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HYDRODYNAMICS OF PRIMORDIAL BLACK HOLE FORMATION

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ENGLISH ABSTRACT

The formation of primordial black holes (PBH) in the case of an expanding Universe was discussed under the assumption that at the early stages the expansion is near to quasi-isotropic. The choice of a quasi-isotropic solution is brought about by the fact that, even though quasi-isotropy does not exist, the particle generation near the singularity should inevitably result in a quasi-isotropic solution.

For simplicity the spherically symmetrical case will be considered below. Then the quasi-isotropic solution is characterized by one function only.

The problem is solved by means of a computer. The computations performed allow the following conclusions to be made:

1. PBH form when there are important deviations from the Fridmanian model. If the deviation from the Fridmanian model can be imaged as a part of the closed world that is smoothly matched with the flat world, the degree of the deviation from the Fridmanian model can be quantitatively characterized by the Lagrange radius of the mentioned matching. If the Lagrange radius is assumed to be the unit at which the maximum of the Euler radius is achieved, our computations show that for the black hole formation it is necessary that the radius of the matching R_m will be 0.85 to 0.9.

2. The hydrodynamical computations made the fact clear that the smoothness of the matching is of great importance; this can be quantitatively characterized by a relative width of the matching Δ with reference to the Lagrangian radius R_m. The smaller Δ, the greater the pressure gradients preventing contraction.

Our computations show that the pressure actually prevents PBH formation decreasing the mass of the hole as compared with that, which should have occurred under the same initial conditions but with zero pressure.

3. To plot the total hydrodynamical picture of accretion it is necessary to compute several dozens of hydrodynamical times. It will be done in another paper. But it is seen even here that the similarity solution typical for catastrophic accretion cannot be obtained. It is explained by the matter ejection from the disturbed region due to pressure gradients; as a result, the size of a black hole, if it can nevertheless form, will be much smaller than the horizon and the formation moment.

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1. Introduction /5*

In 1966, Ya. B. Zel'dovich and I. D. Novikov [1-3], and then in 1971 Hawking [4] pointed out the possibility of the formation of black holes at the very beginning of cosmic expansion from primary matter. Later on, many works discussed the problem of primordial black holes (PBH) (see, for example, [5-7]).

Interest in PBH increased particularly after Hawking's discovery [8] of quantum evaporation of small mass black holes because PBH can have small masses. The Hawking process is important, on the one hand, for the physics of the early stages of expansion of the Universe and, on the other hand, as a possible method of detecting PBH in today's Universe [9-12].

The following two problems are basic for the theory of PBH:

1) What must the deviations be from the Fridman cosmic model at the beginning of expansion so that PBH will form?

2) How will accretion of matter on the PBH formed occur?

Both problems were posed and considered in the very first works on PBH [1,2,3]. In these and succeeding works, it was pointed out that the complete answer to both questions required a numerical calculation on a computer. Besides PBH in literature, white holes are considered as well. The importance of quantum processes close to singularity and the process of accretion [1-3] converting a white hole into a singular black hole was pointed out [13] for these objects. /6

*Numbers in the margin indicate pagination in the foreign text.

This article clarifies a description of the method and results of calculation of the hydrodynamics of the phenomena indicated using an EVM in the simplest hypothesis as to the spherical symmetry of the processes considered.

2. Posing the Problem

PBH cannot form in the early stage of expansion of the Universe (with the equation of state $P = \varepsilon/3$), if deviation from the Fridman model was small both in density and in metrics. Actually, as Lifshits has pointed out [14], perturbations of density which are small in amplitude $\delta\varepsilon/\varepsilon$ increase only to the values of pertubation of metrics which are assumed to be small. After this, the condition of increase of perturbations breaks down (the linear scale of perturbations becomes smaller than the horizon) and they return to sonic oscillation.

On the other hand, as is well known, deviations from the Fridman model exist which result in the formation of PBH. Actually, let us consider the nearest singular state of a uniform semiclosed world [15-17], matched through a narrow orifice with a uniform flat cosmic model. During evolution of a semiclosed world, the signal from the orifice moving with the speed of sound successfully reaches the internal sections of the semiclosed world where these parts will already be at the stage of contraction under an intrinsic gravitational radius and consequently, as is well known, they form a black hole.

The question arises: what must the critical value of deviation from the Fridman model be during which a black hole forms and what is the smallest deviation where the hole does not form. This problem is solved in our work. It will be formulated more precisely later on. We are also interested in the hydrodynamic processes which accompany the formation of PBH. /7

Thus, we will consider some deviations from the Fridman model in the initial state. Selection was made of initial conditions of close singularity very large in a general case (see [18,19]). However, it is important to note that the quantum processes of close singularity sharply limit the possible initial conditions. As has been pointed out in [20,21], immediately after completion of quantum phenomena (that is, after the moment $t = t_{p\ell} = \sqrt{\hbar G/c^5}$ the solution of the OTO[1] equation must be quasi-isotropic [22,23]. This solution is characterized by the fact that close to singularity, all of the elements of matter expand according to the same law $\varepsilon \approx \frac{3}{32\pi G t^2}$ (for $P = \frac{1}{3}\varepsilon$), as is true in the Fridman model. At the same time, the spatial cross section t = const can be a random distortion method. Assignment of this distorted cross section is a general initial condition for a quasi-isotropic solution. In the case we considered of spherical symmetry of a curve of three dimensional space, it is described by a single function of the radial coordinate. Thus, we will consider the condition for PBH occurring depending on assignment of this function.

The process of formation of PBH will be considered[2] depending on the amplitude of deviation from the Fridman model of close singularity (origin of expansion) and to the profile of this /8
deviation (however, staying within the framework of spherical symmetry). We will prescribe a deviation close to singularity in the form of a spherical field with the accompanying space of a constant positive curve -- that is, the perturbed field corresponds to part of a closed Fridman model. The amplitude

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[1] General theory of relativity.

[2] The time of formation of a black hole with mass M and the time /7
during which this black hole then vaporizes, involve an order of magnitude of the relationship $t_{vap}/t_{frm} \approx t_{frm}/t_{p\ell}$, where $t_{p\ell} \approx 5\cdot10^{-5}$d -- the Planck time. From this it is apparent that $t_{vap} >> t_{frm}$, if $M >> 10^{-5}$g (that is, the Planck mass). Consequently, in hydrodynamic calculation, we correctly do not consider the quantum processes of vaporization of black holes.

of deviation can be described with the number which characterizes /8
the part of the closed space of a constant positive curve which we have taken (see paragraph 4). This part of a closed world through a transition field is matched to the flat Fridman model. The width of the transition field, as our calculations have shown, will be the second significant parameter of the problem.

Here it is assumed that within the perturbed field the solution is precisely described by the Fridman model. Thus, perturbation is such that the full mass of substance within the perturbed field is precisely equal to the mass which would occur in this field in an unperturbed Fridman model.

3. Initial Equations

As has been pointed out above, we will consider a spherical problem. We will write the spherically symmetric metric

$$ds^2 = c^2 e^{\sigma} dt^2 - e^{\omega} dR^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) . \qquad (1)$$

Let us select the accompanying system of calculation. Then the R-Lagrange radius. Here we will describe the Einstein system of equations for the metric (1) (see [24]). We note only that this system has a first interval found in the work of Podurets [25] and along with the laws of conservation ($T^k_{i;k} = 0$) equivalent to the following simpler system:

(2) /9

$$m = \frac{1}{2}\left(c^2 r / G\right)\left(1 + e^{-\sigma} \dot{r}^2 / c^2 - e^{-\omega} r'^2\right)$$

(3)

$$\dot{m} = - \frac{4\pi}{c^2 p r^2 \dot{r}}$$

(4)

$$m' = \frac{4\pi}{c^2 \varepsilon r^2 r'}$$

$$\sigma' = -2P'(P+\varepsilon) \tag{5}$$

$$\dot{\omega} = -2\dot{\varepsilon}/(P+\varepsilon) - 4\dot{r}/r \tag{6}$$

Here m -- is mass included within the Lagrange radius R, ε = density of energy, P is pressure.

Numerical calculation of the system of equations (2-6) as Podurets proposed [25] appears to be ineffective for the problems which we have selected to solve. The fact is that during formation of black holes in an expanding Universe, in the central field of perturbation, expansion of matter is changed by compression. Near to this moment in time (different for different R) $\dot{r}$ = 0 and equation (2) has a root characteristic. Inasmuch as this moment was unknown earlier, it appears more convenient to increase the magnitude of the system (2-6) and obtain a new system free of root characteristics.

Let us convert the system (2-6). Incidentally, we will introduce dimensionless values of $\tilde{r}$, $\tilde{t}$, $\tilde{m}$, $\tilde{\varepsilon}$, connected to r, t, m and ε in the following way

$$\begin{cases} r = r_g \cdot \tilde{r}, \quad t = \dfrac{r_g}{c} \cdot \tilde{t}, \quad m = \dfrac{c^2}{2Gr_g} \cdot \tilde{m}, \\ \varepsilon = \dfrac{3}{8\pi} \cdot \dfrac{c^4}{Gr_g^2} \cdot \tilde{\varepsilon}. \end{cases} \tag{7}$$

Here r_g is the gravitational radius corresponding to characteristic mass. Later on we will omit the symbol "~" remembering that we are working with dimensionless values. /10

Let us introduce the value u related to $\dot{r}$ by the relationship:

$$e^{-\sigma}\dot{r}^2 = u^2.$$

u -- has a concept of physical velocity of expansion or contraction of the Lagrange surface R = const.

From 2-6), differentiating (2) in time, we obtain the following system:

$$\dot{u} = -\frac{1}{2} e^{\sigma/2}\left(\frac{2e^{-\omega}P'r'}{P+\varepsilon} + \frac{m}{r^2} + 3Pr\right) \tag{8}$$

$$\dot{r} = e^{\sigma/2} u \tag{9}$$

$$\dot{m} = -3Pr^2\dot{r} \tag{10}$$

$$\varepsilon = \frac{\partial m}{\partial r^3} \tag{11}$$

Equation (8) (see also [26]) is a relativistic analogy of the second law of Newton. On the left is acceleration and on the right -- force. The first member is force involving the gradient of pressure and the two succeeding members describe gravitational attraction.

We will assume that pressure P depends only on ε (this hypothesis is not true for a model of stars but is completely suitable for the early stages of expansion of the Universe), and then the equation of state can be written in the form

$$P = [\gamma(\varepsilon) - 1]. \tag{12}$$

Integrating (5) and (6), we have:

$$\begin{cases} e^{\sigma} = \left[\dfrac{A(\varepsilon)}{\varepsilon} \right]^2 f(t), & (13) \\ e^{\omega} = \left[\dfrac{\varphi(R)}{r} \right]^4 A^{-2} \left(\dfrac{2}{3\gamma} \right)^{4/\gamma} . & (14) \end{cases}$$

Here

$$A = \exp\left[\int_1^{\varepsilon} \frac{d\varepsilon}{\gamma(\varepsilon)\varepsilon} \right] . \qquad (15)$$

If γ does not depend on ε, then $A = \varepsilon^{1/\gamma}$, $f(t)$ and $\phi(R)$ -- are random functions of time and radius.

We will consider the formation of PBH in a cosmic Fridman model with a flat accompanying space. Let us select the function $f(t)$ in such a way that our accompanying system will have large values of R (where the solution is Fridmanian) simultaneously synchronous, that is

$$\lim_{R \to \infty} e^{\sigma} = 1 \quad . \qquad (16)$$

It is not difficult to point out that for a uniform spatially flat Fridman solution with random values of γ (not depending for simplicity on ε) we have:

$$\varepsilon = \left(\frac{2}{3\gamma t} \right)^2 \qquad \text{with calculation of (7).} \qquad (17)$$

Then from (13) and (15-17) we find $f(t)$:

$$f(t) = \left(\frac{2}{3\gamma t} \right)^{(2/\gamma)(\gamma-1)} . \qquad (18)$$

Selection of the function $\phi(R)$ will be considered in the next paragraph.

Later on we will have to determine the portion of matter entering into a black hole. For this it is necessary to find at each moment of time the boundary of the black hole -- the horizon of the event. As is well known (see [6]), a local criterion for the position of the horizon of the event does not exist and to find it, it is necessary to integrate zero geodesics in the future. In our problem, where the fall of matter into the black hole is important, the position of the horizon of the event at this moment essentially depends on the matter which will fall into the black hole in the future. For an observer who is near a black hole, it is more significant to determine the boundary of the black hole as the visibility horizon, that is, to determine the Lagrange radius corresponding to mass which at a given moment is under its gravitational radius. In this work we will use this determination of the boundary of a black hole. This boundary can be defined from calculation of local properties. Actually (see for example [6]), this boundary corresponds to the condition $\frac{m}{r} = 1$, and moreover, the requirement that the accompanying system be compressed into this boundary. /12

Thus, we will form the final criterion of a black hole thus (using the dimensionless values defined above).

If, for a certain point with a Lagrange radius R, two conditions are simultaneously fulfilled

$$\begin{cases} \frac{m}{r} > 1 \\ u < 1 \end{cases} ; \tag{19}$$

then this point lies under the horizon of visibility of the black hole. The equation of this horizon of visibility is

$$\begin{cases} \frac{m}{r} = 1 \\ u < 0 \end{cases} . \tag{19'}$$

We note that the horizon of the event always lies within the horizon of vicibility and, generally speaking, in direct proximity to it. /13

4. Initial and Boundary Conditions

As was discussed in paragraph 2 we consider that near singularity, expansion occurs in the quasi-isotropic way. In the case of spherical symmetry this means that the solution has only one random function $\phi(R)$ (see (14)). An analytical solution of the system (8-11, 12, 13, 14) or an equivalent system (2-4, 12-14) when $t \to 0$ (in singularity) with quasi-isotropic conditions gives us the following asymptotic solution

$$r = \varphi(R)\left\{t^{\frac{2}{3\gamma}} - \frac{\gamma-1}{2\left[1-\left(\frac{2}{3\gamma}\right)^2\right]}\left[-\frac{\varphi''}{\varphi} + \frac{\gamma}{2(\gamma-1)}\,\frac{1-\varphi'^2}{\varphi^2}\right]t^2\right\}^{-\frac{2}{3\gamma}} + \tag{20}$$

$$+ O\left(t^{4-\frac{4}{3\gamma}}\right),$$

$$\varepsilon = \left(\frac{2}{3\gamma t}\right)^2 + \frac{\gamma}{2+3\gamma}\cdot\left(-\frac{2\varphi''}{\varphi} + \frac{1-\varphi'^2}{\varphi^2}\right)t^{-\frac{4}{3\gamma}} + \tag{21}$$

$$+ O\left(t^{2-\frac{8}{3\gamma}}\right).$$

Here two members of expansion are considered. Near singularity the spherically symmetrical deviation from the Fridman model with a flat accompanying space occurs in the field $0 < R < R_+$. The center of deviation coincides with the origin of the coordinate.

Using expansion (20-21), the initial conditions are prescribed in the following way:

When $R < R_-$, /14

$$\varphi(R) = \sin R\,, \tag{22}$$

This corresponds to a space of a constant positive curve (that is, a closed Fridman model). For a closed world, R changes from 0 to π. Then the maximum of $\phi(R)$ is reached when $R_{max} = \frac{\pi}{2}$.

When $R > R_+ = R_- + \Delta$,

$$\varphi(R) = C + (R - R_+)\,, \tag{23}$$

this corresponds to a spatially flat Fridman solution. C -- is a certain constant determined by the conditions of matching (22) and (23) in an intermediate field $R_- < R < R_+$.

When matching (22) and (23) we require continuity of the third derivatives of function $\phi(R)$, inasmuch as these derivatives enter into equation (8) (in $P' \sim \varepsilon'$, see (21) and in r').

As was pointed out in paragraph 2, we will select perturbation in such a way that outside the field of pertubation when $R > R_+$, the solution would be precisely Fridmanian. Therefore, it is obvious that in the transition field of matching $R_- < R < R_+$ there must be a certain discharge (field of decreased density) so that the excess of mass within R_- will be compensated in comparison with a spatially flat Fridman solution.

We will select time t_o fairly small, as the moment of time for prescribing the initial conditions so that the discarded members of expansion in (20) and in (21) will be small in comparison with the remaining. Let us evaluate the value of the remaining and eliminated members. By selection constant C can lead to smoothness of function $\phi(R)$, that is,

$$[\varphi(R_+) - \varphi(R_-)] \sim \Delta\,. \tag{24}$$

Then the maximum value of the second derivative in the order of magnitude equals /15

$$(\varphi'')_{max} \sim \frac{\varphi'(R_+) - \varphi'(R_-)}{\Delta} \sim \Delta^{-1} . \qquad (25)$$

The first discarded member includes the third derivative producing a value on the order of magnitude equal to

$$\varphi''' \sim \frac{\varphi''}{\Delta} \sim \Delta^{-2} . \qquad (26)$$

From this we find the condition for selection of the initial moment of time depending on the width of matching Δ:

$$t_0^{2-4/3\gamma} << \Delta . \qquad (27)$$

Thus, with the consideration of everything presented above, the initial conditions, and this means the entire solution, are characterized by two values:

1) $B = R_- / R_{max} = \frac{2R_-}{\pi}$,

the ratio of the radius of matching to half the radius of a closed Fridman;

2) Δ -- is the width of matching.

B characterizes the value of perturbation of geometry. The role of the pressure gradient depends on the value of Δ. With small values of Δ the role of the pressure gradient increases.

The fact that the point is fairly distant (large values of R) we will use as the first boundary condition; the wave of relief did not successfully reach this and expansion occurs according to the law (17). The boundary condition on the left boundary also has a very simple form:

$$\begin{cases} m(t) = 0 \\ r(t) = 0 \end{cases} \tag{28}$$

Boundary conditions (28) would remain true at the latest stages of collapse after formation of a black hole when at the center for a finite time an actual physical singularity occurs. The moment of occurrence of singularity is different for different Legrange radii, that is, singularity is not quasi-uniform which it was at first. From equations (2-6) or (8-11) one can obtain an asymptotic solution (Kazner type) which in the accompanying system of computation with our selection of random functions has the form:

$$m \approx \mu(R), \tag{29}$$

$$r \approx \rho(R)\left[t - t_*(R)\right]^n, \tag{30}$$

$$\mathcal{E} \approx \epsilon(R)\left[t - t_*(R)\right]^{1-3n}, \tag{31}$$

where

$$n = \begin{cases} \dfrac{2}{3(2-\gamma)}, & \gamma > 4/3 \\ 1, & \gamma \leqslant 4/3. \end{cases} \tag{32}$$

Thus, after singularity has formed, the left boundary condition continues to be at a certain Lagrange radius R, different from zero. This radius R is determined from the condition so that numerical solution is close to an asymptotic solution (29-31). During further collapse, as the new elements of matter drop to singularity, the position where the left boundary condition remains moves toward a large Lagrange radius.

5. A Method of Numerical Calculation

For numerical solution of a system of equations (8-11) the difference method was used similar to the basic characteristics in the methods used earlier for calculating gravitational collapse [26-27]. Therefore, we will not present here the entire system of different equations but will limit ourselves only to a brief description of its basic peculiarities. The entire integral of change of the Lagrange coordinates $0 \leq R \leq R_1$ was broken up into 150 integrals and then the step of the break up ΔR was made smaller in the internal part and particularly close to the matching location of a closed world with the Fridman and was smoothly increased in the external part of the integral of change of R considered. In the difference equations, a method was used of "whole" and "semiwhole" calculation units according to space and time [28]. In the whole units, according to space, u, r, m were calculated and in the semiwhole -- P, ε and depending on them e^{σ} and A; moreover, u was calculated in semiwhole units according to time, and all of the remaining values -- in whole units according to time.

Each alternating step in time began with a calculation of u from the difference analog of equation (8), then from equation (9) and (10) sequentially distributions were calculated according to space of r and m, and finally, the density of energy was found from the difference equation:

$$\varepsilon^{n+1}_{j-1/2} = \frac{m^{n+1}_{j} - m^{n+1}_{j-1}}{(r^3)^{n+1}_{j} - (r^3)^{n+1}_{j-1}}, \quad (j = 1, 2, \ldots, J1),$$

which corresponds to (11); here j -- is the number of the spatial calculation interval, and n -- is the number of the time calculation interval; J1 = 150 -- the whole number of space calculated intervals. The values of density of energy in the flatter semi- /18 whole unit $\varepsilon_{J1+1/2}$ is defined from the boundary when $R = R_1$.

The difference system considered here was tested in sample calculations of gravitational collapse close to free fall, in calculations of Fridman expansion and Fridman expansion with small perturbations in the form of sonic waves. Then it was clear that the process of gravitational collapse is reproduced very well by a numerical method, that is, the different calculations of perturbation rapidly attenuate as the central density increases. We note here than in the work of May and White [26] and Schwarts [27] only gravitational collapse was studied numerically. However, the series of calculations of the Fridman expansion (including the perturbed) showed that this numerical reproduction requires a more accurate approach inasmuch as the absolute value of calculated perturbations of density decrease somewhat more slowly than the density itself and therefore relative errors gradually increase. So that one can calculate the stage of expansion fairly far, it is desirable to significantly decrease the step of integration in time in comparison with its maximum allowable value determined by the condition of numerical stability of Kurant.

We used the following requirement as the criterion of selection of a variable step of integration according to time Δt: the ratio $\Delta\varepsilon/\varepsilon$ does not exceed the prescribed fixed value $\xi \ll 1$ in all calculated spatial integrals ($\Delta\varepsilon$ -- is change of ε for one step in time). If, after completion of the alternating step of integration, $\Delta\varepsilon/\varepsilon > \gamma$ appeared, although for one spatial interval, then Δt decreased and the subsequent step was recalculated. It is convenient to use the integral of equations (8-11) /19 for control of precision:

$$u^2 + 1 - e^{-\omega} \dot{r}^2 - \frac{m}{r} = 0 \tag{33}$$

(see equation (2)). The difference of this expression from zero by several percentage points in relation to the component making the maximum contribution to the left part (33) can be considered fully exemplary. It was apparent that for the class of problems considered in this work this condition is fulfilled when $\xi < 2 \cdot 10^{-3}$. The results presented here of calculations were obtained using the values $\xi = 10^{-4} - 10^{-3}$.

6. A Description of Results of Calculation

The results of calculation of the process of formation of PBH depend on two parameters -- amplitude of deviation from a flat Fridman model characterized by R_-, and the width of matching Δ.

As has already been discussed, in the field of matching there is a certain rarefaction. In the opposite case, outside there would not be a flat Fridman solution and one would have a perturbation of the $\frac{\Delta M}{r}$ type where ΔM -- is the excess mass (see in more detail [6]). Therefore, it is obvious that relative perturbation of density when $R_- < R < R_+$ is larger than the matched.

As calculation showed, in the case of a very narrow matching, sometimes shock waves occur and solution of the problem in the end requires the introduction of viscosity. Figure 1 shows the evolution of this perturbation in the case where the width of matching $\Delta = 0.1R_-$, $R_- = 1$. Later on we will concentrate our attention on those cases where shock waves do not occur. Let us first select a certain fixed ratio of the width of matching to the radius of matching; let us say $\frac{\Delta}{R_-} = 0.5$ and we will study the following question: at what values of R_- does the black hole form and at what values does it not form. It is clear that the smaller the values of R_-, the smaller the perturbation metric is /20 and with a fairly small R_-, the black hole, as is well known, cannot form and perturbation must be transformed to sonic oscillation.

In Figure 2, $R_- = \frac{3\pi}{8}$, that is, it amounts to 75% of $R_{max} = \frac{\pi}{2}$. It is apparent from the drawing that the process occurs in the following way. At first, a general cosmic expansion occurs. The internal field expands more slowly that the external. From the field of matching to the center of perturbation and outside it there are waves of rarefaction. At a certain moment in time, in the internal field, a shift in expansion to contraction occurs. During contraction, the pressure gradient increases so much that the central core is ejected and a wave of compression occurs outside. In this case the perturbation of the metric, or, in other words, the gravitational field, was fairly large and the formation of a black hole did not take place. Perturbation was changed to sonic oscillations.

In Figure 3 also a case is shown where a black hole does not form. Here $R_- = 0.8R_{max}$. But perturbation of the metric is larger in this case and therefore the large and maximally allowable density of energy occurs in the center of perturbation. The ratio of density of energy at the center of perturbation to density of energy outside perturbation reaches a maximum of 10^2, that is, the problem is essentially nonlinear. Later on, perturbation is converted more and more to sonic waves.

A qualitatively different picture is shown in Figure 4 ($R_- = 0.85R_{max}$). We see here unlimited compression of the core. The central density of energy tends toward the infinite. In order to be sure in this case a black hole actually forms, let us turn to Figure 5. In this drawing the ratio $\frac{m}{r}$ is shown. In those fields where expansion occurs, r is increased and m, due to the forces of pressure, decreases. Therefore, the value $\frac{m}{r}$ drops with expansion. When expansion changes to compression, then obviously the ratio $\frac{m}{r}$ increases. If the pressure gradient /21
does not successfully become compression at the moment when $\frac{m}{r}$ reaches 1 (dashed line on Figure 4) then a black hole unavoidably occurs inasmuch as, in this case, both of the conditions for the

criteria of a black hole are present (see paragraph 3, formula (19)). The moment of formation of a black hole in our dimensionless units equals $t = 2.39$. The ratio of the mass of the black hole at the moment of formation to the mass of the entire perturbed field at this moment equals 20%.

Thus, we can conclude that when $\Delta/R_- = 0.5$, the black hole is formed only in the case where mass of a perturbed field is comparable to half the mass of a closed world, that is, R_- is close to $R_{max} = \frac{\pi}{2}$. The quantitative result of this is

$$0.8 < R_- / R_{max} < 0.85 \tag{34}$$

when $\frac{\Delta}{R_-} = 0.5$.

In Figures 6 and 7, similar relationships are presented for a case when $\frac{R_-}{R_{max}} = 0.9$. At this time, gravitational forces even with a certain reserve exceed the forces of the pressure gradient. Therefore, the formation of a black hole occurs much earlier: $t = 1.87$. And the ratio of the mass of a black hole to the mass of a perturbed field is larger at this moment than in the preceding case and equals approximately 30%. The latter indicates the fact that the force of pressure increased from the central field with a smaller mass than the preceding case.

Now we will go on to study the question of how significant the matching width is for formation of a black hole. Let us take $\frac{R_-}{R_{max}} = 0.85$, that is, the minimum R_- at which a black hole forms and a decreased width in matching to $\Delta = 0.3R_-$. Then the role of the pressure gradient increases and as is shown in Figure 8, the black holes do not form. The pressure gradient success- /22 fully leads to dissipation of the core earlier than a black hole is successfully formed.

However, when $R_- = 0.9R_{max}$ and $\Delta = 0.3R_-$ and $\Delta = 0.2R_-$, the black hole is formed as when $\Delta = 0.5R_-$, but the moment of formation of the black hole ensues later than when $\Delta = 0.5R_-$; the ratio of the mass of the hole formed to the mass of the perturbed region is smaller than in the cases indicated above. (When $\frac{R_-}{R_{max}} = 0.9$, the black hole does not form even when $\Delta = 0.1R_-$.)

All of the calculated variations are presented together in Figure 9. The radius of matching is applied along the axis of the abcissa and the width of matching along the axis of the ordinate; the "+" corresponds to the formation of a black hole, and "O" to the occurrence of sonic oscillation. The dashed curve indicates the approximate boundary of the field of the parameters indicated where black holes occur.

The Table presents the most important quantitative results for certain variations of calculation when a black hole formed.

Thus, the calculations made make it possible to draw the following conclusions.

PBH form only with very large deviations in the Fridman model corresponding to $\frac{R_-}{R_{max}} \approx 0.85$-$0.9$. The width of the transition Δ affects the formation of PBH. The smaller Δ, the more significant is the role of pressure gradients, making it difficult for PBH to form.

Before making numerical calculations, the role of pressure in forming PBH should be evaluated by constructing stationary [3] or selfmodeling [6] solutions. The hypothesis was presented that pressure can facilitate accretion of gas by the by the occurrence of PBH and result in a significant increase in their mass. Carr and Hawking [6] pointed out that a selfmodeling solution does not exist which leads to catastrophic accretion

of the PBH matter during which dimensions of the PBH increase /24
like the cosmic horizon. Our calculations indicate that pressure actually interferes strongly with the formation of PBH, decreasing the mass of PBH in comparison with the mass which existed in the PBH under the initial perturbation but with the absence of pressure $P = 0$. Actually, on the spatial section t = const near singularity, density of energy in the perturbed field $R < R_-$ is larger than away from the Fridman model and the gradient of pressure force on R_- directed outward attempts to eject the mass. In the transition field $R_- < R_- < R_+$, density is minimum and on its external boundary R_+ pressure gradient is directed inward causing accretion. However, this phenomenon is less significant during the formation of PBH than the gradients described above for R_- which leads to "efflux" of the mass from the field of perturbation. For constructing a complete hydrodynamic picture of accretion on PBH (without being limited by the hypothesis of selfmodeling as in [6]), it is necessary to calculate the process fairly far in time on the order of 10rg/s. This we will do in a separate work. But right now one can say during the formation of PBH its dimension is much smaller than the cosmic horizon (the mass of the black hole at the moment of formation amounts to a total of only a few percentage points of the mass included within the cosmic horizon (see Table). Therefore, as was pointed out already in [1,2], catastrophic accretion does not occur and this conclusion coincides with the analysis of [5], made for selfmodeling solutions.

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TABLE

PHYSICAL CHARACTERISTICS OF THE PROCESS OF FORMATION OF PBH /23

$\beta = R_- / R_{max}$	Δ	i	t_i	m_-	m_+	m_r	m
0,85	0,5	0	0,0I	2,3I	6,I9	0,02	0,00
		I	0,62	0,48	I,0I	I,I2	0,00
		2	2,39	0,33	0,77	4,9I	0,I6
		3	3,I7	0,42	0,74	6,30	0,32
0,9	0,5	0	0,0I	2,42	6,45	0,02	0,00
		I	0,62	0,5I	I,09	I,I6	0,00
		2	I,86	0,48	0,88	3,79	0,26
		3	2,76	0,49	0,84	5,55	0,46
	0,3	0	0,0I	2,42	4,74	0,02	0,00
		I	0,62	0,52	0,86	I,I6	0,00
		2	2,42	0,45	0,67	[illegible]30	0,I4
		3	3,I2	0,44	0,65	[illegible],29	0,28
	0,2	0	0,0I	2,42	4,05	0,02	0,00
		I	0,62	0,54	0,78	I,I6	0,00
		2	3,3I	0,42	0,56	6,59	0,09
		3	3,80	0,4I	0,55	8,26	0,I5

[Commas in the tabulated material are equivalent to decimals.]

Symbols:

m_- -- mass within the Lagrange radius R_- (mass of part of a closed Fridman world);

m_+ -- mass within R_+ (mass of total perturbation);

m_r -- mass included within the cosmic horizon;

$m_{b.h.}$ -- mass of a black hole (mass included within the visibility horizon);

i = 0 -- initial moment; i = 1 -- beginning of compression; i = 2 -- moment of formation of a black hole; i = 3 -- end of calculation.

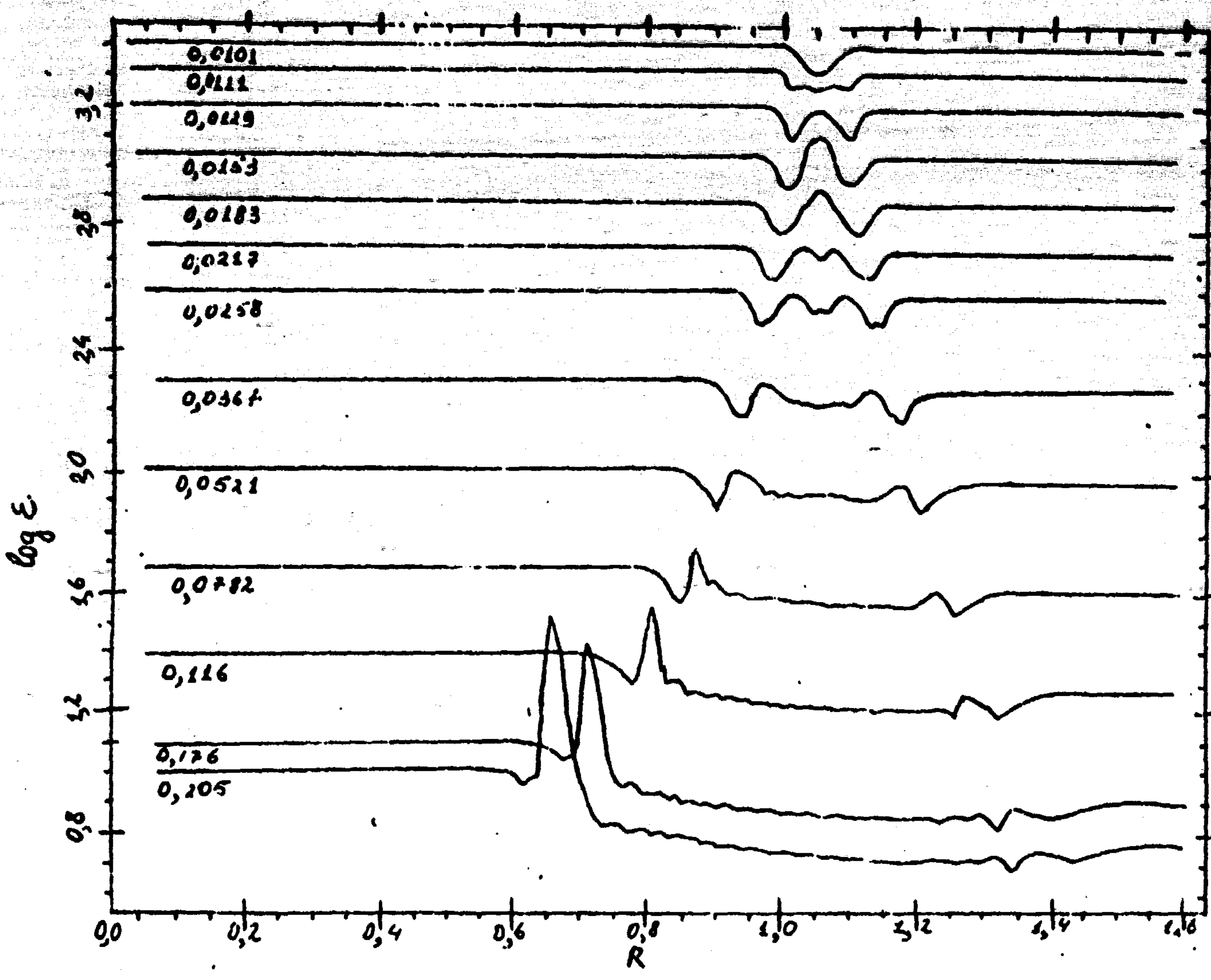

Figure 1. The occurrence of a shock wave. The Lagrange radius R is applied along the axis of the abcissa, the logarithm of density along the ordinate axis. Different curves correspond to different moments of time. Here $R_- = 1$, $\Delta = 0.1$.

[Commas in this figure and in the following figures are equivalent to decimal points.]

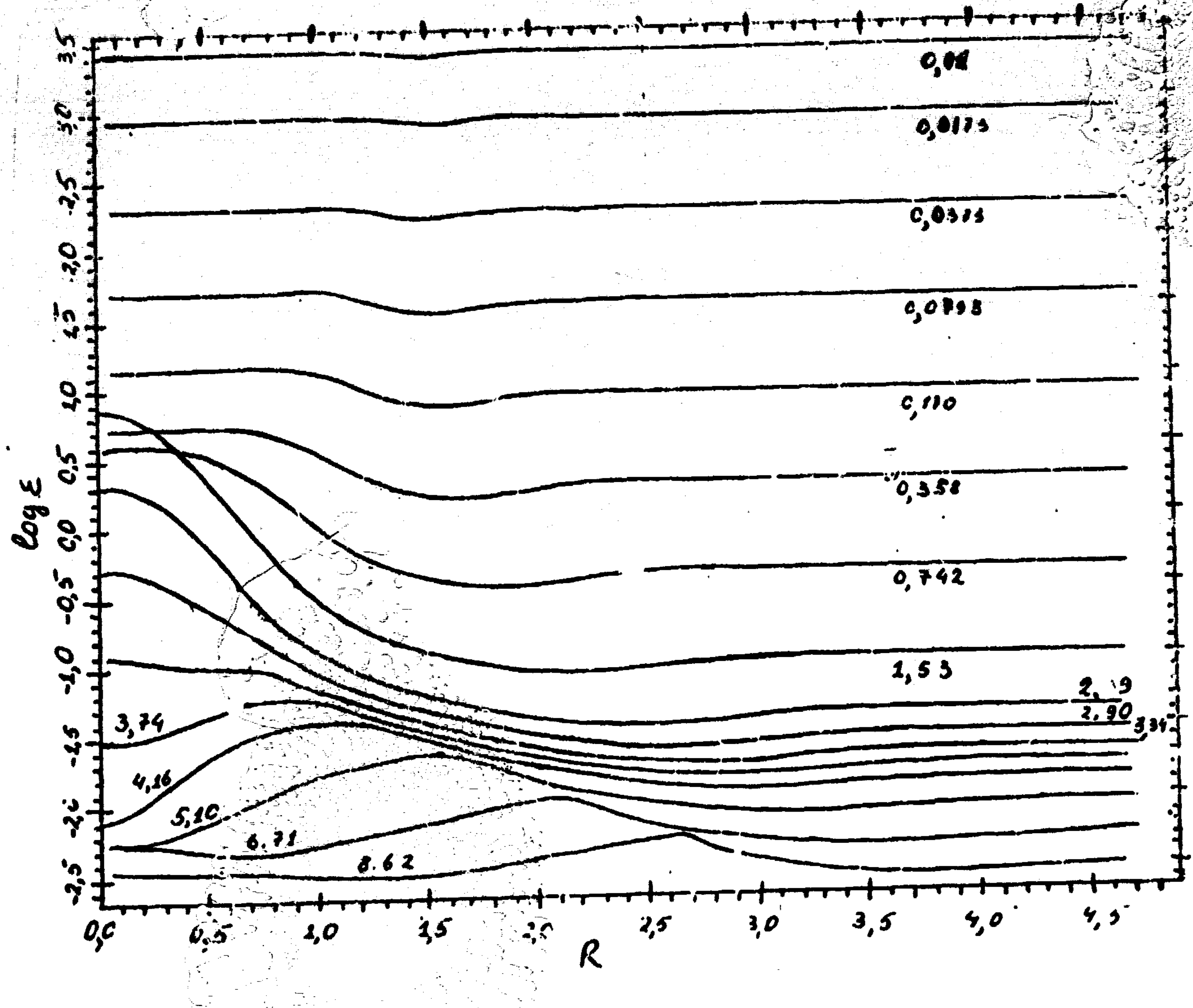

Figure 3. Occurrence of a sonic wave, coming from the center. $R_{-} = 0.75 \cdot R_{max}$, $\Delta = 0.5 \cdot R_{-}$. PBH do not form.

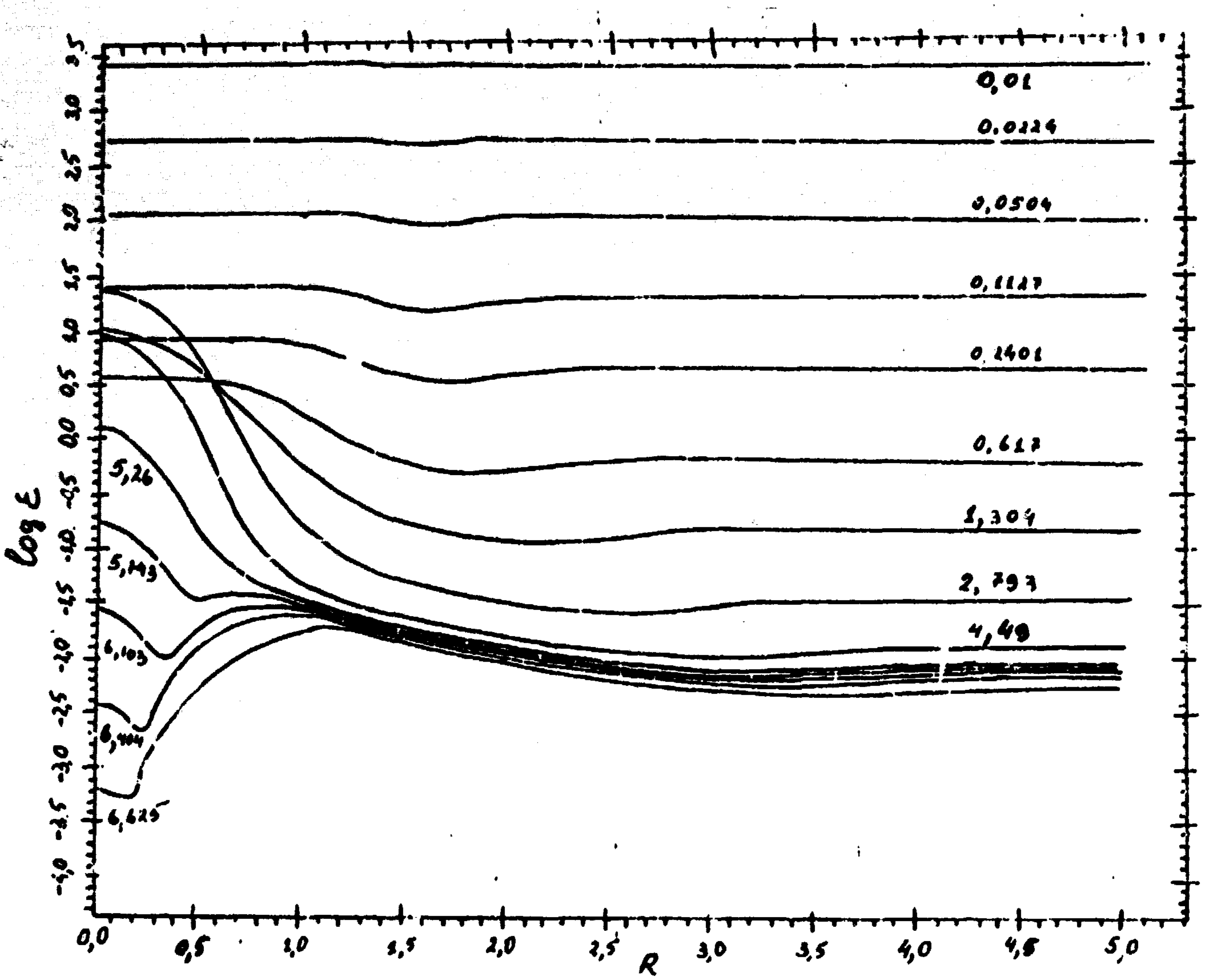

Figure 3. The same as in Figure 2 for $R_- = 0.8\ R_{max}$, $\Delta = 0.5 \cdot R_-$. PBH do not form.

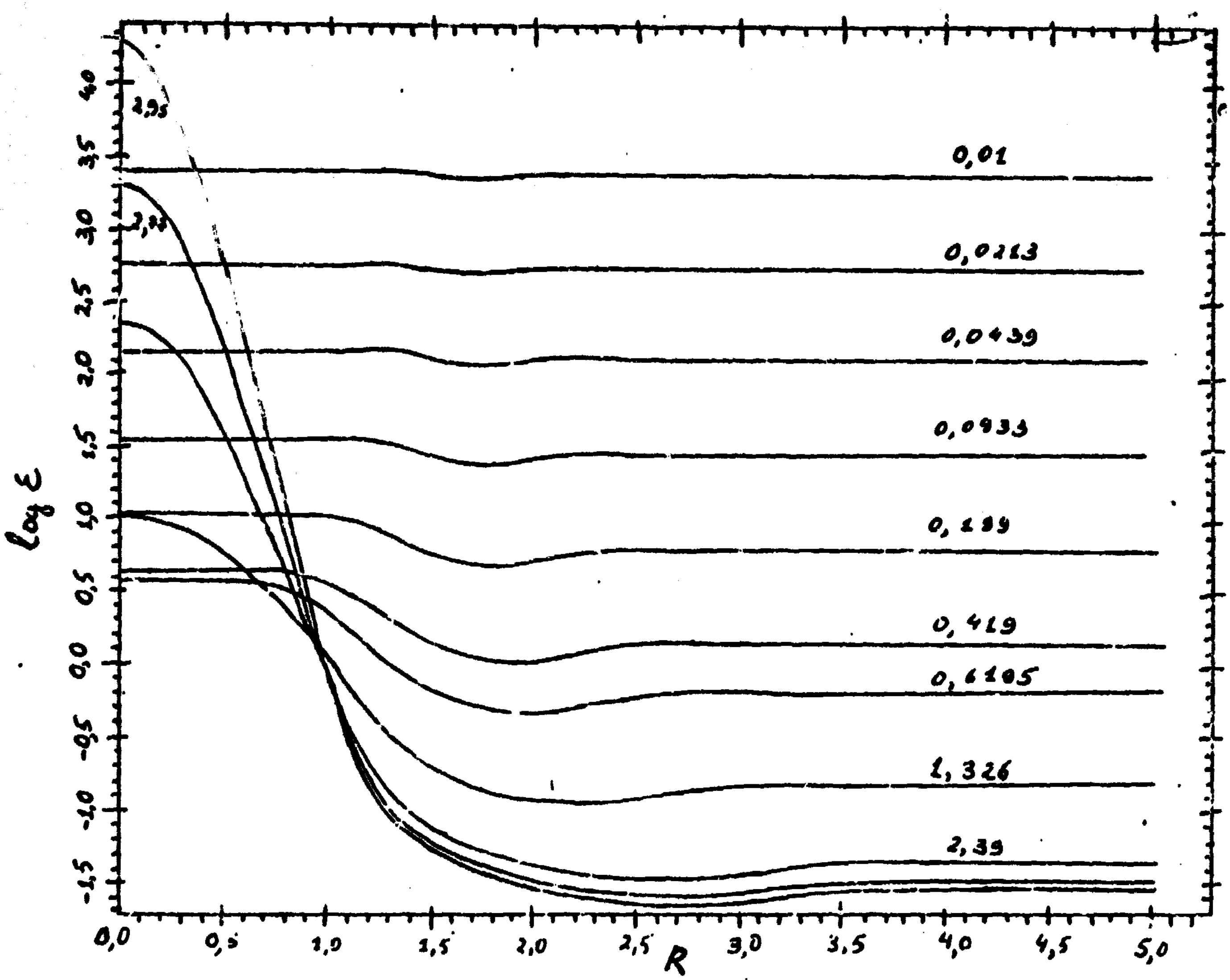

Figure 4. The occurrence of PBH. $R_- = 0.85 \cdot R_{max}$, $\Delta = 0.5 \cdot R_-$.

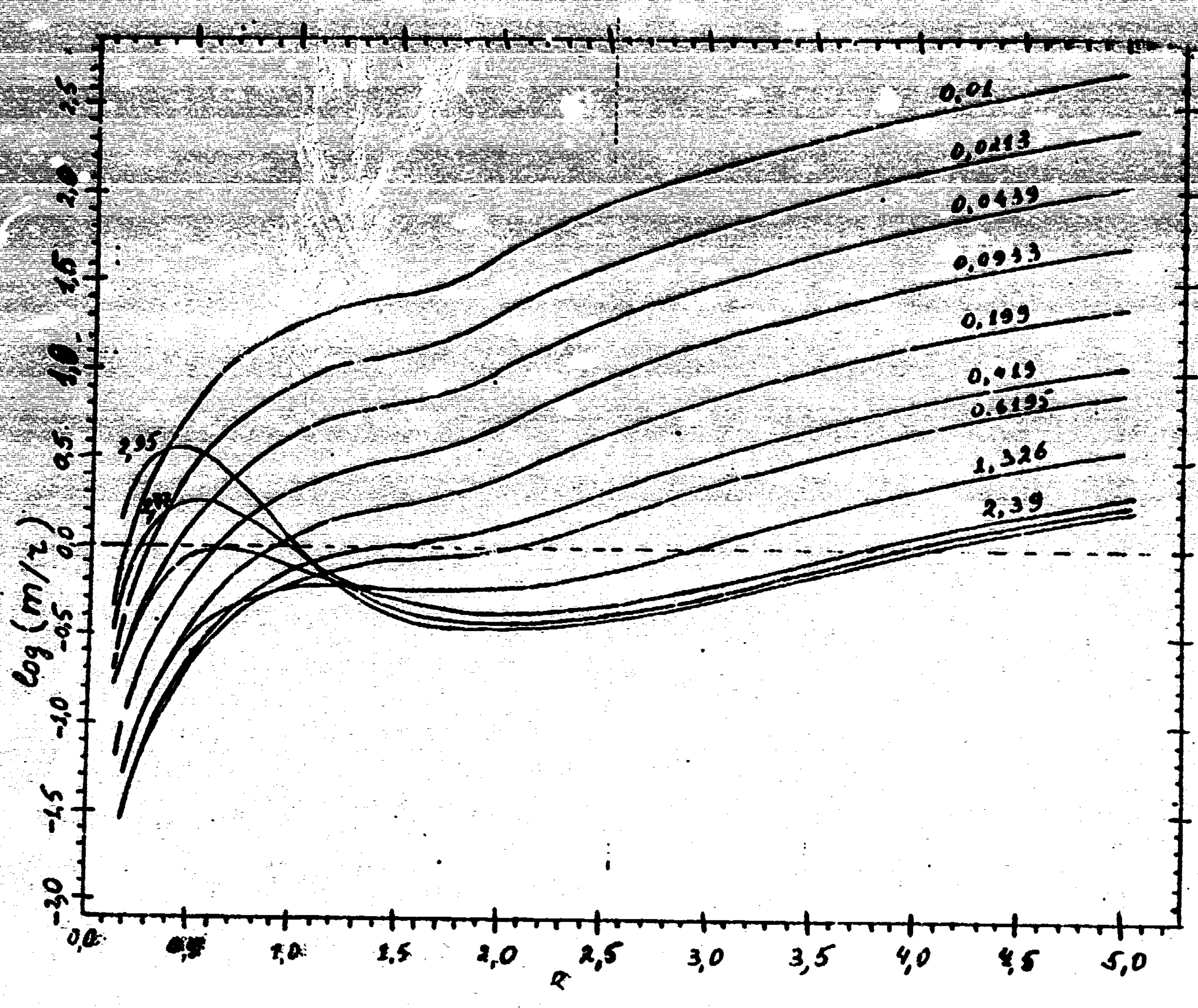

Figure 5. Criteria of PBH. The Lagrange radius is applied on the axis of the abcissa and the ratio m/r on the ordinate axis. The moment of formation of a PBH corresponds to the tangency of the curve with a dashed horizontal.

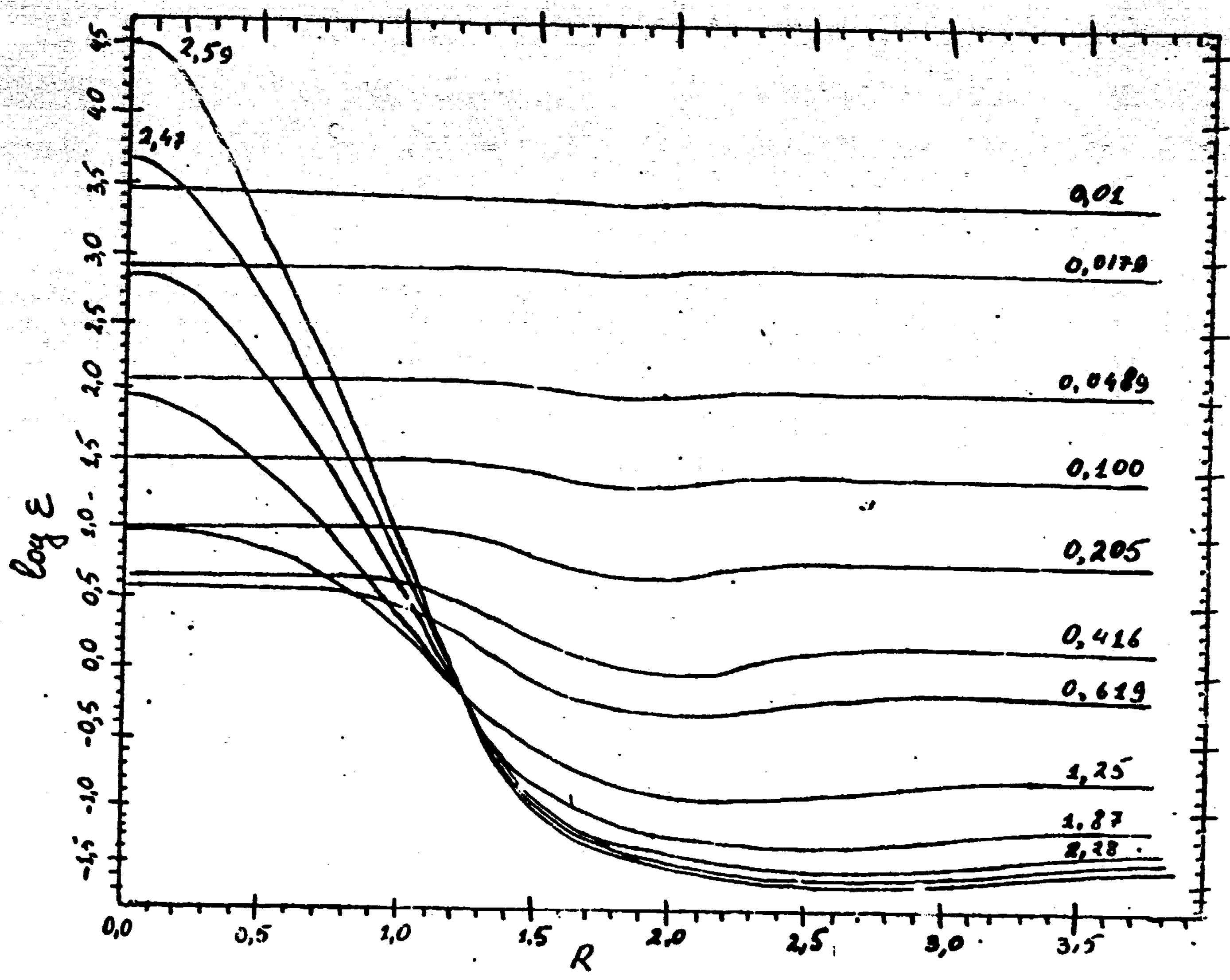

Figure 6. The same as in Figure 4 for $R_- = 0.9 \cdot R_{max}$, $\Delta = 0.5 \cdot R_-$.

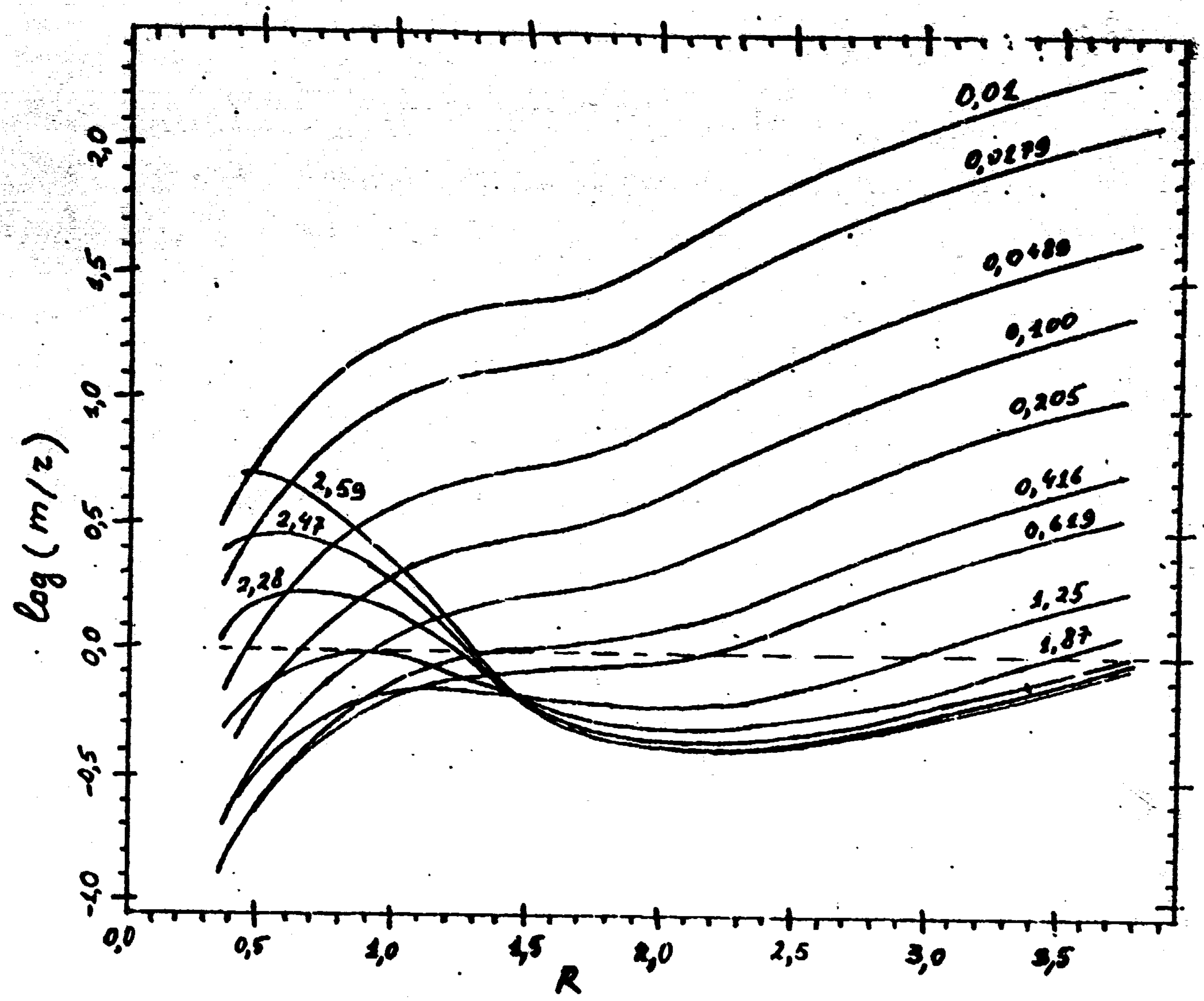

Figure 7. The same as in Figure 5, for $R_{-} = 0.9 \cdot R_{max}$, $\Delta = 0.5 \cdot R_{-}$.

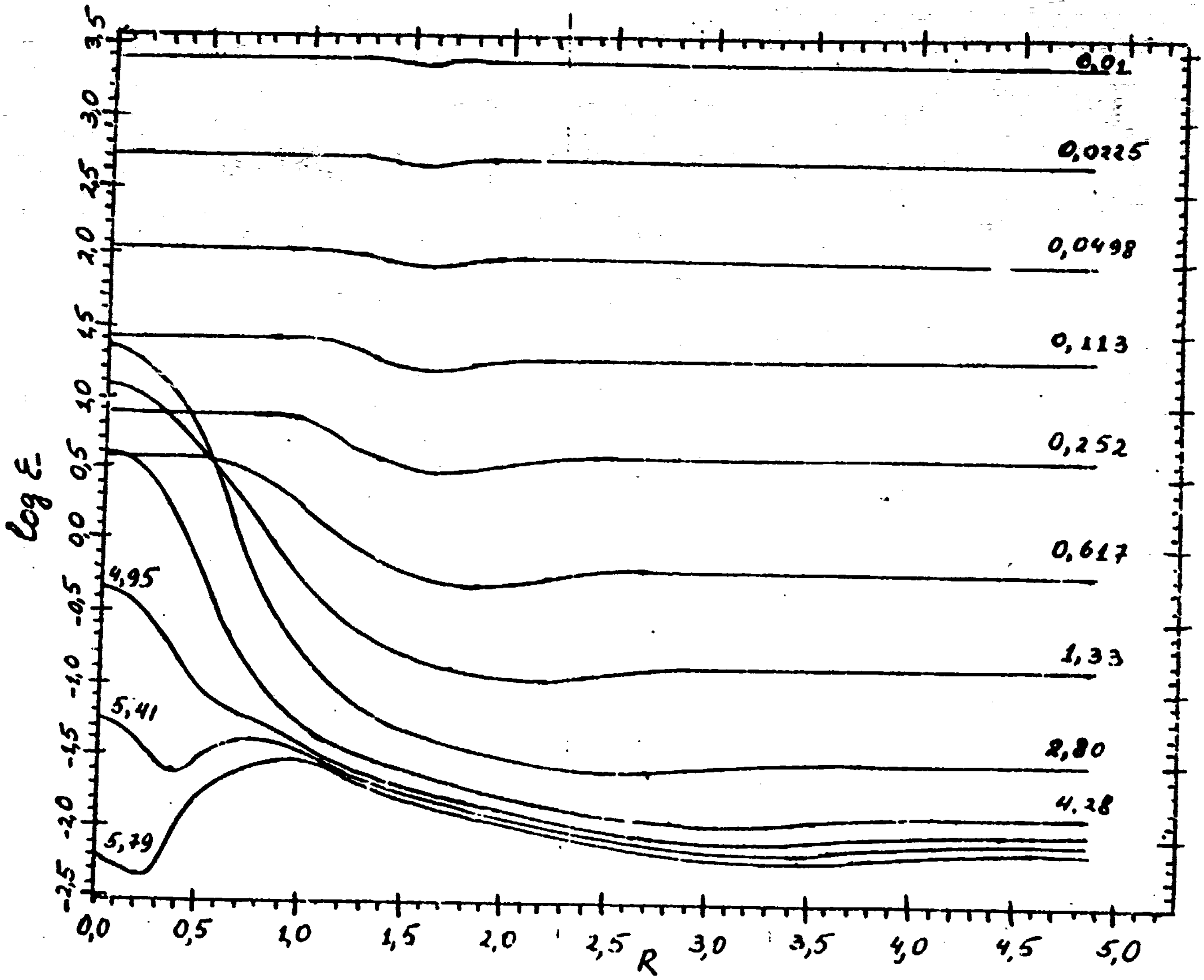

Figure 8. The same as for Figure 2 for $R_{-} = 0.85 \cdot R_{max}$, $\Delta = 0.3 \cdot R_{-}$.

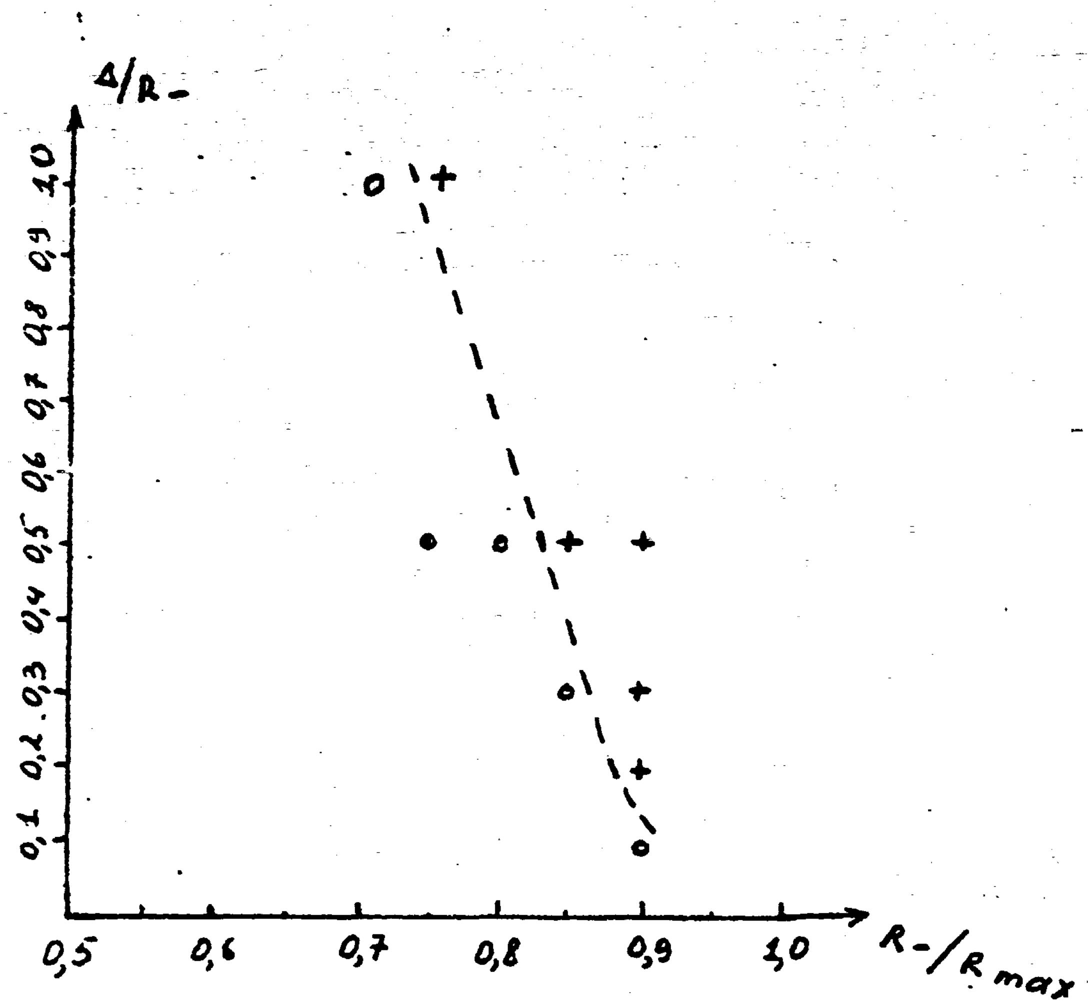

Figure 9. Conditions for the occurrence of PBH. R_-/R_{max} is applied along the axis of the abcissa and Δ/R_- on the ordinate axis. All of the calculated variations are presented in the drawing. "O" -- means the PBH does not form, "+" -- PBH do form. The dashed curve shows the approximate boundary of the field of parameters during which PBH occur.

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