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Postseismic Viscoelastic Deformation and Stress

Part 2: Stress Theory and Computation; Dependence of Displacement, Strain, and Stress on Fault Parameters

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ABSTRACT

This paper reports on a viscoelastic model for deformation and stress associated with earthquakes. The model consists of a rectangular dislocation (strike-slip fault) in a viscoelastic layer (lithosphere) lying over a viscoelastic half-space (asthenosphere). The first part of the paper contains an analysis of the time-dependent surface stresses. The model predicts that near the fault a significant fraction of the stress that was reduced during the earthquake may be recovered by viscoelastic softening of the lithosphere. By contrast, the strain shows very little change near the fault. The model also predicts that the stress changes associated with asthenospheric flow extend over a broader region than those associated with lithospheric relaxation even though the peak value is less. The second part of the paper studies the dependence of the displacements, strains, and stresses on fault parameters. Peak values of strain and stress drop increase with increasing fault height and decrease with fault depth. Under many circumstances postseismic strains and stresses show an increase with decreasing depth to the lithosphere-asthenosphere boundary. Values of the strain and stress at distant points from the fault increase with fault area but are relatively insensitive to fault depth.

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I. INTRODUCTION

This paper is devoted first to a consideration of the postseismic stresses associated with earthquakes using a viscoelastic model of the relaxation processes. It also considers the dependence of the displacements, strains, and stresses on fault parameters. The mathematical model consists of a vertical rectangular dislocation located in a viscoelastic layer which lies over a viscoelastic half-space with different rheological properties. Figure 1 shows the relevant features of this model which was used in previous work (Cohen, 1979b; herein referred to as Part 1. See also Cohen, 1979a) to calculate postseismic displacements and strains due to earthquakes along strike-slip faults. The upper layer, of thickness H, is meant to represent the lithosphere. Its deviatoric rheology is modeled by a standard viscoelastic solid; its dilatational rheology will generally be taken to be elastic (model DILE), although we have also considered the standard viscoelastic solid case. The half-space represents the asthenosphere for which only the deviatoric response needs to be considered. This we have taken to be a Maxwell substance with a response time much longer than that of the lithosphere. The dislocation which models the fault has length, 2L, is located at a mean depth, δ, has a height, Δh. The size of the dislocation at the time of the earthquake is U_o. Additional details about this model were discussed in Part 1 where we also presented the computed displacements and strains. These strains are used in the present paper to compute coseismic and postseismic stresses taking into account the assumed viscoelastic properties of the earth. The analysis and numerical results will indicate that the patterns of postseismic stress and strain may be considerably different. In particular the postseismic shear stress recovery near the fault following an earthquake may be large even when the strain change is small. Thus postseismic deformations may be due to both viscoelastic changes in rigidity and postseismic fault motion caused by stress recovery.

II. STRESS CONSIDERATIONS

The evaluation of the coseismic and postseismic stresses starts with a consideration of the stress, σ, – strain, ϵ, equation for a homogeneous isotropic elastic body, viz.,

$$\sigma_{ij} = 2\mu_1 \epsilon_{ij} + \left(k_1 - \frac{2}{3}\mu_1 \right) \Delta \, \delta_{ij} \tag{1}$$

where μ_1 and k_1 are the shear and bulk modulus, Δ is the dilatation ($\Delta = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$) and $\delta_{ij} = 1$ if $i = j$ or $\delta_{ij} = 0$ if $i \neq j$. To determine the surface stress for viscoelastic material we adopt the following procedure. First attention is focused on three times. The first of these is t_0, the time immediately after the earthquake; the second t_ℓ is a time long compared to the lithosphere viscoelastic relaxation time and short compared to the asthenosphere relaxation time; the third t_a is a time long compared to the asthenosphere relaxation time. The viscoelastic strains at each of these were calculated in Part I where we argued that the viscoelastic displacement equations can be obtained from a knowledge of the elastic displacement equations by replacing the elastic moduli by moduli appropriate to the viscoelastic case. Similarly if the viscoelastic strains are known then the correspondence principle (Flugge, 1967) and the viscoelastic constitutive equations indicate that the stresses may be computed from Equation (1) provided μ_1 and k_1 are replaced by the appropriate effective moduli. Specifically the constitutive equation for the deviatoric behavior of the lithosphere is

$$\sigma + \frac{\eta}{\mu_a + \mu_b}\dot{\sigma} = \frac{\mu_a \mu_b}{\mu_a + \mu_b}\epsilon + \frac{\eta \mu_a}{\mu_a + \mu_b}\dot{\epsilon} \tag{2}$$

At t_0 there is a sudden change in stress and strain so terms in $\dot{\sigma}$ and $\dot{\epsilon}$ are large compared to those in σ and ϵ. Thus

$$\Delta\sigma(t_0) = \mu_a \Delta\epsilon(t_0) \tag{3}$$

Thus the effective elastic shear modulus of the lithosphere at time t_0 is, from the correspondence principle,

$$\mu_1(t_0) = \frac{\mu_a}{2} \tag{4}$$

For times t_ℓ and t_a the terms in σ and ϵ dominate the terms involving the time derivatives so

$$\sigma(t_\ell \text{ or } t_a) = \frac{\mu_a}{1+\beta}\,\epsilon(t_\ell \text{ or } t_a) \tag{5}$$

where $\beta = \mu_a/\mu_b$. In this case

$$\mu_1(t_\ell \text{ or } t_a) = \frac{1}{2}\,\frac{\mu_a}{1+\beta} \tag{6}$$

The dilational stress–strain law of the lithosphere is

$$\sigma = k\epsilon \tag{7}$$

So from the correspondence principle (and at all times)

$$k_1 = \frac{k}{3} \tag{8}$$

With these results we find

$$\frac{\sigma_{ij}(t_0)}{\mu_a} = \epsilon_{ij}(t_0) + \frac{1}{3}\left(\frac{k}{\mu_a} - 1\right)\Delta(t_0)\delta_{ij} \tag{9a}$$

$$\frac{\sigma_{ij}(t_\ell)}{\mu_a} = \frac{\epsilon_{ij}(t_\ell)}{1+\beta} + \frac{1}{3}\left(\frac{k}{\mu_a} - \frac{1}{1+\beta}\right)\Delta(t_\ell)\delta_{ij} \tag{9b}$$

$$\frac{\sigma_{ij}(t_a)}{\mu_a} = \frac{\epsilon_{ij}(t_a)}{1+\beta} + \frac{1}{3}\left(\frac{k}{\mu_a} - \frac{1}{1+\beta}\right)\Delta(t_a)\delta_{ij} \tag{9c}$$

In our numerical computations we have taken $\dfrac{k_1(t_0)}{\mu_1(t_0)} = \dfrac{2}{3}\dfrac{k}{\mu_a} = \dfrac{5}{3}$ corresponding to a Poisson's ratio of 0.25 and have assumed $\beta = 3/2$. The evaluation of the surface shear stress, σ_{12}, is obtained directly from Equation (1) using the shear strains computed in Part 1 with the assumption that the instantaneous shear moduli of the lithosphere and asthenosphere are equal. The evaluation of the surface normal stresses, σ_{11} and σ_{22}, requires knowledge of the dilatation $\Delta = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$. The horizontal strains, ϵ_{11} and ϵ_{22}, were evaluated in Part 1 under the condition that at the earth's surface $\sigma_{33} = 0$. Inserting this condition in Equation (9) permits evaluation of ϵ_{33} (and hence Δ, σ_{11} and σ_{22}) in terms of ϵ_{11} and ϵ_{22}. Explicitly

$$\epsilon_{33}(t_0) = -\frac{\left(\frac{k}{\mu_a} - 1\right)}{\left(\frac{k}{\mu_a} + 2\right)} \left[\epsilon_{11}(t_0) + \epsilon_{22}(t_0)\right] \tag{10a}$$

$$\epsilon_{33}(t_\ell \text{ or } t_a) = -\left[\frac{\frac{k}{\mu_a}(1+\beta) - 1}{\frac{k}{\mu_a}(1+\beta) + 2}\right] \left[\epsilon_{11}(t_\ell \text{ or } t_a) + \epsilon_{22}(t_\ell \text{ or } t_a)\right] \tag{10b}$$

III. STRESS COMPUTATIONS

The earthquake related stresses have been evaluated at the three times t_0, t_ℓ, and t_a. The results shown in Figures 2–4 are for the same conditions used in computing the displacements and strains in Part I namely $L = H = 4\Delta h = 8\delta$. The shear stresses $\sigma_{12}(t_0)$, $\sigma_{12}(t_\ell)$, and $\sigma_{12}(t_a)$ are proportional to the corresponding shear strains with the proportionality constant being μ_a at t_0 and $\frac{\mu_a}{1+\beta}$ at t_ℓ and t_a. The contribution to the stress due to lithospheric relaxation can be analyzed by defining a shear modulus difference by

$$\Delta\mu = \frac{\mu_a}{1+\beta} - \mu_a = -\frac{\beta\mu_a}{1+\beta} \tag{11}$$

Then

$$\Delta\sigma_{12} = \sigma_{12}(t_\ell) - \sigma_{12}(t_0) = \frac{\mu_a}{1+\beta}\Delta\epsilon_{12} + \Delta\mu\epsilon_{12}(t_0) \tag{12}$$

where

$$\Delta\epsilon_{12} = \epsilon_{12}(t_\ell) - \epsilon_{12}(t_0) \tag{13}$$

The change in the shear stress due to lithospheric relaxation is composed of two components. One contains the change in strain, the other the change in rigidity. Of course these components are coupled to one another since the strain changes are also determined by the changes in lithospheric rigidity. It is interesting to compare the stress changes with the corresponding strain changes. Such a comparison is shown as a function of distance from the fault along the y axis in Figure 5. At many points, particularly near the fault, $\Delta\epsilon_{12}$ is small and $\Delta\sigma_{12}$ is dominated by $\Delta\mu$. At the fault, for example, $\left|\frac{\Delta\epsilon_{12}}{\epsilon_{12}(t_0)}\right| \lesssim 10^{-2}$ but $\left|\frac{\Delta\sigma_{12}}{\sigma_{12}(t_0)}\right| \sim 0.6$. Thus there is a significant postseismic stress recovery even though the strain change is small. Clocely related arguments concerning stress recovery following an earthquake have been advanced by Dieterich (1972), Cohen (1978), and Yamashita (1979). As for the changes occurring as the asthenosphere relaxes, they are proportional to the strain changes since

$$\sigma_{12}(t_a) - \sigma_{12}(t_\ell) = \frac{\mu_a}{1 + \beta} [\epsilon_{12}(t_a) - \epsilon_{12}(t_\ell)] \tag{14}$$

When these results are combined with those of Part 1, the following picture of postseismic phenomena emerges. In the short term and near the fault the direct postseismic deformations due to rigidity changes are small. Since, however, the stress recovery may be significant, postseismic deformation may be due to aseismic afterslip on the fault. The mechanisms for such afterslip may include stress recovery along with time dependent friction (Dietrich, 1972), space dependent friction (Cohen, 1978), fatigue failure (Scholz, 1972), etc. Far away from the fault the effects of afterslip are small unless the source is very deep. Thus on a relative basis distant postseismic deformations seem to be more attributable to the softening of the lithosphere rigidity. As for the long term postseismic deformations the model attributes them to the flow of the asthenosphere. The stresses extend over a broader zone than they do in the case of the lithosphere relaxation. The relative senses of the strains and stresses due to the instantaneous response, the lithospheric relaxation, and the asthenospheric flow may differ from one another and they vary with the source parameters and observation point.

IV. DISPLACEMENT, STRAIN, AND STRESS VARIATIONS WITH FAULT PARAMETERS

The calculated values of the viscoelastic displacements, strains, and stresses are dependent on the choice of model parameters that are introduced into the numerical evaluations. These parameters can be grouped into two groups. The first group is comprised of the material parameters, namely the elastic moduli and viscosities. The second group is composed of geometric factors including the fault dislocation, U_0, the lithospheric thickness, H, the coordinates of the observation point under consideration, and the parameters defining the fault: the length, 2L, the depth, δ, and the height Δh. In this section we examine some aspects of the dependence of the deformations and stresses on the geometric factors. To restrict the discussion to a reasonable length it is convenient to examine how the results vary with distance from the fault along the y axis. The non-vanishing quantities along this axis are the parallel displacements, u(y,t) the shear strains, $\epsilon_{12}(y,t)$, and the shear stresses, $\sigma_{12}(y,t)$. Specifically, we consider the initial values: $u(t_0)$, $\epsilon_{12}(t_0)$, $\sigma_{12}(t_0)$, the differences between quantities at times t_ℓ and t_0: $u(t_\ell) - u(t_0)$, $\epsilon_{12}(t_\ell) - \epsilon_{12}(t_0)$, $\sigma_{12}(t_\ell) - \sigma_{12}(t_0)$, and the differences between quantities at times, t_a and t_ℓ: $u(t_a) - u(t_\ell)$, $\epsilon_{12}(t_a) - \epsilon_{12}(t_\ell)$, $\sigma_{12}(t_a) - \sigma_{12}(t_\ell)$. As before, it is convenient to normalize the results to a unit value of fault slip leaving as independent parameters, δ, Δh, H and L. The following paragraphs summarize general tendencies we have noted for the dependence of the peak values and distant values of deformation and stress on the fault parameters. Our remarks are based on calculations with parameters in the ranges $5 \leqslant \delta \leqslant 15$, $10 \leqslant \Delta h \leqslant 30$, $20 \leqslant H \leqslant 100$, $50 \leqslant L \leqslant 250$ where all numbers are in kilometers. It should be noted that in some cases there are exceptions to the general tendencies we discuss.

For ruptures that penetrate the surface, the peak values (in an absolute value sense) of the following quantities occur at the fault: $u(t_0)$, $\epsilon_{12}(t_0)$, $\sigma_{12}(t_0)$, $\epsilon_{12}(t_\ell) - \epsilon_{12}(t_0)$, $\sigma_{12}(t_\ell) - \sigma_{12}(t_0)$, $\epsilon_{12}(t_a) - \epsilon_{12}(t_\ell)$, $\sigma_{12}(t_a) - \sigma_{12}(t_\ell)$. The peak values of $u(t_\ell) - u(t_0)$ and $u(t_a) - u(t_\ell)$ occur at some distance from the fault. For ruptures that do not penetrate the surface some caution must be exercised. For example, in this case, $\sigma_{12}(t_0)$ is positive, indicating a stress rise, at the surface point above the fault. We take as a maximum value of the surface stress drop the most negative value of $\sigma_{12}(t_0)$ which occurs at some value $y > 0$. Similar remarks apply to some of the other quantities we discuss below. Variations in δ, Δh, L, and H have the following general effects:

Depth, δ

The quantities u, ϵ_{12} and σ_{12} at time t_0 and the differences between these quantities at times t_ℓ and t_0 decrease with increasing δ because deeper sources in the lithosphere produce smaller peak surface effects. The differences between these quantities at times t_a and t_ℓ increase with δ due to the reduced distance to the asthenosphere with the attendant greater lithosphere-asthenosphere interaction.

Fault Height, Δh

All the peak displacements, strains, and stresses increase with Δh due to the increased source size.

Length, L

For ruptures that break the surface, the peak displacement occurs at the fault and is a boundary condition independent of L. For buried faults, the peak coseismic displacement tends to decrease somewhat with length. In addition the coseismic strain and stress either decrease with or are independent of fault length. The differences between the deformation and stress quantities at times t_a and t_ℓ increase with L, presumably due to the greater lithosphere-asthenosphere coupling with the larger fault length. The quantity $u(t_\ell)-u(t_0)$ also increases with increasing L. By contrast $\epsilon_{12}(t_\ell)-\epsilon_{12}(t_0)$ decreases with increasing L. So does $\sigma_{12}(t_\ell)-\sigma_{12}(t_0)$ but the dependence is weak in this latter case.

Lithospheric Thickness, H

The values of displacement, strain, and stress at time t_0 are independent of H due to the assumption $\mu_1(t_0) = \mu_2(t_0)$. This assumption leads to the condition that the interface between the lithosphere and asthenosphere is not sensed at the time of the earthquake and the earth responds as an elastic half-space. By contrast, peak values of all differences between displacements, strains, and stresses at times t_a and t_ℓ decrease with increasing H due to the increased distance between the fault and the lithosphere-asthenosphere boundary. The quantities $u(t_\ell) - u(t_0)$ and $\epsilon_{12}(t_\ell) - \epsilon_{12}(t_0)$ also decrease with increasing H. By contrast $\sigma_{12}(t_\ell) - \sigma_{12}(t_0)$ is affected very little by changes in H.

In a manner mimicing the preceding discussion we can discuss the dependence of the displacements, strains, and stresses of the fault parameters at distant points from the fault. To do this, we must first note that at distant points ($y > L, H$) there are several phase changes in ϵ_{12} and σ_{12} compared to their peak values. Specifically $\epsilon_{12}(t_0)$ and $\sigma_{12}(t_0)$ are positive and $\sigma_{12}(t_\ell) - \sigma_{12}(t_0)$ is generally negative. In addition to our general comments below, we note two very simple relationships that follow from the displacement and strain equations for y large:

$$u(t_0) = \frac{2L\Delta h}{4\pi y^2} = \frac{A}{4\pi y^2} \tag{15}$$

$$\epsilon_{12}(t_0) = \frac{2L\Delta h}{4\pi y^3} = \frac{A}{4\pi y^3} \tag{16}$$

where A is the area of the fault.

Depth, δ

To a fair approximation u, ϵ_{12}, and σ_{12} are independent of depth unless δ approaches H. Since at large distances from the fault the exact depth of the source matters little so long as the depth is shallow. The most sensitive dependence of the distant deformation variables on δ is that of $u(t_a) - u(t_\ell)$.

Fault Height, Δh

Increasing the fault height increases u, ϵ_{12}, σ_{12} and the corresponding time differences.

Fault Length, L

Increasing the fault length increases u, ϵ_{12}, σ_{12} and the corresponding time differences.

Lithospheric Thickness, H

Based on the previously discussed assumption that $\mu_1(t_0)=\mu_2(t_0)$ the lithospheric thickness does not effect $u(t_0)$, $\epsilon_{12}(t_0)$, or $\sigma_{12}(t_0)$. The quantities $u(t_\ell) - u(t_0)$, $\epsilon_{12}(t_\ell) - \epsilon_{12}(t_0)$, $u(t_a) - u(t_\ell)$, $\epsilon_{12}(t_a) - \epsilon_{12}(t_\ell)$, and $\sigma_{12}(t_a) - \sigma_{12}(t_\ell)$ generally decrease with increasing H as do the corresponding peak values. The quantity $\sigma_{12}(t_\ell) - \sigma_{12}(t_0)$ either decreases with increasing H or shows little change depending on whether the change in μ, or in ϵ_{12} is controlling the change in σ_{12}.

V. SUMMARY

This paper has considered a model of the surface stresses associated with viscoelastic relaxation of the lithosphere and asthenosphere following an earthquake. Near the fault there is a postseismic stress rise whose magnitude can be a significant fraction of the stress reduction due to the earthquake. This stress recovery is due to a change in the rigidity of the lithosphere and is accompanied by only small changes in strain. By contrast the stress changes associated with asthenospheric flow mimic the strain changes. The amplitude of these changes is small compared to the peak values of the stress changes associated with lithospheric relaxation, but the spatial extent of the changes is broader.

The paper has also considered the dependence of coseismic and postseismic surface displacements, strains, and stresses on fault depth, height, and length (strike–slip faults) and on lithospheric thickness. Among the findings are that the peak values of the deformations and stresses increase with increasing fault height and decrease with fault depth. Distant deformations and stresses increase with fault area but are relatively insensitive to the source depth so long as the depth remains sufficiently shallow. Values of the postseismic deformations and stresses associated with asthenospheric flow are increased by reduced values of lithospheric thickness.

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FIGURE CAPTIONS

Figure 1. Viscoelastic Model for Deformation and Stress Due to an Earthquake Along a Strike-Slip Fault

Figure 2. Shear Stress, $\sigma_{12}(t)$. Fault extends along x axis from -L to +L. Tick mark spacing = 2L.

Figure 3. Normal Stress, $\sigma_{11}(t)$

Figure 4. Normal Stress, $\sigma_{22}(t)$

Figure 5a. Shear Stress, $\sigma_{12}(t)$, Versus Distance From Fault, y

5b. Shear Strain, $\epsilon_{12}(t)$, Versus Distance From Fault, y

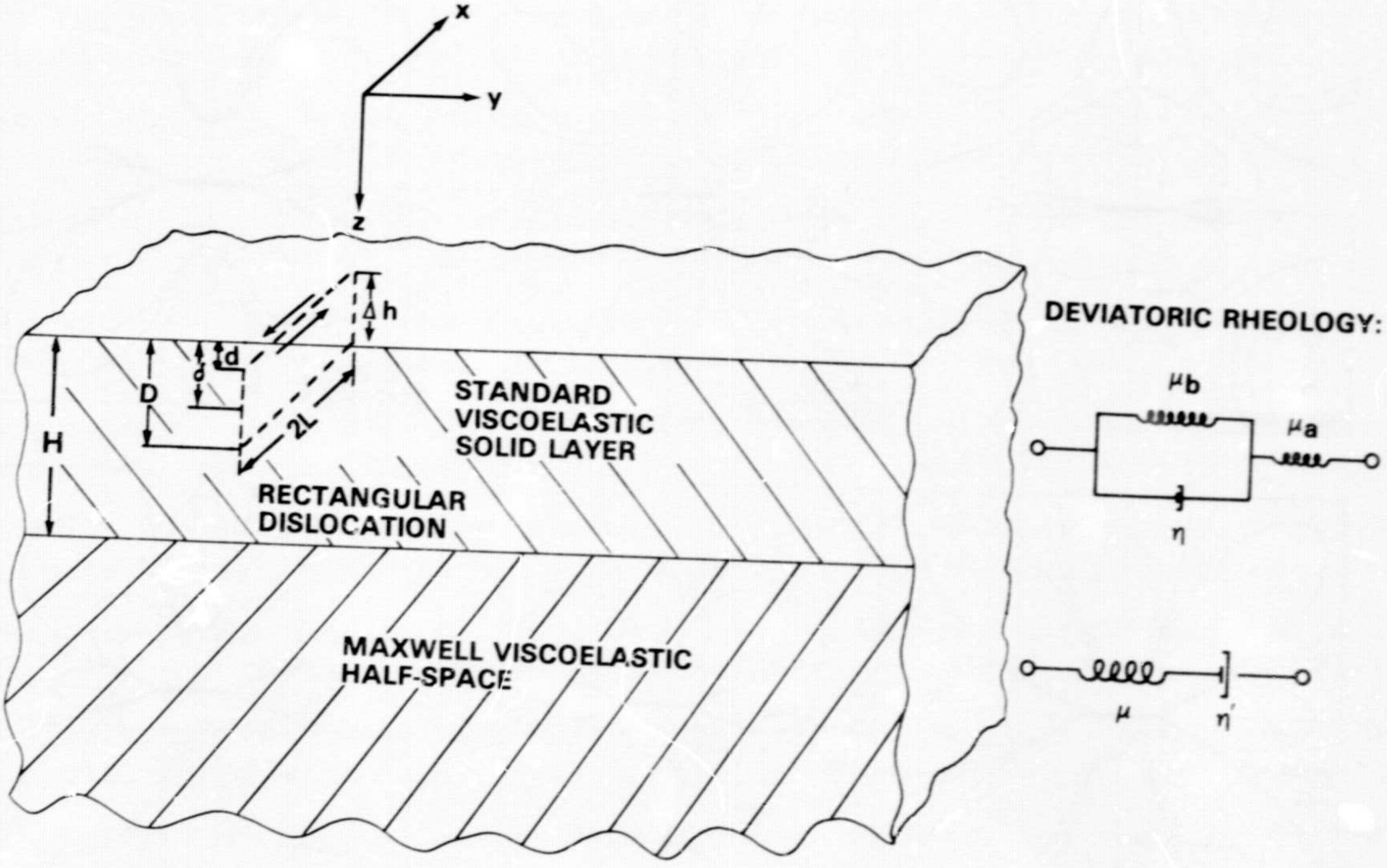

Figure 1. Viscoelastic Model for Deformation and Stress Due to an Earthquake Along a Strike-Slip Fault

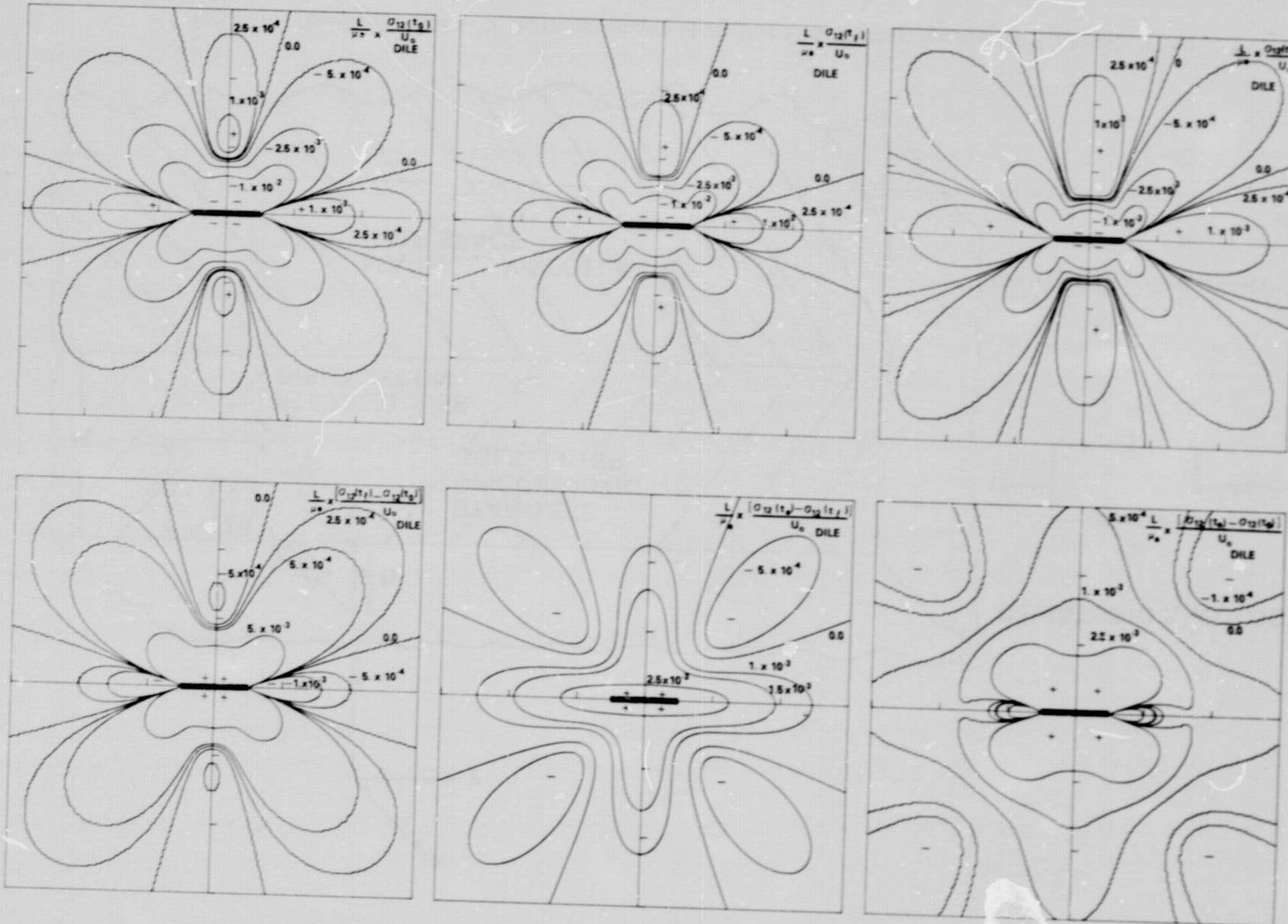

Figure 2. Shear Stress, $\sigma_{12}(t)$. Fault extends along x axis from –L to +L. Tick mark s[illegible] = 2L.

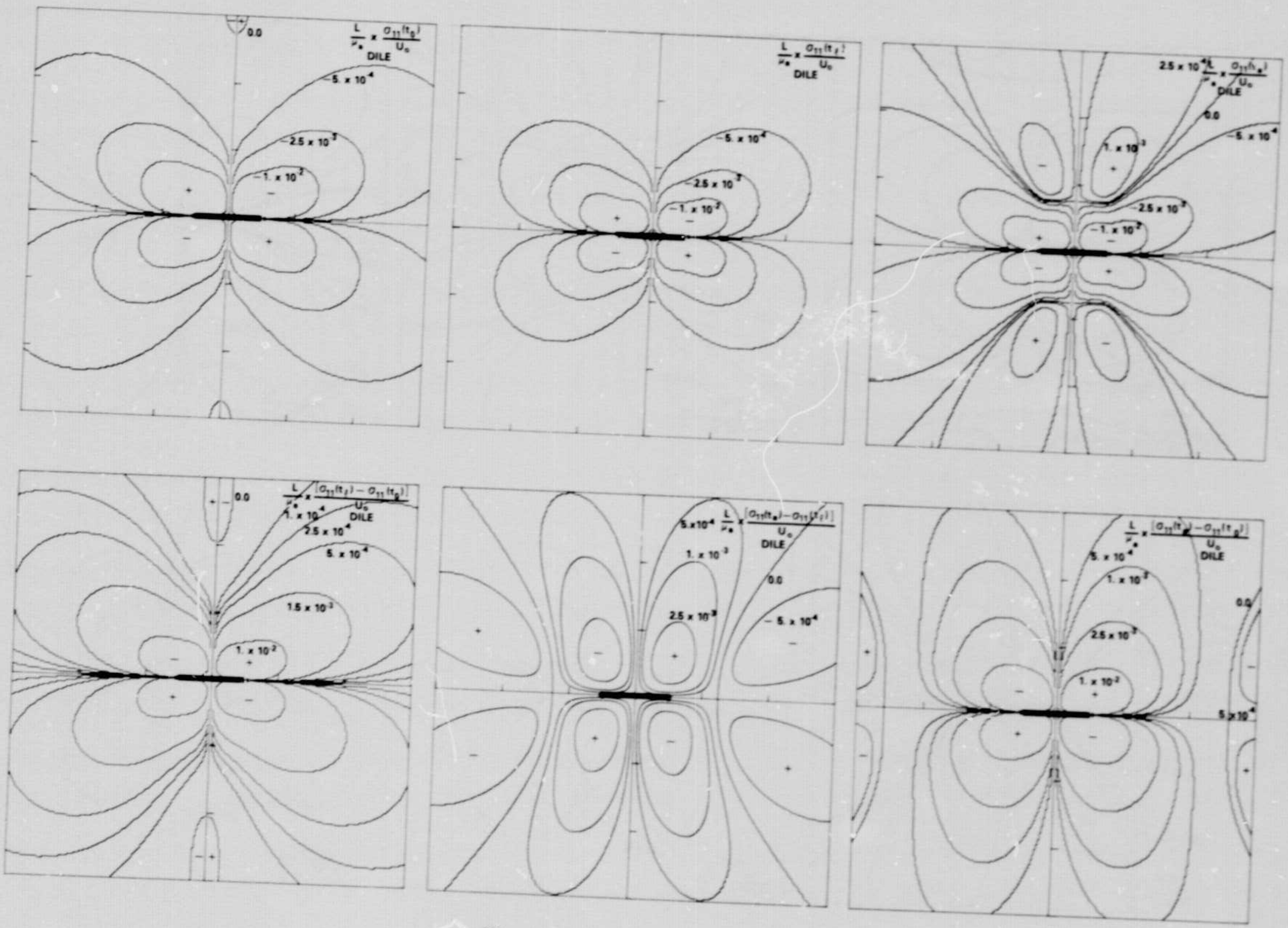

Figure 3. Normal Stress, σ_{11} (t)

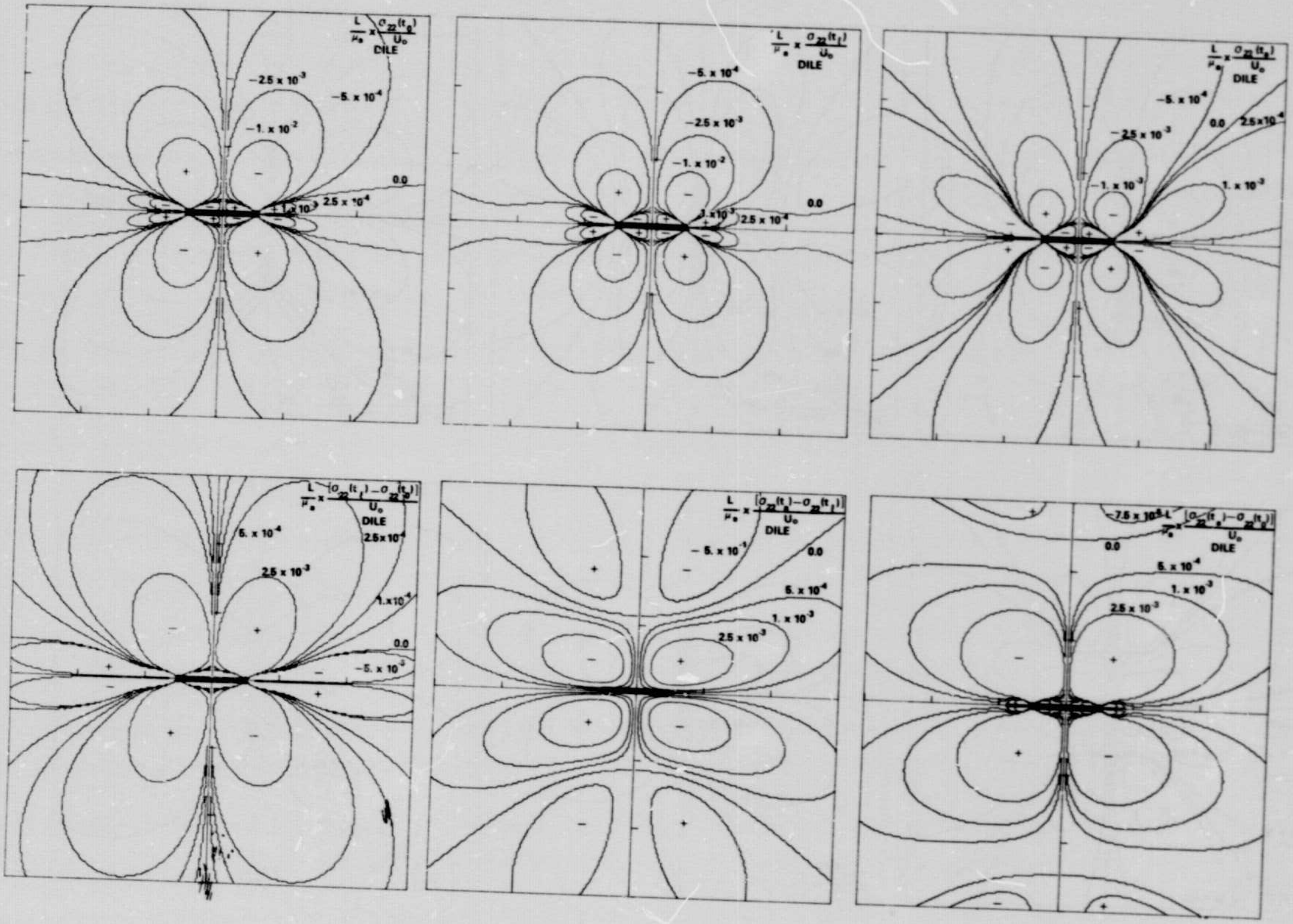

Figure 4. Normal Stress, $\sigma_{22}(t)$

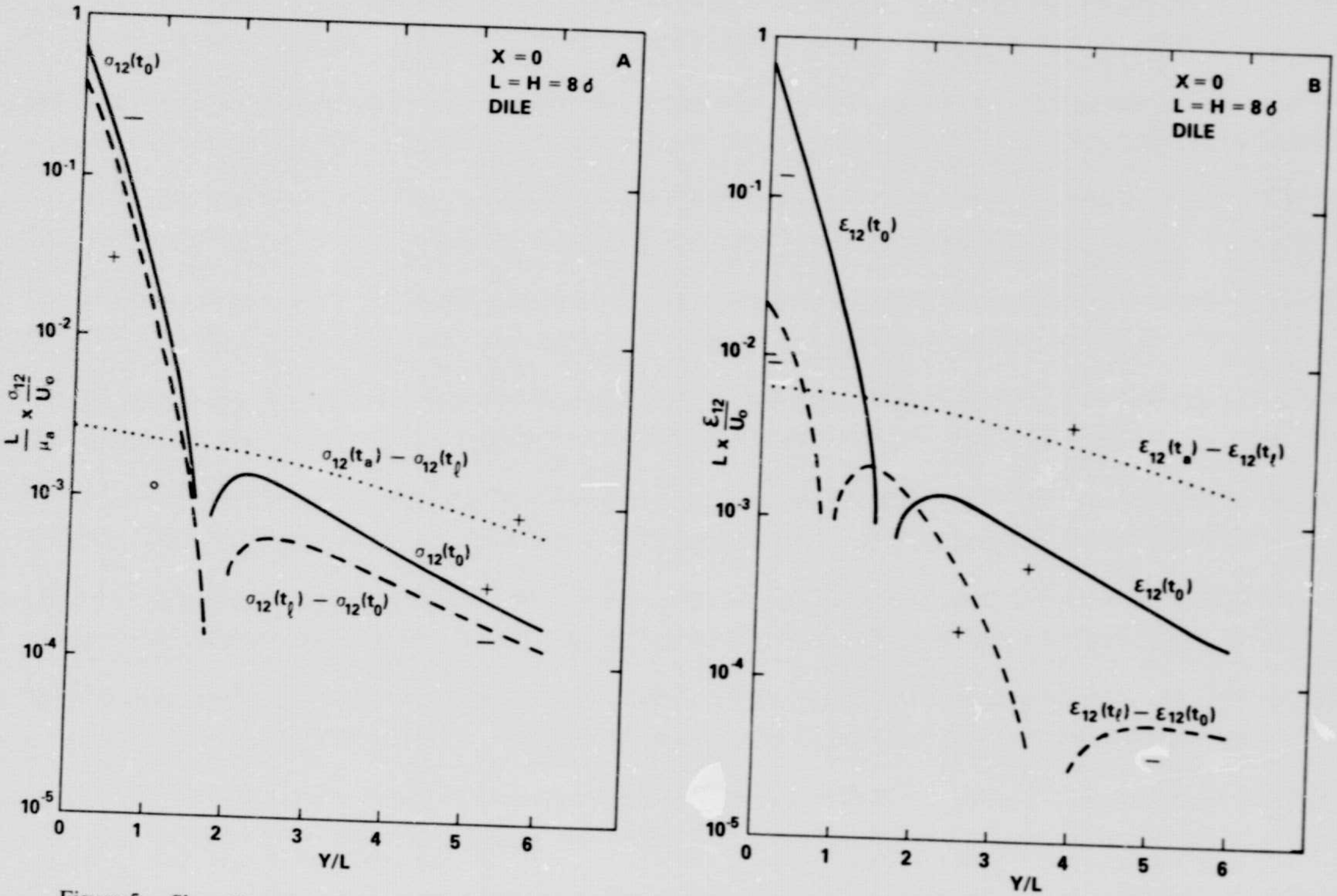

Figure 5a. Shear Stress, $\sigma_{12}(t)$, Versus Distance From Fault, y; 5b. Shear Strain $\epsilon_{12}(t)$, Versus Distance From Fault, y