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NASA TECHNICAL MEMORANDUM NASA TM - 75920

ALGORITHMS FOR LOCATING CELESTIAL SOURCES OF X-RAYS AND GAMMA FLARES WITH THE AID OF SEVERAL SPACECRAFT

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(NASA-TM-75920) ALGORITHMS FOR LOCATING CELESTIAL SOURCES OF X-RAYS AND GAMMA FLARES WITH THE AID OF SEVERAL SPACECRAFT (National Aeronautics and Space Administration) 15 p HC A02/MF A01 CSCL 03B G3/93

N80-10991

Unclas 45876

Translation of "Algoritmy lokalizatsii nebesnykh istochnikov rentgenovskikh i gamma-vspleskov s pomoshch'yu neskol'kikh kosmicheskikh apparatov", Academy of Sciences USSR, Institute of Space Research, Report Pr-344, Moscow, 1977, pp 1-14.

12345678910111213141516171819202122232425262728293031
NOV 1979
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 OCTOBER 1979

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-75920		
4. Title and Subtitle Algorithms for Locating Celestial Sources of X-rays and Gamma Flares With the Aid of Several Spacecraft		5. Report Date October 1979 6. Performing Organization Code
7. Author(s) G.A. Mersov		8. Performing Organization Report No. 10. Work Unit No.
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, CA 94063		11. Contract or Grant No. NASW 3199 13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address NATIONAL AERONAUTICS OF SPACE ADMINISTRATION, Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

"Algoritmy lokalizatsii nebesnykh istochnikov rentgenovskikh i gamma-vspleskov s pomoshch' yu neskol' kikh ksomicheskikh apparatov,"Academy of Sciences USSR, Institute of Space Research, Report Pr- 344, Moscow, 1977 pp 1-14.

16. Abstract

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17. Key Words (Selected by Author(s))	18. Distribution Statement
	Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	15	

An algorithm is suggested for defining the coordinants of X-ray and Gamma ray radiation sources by measuring the time lag of the transmission of radiation flares from various points of space in which spacecraft are located. Instances where the 2-x, 3-x and 4x spacecraft are used.

I. The Problem of Localization

A sudden and intermittent increase in the intensity of x-ray or gamma radiation which is detected by spacecraft is commonly referred to as x-ray or gamma-ray flares [1]. Flares are observed comparatively seldom. Therefore, when registering them in n various points (which we will call reference points) M_i, $i = \overline{0, n-1}$, one may subesequently indentify "temporal spectra" of S_i (i.e. the dependence of the radiations intensity on time), which pertain to one and the same flare. Using statistical methods considered in [2], the size of the time lag τ_k, $k = \overline{1, n-1}$ of the flare's transmission in points M_k (as compared to point M_0, which is taken for the original), may be determined by these spectra.

The rates of time lags are used for the localization of a flare's source, i.e. for determining three coordinants of the source $\bar{r}_\gamma$ or only of the direction towards the source $\bar{\ell}_\gamma$, if it is presumed that the source may be considered infinitely remote. Together with $\bar{r}_\gamma$ and $\bar{\ell}_\gamma$ it is also necessary to determine the statistical characterisitics of their errors $\Delta\bar{r}_\gamma$ and $\Delta\bar{\ell}_\gamma$. The methods of locating sources are similar to the methods of hyperbolic navigation (Loran's system) [3]. In view of the fact that the solution to the problem of localization may not turn out to be simple, it is advisable to use additional information on the orientation of the axes of the detectors which register the flares. If the diagrams of the detectors' directionality are not spherical, then the information may be essential for uncovering the reason that the solution is not simple.

Thus it is supposed that when locating the sources the following initial data is used:

a) the rate of time lags τ_k, $k = \overline{1, n-1}$ and the covariance matrix of their error K_τ which is determined by the spectra S_i $(i = \overline{0, n-1})$ [2], and also the moment of Moscow (or Greenwich) time t_0 of the flare's trans-

-mission in the point M_0 (since the flare is not impulsive then t_0 corresponds to some representative point of the spectrum S_0, its origin, for example);

b) the radii-vectors $\bar{r}_i$ of the points M_i, ($i = 0, n - 1$)and the covariance matrixes K_{r_i} of their errors $\Delta\bar{r}_i$ at the moments $t_i = t_0 + \tau_i$ (τ_0 will be made to equal zero) in some system of coordinants OXYZ;

c) the single vectors of direction of the axes of the diagrams of the detectors' directionalities $\bar{d}_i$ in points M_i at the moments t_i in the same system of coordinants OXYZ, and also the angles of semi-solution of the diagrams of directionalities β_i, $i = \overline{0, n - 1}$.

2. Basic Correlations

The quantities τ_k will be considered, when it is convenient, as constitutes of the vector $\bar{\tau}$. All incidental vectors $\Delta\bar{\tau}$ and $\Delta\bar{r}_i$ will be considered uncorrelated between themselves, and their universal means as equal to zero:

$$E(\Delta\bar{\tau})=\bar{0}, \quad E(\Delta\bar{r}_i)=\bar{0}, \quad i=\overline{0,n-1}$$
$$E(\Delta\bar{\tau}\cdot\Delta\bar{r}_i^T)=0, \; E(\Delta\bar{r}_i\cdot\Delta\bar{r}_j^T)=0, \; i\neq j, \; i,j=\overline{0,n-1} \tag{1}$$

where "T" denotes transportation, and "E" dentoes the operation of the universal mean.

We will transfer the origin of the system OXYZ to point M_0 and designable the system $o\xi\eta\zeta$. The radii-vectors $\bar{\rho}_k$ of point M_k in the system $o\xi\eta\zeta$ will equal

$$\bar{\rho}_k = \bar{r}_k - \bar{r}_0 \,, \qquad k = \overline{1,\ n-1}\,, \tag{2}$$

and for the covariance matrixes $K_{\rho_{kk}}$ of their errors $\Delta\bar{\rho}_k$ in accordance with (I), one may write

$$K_{\rho_{kk}} = K_{r_k} + K_{r_0} \,, \quad k = \overline{1,\ n-1}\,. \tag{3}$$

It should be noted that the errors $\Delta\bar{\rho}_k$, as opposed to the errors $\Delta\bar{r}_i$, are correlated, since

$$K_{\rho_{k\ell}} = E\left(\Delta\bar{\rho}_k \cdot \Delta\bar{\rho}_\ell^T\right) = K_{r_0}\,. \tag{4}$$

The single vector $\bar{\ell}_\gamma$ of direction towards an infinite remote source will be exactly the same in both systems, and the radius-vector of a source which is located in a finite distance will equal

$$\bar{\rho}_\gamma = \bar{r}_\gamma - \bar{r}_0\,.$$

We will suppose that the velocity of radiation propogation C is constant and the wave of radiation for infinitely remote sources is flat, and for a source located in a finite distance it is spherical.

Then, in the case of an infinitely remote source the quantities $\bar{\tau}$, $\bar{\rho}_k$ and $\bar{\ell}_\gamma$ will be bound by the correlations

$$\text{I}\quad \begin{cases} R^T\bar{\ell}_\gamma + C\bar{\tau} = 0, & (5)\\ |\bar{\ell}_\gamma| = 1, & (6)\\ \bar{\ell}_\gamma^{\,T}\bar{d}_i \geq \cos\beta_i\,, \quad i = \overline{1,\ n-1}\,, & (7)\end{cases}$$

where R is the matrix whose columns are $\bar{\rho}_k$, $k = \overline{\text{I},\ n - \text{I}}$.

In the case of a source within a finite distance, the quantities τ_k, $\bar{\rho}_k$ and $\bar{\rho}_\gamma$ will be bound by the correlations

$$\text{II}\quad \begin{cases} |\bar{\rho}_k - \bar{\rho}_\gamma| - |\bar{\rho}_\gamma| = C\tau_k\,, \quad k = \overline{1,\ n-1}\,, & (8)\\ \bar{\rho}_\gamma^{\,T}\bar{d}_i \geq \rho_\gamma\cos\beta_i\,, \qquad i = \overline{0,\ n-1}\,. & (9)\end{cases}$$

Algorithms of the definition $\bar{\ell}_\gamma$ and $\bar{r}_\gamma$, with the help of systems I and II, repsectively, will be considered for the cases of $n = 2, 3, 4$. In each of these cases we will suppose that

$$\text{rank}\,(D) = n - 1 \tag{10}$$

where

$$D = R^T R\ . \tag{11}$$

If the conditions (10) is not fulfilled, then it is necessary to exclude one of the points M_k and consider the problem with smaller numbers of reference points.

It should be noted that when $n > 2$ the solutions of systems I and II can depend on the choice of the original reference point M_0 of M_i. We will consider, for example, the case where $n = 3$. Let τ_{ij} be the rate of time lag of the transmission of a flare in point M_i, as compared to the point M_j, which is determined as a result of evaluating the spectra S_i and S_j. Depending on the choice of the initial point three sets of rates of time lags may be obtained: τ_{10} and τ_{20}; and τ_{01}; τ_{21}; τ_{02} and τ_{12}. Since for any method of evaluating spectra the condition $\tau_{ij} = -\tau_{ji}$, (12) should be fulfilled, only three quantities of τ_{ij} can be independent. For example τ_{10}, τ_{02} and τ_{21}, which for convenience we will indicate as $\tau^{(1)}$, $\tau^{(2)}$ and $\tau^{(3)}$, respectively (if this is compared to quantities introduced earlier, then $\tau_1 = \tau^{(1)}$, $\tau_2 = -\tau^{(2)}$) If the condition

$$\alpha \equiv \tau^{(1)} + \tau^{(2)} + \tau^{(3)} = 0\,, \tag{13}$$

is fulfilled, then, as is easily seen, the solutions of systems I and II do not depend on the choice of the original point. The condition (13) will be fulfilled, evidently, in the case that the spectra S_0 S_1 and S_2 are identical. Usually condition (13) is not fulfilled and then the difference of the quantity α from zero will characterize the accuracy of the definition of $\tau^{(k)}$. Another way of using condition (13) is by bringing it into the interpretation of spectra with the definition of $\tau^{(k)}$. In this case all spectra will be processed together. Specifically, if $\tau^{(k)}$ is defined as the points of the extremum of several functions $\varphi_k(\tau)$, i.e. if

$$\frac{d\varphi_k(\tau)}{d\tau}\Big|_{\tau \neq \tau_k} = 0 \quad , \qquad b_k \equiv \frac{d^2\varphi_k(\tau)}{d\tau^2}\Big|_{\tau = \tau_k} > 0 \; (\text{or } < 0), \quad (14)$$

then the condition (13) may be considered by introducing amendments into the quantities $\tau^{(k)}$. It may be shown that the quantities $\hat{\tau}^{(k)}$, which guarantee the attainment of the conditional extremum of functions $\varphi_k(\tau)$ (i.e. by calculating condition (13)), in a linear approach will equal

$$\hat{\tau}^{(k)} = \tau^{(k)} - \alpha / b_k (b_1^{-1} + b_2^{-1} + b_3^{-1}), \; k = 1,2,3 \,. \qquad (15)$$

If all b_k are equal amongst themselves, then

$$\hat{\tau}^{(k)} = \tau^{(k)} - \alpha/3 \,, \quad k = 1,2,3 \,. \qquad (16)$$

These equations may be applied if b_k are not know.

3. The Case of Two.

The solution to system I will be the vector $\bar{\ell}_\gamma$, which equals

$$\bar{\ell}_\gamma = (\bar{\rho}_1/\rho_1)\cos\alpha + (\bar{\varkappa}/\varkappa)\sin\alpha \,, \qquad (17)$$

where

$$\cos\alpha = -c\tau_1/\rho_1 \,, \quad 0 \leqslant \alpha \leqslant \pi \,, \qquad (18)$$

and $\bar{\varkappa}$ is the random vector, orthogonal $\bar{\rho}_1$:

$$\bar{\varkappa}^T \bar{\rho}_1 = 0 \,. \qquad (19)$$

In the celestial sphere on assemble of solutions of $\bar{\ell}_\gamma$ form the circumference of radius α with the center at point $\bar{\rho}_1/\rho_1$.

Errors of measurements $\Delta\tau_1$ and $\Delta\bar{\rho}_1$, lead to the error $\Delta\bar{\ell}$ of the definition of $\bar{\ell}_\gamma$, which is directed along the normal to this circumference at point $\bar{\ell}_\gamma$. The root-mean-square declination σ_ℓ of the error $\Delta\ell$ depends upon the assumed quantity $\bar{\ell}_\gamma$ and equals [4] :

$$\sigma_\ell = \sigma_S/(\rho_1 \sin\alpha), \tag{20}$$

where

$$\sigma_S = (c^2 K_\tau + \bar{\ell}_\gamma{}^T K_{\rho_{11}} \bar{\ell}_\gamma)^{1/2}. \tag{21}$$

Since the dimension of the matrix K_τ in this case equals I, K_τ is simply the dispersion of error $\Delta\tau_1$.

The assemble of solutions of system II forms a surface which is the hyperboloid of the rotation whose focus lies in points M_0 and M_1. The semi-major axis of the rotation equals $|c\tau_1|/2$. When $\tau_1 > 0$ the solutions are the points of that cavity of the hyperboloid which encompasses the focus M_1 and when $\tau_1 < 0$ the solutions are the points of that cavity which encompass the focus M_0. If $\tau_1 = 0$, then the hyperboloid degenerates into a plane which is normal to $\bar{\rho}_1$ and transient through its center. In consequence of the indeterminacy of the solution of system II, it is more expedient to solve the problem, which is reversed with respect to the problem of localization: the verification of the correspondence to system II of the coordinants of some cosmological objects which could be flare sources.

If $\tilde{\bar{\rho}}_\gamma$ is the radius-vector of such a "suspicious" object, then a rate of time lag $\tilde{\tau}_1$, which corresponds to this object, can be determined:

$$\tilde{\tau}_k = \left(|\rho_k - \tilde{\bar{\rho}}_\gamma| - |\tilde{\bar{\rho}}_\gamma|\right)/c, k = 1 . \tag{22}$$

The hypothesis that a region of space is a "suspected" flare source can be assumed by fulfilling the condition

$$|\tau_1 - \tilde{\tau}_1| / \sigma_\tau \leqslant K_\delta , \tag{23}$$

where K_δ depends upon the assumed probability of δ to repudiate the correct hypothesis (the level of significance). If the law of distribution of the error $\Delta\tau_1$, is normal, then the quantity K_δ will be determined by the equation

$$\mathrm{erf}\left(K_\delta/\sqrt{2}\right) = 1 - \delta . \tag{24}$$

4. <u>The Case of Three</u>

In correspondence [4] with the solution of system I, vector $\bar{\ell}_\gamma$, will equal

$$\bar{\ell}_\gamma = ([\bar{\varkappa} \times \bar{\rho}_2]\, c\tau_1 + [\bar{\rho}_1 \times \bar{\varkappa}]c\tau_2 \pm \bar{\varkappa}\sin\varphi)/\varkappa^2 , \tag{25}$$

where

$$\bar{\varkappa} = \bar{\rho}_1 \times \bar{\rho}_2 , \tag{26}$$

$$\cos\varphi = \left(c\bar{\tau}^T D^{-1} c\bar{\tau}\right)^{1/2} , \quad 0 \leqslant \varphi \leqslant \pi/2 . \tag{27}$$

The expression (27) defines the size of the angle φ between the directions towards the source and the reference plane which passes through points M_0, M_1 and M_2. The condition $|\cos\varphi| \leq 1$ defines the region of possible values of $\bar{\tau}$. The region is limited by the ellipse

$$c\bar{\tau}^T D^{-1} c\bar{\tau} = 1 , \tag{28}$$

which is written into the square $|c\tau_1| \leqslant \rho_1$ and $|c\tau_2| \leqslant \rho_2$. On the border of the region (28) there exists a single solution $\bar{\ell}_\gamma$, which lies in the reference plane. Within this area there exist two solutions of $\bar{\ell}_\gamma$, which are symmetrical relative to the reference plane. For uncovering the non-simplicity it is necessary to appropriate, in this case, condition (7).

If $\bar{\ell}_\gamma$ does not lie in the reference plane, i.e.

$$\bar{\ell}_\gamma^T \bar{\varkappa} \neq 0 , \tag{29}$$

then the covariance matrix K_ℓ of the error $\Delta\bar{\ell}$ is determined in the following manner [4] :

$$K_\ell = S\,(c^2 K_\tau + K)\,S^T , \tag{30}$$

where S is the matrix of the dimension 3 X 2, whose columns are equal to:

$$\bar{S}_1 = [\bar{\ell}_\gamma \times \bar{\rho}_2] / \bar{\ell}^T \bar{\varkappa} , \quad \bar{S}_2 = [\bar{\rho}_1 \times \bar{\ell}_\gamma] / \bar{\ell}^T \bar{\varkappa} , \tag{31}$$

and K is the matrix of dimension 2 X 2, whose elements equal:

$$[K]_{k\ell} = \bar{\ell}_\gamma^T K_{\rho_{k\ell}} \bar{\ell}_\gamma , \quad k, \ell = 1, 2 , \tag{32}$$

where the covariance matrixes $K_{\rho_{k\ell}}$ are determined in accordance with (3) and (4).

At least one natural value of the matrix K_ℓ is equal to zero. The corresponding natural vector is collinear to $\bar{\ell}_\gamma$. Thus, K_ℓ defines the dispersion of $\Delta\bar{\ell}$ in the plane which is tangent to the celestial sphere at point $\bar{\ell}_\gamma$. If $\bar{\ell}_\gamma$ lies in the reference plane, that is

$$\bar{\ell}_\gamma^T \bar{\varkappa} = 0 , \tag{33}$$

then the product of the error $\Delta\bar{\ell}$, which is orthogonal to the reference plane, turns out to be indeterminant. The root-mean-square declination of the projections of error $\Delta\bar{\ell}$ on the reference plane can be determined in the same manner as in the case where $n = 2$ (20).

For a source located within a finite distance, system II determines an assemble of possible solutions which form a line of of intersection of two hyperboloids of the rotation. As in the case where $n = 2$, in this instance it is most expedient to solve the reversed problem, determining in accordance with (22) the quantities $\tilde{\tau}_1$ and $\tilde{\tau}_2$ for the coordinants of the supposed sources of flares and checking condition (23) for each quantity $\tilde{\tau}_k$.

5. The Case of Four

When measurement errors are lacking the solution of system II would be the vector

$$\bar{S} = -R^{T^{-1}} c\bar{\tau} \,. \tag{34}$$

The influence of the measurement errors leads to the failure of condition (6). Therefore, in the capacity of $\bar{\ell}_\gamma$ the normalized vector can be used:

$$\bar{\ell}_\gamma = \bar{S}/S \,. \tag{35}$$

However, it is more convenient to use the more general algorithm, which determines the solution of system II, and in a particular instance the solution of system I also. In accordance with the algorithm [4] the distance to a source ρ_γ is determined from the quadratic equation

$$|\mu_\gamma \bar{P} + \bar{S}| = 1 \,, \tag{36}$$

$$\mu_\gamma \equiv 1/\rho_\gamma \,, \tag{37}$$

$$\bar{P} = R^{T^{-1}} \bar{q} \,, \tag{38}$$

and the constituents q_k vector $\bar{q}$ equal (39)

$$q_k = (\rho_k^2 - c^2\tau_k^2)/2 \,, \quad k = 1, 2, 3.$$

Equation (36) has a single positive solution $\mu_\gamma > 0$ when $S < I$, i.e., if vector $c\bar{\tau}$ lies within the ellipsoid

$$c\bar{\tau}^{T} D^{-1} c\bar{\tau} = 1. \tag{40}$$

When S=I, i.e. on the verification of the ellispoid, there exists two solutions. One of them corresponds to an infinitely remote source ($\mu_\gamma = 0$), and the other corresponds to a source located at a distance

$$\rho_\gamma = -P^2 / 2\bar{P}^T \bar{S}. \tag{41}$$

When S=I both solutions (36) may be positive. Each solution of μ_γ determines the direction towards a source at point M_0 and equals

$$\bar{\ell}_0 = \mu_\gamma \bar{P} + \bar{S}. \tag{42}$$

The accuracy of determining the distance to a source depends on the position of the source relative to the reference points. For the case where the distance to a source significantly exceeds the distance between reference points, a simple expression for the root-mean-square declination σ_ρ of the error of determining the distance can be obtained:

$$\sigma_\rho \leqslant \sigma_S \, \rho_\gamma^2 / \rho^2_{min}, \qquad \sigma_S = 2\left(\sigma^2_{\tau_{max}} + 2\,\sigma^2_{r_{max}}\right)^{1/2}, \tag{43}$$

where ρ_{min} is the least of the distance ρ_1 ; ρ_2 , ρ_3 ; $\sigma^2_{\tau_{max}}$ is maximum natural value of the matrix K_τ ; and $\sigma^2_{r_{max}}$ is the greatest of the maximum natural calues of the matrixes K_{r_i} .

If it is demanded that σ_ρ be N times less than the determined distance ρ_γ , then an expression for the greatest distance to sources which can be determined with the fullfillment of the following condition can be obtained:

$$\rho_{\gamma\,max}(\underset{[parsec]}{\text{парсек}}) \leqslant \rho_{min}(A.E)/N \cdot \Delta, \tag{44}$$

where Δ is the relationshi σ_s/ρ_{min}, expressed in angular seconds.

6. Conclusion

For the localization of infinitely remote sources it is sufficient to register a flare at three spacecrafts.
The bivalency arising in this case can be eliminated with the use of directed detectors. However it is impossible to distinquish a flat-wave from a spherical. Therefore, the analysis of registrations of flares from nearby sources will determine the direction, which can differ considerably from the direction towards the true source. Thus, when using three sc it is necessary to verify (the reverse problem) the possibility of flare radiation from "suspicious" areas of space. The identification of wave form and the subsequent determination of the direction towards an infinitely remote_source or coordinants of nearby sources are possible only with the use of four sc. In all cases of localization there occurs the degeneration of the measurement system when the source and any two reference points are located on one straight line. During the working of the problem it is necessary to exclude one of these points and conduct the localization using the remaining points.

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