plot of the Lagrangian projectile zoning, and the velocity vectors in the Eulerian target material grid at 3 times encompassing the first microsecond of the impact. All three of the shock processes discussed above are seen in these plots. At 1 μsec the projectile has been completely engulfed by the target, although it has not yet penetrated to a depth equal to one projectile diameter, due to the shock deformation of the projectile itself.

Over the next few microseconds, the projectile continues to deform, tending towards the "classical" crescent shape. The distortion becomes so severe at at 2.57 μsec the Lagrangian projectile grid folds over on itself in the calculation (See Figure 5.2b). At this time the shock wave is fairly well developed, with a peak pressure of approximately 9 GPa. The shock wave has been significantly influenced by the free surface, and a rarefaction wave has developed directly behind the initial shock.
Figure 5.1  Lagrangian grid of the aluminum projectile and vector velocity plots in the Eulerian grid containing the plasticene clay target, depicting the early time jetting of target material and projectile deformation at (a) 0.30 μsec, (b) 0.55 μsec, and (c) 1.02 μsec. (The horizontal line through the center of each frame shows the pre-impact location of the target surface.)
Figure 5.2 Material boundary, pressure contour, and vector velocity. Plots at the time when the projectile is rezoned from Lagrangian to Eulerian coordinates (Calculation 1).
wave. (These effects can be seen in the pressure contour plot, Figure 5.2a). At this point the projectile has penetrated to a maximum depth of approximately 1.4 initial projectile diameters.

At 2.57 μsec it was necessary to rezone the projectile to include the aluminum as a second material in the Eulerian grid. The purpose of using a Lagrangian grid initially was to maintain an accurate transfer of energy and momentum in the first few microseconds (during that time there would be a certain amount of mass diffusion if the projectile were zoned in Eulerian coordinates. However, at 2.57 μsec, the projectile was so severely distorted that it could not be handled in Lagrangian coordinates any longer. A tape dump at a slightly earlier time, 2.45 μsec, was used to perform the rezone for calculation 1.

As the calculation was continued to 4.1 μsec (Figure 5.3) the appearance of small lip of target material above the original target surface was noted. The projectile had progressed at this time to 1.8 projectile diameters; the shock wave to 3 projectile diameters. It should be noted that the projectile still retained about 20 percent of its original momentum (see Section 5.2) so that the plasticene clay directly below the projectile was still being significantly overdriven. The shock wave was maturing and spreading, with a peak pressure at this point of approximately 3.5 GPa (35 Kbar). This is already less than the pressure required to fully vaporize the plasticene clay (6 GPa).

At 6.02 μsec (see Figure 5.4), the transient crater lip had started to grow to a significant height. The one new significant phenomenon shown here is that the vector velocities directly behind the main shock are slightly smaller than those at the shock front, at least for regions not located directly beneath
Figure 5.3 Contour and velocity vector plots (Calculation 1) at 4.1 μsec.
Figure 5.4 Contour and velocity vector plots (Calculation 1) at 6.02 usec.
the projectile. The effort is seen a little more clearly in Figure 5.5 (10 μsec). This has implications concerning the development of the cratering flow field, which will be discussed later. The projectile at this time had traveled two projectile diameters into the target, while the shock wave had traveled 3.8 projectile diameters. The peak pressure at this point was approximately 2.0 GPa (20 Kbar).

At 10 μsec (see Figure 5.5) the shock wave had traveled to 5.5 projectile diameters while the projectile had traveled 2.5 projectile diameters. The peak pressure in the shock wave at this point was approximately 1.1 GPa. The pressure contour plot shows a series of rarefaction waves behind the primary shock due to the continuous interaction of the outgoing shock wave with the target free surface. This effect has also been seen in the impact calculations of O'Keefe and Ahrens (10).

At 15.0 and 18.0 μsec (Figures 5.6 and 5.7), the flow field is developing behind the propagating shock wave. The maximum shock pressure continues to attenuate and reaches approximately 0.4 GPa (4 Kbar) at 18 μsec (Figure 5.7).

In all of the above figures (Figures 5.2 to 5.7) the large velocity vectors above the target surface and inside the transient crater lip represent the magnitude and direction of plasticene clay which was vaporized by the projectile impact. This material is of very low density, and the pressures associated with it are much lower than the shock pressures in the plasticene below the target surface. A downward-directed flow of vaporized plasticene filled the region behind the projectile as it moved further into the target; an upward-directed flow represents plasticene escaping into the vacuum.
Figure 5.5 Contour and velocity vector plots (Calculation 1) at 10.0 μsec.
Figure 5.6 Contour and velocity vector plots (Calculation 1) at 15.0 \textmu s ec.
Figure 5.7 Contour and velocity vector plots (Calculation 1) at 18.0 μsec.
For reference, Figure 5.8 summarizes the maximum depth of projectile penetration vs time for both calculations 1 and 2 (which generally gave identical results at these early times). A careful plot of the projectile position before and after each rezone revealed that those rezones had no effect on that position. Figure 5.9 gives the maximum shock pressure vs range in the target for a radial down the symmetry axis ($\theta = 0^\circ$) and along a radial at $\theta = 45^\circ$ from that axis.

5.2 ENERGY AND MOMENTUM TRANSFER TO THE TARGET

The first 18 $\mu$sec of the impact encompass the time when most of the energy of the aluminum projectile is transferred to the plasticene clay target. Initially all of the energy involved in the impact (5.4 KJ) was assumed stored in the projectile as kinetic energy (heating of the projectile inside the vertical gun was ignored). During the early stages of the impact, the projectile lost most of this kinetic energy, as shown in Figure 5.10, while gaining some internal energy (~5 percent*) due to shock heating. The end result was that the total energy contained in the projectile decreased very rapidly during the first 6 $\mu$sec following impact, than leveled off at about 5.6 percent. The projectile kinetic energy continued to decrease rapidly after 6 $\mu$sec, and became negligible ($\leq$1 percent) after about 7 $\mu$sec. Of the energy transferred to the plasticene clay target (94.4 percent), 56 percent was kinetic energy, and 38.4 percent was internal energy (at 18 $\mu$sec). The energy transferred to the target was responsible for the shock processes which occurred in the target, some of which were discussed and shown pictorially in the previous section. The target internal energy was held in shock heated, melted or vaporized clay; the kinetic energy was held in the cratering flow field, and the shock front, and in the vaporized plasticene above the target surface.

*All energies discussed here are given as percentages of the total impact energy, 5.4 KJ.
Figure 5.8 Maximum depth of projectile penetration into the plasticene clay target for Calculations 1 and 2.
Figure 5.9 Maximum shock pressure in target vs range for $\theta = 0$ degrees and 45 degrees (Calculations 1 and 2).
Figure 5.10 Energy partitioning for early times.
We wanted in particular to know how much energy was expended in vaporizing the clay. The specific internal energy corresponding to the shock vaporization pressure of plasticene clay (about 6 GPa, Section IV) is about 700 Joules/cc. A special editing routine was built to sum up the total mass and energy of plasticene cells whose internal energy was greater than 700 J/cc. The results of this analysis over the first 3 μsec of calculation 1 are shown in Figure 5.11; the energy in vaporized clay reaches a maximum value of 5.1 KJ (28 percent) at a time of 1.6 μsec, and then decreases, due to adiabatic expansion of the vaporized clay. This time corresponds to the time when the maximum shock pressure induced in the target by the impact falls below 6 GPa. The corresponding total mass of vaporized plasticene was found to be about 2.75 g, or over 9 times the projectile mass.

A value of the internal energy required to melt plasticene clay could not be found. An attempt to measure this value will be made as part of the upcoming experimental effort (See Section I).

Momentum transfer from the projectile to the target (Figure 5.12) occurred at a slower rate than did the energy transfer. The total momentum remaining in the projectile after the projectile kinetic energy has dropped to less than 1 percent is 8 percent of the initial momentum. Being associated with the projectile, it is located near, or at, the bottom of the transient crater, as can be seen in the pictorial figures of the previous section. Thus, although the remaining amount of momentum in the projectile is small, it allows the projectile to continue to drive a small portion of the cratering flow field directly beneath it. This phenomenon affected our cratering flow field analysis results, which will be discussed in the next part of this section. Table 5.1 summarizes the time scales of the energy and momentum coupling processes discussed in this section.
Figure 5.11 Energy partitioning in the plasticene clay target over the first 3 μsec (Calculation 1).
Figure 5.12 Projectile momentum summary plot.
Table 5.1  Summary of the time domains of energy and momentum coupling processes.

<table>
<thead>
<tr>
<th>Process</th>
<th>Time sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vaporization of target material</td>
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</tr>
<tr>
<td>Shock heating of the projectile</td>
<td></td>
</tr>
<tr>
<td>Energy transfer from projectile to target</td>
<td></td>
</tr>
<tr>
<td>Momentum transfer from projectile to target</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
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<tbody>
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<td></td>
<td></td>
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<tr>
<td>Shock heating of the projectile</td>
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<tr>
<td>Energy transfer from projectile to target</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Momentum transfer from projectile to target</td>
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</tr>
</tbody>
</table>
5.3 CRATERING FLOW FIELD ANALYSIS

A substantial effort was spent in the analysis of the cratering flow field developed in the impact cratering calculations. We defined the cratering flow field in the same manner as it was defined for explosion cratering calculations (See Section 3.1). Because the early time calculations were performed in Eulerian coordinates, it was necessary to define a transient crater boundary behind the initial outgoing shock front. The 1 g/cc density contour was used to define the wall of the transient crater boundary because it was the best estimate of boundary line between vaporized and non-vaporized target material.

From the pictorial figures given in Section 5.1, it was seen that the shock wave emanating from the point of impact was approximately hemispherical, but that the center of the hemisphere was below the initial target surface, and not at the surface as was the case for surface-burst explosion cratering calculations. The cratering flow field was also seen to be centered at approximately the same point. The fact that this "center of energy" is below the target surface was expected because of the initial directedness of the projectile momentum.

The basic equations of Maxwell's Z-Model can be used to analyze the cratering flow field, whether or not the Z-Model itself is valid. In particular, we wanted to see if the radial velocity field attenuated with range according to a power law, as was shown for surface burst cratering calculations (Equation 3.1).

For the above to be true, it is first necessary that the outgoing shock wave interact with the target surface (thereby including the effects of surface rarefactions within the cratering flow field); and secondly that steady state flow be established. Until the momentum and energy of the projectile is completely transferred to the target, there can be no steady state flow. Therefore, the calculations were examined at the latest possible time (18 μsec).
Our analysis to date has shown the following:

1. The flow field center appears independent of material strength and is beginning to be established at the latest time of the current calculation (18 μsec).

2. When all the projectile momentum has been deposited, a flow field center is established at some depth below the original target surface near the geometrical center determined from pictorial contour plots.

3. A true flow field center does not exist until all (nearly all) the projectile momentum is deposited, i.e., a and Z are not constant with θ, and Z < 2 for some regions.

The first conclusion was expected, and is explained by the fact that the shock pressures developed at early times far exceed the yield strength of the target material. The effects of material strength will be seen during the later stages of the cratering process: as shown in Section 5.4, the higher strength target is expected to yield a smaller final crater.

Our second conclusion was supported by analyzing the cratering flow field at 18 μsec using our special Z-Model editing subroutine. Almost all of the momentum (and energy) had been transferred to the target at that time. Our search for a flow field center for which Equation 3.1 would be valid centered on an on-axis region of 0.4 to 0.8 cm depth, which was within the range of the rough geometric center of the emanating shock wave. The coordinate system is illustrated in Figure 5.13a. At a depth of 6.5 mm, it was found that \( \hat{R} \) did decay with \( R^{-Z} \) as is shown in Figure 5.12b for \( θ = 30^\circ \). For values of \( θ \) spanning 0 to 70 degrees,
Figure 5.13 Maxwell's Z-model applied to a calculation of the impact of a 0.3 g aluminum projectile into a plasticene clay target at a velocity of 6 km/sec at a time of 18 μsec. a) Vector velocity plot in cratering flow field region, and spherical polar coordinates centered beneath the original target surface; b) plot of $\dot{R}$ vs $R$ for $\theta = 30$ degrees with the values $\alpha = 76.0 \text{ mm}^2/\mu\text{sec}$ and $Z = 2.017$ obtained from a fit to the calculated radial velocity field.
it was found that $\alpha$ was almost constant, with an average value $(\bar{\alpha})$ of $0.084 \text{ cm}^{Z+1}/\mu\text{sec}$ (Figure 5.14a). A similar plot of $Z$ vs $\theta$ (Figure 5.14b) shows that $Z$ is not as independent of angle as is $\alpha$, but some degree of variation with $\theta$ is expected. The key points are that a flow field center is established, that the velocity attenuation within that flow field is describable by Maxwell's $Z$-Model, and that the center is roughly the geometric "center of energy" of the impact.

The center picked is equivalent to about 1.1 initial projectile diameters. Because almost all of the projectile momentum has been transferred to the target, we suspect that the center will not move much deeper into the target at later times, but cannot at this time rule out a deeper flow field center (even though it contains no appreciable momentum, the projectile by its mere presence can continue to affect the cratering flow field beneath it). Further, we suspect that the location of the flow field center is very much dependent on its initial momentum (with energy held constant). The location of that center then becomes another required parameter for the $Z$-Model, in addition to $\alpha$ and $Z$.

Our third conclusion was supported by attempting to apply Equation 3.1 at earlier times in the calculation (5-12 $\mu\text{sec}$). With the chosen flow field center, we examined the earlier projectile momentum on a localized area of the cratering flow field. We found that for times less than 12 $\mu\text{sec}$, the directed momentum of the projectile overdrove the cratering flow field in the target region directly beneath it. This resulted in values of $Z$ less than 2 over a considerable range of $\theta$ for the chosen flow center. This is shown in Figure 5.15 for a time of 10 $\mu\text{sec}$. A minimum value of $Z$ was achieved for the value of $\theta = \theta_{\text{min}}$, which we found
Figure 5.14 These plots show the calculated values of $\alpha$ (a) and $\bar{z}$ (b) vs $\theta$ for a flow field center taken at a depth of 6.5 mm at a time of 18 $\mu$sec. The values of $\alpha$ and $\bar{z}$ were averaged over all values of $\theta$ and these values are shown as dotted lines on the respective plots. The standard deviations of $\bar{\alpha}$ and $\bar{\bar{z}}$ were also computed and are given in the plots.
Figure 5.15 These plots show the effect of the projectile on the early time Z flow field. a) A plot of Z vs θ at t = 10 μsec shows values of Z < 2, with Z achieving a minimum at θ_{min} = 35°. b) The angle subtended by the projectile (β) at 10 μsec is also about 35 degrees. c) A plot of β vs θ_{min} shows that a direct correlation holds during the early stages of the impact process.
to be directly correlatable with the maximum angle (β) subtended by the projectile from the flow field center. This is shown for that same time, 10 μsec, in Figure 5.15b. We also found this correlation between θmin and β held at earlier times (Figure 5.15c). For times less than 5 μsec, the actual position of the projectile was above 0.65 cm, and it was not possible to carry out this analysis. Thus, we found that the values of Z less than 2 were directly correlatable with this region where the projectile was overdriving the target.

We have shown here only that a cratering flow field with initial characteristics describable by Maxwell's Z-Model has been generated. The further assumption of incompressible flow (Equation 3.2) will be checked as the calculations are continued. We expect this assumption to hold, however, because of the basic similarity between the shock processes developed in the target in near-surface explosion and impact cratering.

5.4 FURTHER RESULTS AND THEIR IMPLICATIONS

The impact cratering calculations have been carried to later times than have been reported in previous sections of this report. Figure 5.16 shows a grid plot from calculation 1 at a time of 100 μsec*. The formation of a substantial crater lip is seen

* To run the calculation to times past 18 μsec, a Lagrangian grid was coupled to the Eulerian grid, and the coupled Eulerian-Lagrangian treatment in 2DELK is used to carry the calculation out. Details of the coupling process will be discussed in a later report. It is important to note that the Lagrangian-Eulerian interface is not the boundary of the transient crater. However, most of the cratering flow field does reside in the Lagrangian part of the grid (Figure 5.16a).
Figure 5.16 (a) Shows the Lagrange grid in comparison to the original projectile, $t = 97.7 \mu s$; (b) Velocity vector plot shows the development of the flow field, $t = 97.7 \mu s$. 
above the original target surface. Analysis of this lip and its growth and continuing development will also be part of the second year effort.

Vector velocity plots are shown in Figure 5.16b. The predominately radial motion seen at 18 µsec (See Figure 5.15a) has become upward and outward motion, but the figure shows a very orderly process within the cratering flow field. This type of motion is predicted by Maxwell's Z-model for values of Z greater than 2. For a given α and Z, the motion of a mass element as a function of time can be calculated and compared with the motion calculated during the impact calculation. It is particularly useful to perform the calculation in Lagrangian coordinates, because it is so much easier to follow the motion of a particular mass element. It is also easier to check the other basic assumption of Maxwell's Z-Model, that is, the assumption of incompressible flow (Equation 3.2) within the cratering flow field in Lagrangian coordinates. The above checks and analyses will be performed over a time range from before 18 µsec to at least 0.5 msec as part of next year's effort.

Using values of α and Z, the final crater dimensions from the impact cratering calculations can be estimates, using Equations 5.1 and 5.2 given below. Here R is the crater radius (cm) and D is the crater depth (cm) [30]:

\[ D = \alpha^{1/Z} \gamma_a^{1/2Z} + D_0 \]  

\[ \gamma_a = \frac{0.5\rho/(Z-1)}{\frac{6ZY}{(Z+1)^2} \left\{ \frac{1}{4} \right\}^{2Z/(Z+1)}} + \frac{D_0 g}{(Z+2) \left\{ \frac{1}{2} \right\}^{(2Z+1)/(Z+2)}} \]
\[ R = a^{1/2} \gamma_b^{1/2} \]  

\[ \gamma_b = \frac{0.2p \left[ 1+(Z-2) \right]^{2/3}}{1.65ZY} \left( \frac{1}{Z+1} + \frac{g}{(Z+2)} \right) \]  

The value of the factor \( \epsilon \) in Equation 5.2b depends on the value of \( Z \) (Table 5.2).

Both Equations 5.1 and 5.2 are implicit, and if all constants are known, \( R \) and \( D \) can be calculated by an implicit iteration process. A simpler method for obtaining \( R \) and \( D \) was developed by taking Equation 5.1 and 5.2 and solving for \( Y(D) \) and \( Y(R) \), respectively. If the target strength is known, values of \( R \) and \( D \) can be entered into the equations until the correct value of \( Y \) is achieved. The advantage of solving the equation in this manner is that the iteration can be performed explicitly.
Table 5.2 The value of $\varepsilon$ as a function of $Z$ (Reference 30)

<table>
<thead>
<tr>
<th>$Z$</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0.</td>
</tr>
<tr>
<td>2.5</td>
<td>0.158</td>
</tr>
<tr>
<td>3.0</td>
<td>0.268</td>
</tr>
<tr>
<td>3.5</td>
<td>0.349</td>
</tr>
<tr>
<td>4.0</td>
<td>0.41</td>
</tr>
</tbody>
</table>
It was found that the data given in Table 5.1 was adequately represented by:

\[ e = -0.914 + 0.583Z - 0.063Z^2 \]  

(5.3)

This allowed calculation of \( e \) for any value of \( Z \) between 2 and 4.

Using the values of \( a \) and \( Z \), and taking

\[ \rho = 1.69 \text{ g/cc}, \]
\[ g = 9.8 \times 10^{-10} \text{ cm/μsec}^2, \] and
\[ D_0 = 0.65 \text{ cm}, \]

Equations 5.1 and 5.2 can be solved for \( R \) and \( D \) for \( Y = 0.5 \) bars and 1.5 bars (\( e = 0.0356 \) from Equation 5.3). The results are given in Table 5.3.

Table 5.3 Estimate of crater dimensions from the analytic equations of Maxwell's Z-Model for low and high strength target impact calculations.

<table>
<thead>
<tr>
<th>Clay Strength</th>
<th>( R(\text{cm}) )</th>
<th>( D(\text{cm}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 KPa (0.5 bar)</td>
<td>11.6</td>
<td>11.2</td>
</tr>
<tr>
<td>150 KPa (1.5 bar)</td>
<td>8.9</td>
<td>8.8</td>
</tr>
</tbody>
</table>
Because Z is very close to 2, the craters are predicted to be bowl-shaped with \( R \approx D \). The effect of increasing target shear strength is to reduce the crater dimensions. Further calculation, plus the results of the experimental program, will of course refine the above estimates.
REFERENCES


APPENDIX

PAPER TO BE PUBLISHED IN THE PROCEEDINGS OF THE TENTH LUNAR AND PLANETARY SCIENCE CONFERENCE

NASA Johnson Space Flight Center
Houston, Texas

March 19-23, 1979
CALCULATIONAL INVESTIGATION OF IMPACT CRATERING DYNAMICS:

EARLY TIME MATERIAL MOTIONS

May 31, 1979
(Revised Manuscript No. 3022,
Conference No. 10)

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ABSTRACT

Early time two-dimensional finite difference calculations of laboratory-scale hypervelocity (6 km/sec) impact of 0.3 g spherical 2024 aluminum projectiles into homogeneous plasticene clay targets were performed and the resulting material motions analyzed. Results show that the initial jetting of vaporized target material is qualitatively similar to experimental observation. The velocity flow field developed within the target is shown to have features quite similar to those found in calculations of near-surface explosion cratering. Specific application of Maxwell's analytic Z-Model (developed to interpret the flow fields of near-surface explosion cratering calculations), shows that this model can be used to describe the flow fields resulting from the impact cratering calculations, provided that the flow field center is located beneath the target surface, and that application of the model is made late enough in time that most of the projectile momentum has been dissipated.

INTRODUCTION

Calculations can facilitate understanding of the dynamics of cratering processes which have been studied in detail in laboratory-scale impact experiments (Gault et al., 1968; Oberbeck, 1971; Gault and Wedekind, 1977; Moore, 1976). Relatively few impact calculations have been performed, and fewer still have
directly addressed impacts at scales for which experimental and theoretical results can be compared. Bjork (1961) and Bryan et al. (1978) calculated a terrestrial impact crater, Meteor Crater in Arizona. O'Keefe and Ahrens (1975) emphasized the early-time shock effects of a large meteorite impact at 15 km/sec into a material thought to be typical of the early lunar crust. Similar calculations of small-scale impact (O'Keefe and Ahrens, 1978) examined scaling of the above results and, through ballistic extrapolation, the ejecta distribution assuming lunar gravity (O'Keefe and Ahrens, 1976). O'Keefe and Ahrens (1977) also showed the dependence of the early-time partitioning of energy on impact velocity in the range of 5 to 45 km/sec.

The present calculational effort examines the fundamental impact dynamics in a uniform, non-geologic material at impact velocities achievable in laboratory-scale experiments. Impact shock effects in the target and the resultant initial material motions are computed and examined during the compression stage and the early part of the excavation stage of laboratory-scale impact crater formation, as described empirically by Gault et al. (1968). The compression stage described the events associated with the initial contact between the projectile and the target. This included the shock compression of the target material directly below the projectile and the subsequent hydrodynamic motion of that material. This stage was defined to terminate at approximately the time of complete engulfment of the projectile by the
target. The excavation stage then begins. It described the basic shock wave geometry generated in the target and the resulting motions of the target material. It was empirically observed that the initial dominantly radial motions (centered close to the point of impact) were subsequently modified by a continuous fan of rarefaction waves originating at the target surface. These waves caused an upward velocity component to develop in the shocked target material which led eventually to the orderly ejection of target material from the impact area. The end result of the excavation stage was a bowl-shaped depression; this is also known as the final stage of development of the transient crater.

During the compression stage the present effort discusses the dynamics of the initial contact between the projectile and the target from a calculational viewpoint. Energy and momentum transfer from the projectile to the target (quantities not directly observable in impact experiments) are discussed, as are the processes of shock heating and vaporization. The latter are relevant because of the specific target and projectile materials chosen.

Only the initial radial motions during the excavation stage of Gault et al. (1968) are examined. An analytic model developed by Maxwell (1977) to describe the explosive cratering excavation process is applied to the impact cratering calculations. This model, the Z-Model, implies an orderly ejection of material from the cratering region in a manner very similar to that observed empirically by Gault et al. (1968).
INITIAL CONDITIONS, MATERIALS, AND MATERIAL MODELS

Figure 1 summarizes the initial conditions for the calculations. The calculations simulate the impact of 6 mm diameter 2024 aluminum projectiles into plasticene (oil base) clay targets. Axial symmetry and terrestrial gravity are assumed. It is also assumed that the impacts occur in a vacuum chamber. Two calculations were performed in which only the clay strength was varied.

The present effort is directed toward a better understanding of cratering dynamics, and not toward simulation of a specific planetary impact event. For this purpose plasticene clay has properties which make it rather unique as a target material. It has a simple equation of state description, primarily because it contains no air voids. Its shear strength can also be simply characterized by a von Mises (Maxwell and Reaugh, 1972) or Mohr-Coulomb (Holsapple and Schmidt, 1979) criterion, and the strength magnitude can be changed by varying the ambient temperature. Finally, it is readily available, and this should allow close integration of this calculational effort with future experimental studies.

Static material properties determined for plasticene clay (Christensen et al., 1968) are: an initial density of 1.69 Mg/m$^3$, a compressional wave velocity of 1.4 m/msec, a shear wave velocity of 0.475 m/msec, and a Poisson's ratio of 0.435.
The equation of state for plasticene clay (Maxwell and Reaugh, 1972) is based on Hugoniot data from Christensen et al. (1968) presented in Table 1. From these data the relative volumes and internal energies were calculated using Hugoniot relationships (see for example McQueen et al., 1970). The equation of state form which matches these states gives the pressure $P$ (GPa), as a function of compression, $V$ [where $V = V_o \cdot \rho / \rho_o$ (Mg/m$^3$) is the initial density and $\rho$ (Mg/m$^3$) is the current density]; compressibility, $\mu$ [where $\mu = (1-V) \cdot V$]; and energy density, $E$ (GJ/m$^3$):

$$
P = 2.8\mu + 40.7\mu^2 - 36.0\mu^3 + 1.7 \frac{E}{V} \quad \mu \geq 0
$$

$$
P = 1.7 \frac{E}{V} \quad \mu < 0
$$

This form incorporates the initial (zero pressure) bulk modulus of 2.8 GPa, provides the required fit to the Hugoniot data, and allows for reasonable extrapolation to pressures greater than those covered by experimental data ($0<P<21.4$ GPa).

A von Mises failure envelope was used in the calculations reported here; the actual von Mises strength was found to be dependent on the ambient clay temperature, $T$ (Maxwell et al., 1972), so two calculations with differing strengths were performed. One calculation used the estimated low strength ($50$ kPa, $T = 32.2^\circ C$) and the other used a higher strength ($150$ kPa). We did not expect the results of the two calculations to differ greatly during the compression stage or the early part of the excavation stage discussed here, when the impact-induced shock pressures greatly
exceeded the clay strengths. Explosive cratering experiments in plasticene clay (Maxwell et al., 1972) revealed, however, that the final crater volume increased by a factor of three as the ambient clay temperature increased from 17.5 to 32.2°C. Thus, when we examine (in the future) the calculational results near the end of the excavation stage, we expect significant differences to appear.

The equation of state for 2024 aluminum was a Hugoniot fit reported by van Thiel (1977) based on the Hugoniot measurements of McQueen et al. (1970):

\[ P(\text{GPa}) = \frac{\rho_0 C_0^2 \mu (1+\mu)}{[1+\mu(1-s)]^2} \quad 8.4 \leq P \leq 109 \text{ GPa}, \quad (2) \]

where \( \rho_0 = 2.783 \text{ Mg/m}^3; \ C_0 \) (the bulk sound speed) = 5.343 m/msec and \( s = 1.325. \) The maximum strength of the aluminum Mohr-Coulomb failure envelope used [320 MPa, van Thiel (1977)] is much greater than the clay strengths.

The one-dimensional pressure state generated behind the shock during the impact of the aluminum sphere into the plasticene clay target at 6 km/sec was determined from Equations 1 and 2 using the method of Gault and Heitowitz (1963). The calculated pressure of 50 GPa (Figure 2) is not sufficient to shock melt aluminum, as a shock pressure of 60 GPa is required to cause incipient melting (Gehring, 1970). Maxwell and Reaugh (1972) estimated that
incipient vaporization of the volatile constituents of the plasticene clay begins at a pressure of about 6 GPa. This latter pressure is substantially lower than 50 GPa, indicating that vaporization of the clay target will occur during the compression stage.

CALCULATIONAL PROCESS AND RESULTS OF INITIAL IMPACT

Figure 3 shows vector velocities in the target material during the compression stage and also illustrates the initial part of the calculational process. The two-dimensional computer code used, PISCES 2D ELK (Hancock, 1976), permitted initial modeling of the projectile using a Lagrangian finite difference approximation and the target using an Eulerian approximation. As the projectile moved into the target, it interacted continuously along its outer boundary with the target. In the computer code treatment the Lagrangian grid literally "moves through" the Eulerian grid. Material cannot occupy simultaneously the same point in space in both grids, however, and the Lagrangian material predominates. Thus the Eulerian target material gets pushed out of the way by the impacting Lagrangian projectile; momentum and energy are transferred continuously, and a shock wave is formed in the target. The Lagrangian projectile zone size is shown explicitly. The zone size in the Eulerian target region was initially 0.3 mm square, and thereby spanned the initial projectile radius with ten radial zones.
A maximum pressure of 42.1 GPa was observed in the calculations at the earliest time when the Eulerian cell variables were printed out after the initial impact (0.115 μsec). It occurred close to the symmetry axis directly below the projectile. The target region spanning the projectile contact radius was all highly compressed, and a shock wave was quickly formed in the clay. By 0.3 μsec (Figure 3a), the presence of the free surface beyond the projectile contact radius caused rarefaction waves to relieve the shock pressures in the clay at the point of projectile contact with the original target surface; but in doing so a jet of vaporized clay was formed. The vector velocities of Figure 3 above the target surface depict the magnitude and direction of the jet. The maximum jet velocities (10.0 km/sec) occurred in the lowest density jetted material (1-2 × 10⁻⁶ Mg/m³), but that with the highest specific internal energy. The density cutoff value in each Eulerian cell was 10⁻⁶ Mg/m³; lowering the cutoff value to 10⁻⁷ Mg/m³ resulted in a maximum jet velocity of 10.7 km/sec, or only about 7 percent greater. The density associated with this slightly faster velocity was 2-3 × 10⁻⁷ Mg/m³.

At times up to 1 μsec (Figure 3c) the density of the jetted material increased in the direction of the target surface. At heights of 1 mm above the surface, the jetted material had a density of approximately 1 Mg/m³, and a velocity an order of magnitude less than the maximum jet velocity. This relatively
stationary material was in the form of a "lip" which controlled the direction of subsequently vaporized and jetted clay. The effect of the control is seen most clearly in Figure 3c, where velocity vectors beyond the lip position are actually pointed in the direction of the target surface. It is also clear from Figure 3c that by the time of complete engulfment of the projectile by the target (approximately the end of the compression stage) the projectile had been severely deformed.

Gault et al. (1968) similarly described the onset of target material jetting early in the compression stage, as observed in laboratory-scale impacts into metals and natural materials. They noted that jetting velocities which are higher than the original impact velocity were also predicted by theory. The maximum calculated jetting velocities in the present effort (10.7 km/sec, or 1.8 times the original impact velocity) were in qualitative agreement with their observations. Due to materials differences, however, an exact comparison of the magnitude of their theoretically predicted jetting velocities with the calculationally derived velocities could not be made.

Vaporization of target material in the region near the impact continued to a time of about 1.5 μsec, when the shock pressures induced in the target fell to below 6 GPa. It was determined that a mass of approximately 2.75 g of target material (over nine times the projectile mass) was vaporized and that this mass
contained about 1.5 kJ of internal energy (roughly 28 percent of the total impact energy). This mass of vaporized target material is very much greater than those reported by O'Keefe and Ahrens (1977) for gabbroic anorthosite or iron projectiles impacting gabbroic anorthosite targets at 7.5 km/sec. This is because both iron and gabbroic anorthosite require higher shock vaporization pressure levels than can be generated by an impact at 7.5 km/sec. The vaporized target masses in our calculations are comparable to the vaporized target masses of O'Keefe and Ahrens (1977) for gabbroic anorthosite impacting gabbroic anorthosite at 30 to 45 km/sec and for iron impacting gabbroic anorthosite at 15 to 30 km/sec.

By 2.6 μsec after initial contact the projectile had penetrated to a depth greater than its original diameter into the target, and had become severely distorted (Figure 4). The plasticene clay lip above the original target surface grew substantially and is plotted specifically in Figure 4 as a contour line marking a density of 1 Mg/m³. The velocity vectors on the side of this contour in the direction of the target are generally moving uniformly radially outward from a point on-axis slightly behind the projectile. This region contained solid and some melted clay. The velocity vectors on the other side of the contour, inside the "hole" carved by the projectile and above the target surface, were moving in various directions, and the associated target material had a density much lower than 1 Mg/m³ and was totally vaporized.
At 2.6 \( \mu \text{sec} \) the Lagrangian grid was discarded and the calculation continued using only the Eulerian grid, which then contained the aluminum projectile as well as the plasticene clay. The calculations were continued to a total time of 18 \( \mu \text{sec} \) in the present effort. The total calculated time seems very short; however, it must be kept in mind that these are simulations of laboratory-sized impacts. Using simple cube-root scaling of total energy and comparing the total calculated time scaled to the Johnie Boy nuclear explosive event yield (0.5 \( \text{kt} \), or 2.0 \( \text{TJ} \)) gives 13 \( \mu \text{sec} \). As reported by Orphal (1977a), the early time coupling of the Johnie Boy nuclear device energy to the alluvium was complete by 7 \( \mu \text{sec} \), and the wall of the transient crater chosen at 7.84 \( \mu \text{sec} \). Thus our impact calculations had been carried to scaled times comparable to the early times of the Johnie Boy calculation.

**MOMENTUM AND ENERGY TRANSFER TO THE TARGET**

Treating the projectile at very early times in Lagrangian coordinates and using very small zones in the Eulerian target region allowed careful monitoring of the early energy and momentum transfer to the target as a function of penetration depth (the depth of that part of the projectile which had penetrated the deepest into the target) when the rate of transfer was the most rapid (Figure 5). For reference, a plot of the projectile penetration depth vs time is also given. With the projectile mass and impact velocity used in the calculations, momentum transfer to the target occurred at a slower rate than did energy transfer.
At 18 μsec, the projectile had penetrated to a depth of over 3 initial projectile diameters and no longer contained any appreciable kinetic energy. A small amount of momentum (~3 percent of the initial projectile momentum) was still retained by the projectile.

Although energy was rapidly coupled to the target, not all of it was available for excavation. Energy was absorbed through shock heating of the projectile (approximately 5 percent of the total impact energy, Figure 5c), and more importantly through shock vaporization of the target material. The shock vaporized material escaped easily into the vacuum above the original target surface and no longer played a role in the excavation process.

At the approximate end of the compression stage of Gault et al. (1968), which occurs when the projectile penetrates the target to a depth equal to its original diameter, 70 percent of the total energy, but only 50 percent of the total momentum, had been transferred to the target. Thus, for the laboratory-scale impacts simulated by our calculations, the energy and momentum transfer process continues during the early part of the excavation stage.

The 1 Mg/m³ density contour (e.g., Figure 4) defined the wall of the transient crater because it was the best estimate of the boundary line between vaporized and non-vaporized target
material. The kinetic energy coupled beyond this contour at later times (10 to 18 μsec) constituted 30 to 37 percent of the total impact energy. This was incorporated in the shock wave and in the region behind it and represented a maximum percentage of the initial impact energy available for excavation.

CRATERING FLOW FIELD ANALYSIS

The cratering flow field has been defined previously for explosion cratering (Orphal, 1977b) as the particle velocity flow field existing in the region of the transient crater but behind the initial out-going shock front (Figure 6). Maxwell (1977) and Orphal (1977b) described an analytic model called Maxwell's Z-Model that was developed to describe the cratering motions observed in near-surface explosion cratering calculations. After the initial explosion, a shock wave propagates through the target material approximately spherically from the detonation point and decays rapidly. After the shock front passes, the shocked material retains a residual velocity which, subsequently modified by surface rarefactions and by the effects of material strength and gravity, produces the final crater at much later times. Maxwell found for near-surface explosion calculations that an early-time crater growth was nearly hemispherical. In spherical polar coordinates taken about the on-axis detonation point, the flow field could be described by

$$\dot{R} = \alpha(t) R^{-Z}$$

(3)
where \( R \) is the radial velocity of the flow field, \( a \) is a time-dependent coupling term describing the flow field strength, and \( Z \) defines the rate of velocity decay with range, \( R \). Maxwell (1977) also observed that the density in the cratering flow-field region was approximately constant, yielding incompressible flow:

\[
\nabla \cdot U(R, \theta) = 0 \tag{4}
\]

where \( U(R, \theta) \) is the vector velocity of the flow field. Combining Equations 3 and 4 permits derivation of the full equation of motion of a mass element within the cratering flow field:

\[
U(R, \theta) = \ddot{R} \hat{R} + \dot{R}(Z-2)\tan(\theta/2)\hat{\theta}, \tag{5}
\]

where \( \hat{R} \) and \( \hat{\theta} \) are unit vectors in spherical polar coordinates. This can be integrated to give the mass element motion as a function of time. Gravity and material strength effects can also be included. It is clear from Equation 5 that for \( Z = 2 \), \( U \) is radial at all points, yielding an irrotational flow field. The corresponding physical case is the purely radial velocity field resulting from the explosion of a spherical source in an homogeneous medium of infinite extent in all directions. Maxwell (1977) found for near-surface explosion cratering calculations that \( Z \approx 2 \) for \( \theta = 0^\circ \), that \( Z \approx 2.7 \) for \( 30^\circ \leq \theta \leq 60^\circ \); and that \( Z \gtrsim 4 \) for \( \theta \gtrsim 75^\circ \). An average value of \( Z \approx 3 \) was found to be representative of the
entire cratering flow field produced by a near-surface explosion. Values of $Z > 2$ lead to flow fields which are rotational in the direction of the surface (e.g. Orphal, 1977b). The rotation leads to the eventual ejection of material below the ground plane in a very orderly manner, beginning with material closest to the wall of the transient cavity. Total energy, material strength and gravity limit the total mass ejected, and also control the crater depth (e.g., O'Keefe and Ahrens, 1979).

We investigated the applicability of Maxwell's Z-Model to impact cratering, using our calculations simulating laboratory-scale impact. Because the impact cratering process is initially very different from the explosion cratering process, we felt it was particularly necessary to determine that Equation 3 was valid. Figure 7a reveals that the wall of the transient crater at 18 μsec was nearly hemispherical about a point centered on-axis and 1.08 initial projectile diameters beneath the original target surface. In spherical polar coordinates taken about this point, the residual velocities within the cratering flow field were found to decay in the manner consistent with Equation 3 (Figure 7b). Secondary shocks behind the main shock front obscured the attenuation of $\dot{R}$ with $R$, and the actual calculation of $\alpha$ and $Z$ was performed only in the region between the dotted lines ($15 < R < 23$ mm).
The computer routine developed to perform the cratering flow field Z-Model analysis calculated $R$ and $R$ about the flow center for 18 μsec at $\theta$ increments of $5^\circ$, and $\alpha$ and $Z$ were plotted as a function of $\theta$ (Figure 8). It is seen that $\alpha$ is nearly independent of $\theta$ in a manner similar to that observed in explosion calculations.

Various depths of the flow center were tried to determine if the true center was indeed at a depth equal to 1.08 initial projectile diameters at 18 μsec. It was found that the chosen flow center first, minimized the standard deviation of $\alpha$ for $0 < \theta < 70^\circ$; and secondly, gave the most regular variation of $Z$ with $\theta$. As expected, our calculation of low and higher strength targets gave identical results at this early time. Oberbeck (1971) found, for impact of aluminum projectiles into noncohesive quartz sand targets at 2 km/sec, that the resulting crater was the same as an equivalent energy explosion-generated crater in the same material when the explosive was buried at a depth of 6.3 ± 2 mm beneath the target surface. In the framework of the Z-Model, this empirical result manifests itself in a cratering flow field centered below the target surface.

For times less than about 12 μsec, the directed momentum of the projectile overdrove the cratering flow field in the target region beneath the projectile. This initial directedness of momentum (and kinetic energy) in an impact is a basic difference
between an impact and an explosion event. Application of the Z-Model at these times using the 18 μsec flow field center yielded values of $Z < 2$ (Figure 9a) over a considerable range of $\theta$. A minimum value of $Z$ was achieved for $\theta = \theta_{\text{min}}$, which we found to be directly correlatable with the maximum angle ($\beta$) subtended by the projectile from the flow field center (Figure 9b). This correlation also held at earlier times (Figure 9c) as long as the actual position of the projectile was below the flow center. Values of $Z < 2$ have not been seen in explosion cratering and the direct correlation of this effect with the projectile leads us to conclude that it is probably unique to impact cratering. By Equation 5 this leads to a flow field which is rotational in the downward direction, i.e., into the target. In this context, however, the effect is simply the manifestation of the almost completely expended projectile driving a continually more localized portion of the cratering flow field.

CONCLUSIONS

Major results from the two early time finite difference calculations of spherical aluminum projectile impact at 6 km/sec into homogeneous plasticene clay targets with differing von Mises material yield strengths are:

1. The magnitude of the target material jetting velocity which occurs upon projectile impact is in qualitative agreement with experimental observation.
2. The velocity field developed within the target is strikingly similar to those developed in near-surface explosion cratering calculations. This similarity has been quantified by extending Maxwell's Z-Model of explosion cratering to impact cratering.

Plasticene clay is a uniform material which is characterized by a simple material model and which is also readily available. Thus it is ideal for studies of basic cratering phenomenology employing either cratering calculations as in the present effort, or laboratory-scale experiments. It is probably not a good simulant of materials existing on lunar or planetary surfaces. Calculations were performed to an early time (18 μsec) for two von Mises strengths differing by a factor of 3; no differences in the computed results were seen and none were expected. At much later times during the cratering process, we do expect significant differences to develop, based on data from past laboratory-scale explosion cratering experiments.

We have carefully analyzed the impact-induced material motions, capturing first the initial jetting, then the deformation of the projectile and finally the response of the clay within the target. The major assumption of Maxwell's Z-Model (and the one which would not necessarily be valid for impact cratering), namely the regular power law decay of $\dot{R}$ with $R$, was found to be
valid at 18 μsec, provided that the center of the model coordinate system was chosen at a specific depth below the original target surface. This result differs from the surface-centered coordinate system employed in near-surface explosion cratering work, and is a consequence of the initial directedness of projectile momentum and kinetic energy. A second assumption, that of incompressible flow within the cratering flow field, will be tested as the calculations are continued; however, because the shock processes are the same in both explosion and impact events, we expect that this assumption will be valid.

The specific depth of the flow field center is suspected to depend strongly on the projectile material, and its initial energy and momentum. The projectile was not melted during the impact, but a significant amount of target material was vaporized in our calculations. For this case it was found that the center was located at a depth of 1.08 initial projectile diameters (6.5 mm) at 18 μsec. Further slight movement downward may occur as the calculations are continued. Calculational studies should be performed varying the projectile material and mass at constant initial kinetic energy to show explicitly the variation of the depth of the center with these quantities.

Acknowledgments - This work was supported by the National Aeronautics and Space Administration under Contract No. NASW-3168. We appreciate the critical review comments of J. Bryan and S. Croft on the manuscript.
REFERENCES


Table 1. Plasticene Hugoniot data from Christensen et al. (1968)

<table>
<thead>
<tr>
<th>Measured Shock Velocity $U_s$ (km/sec)</th>
<th>Calculated Particle Velocity $U_p$ (km/sec)</th>
<th>Measured Pressure $P$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.7</td>
<td>0.91</td>
<td>5.75</td>
</tr>
<tr>
<td>5.6</td>
<td>2.26</td>
<td>21.4*</td>
</tr>
</tbody>
</table>

*Represents the average of two measurements.
Figure 1 Summary of calculation initial conditions.

Figure 2 Intersection of curves defining the one-dimensional theoretical pressure state occurring behind the shock front for an aluminum projectile with a velocity of 6 km/sec impacting into a plasticene clay target. The two clay experimental data points of Christensen et al. (1968) are also shown.

Figure 3 Lagrangian grid of the aluminum projectile and vector velocity plots in the Eulerian grid containing the plasticene clay target, depicting the early time jetting of target material and projectile deformation at (a) 0.30 μsec, (b) 0.55 μsec, and (c) 1.02 μsec. The horizontal line through the center of each frame shows the pre-impact location of the target surface.

Figure 4 Vector velocity plot in target material, and plot of projectile Lagrangian grid at 2.6 μsec (the high velocity target material above the 1 Mg/m³ density contour is shock vaporized and ρ< 1 Mg/m³).

Figure 5 Projectile penetration history (a); and momentum (b) and energy (c) transfer from the projectile to the target versus penetration depth.

Figure 6 This schematic representation shows the cratering flow field region as defined for near-surface explosion cratering calculations.
Figure 7  Maxwell's Z-Model applied to a calculation of the impact of a 0.3 g aluminum projectile into a plasticene clay target at a velocity of 6 km/sec at a time of 18 μsec. a) Vector velocity plot in cratering flow field region, and spherical polar coordinates centered beneath the original target surface; b) Plot of $\dot{R}$ vs $R$ for $\theta = 30^\circ$ with the values $a = 76.0 \text{ mm}^{2+1}/\mu\text{sec}$ and $Z = 2.017$ obtained from a fit to the calculated radial velocity field.

Figure 8  These plots show the calculated values of $a$ (a) and $Z$ (b) versus $\theta$ for a flow-field center taken at a depth of 6.5 mm at a time of 18 μsec. The values of $a$ and $Z$ were averaged over all values of $\theta$ and these values are shown as dotted lines on the respective plots. The standard deviations of $\bar{a}$ and $\bar{Z}$ were also computed and are given in the plots.

Figure 9  These plots show the effect of the projectile on the early time Z flow field. a) A plot of $Z$ vs $\theta$ at $t = 10$ μsec shows values of $Z < 2$, with $Z$ achieving a minimum at $\theta_{\text{min}} = 35^\circ$. b) The angle subtended by the projectile ($\beta$) at 10 μsec is also about $35^\circ$. c) A plot of $\beta$ vs $\theta_{\text{min}}$ shows that a direct correlation holds during the early stages of the impact process.
Figure 1
Figure 2
Figure 4
Figure 5
Figure 6
Figure 8

(a) $\alpha = 0.084 \pm 0.009$

(b) $\bar{Z} = 2.11 \pm 0.17$
Figure 9

(a) Graph showing $Z$ vs $\theta$, degrees for $t = 10 \mu\text{sec}$ with $\theta_{\text{min}} = 35^\circ \pm 4^\circ$

(b) Graph showing depth vs range (mm) with $\beta = 35^\circ$ and walls of transient cavity

(c) Graph showing $\beta$ vs $\theta_{\text{min}}$ for various times: 5 $\mu\text{sec}$, 6.3 $\mu\text{sec}$, 8 $\mu\text{sec}$, 10 $\mu\text{sec}$, 12 $\mu\text{sec}$