THE VALUE OF VOLUME AND GROWTH MEASUREMENTS IN TIMBER SALES MANAGEMENT OF THE NATIONAL FORESTS

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ABSTRACT

This paper summarizes work performed in the estimation of gross social value of timber volume and growth rate information used in making regional harvest decisions in the National Forest System. A model was developed to permit parametric analysis, thus providing measures of gross benefit of an information system as a function of system capability. The problem is formulated as one of finding optimal inventory holding patterns. Public timber management differs from other inventory holding problems in that the inventory, itself, generates value over time in providing recreational, aesthetic and environmental goods. "Non timber" demand (value accruing from unmarketed forest goods) estimates are inferred from past Forest Service harvest and sales levels. The solution requires a description of the harvest rates which maintain the optimum inventory level. Using NASA-supported estimates of LANDSAT capabilities, gross benefits of the LANDSAT systems are estimated by comparison with Forest Service information gathering models. Gross annual benefits are estimated to be $5.9 million for the MSS system and $7.2 million for the TM system.

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1. INTRODUCTION

The Forest Service of the USDA maintains an information system which delivers forest resource information for use in determining socially optimal harvest and investment levels. But the actual accuracy specifications of this information system were set without economic analysis of the value of these information products. The advent of remote sensing systems, and the possibility of significantly reduced costs of information gathering, throws open the question as to how much information should be bought to assist public forest management. In particular, does the decision maker have a negatively sloped demand curve such that, given cheaper information, he will demand more of it? The answer to this question is important to a rational approach to determining optimal Forest Service information system specifications, but it was used in the study reported here to estimate the economic benefits of a LANDSAT system as applied to Forest Service timber harvest and sales operations.

This paper discusses an analytic approach to the value of information used in making selected, important decisions in U.S. Forest Service management of the National Forest System. The study reported here (more details are supplied in the source report, Reference 1) was conducted for the purpose of estimating the social value of selected types of information when used in forest management. The work was funded by the National Aeronautics and Space Administration (NASA) in support of their LANDSAT program and is intended for use in guiding the selection of socially optimal information systems designed by NASA. The work in this report represents only a fraction of the economic work developed in support of the ERS program which describes the gross social value of information systems as a function of their capabilities and is used in combination with efforts, such as General Electric's TOSS study describing system cost as a function of capability, in order to locate the optimum ERS system attributes.

Due to time and budget constraints, it was not possible to evaluate the use of information in all forests, by all managers, for all decisions. Rather, some best subset of United States forest management was selected which would permit a description of the major benefits while still satisfying the problem constraints. The management of the National Forest System (NFS) lands by the United States Forest Service (USFS) was selected for several reasons:
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1. It represents a very large, unified management system.

2. The necessary data--sales, values, growth rates, current information system characteristics--are publicly available.

3. It is reasonable to expect that the current information about NFS lands is less than that about the more intensively managed, private industry lands.

The decisions modeled are timber harvest/sales decisions at the "regional" level. The regions chosen correspond to those designated by the Forest Service in their 1973 Timber Outlook study, with the exception that the Pacific Coast regions are further subdivided to separate "eastside" timber from "westside" timber, i.e., timber which gets shipped inland versus that which gets shipped to the coastal area. The breakdown of these regions is shown in Figure 1. From all appearances, it seems that the Forest Service's own information system is designed to produce information for this level of management. Intraregional decisions involving less aggregated areas were recognized as important, but were not studied. Corresponding benefits, thus, were not estimated; however, it is believed that benefits from information used in the less aggregated decisions are smaller in nature. Other decisions--investments in planting, thinning, fertilization and, notably, protection--will also find value from the use of remotely sensed information. However, these too were not studied.

Hardwood timber was not modeled, nor were holdings in the northeastern United States, due to their relatively small contribution to the entire NFS output.

Finally, the only types of information studied were timber volume inventory and growth measurements. It was originally intended that the detection of forest stress also be included, but the lack of data and the extreme complexity of the subject prohibited this.

The basic problem has been set up as one of optimal inventory control. The state variable, not surprisingly, is the standing inventory volume of softwood trees in the National Forest System, region by region. It is changed, period by period, by additions of net growth (growth minus mortality) and deletions for timber harvests and sales. The regional timber harvest/sales decision is
Figure 1  Timber Demand Regions and Timber Production Regions used in this study
the only decision variable given to the NFS manager in this model. The problem has a peculiarity in the structure of the value function in that value is derived either from timber harvested or from standing inventory, per se. This latter value, which we are calling non-timber value, comes from the desirable attributes of standing forests when used for recreational, wildlife habitat, aesthetics and soil and water conservation and from stated national priorities of maintaining forests for future generations.

Due to the intentionally decentralized nature of Forest Service decision making, most operations are decided upon and performed by individuals directly in contact with the forest resource. For these decisions, then, information gathering is an automatic process. However, decisions about aggregate levels of harvests must be made by increasingly centralized authorities who can take into account the broader regional and national consequences of the decisions. These decision makers are dependent upon information systems in order to make rational choices. Much of this information comes in the form of written and oral intra-service reports; for example, about the productive capacities of the different districts, forests and regions. The Forest Service uses another method of providing accurate resource information to the more centralized decision makers. The National Forest Inplace Inventory is performed on NFS lands while another survey is performed on non-NFS lands by the Resources Evaluation (formerly Forest Survey) Branch of the Forest Service. Methodologies for these surveys are outlined in detail in the Forest Survey Handbook (Forest Service Handbook #4813). More about the Forest Service inventory system is discussed in Section 4.2.

2. MODELING THE SYSTEM

For this study, we have formulated a simple, very aggregate level analysis of National Forest management. In this section a nonstochastic model is discussed and its equations derived; the consequences of uncertainty are studied in Section 5.

The State Variables and State Transformation

The (relevant) state of nature at any point in time is considered to be exhaustively defined by the total regional inventory of standing timber; i.e., this state variable, \( x_t \), measured in cubic feet of timber, represents the state of the system.
The state is changed over time by controlled and uncontrolled processes as described in the state transformation function. Thus,

$$x_{t+1} = T(x_t, y_t)$$

where $y_t$ represents the decisions made during the $t^{th}$ time period and is called the decision or control variable. In our nonstochastic system, $x_t$ changes according to net timber growth and timber harvest, the latter being the control variable. Although several different decision variables are available to forest managers (e.g., stand improvement, fertilization, pest control reforestation), only timber harvest is used in this analysis; this is felt to be the most important decision at the regional level.

At the level of aggregation with which we are dealing, net growth is best described as a percentage of total volume, such that our state transformation function has the form:

$$x_{t+1} = mx_t - y_t$$  \hspace{1cm} (2.1)

In this study, time periods are one year in length, roughly corresponding with the annual budgetary process by which the Forest Service receives funds for its activities and thus can schedule its operations, such as timber harvest and sales. Ours is an assumed steady-state expression of the state transformation function: the parameter $m$ is not time dependent.

In our model, we have assumed equivalence between timber sales and harvests. In the real world, the Forest Service has direct control over sales only, but in order to avoid expressing complex (and probably arbitrary) relationships between sales and harvests, we have assumed their equivalence. Alternatively, we have assumed that timber harvest is the direct Forest Service control variable.

The Incremental Value Function

The total social value (using the Marshallian concept of the area under the demand curve) obtained from our forest system derives from two sources: Timber value results from timber harvests, while nontimber value comes from the standing inventory of
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trees. Our incremental value function, thus, has two arguments: \( v(x_t, y_t) \).

In this study, we have estimated the demand for these two commodities \( x \) and \( y \), assuming linear demand curves. There are three reasons for the assumption of linearity:

1. The present state of econometric art is rarely able to provide details concerning a more complex demand function. The parameters of a linear demand curve can be directly obtained from an estimate of price elasticity and equilibrium price and quantity.

2. Our derivation of the value of information will only be concerned with small portions of the demand curve around the existing equilibrium. Thus, linear approximations of the curve about that point are very adequate.

3. This formulation leads to a quadratic value function, thus permitting a very simple optimization to be performed.

Our incremental value function, then, takes on the form:

\[
v(x_t, y_t) = ax_t^2 + bx_t + cy_t^2 + dy_t.\tag{2.2}
\]

This obviously represents a simplification of the real world and it is recognized as such. There are certainly many features which the Forest Service considers—the age of the timber, perhaps—but we consider this abstraction to be the best and most efficient expression at this level of aggregation. Additionally, we have assumed that the parameters, both of the transformation and incremental value functions, are not time related. This may be an important point if society keeps demanding more timber and non-timber forest products relative to other economic goods, as indicated by the recent past. Estimates of the social value of information would then be underestimated.

The Optimization

For the purposes of the insight and efficiency of calculation, we have chosen to express the problem in a dynamic programming formulation. We do not attempt to study the entire dynamic process of timber harvest decisions as a unit; rather we analyze
Substituting Eq. 2.1 into 2.3, we obtain
\[ V_t(x_t) = \max_{y_t} \{ v(x_t, y_t) + V_{t+1} [mx_t - y_t] \}. \]  
(2.5)

Inserting 2.4 into 2.5 and dropping the "t" subscript:
\[ V(x) = \max_y \{ v(x,y) + r [q(mx - y)^2 + 1(mx - y) + k] \} \]  
(2.6)

Where \( r \) is the one period discount factor: \( r = \frac{1}{1+i} \); \( i \) is the annual discount rate. Expanding 2.6 in terms of \( y \) and inserting 2.2:
\[ V(x) = \max_y \left( cy^2 + rqy^2 + dy - 2rq mx y - rly + ax^2 + bx + rqm x^2 + rlm x + rk \right). \]  
(2.7)

We have expressed \( V(x) \) in terms of a quadratic in \( y \). Using the algebraic maximum, we find that the optimal \( y \) is
\[ y = \frac{d - 2rqm x - rl}{2(c + rq)} \]  
(2.8)

and
\[ V(x) = ax^2 + bx + rqm x^2 + rlm x + rk - \frac{[d - 2rqm x - rl]^2}{2(c + rq)} \]  
(2.9)

Let us use the following
\[ s = - rqm \quad t = 1/2 (d-rl) \quad h = \frac{1}{c + rq} \]
Thus
\[ V(x) = ax^2 + bx + rqm x^2 + rlm x + rk - h(sx + t)^2. \]  
(2.10)
them in sequence, one at a time, as they occur in nature. The application of a deceptively simple principle permits this formulation; the optimality principle is:

An optimal policy has the property that, whatever the initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.4

Elsewhere, the optimality principal has been appropriately described:

Although the principle of optimality seems both obvious and simple, it can more appropriately be described as powerful, subtle and elusive.5

The interested reader is referred to the two texts referenced for further explication of dynamic programming.

We have already introduced the incremental value function. There is another value function of importance here: the maximum (present value) possible return, \( V_t(x_t) \), over the future of the process from the current state, \( x_t \). The optimality principle now leads to the following quantitative statement:

\[
V_t(x_t) = \max_{y_t} \{ v(x_t, y_t) + V_{t+1}[T(x_t, y_t)] \} \quad (2.3)
\]

This is a very important step, for we have obtained our expression for the total value function (discounted present value of all future harvest and nontimber value flows, optimal decisions assumed) expressed only in terms of \( x_t \) and \( y_t \). Since \( y_t \) is assumed optimally chosen in the total value function, it will be possible to express \( V_t(\cdot) \) without using \( y_t \) as an argument. Since the incremental value function, \( v(x_t, y_t) \), is quadratic in \( x \), the total value function, \( V_t(x_t) \) is quadratic as well and can be expressed as:

\[
V_t(x_t) = q x_t^2 + 1 x_t + k. \quad (2.4)
\]

The rest of this section will be devoted to deriving expressions for \( q, l \) and \( k \). Those who are not interested in this derivation may move on to the next section without loss of continuity.
Collecting in terms of $x$

$$V(x) = ax^2 + rqm^2 - hsm^2 + bx + 2rqmx + rmlx - 2hstx + rk - ht^2.$$  \hspace{1cm} (2.11)

We now have expressions for the parameters in the quadratic of Eq. 2.4, where

\begin{align*}
q &= a + rqm^2 - hsm^2 \\
1 &= b + 2rqmz + rml - 2hst \\
k &= rk - ht^2.
\end{align*}

Substituting back for $h$, $s$ and $t$:

\begin{align*}
q &= a + rqm^2 - \frac{(-rqm)^2}{c + rq} \\
cq + rq^2 &= ac + arq + crqm^2 + r^22^2m^2 - r^22^2m^2 \\
rq^2 + (c - ar -crm^2)q - ac &= 0 \\
q &= \frac{ar + crm^2 - c \pm \sqrt{(c - ar -crm^2)^2 + 4rac}}{2r} \\
l &= b + rml - hs (d-rl) = b + (rm + hsr) l - hsd \\
\frac{b - hsd}{1 - rm - hsr} &= \frac{b + rqmd}{c+rq} \\
1 - rm + \frac{r^2qm}{c+rq} &= \frac{(d - rl)^2}{4(c+rq)} \\
k &= rk - \frac{(d - rl)^2}{4(c+rq)}(r-1)
\end{align*}
We are now able to specify the total value of a forest system given an initial state volume and the other parameters and assuming optimal decision making. This formulation, with some changes to account for stochastic processes and imperfect information, will be used in Section 5 to derive an expression for the value of information.

3. FOREST VALUES

The Value of Timber Outputs

The NFS system has been broken up into three different demand regions for derivation of timber demand elasticities (Fig. 1). The West Coast, Western Pine and Southern Pine regions were selected for the same reasons used by Adams.6 Exports are seen as a significant influence to the West Coast market. The Southern Pine market provides considerable supplies to the populous northeast. The Western Pine region is somewhat geographically isolated, separated by the coastal mountains from the West Coast market and by the Great Plains from the East.

Two-stage least squares techniques were used to estimate the supply of NFS stumpage and the demand for NFS stumpage equations. NFS supply was theorized as responsive to prices, but, due to the lack of rapid adaptability of the supply structure, we propose that supply is responsive primarily to prices the previous year and before. This hypothesis was subjected to significance tests, and was, in general, borne out, with some exceptions. The following prices were tested for significance in contributing to regional NFS supply: regional stumpage prices, regional lumber prices and regional plywood prices. Additionally, last year's NFS supply and private stumpage supply were tested. For the West Coast Market, exports were also tested. Price was considered more responsive and was tested against the same variables, with the inclusion of current year levels and current year NFS supply.

Our estimate for the Southern Pine Region and the West Coast region are adequate; the regressions are significant at the 90 and 97.5 percent confidence levels, respectively. However, the regression for the Western Pine region is clearly not significant and the results based upon it should be reworked if a superior estimate becomes available. The following demand elasticity estimates result
Southern Pine: -7.790
West Coast : -5.228
Western Pine : -1.0932.

All the coefficients estimated have signs consistent with what economic theory would lead one to expect and there were no surprises in the magnitudes. The elasticities are in general agreement with demand elasticities previously estimated for NFS timber for the entire country. They also are explainable in relation to each other: Southern Pine elasticity is very high due to the small share of the market held there by the National Forests, while we see a relatively inelastic demand in the Western Pine region because the Forest Service is the dominant supplier in that region. On the West Coast, Forest Service timber represents a relatively large share of the market, but its influence is dampened due to the always available export market.

The Value of Nontimber Outputs

Our analysis of Section 2 requires the formulation of a value function for nontimber forest outputs. In this section, we describe the nature of these outputs and the way in which their value can be estimated. Particular attention is paid to the derivation of the important parameter, marginal social value or social price.

As discussed earlier, the Forest Service is mandated to recognize outputs from National Forests other than timber, managing all the resources "in the combination that will best meet the needs of the American people..." Specifically, the Forest Service must consider forage, wildlife, recreation and aesthetics, soil and water conservation, and minerals. In the Multiple Use--Sustained Yield Act, management for the purpose of "greatest dollar return" is explicitly rejected as an overall objective.

Other objectives of National Forest management are expressed in the relevant congressional statutes and USFS statements. Significantly, the National Forests must be managed so that harvest occurs at an indefinitely sustainable rate; the purpose of this is to guarantee timber supplies for future generations. It also is clear that Forest Service managers are concerned with issues at the local levels, such as employment and price stability.
The central point is that there exist outputs or objectives in National Forest management other than timber outputs and, significantly, that the production of these (or some of these) outputs conflicts with optimum timber production from a maximized net revenue approach. Management for these alternative purposes is given authorization in the Multiple Use—Sustained Yield Act of 1960. The fact that such considerations do conflict with revenue maximization is acknowledged by USFS personnel who admit that, were it not for these nontimber objectives, the National Forests would be liquidated. Thus, we see that there exist alternative, nonmarket values from standing forest inventory and that harvesting for the achievement of timber value requires that some nonmarket values be foregone.

Not all the alternative objectives mentioned directly conflict with timber production, but there clearly exists some set of objectives which does conflict, and the existence of which leads to increased forest inventory holdings and decreased harvests. It is this set which we are calling, simply, "nontimber" outputs and which is the source of nontimber values from standing forest inventory. These outputs lead to a positive-valued "b" in Eq. 2.2. Whereas timber is a marketable good, most of the "nontimber" outputs are not. We thus face a problem of finding an objective method of estimating "a" and "b" in Eq. 2.2. "c" and "d" have been analyzed by looking at the market for timber with standard econometric tools, but we yet require some means to quantitatively compare the timber and nontimber values.

The inability to properly price nonmarket goods has long been a thorn in the side of applied economics. Few real economic problems have impacts only in the marketplace. In the 1950s and early 1960s, a great deal of interest was aroused concerning income distribution and equity. Many public investment projects had income distribution impacts and could not be fairly treated solely by analyses of economic efficiency. More recently, production externalities, such as pollution, have pointed out the value of clean environments and the problems associated with unappropriated environmental property rights. The lack of economic tools to handle the problem of nonmarket evaluation has been a source of frustration to many economists and, in our objectivist society, has led in some cases to biases in favor of economic efficiency and against the alternative objectives.
Little has been accomplished in the development of techniques for shadow pricing. According to Margolis7, "[The] basic question asked by the analyst when he searches for shadow price is: what would the users of the public output be willing to pay." Two general techniques have evolved in shadow pricing, but each has very limited application. The first applies when the public output is used as an intermediate good; the value of the good to its user is then the amount that it increases the user's net income. This technique has been applied to the evaluation of public water projects, particularly in providing water for agricultural purposes. This approach is of limited applicability in that only a small number of publicly produced goods can be considered intermediate outputs. Additionally, in many cases, the good will be used in private production processes and an appropriate market for the good can be easily established. A second general technique for shadow pricing is to identify an already existing market for substitute goods. This appeal to market information can prove very accurate, depending upon the degree of substitutability. This approach could be used, for example, in evaluating education programs. Obviously, the limitation of this method involves products where close substitutes in the marketplace do not exist, e.g., environmental quality. In some cases involving environmental goods in residential or business areas, the social value of superior environmental conditions can be imputed from differences in property values, but it is extremely difficult to isolate the causes of the price changes, and property values respond only very slowly over time. In certain instances, it may be possible to develop highly situation-specific methods for evaluating nonmarket goods. An example of this has evolved in shadow pricing publicly supplied recreation facilities. Inspired by a letter written to the Chief of the Forest Service by Harold Hotelling, Clawson evaluated the willingness-to-pay of users by determining their travel costs in reaching the recreation site.8 Such an approach only yields a lower bound figure—the user may still have come even if the site was further away—but it is a very objective evaluative method when travel costs are properly measured. Such ingenious techniques may be forthcoming for other applications, but again, they are apt to be highly specific.

In 1966, Maass proposed an original way to address the nonmarket evaluation problem, but his suggestion has not been well received by economists. Maass's central theme was that nonefficiency social objectives are part of the social welfare function, and that they could be and are evaluated by Congress. Maass demonstrated that Congress was capable of achieving this result and
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documented the way in which this was accomplished. The legislature, in considering programs, is given the costs and expected results of alternative programs and is asked to choose among them. Implicit in its decision is a relative valuation of the net social benefits of alternative projects. The implication of this proposal is that Congress fairly shadow prices (some) nonmarket goods and that these prices can be carefully used by analysts for future reference.³ Maass has been criticized by at least two economists, Havemen¹⁰ and Haefele¹¹ (some of these criticisms are referred to below), but these criticisms tend to miss the point of Maass’s proposal. (See source document--Reference 1, pp. 57-61--for further details.) It does seem, however, that Maass has underemphasized the most important justification for his proposal. This is simply that our political/economic/social system has explicitly designated Congress as the proper representative of the individual’s and society’s unappropriated property rights. Looked at from this perspective, the economist is tempted to view all laws as existing in order to evaluate these unappropriated rights relative to other social objectives--maximum GNP or other, nonefficiency goals. When a community taxes and appropriates money to support a law enforcement agency, for example, it has said (by whatever mechanisms of choice it uses) that, as a whole, the community values the expected order resulting from the agency’s activities over and above the money which achieves it. Surely cost-effectiveness is an issue, as Haveman points out, but the social mechanism has placed an implied value upon the nonmarket good, social order, and this implication may be useful to the economic analyst. Certainly, there exist dissenters in the social decision, but if they are freely participating in the community, we must consider their behavior as rational and utility-maximizing. Margolis has pointed out that there may be differences between the conception of social value of the economist and the politician. Nonetheless, the politically derived evaluation can correctly be considered an approximation of the social value estimate for which the economist searches. And to a significant extent, the economist will search in vain for other mechanisms for evaluating nonmarket goods, such as and clean environments, because society has already designated a proper and acceptable place for such claims to be recognized and evaluated.

The thrust of this approach is towards the consideration of nonefficiency social objectives as goods, comparable to those which are privately traded; the political processes replace the
marketplace as evaluative mechanisms for these nonmarket goods. In this scheme, Congress performs a necessary economic function. Congress is seen as an institution which organizes and (coherently) presents the public sentiment regarding its unappropriated rights. It then presents to the Executive relative valuations of social objectives, which the Executive then uses as part of its objective functions for performing optimization over the resource flows which it controls. The Judiciary performs the function of assuring that the congressional evaluations are consistent with the agreed-upon basis for social activity—the Constitution. A simplistic representation of this process shown in Fig. 2. The Executive is the final actor because it can react more quickly to the changing resource base; the resource system (consisting largely of the physical, international and domestic social environment) is presumed more dynamic than the social welfare function.

Congress is sometimes criticized for the vagueness of its statutes. But often the intent of the statute is not to specify that some activity should be performed; rather the intent is to indicate a social preference for one type of output over another. The best way for balancing these outputs is left up to the Executive. (Other forms and directions of communication exist among the relevant bodies, but these are not significant to our thesis.)

Public forest management appears to be an excellent example of the application of this technique. When the Executive proposes to Congress a set of alternative projects where the central difference among the projects is the rate of timber harvest from public lands, the Executive is asking Congress to specify its preference between timber and nontimber outputs from National Forests. (See, for example, the alternative program goals proposed by the Forest Service in 1975.) When Congress chooses a timber harvest rate which is below the most efficient rate from the point of market economics, as it has consistently done, we can correctly infer that Congress sees value from standing forest inventory, per se.

The arithmetic of this problem can be simply demonstrated. Let us imagine a publicly owned resource system which produces two outputs, x and y. Let us further specify that the production of x and y is inversely related:

\[ y = a \times, a < 0. \]
Figure 2 Schematic of Market and Nonmarket Evaluation Procedures
Product x is traded on the market, while there exists no market for y. Based on market information, the public management authority knows that it faces a linear demand for x, such that the net social value accruing from the production of x is $px - bx^2$. Were management solely a market decision maker, it would choose to produce at the optimum value of x:

$$x = \frac{P}{2b},$$

since y does not enter into the objective function. Let us now say that we can observe that management is only producing x at the rate of $P_c - c$ and that management has informed us that the production of y does, indeed, enter into its objective function. What can we infer from this situation? If management is still optimizing, we can work backwards in a Lagrangian format to find the marginal net value of y, i.e., its shadow price. Let us call $m(y)$ the marginal net value of y. Our total value function now reads:

$$v = px - bx^2 + \int m(y) \, dy$$

which is to be maximized subject to the constraint

$$y = ax.$$

In the Lagrangian format, we set

$$z = v + \lambda (y - ax)$$

$$\frac{\partial z}{\partial x} = p - 2bx - a\lambda = 0$$

$$\frac{\partial z}{\partial y} = m(y) + y = 0$$

$$\frac{\partial z}{\partial \lambda} = y - ax = 0.$$ 

Recalling that $x = \frac{P}{2b} - c$, we now have (with the partials of z) a system of four equations in four unknowns and can see that

$$m(y) = ac - \frac{8P}{2b}.$$
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at the optimum. Haefele was correct; we do not have the entire value function for \( y \), but we do have a most important component of it: the shadow price (or marginal net value) at equilibrium.

What are the real difficulties in applying this principle in evaluating nonmarket goods? Theoretically, there are several. The first has already been pointed out by Margolis, i.e., we have obtained a politician's measure of marginal value, not an economist's. There are no doubt practical differences between these measures, but we feel that the former represents a good approximation of the latter. A second basic problem is that of optimality in public decision making. The derivation of the value for \( m(y) \) above assumed optimal policy on the part of public management. But almost any individual can find fault with some aspect of government policy and many can find inconsistencies. It is our premise that these problems stem from informational difficulties and not from irrationality, that a congressional committee is practically as capable of expressing consistent preferences as is an individual. One must judge congressional activity on its intent and not on its results and judge that clearly inconsistent and irrational results were not intended. To say that a politician's preferences are nonoptimal is to say that he is not properly representing his constituency. In general, though, the people have freely elected their representatives as those who best serve their needs, subject to the constraints of the electoral process.

Extension of this line of reasoning, however, quickly leads to the position that all congressional activity is optimal, a position that is uncomfortable to maintain. And yet, this is the position typically advocated by economists with regard to the activity of private enterprise, so that its extension to the public management domain does not seem grossly unreasonable.

However, since none of this proves the optimality of congressional activity, application of the discussed principle should be used with a grain of salt.

On the practical side, there are also problems in applying this principle. It may prove impossible to construct a realistic social value function that is simple enough to work with, although this is what we have attempted in the case of public timber by showing the competing nontimber values expressing themselves solely through the timber volume argument. Additionally, we have stressed that it is the intent of congressional activity which must be used
for analysis; although this intent is typically indicated in statutes, the wording of these may be too vague to be used in quantitative analyses. These problems have been hinted at by Haefele.

It may be commented that the approach discussed is simplistic and this may be so. Nonetheless, when applied while acknowledging the theoretic caveats mentioned, this approach can provide measures of shadow prices which economists have been unable to find by other means—and, as we have pointed out, measures which the economists may find impossible to obtain by other means. These cannot be thought of as extremely precise measures, but they can be properly used as approximations of shadow prices. This technique should prove useful to the analyst when applied to environmental amenity goods and to publicly supplied, nonmarket commodities. It may also be beneficial to make use of the Maass principle in direct dealing with Congress; demonstrating the economic implications of their activity could lead to greater consistency in congressional authorizations and mandates. Finally, this approach may give economists greater insight into political activities and the economic roles played by the governmental branches.

Estimating the Nontimber Value Function

Having already mentioned the sources of nontimber value and described the technique employed to estimate these values, we present here our estimates of the nontimber value function. Our procedure is simply to use the model developed in Section 2, manipulating the input shadow price until the model produces harvest decisions comparable to those produced in reality by the Forest Service decisions. Table 1 presents the estimated shadow prices obtained by this technique.

Application to NFS timber management of the methods outlined above is not totally original. In considering management techniques for old-growth Douglas Fir stands, Rickard, Hughes and Newport have written:

An "objective" method, relying on precedent for valuing nondollar yields in dollar terms, uses "shadow prices." Shadow prices of the nondollar yields of a specific alternative might be estimated as equivalent to the dollar value foregone by using the same alternative in another instance to accomplish the same purpose. For example,
### Table 1: Estimated Nontimber Prices in National Forests Compared with Timber Prices, 1974

<table>
<thead>
<tr>
<th>Region</th>
<th>Nontimber Price</th>
<th>Timber Price</th>
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<td></td>
<td>(1967 $ per Cubic Foot of Standing Timber)</td>
<td>(1967 $ per Cubic Foot of Harvested Timber)</td>
</tr>
<tr>
<td>Southern Pine</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand Region</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R1</td>
<td>0.00290</td>
<td>0.1812</td>
</tr>
<tr>
<td>R2</td>
<td>0.00377</td>
<td>0.2068</td>
</tr>
<tr>
<td>R3</td>
<td>0.00120</td>
<td>0.2390</td>
</tr>
<tr>
<td>R4</td>
<td>0.00241</td>
<td>0.2462</td>
</tr>
<tr>
<td>Western Pine</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand Region</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R5</td>
<td>0.01270</td>
<td>0.1486</td>
</tr>
<tr>
<td>R6</td>
<td>0.01160</td>
<td>0.1286</td>
</tr>
<tr>
<td>R7</td>
<td>0.02804</td>
<td>0.1578</td>
</tr>
<tr>
<td>R8</td>
<td>0.01594</td>
<td>0.2399</td>
</tr>
<tr>
<td>West Coast</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand Region</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R9</td>
<td>0.02169</td>
<td>0.4209</td>
</tr>
<tr>
<td>R10</td>
<td>0.02585</td>
<td>0.4209</td>
</tr>
<tr>
<td>R11</td>
<td>0.00365</td>
<td>0.0517</td>
</tr>
</tbody>
</table>

A two-cut shelterwood system may have been used successfully to treat a roadside area. Clearcutting the same area would have yielded $400 per acre more in present net worth. Hence, the worth of the difference in nondollar yields between the two-cut shelterwood and clearcutting might be taken as at least $400 per acre. This value is then compared with the difference in dollar value yield between the two alternatives. The problem here is that use of the shadow price presumes that the previous decision was a good decision, and that the nondollar yield was actually equivalent to the dollar yield given up.

It has already been noted that this method does not produce the entire value function, only the marginal net value of nontimber goods (standing timber). Another parameter is required to indicate the shape of the function: the elasticity of demand for nontimber goods. (We are using a linear demand estimate for nontimber goods, as we did for timber.) For the three timber demand regions, we consistently used a nontimber demand elasticity of -1. It is difficult to develop an objective argument to support any selection of this parameter. We feel, however, that the behavior of the public and the public resource managers
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indicates an inelastic demand for nontimber goods. That is, even though timber goods are valued at a much higher rate than non-
timber goods at the current market equilibrium--greater by at
least an order of magnitude--there is strong resistance to move-
ments to increase timber harvests. This is evidence that the
nontimber demand elasticity may be considerably less elastic than
that for timber demand and, thus, we have used a value of -1.
Further evidence in support of a relatively inelastic nontimber
demand is that NFS lands frequently provide a sole source (or
nearly so) of nontimber goods.

As presented in Section 2, our regional nontimber value func-
tion has only one argument, standing timber inventory. This is a
very simplistic expression of the function. Other factors such as
the health, age, site class and species composition of the timber
stands, and perhaps land area, would all enter into the true
function. However, we feel the expression used to be adequate at
this level of abstraction.

Our use of the nontimber value function, as in the timber
value function, assumes a static state. While this is probably
not true--public attitudes about nonmarket, amenity goods have
been changed recently--resource constraints did not permit an
examination into the dynamics of the situation.

Public decision makers may have recently been exhibiting a
type of risk averseness in determining timber harvest levels from
National Forests. Spurred by public outcry at the effects of some
timber harvests on government lands, legislators may be reluctant
to permit larger, and otherwise optimal, harvests in response to
uncertainty regarding the biological (and social) consequences of
such harvests. If this is the case, public managers should be
willing to divert some resources to gain more information about
the current timber situation. However, such risk averseness would
manifest itself in our analysis in an estimated nontimber shadow
price greater than the actual, since we have employed an expected
value maximization decision rule. These two effects will offset
each other, but may also produce results with an indeterminate,
but small, bias.

4. THE SUPPLY SIDE OF INFORMATION

Various forest managers have developed information systems to
provide the data necessary for decision making. Two such systems
are discussed in this section.
Information Production in the National Forests

Management of the National Forests is carried out by the Timber Management Division of the National Forest System within the Forest Service. They produce information, for their own management purposes, of the type we are interested, in the "stage 1 in-place inventory". These inventories are performed in all NFS regions in ten-year cycles and make use of aerial photography and detailed measurements on permanent ground plots. Details of their procedures are outlined in the Forest Survey Handbook of the Forest Service (FSH 4813.1). Table 2 shows the attributes of the USFS information systems in which we are most interested. Along with the frequency of measurement of every ten years, the volume and growth errors shown in Table 2 provide a complete description (for our purposes) of the existing Forest Service information system. The relationship between the specified sampling errors shown in Table 2 (these represent normalized percentage errors) and the percentage sampling errors is:

<table>
<thead>
<tr>
<th>Table 2 Specified Sampling Error (SE) in National Forest Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Section</strong></td>
</tr>
<tr>
<td>East</td>
</tr>
<tr>
<td>West</td>
</tr>
</tbody>
</table>

1. Per 1 billion cubic feet of growing stock on commercial forest land. Error be achieved as closely as practicable.
2. Per 1 million acres of commercial forest land. This is maximum allowable error.
3. Per 1 million acres of noncommercial forest land. This is maximum allowable error.
4. Per 1 billion cubic feet of annual timber cut from growing stock on commercial forest land. Error to be achieved as closely as practicable.
5. Per 1 billion cubic feet of net annual growth of growing stock on commercial forest land. Error to be achieved as closely as practicable.

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\[ e = \frac{(SE) \sqrt{\text{Specified volume or area}}}{\sqrt{\text{Total volume or area in unit}}} \]

where

- \( e \) = allowable percentage sampling error
- \( SE \) = specified (or normalized) percentage sampling error

The specified volume or area refers to one billion cubic feet or one million acres, whichever is relevant, as shown in Table 2.

**LANDSAT Capabilities in Providing Forest Resource Information**

Although the modeling in this study is performed parametrically to permit examination of any appropriately defined information system, we present here NASA-estimated capability of the LANDSAT (formerly ERTS) Multi-Spectral Scanner (MSS) and Thematic Mapper (TM) systems. Given these inputs, it will be possible to attribute gross benefits to the sensing system when compared with the existing Forest Service system.

The primary piece of research was performed by the Remote Sensing Research Program (RSRP) at the University of California, Berkeley. This study showed that a LANDSAT MSS system, coupled with low altitude aircraft imagery and ground plot studies, could provide significantly increased capability over the existing Forest Service information system.\(^{14}\) The study was performed in the Quincy Ranger District in the Plumas National Service (northern California) with the cooperation of the Forest Service. By greater capability, we mean greater accuracy at equal cost or cost savings with equivalent accuracy.

Figure 3 summarizes RSRP's results. RSRP reports that the LANDSAT MSS system can achieve equal accuracy with a 44 percent cost savings or a 10 percent decrease in sampling variance at equal costs. In another study on the Sam Houston National Forest in Texas, RSRP was not able to demonstrate improved capabilities from the LANDSAT MSS system due to the fact that the Southern Pine stands there are considerably less structured than those of the Plumas National Forest.\(^{15}\)
VALUE OF MEASUREMENTS--TIMBER MANAGEMENT

Figure 3 RSRP Cost-Capability Comparison of Timber Inventory Sampling System
(Source: Reference 14)
Communications with NASA technical staff have indicated that the Plumas results should extend to all western forests and that they expect the increased spectral and spatial resolution associated with the planned TM system to extend the results achieved by RSRP in Plumas to Southern Pine and other less structured stands.16

Thus, we attribute to the MSS system a 10 percent reduced sampling variance or 44 percent increased frequency (coming from cost savings) over the current Forest Service system in the Western Pine and West Coast demand regions and no improvement in the Southern Pine demand region. We attribute to the TM system the same increase in capability extended to the South.

5. MODELING THE EFFECTS OF UNCERTAINTIES

In this section we reformulate the expressions for q, 1 and k found in Section 2 incorporating the effects of uncertainty. We then derive an expression for the uncertainty in knowing the status of the state variable as a function of time.

Deriving the Total Value Function with a Stochastic System

Our procedure will parallel the presentation in Section 2, but we will show the new expressions only at the key points.

An important difference between this formulation and that presented previously is that the variable \( x_t \), our state variable, is now the expected value or best estimate of standing timber volume, instead of standing timber volume itself. This difference should be remembered. If we define real timber inventory as \( \tilde{x}_t \), then

\[
\tilde{x}_t = x_t - \epsilon_t
\]

where \( \epsilon_t \) is a random element describing the difference between the estimated and actual volume. We are defining our information system as producing unbiased estimates and so \( E(\epsilon_t) = 0 \), with \( E(\epsilon_t^2) = \sigma^2 \). A second stochastic element enters into this formulation; due to uncertainty in timber growth rates and random shocks to growth, temporal movements from one state variable to the next occur in a stochastic way. We express our new state transformation function as:
VALUE OF MEASUREMENTS—TIMBER MANAGEMENT

\[ x_{t-1} = m_t x_t + \phi_t - y_t \]  
(5.1)

with \( m_t \) now being defined similar to \( x_t \) as one plus the estimated percentage rate of growth of timber. \( \phi_t \) is defined as the stochastic term representing the difference between our estimate of the state variable at time \( t \) and what the state variable will be at time \( t+1 \) where \( \hat{x}_{t+1} \) is our estimate of \( x_{t+1} \) at time \( t \), \( \phi_t = x_{t+1} - \hat{x}_{t+1} \). Again assuming an unbiased information system, we define \( E[\phi_t] = 0 \) and \( E[\phi_t^2] = \sigma_{\phi,t}^2 \).

As it seems that nontimber values should derive from actual standing timber volume and not our estimate of it, the incremental value function takes on a slightly different form because we have defined our state variable differently:

\[ v_t (x_t, y_t) = a (x_t + \varepsilon_t)^2 + b (x_t + \varepsilon_t) + cy_t^2 + dy_t \]  
(5.2)

Following the development in Section 2, Eq. 2.5 now is expressed as:

\[ v_t (x_t) = \max \left\{ y_t \left[ v_t (x_t, y_t) + v_{t+1} [m_t x_t + \phi_t - y_t] \right] \right\} \]  
(5.3)

where the bar over the expression in braces indicates the mean value of that expression. This says that our decision rule is to maximize the expected value of present and future resource flows; the underlying assumption is that the decision makers (government) are risk neutral.

All \( x, y \) and \( \varepsilon \) terms will be referenced to time period \( t \); so dropping the "tilde" subscript on \( x, y \) and \( \varepsilon \), but keeping it on \( q, l, k, m \) and \( \phi \), we insert Eq. 2.4 and 5.2 into 5.3:

\[ v_t (x) = \max \left\{ \frac{y^2 + r q_{t+1} y^2 + dy - 2 r q_{t+1} (m_t x + \phi)}{(m_t x + \phi) + r k_{t+1}} \right\} \]  
(5.4)
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where \( r \) is defined as before. Recalling our definition of the stochastic terms \( \varepsilon \) and \( \phi \), we take the expected value of the expression inside the braces:

\[
\max_{\varepsilon} V_t(x) = y \left( cy^2 + rq_{t+1}y^2 + dy - 2rq_{t+1}m_txy - rl_{t+1}y + ax^2 \right.
\]

\[
+ a \sigma^2_{\varepsilon,t} + bx + rq_{t+1}m^2_{t}x + rq_{t+1} \sigma^2_{\phi,t} + rl_{t+1}m_{t}x
\]

\[
+ rk_{t+1} \right)
\]

obtaining the algebraic maximum we find

\[
y = \frac{d - 2rq_{t+1}m_{t}x - rl_{t+1}}{2(c + rq_{t+1})}
\]

We see by comparing Equations 2.8 and 5.6 that the addition of uncertainty to a system does not change the optimal first period decision. However, the expression for total value does change:

\[
V_t(x) = ax^2 + a \sigma^2_{\varepsilon,t} + bx + rq_{t+1}m^2_{t}x + rq_{t+1} \sigma^2_{\phi,t} + rl_{t+1}m_{t}x
\]

\[
+ rk_{t+1} - \frac{[d - 2rq_{t+1}m_{t}x - rl_{t+1}]^2}{4(c + rq_{t+1})}
\]

Uncertainty expressed in positive values for \( \sigma^2_{\varepsilon,t} \) and \( \sigma^2_{\phi,t} \) reduces the total value since both \( a \) and \( q \) must be negative ("law" of diminishing returns).

Finally obtaining the desired expressions for \( q, l \) and \( k \), we have

\[
q = \frac{ar + cr^{2}_{m,t} - c - \sqrt{(c - ar - cr^{2}_{m,t})^2 + 4rac}}{2r}
\]

\[
l = \frac{b + \frac{drqm}{c+rq}}{1 - r - \frac{r^{2}qm}{c+rq}}
\]

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\[ k_t = a \sigma^2_{\varepsilon,t} + rq \sigma^2_{\phi,t} + rk_{t+1} - \frac{\left(-\frac{1}{2}(d-r1)\right)^2}{c+rq} \]

If we were to assume that the sign in front of the radical in the expression for \( q \) was positive, we can see that it would be (in principle) possible to find a value for "a" small enough to force a positive \( q \) \((r > 0)\). Since we can say that this is not the case based on theoretic grounds--the "law" of diminishing returns--we can safely say that the sign in front of the radical must be negative, as is shown above.

\( k_t \) is time-dependent, since \( \phi_t, \sigma^2_{\phi,t}, \varepsilon_t \) and \( \sigma^2_{\varepsilon,t} \) are not steady-state. The derivation of expressions for these terms is the subject of the next section.

Modeling the Information System

We will exhaustively define any forest resource information system by three parameters: \( j, \sigma^2_{\phi}, \sigma^2_{\varepsilon} \). Before we define these parameters, we must specify the transformation function for actual timber inventory, \( \tilde{x} \):

\[ \tilde{x}_{t+1} = (\tilde{m} + \psi_t) \tilde{x}_t - y_t \]  \hspace{1cm} (5.8)

where \((\tilde{m} - 1)\) is the long run average percentage growth rate of timber and \( \psi_t \) is a stochastic term representing random shocks to the growth rate and where \( E[\psi_t] \equiv 0 \) and \( E[\psi_i, \psi_j] \equiv \sigma^2_{\psi} \) for \( i = k \) and \( \equiv 0 \) for \( i \neq k \).

\( j \) refers to the number of decision intervals (referenced by the subscript \( t \)) between successive measures and is the inverse of the measurement frequency. Measurement intervals will be referenced using the subscript \( n \); the time of the \( n \)th measurement (the beginning of the \( n \)th measurement period) will always correspond with the beginning of the \( jn \)th decision period. Decision periods have been defined as one year in length.

Let us define \( \theta_n \), the timber volume measurement error, as

\[ \theta_n = \tilde{x}_{nj} - \tilde{x}_{nj} \]  \hspace{1cm} (5.9)
\( \theta \)'s (as well as \( \tau \) defined later) occur every \( j \) decision periods.

\( E[\theta] = 0 \) and \( E[\theta^2] = \sigma_\theta^2 \). \( \tilde{x} \) is defined as the measurement of volume produced at the beginning of the \( n \)th measurement period. The process producing \( \theta \) is defined as a steady-state process, thus

\[
\sigma_{\theta, n}^2 = \sigma_{\theta, n+1}^2 = \ldots
\]

\( \tau \) is growth measurement error:

\[
\tau = \tilde{m} + \psi_j n - m_j n
\]

where \( E[\tau_j] = 0 \), \( E[\tau^2] = \sigma_\tau^2 \) and \( \sigma_{\tau, n}^2 = \sigma_{\tau, n+1}^2 = \ldots \). Again, every \( j \) decision periods \( \tilde{m} \) comes from the measurement system.

Our objective is to find expressions for the vectors \( \varepsilon_{t+1} \), \( \varepsilon_{t+2} \), ..., thus obtaining expressions for \( \sigma_{\varepsilon, t}^2 \), \( \sigma_{\varepsilon, t+1}^2 \), ... for use in our definition of \( k_t \) derived in the last section. Defining \( \varepsilon_t \) as

\[
\varepsilon_t = \bar{x}_t - x_t.
\]

We will start off the information process by saying that

\[
x_o = \bar{x}_o = \bar{x}_o - \varepsilon_o.
\]

Using the transformation functions, 5.1 and 5.9, and 5.10, we derive

\[
\varepsilon_1 = (\tilde{m} + x_1) \bar{x}_o - (\tilde{m} + \psi_1 + \tau_1) x_o
\]

\[
= (\tilde{m} + \psi_1) (\bar{x}_o - x_o) - \tau_1 (\bar{x}_o - \theta_o)
\]

\[
= (\tilde{m} + \psi_1 - \tau_1) \theta_1 + \tau_1 \bar{x}_o
\]

and

\[
\sigma_{\varepsilon, 1}^2 = (\tilde{m}^2 + \sigma_x^2 + \sigma_\psi^2) \sigma_{\varepsilon, o}^2 + \sigma_\tau^2 \bar{x}_o^2
\]

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This expression will generalize to:

\[ \sigma_{\varepsilon,t}^2 = (\bar{m} + \sigma_x^2 + \sigma_{\varepsilon,t-1}^2) \sigma_{\varepsilon,t-1}^2 + \sigma_{\varepsilon,t-1}^2 \bar{x}_{t-1}^2 \]  

(5.12)

requiring the specification of \( \sigma_{\varepsilon,t-1}^2 \) and \( \bar{x}_{t-1} \) and thus necessitating iterative techniques. \( \varepsilon_t \) proceeds from \( \varepsilon_{t-1} \) in a straightforward manner until the time of a new measurement, and new information is used to produce \( \bar{x} \). We then must find an expression for \( \bar{x}_{n_j} \). It could be simply defined as \( \bar{x}_{n_j} \), the new measurement, but this would not be making use of information already available from past measurements. Let us define \( \bar{x}_{n_j} \) as

\[ x_{n_j} = \alpha \bar{x}_{n_j} + (1 - \alpha) \bar{x}_{n_j} \]  

(5.13)

where \( \bar{x}_{n_j} \) is defined as the forecast of \( \bar{x}_{n_j} \) from the time \( n_j-1 \) and where \( \alpha_n \) is optimally selected so as to minimize the variance of \( \bar{x}_{n_j} - \bar{x}_{n_j} \). This procedure will provide an estimate of \( \bar{x}_{n_j} \) superior to \( \bar{x}_{n_j} \) when the optimal \( \alpha_n < 1 \). From 5.13

\[ \bar{x}_{n_j} - \bar{x}_{n_j} = \alpha_n (\bar{x}_{n_j} - \bar{x}_{n_j}) + (1 - \alpha_n) (\bar{x}_{n_j} - \bar{x}_{n_j}) \]

from 5.9. \( \varepsilon_{n_j} \) is defined as \( \bar{x}_{n_j} - \bar{x}_{n_j} \).

\[ \text{Var} [\bar{x}_{n_j} - \bar{x}_{n_j}] = \alpha_n^2 \sigma_\varepsilon^2 + (1 - \alpha_n)^2 \sigma_{\varepsilon,n_{n-1}}^2 \]

where we can derive \( \sigma_{\varepsilon,n_{n-1}}^2 \) from 5.12. To find the optimal \( \alpha_n \):

\[ \frac{\partial \text{Var} [\bar{x}_{n_j} - \bar{x}_{n_j}]}{\partial \alpha_n} = 2 \alpha_n \sigma_\varepsilon^2 - 2(1 - \alpha_n) \sigma_{\varepsilon,n_{n-1}}^2 = 0 \]
and
\[
\alpha_n = \frac{\sigma^2_{\varepsilon,nj-1}}{\sigma^2_\theta + \sigma^2_{\varepsilon,nj-1}} \tag{5.14}
\]

We thus obtain our estimate for \(x_{nj}\) (Eq. 5.13).

To find \(\sigma^2_{\varepsilon,nj}\):
\[
\varepsilon_{nj} = \tilde{x}_{nj} - x_{nj} = \tilde{x}_{nj} - \alpha_n \tilde{x}_{n} - (1 - \alpha_n) x_{nj}
\]
\[
= \tilde{x}_{nj} - \alpha_n (\tilde{x}_{nj} - \theta_n) - (1 - \alpha_n) (\tilde{x}_{nj} - \tilde{x}_{n})
\]
\[
= \alpha_n \theta_n + (1 - \alpha_n) \varepsilon_{nj}
\]
\[
\sigma^2_{\varepsilon,nj} = \alpha_n^2 \sigma^2_\theta + (1 - \alpha_n)^2 \sigma^2_{\varepsilon,nj} \tag{5.15}
\]

To derive \(\sigma^2_{\phi,t'}\), we note that \(\phi_t\) is defined as the difference between \(x_t\) and the forecast of \(x_t\) from \(x_{t-1}\). Without the addition of new information, these forecasts will remain the same such that \(\phi_t = 0\) for \(t \neq nj-1\). Thus we are interested in \(\phi_{nj-1}\) defined as
\[
\phi_{nj-1} = x_{nj} - (m_{nj-1} x_{nj-1} - y_{nj-1})
\]
\[
= (\tilde{x}_{nj} - \varepsilon_{nj}) - (m_{nj-1} (\tilde{x}_{nj-1} - \varepsilon_{nj-1}) - y_{nj-1})
\]

Using 5.10
\[
\phi_{nj-1} = \tilde{x}_{nj} - \varepsilon_{nj} - (\tilde{m} + \psi_{nj-1} - \tau_n) (\tilde{x}_{nj-1} - \varepsilon_{nj-1}) + y_{nj-1}
\]

and using 5.9
\[
\phi_{nj-1} = -\varepsilon_{nj} + \tau_n \tilde{x}_{nj-1} + \varepsilon_{nj-1} - (\tilde{m} + \psi_{nj-1} - \tau_n)
\]

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Thus,
\[ \sigma^2_{\phi, nj} = \sigma^2_{\epsilon, nj} + \sigma^2_{\tau, nj} + \sigma^2_{\tau, nj} (\bar{m}^2 + \sigma^2_{\psi} + \sigma^2_{\tau}) \tag{5.16} \]

All that remains is to find \( \sigma^2_\theta \) and \( \sigma^2_\tau \) (and \( \sigma^2_{\psi} \)) given information system capability parameters of the same basis as those used by the Forest Service, i.e., SE of the type shown in Table 2.

\( \epsilon_v \) is defined as the allowable percentage volume measurement error, or \( \epsilon_v = \frac{\theta}{x_0} \). Thus, using the formula for \( \epsilon \) of Section 4.

\[ \sigma^2_\theta = \epsilon_v^2 x_0^2 = \frac{x_0^2 (SE_v)^2 10^9}{x_0} = x_0 (SE_v)^2 10^9 \]

To derive \( \sigma^2_\tau \), let us define the growth measurement error:

\[ G_n = \bar{q}_n - q_n = (\bar{m} + \psi_{nj} - 1) \bar{x}_{jn} - (1 - m_{nj}) x_{nj} \]

using 5.10 and 5.9

\[ = (\bar{m} + \psi_{nj} - 1) \bar{x}_{jn} - (\bar{m} + \psi_{nj} - 1 - n) (\bar{x}_{nj} - x) \]

\[ = -\tau_n \bar{x}_{nj} + \theta_n (\bar{m} + \psi_{nj} - 1 - \tau_n) \]

\[ = -\tau_n \bar{x}_{nj} + \theta_n (\bar{m} + \psi_{nj} - 1) \]

\[ \theta_n = x_{nj} + \theta_n \]

using \( x_0 \) as an approximation of \( x_{jn} \)

\[ \sigma^2_\tau = \frac{\sigma^2_\theta (1 - 2\bar{m} + \bar{m}^2 + \sigma^2_{\psi} + \sigma^2_{\tau}) + \sigma^2_{\tau}}{x_0^2 + \sigma^2_\theta} \]
Table 3 Estimated Annual Gross Benefits from LANDSAT - MSS and TM Systems in National Forest Timber Sales Management

<table>
<thead>
<tr>
<th>Timber Demand Region</th>
<th>LANDSAT - MSS System</th>
<th>LANDSAT - TM System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>From Increased Measurement Accuracy</td>
<td>From Increased Frequency of Inventory</td>
</tr>
<tr>
<td>Southern Pine</td>
<td>NONE</td>
<td>NONE</td>
</tr>
<tr>
<td>Western Pine</td>
<td>1.96</td>
<td>3.03</td>
</tr>
<tr>
<td>West Coast</td>
<td>1.97</td>
<td>1.69</td>
</tr>
<tr>
<td>TOTAL</td>
<td>5.9</td>
<td></td>
</tr>
</tbody>
</table>

*Estimated benefits based on softwood timber volume and growth measurements only; see text.

b System capability inputs from NASA, based on RSRP 1974 and RSRP 1976.

c Benefits shown for increased accuracy and increased frequency are alternative benefits and are not additive.
where

\[ \sigma_G^2 = \sigma_{G_jn}^2 = \bar{g}_{jn}^2 \left( \text{SE}_G \right)^2 10^9 = (\bar{m} - 1) \bar{x}_0 \left( \text{SE}_G \right)^2 10^9 \]

\( \bar{g}_{jn} \) being defined as the mean value of \( g_{jn} \).

Finally, we need an estimate of \( \sigma_0^2 \). This is a parameter of the biological system, not of the information system, and represents the variability of net timber growth. This parameter enters into the information system in a similar manner as \( \bar{m} \) and is modeled as such--substituting \( \text{SE}_T \) for \( \text{SE}_G \):

\[ S_n = \tilde{g}_n - \bar{g}_n = (\bar{m} + \psi)j_{jn} - (\bar{m} - 1) \tilde{x}_{jn} \]

\[ = \psi_j_{jn} \tilde{x}_{jn} = \psi_j_{jn} \tilde{x}_0 \]

\[ \sigma_s^2 = \sigma_{s_jn}^2 = (\bar{m} - 1) \tilde{x}_0 \left( \text{SE}_T \right)^2 10^9 \]

\[ \sigma_x^2 \tilde{x}_0^2 = (\bar{m} - 1) \tilde{x}_0 \left( \text{SE}_T \right)^2 10^9 \]

\[ \sigma_\psi^2 = \frac{(\bar{m} - 1)(\text{SE}_T)^2 10^9}{\tilde{x}_0} \]

Estimating Gross Benefits from Alternative Information Systems

We are interested in finding the difference in total value, \( V_0(x_0) \), obtained from the use of the baseline Forest Service information, B, and that obtained from an alternative system, A; this value we will refer to as the gross benefit obtained from A. We note that the only differences between systems A and B are the information capability inputs: \( \sigma_A^2 \), \( \sigma_T^2 \) and \( j \). Further, we see that our expressions for \( q \) and \( l \) do not contain these parameters so that we can define gross benefits from A as:
K. R. LIETZE

Let us express \( k_0 \) as

\[
    k_0 = H_0 + rk_1
\]

where

\[
    H_t = a \sigma^2_{\varepsilon,t} + rq \sigma^2_{\theta,t} - K
\]

\[
    K = \frac{[-\frac{1}{2} (d - rl)]^2}{c + rq}
\]

\( k_0 \) expands to

\[
    k_0 = H_0 + rH_1 + r^2H_2 + \ldots = \sum_{i=0}^{\infty} ri H_i
\]

\[
    = a \sum_{i=0}^{\infty} ri \sigma^2_{\varepsilon,i} + rq \sum_{i=0}^{\infty} ri \sigma^2_{\theta,i} - K \sum_{i=0}^{\infty} ri
\]

With the last term cancelling out, we can express gross benefits from \( A \) as

\[
    GB(A) = a \sum_{i=0}^{\infty} ri \left[ \sigma^2_{\varepsilon,i}(A) - \sigma^2_{\varepsilon,i}(B) \right] + rq \sum_{i=0}^{\infty} ri \left[ \sigma^2_{\theta,i}(A) - \sigma^2_{\theta,i}(B) \right]
\]

6. ESTIMATED BENEFITS OF THE LANDSAT SATELLITE SYSTEMS

Using the inputs about LANDSAT system capabilities discussed in Section 4, we estimate the annual gross value of a LANDSAT MSS system in National Forest timber sales management to be $5.9 million and $7.2 million for the higher capability LANDSAT TM system. Table 3 summarizes these results. Gross benefits can be expressed as a function of (any realistic value of) information system capabilities: \( j, \sigma^2_{\theta} \) and \( \sigma^2_{G} \).

The reader should bear in mind that the benefits documented here in no way represent the total value of LANDSAT systems in United States timber management as a whole or even in NFS management in particular. We have examined only the use of information in one decision (timber sales/harvest) at one level (the aggregated regional level). We have also only modeled the effects of volume and growth measurement information. While we feel that
<table>
<thead>
<tr>
<th>Region</th>
<th>Annual Loss in Net Growth from Insects and Diseases&lt;sup&gt;3&lt;/sup&gt;</th>
<th>Total Annual Loss in Economic Value from Insects and Disease</th>
<th>Annual Economic Value Effect of Reducing Annual Loss from Insects and Disease by:</th>
<th>0.5%</th>
<th>1.0%</th>
<th>5.0%</th>
<th>10.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Millions of Cubic Feet</td>
<td>Millions of 1975 Dollars</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Southern Pine Region</td>
<td>123.44</td>
<td>71.96</td>
<td>0.342</td>
<td>0.688</td>
<td>3.121</td>
<td>6.313</td>
<td></td>
</tr>
<tr>
<td>Western Pine Region</td>
<td>1907.91</td>
<td>1155.024</td>
<td>6.085</td>
<td>12.182</td>
<td>55.026</td>
<td>110.553</td>
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</tr>
<tr>
<td>West Coast Region</td>
<td>1273.78</td>
<td>671.293</td>
<td>3.855</td>
<td>7.723</td>
<td>31.334</td>
<td>59.420</td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>3406.88</strong></td>
<td><strong>1898.291</strong></td>
<td><strong>10.28%</strong></td>
<td><strong>20.593</strong></td>
<td><strong>89.490</strong></td>
<td><strong>176.285</strong></td>
<td></td>
</tr>
</tbody>
</table>
this study does adequately reflect the benefits from volume and
growth measurement in the NFS, we also feel that substantial fur­
ther benefits would accrue from the use of LANDSAT information in
forest pest monitoring and management; other benefits might come
from utilizing volume and growth measurements at a more local
level and from assisting in timber investment decisions. Another
possibly significant application of satellite-sensed information
is in wildfire detection and wildfire hazards monitoring and map­
ing. All these applications will pertain to timber management
in the private sector as well.

7. The Benefit Potential for Forest Pest Monitoring and Control

An interesting and useful exercise possible with the decision
model developed in this study is to examine the economic effects
of increased net timber growth rates, as obtained by the control
of forest pests. Satellite remote sensing may offer an extremely
important function in the detection of timber afflicted with in­
sects or disease and may lead to reduced losses from these causes.
Average annual losses of timber due to these causes is 11.57 bil­
lion cubic feet17. The synoptic satellite view may prove partic­
ularly useful in identifying areas of infestation where control
would be economic—areas which may have gone undetected without
satellite imagery. In order to illustrate the economic potential
in this application, we have distributed these annual losses among
the different NFS timber supply regions studied and then assessed
the effects upon total economic value from reducing these annual
losses. Table 4 summarizes these results.

The figures in Table 4 in no way indicate the value of sat­
ellite-obtained information in forest pest control, for economic
satellite capabilities have not been demonstrated. Even if such
capabilities were demonstrated, these figures could not be inter­
preted as information benefits because they do not include the
costs of control. The exercise summarized in Table 4 indicates
that almost $2 billion is lost annually to forest insects and
disease and provides an indication of the potential economic
value obtainable by reducing these timber losses.

8. CONCLUDING REMARKS

This study demonstrated that information in public forest
management has true economic value and has calculated the magni­
tude of this value in ways useful for economic justification of
timber resource inventory systems. The gross benefits cited herein only represent a portion of the total benefit attainable by a LANDSAT system when applied to public forest management. Specifically, benefits attributable to improved information in the following areas have not been quantified and are not included in these estimates: pest control, timber investment practices, and harvest/sales decisions at less aggregated decision levels. Further benefits should result from the application of LANDSAT-derived information in these areas in private forestry as well. The gross annual benefit attributable to the use of LANDSAT information harvest/sales management of the National Forest System at the MSS level of capability is estimated to be $5.9 million. At the TM level of capability proposed for LANDSAT Follow-on the estimated benefit is $7.2 million.

References


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