SOME CONSIDERATIONS OF AERODYNAMIC HEATING

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With the contemplation in recent years of flight at ever-increasing Mach numbers, the problem of aerodynamic heating has become increasingly important to the aeronautical engineer. Unfortunately, much of the work on this subject has not been done from the point of view of the aerodynamicist but has been based on the conventions of the varied branches of specialized fields of heat exchange. From considerations, therefore, of this difficulty and of the increasing importance of this subject to the aerodynamicist, a brief review of boundary-layer heating phenomena appears to be desirable at this time.

If air is brought to rest near the surface of an insulated plate and no energy is assumed to be transferred to or from any element of mass, then from the equation for the conservation of energy

$$T + \frac{U^2}{2c_p} = \text{Const} = T_s$$

where T is the local temperature, U is the local velocity, c_p is specific heat at constant pressure, and T_s is the stagnation temperature. Then, the temperature rise from the free stream to the surface ΔT is found to be

$$\Delta T = \Delta T_{ad} = \frac{U^2}{2c_p}$$

This temperature rise is called the adiabatic temperature recovery and is used as a reference temperature rise in most heat-transfer discussions.

The importance of this temperature rise at high Mach numbers is clear if the equation for the stagnation temperature is written

$$T_s = T\left(1 + \frac{M^2}{5}\right)$$

which is the equation relating this adiabatic stagnation temperature T_s to the free-stream temperature T, where M is the stream Mach number. At a Mach number of 5.0 the surface temperature is six times the free-stream temperature, so that the problem of aerodynamic heating requires serious attention.

Consider now the energy per unit area transferred into an element of height dy while the air is brought to rest first by the temperature gradient set up and secondly by the friction work done. The energy transferred into the element by heat conduction per second is, then,

$$- k \frac{\partial^2 T}{\partial y^2} dy$$

and that by frictional work is

$$\mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + u \frac{\partial^2 u}{\partial y^2}\right] dy$$

where k is thermal conductivity, μ is viscosity, and u is the velocity component in the x-direction..

These two effects are of opposite sign and if they are equal in magnitude the energy per element remains constant and the energy equation holds through the boundary layer. If the two effects are assumed equal, then

$$k \frac{\partial^2 T}{\partial y^2} = \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + u \frac{\partial^2 u}{\partial y^2}\right]$$

Now, from the energy equation

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{c_p}\left[\left(\frac{\partial u}{\partial y}\right)^2 + u \frac{\partial^2 u}{\partial y^2}\right]$$

or

$$\frac{\mu c_p}{k} = 1$$

The quantity $\frac{\mu c_p}{k}$ is called the Prandtl number σ and is a measure of the relative magnitude of the friction and heat-conduction effects in an insulated flow. This parameter is very important in all heat-transfer phenomena. If the Prandtl number is 1.0, the temperature recovery on an insulated body is equal to the adiabatic recovery ΔT_{ad}; thus,

$$\Delta T = \Delta T_{ad} = \frac{U^2}{2c_p}$$

The value of the Prandtl number for air has been variously measured and placed at between 0.72 and 0.76 and hence the temperature recovery on an insulated body should be lower than the adiabatic recovery. The ratio of the actual recovery to the adiabatic recovery $\Delta T/\Delta T_{ad}$ is called the temperature-recovery factor.

If the Prandtl number is not equal to 1.0 or if there exists a heat transfer in or out through the surface at which the velocity is zero, then, in order to solve for the temperatures that exist for a particular velocity profile, three differential equations may be set up. The first two are the well-known continuity and momentum equations.

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{1}$$

and

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2}$$

where ρ is density.

The third equation is an expression of the fact that all the energy transferred into an element by frictional work, conduction, and convection must be carried away by these same processes so that the temperature of the element does not increase with time. For simplification, variations of v in the x- and y-directions are neglected as well as variations of u in the x-direction, with the result that

$$k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 = \rho c_p\left\{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right\} \tag{3}$$

In order to solve these three equations Eckert and Drewitz (reference 1) assumed that the continuity equation could be satisfied by the use of a stream function and then, use can be made of the new variables

$$\xi = \frac{1}{2}\,y\sqrt{\frac{u_o}{\nu x}}$$

and

$$\zeta = \frac{\psi}{\sqrt{\nu u_o x}}$$

where u_o is free-stream velocity, ν is kinematic viscosity, and ψ is the stream function. Equations (2) and (3) can be put into convenient form by use of these variables and the solution can then be obtained. The result for the heat transfer per unit area h through the surface, when $0.5 < \sigma < 2$, was

$$h \approx 0.332\, k \sqrt{\frac{u_o}{\nu x}}\, \sqrt[3]{\sigma} \left[\left(T_w - T_o\right) - \frac{u_o^2}{2c_p} \sqrt{\sigma} \right]$$

where T_w is the wall temperature and T_o is free-stream temperature. This result is all that is needed for comparison with the results of experimental measurements on an insulated flat plate, for when $h = 0$,

$$T_w - T_o = \frac{u_o^2}{2c_p} \sqrt{\sigma}$$

and the temperature-recovery factor is

$$\frac{\Delta T}{\Delta T_{ad}} = \sqrt{\sigma}$$

For air with a Prandtl number $\sigma = 0.72$ the theoretical temperature-recovery factor is 0.85.

Figures 1 and 2 show the full solution by Eckert and Drewitz (reference 1) for the local temperature rise and stagnation temperature for the laminar-boundary-layer profile. It may be seen from figure 2 that, since the air near the surface of the plate has a stagnation pressure less than free stream, conservation of energy requires the air in the outer portion of the boundary layer to have a stagnation temperature greater than free stream.

The temperature recoveries measured experimentally on an insulated flat plate are shown in figure 3 as a function of local Reynolds number R. The theoretically determined recovery factor $\frac{\Delta T}{\Delta T_{ad}} = 0.85$ agrees well with the experimental values in the laminar region, but as the Reynolds number increases along the plate, transition occurs and the temperature-recovery factor increases from the laminar value to a value at the beginning of the turbulent layer of 0.90.

Before further discussion is made of the results of this laminar analysis to predict the temperature recoveries about bodies other than flat plates, some discussion should be given to one of the methods of analyzing the heat-transfer characteristics of the turbulent boundary layer.

Figure 4 is an illustration of the type of velocity profile that will be assumed. The method of solution is as follows (reference 3): The laminar sublayer will be assumed to have a linear velocity profile and a parabolic temperature profile $T = A + By + Cy^2$. The local heat and momentum transfers of this layer are determined and, when made to agree with the local heat and momentum transfers of the turbulent layer at the outer edge of the laminar sublayer, a unique solution for the heat transfer through the combined layers results.

For the analysis of the turbulent layer the following equations are at our disposal. They are the continuity equation

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{4}$$

the momentum equation

$$\rho \frac{du}{dt} = - \frac{\partial \tau}{\partial y} \tag{5}$$

and the energy equation

$$\rho \frac{d}{dt}\left(c_p T + \frac{u^2}{2}\right) = - \frac{\partial}{\partial y}(u\tau) - \frac{\partial h}{\partial y} \tag{6}$$

where

$$\tau = \overline{\rho v' l_y} \frac{\partial u}{\partial y}$$

$$= A \frac{\partial u}{\partial y}$$

$$h = c_p \overline{\rho v' l_y} \frac{\partial T}{\partial y}$$

$$= \Lambda \frac{\partial T}{\partial y}$$

The transfer term $\overline{\rho v' l_y}$ is assumed to be equal in both the transfer of heat and momentum; that is, the mixing length l_y in the two cases is the same so that

$$\Lambda = c_p A$$

The momentum and energy equations (equations (5) and (6)) then become

$$\rho \frac{du}{dt} = - \frac{\partial}{\partial y}\left(A \frac{\partial u}{\partial y}\right)$$

$$\rho \frac{d}{dt}\left(c_p T + \frac{u^2}{2}\right) = - \frac{\partial}{\partial y}\left[A \frac{\partial}{\partial y}\left(c_p T + \frac{u^2}{2}\right)\right]$$

and the total energy and the velocity satisfy the same linear differential equation, a significant fact first pointed out by Crocco. (See reference 3.) All boundary conditions may be satisfied if

$$c_p T + \frac{u^2}{2} = au + b$$

where a and b are constants independent of y.

Frankel used the foregoing procedure to obtain the following result for the heat transfer through a turbulent boundary layer:

$$\frac{h_w}{\tau_w} = \frac{c_p\left(T_o - T_w\right) + \frac{u_o^2}{2} + \frac{u_1^2}{2}(\sigma - 1)}{u_o + u_1(\sigma - 1)}$$

Again, if the case of an insulated plate is considered, the temperature recovery is

$$\Delta T = \frac{u_o^2}{2c_p} - \frac{(1 - \sigma)u_1^2}{2c_p}$$

or

$$\frac{\Delta T}{\Delta T_{ad}} = 1 - (1 - \sigma)\left(\frac{u_1}{u_o}\right)^2$$

From the work done on the turbulent boundary layer at low speeds the value of the square of the ratio u_1/u_o is found to be proportional to the friction stress at the wall divided by twice the dynamic pressure

$$\left(\frac{u_1}{u_o}\right)^2 = 135 \frac{\tau_w}{2q}$$

The temperature-recovery factor for air becomes

$$\frac{\Delta T}{\Delta T_{ad}} = 1 - 37.8\,\frac{\tau_w}{2q}$$

Figure 5 is a plot of this relationship for values of $\tau_w/2q$ usually encountered. The results are found to be of the right order of magnitude for the temperature-recovery factors of turbulent layers. Experimental measurements on turbulent layers have given temperature-recovery factors ranging from 0.89 to 0.93 but no such linear dependence as is indicated by the equation has been shown experimentally. Research is needed to determine the proper relationship between u_l and u_o at high speeds as the use of the low-speed relationship at high Mach numbers is not at all logical.

The next step is to determine whether it is possible to apply the results just obtained for insulated flat plates to the prediction of temperatures in insulated bodies of other shapes.

If the velocity distribution about a body is known, the local temperature distribution outside the boundary layer can be found; each element of the body is then assumed to have the flat-plate recovery factor based on its own local conditions. The temperature rise above local temperature for a laminar boundary layer is then

$$\Delta T_l = 0.85\,\frac{{u_l}^2}{2c_p}$$

and the local recovery factor will be 0.85. The recovery factor based on free-stream temperature and the adiabatic recovery of the free-stream velocity is

$$\frac{\Delta T_o}{\Delta T_{ad_o}} = 1 - 0.15\left(\frac{u_l}{u_o}\right)^2$$

Figure 6 shows this last recovery factor $\Delta T_o/\Delta T_{ad_o}$ as measured around a circular cylinder at a Mach number of 0.526 and a Reynolds number of 1.81×10^5. The only part of these data that can be compared with our analysis are those obtained at stations less than 80° from the leading edge, because at larger angles the vortex street shed by the body completely alters the phenomena with the result that surface temperatures are much lower. When these data are converted into the form $\Delta T_l/\Delta T_{ad_l}$ (the dashed line) the agreement with the flat-plate results is good except in the region near the stagnation point. This result seems to indicate that it is permissible to use flat-plate results to predict temperature distributions over insulated bodies of different shape.

Finally, the measurements on this cylinder over a large range of Mach number indicate that the theoretical prediction - namely, that the local temperature-recovery factor is independent of Mach number - is correct for all moderate Mach numbers. This result is shown in figure 7, which is a plot of the local temperature-recovery factor at a station 70° from the leading edge of the cylinder for a range of free-stream Mach numbers. Since the local Mach numbers are well in excess of unity, these data indicate that the local recovery factor is independent of Mach number up to local Mach numbers approaching 2.0.

These results indicate that the theoretical analysis of the laminar boundary layer on a flat plate presented is an adequate tool for predicting the temperature recoveries on the surfaces of insulated bodies moving at high speeds. It may also be used for calculating moderate heat transfers, but the theory fails if the heat transfer is of a magnitude large enough to change appreciably the common laminar-boundary-layer profile of figures 1 and 2.

The analysis of the turbulent boundary layer indicates that the temperature-recovery factor of an insulated flat plate depends upon the friction stress at the wall and that experimentally it is desirable to measure this quantity simultaneously with the temperature-recovery factor. Certainly further research is needed on the nature and extent of the laminar sublayer of the turbulent boundary layer at high speeds.

Finally, it must be pointed out that the methods of analysis presented herein are not the most refined available to the specialist in the field of heat transfer today (see references 4 to 6) but are presented because they represent the basic methods of approach and serve as an introduction to the problems of aerodynamic heating.

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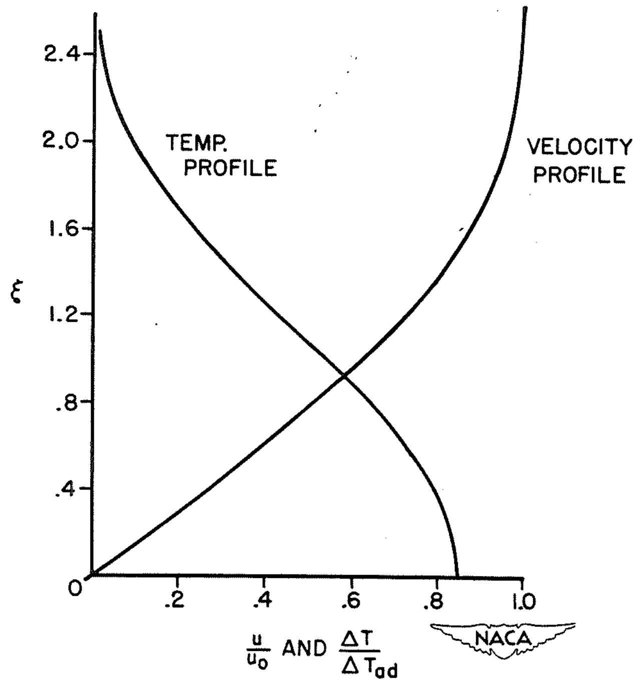

Figure 1.- Local temperature-recovery factor and velocity profile for laminar boundary layer.

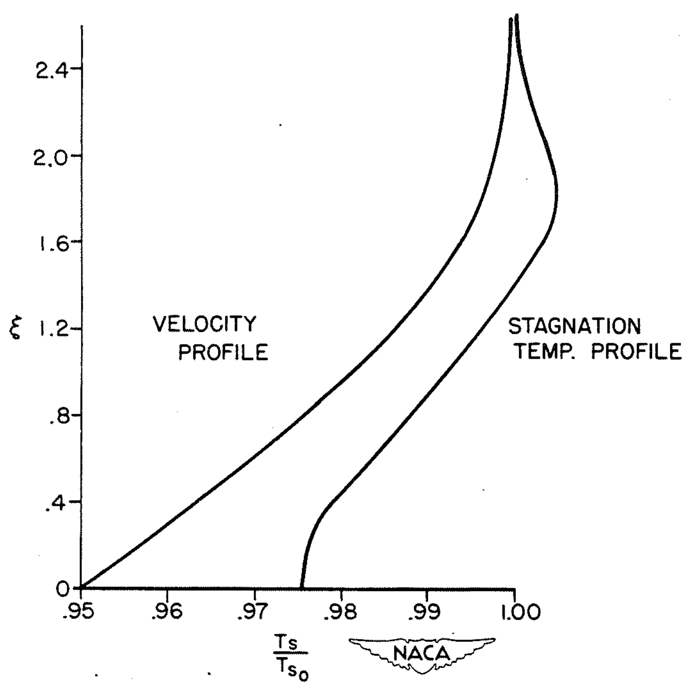

Figure 2.- Stagnation-temperature ratio and velocity profile for laminar boundary layer.

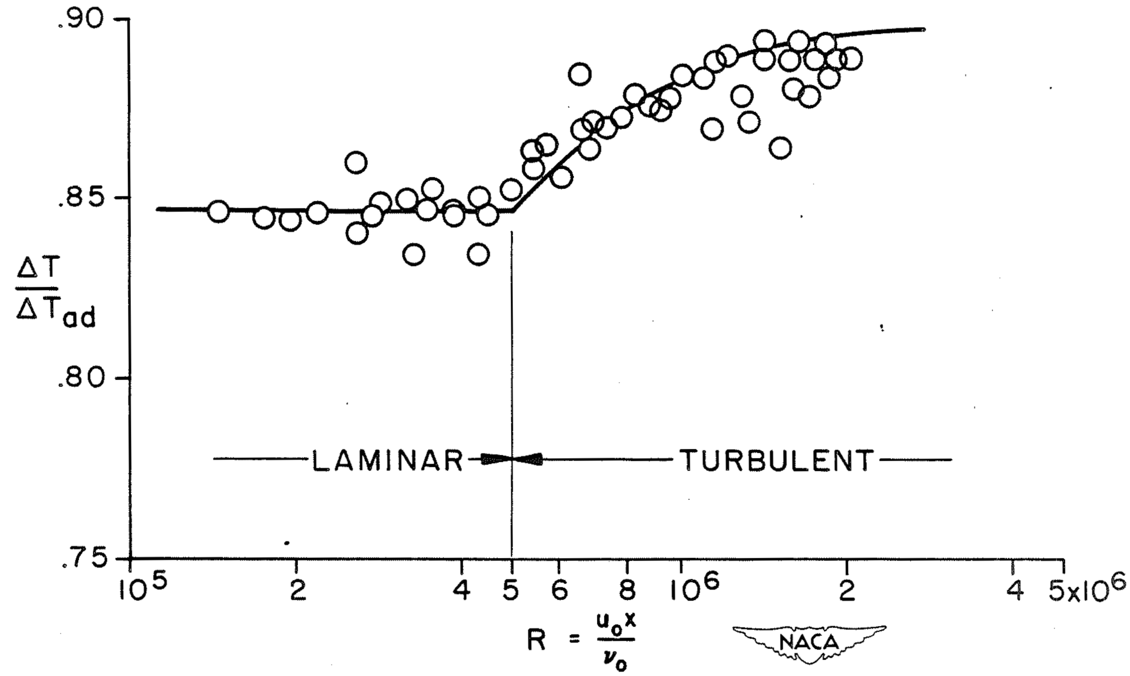

Figure 3.- Temperature recoveries on a flat plate.

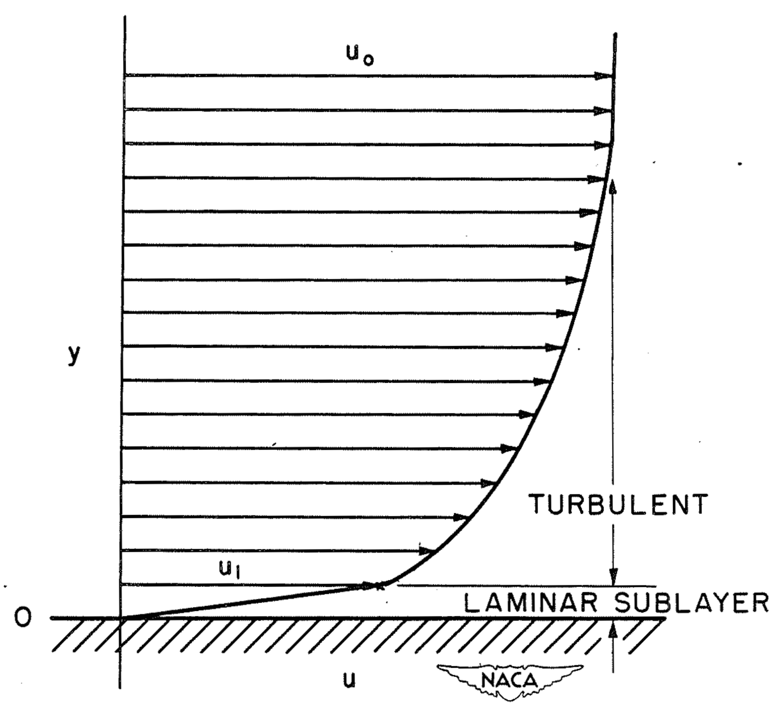

Figure 4.- Assumed turbulent-velocity profile.

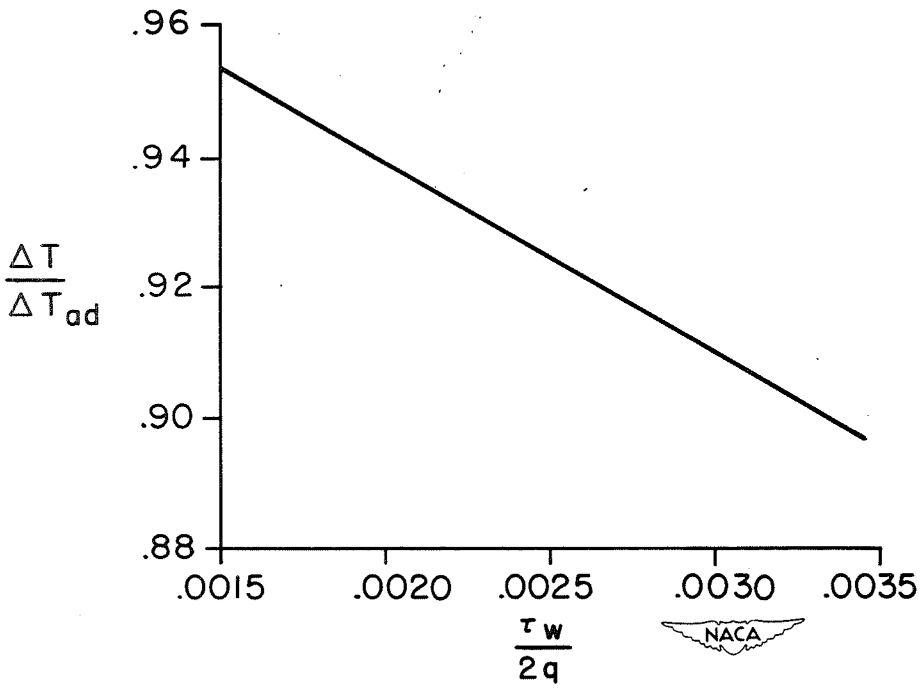

Figure 5.- Theoretical temperature recovery for turbulent boundary layer.

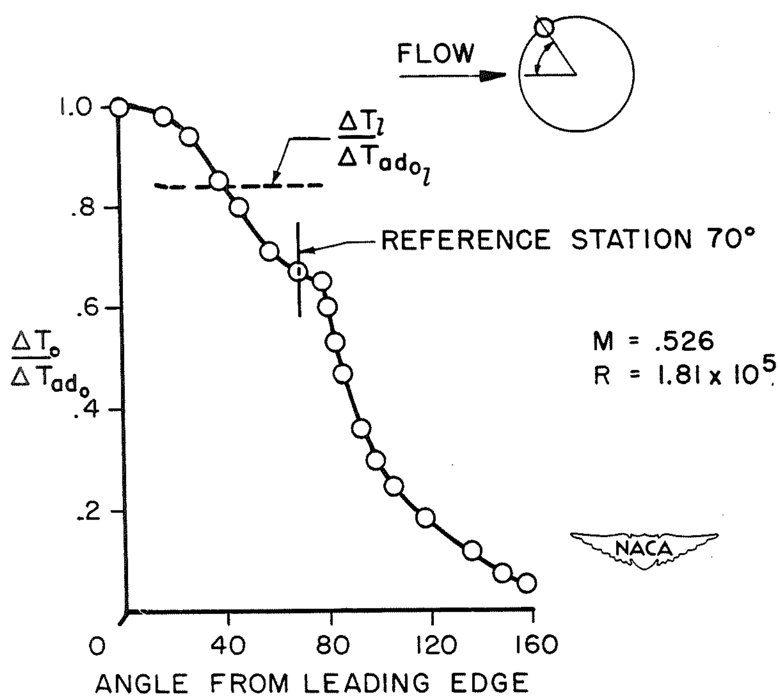

Figure 6.- Temperature recovery on circular cylinder.

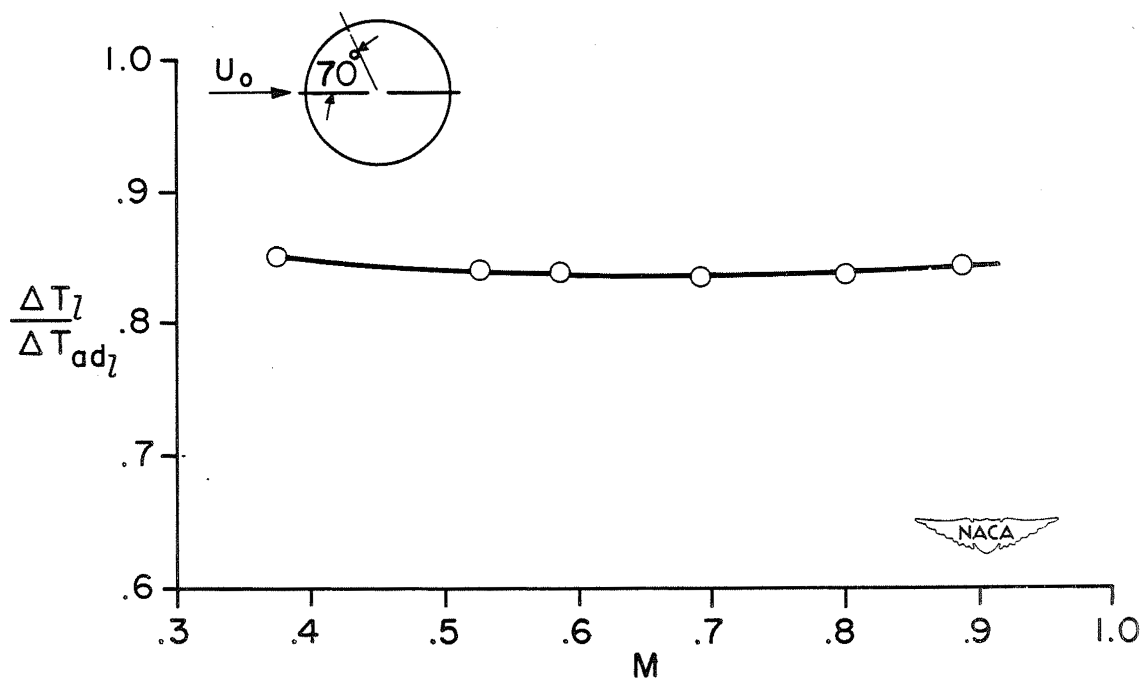

Figure 7.- Temperature recovery at 70° station on circular cylinder at various free-stream Mach numbers.