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SUMMARY

This paper encompasses that part of error analysis which deals with errors resulting from the instrumentation used in measuring pertinent wind-tunnel parameters. The parameters, for this discussion and analysis, are limited to those required for the wind-tunnel model measurements. The pertinent parameters, their standard deviations, and the theoretical derivation of them, are given. BASIC programs and plots for the standard deviations of dynamic pressure, Mach number, and Reynolds number are included for the National Transonic Facility. A literature search was made back to 1934 and it was found that information of this kind is almost nonexistent.

INTRODUCTION

In any successful research endeavor, it should be possible to experimentally validate the results. In addition, the same or other experimenters duplicating the parameters of the experiment should be able to get the same results within experimental error. When this cannot be achieved, it must be concluded that the theories or formulations that were to be validated were incorrect or that large errors of some kind entered into the experiment, which prevents duplication of results within the experimental error.

In numerous reported wind-tunnel measurements, it is indicated or can be inferred that correlation of measurements between similar tests has not reached a level of desired consistency. The purpose of this paper is to determine the extent to which the accuracy and precision of the measuring instrumentation contributes to the total error. Though it is not dealt with herein, the two other types of errors are those introduced in establishing and measuring pertinent tunnel parameters and those entering into the acquisition and reduction of data.

Before any results can be compared, it is imperative that their accuracies and precisions be reported in a consistent manner. The major experimental errors are either systematic or random (accidental). Although systematic errors do exist and occur, they are not statistical and can be corrected or eliminated. Therefore, they are not further discussed in this paper. This work assumes that the errors are random (accidential), which means that they are variable in magnitude, follow no pattern in occurrence, and can be either negative or positive. General references 1, 2, and 3 show that the arithmetic mean of the measured value is the one that most closely approaches the true value of the measured quantity. The standard deviation, which is the root-mean-square value of the deviations, best describes the scatter of the measured values from the mean value. In reporting any experimentally measured results, therefore, the accuracy (arithmetic mean) and the standard deviation of the mean value $\sigma/\sqrt{n}$ should be given or the mean and σ preferably with n included. When graphing these results, either set of values should also be shown. In the following pages and in appendix A it is shown how to obtain the standard deviations of significant parameters of wind-tunnel measurements. Knowing the number of observations taken to obtain the standard deviation, the standard deviation of the mean value is obtained. From these and their mean values, it can be determined where accurate and precise measurements are more critical and necessary. This may require more research and development to advance the state of the instrument art as well as the other areas involved in the final wind-tunnel data accumulation.

Since the current problem is a manifold problem, this paper deals only with the accuracy and precision of the measuring instrumentation being used in wind-tunnel testing at present, the theoretical analyses which show where the errors are produced, their criticality and how they combine, the development of the second-order strain-gauge balance interactions and how they combine, and why higher-order terms are ignored. Then, using the parameters for the National Transonic Facility (NTF) at Langley Research Center and the instrument errors for the instruments used in this facility in the derived formulas, the following standard deviations were obtained: σ(Reynolds number), σ(Mach number), σ(dynamic pressure), σ(normal-force model), σ(axial-force model), σ(pitching-moment model), σ(yawing-moment model), σ(rolling-moment model), and subsidiary sigma values for items such as normal-force coefficient, side-force coefficient, lift and drag coefficients, etc. Programs have been written (see appendix B) for the Hewlett-Packard 9830A calculator for obtaining σ(Mach number), σ(Reynolds number), and σ(dynamic pressure), and results for the NTF have been plotted. Assuming the values for the parameters and the instrumentation for the NTF remain unchanged, potential users of the NTF should find the error results of this report useful. If changes in the values do occur, these can readily be taken care of in the data section of the Hewlett-Packard program.

Use of trade names or names of manufacturers in this report does not constitute an official endorsement of such products or manufacturers, either expressed or implied, by the National Aeronautics and Space Administration.

SYMBOLS

A reference area

A_b local area (base) where Δp acts across base of model

B_i balance output including second-order terms; for example, B_1 is balance normal-force output; see equation (11) $(i = 1, 2, \ldots, 6)$

b reference length

C_A axial-force coefficient

C_{AB} base axial-force coefficient

C_D drag coefficient

C_{DB} base drag-force coefficient

C_L lift coefficient

C_l rolling-moment coefficient

C_m pitching-moment coefficient

C_N normal-force coefficient

C_n yawing-moment coefficient

C_p pressure coefficient

C_Y side-force coefficient

$\bar{c}$ mean geometric chord

c_v specific heat at constant volume

F_A axial-force component on model

F_N normal-force component on model

F_Y side-force component on model

$K_{i,j}$ jth component factor interacting to contribute to ith component; for example, $K_{1,2}$ is side-force component factor interacting to contribute first-order terms to normal force; see table I ($i = 1, 2, \ldots, 6$; $j = 1, 2, \ldots, 27$)

ℓ reference length

M Mach number

M_X rolling moment

M_Y pitching moment

M_Z yawing moment

n number of observations

p static pressure

p_b base static pressure

p_l local static pressure

p_t stagnation pressure

p_∞ free-stream static pressure

q dynamic pressure

q_∞ free-stream dynamic pressure

R Reynolds number

S reference surface

T absolute temperature

T_t total temperature

α angle of attack

β sideslip angle

γ ratio of specific heat of tunnel gas at constant pressure to that at constant volume

μ_t dynamic viscosity

$\sigma(\ )$ standard deviation

There are six balance components (F_A, F_N, F_Y, M_X, M_Y, and M_Z). A superscript "(2)" on any of these subscripts indicates the second-order term of that component. For example, $F_{N(2)}$ is the second-order term of the normal-force component. Any two of these components together indicate the interaction of the second term upon the first. For example, F_NF_Y is the cross term whereby the side-force component contributes to the normal-force output.

ERROR ANALYSIS

When considering the following sections, it should be understood that small errors in the independent variables are assumed in the resulting formulas. If the errors become so large that second- or higher-order terms are not negligible, then the errors are derived by using Δ values and not exact differentials. It is assumed that random errors are being dealt with whose magnitudes are variable and whose occurrence is disordered. It is also assumed that their distribution is normal or Gaussian if systematic errors are removed and good sampling practices employed. In those equations that are applicable, the values used for the constants are for air. Where different substances are used, the appropriate constants for that substance should be used.

The general texts on errors given in the references show that the mean value of a measured quantity (assuming random errors only) approaches the true value of that quantity. If systematic (bias) errors exist, then the mean value becomes displaced from its true value and results in inaccuracy. Systematic errors are not statistical and by definition can be eliminated when they are discovered. The general reference texts also show that the root-mean-square value of the measured quantity equals its standard deviation from the mean value. The mean value and the standard deviation can thus define the accuracy and precision of a random measurement and should always be used together when this is done. If a function $f(x,y,z)$ is dependent on the three independent variables x, y, and z, the standard deviation may be obtained by taking the exact differential of the function, e.g.,

$$df = \frac{\partial f}{\partial x}\,\delta x + \frac{\partial f}{\partial y}\,\delta y + \frac{\partial f}{\partial z}\,\delta z \tag{1}$$

If df is the deviation from f, then the right side of equation (1) must be the sum of the deviations of the three independent variables. Using superposition and the definition of σ, then

$$\sigma(f) = \left\{\left[\frac{\partial f}{\partial x}\,\sigma(x)\right]^2 + \left[\frac{\partial f}{\partial y}\,\sigma(y)\right]^2 + \left[\frac{\partial f}{\partial z}\,\sigma(z)\right]^2\right\}^{1/2} \tag{2}$$

The mean value of f is as follows: $\bar{f} = \sum_{n=1}^{n} \frac{f(x,y,z)}{n}$.

This section is a summary of the standard deviations which are valid over all Mach numbers and assume the use of differentials (small errors). The derivations for these values as well as the remainder of the balance output errors are shown in appendix A.

Standard deviation for Mach number (ref. 4):

$$M^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_t - p}{p} + 1\right)^{(\gamma-1)/\gamma} - 1\right] \tag{3}$$

$$\sigma(M) = \left(\frac{p_t}{p}\right)^{-1/\gamma}\left(\frac{1}{M\gamma p}\right)\left[\sigma(p_t)^2 + \left(\frac{p_t}{p}\right)^2 \sigma(p)^2\right]^{1/2} \tag{4}$$

$$\frac{\sigma(M)}{M} = \left(\left\{\frac{\gamma - 1}{2\gamma\left[1 - \left(\frac{p}{p_t}\right)^{(\gamma-1)/\gamma}\right]}\right\}^2 \left\{\left[\frac{\sigma(p_t)}{p_t}\right]^2 + \left[\frac{\sigma(p)}{p}\right]^2\right\}\right)^{1/2} \tag{5}$$

Standard deviation for dynamic pressure (ref. 4):

$$q = \frac{\gamma}{2} pM^2 \tag{6}$$

$$\sigma(q) = \left\{ \left[\frac{\gamma M^2}{2} \sigma(p) \right]^2 + \left[\gamma p M \sigma(M) \right]^2 \right\}^{1/2} \tag{7}$$

$$\frac{\sigma(q)}{q} = \left\{ \left[\frac{\sigma(p)}{p} \right]^2 + \left[\frac{2\sigma(M)}{M} \right]^2 \right\}^{1/2} \tag{8}$$

Standard deviation for Reynolds number per meter:

$$\frac{R}{\ell} = 4.790 \times 10^4 \left[\frac{\left(\frac{p_t}{p}\right)^{-4/7} - \left(\frac{p_t}{p}\right)^{-6/7}}{T_t} 5 \right]^{1/2} \left(p_t{}^{5/7} p^{2/7} + \frac{110.3 p_t}{T_t} \right) \tag{9}$$

$$\sigma \frac{R}{\ell} = \left((4.790 \times 10^4)^2 \left\{ \left(\left\{ \left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7} \right] 5 \right\}^{1/2} \right. \right. \right.$$

$$\left. \times (-1) \left[\left(\frac{p}{p_t}\right)^{2/7} \frac{T_t + 220.6}{T_t{}^3} \right] p_t \, \sigma(T_t) \right)^2 + \left[\left(\left\{ \left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7} \right] 5 \right\}^{1/2} \right. \right.$$

$$\times \frac{\frac{5}{7}\left(\frac{p}{p_t}\right)^{2/7} T_t + 110.3}{T_t{}^2} + \frac{\left(\frac{p}{p_t}\right)^{2/7} T_t + 110.3}{T_t{}^2} \frac{p_t}{p} \frac{5}{2} \left\{ \left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7} \right] 5 \right\}^{-1/2}$$

(Equation continued on next page)

$$\times \frac{-4\left(\frac{p}{p_t}\right)^{11/7} + 6\left(\frac{p}{p_t}\right)^{13/7}}{7}\Bigg)\sigma(p_t)\Bigg]^2 + \Bigg[\Bigg(\Bigg\{\left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7}\right]5\Bigg\}^{1/2} \frac{2}{7T_t}\left(\frac{p}{p_t}\right)^{-5/7}$$

$$+ \frac{\left(\frac{p}{p_t}\right)^{2/7} T_t + 110.3}{T_t^2} \; \frac{p_t}{p}\,\frac{5}{2}\left\{\left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7}\right]5\right\}^{-1/2}$$

$$\times \frac{4\left(\frac{p}{p_t}\right)^{4/7} p^{6/7} - 6\left(\frac{p}{p_t}\right)^{6/7}}{7}\Bigg)\sigma(p)\Bigg]^2\Bigg\}\Bigg)^{1/2} \qquad (10)$$

For an interaction correction of a six-component strain-gauge balance, it is assumed that each component contributes second-order terms and that the second-order terms are small. Taking two components at a time to obtain the second-order terms (refs. 5 and 6), results in $6C_2 = \frac{6!}{2!(6-2)!} = 15$. Fifteen cross terms plus 6 first-order terms plus 6 squared terms result in 27 terms for each of the 6 model components obtained from the strain-gauge balances. These terms are listed in table I. The symbolization (see table I) of a contribution of side force, for example, on the normal force is $K_{1,2}$ times F_Y for the first-order term and $K_{1,8}$ times F_Y for the second-order term. This notation is more easily grasped in the following matrix equations. Terms involving higher than second order are usually too small to be significant and will be ignored. The balance output column matrix is

$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix} = \begin{bmatrix} \text{Normal-force output from balance} \\ \text{Side-force output from balance} \\ \text{Axial-force output from balance} \\ \text{Pitching-moment output from balance} \\ \text{Yawing-moment output from balance} \\ \text{Rolling-moment output from balance} \end{bmatrix} = \left[B_i\right] \quad (i = 1, 2, \ldots, 6) \qquad (11)$$

The force-moment equations for a six-component strain-gauge balance are

$$
\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix}
=
\begin{bmatrix}
1 & K_{1,2} & K_{1,3} & K_{1,4} & K_{1,5} & K_{1,6} \\
K_{2,1} & 1 & K_{2,3} & K_{2,4} & K_{2,5} & K_{2,6} \\
K_{3,1} & K_{3,2} & 1 & K_{3,4} & K_{3,5} & K_{3,6} \\
K_{4,1} & K_{4,2} & K_{4,3} & 1 & K_{4,5} & K_{4,6} \\
K_{5,1} & K_{5,2} & K_{5,3} & K_{5,4} & 1 & K_{5,6} \\
K_{6,1} & K_{6,2} & K_{6,3} & K_{6,4} & K_{6,5} & 1
\end{bmatrix}
\begin{bmatrix} F_N \\ F_Y \\ F_A \\ M_Y \\ M_Z \\ M_X \end{bmatrix}
+
\begin{bmatrix}
K_{1,7} & K_{1,8} & K_{1,9} & K_{1,10} & K_{1,11} & K_{1,12} & K_{1,13} & K_{1,14} & K_{1,15} & K_{1,16} & K_{1,17} & K_{1,18} & K_{1,19} & K_{1,20} & K_{1,21} & K_{1,22} & K_{1,23} & K_{1,24} & K_{1,25} & K_{1,26} & K_{1,27} \\
K_{2,7} & K_{2,8} & K_{2,9} & K_{2,10} & K_{2,11} & K_{2,12} & K_{2,13} & K_{2,14} & K_{2,15} & K_{2,16} & K_{2,17} & K_{2,18} & K_{2,19} & K_{2,20} & K_{2,21} & K_{2,22} & K_{2,23} & K_{2,24} & K_{2,25} & K_{2,26} & K_{2,27} \\
K_{3,7} & K_{3,8} & K_{3,9} & K_{3,10} & K_{3,11} & K_{3,12} & K_{3,13} & K_{3,14} & K_{3,15} & K_{3,16} & K_{3,17} & K_{3,18} & K_{3,19} & K_{3,20} & K_{3,21} & K_{3,22} & K_{3,23} & K_{3,24} & K_{3,25} & K_{3,26} & K_{3,27} \\
K_{4,7} & K_{4,8} & K_{4,9} & K_{4,10} & K_{4,11} & K_{4,12} & K_{4,13} & K_{4,14} & K_{4,15} & K_{4,16} & K_{4,17} & K_{4,18} & K_{4,19} & K_{4,20} & K_{4,21} & K_{4,22} & K_{4,23} & K_{4,24} & K_{4,25} & K_{4,26} & K_{4,27} \\
K_{5,7} & K_{5,8} & K_{5,9} & K_{5,10} & K_{5,11} & K_{5,12} & K_{5,13} & K_{5,14} & K_{5,15} & K_{5,16} & K_{5,17} & K_{5,18} & K_{5,19} & K_{5,20} & K_{5,21} & K_{5,22} & K_{5,23} & K_{5,24} & K_{5,25} & K_{5,26} & K_{5,27} \\
K_{6,7} & K_{6,8} & K_{6,9} & K_{6,10} & K_{6,11} & K_{6,12} & K_{6,13} & K_{6,14} & K_{6,15} & K_{6,16} & K_{6,17} & K_{6,18} & K_{6,19} & K_{6,20} & K_{6,21} & K_{6,22} & K_{6,23} & K_{6,24} & K_{6,25} & K_{6,26} & K_{6,27}
\end{bmatrix}
\begin{bmatrix} F_N^2 \\ F_Y^2 \\ F_A^2 \\ M_Y^2 \\ M_Z^2 \\ M_X^2 \\ F_N F_Y \\ F_N F_A \\ F_N M_Y \\ F_N M_Z \\ F_N M_X \\ F_Y F_A \\ F_Y M_Y \\ F_Y M_Z \\ F_Y M_X \\ F_A M_Y \\ F_A M_Z \\ F_A M_X \\ M_Y M_Z \\ M_Y M_X \\ M_Z M_X \end{bmatrix}
\tag{12}
$$

If we let B equal the matrix in the left column, Y equal the matrix in the middle column, and Z equal the matrix in the right column of equations (12), and let K equal the 6×6 matrix and K_1 equal the 6×21 matrix, then

$$B = KY + K_1 Z$$

Since B equals the output from the strain-gauge balances and Y equals the actual loads applied to the balance, solving for Y yields

$$K^{-1}KY = K^{-1}B - K^{-1}K_1 Z$$

Since $K^{-1}K$, if K^{-1} exists, is the identity or unity matrix $K^{-1}K = [1]$ and $Y = K^{-1}B - K^{-1}K_1 Z$. From this, $\sigma(Y)$ can be obtained, which is the same as $\sigma(F_N)$, $\sigma(F_Y)$, $\sigma(F_A)$, $\sigma(M_Y)$, $\sigma(M_Z)$, or $\sigma(M_X)$.

Since obtaining Y is prolonged and tedious and since it, as well as $\sigma(Y)$ is not available as yet, the development in this paper proceeds with the outputs of the strain-gauge balance components, matrix B and $\sigma(B)$, that is, $\sigma(B_1)$, $\sigma(B_2)$, $\sigma(B_3)$, $\sigma(B_4)$, $\sigma(B_5)$, and $\sigma(B_6)$. From this it is shown how to obtain the extrema and thus the errors of the Y matrix. As shown in equation (13), $\sigma(B_1)/B_1 = f(F_N, F_Y, F_A, M_Y, M_Z, M_X$, the interaction coefficients, and measurement errors). Since the only time F_N, F_Y, F_A, M_Y, M_Z, and M_X are known is at balance calibration, the K factors are obtained using known loads having known load standard deviations and using statistical methods for arriving at the expectation values for K and $\sigma(K)$. The measured values of K are obtained by applying the loads singly and in combination and observing the effects on and the values of the outputs of interest. Once this information is obtained, wind-tunnel measurements can be made. For example, suppose measurements of the output of the normal component of the balance are made. If the loads on the model are not known, all of them may be assumed to be acting. Each balance component output is then multiplied by its sensitivity constant which was obtained during balance calibration. This yields the uncorrected balance outputs; i.e., $C_N B_1$, $C_Y B_2$, $C_A B_3$, $C_m B_4$, $C_n B_5$, and $C_l B_6$. These outputs are then corrected by taking the products of the interaction coefficients and their uncorrected balance outputs and subtracting them from $C_N B_1$ which yields a corrected B_1. Corrected B_2, B_3, B_4, B_5, and B_6 are obtained in the same manner. The process is reiterated until the corrected balance outputs converge to a value which is within 1 percent of their last value. With these values, the standard deviations of the balance outputs can be found by using equation (13) in the body of the paper and equations (A21) to (A25) in appendix A. Once the corrected loads are determined, the calibration curve shows the corresponding balance outputs. The positive and negative standard deviations can be projected onto the calibration and in turn onto the axis for the actual load values. For linear calibrations, these projections would show the load standard deviations. For nonlinear calibrations, these projections would show the extrema from which the errors are obtained. It is important to note that all of the interaction coefficients K, excluding systematic errors, have random errors associated with their measurement, and for this reason they are treated as random variables because they can vary between the extremes set by their error limits.

The standard deviations of B_1, B_2, B_3, B_4, B_5, and B_6 are typified by equation (13). The derivation for $\sigma(B_1)/B_1$, in which all the K factors are considered to be random variables, is as follows:

$$\frac{\sigma(B_1)}{B_1} = \Bigg\{(1 + 2F_N K_{1,7} + K_{1,13}F_Y + K_{1,14}F_A + K_{1,15}M_Y + K_{1,16}M_Z + K_{1,17}M_X)^2$$

$$\times \left(\frac{F_N}{B_1}\right)^2\left[\frac{\sigma(F_N)}{F_N}\right]^2 + (K_{1,2} + 2F_Y K_{1,8} + F_N K_{1,13} + F_A K_{1,18} + M_Y K_{1,19}$$

$$+ M_Z K_{1,20} + M_X K_{1,21})^2\left(\frac{F_Y}{B_1}\right)^2\left[\frac{\sigma(F_Y)}{F_Y}\right]^2 + (K_{1,3} + 2F_A K_{1,9} + F_N K_{1,14} + F_Y K_{1,18}$$

$$+ M_Y K_{1,22} + M_Z K_{1,23} + M_X K_{1,24})^2\left(\frac{F_A}{B_1}\right)^2\left[\frac{\sigma(F_A)}{F_A}\right]^2 + (K_{1,4} + 2M_Y K_{1,10}$$

$$+ F_N K_{1,15} + F_Y K_{1,19} + F_A K_{1,22} + M_Z K_{1,25} + M_X K_{1,26})^2\left(\frac{M_Y}{B_1}\right)^2\left[\frac{\sigma(M_Y)}{M_Y}\right]^2$$

$$+ (K_{1,5} + 2M_Z K_{1,11} + F_N K_{1,16} + F_Y K_{1,20} + F_A K_{1,23} + M_Y K_{1,25} + M_X K_{1,27})^2$$

$$\times \left(\frac{M_Z}{B_1}\right)^2\left[\frac{\sigma(M_Z)}{M_Z}\right]^2 + (K_{1,6} + 2M_X K_{1,12} + F_N K_{1,17} + F_Y K_{1,21} + F_A K_{1,24}$$

$$+ M_Y K_{1,26} + M_Z K_{1,27})^2\left(\frac{M_X}{B_1}\right)^2\left[\frac{\sigma(M_X)}{M_X}\right]^2 + \left(\frac{F_Y K_{1,2}}{B_1}\right)^2\left[\frac{\sigma(K_{1,2})}{K_{1,2}}\right]^2 + \left(\frac{F_A K_{1,3}}{B_1}\right)^2$$

$$\times \left[\frac{\sigma(K_{1,3})}{K_{1,3}}\right]^2 + \left(\frac{M_Y K_{1,4}}{B_1}\right)^2\left[\frac{\sigma(K_{1,4})}{K_{1,4}}\right]^2 + \left(\frac{M_Z K_{1,5}}{B_1}\right)^2\left[\frac{\sigma(K_{1,5})}{K_{1,5}}\right]^2 + \left(\frac{M_X K_{1,6}}{B_1}\right)^2$$

(Equation continued on next page)

$$\times \left[\frac{\sigma(K_{1,6})}{K_{1,6}}\right]^2 + \left(\frac{F_N{}^2 K_{1,7}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,7})}{K_{1,7}}\right]^2 + \left(\frac{F_Y{}^2 K_{1,8}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,8})}{K_{1,8}}\right]^2$$

$$+ \left(\frac{F_A{}^2 K_{1,9}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,9})}{K_{1,9}}\right]^2 + \left(\frac{M_Y{}^2 K_{1,10}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,10})}{K_{1,10}}\right]^2 + \left(\frac{M_Z{}^2 K_{1,11}}{B_1}\right)^2$$

$$\times \left[\frac{\sigma(K_{1,11})}{K_{1,11}}\right]^2 + \left(\frac{M_X{}^2 K_{1,12}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,12})}{K_{1,12}}\right]^2 + \left(\frac{F_N F_Y K_{1,13}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,13})}{K_{1,13}}\right]^2$$

$$+ \left(\frac{F_N F_A K_{1,14}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,14})}{K_{1,14}}\right]^2 + \left(\frac{F_N M_Y K_{1,15}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,15})}{K_{1,15}}\right]^2 + \left(\frac{F_N M_Z K_{1,16}}{B_1}\right)^2$$

$$\times \left[\frac{\sigma(K_{1,16})}{K_{1,16}}\right]^2 + \left(\frac{F_N M_X K_{1,17}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,17})}{K_{1,17}}\right]^2 + \left(\frac{F_Y F_A K_{1,18}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,18})}{K_{1,18}}\right]^2$$

$$+ \left(\frac{F_Y M_Y K_{1,19}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,19})}{K_{1,19}}\right]^2 + \left(\frac{F_Y M_Z K_{1,20}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,20})}{K_{1,20}}\right]^2 + \left(\frac{F_Y M_X K_{1,21}}{B_1}\right)^2$$

$$\times \left[\frac{\sigma(K_{1,21})}{K_{1,21}}\right]^2 + \left(\frac{F_A M_Y K_{1,22}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,22})}{K_{1,22}}\right]^2 + \left(\frac{F_A F_Y K_{1,23}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,23})}{K_{1,23}}\right]^2$$

$$+ \left(\frac{F_A M_X K_{1,24}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,24})}{K_{1,24}}\right]^2 + \left(\frac{M_Y M_Z K_{1,25}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,25})}{K_{1,25}}\right]^2 + \left(\frac{M_Y M_X K_{1,26}}{B_1}\right)^2$$

$$\left. \times \left[\frac{\sigma(K_{1,26})}{K_{1,26}}\right]^2 + \left(\frac{M_Z M_X K_{1,27}}{B_1}\right)^2 \left[\frac{\sigma(K_{1,27})}{K_{1,27}}\right]^2 \right\}^{1/2} \tag{13}$$

The derivations for $\sigma(B_2)/B_2$, $\sigma(B_3)/B_3$, $\sigma(B_4)/B_4$, $\sigma(B_5)/B_5$, and $\sigma(B_6)/B_6$ are given in appendix A.

Equations (12) are the force-moment equations for B_1, B_2, B_3, B_4, B_5, and B_6. At this point, since we have the parameters that define measurements of interest to wind-tunnel researchers, these are summarized.

Body-axis measurements (balance axis) if balance is aligned with model

Axial-force coefficient = F_A/qS

Normal-force coefficient = F_N/qS

Side-force coefficient = F_Y/qS

Pitching-moment coefficient = $M_Y/qS\bar{c}$

Rolling-moment coefficient = M_X/qSb

Yawing-moment coefficient = M_Z/qSb

Angle measurements

Sting angle

Angle of roll

Angle of sideslip

Angle of attack

Stability-axis coefficients

Drag coefficient = $f(C_A,\alpha,C_{DB}) = C_D = C_A \cos \alpha + C_N \sin \alpha$

Lift coefficient = $f(C_N,\alpha,C_{DB}) = C_L = C_N \cos \alpha - C_A \sin \alpha$

Side-force coefficient = $f(C_Y,\beta)$

Pitching-moment coefficient = $f(C_m)$

Rolling-moment coefficient = $f(C_l,\alpha) = C_l \cos \alpha + C_n \sin \alpha$

Yawing-moment coefficient = $f(C_n,\alpha) = C_n \cos \alpha - C_l \sin \alpha$

Base axial-force coefficient = $A_b(p_b - p_\infty)/qS$

Base drag-force coefficient = $f(C_{AB},\alpha) = A_b(p_b - p_\infty) \cos \alpha/qS$

Lift-drag ratio L/D

Plot of C_L versus C_D

$C_p = (p_l - p_\infty)/q_\infty$

Using the preceding list, the standard deviations for these coefficients are enumerated in the remaining equations. Where it is believed to be necessary, the full derivation for obtaining them is given in appendix A.

$$\frac{\sigma(\text{Axial-force coefficient})}{\text{Axial-force coefficient}} = \frac{\sigma(C_A)}{C_A} = \left\{\left[\frac{\sigma(F_A)}{F_A}\right]^2 + \left[\frac{\sigma(q)}{q}\right]^2 + \left[\frac{\sigma(S)}{S}\right]^2\right\}^{1/2} \tag{14}$$

$\dfrac{\sigma(S)}{S}$ is self-explanatory.

$$\frac{\sigma(\text{Normal-force coefficient})}{\text{Normal-force coefficient}} = \frac{\sigma(C_N)}{C_N} = \left\{\left[\frac{\sigma(F_N)}{F_N}\right]^2 + \left[\frac{\sigma(q)}{q}\right]^2 + \left[\frac{\sigma(S)}{S}\right]^2\right\}^{1/2} \tag{15}$$

$$\frac{\sigma(\text{Side-force coefficient})}{\text{Side-force coefficient}} = \frac{\sigma(C_Y)}{C_Y} = \left\{\left[\frac{\sigma(F_Y)}{F_Y}\right]^2 + \left[\frac{\sigma(q)}{q}\right]^2 + \left[\frac{\sigma(S)}{S}\right]^2\right\}^{1/2} \tag{16}$$

$$\frac{\sigma(\text{Pitching-moment coefficient})}{\text{Pitching-moment coefficient}} = \frac{\sigma(C_m)}{C_m} = \left\{\left[\frac{\sigma(M_Y)}{M_Y}\right]^2 + \left[\frac{\sigma(q)}{q}\right]^2 + \left[\frac{\sigma(S)}{S}\right]^2 + \left[\frac{\sigma(\bar{c})}{\bar{c}}\right]^2\right\}^{1/2} \tag{17}$$

$$\frac{\sigma(\text{Rolling-moment coefficient})}{\text{Rolling-moment coefficient}} = \frac{\sigma(C_l)}{C_l} = \left\{\left[\frac{\sigma(M_X)}{M_X}\right]^2 + \left[\frac{\sigma(q)}{q}\right]^2 + \left[\frac{\sigma(S)}{S}\right]^2 + \left[\frac{\sigma(b)}{b}\right]^2\right\}^{1/2} \tag{18}$$

$$\frac{\sigma(\text{Yawing-moment coefficient})}{\text{Yawing-moment coefficient}} = \frac{\sigma(C_n)}{C_n} = \left\{\left[\frac{\sigma(M_Z)}{M_Z}\right]^2 + \left[\frac{\sigma(q)}{q}\right]^2 + \left[\frac{\sigma(S)}{S}\right]^2 + \left[\frac{\sigma(b)}{b}\right]^2\right\}^{1/2} \tag{19}$$

The standard deviations for the angle measurements are not derived but measured. The technique for obtaining the angles is to use an accelerometer employing a null servo system. By this means a strut angle can be measured to a relative precision of $2\sigma = \pm 0.02^{\circ}$ and a model angle of $2\sigma = \pm 0.03^{\circ}$. The accelerometer must be isolated from vibration which has an adverse effect on it. Tunnel investigators would like to obtain $2\sigma = \pm 0.01^{\circ}$. This is possible to obtain but is not routinely feasible at present.

$$\sigma(C_D) = \left\{\left[\frac{\sin\alpha}{(p_t - p)S}\right]^2 \sigma(F_N)^2 + \left[\frac{\cos\alpha}{(p_t - p)S}\right]^2 \sigma(F_A)^2 + C_L^2\,\sigma(\alpha)^2 + \left(\frac{C_D}{p_t - p}\right)^2 \times \sigma(p_t)^2 + \left(\frac{C_D}{p_t - p}\right)^2 \sigma(p)^2 + \left(\frac{C_D}{S}\right)^2 \sigma(S)^2\right\}^{1/2} \tag{20}$$

$$\frac{\sigma(C_D)}{C_D} = \left\{\left[\frac{\sin\alpha F_N}{C_D(p_t - p)S}\right]^2\left[\frac{\sigma(F_N)}{F_N}\right]^2 + \left[\frac{\cos\alpha F_A}{C_D(p_t - p)S}\right]^2\left[\frac{\sigma(F_A)}{F_A}\right]^2 + \left(\frac{C_L\alpha}{C_D}\right)^2\left[\frac{\sigma(\alpha)}{\alpha}\right]^2 + \left(\frac{p_t}{p_t - p}\right)^2\left[\frac{\sigma(p_t)}{p_t}\right]^2 + \left(\frac{p}{p_t - p}\right)^2\left[\frac{\sigma(p)}{p}\right]^2 + \left[\frac{\sigma(S)}{S}\right]^2\right\}^{1/2} \tag{21}$$

$$\sigma(C_L) = \left\{\left[\frac{\cos\alpha}{(p_t - p)S}\right]^2 \sigma(F_N)^2 + \left[\frac{\sin\alpha}{(p_t - p)S}\right]^2 \sigma(F_A)^2 + C_D^2\,\sigma(\alpha)^2 + \left(\frac{C_L}{p_t - p}\right)^2 \times \sigma(p_t)^2 + \left(\frac{C_L}{p_t - p}\right)^2 \sigma(p)^2 + \left(\frac{C_L}{S}\right)^2 \sigma(S)^2\right\}^{1/2} \tag{22}$$

$$\frac{\sigma(C_L)}{C_L} = \left\{\left[\frac{\cos\alpha F_N}{C_L(p_t - p)S}\right]^2\left[\frac{\sigma(F_N)}{F_N}\right]^2 + \left[\frac{\sin\alpha F_A}{C_L(p_t - p)S}\right]^2\left[\frac{\sigma(F_A)}{F_A}\right]^2 + \left(\frac{C_D\alpha}{C_L}\right)^2\left[\frac{\sigma(\alpha)}{\alpha}\right]^2 + \left(\frac{p_t}{p_t - p}\right)^2\left[\frac{\sigma(p_t)}{p_t}\right]^2 + \left(\frac{p}{p_t - p}\right)^2\left[\frac{\sigma(p)}{p}\right]^2 + \left[\frac{\sigma(S)}{S}\right]^2\right\}^{1/2} \tag{23}$$

Of course, other factors contributing independently to any of these standard deviations, such as the error in C_D and C_L due to grit effect, internal friction effects, etc., must be added in. F_N, F_A, α, p_t, p, and S may be dependent on other parameters, so their errors would be formulated on more fundamental parameters. This continues until the last desired quantity is formulated in as fundamental a system as desired. The latter is usually mass, length, time, temperature, and charge. This would be the suggested error-analysis format recommended for use in instrument work. For wind-tunnel application or for any other specialized field application, the error-analysis format would be stated in terms of basic parameters and could be presented thusly:

$$\sigma(C_D) = \left\{\left[\frac{(\sin\alpha)\sigma(F_N)}{qS}\right]^2 + \left[\frac{(\cos\alpha)\sigma(F_A)}{qS}\right]^2 + C_L{}^2\,\sigma(\alpha)^2 + \left[\frac{C_D\,\sigma(q)}{q}\right]^2\right\}^{1/2} \tag{24}$$

where

$$\left[\frac{C_D\,\sigma(q)}{q}\right]^2 = \left\{C_D\left[\frac{\sigma(p)}{p}\right]^2 + \frac{2C_D\,\sigma(M)}{M}\right\}^2$$

$$\sigma(C_L) = \left(\left(\frac{\cos\alpha}{qS}\right)^2\sigma(F_N)^2 + \left(\frac{\sin\alpha}{qS}\right)^2\sigma(F_A)^2 + C_D{}^2\,\sigma(\alpha)^2 + C_L{}^2\right.$$

$$\left.\times\left\{\left[\frac{\sigma(p)}{p}\right]^2 + \left[\frac{2\sigma(M)}{M}\right]^2\right\}\right)^{1/2} \tag{25}$$

$$\frac{\sigma(C_Y)}{C_Y}\ \text{(Same as eq. (16))} \tag{26}$$

$$\frac{\sigma(C_m)}{C_m} = \left\{\left[\frac{\sigma(M_Y)}{M_Y}\right]^2 + \left[\frac{\sigma(q)}{q}\right]^2 + \left[\frac{\sigma(S)}{S}\right]^2 + \left[\frac{\sigma(\bar{c})}{\bar{c}}\right]^2\right\}^{1/2} \tag{27}$$

$$\frac{\sigma(C_l)}{C_l} = \left[\left(\frac{M_X \sin\alpha}{qSb}\right)^2 \sigma(\alpha)^2 + \left(\frac{\cos\alpha}{qSb}\right)^2 \sigma(M_X)^2 + \left(\frac{M_X \cos\alpha}{q^2 Sb}\right)^2 \sigma(q)^2 + \left(\frac{M_X \cos\alpha}{qSb^2}\right)^2\right.$$

$$\left. \times\ \sigma(b)^2 + \left(\frac{M_X \cos\alpha}{qbS^2}\right)^2 \sigma(S)^2\right]^{1/2} \tag{28}$$

$$\frac{\sigma(C_l)}{C_l} = \left\{\left[\frac{\sigma(M_X)}{M_X}\right]^2 + 2\left[\frac{\sigma(q)}{q}\right]^2 + 2\left[\frac{\sigma(A)}{A}\right]^2 + 2\left[\frac{\sigma(b)}{b}\right]^2 + (\alpha \tan\alpha)^2\left[\frac{\sigma(\alpha)}{\alpha}\right]^2\right.$$

$$\left. + \left[\frac{\sigma(M_Z)}{M_Z}\right]^2 + (\alpha \cot\alpha)^2\left[\frac{\sigma(\alpha)}{\alpha}\right]^2\right\}^{1/2} \tag{29}$$

$$\sigma(C_n) = \left[\left(\frac{M_Z \sin\alpha}{qAb}\right)^2 \sigma(\alpha)^2 + \left(\frac{\cos\alpha}{qSb}\right)^2 \sigma(M_Z)^2 + \left(\frac{M_Z \cos\alpha}{q^2 Sb}\right)^2 \sigma(q)^2 + \left(\frac{M_Z \cos\alpha}{qS^2 b}\right)^2\right.$$

$$\times\ \sigma(S)^2 + \left(\frac{M_Z \cos\alpha}{qAb^2}\right)^2 \sigma(b)^2 + \left(\frac{M_X \cos\alpha}{qSb}\right)^2 \sigma(\alpha)^2 + \left(\frac{\sin\alpha}{qSb}\right)^2 \sigma(M_X)^2$$

$$\left. + \left(\frac{M_X \sin\alpha}{q^2 Sb}\right)^2 \sigma(q)^2 + \left(\frac{M_X \sin\alpha}{qS^2 b}\right)^2 \sigma(A)^2 + \left(\frac{M_X \sin\alpha}{qSb^2}\right)^2 \sigma(b)^2\right]^{1/2} \tag{30}$$

$$\frac{\sigma(C_n)}{C_n} = \left\{\left[\frac{\sigma(M_Z)}{M_Z}\right]^2 + \alpha^2(\tan\alpha + \cot\alpha)^2\left[\frac{\sigma(\alpha)}{\alpha}\right]^2 + 2\left[\frac{\sigma(b)}{b}\right]^2\right\}^{1/2} \tag{31}$$

$$\sigma(C_{AB}) = \left\{\left(\frac{A_b}{qS}\right)^2 \sigma(p_\infty)^2 + \left(\frac{A_b}{qS}\right)^2 \sigma(p_b)^2 + \left(\frac{p_b - p_\infty}{qS}\right)^2 \sigma(A_b)^2 + \left[\frac{A_b(p_b - p_\infty)}{q^2 S}\right]^2 \sigma(q)^2 \right.$$

$$\left. + \left[\frac{A_b(p_b - p_\infty)}{qS^2}\right]^2 \sigma(S)^2 \right\}^{1/2} \tag{32}$$

$$\frac{\sigma(C_{AB})}{C_{AB}} = \left\{\left[\frac{\sigma(A_b)}{A_b}\right]^2 + \left(\frac{p_b}{p_b - p_\infty}\right)^2 \left[\frac{\sigma(p_b)}{p_b}\right]^2 + \left(\frac{p_\infty}{p_b - p_\infty}\right)^2 \left[\frac{\sigma(p_\infty)}{p_\infty}\right]^2 \right.$$

$$\left. + \left[\frac{\sigma(q)}{q}\right]^2 + \left[\frac{\sigma(S)}{S}\right]^2 \right\}^{1/2} \tag{33}$$

$$\sigma(C_{DB}) = \left\{\left[\frac{A_b(p_b - p_\infty) \sin \alpha}{qS}\right]^2 \sigma(\alpha)^2 + \left(\frac{A_b \cos \alpha}{qS}\right)^2 \sigma(p_b)^2 + \left(\frac{A_b \cos \alpha}{qS}\right)^2 \sigma(p_\infty)^2 \right.$$

$$+ \left[\left(\frac{p_b - p_\infty}{qS}\right) \cos \alpha\right]^2 \sigma(A_b)^2 + \left[\frac{A_b(p_b - p_\infty) \cos \alpha}{q^2 S}\right]^2 \sigma(q)^2$$

$$\left. + \left[\frac{A_b(p_b - p_\infty)}{qS^2}\right]^2 (\cos \alpha)^2 \sigma(S)^2 \right\}^{1/2} \tag{34}$$

$$\frac{\sigma(C_{DB})}{C_{DB}} = \left\{\left[\frac{\sigma(A_b)}{A_b}\right]^2 + \left(\frac{p_b}{p_b - p_\infty}\right)^2 \left[\frac{\sigma(p_b)}{p_b}\right]^2 + \left[\frac{\sigma(p_\infty)}{p_\infty}\right]^2 \left(\frac{p_\infty}{p_b - p_\infty}\right)^2 \right.$$

$$\left. + (\alpha \tan \alpha)^2 \left[\frac{\sigma(\alpha)}{\alpha}\right]^2 + \left[\frac{\sigma(q)}{q}\right]^2 + \left[\frac{\sigma(S)}{S}\right]^2 \right\}^{1/2} \tag{35}$$

$$\sigma(L/D) = \left\{\left[\frac{F_A{}^2 + F_N{}^2}{(F_A \cos\alpha + F_N \sin\alpha)^2}\right]^2 \sigma(\alpha)^2 + \left[\frac{F_A}{(F_A \cos\alpha + F_N \sin\alpha)^2}\right]^2 \sigma(F_N)^2\right.$$

$$\left. + \left[\frac{F_N}{(F_A \cos\alpha + F_N \sin\alpha)^2}\right]^2 \sigma(F_A)^2\right\}^{1/2} \tag{36}$$

$$\frac{\sigma(L/D)}{L/D} = \left\{\left[\frac{(F_A{}^2 + F_N{}^2 + 4F_AF_N \cos\alpha \sin\alpha)\alpha}{(F_N \cos\alpha - F_A \sin\alpha)(F_A \cos\alpha + F_N \sin\alpha)}\right]^2\left[\frac{\sigma(\alpha)}{\alpha}\right]^2\right.$$

$$+ \left[\frac{(F_N - 2F_A \sin\alpha \cos\alpha)F_A}{(F_N \cos\alpha - F_A \sin\alpha)(F_A \cos\alpha + F_N \sin\alpha)}\right]^2\left[\frac{\sigma(F_A)}{F_A}\right]^2$$

$$\left. + \left[\frac{F_AF_N}{(F_N \cos\alpha - F_A \sin\alpha)(F_A \cos\alpha + F_N \sin\alpha)}\right]^2\left[\frac{\sigma(F_N)}{F_N}\right]^2\right\}^{1/2} \tag{37}$$

$$\sigma(C_p) = \left\{q_\infty{}^{-2}\left[\sigma(p_l)\right]^2 + q_\infty{}^{-2}\,\sigma(p_\infty)^2 + \left[\frac{(p_l - p_\infty)}{q_\infty{}^2}\right]^2 \sigma(q_\infty)^2\right\}^{1/2} \tag{38}$$

$$\frac{\sigma(C_p)}{C_p} = \left\{\left(\frac{p_l}{p_l - p_\infty}\right)^2\left[\frac{\sigma(p_l)}{p_l}\right]^2 + \left(\frac{p_\infty}{p_l - p_\infty}\right)^2\left[\frac{\sigma(p_\infty)}{p_\infty}\right]^2 + \left[\frac{\sigma(q_\infty)}{q_\infty}\right]^2\right\}^{1/2} \tag{39}$$

Some of these formulations can be used immediately and in a general way. As an example, the National Transonic Facility (NTF), a cryogenic tunnel using nitrogen is being built at this Center. Tentatively, the following parameter limits are being used for its design:

Specific heat of nitrogen γ_{N_2} at one atmosphere . . . . 1.4 to 1.45

Specific heat of nitrogen at nine atmospheres 1.74

Stagnation pressure p_t from $M = 0.2$ to 1.0, N/m^2 . . . . 8.963×10^5

Mach number . . . . 0.2 to 1.2

R/ℓ, m^{-1} . . . . 0 to 3.94×10^8

q, N/m^2 absolute . . . . 0 to 3.352×10^5

The total temperature varies from 339 K to the temperature of liquid nitrogen for the value of p_t at that Mach number.

Using the preceding information, programs were written in BASIC and run on the Hewlett-Packard 9830A calculator for $\sigma(M)$, $\sigma(q)$, and $\sigma(R/\ell)$. The programs are in appendix B and the plots of the standard deviations are shown in figures 1 to 4. These standard deviations are due to instrument errors only, and other independent sources contributing to the error of these variables must be included to obtain their total standard deviation.

DESCRIPTION OF INSTRUMENTATION

Some of the basic instrumentation types whose parameters are used in this error analysis are described. The error values are real and were obtained from the manufacturer's error-band quotation and from data obtained from the instruments.

The acoustic manometer is essentially a U-tube mercury manometer. (See fig. 5.) A sound signal is simultaneously sent through both legs of the manometer from which the differential mercury height, and thus the differential pressure, is obtained. This manometer is used over the following ranges, and the errors given by the manufacturers are shown as 3σ values:

1.034×10^5 N/m^2 < p_t < 2.758×10^5 N/m^2; $3\sigma = 2.069 \times 10^1$ N/m^2

4.826×10^4 N/m^2 < p < 2.758×10^5 N/m^2; $3\sigma = 2.069 \times 10^1$ N/m^2

2.758×10^5 N/m^2 < p_t < 4.826×10^5 N/m^2; $3\sigma = 4.137 \times 10^1$ N/m^2

1.931×10^5 N/m^2 < p < 4.826×10^5 N/m^2; $3\sigma = 4.137 \times 10^1$ N/m^2

For higher pressures, a fused-quartz Bourdon tube gauge is used. (See fig. 6.) As the tube is deflected by a change in pressure, it is restored electromagnetically to a null position, thus minimizing errors. The current required to hold the tube in its null position is a measure of the pressure. Its 3σ error is 0.006 percent of full scale plus 0.012 percent of the reading. In normal tunnel operations, for pressures greater than 4.826×10^5 N/m^2, only one fused-quartz Bourdon tube type of transducer is used whose full scale exceeds the estimated total tunnel pressure. In the NTF, however, two will be used with full scales of 6.895×10^5 N/m^2 and 1.034×10^6 N/m^2. A sensor will activate a switch which will cut the lower pressure instrument off just before its full scale is reached. The data will be marked at the same time to identify the recording transducer. The full scales of the fused-quartz Bourdon tube pressure transducers start at 6.895×10^4 N/m^2, and increase in multiples of ten. There are also pressure transducers with full scales which start at 1.034×10^5 N/m^2 and increase in multiples of ten. In fact, using three pressure capsules and staying within the pressure limits of these capsules, any full scale may be obtained by adjustment of the electronics.

Acoustic manometers have at least three times the precision of the fused-quartz Bourdon tube type, but since their price is more than three times as high, they are used only when warranted by the precision requirements. (Acoustic manometers have almost the accuracy and precision of a primary standard.) In tunnel work, acoustic manometers are used for measuring pressures below 4.826×10^5 N/m^2. For higher pressures, fused-quartz Bourdon tube pressure transducers are available with full scales of 5.171×10^5 N/m^2, 6.895×10^5 N/m^2, and 1.034×10^6 N/m^2. The last value is sufficient for tunnel work. However, these instruments can be obtained at very much higher full scales. By the same token, if the accuracy and precision of the fused-quartz transducer are adequate, they can also be obtained with full scales of 6.895×10^4 N/m^2, 1.034×10^5 N/m^2, 2.068×10^5 N/m^2, and 3.447×10^5 N/m^2. This is also essentially a static-type transducer.

Stagnation temperature measurements are made with platinum resistance thermometers. (See fig. 7.) The operating range of these thermometers is from 8 to 533 K. In steady-state operation, $\sigma = 0.3$ K.

The six-component strain-gauge-balance calibration fixture is shown in figure 8. This fixture, as well as manual loading, is used to calibrate the strain-gauge balances and obtain the coefficients for the interactions and second-order terms.

RESULTS AND DISCUSSION

This discussion is limited to some typical results rather than all the results that can be obtained and is used to illustrate what can be done. It is based on parametric data applicable to the NTF. In figure 1, which is a plot of $\sigma(M)$ versus Mach number, $\sigma(M)$ is dependent on the pressure parameter. By using basic relationships and dimensional analysis, and by following procedures shown in appendix A, $\sigma(M)$ can be written with dependency on other parameters. The sharp breaks in the curves of figures 1 and 2 occur because different types of pressure instruments, as well as the same type with different full scales, are used. Since the errors are a function of the full scale of the instruments used and/or their indicated value, a break occurs in the error curve when the pressure transducer is changed. Since these error calculations are based on a fixed total pressure, the breaks appear when the static pressure, which is a function of total pressure and Mach number, changes to a value requiring a change of transducer range with a corresponding change in error values. It is apparent that the greatest absolute errors in Mach number occur at the low Mach numbers. This is apparent from the relationships in equation (4) and

$p = p_t(1 + M^2/5)^{-7/2}$, and from the errors in p and p_t at different Mach numbers. It would be preferable not to try to interpolate values for pressures not plotted, because of the instrument error changes with pressure range. Instead, curves should be plotted for any additional total pressures of interest. The previous comments also apply to figure 2, which is a plot of $\sigma(q)$ versus Mach number for various total pressures. In figure 3, which is a plot of $\sigma(R/\ell)$ versus Mach number at a temperature of 77.8 K, the greatest absolute errors are encountered at the highest Mach numbers and increase with higher total pressures.

It is profitable to examine the standard-deviation equations in the body of the paper because they are algebraic. Discrete points of error buildup can thus be pinpointed to large multiplying coefficients. These points can be verified with actual numerical values. Time can then be more efficiently allotted for finding the means of eliminating or reducing these large sources of error. Examination of equations (13) to (18) shows that second-order terms as well as terms produced by interactions increase the errors in the strain-gauge-balance outputs. In addition, the errors made in measuring the interaction coefficients increase this error still more. Errors of this type result in nonlinear calibrations. There are other causes for producing nonlinear calibrations but these can usually be isolated. When nonlinearities attributable to the aforementioned causes are found, the production sources should be attenuated within the bounds of desired accuracy and precision and the limits imposed by the state of the art.

Figure 4 shows that the standard deviation obtained for Mach numbers using only fused-quartz Bourdon tube pressure transducers of different full scales gives 2 to 3.5 times the error obtained in figure 1, where different types of pressure transducers were used. These larger errors still fall within acceptable limits for most research testing.

Equations (13) and (24) to (28) give the relative errors for the strain-gauge-balance outputs. Although this may be the desired end information for some researchers, the actual loads and their errors are probably the more desired information. This information can be obtained from the calibration curves by finding the load corresponding to the balance output and applying the sigma values to obtain the extrema of the loads. The explicit, theoretical solution for the actual loads is omitted in this paper because it is believed that it more properly belongs in a paper dealing with the strain-gauge balances alone. These same equations show also that since the K coefficients appear everywhere as multipliers and are used in the iteration process, it is necessary that they be determined with a high degree of confidence in accuracy and precision.

CONCLUDING REMARKS

The work herein represents a beginning to a systematic approach to error analysis for measurement instrumentation for the National Transonic Facility (NTF), which is located at the Langley Research Center. This work can be easily applied to any wind-tunnel measurements. It was long overdue in that a thorough search of the literature back to 1934 revealed almost no such work.

Also, the work that was found constituted only a very small part of the effort being undertaken. Again, it is very simple to apply the error analysis of this paper to almost any instrument measurements. However, throughout this work random errors are assumed; that is, there is just as much likelihood of the errors being negative as positive. They are variable in magnitude and disordered in occurrence. It is also assumed that systematic errors are removed and good sampling practices employed.

Pertinent wind-tunnel parameters, their standard deviations, and their theoretical derivation are given as are BASIC language computer programs and plots for obtaining the standard deviations of Mach number, dynamic pressure, and Reynolds number versus Mach number for the NTF. These standard deviations are for the instrument contribution only. It can be determined from the equations and graphs which parameters are producing the largest errors or undesirable errors and take the necessary steps for minimizing them or making them insignificant.

Finally, the author has noted that in many technical writings and investigations, the measurement errors are not reported, or are erroneously reported, partial errors are reported as total errors, or for whatever reason, the errors are glossed. The author hopes that this work, in some way, will encourage investigators to analyze their measurements, isolate and eliminate errors that have no role in statistical measurement, find the causes of and minimize their statistical errors when possible, and, finally, report their findings (their measured values) with their appropriate related errors.

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APPENDIX A

DERIVATIONS OF STANDARD DEVIATIONS OF PERTINENT WIND-TUNNEL PARAMETERS

Derivation of $\sigma(M)$ for All Mach Numbers

$$M^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_t - p}{p} + 1\right)^{(\gamma-1)/\gamma} - 1\right] \quad \text{(From ref. 4, p. 4, eq. (44))} \tag{A1}$$

$$M = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_t - p}{p} + 1\right)^{(\gamma-1)/\gamma} - 1\right]} \tag{A2}$$

$$dM = \frac{1}{2}\left[\frac{2}{\gamma - 1}\left(\frac{p_t - p}{p} + 1\right)^{(\gamma-1)/\gamma} - 1\right]^{-1/2} \frac{2}{\gamma - 1}\left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{p_t - p}{p} + 1\right)^{[(\gamma-1)/\gamma]-1}$$

$$\times \frac{1}{p}\,dp_t + \frac{1}{2}\left[\frac{2}{\gamma - 1}\left(\frac{p_t - p}{p} + 1\right)^{(\gamma-1)/\gamma} - 1\right]^{-1/2}\left(\frac{2}{\gamma - 1}\right)\left(\frac{\gamma - 1}{\gamma}\right)$$

$$\times \left(\frac{p_t - p}{p} + 1\right)^{[(\gamma-1)/\gamma]-1}\left[\frac{-p - (p_t - p)}{p^2}\right]dp \tag{A3}$$

$$dM = \frac{1}{2M}\,\frac{2}{\gamma}\left(\frac{p_t - p}{p} + 1\right)^{-1/\gamma}\frac{1}{p}\,dp_t + \frac{1}{2M}\,\frac{2}{\gamma}\left(\frac{p_t - p}{p} + 1\right)^{-1/\gamma}\left(\frac{-p_t}{p^2}\right)dp \tag{A4}$$

$$dM = \frac{1}{2M}\left[\frac{2}{\gamma}\left(\frac{p_t - p}{p} + 1\right)^{-1/\gamma}\frac{1}{p}\right]\left(dp_t - \frac{p_t}{p}\,dp\right) \tag{A5}$$

$$dM = \frac{1}{2M}\left[\frac{2}{\gamma p}\left(\frac{p_t}{p}\right)^{-1/\gamma}\right]\left[p_t\left(\frac{dp_t}{p_t} - \frac{dp}{p}\right)\right] \tag{A6}$$

$$dM = \frac{1}{M}\,\frac{p_t}{\gamma p}\left(\frac{p_t}{p}\right)^{-1/\gamma}\left(\frac{dp_t}{p_t} - \frac{dp}{p}\right) = \frac{p_t^{[(1-1)/\gamma]}}{M\gamma p^{[(1-1)/\gamma]}}\left(\frac{dp_t}{p_t} - \frac{dp}{p}\right)$$

$$= \frac{p_t^{-1/\gamma}dp_t}{M\gamma p^{[(1-1)/\gamma]}} - \frac{p_t^{[(1-1)/\gamma]}dp}{M\gamma p^{[(2-1)/\gamma]}} \tag{A7}$$

$$\frac{dM}{M} = \frac{p_t^{[(1-1)/\gamma]}}{p^{[(1-1)/\gamma]}}\,\frac{\left(\dfrac{dp_t}{p_t} - \dfrac{dp}{p}\right)}{\gamma\left\{\dfrac{2}{\gamma - 1}\left[\left(\dfrac{p_t - p}{p} + 1\right)^{(\gamma-1)/\gamma} - 1\right]\right\}}$$

$$= \frac{\dfrac{\gamma - 1}{\gamma}\left(\dfrac{dp_t}{p_t} - \dfrac{dp}{p}\right)}{2\left[\left(\dfrac{p_t}{p}\right)^{(\gamma-1)/\gamma} - 1\right]\left(\dfrac{p}{p_t}\right)^{(\gamma-1)/\gamma}} \tag{A8}$$

$$\frac{dM}{M} = \frac{\dfrac{\gamma - 1}{\gamma}\left(\dfrac{dp_t}{p_t} - \dfrac{dp}{p}\right)}{2\left[1 - \left(\dfrac{p}{p_t}\right)^{(\gamma-1)/\gamma}\right]} \tag{A9}$$

APPENDIX A

$$\frac{\sigma(M)}{M} = \sqrt{\left\{\left(\frac{\gamma - 1}{2\gamma}\right)\left[1 - \left(\frac{p}{p_t}\right)^{(\gamma-1)/\gamma}\right]^{-1}\right\}^2 \left\{\left[\frac{\sigma(p_t)}{p_t}\right]^2 + \left[\frac{\sigma(p)}{p}\right]^2\right\}} \tag{A10}$$

From equation (7)

$$\sigma(M) = \sqrt{\frac{\left(p_t^{-2/\gamma}\right)\sigma(p_t)^2}{\left\{M\gamma p^{[(1-1)/\gamma]}\right\}^2} + \frac{\left\{p_t^{[(1-1)/\gamma]}\right\}^2 \sigma(p)^2}{\left\{M\gamma p^{[(2-1)/\gamma]}\right\}^2}}$$

$$= \sqrt{\frac{p^2\left(p_t^{-2/\gamma}\right)\sigma(p_t)^2 + \left\{p_t^{[(1-1)/\gamma]}\right\}^2 \sigma(p)^2}{\left\{M\gamma p^{[(2-1)/\gamma]}\right\}^2}}$$

$$= \frac{1}{M\gamma p}\sqrt{\left(\frac{p_t}{p}\right)^{-2/\gamma} \sigma(p_t)^2 + \left(\frac{p_t}{p}\right)^2 \left(\frac{p_t}{p}\right)^{-2/\gamma} \sigma(p)^2}$$

$$= \left(\frac{p_t}{p}\right)^{-1/\gamma}\left(\frac{1}{M\gamma p}\right)\sqrt{\sigma(p_t)^2 + \left(\frac{p_t}{p}\right)^2 \sigma(p)^2} \tag{A11}$$

Derivation of q Over Compressible and Incompressible Range

$$q = \frac{\gamma p M^2}{2} \qquad \text{(From ref. 4, p. 4, eq. (31b))} \tag{A12}$$

$$\ln q = (\ln \gamma + \ln p + 2 \ln M) - \ln 2 \tag{A13}$$

$$\frac{d(q)}{q} = \frac{\delta p}{p} + \frac{2\delta M}{M} \tag{A14a}$$

$$\frac{\sigma(q)}{q} = \sqrt{\left[\frac{\sigma(p)}{p}\right]^2 + \left[\frac{2\sigma(M)}{M}\right]^2} \qquad \text{(A14b)}$$

$$\sigma(q) = \sqrt{\left[\gamma p M \sigma(M)\right]^2 + \left[\frac{\gamma M^2}{2}\,\sigma(p)\right]^2} \qquad \text{(A15)}$$

Derivations of Reynolds Number in U.S. Customary Units

$$\left.\begin{aligned}
\frac{R}{\ell} &= \frac{p_t M}{\mu_t}\sqrt{\frac{\gamma}{(\gamma-1)c_v T_t}}\left(\frac{T_t}{T}\right)^{(\gamma-2)/(\gamma-1)}\left(\frac{\frac{T}{T_t} + \frac{198.6}{T_t}}{1 + \frac{198.6}{T_t}}\right) \quad \text{(From ref. 4, p. 19, eq. (B3))}\\
\frac{R}{\ell} &= \frac{p_t}{\mu_t}\left\{\left[\left(\frac{p_t}{p}\right)^{2/7} - 1\right]5\right\}^{1/2}\sqrt{\frac{1.4}{(1.4-1)\,4290\,\frac{\text{ft}^2\ T_t}{\text{sec}^2\text{-}^{\circ}\text{R}}}}\left(\frac{T_t}{T}\right)^{(1.4-2)/(1.4-1)}\left(\frac{\frac{T}{T_t} + \frac{198.6}{T_t}}{\frac{T_t + 198.6}{T_t}}\right)\\
\frac{R}{\ell} &= \frac{p_t\left\{\left[\left(\frac{p_t}{p}\right)^{2/7} - 1\right]5\right\}^{1/2}}{2.270\left(\frac{T_t^{3/2}}{T_t + 198.6}\right)\times 10^{-8}\,\frac{\text{lb-sec}}{\text{ft}^2}}\left(\frac{2.8563\times 10^{-2}}{T_t^{1/2}}\,\frac{\text{sec}}{\text{ft}}\right)\left[\left(\frac{p_t}{p}\right)^{2/7}\right]^{-3/2}\\
&\quad \times \frac{\left(\frac{p}{p_t}\right)^{2/7} + \frac{198.6}{T_t}}{T_t + 198.6}\,T_t
\end{aligned}\right\}$$

(A16)

(Equation (A16) continued on next page)

$$\left.\begin{aligned}
\frac{R}{\ell} &= \frac{p_t\left\{\left[\left(\dfrac{p_t}{p}\right)^{2/7} - 1\right]5\right\}^{1/2}(1.2583 \times 10^6)\,\mathrm{ft}}{T_t\,(^{\circ}R)\,\mathrm{lb}\left(\dfrac{p_t}{p}\right)^{-3/7}\left[\left(\dfrac{p}{p_t}\right)^{2/7} + \dfrac{198.6}{T_t}\right]} \\
\frac{R}{\ell} &= 1.2583 \times 10^6\left\{\left[\left(\frac{p_t}{p}\right)^{-4/7} - \left(\frac{p_t}{p}\right)^{-6/7}\right]5\right\}^{1/2}\frac{p_t}{T_t}\left[\left(\frac{p}{p_t}\right)^{2/7} + \frac{198.6}{T_t}\right]^{\circ}R\,\frac{\mathrm{ft}}{\mathrm{lb}}
\end{aligned}\right\} \quad \text{(A16)}$$

The only noncancelable dimensional item is p_t/T_t. If this item is in $\mathrm{lb/ft^2/^{\circ}R}$, then Reynolds number is in $\mathrm{ft^{-1}}$. In the metric system, the derivation proceeds in the same manner. The 198.6 is a temperature in degrees Rankine and must be changed to degrees Kelvin to get a dimensionless ratio. This conversion yields

$$\frac{R}{\ell} = 1.2583 \times 10^6\left\{\left[\left(\frac{p_t}{p}\right)^{-4/7} - \left(\frac{p_t}{p}\right)^{-6/7}\right]5\right\}^{1/2}\left(\frac{p_t}{T_t}\right)\left[\left(\frac{p}{p_t}\right)^{2/7} + \frac{198.6}{T_t}\left(\frac{5}{9}\right)\mathrm{K/^{\circ}R}\right]\frac{\mathrm{ft}}{^{\circ}R\ \mathrm{lb}}$$

$$\times \frac{1}{4.4482\ \mathrm{N/lb}}(0.3048\ \mathrm{m/ft}) \quad \text{(A17)}$$

Since reference 4 used the U.S. system and the constants contain degrees Rankine, when metric is used, an uncompensated T_t in the denominator is in degrees Kelvin and must be changed to degrees Rankine to correlate. Therefore,

$$\frac{R}{\ell} = \frac{8.6221 \times 10^4}{\dfrac{9}{5}\,\dfrac{^{\circ}R}{K}}\left\{\left[\left(\frac{p_t}{p}\right)^{-4/7} - \left(\frac{p_t}{p}\right)^{-6/7}\right]5\right\}^{1/2}\left(\frac{p_t^{5/7}p^{2/7}}{T_t} + \frac{110.33p_t}{T_t^{2}}\right)\mathrm{m^{-1}}$$

Derivation of Reynolds Number in SI Units

$$d\,\frac{R}{\ell} = 4.7901 \times 10^4\left\{\left[\left(\frac{p_t}{p}\right)^{-4/7} - \left(\frac{p_t}{p}\right)^{-6/7}\right]5\right\}^{1/2}\left[\frac{-p_t^{5/7}p^{2/7}}{T_t^{2}} - \frac{2(110.33p_t)}{T_t^{3}}\right]\delta T_t$$

(Equation (A18) continued on next page)

$$+ 4.7901 \times 10^4 \left(\left\{\left[\left(\frac{p_t}{p}\right)^{-4/7} - \left(\frac{p_t}{p}\right)^{-6/7}\right]5\right\}^{1/2} \left[\left(\frac{p_t}{p}\right)^{-2/7}\left(\frac{5}{7T_t}\right) + \frac{110.33}{T_t^2}\right]\right.$$

$$+ \left(\frac{p_t^{5/7}p^{2/7}}{T_t} + \frac{110.33p_t}{T_t^2}\right)\left(\frac{1}{2}\right)\left\{\left[\left(\frac{p_t}{p}\right)^{-4/7} - \left(\frac{p_t}{p}\right)^{-6/7}\right]5\right\}^{-1/2} (5)\left[-\frac{4}{7}\left(\frac{p_t}{p}\right)^{-11/7} \frac{1}{p}\right.$$

$$\left.\left. + \frac{6}{7}\left(\frac{p_t}{p}\right)^{-13/7} \frac{1}{p}\right]\right)\delta p_t + 4.7903 \times 10^4 \left(\left\{\left[\left(\frac{p_t}{p}\right)^{-4/7} - \left(\frac{p_t}{p}\right)^{-6/7}\right]5\right\}^{1/2}\right.$$

$$\times \left[\left(\frac{p_t}{p}\right)^{5/7}\left(\frac{2}{7T_t}\right)\right] + \left(\frac{p_t^{5/7}p^{2/7}}{T_t} + \frac{110.33p_t}{T_t^2}\right)\left(\frac{1}{2}\right)\left\{\left[\left(\frac{p_t}{p}\right)^{-4/7} - \left(\frac{p_t}{p}\right)^{-6/7}\right]5\right\}^{-1/2}$$

$$\left.\times (5)\left[\frac{4}{7}\left(\frac{p_t}{p}\right)^{-4/7} p^{-1} - \frac{6}{7}\left(\frac{p_t}{p}\right)^{-6/7} p^{-1}\right]\right)\delta p \qquad \text{(A18)}$$

$$d\,\frac{R}{\ell} = 4.7901 \times 10^4 \left[\left(\left\{\left[\left(\frac{p_t}{p}\right)^{-4/7} - \left(\frac{p_t}{p}\right)^{-6/7}\right]5\right\}^{1/2} \left\{-\left[\left(\frac{p}{p_t}\right)^{2/7} T_t\right] - 220.66\right\}\frac{p_t}{T_t^3}\right)\delta T_t\right.$$

$$+ \left(\left\{\left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7}\right]5\right\}^{1/2} \left[\frac{5}{7T_t}\left(\frac{p}{p_t}\right)^{2/7} + \frac{110.33}{T_t^2}\right]\right.$$

$$+ \left[\frac{\left(\frac{p}{p_t}\right)^{2/7} T_t + 110.33}{T_t^2}\right](p_t)\left(\frac{5}{2}\right)\left\{\left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7}\right]5\right\}^{-1/2} \left[-\frac{4}{7}\left(\frac{p}{p_t}\right)^{11/7}\right.$$

(Equation (A19) continued on next page)

$$+ \left.\left.\frac{6}{7}\left(\frac{p}{p_t}\right)^{13/7}\right]\frac{1}{p}\right)\delta p_t + \left(\left\{\left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7}\right]5\right\}^{1/2}\left(\frac{p}{p_t}\right)^{-5/7}\left(\frac{2}{7T_t}\right)\right.$$

$$+ \left[\frac{\left(\frac{p}{p_t}\right)^{2/7} T_t + 110.33}{T_t^{\,2}}\right](p_t)\left(\frac{5}{2}\right)\left\{\left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7}\right]5\right\}^{-1/2}\left[\frac{4}{7}\left(\frac{p}{p_t}\right)^{4/7}\right.$$

$$\left.\left.\left. - \frac{6}{7}\left(\frac{p}{p_t}\right)^{6/7}\right]p^{-1}\right)\delta p\right] \tag{A19}$$

$$\sigma\frac{R}{\ell} = \left\{(4.7901 \times 10^4)^2\left[\left(\left(\left\{\left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7}\right]5\right\}^{1/2}(-1)\left[\frac{\left(\frac{p}{p_t}\right)^{2/7} T_t + 220.66}{T_t^{\,3}}\right]\right.\right.\right.\right.$$

$$\left.\times\, p_t\sigma(T_t)\right)^2 + \left[\left(\left(\left\{\left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7}\right]5\right\}^{1/2}\left[\frac{\frac{5}{7}\left(\frac{p}{p_t}\right)^{2/7} T_t + 110.33}{T_t^{\,2}}\right]\right.\right.\right.$$

$$+ \left[\frac{\left(\frac{p}{p_t}\right)^{2/7} T_t + 110.33}{T_t^{\,2}}\right]\left(\frac{p_t}{p}\right)\left(\frac{5}{2}\right)\left\{\left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7}\right]5\right\}^{-1/2}$$

(Equation (A20) continued on next page)

$$\times \frac{-4\left(\frac{p}{p_t}\right)^{11/7} + 6\left(\frac{p}{p_t}\right)^{13/7}}{7}\Bigg)\sigma(p_t)\Bigg]^2 + \left(\left\{\left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7}\right]5\right\}^{1/2}\right.$$

$$\times \left(\frac{p}{p_t}\right)^{-5/7}\left(\frac{2}{7T_t}\right) + \left[\frac{\left(\frac{p}{p_t}\right)^{2/7} T_t + 110.33}{T_t^2}\right]\left(\frac{5}{2}\right)\left(\frac{p_t}{p}\right)$$

$$\left.\times \left\{\left[\left(\frac{p}{p_t}\right)^{4/7} - \left(\frac{p}{p_t}\right)^{6/7}\right]5\right\}^{-1/2}\left[\frac{4\left(\frac{p}{p_t}\right)^{4/7} - 6\left(\frac{p}{p_t}\right)^{6/7}}{7}\right]\sigma(p)\Bigg)^2\Bigg]\right\}^{1/2} \qquad \text{(A20)}$$

Derivation of Remaining Standard Deviation of Strain-Gauge Balance Outputs

$$\frac{\sigma(B_2)}{B_2} = \Bigg(\Bigg\{(K_{2,1} + 2F_N K_{2,7} + F_Y K_{2,13} + F_A K_{2,14} + M_Y K_{2,15} + M_Z K_{2,16} + M_X K_{2,17})^2$$

$$\times \left(\frac{F_N}{B_2}\right)^2\left[\frac{\sigma(F_N)}{F_N}\right]^2\Bigg\} + \Bigg\{(1 + 2F_Y K_{2,8} + F_A K_{2,18} + M_Y K_{2,19} + M_Z K_{2,20}$$

$$+ M_X K_{2,21} + F_N K_{2,13})^2\left(\frac{F_Y}{B_2}\right)^2\left[\frac{\sigma(F_Y)}{F_Y}\right]^2\Bigg\} + \Bigg\{(K_{2,3} + 2F_A K_{2,9} + F_N K_{2,14}$$

$$+ F_Y K_{2,18} + M_Y K_{2,22} + M_Z K_{2,23} + M_X K_{2,24})^2\left(\frac{F_A}{B_2}\right)^2\left[\frac{\sigma(F_A)}{F_A}\right]^2\Bigg\}$$

(Equation (A21) continued on next page)

$$+ \left\{ (K_{2,4} + 2M_Y K_{2,10} + F_N K_{2,15} + F_Y K_{2,19} + F_A K_{2,22} + M_Z K_{2,25} + M_X K_{2,26})^2 \right.$$

$$\left. \times \left(\frac{M_Y}{B_2}\right)^2 \left[\frac{\sigma(M_Y)}{M_Y}\right]^2 \right\} + \left\{ (K_{2,5} + 2M_Z K_{2,11} + F_N K_{2,16} + F_Y K_{2,20} + F_A K_{2,23} \right.$$

$$\left. + M_Y K_{2,25} + M_X K_{2,27})^2 \left(\frac{M_Z}{B_2}\right)^2 \left[\frac{\sigma(M_Z)}{M_Z}\right]^2 \right\} + \left\{ (K_{1,16} + 2M_X K_{2,12} + F_N K_{2,17} \right.$$

$$\left. + F_Y K_{2,21} + F_A K_{2,24} + M_Y K_{2,26} + M_Z K_{2,27})^2 \left(\frac{M_X}{B_2}\right)^2 \left[\frac{\sigma(M_X)}{M_X}\right]^2 \right\}$$

$$+ \left(\frac{F_N K_{2,1}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,1})}{K_{2,1}}\right]^2 + \left(\frac{F_A K_{2,3}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,3})}{K_{2,3}}\right]^2 + \left(\frac{M_Y K_{2,4}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,4})}{K_{2,4}}\right]^2$$

$$+ \left(\frac{M_Z K_{2,5}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,5})}{K_{2,5}}\right]^2 + \left(\frac{M_X K_{2,6}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,6})}{K_{2,6}}\right]^2 + \left(\frac{F_N^2 K_{2,7}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,7})}{K_{2,7}}\right]^2$$

$$+ \left(\frac{F_Y^2 K_{2,8}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,8})}{K_{2,8}}\right]^2 + \left(\frac{F_A^2 K_{2,9}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,9})}{K_{2,9}}\right]^2 + \left(\frac{M_Y^2 K_{2,10}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,10})}{K_{2,10}}\right]^2$$

$$+ \left(\frac{M_Z^2 K_{2,11}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,11})}{K_{2,11}}\right]^2 + \left(\frac{M_X^2 K_{2,12}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,12})}{K_{2,12}}\right]^2$$

$$+ \left(\frac{F_N F_Y K_{2,13}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,13})}{K_{2,13}}\right]^2 + \left(\frac{F_N F_A K_{2,14}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,14})}{K_{2,14}}\right]^2$$

$$+ \left(\frac{F_N M_Y K_{2,15}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,15})}{K_{2,15}}\right]^2 + \left(\frac{F_N M_Z K_{2,16}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,16})}{K_{2,16}}\right]^2$$

(Equation (A21) continued on next page)

$$+ \left(\frac{F_N M_X K_{2,17}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,17})}{K_{2,17}}\right]^2 + \left(\frac{F_Y F_A K_{2,18}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,18})}{K_{2,18}}\right]^2$$

$$+ \left(\frac{F_Y M_Y K_{2,19}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,19})}{K_{2,19}}\right]^2 + \left(\frac{F_Y M_Z K_{2,20}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,20})}{K_{2,20}}\right]^2$$

$$+ \left(\frac{F_Y M_X K_{2,21}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,21})}{K_{2,21}}\right]^2 + \left(\frac{F_A M_Y K_{2,22}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,22})}{K_{2,22}}\right]^2$$

$$+ \left(\frac{F_A M_Z K_{2,23}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,23})}{K_{2,23}}\right]^2 + \left(\frac{F_A M_X K_{2,24}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,24})}{K_{2,24}}\right]^2$$

$$+ \left(\frac{M_Y M_Z K_{2,25}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,25})}{K_{2,25}}\right]^2 + \left(\frac{M_Y M_X K_{2,26}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,26})}{K_{2,26}}\right]^2$$

$$+ \left(\frac{M_Z M_X K_{2,27}}{B_2}\right)^2 \left[\frac{\sigma(K_{2,27})}{K_{2,27}}\right]^2 \Bigg)^{1/2} \tag{A21}$$

$$\frac{\sigma(B_3)}{B_3} = \Bigg(\Bigg\{(K_{3,1} + 2F_N K_{3,7} + F_Y K_{3,13} + F_A K_{3,14} + M_Y K_{3,15} + M_Z K_{3,16} + M_X K_{3,17})^2$$

$$\times \left(\frac{F_N}{B_3}\right)^2 \left[\frac{\sigma(F_N)}{F_N}\right]^2\Bigg\} + \Bigg\{(K_{3,2} + 2F_Y K_{3,7} + F_N K_{3,13} + F_A K_{3,18} + M_Y K_{3,19}$$

$$+ M_Z K_{3,20} + M_X K_{3,21})^2 \left(\frac{F_Y}{B_3}\right)^2 \left[\frac{\sigma(F_Y)}{F_Y}\right]^2\Bigg\} + \Bigg\{(1 + 2F_A K_{3,9} + F_N K_{3,14} + F_Y K_{3,18}$$

(Equation (A22) continued on next page)

$$+ M_YK_{3,22} + M_ZK_{3,23} + M_XK_{3,24})^2\left(\frac{F_A}{B_3}\right)^2\left[\frac{\sigma(F_A)}{F_A}\right]^2\Bigg\} + \Bigg\{(K_{3,4} + 2M_YK_{3,10}$$

$$+ F_NK_{3,15} + F_YK_{3,19} + F_AK_{3,22} + M_ZK_{3,23} + M_XK_{3,24})^2\left(\frac{M_Y}{B_3}\right)^2\left[\frac{\sigma(M_Y)}{M_Y}\right]^2\Bigg\}$$

$$+ \Bigg\{(K_{3,5} + 2M_YK_{3,11} + F_NK_{3,16} + F_YK_{3,20} + F_AK_{3,23} + M_YK_{3,25}$$

$$+ M_XK_{3,27})^2\left(\frac{M_Z}{B_3}\right)^2\left[\frac{\sigma(M_Z)}{M_Z}\right]^2\Bigg\} + \Bigg\{(K_{3,6} + 2M_XK_{3,12} + F_NK_{3,17}$$

$$+ F_YK_{3,21} + F_AK_{3,24} + M_YK_{3,26} + M_ZK_{3,27})^2\left(\frac{M_X}{B_3}\right)^2\left[\frac{\sigma(M_X)}{M_X}\right]^2\Bigg\}$$

$$+ \left(\frac{F_NK_{3,1}}{B_3}\right)^2\left[\frac{\sigma(K_{3,1})}{K_{3,1}}\right]^2 + \left(\frac{F_YK_{3,2}}{B_3}\right)^2\left[\frac{\sigma(K_{3,2})}{K_{3,2}}\right]^2 + \left(\frac{M_YK_{3,4}}{B_3}\right)^2\left[\frac{\sigma(K_{3,4})}{K_{3,4}}\right]^2$$

$$+ \left(\frac{M_ZK_{3,5}}{B_3}\right)^2\left[\frac{\sigma(K_{3,5})}{K_{3,5}}\right]^2 + \left(\frac{M_XK_{3,6}}{B_3}\right)^2\left[\frac{\sigma(K_{3,6})}{K_{3,6}}\right]^2 + \left(\frac{F_N^2K_{3,7}}{B_3}\right)^2\left[\frac{\sigma(K_{3,7})}{K_{3,7}}\right]^2$$

$$+ \left(\frac{F_Y^2K_{3,8}}{B_3}\right)^2\left[\frac{\sigma(K_{3,8})}{K_{3,8}}\right]^2 + \left(\frac{F_A^2K_{3,9}}{B_3}\right)^2\left[\frac{\sigma(K_{3,9})}{K_{3,9}}\right]^2 + \left(\frac{M_Y^2K_{3,10}}{B_3}\right)^2\left[\frac{\sigma(K_{3,10})}{K_{3,10}}\right]^2$$

$$+ \left(\frac{M_Z^2K_{3,11}}{B_3}\right)^2\left[\frac{\sigma(K_{3,11})}{K_{3,11}}\right]^2 + \left(\frac{M_X^2K_{3,12}}{B_3}\right)^2\left[\frac{\sigma(K_{3,12})}{K_{3,12}}\right]^2$$

$$+ \left(\frac{F_NF_YK_{3,13}}{B_3}\right)^2\left[\frac{\sigma(K_{3,13})}{K_{3,13}}\right]^2 + \left(\frac{F_NF_AK_{3,14}}{B_3}\right)^2\left[\frac{\sigma(K_{3,14})}{K_{3,14}}\right]^2$$

(Equation (A22) continued on next page)

$$+\left(\frac{F_N M_Y K_{3,15}}{B_3}\right)^2\left[\frac{\sigma(K_{3,15})}{K_{3,15}}\right]^2+\left(\frac{F_N M_Z K_{3,16}}{B_3}\right)^2\left[\frac{\sigma(K_{3,16})}{K_{3,16}}\right]^2$$

$$+\left(\frac{F_N M_X K_{3,17}}{B_3}\right)^2\left[\frac{\sigma(K_{3,17})}{K_{3,17}}\right]^2+\left(\frac{F_Y F_A K_{3,18}}{B_3}\right)^2\left[\frac{\sigma(K_{3,18})}{K_{3,18}}\right]^2$$

$$+\left(\frac{F_Y M_Y K_{3,19}}{B_3}\right)^2\left[\frac{\sigma(K_{3,19})}{K_{3,19}}\right]^2+\left(\frac{F_Y M_Z K_{3,20}}{B_3}\right)^2\left[\frac{\sigma(K_{3,20})}{K_{3,20}}\right]^2$$

$$+\left(\frac{F_Y M_X K_{3,21}}{B_3}\right)^2\left[\frac{\sigma(K_{3,21})}{K_{3,21}}\right]^2+\left(\frac{F_A M_Y K_{3,22}}{B_3}\right)^2\left[\frac{\sigma(K_{3,22})}{K_{3,22}}\right]^2$$

$$+\left(\frac{F_A M_Z K_{3,23}}{B_3}\right)^2\left[\frac{\sigma(K_{3,23})}{K_{3,23}}\right]^2+\left(\frac{F_A M_X K_{3,24}}{B_3}\right)^2\left[\frac{\sigma(K_{3,24})}{K_{3,24}}\right]^2$$

$$+\left(\frac{M_Y M_Z K_{3,25}}{B_3}\right)^2\left[\frac{\sigma(K_{3,25})}{K_{3,25}}\right]^2+\left(\frac{M_Y M_X K_{3,26}}{B_3}\right)^2\left[\frac{\sigma(K_{3,26})}{K_{3,26}}\right]^2$$

$$\left.+\left(\frac{M_Z M_X K_{3,27}}{B_3}\right)^2\left[\frac{\sigma(K_{3,27})}{K_{3,27}}\right]^2\right)^{1/2} \tag{A22}$$

$$\frac{\sigma(B_4)}{B_4}=\left(\left\{(K_{4,1}+2F_Y K_{4,8}+F_N K_{4,13}+F_A K_{4,18}+M_Y K_{4,19}+M_Z K_{4,16}+M_X K_{4,21})^2\right.\right.$$

$$\left.\times\left(\frac{F_N}{B_4}\right)^2\left[\frac{\sigma(F_N)}{F_N}\right]^2\right\}+\left\{(K_{4,2}+2F_Y K_{4,8}+F_N K_{4,13}+F_A K_{4,18}+M_Y K_{4,19}\right.$$

(Equation (A23) continued on next page)

$$+ M_Z K_{4,20} + M_X K_{4,21})^2 \left(\frac{F_Y}{B_4}\right)^2 \left[\frac{\sigma(F_Y)}{F_Y}\right]^2 \Bigg\} + \Bigg\{(K_{4,3} + 2F_A K_{4,9} + F_N K_{4,14}$$

$$+ F_Y K_{4,18} + M_Y K_{4,22} + M_Z K_{4,23} + M_X K_{4,24})^2 \left(\frac{F_A}{B_4}\right)^2 \left[\frac{\sigma(F_A)}{F_A}\right]^2 \Bigg\}$$

$$+ \Bigg\{(1 + 2M_Y K_{4,10} + F_N K_{4,15} + F_Y K_{4,19} + F_A K_{4,22} + M_Z K_{4,25}$$

$$+ M_X K_{4,26})^2 \left(\frac{M_Y}{B_4}\right)^2 \left[\frac{\sigma(M_Y)}{M_Y}\right]^2 \Bigg\} + \Bigg\{(K_{4,5} + 2M_Z K_{4,11} + F_N K_{4,16} + F_Y K_{4,20}$$

$$+ F_A K_{4,23} + M_Y K_{4,25} + M_Z K_{4,27})^2 \left(\frac{M_Z}{B_4}\right)^2 \left[\frac{\sigma(M_Z)}{M_Z}\right]^2 \Bigg\} + \Bigg\{(K_{4,6} + 2M_X K_{4,12}$$

$$+ F_N K_{4,17} + F_Y K_{4,21} + F_A K_{4,24} + M_Y K_{4,26} + M_Z K_{4,27})^2 \left(\frac{M_X}{B_4}\right)^2 \left[\frac{\sigma(M_X)}{M_X}\right]^2 \Bigg\}$$

$$+ \left(\frac{F_N K_{4,1}}{B_4}\right)^2 \left[\frac{\sigma(K_{4,1})}{K_{4,1}}\right]^2 + \left(\frac{F_Y K_{4,2}}{B_4}\right)^2 \left[\frac{\sigma(K_{4,2})}{K_{4,2}}\right]^2 + \left(\frac{F_A K_{4,3}}{B_4}\right)^2 \left[\frac{\sigma(K_{4,3})}{K_{4,3}}\right]^2$$

$$+ \left(\frac{M_Z K_{4,5}}{B_4}\right)^2 \left[\frac{\sigma(K_{4,5})}{K_{4,5}}\right]^2 + \left(\frac{M_X K_{4,6}}{B_4}\right)^2 \left[\frac{\sigma(K_{4,6})}{K_{4,6}}\right]^2 + \left(\frac{F_N^2 K_{4,7}}{B_4}\right)^2 \left[\frac{\sigma(K_{4,7})}{K_{4,7}}\right]^2$$

$$+ \left(\frac{F_Y^2 K_{4,8}}{B_4}\right)^2 \left[\frac{\sigma(K_{4,8})}{K_{4,8}}\right]^2 + \left(\frac{F_A^2 K_{4,9}}{B_4}\right)^2 \left[\frac{\sigma(K_{4,9})}{K_{4,9}}\right]^2 + \left(\frac{M_Y^2 K_{4,10}}{B_4}\right)^2 \left[\frac{\sigma(K_{4,10})}{K_{4,10}}\right]^2$$

$$+ \left(\frac{M_Z^2 K_{4,11}}{B_4}\right)^2 \left[\frac{\sigma(K_{4,11})}{K_{4,11}}\right]^2 + \left(\frac{M_X^2 K_{4,12}}{B_4}\right)^2 \left[\frac{\sigma(K_{4,12})}{K_{4,12}}\right]^2$$

(Equation (A23) continued on next page)

$$+\left(\frac{F_N F_Y K_{4,13}}{B_4}\right)^2\left[\frac{\sigma(K_{4,13})}{K_{4,13}}\right]^2 + \left(\frac{F_N F_A K_{4,14}}{B_4}\right)^2\left[\frac{\sigma(K_{4,14})}{K_{4,14}}\right]^2$$

$$+\left(\frac{F_N M_Y K_{4,15}}{B_4}\right)^2\left[\frac{\sigma(K_{4,15})}{K_{4,15}}\right]^2 + \left(\frac{F_N M_Z K_{4,16}}{B_4}\right)^2\left[\frac{\sigma(K_{4,16})}{K_{4,16}}\right]^2$$

$$+\left(\frac{F_N M_X K_{4,17}}{B_4}\right)^2\left[\frac{\sigma(K_{4,17})}{K_{4,17}}\right]^2 + \left(\frac{F_Y F_A K_{4,18}}{B_4}\right)^2\left[\frac{\sigma(K_{4,18})}{K_{4,18}}\right]^2$$

$$+\left(\frac{F_Y M_Y K_{4,19}}{B_4}\right)^2\left[\frac{\sigma(K_{4,19})}{K_{4,19}}\right]^2 + \left(\frac{F_Y M_Z K_{4,20}}{B_4}\right)^2\left[\frac{\sigma(K_{4,20})}{K_{4,20}}\right]^2$$

$$+\left(\frac{F_Y M_X K_{4,21}}{B_4}\right)^2\left[\frac{\sigma(K_{4,21})}{K_{4,21}}\right]^2 + \left(\frac{F_A M_Y K_{4,22}}{B_4}\right)^2\left[\frac{\sigma(K_{4,22})}{K_{4,22}}\right]^2$$

$$+\left(\frac{F_A M_Z K_{4,23}}{B_4}\right)^2\left[\frac{\sigma(K_{4,23})}{K_{4,23}}\right]^2 + \left(\frac{F_A M_X K_{4,24}}{B_4}\right)^2\left[\frac{\sigma(K_{4,24})}{K_{4,24}}\right]^2$$

$$+\left(\frac{M_Y M_Z K_{4,25}}{B_4}\right)^2\left[\frac{\sigma(K_{4,25})}{K_{4,25}}\right]^2 + \left(\frac{M_Y M_X K_{4,26}}{B_4}\right)^2\left[\frac{\sigma(K_{4,26})}{K_{4,26}}\right]^2$$

$$\left.+\left(\frac{M_Z M_X K_{4,27}}{B_4}\right)^2\left[\frac{\sigma(K_{4,27})}{K_{4,27}}\right]^2\right)^{1/2} \qquad \text{(A23)}$$

$$\frac{\sigma(B_5)}{B_5} = \Bigg(\Bigg\{(K_{5,1} + 2F_N K_{5,7} + F_Y K_{5,13} + F_A K_{5,14} + M_Y K_{5,15} + M_Z K_{5,16} + M_X K_{5,17})^2$$

$$\times \left(\frac{F_N}{B_5}\right)^2 \left[\frac{\sigma(F_N)}{F_N}\right]^2\Bigg\} + \Bigg\{(K_{5,2} + 2F_Y K_{5,8} + F_N K_{5,13} + F_A K_{5,18} + M_Y K_{5,19}$$

$$+ M_Z K_{5,20} + M_X K_{5,21})^2 \left(\frac{F_Y}{B_5}\right)^2 \left[\frac{\sigma(F_Y)}{F_Y}\right]^2\Bigg\} + \Bigg\{(K_{5,3} + 2F_A K_{5,9} + F_N K_{5,14}$$

$$+ F_Y K_{5,18} + M_Y K_{5,22} + M_Z K_{5,23} + M_X K_{5,24})^2 \left(\frac{F_A}{B_5}\right)^2 \left[\frac{\sigma(F_A)}{F_A}\right]^2\Bigg\}$$

$$+ \Bigg\{(K_{5,4} + 2M_Y K_{5,10} + F_N K_{5,15} + F_Y K_{5,19} + F_A K_{5,22} + M_Z K_{5,23}$$

$$+ M_X K_{5,24})^2 \left(\frac{M_Y}{B_5}\right)^2 \left[\frac{\sigma(M_Y)}{M_Y}\right]^2\Bigg\} + \Bigg\{(1 + 2M_Z K_{5,11} + F_N K_{5,16} + F_Y K_{5,20}$$

$$+ F_A K_{5,23} + M_Y K_{5,25} + M_X K_{5,27})^2 \left(\frac{M_Z}{B_5}\right)^2 \left[\frac{\sigma(M_Z)}{M_Z}\right]^2\Bigg\} + \Bigg\{(K_{5,6} + 2M_X K_{5,12}$$

$$+ F_N K_{5,17} + F_Y K_{5,21} + F_A K_{5,24} + M_Y K_{5,26} + M_Z K_{5,27})^2 \left(\frac{M_X}{B_5}\right)^2 \left[\frac{\sigma(M_X)}{M_X}\right]^2\Bigg\}$$

$$+ \left(\frac{F_N K_{5,1}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,1})}{K_{5,1}}\right]^2 + \left(\frac{F_Y K_{5,2}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,2})}{K_{5,2}}\right]^2 + \left(\frac{F_A K_{5,3}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,3})}{K_{5,3}}\right]^2$$

$$+ \left(\frac{M_Y K_{5,4}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,4})}{K_{5,4}}\right]^2 + \left(\frac{M_X K_{5,6}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,6})}{K_{5,6}}\right]^2 + \left(\frac{F_N^2 K_{5,7}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,7})}{K_{5,7}}\right]^2$$

(Equation (A24) continued on next page)

$$+ \left(\frac{F_Y{}^2 K_{5,8}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,8})}{K_{5,8}}\right]^2 + \left(\frac{F_A{}^2 K_{5,9}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,9})}{K_{5,9}}\right]^2 + \left(\frac{M_Y{}^2 K_{5,10}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,10})}{K_{5,10}}\right]^2$$

$$+ \left(\frac{M_Z{}^2 K_{5,11}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,11})}{K_{5,11}}\right]^2 + \left(\frac{M_X{}^2 K_{5,12}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,12})}{K_{5,12}}\right]^2$$

$$+ \left(\frac{F_N F_Y K_{5,13}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,13})}{K_{5,13}}\right]^2 + \left(\frac{F_N F_A K_{5,14}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,14})}{K_{5,14}}\right]^2$$

$$+ \left(\frac{F_N M_Y K_{5,15}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,15})}{K_{5,15}}\right]^2 + \left(\frac{F_N M_Z K_{5,16}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,16})}{K_{5,16}}\right]^2$$

$$+ \left(\frac{F_N M_X K_{5,17}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,17})}{K_{5,17}}\right]^2 + \left(\frac{F_Y F_A K_{5,18}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,18})}{K_{5,18}}\right]^2$$

$$+ \left(\frac{F_Y M_Y K_{5,19}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,19})}{K_{5,19}}\right]^2 + \left(\frac{F_Y M_Z K_{5,20}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,20})}{K_{5,20}}\right]^2$$

$$+ \left(\frac{F_Y M_X K_{5,21}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,21})}{K_{5,21}}\right]^2 + \left(\frac{F_A M_Y K_{5,22}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,22})}{K_{5,22}}\right]^2$$

$$+ \left(\frac{F_A M_Z K_{5,23}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,23})}{K_{5,23}}\right]^2 + \left(\frac{F_A M_X K_{5,24}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,24})}{K_{5,24}}\right]^2$$

(Equation (A24) continued on next page)

$$+ \left(\frac{M_Y M_Z K_{5,25}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,25})}{K_{5,25}}\right]^2 + \left(\frac{M_Y M_X K_{5,26}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,26})}{K_{5,26}}\right]^2$$

$$+ \left(\frac{M_Z M_X K_{5,27}}{B_5}\right)^2 \left[\frac{\sigma(K_{5,27})}{K_{5,27}}\right]^2\Bigg)^{1/2} \tag{A24}$$

$$\frac{\sigma(B_6)}{B_6} = \Bigg(\bigg\{(K_{6,1} + 2F_N K_{6,7} + F_Y K_{6,13} + F_A K_{6,14} + M_Y K_{6,15} + M_Z K_{6,16} + M_X K_{6,17})^2$$

$$\times \left(\frac{F_N}{B_6}\right)^2 \left[\frac{\sigma(F_N)}{F_N}\right]^2\bigg\} + \bigg\{(K_{6,2} + 2F_Y K_{6,8} + F_N K_{6,13} + F_A K_{6,18} + M_Y K_{6,19}$$

$$+ M_Z K_{6,20} + M_X K_{6,21})^2 \left(\frac{F_Y}{B_6}\right)^2 \left[\frac{\sigma(F_Y)}{F_Y}\right]^2\bigg\} + \bigg\{(K_{6,3} + 2F_A K_{6,9} + F_N K_{6,14}$$

$$+ F_Y K_{6,18} + M_Y K_{6,22} + M_Z K_{6,23} + M_X K_{6,24})^2 \left(\frac{F_A}{B_6}\right)^2 \left[\frac{\sigma(F_A)}{F_A}\right]^2\bigg\}$$

$$+ \bigg\{(K_{6,4} + 2M_Y K_{6,10} + F_N K_{6,15} + F_Y K_{6,19} + F_A K_{6,22} + M_Z K_{6,23}$$

$$+ M_X K_{6,24})^2 \left(\frac{M_Y}{B_6}\right)^2 \left[\frac{\sigma(M_Y)}{M_Y}\right]^2\bigg\} + \bigg\{(K_{6,5} + 2M_Z K_{6,11} + F_N K_{6,16} + F_Y K_{6,20}$$

$$+ F_A K_{6,23} + M_Y K_{6,25} + M_X K_{6,27})^2 \left(\frac{M_Z}{B_6}\right)^2 \left[\frac{\sigma(M_Z)}{M_Z}\right]^2\bigg\} + \bigg\{(1 + 2M_X K_{6,12}$$

$$+ F_N K_{6,17} + F_Y K_{6,21} + F_A K_{6,24} + M_Y K_{6,26} + M_Z K_{6,27})^2 \left(\frac{M_X}{B_6}\right)^2 \left[\frac{\sigma(M_X)}{M_X}\right]^2\bigg\}$$

(Equation (A25) continued on next page)

$$+\left(\frac{F_N K_{6,1}}{B_6}\right)^2\left[\frac{\sigma(K_{6,1})}{K_{6,1}}\right]^2 + \left(\frac{F_Y K_{6,2}}{B_6}\right)^2\left[\frac{\sigma(K_{6,2})}{K_{6,2}}\right]^2 + \left(\frac{F_A K_{6,3}}{B_6}\right)^2\left[\frac{\sigma(K_{6,3})}{K_{6,3}}\right]^2$$

$$+\left(\frac{M_Y K_{6,4}}{B_6}\right)^2\left[\frac{\sigma(K_{6,4})}{K_{6,4}}\right]^2 + \left(\frac{M_Z K_{6,5}}{B_6}\right)^2\left[\frac{\sigma(K_{6,5})}{K_{6,5}}\right]^2 + \left(\frac{F_N^2 K_{6,7}}{B_6}\right)^2\left[\frac{\sigma(K_{6,7})}{K_{6,7}}\right]^2$$

$$+\left(\frac{F_Y^2 K_{6,8}}{B_6}\right)^2\left[\frac{\sigma(K_{6,8})}{K_{6,8}}\right]^2 + \left(\frac{F_A^2 K_{6,9}}{B_6}\right)^2\left[\frac{\sigma(K_{6,9})}{K_{6,9}}\right]^2 + \left(\frac{M_Y^2 K_{6,10}}{B_6}\right)^2\left[\frac{\sigma(K_{6,10})}{K_{6,10}}\right]^2$$

$$+\left(\frac{M_Z^2 K_{6,11}}{B_6}\right)^2\left[\frac{\sigma(K_{6,11})}{K_{6,11}}\right]^2 + \left(\frac{M_X^2 K_{6,12}}{B_6}\right)^2\left[\frac{\sigma(K_{6,12})}{K_{6,12}}\right]^2$$

$$+\left(\frac{F_N F_Y K_{6,13}}{B_6}\right)^2\left[\frac{\sigma(K_{6,13})}{K_{6,13}}\right]^2 + \left(\frac{F_N F_A K_{6,14}}{B_6}\right)^2\left[\frac{\sigma(K_{6,14})}{K_{6,14}}\right]^2$$

$$+\left(\frac{F_N M_Y K_{6,15}}{B_6}\right)^2\left[\frac{\sigma(K_{6,15})}{K_{6,15}}\right]^2 + \left(\frac{F_N M_Z K_{6,16}}{B_6}\right)^2\left[\frac{\sigma(K_{6,16})}{K_{6,16}}\right]^2$$

$$+\left(\frac{F_N M_X K_{6,17}}{B_6}\right)^2\left[\frac{\sigma(K_{6,17})}{K_{6,17}}\right]^2 + \left(\frac{F_Y F_A K_{6,18}}{B_6}\right)^2\left[\frac{\sigma(K_{6,18})}{K_{6,18}}\right]^2$$

$$+\left(\frac{F_Y M_Y K_{6,19}}{B_6}\right)^2\left[\frac{\sigma(K_{6,19})}{K_{6,19}}\right]^2 + \left(\frac{F_Y M_Z K_{6,20}}{B_6}\right)^2\left[\frac{\sigma(K_{6,20})}{K_{6,20}}\right]^2$$

(Equation (A25) continued on next page)

$$+\left(\frac{F_Y M_X K_{6,21}}{B_6}\right)^2\left[\frac{\sigma(K_{6,21})}{K_{6,21}}\right]^2+\left(\frac{F_A M_Y K_{6,22}}{B_6}\right)^2\left[\frac{\sigma(K_{6,22})}{K_{6,22}}\right]^2$$

$$+\left(\frac{F_A M_Z K_{6,23}}{B_6}\right)^2\left[\frac{\sigma(K_{6,23})}{K_{6,23}}\right]^2+\left(\frac{F_A M_X K_{6,24}}{B_6}\right)^2\left[\frac{\sigma(K_{6,24})}{K_{6,24}}\right]^2$$

$$+\left(\frac{M_Y M_Z K_{6,25}}{B_6}\right)^2\left[\frac{\sigma(K_{6,25})}{K_{6,25}}\right]^2+\left(\frac{M_Y M_X K_{6,26}}{B_6}\right)^2\left[\frac{\sigma(K_{6,26})}{K_{6,26}}\right]^2$$

$$\left.+\left(\frac{M_Z M_X K_{6,27}}{B_6}\right)^2\left[\frac{\sigma(K_{6,27})}{K_{6,27}}\right]^2\right)^{1/2} \tag{A25}$$

APPENDIX B

TYPICAL BASIC PROGRAMS FOR OBTAINING SOME STANDARD DEVIATIONS FOR WIND-TUNNEL MEASUREMENTS

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10 PRINT "PROGRAM FOR SIGMA(M) VS.M AT VARIOUS STAGNATION PRESSURES USING"
20 PRINT "DIFFERENT TYPES OF PRESSURE TRANSDUCERS. VALID FOR ALL MACH NUMBERS."
30 PRINT "FOR USE ON THE HEWLITT-PACKARD MODEL 9830(BASIC CALCULATOR)."
40 PRINT
50 PRINT
60 PRINT "                                    FILE 3"
70 PRINT
80 PRINT "THIS PROGRAM WAS WRITTEN IN I.S.U. FEB.28,1979 BY E.RIND"
90 PRINT "PROGRAM FOR SIGMA(M)VS.M VALID FOR ALL MACH NUMBERS."
100 PRINT "Y=SIGMA(M)"
110 PRINT "F=FULL SCALE ON PRESSURE GAUGE USED"
120 PRINT "P1=STAGNATION PRESSURE;N/M↑2"
130 PRINT "P2=STATIC PRESSURE;N/M↑2"
140 PRINT "S1=SIGMA(P1);N/M↑2"
150 PRINT "S2=SIGMA(P2);N/M↑2"
160 PRINT "G=GAMMA"
170 SCALE 0,1.2,0,3.5E-04
180 A=4.826E+05
190 B=6.895E+05
200 C=1.034E+06
210 D=2.758E+05
220 H=1.931E+05
230 J=1.034E+05
240 K=4.826E+04
250 N=2.068E+05
260 G=1.4
270 G1=1/G
280 READ P1
290 PRINT
300 PRINT "P1="P1"N/M↑2;STAGNATION PRESSURE"
310 PRINT
320 FOR M=0.2 TO 1.2 STEP 0.1
330 P2=P1*(1+M*M/5)↑(-7/2)
340 IF P1>A AND P2>A AND P1 <= B AND P2 <= B THEN 380
350 IF P1>B OR P2>B THEN 360
360 F=C
370 GOTO 390
380 F=B
390 S2=(6E-05*F+P2*1.2E-04)/3
400 S1=(6E-05*F+P1*1.2E-04)/3
410 IF P1>A THEN 500
420 IF P1 <= A AND P2 <= A AND P1 >= D AND P2 >= H THEN 480
430 IF P1<D AND P2 <= D AND P1 >= J AND P2 >= K THEN 450
440 IF P1<J OR P2<K THEN 450
450 S2=6.895
460 S1=6.895
470 IF S2 OR S1=6.895 THEN 500
480 S2=13.79
490 S1=13.79
500 Y=(P1/P2)↑(-G1)*(1/(M*G*P2))*SQR(S1↑2+(P1/P2)↑2*S2↑2)
510 PRINT "M="M;"Y="Y;"P1="P1;"P2="P2;"G1="G1;"G="G,"S1="S1;"S2="S2
520 GOTO 540
530 PLOT M,Y
540 NEXT M
550 PEN
560 DATA 1.035E+05,2.068E+05,2.758E+05,3.447E+05,4.137E+05,4.826E+05,6.205E+05
570 DATA 6.895E+05,8.274E+05,8.963E+05,1.034E+06
580 GOTO 280
590 END
600 PRINT "THIS PROGRAM WAS RUN BY                           ON
```

APPENDIX B

```
10 PRINT "PROGRAM FOR SIGMA(M) VS. M FOR DIFFERENT STAGNATION PRESSURES"
20 PRINT "USING VARIOUS FULL SCALE RUSKA PRESSURE TRANSDUCERS.VALID FOR ALL MACH
30 PRINT "NOS. FOR USE ON THE HEWLITT-PACKARD MODEL 9830 (BASIC) CALCULATOR."
40 PRINT
50 PRINT
60 PRINT "                                          FILE 7"
70 PRINT
80 PRINT "THIS PROGRAM WAS WRITTEN BY E.RIND MAY 17,1979."
90 PRINT "PROGRAM FOR SIGMA(M)VS.M USING VARIOUS FULL SCALE RUSKA PRESSURE"
100 PRINT "TRANSDUCERS. VALID FOR ALL VALUES OF MACH NUMBER."
110 PRINT "ALL UNITS ARE IN I.S.U."
120 PRINT "Y=SIGMA(M)"
130 PRINT "P1=STAGNATION PRESSURE-N/M↑2"
140 PRINT "P2=STATIC PRESSURE-N/M↑2"
150 PRINT "S1=SIGMA(P1)"
160 PRINT "S2=SIGMA(P2)"
170 PRINT "M=MACH NUMBER"
180 PRINT "G=GAMMA=CSUBP/CSUBV"
190 SCALE 0,1.2,0,0.00133
200 G=1.4
210 G1=1/G
220 READ P1,F,F1
230 PRINT
240 PRINT
250 PRINT "P1="P1"STAGNATION PRESS.-N/M↑2","F="F,"F1="F1
260 PRINT
270 FOR M=0.2 TO 1.2 STEP 0.2
280 P2=P1*(1+M*M/5)↑(-7/2)
290 DATA 8.963E+05,1.034E+06,1.034E+06,8.963E+05,1.034E+06,6.895E+05
300 DATA 8.963E+05,1.034E+06,3.447E+05
310 IF P1=8.963E+05 AND F=1.034E+06 AND F1=1.034E+06 THEN 590
320 IF P1=8.963E+05 AND F=1.034E+06 AND F1=6.895E+05 THEN 610
330 IF P1=8.963E+05 AND F=1.034E+06 AND F1=3.447E+05 THEN 640
340 REM THE TOTAL PRESSURE IN THE TUNNEL FOR THE FOLLOWING IS 5.171E5 N/M↑2 ABS.
350 DATA 5.171E+05,1.034E+06,6.895E+05,5.171E+05,6.895E+05,6.895E+05
360 DATA 5.171E+05,6.895E+05,3.447E+05,5.171E+05,6.895E+05,2.068E+05
370 IF P1=5.171E+05 AND F=1.034E+06 AND F1=6.895E+05 THEN 610
380 IF P1=5.171E+05 AND F=6.895E+05 AND F1=6.895E+05 THEN 670
390 IF P1=5.171E+05 AND F=6.895E+05 AND F1=3.447E+05 THEN 700
400 IF P1=5.171E+05 AND F=6.895E+05 AND F1=2.068E+05 THEN 730
410 REM THE TOTAL PRESSURE IN THE TUNNEL FOR THE FOLLOWING IS 2.068E5 N/M↑2 ABS.
420 DATA 2.068E+05,1.034E+06,3.447E+05,2.068E+05,6.895E+05,2.068E+05
430 DATA 2.068E+05,6.895E+05,1.034E+05
440 DATA 2.068E+05,3.447E+05,2.068E+05,2.068E+05,3.447E+05,1.034E+05
450 DATA 2.068E+05,2.068E+05,2.068E+05,2.068E+05,2.068E+05,1.034E+05
460 IF P1=2.068E+05 AND F=1.034E+06 AND F1=3.447E+05 THEN 640
470 IF P1=2.068E+05 AND F=6.895E+05 AND F1=2.068E+05 THEN 730
480 IF P1=2.068E+05 AND F=6.895E+05 AND F1=1.034E+05 THEN 750
490 IF P1=2.068E+05 AND F=3.447E+05 AND F1=2.068E+05 THEN 790
500 IF P1=2.068E+05 AND F=3.447E+05 AND F1=1.034E+05 THEN 820
510 IF P1=2.068E+05 AND F=2.068E+05 AND F1=2.068E+05 THEN 850
520 IF P1=2.068E+05 AND F=2.068E+05 AND F1=1.034E+05 THEN 870
530 REM THE TOTAL PRESSURE IN THE TUNNEL FOR THE FOLLOWING IS 1.034E5 N/M↑2 ABS.
540 DATA 1.034E+05,2.068E+05,2.068E+05,1.034E+05,2.068E+05,1.034E+05
550 DATA 1.034E+05,1.034E+05,1.034E+05
560 IF P1=1.034E+05 AND F=2.068E+05 AND F1=2.068E+05 THEN 850
570 IF P1=1.034E+05 AND F=2.068E+05 AND F1=1.034E+05 THEN 870
580 IF P1=1.034E+05 AND F=1.034E+05 AND F1=1.034E+05 THEN 900
590 S1=S2=137.9/2
600 GOTO 910
610 S1=137.9/2
620 S2=96.53/2
630 GOTO 910
640 S1=137.9/2
650 S2=55.16/2
```

FILE 7 CONTINUED

```
660 GOTO 910
670 S1=S2=96.53/2
680 GOTO 910
690 STOP
700 S1=96.53/2
710 S2=55.16/2
720 GOTO 910
730 S1=96.53/2
740 S2=34.48/2
750 GOTO 910
760 S1=96.53/2
770 S2=20.68/2
780 GOTO 910
790 S1=55.16/2
800 S2=34.48/2
810 GOTO 910
820 S1=55.16/2
830 S2=20.68/2
840 GOTO 910
850 S1=S2=34.48/2
860 GOTO 910
870 S1=34.48/2
880 S2=20.68/2
890 GOTO 910
900 S1=S2=20.68/2
910 Y=(P1/P2)↑(-G1)*(1/(M*G*P2))*SQR(S1↑2+(P1/P2)↑2*S2↑2)
920 PRINT "M="M;"Y="Y;"P1="P1;"F="F;"P2="P2;"F1="F1,"S1=";1,"S2="S2;
930 PRINT "G1="G1;"G="G
940 PLOT M,Y
950 NEXT M
960 PEN
970 GOTO 220
980 PRINT
990 END
1000 PRINT "THIS PROGRAM WAS RUN BY                    ON
```

APPENDIX B

```
10 PRINT "THIS IS THE PROGRAM FOR SIGMA (Q)VS.M FOR DIFFERENT STAGNATION PRESS-
20 PRINT "URES AND DIFFERENT TYPES OF PRESSURE TRANSDUCERS.VALID FOR ALL MACH"
30 PRINT "NUMBERS.IN I.S.U. (METRIC SYSTEM). FOR USE ON THE HEWLETT-PACKARD"
40 PRINT "MODEL 9830 (BASIC) CALCULATOR."
50 PRINT
60 PRINT
70 PRINT "                                         FILE 8"
80 PRINT
90 PRINT "PROGRAM FOR SIGMA(Q) VS. M IN I.S.U.BY E.RIND AUG15,1979"
100 PRINT "USING FORMULAS APPLICABLE OVER ALL MACH NUMBERS"
110 PRINT
120 PRINT "Y=SIGMA (Q)"
130 PRINT "F=FULL SCALE ON PRESSURE GAUGE USED"
140 PRINT "P1=STAGNATION PRESSURE, N/M↑2"
150 PRINT "P2=STATIC PRESSURE, N/M↑2"
160 PRINT "S1=SIGMA(P1), N/M↑2"
170 PRINT "S2=SIGMA(P2), N/M↑2"
180 PRINT "S3=SIGMA(M)"
190 PRINT "Q=THE DYNAMIC PRESSURE, N/M↑2"
200 SCALE 0,1.2,0,91
210 A=1.034E+05
220 B=2.758E+05
230 C=3.447E+05
240 D=4.826E+05
250 H=6.895E+05
260 J=8.963E+05
270 K=1.034E+06
280 N=1.931E+05
290 U=4.826E+04
300 G=1.4
310 G1=1/G
320 READ P1
330 PRINT
340 PRINT "P1="P1"N/M↑2; STAGNATION PRESSURE"
350 PRINT "(Y,P1,P2,S1,S2,AND S3 ARE IN N/M↑2)"
360 PRINT
370 FOR M=0.2 TO 1.2 STEP 0.5
380 P2=P1*(1+M*M/5)↑(-7/2)
390 Q=G/2*P2*M↑2
400 IF P1>D AND P2>D AND P1 <= H AND P2 <= H THEN 440
410 IF P1>H AND P2>H THEN 420
420 F=K
430 GOTO 450
440 F=H
450 S2=(6E-05*F+P2*1.2E-04)/3
460 S1=(6E-05*F+P1*1.2E-04)/3
470 IF P1>D THEN 570
480 IF P1 <= D AND P2 <= D AND P1 >= B AND P2 >= N THEN 550
490 IF P1<B AND P2<B AND P1 >= A AND P2 >= U THEN 510
500 IF P1<A OR P2<U THEN 510
510 S2=6.895
520 S1=6.895
530 IF S2=6.895 THEN 570
540 IF S1=6.895 THEN 570
550 S2=13.789
560 S1=13.789
570 S3=(P1/P2)↑(-G1)*(1/(M*G*P2))*SQR(S1↑2+(P1/P2)↑2*S2↑2)
580 Y=SQR(((G/2)*M↑2*S2)↑2+(G*P2*M*S3)↑2)
590 PLOT M,Y
600 PRINT "M="M;"Y="Y;"P1="P1;"P2="P2;"S1="S1,"S2="S2;"S3="S3
610 PEN
620 NEXT M
630 DATA 1.034E+05,2.758E+05,3.477E+05,4.826E+05,6.895E+05,8.963E+05,1.034E+06
640 GOTO 320
650 END
660 PRINT "THIS PROGRAM WAS RUN BY                          ON"
```

APPENDIX B

```
10 PRINT "THIS IS THE PROGRAM FOR SIGMA (R/L) VS. MACH NUMBER FOR VARIOUS "
20 PRINT "STAGNATION PRESSURES USING DIFFERENT TYPES OF PRESSURE TRANSDUCERS."
30 PRINT "VALID FOR ALL MACH NUMBERS. FOR USE ON THE HEWLETT-PACKARD MODEL 9830"
40 PRINT "(BASIC) CALCULATOR."
50 PRINT
60 PRINT
70 PRINT "                                        FILE 5"
80 PRINT
90 PRINT "THIS PROGRAM WAS WRITTEN BY E.RIND MARCH 7,1979."
100 PRINT "PROGRAM FOR SIGMA(R/L)VS.MACH NO. IN I.S.U. FOR ALL VALUES OF MACH NO
110 PRINT "P1=STAGNATION PRESSURE,N/M↑2"
120 PRINT "P2=STATIC PRESSURE,N/M↑2"
130 PRINT "S1=SIGMA(P1),N/M↑2"
140 PRINT "S2=SIGMA(P2),N/M↑2"
150 PRINT "Q= DYNAMIC PRESSURE,N/M↑2"
160 PRINT "T=ABSOLUTE TOTAL TEMPERATURE,(DEG. KELVIN)"
170 A=1.034E+05
180 B=2.758E+05
190 C=3.447E+05
200 D=4.826E+05
210 H=6.895E+05
220 J1=8.963E+05
230 K=1.034E+06
240 N=1.931E+05
250 U=4.826E+04
260 PRINT "ALL TEMPERATURES MUST BE IN DEG. KELVIN."
270 PRINT "Y=SIGMA (R/L)"
280 REM T1=77.78 DEG. KELVIN=140 DEG. RANKINE
290 REM T2=111.1 DEG. KELVIN=200 DEG. RANKINE
300 REM T3=166.7 DEG. KELVIN=300 DEG. RANKINE
310 REM T4=222.2 DEG. KELVIN=400 DEG. RANKINE
320 REM T5=277.8 DEG. KELVIN=500 DEG. RANKINE
330 REM T6=333.3 DEG. KELVIN=600 DEG. RANKINE
340 REM T7=338.9 DEG. KELVIN=610 DEG. RANKINE
350 PRINT "F=FULL SCALE ON PRESSURE GAUGE USED"
360 REM (R/L)=1.2583E06*P1*M/T↑2*(1+M*M/5)↑(-1.5)*((1+M*M/5)↑(-1)*T+198.6)
370 REM (R/L)=1.2583E06*SQR(((P1/P2↑(-4/7)-(P1/P2)↑(-6/7))*5)
380 REM *(P1↑(5/7)*P2↑(2/7)/T+198.6*P1/T↑2
390 PRINT "S1=SIGMA(P1);S2=SIGMA(P2);S3=SIGMA(T);T=TEMP,DEG.KELVIN;M=MACH NO."
400 PRINT "G=GAMMA"
410 REM S3=0.5 DEG./3(DEG.C)=0.9/3 DEG. F=0.3 DEG. RANKINE
420 G=1.4
430 G1=1/G
440 SCALE 0,1.2,0,4.9E+06
450 G2=G1-1
460 PRINT "W=REYNOLD'S NUMBER/L"
470 FOR J=1 TO 7
480 READ T
490 FOR I=1 TO 7
500 READ P1
510 PRINT
520 PRINT "P1="P1"N/M↑2,STAGNATION PRESSURE","T="T"TEMP,DEG. KELVIN"
530 PRINT
540 FOR M=0.2 TO 1.2 STEP 0.5
```

APPENDIX B

FILE 5 CONTINUED

```
550 P2=P1*(1+M*M/5)↑(-7/2)
560 Q=P2*(1+M*M/5)↑(7/2)-P2=P1-P2
570 IF P1>D AND P2>D AND P1 <= H AND P2 <= H THEN 610
580 IF P1>H AND P2>H THEN 590
590 F=K
600 GOTO 620
610 F=H
620 S2=(6E-05*F+P2*1.2E-04)/3
630 S1=(6E-05*F+P1*1.2E-04)/3
640 IF P1 <= D AND P2 <= D AND P1 >= B AND P2 >= N THEN 700
650 IF P1<B AND P2 <= B AND P1>A AND P2>U THEN 670
660 IF P1<A OR P2<U THEN 670
670 S2=6.895
680 S1=6.895
690 IF S1 OR S2=6.895 THEN 720
700 S2=13.789
710 S1=13.789
720 S3=0.167
730 DATA 77.78,1.034E+05,2.758E+05,3.447E+05,4.826E+05,6.895E+05,8.963E+05
740 DATA 1.034E+06
750 DATA 111.1,1.034E+05,2.758E+05,3.447E+05,4.826E+05,6.895E+05,8.963E+05
760 DATA 1.034E+06
770 DATA 166.7,1.034E+05,2.758E+05,3.447E+05,4.826E+05,6.895E+05,8.963E+05
780 DATA 1.034E+06
790 DATA 222.2,1.034E+05,2.758E+05,3.447E+05,4.826E+05,6.895E+05,8.963E+05
800 DATA 1.034E+06
810 DATA 277.8,1.034E+05,2.758E+05,3.447E+05,4.826E+05,6.895E+05,8.963E+05
820 DATA 1.034E+06
830 DATA 333.3,1.034E+05,2.758E+05,3.447E+05,4.826E+05,6.895E+05,8.963E+05
840 DATA 1.034E+06
850 DATA 338.9,1.034E+05,2.758E+05,3.447E+05,4.826E+05,6.895E+05,8.963E+05
860 DATA 1.034E+06
870 L1=((P2/P1)↑(4/7)-(P2/P1)↑(6/7))*5
880 W=4.7903E+04*(SQRL1/T)*(P1↑(5/7)*P2↑(2/7)+110.3*P1/T)
890 REM W=REYNOLD'S NUMBER/L
900 Y1=(SQRL1*(((P2/P1)↑(2/7)*T+220.6)/T↑3)*P1*S3)↑2
910 Y2=SQRL1*(((5/7)*(P2/P1)↑(2/7)*T+110.3)/T↑2)
920 Y3=(((P2/P1)↑(2/7)*T+110.3)/T↑2)*(P1/P2)*(5/2)*(1/SQRL1)
930 Y4=((-4*(P2/P1)↑(11/7)+6*(P2/P1)↑(13/7))/7)
940 Y5=((Y2+Y3*Y4)*S1)↑2
950 Y6=SQRL1*(2/(7*T))*(P1/P2)↑(5/7)
960 Y7=(((P2/P1)↑(2/7)*T+110.3)/T↑2)*(P1/P2)*(5/2)*(1/SQRL1)
970 Y8=(4*(P2/P1)↑(4/7)-6*(P2/P1)↑(6/7))/7
980 Y9=((Y6+Y7*Y8)*S2)↑2
990 Y=SQR((4.7903E+04)↑2*(Y1+Y5+Y9))
1000 PRINT M,Y,W,P1,P2,S1,S2,S3,T
1010 PRINT L1,Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9,Y,W
1020 PLOT M,Y
1030 PEN
1040 NEXT M
1050 NEXT I
1060 NEXT J
1070 REM A=1.034E+05N/M↑2=2160#/FT↑2
1080 REM B=2.758E+05N/M↑2=5760#/FT↑2
1090 REM C=3.447E+05N/M↑2=7200#/FT↑2
1100 REM D=4.826E+05N/M↑2=10080#/FT↑2
1110 REM H=6.895E+05N/M↑2=14400#/FT↑2
1120 REM J1=8.963E+05N/M↑2=18720#/FT↑2
1130 REM K=1.034E+06N/M↑2=21600#/FT↑2
1140 REM N=1.931E+05N/M↑2=4032#/FT↑2
1150 REM U=4.826E+04N/M↑2=1008#/FT↑2
1160 END
1170 PRINT "THIS PROGRAM WAS RUN BY                    ON
```

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5. Cook, T. A.: A Note on the Calibration of Strain Gauge Balances for Wind Tunnel Models. Tech Note No. Aero. 2631, British R.A.E., Dec. 1959.

6. Smith, David L.: An Efficient Algorithm Using Matrix Methods To Solve Wind-Tunnel Force-Balance Equations. NASA TN D-6860, 1972.

TABLE I.- LOAD CORRESPONDING TO A GIVEN SUBSCRIPT

Load	Load denoted by subscript -	Load	Load denoted by subscript -
F_N	1	$F_N M_Y$	15
F_Y	2	$F_N M_Z$	16
F_A	3	$F_N M_X$	17
M_Y	4	$F_Y F_A$	18
M_Z	5	$F_Y M_Y$	19
M_X	6	$F_Y M_Z$	20
F_N^2	7	$F_Y M_X$	21
F_Y^2	8	$F_A M_Y$	22
F_A^2	9	$F_A M_Z$	23
M_Y^2	10	$F_A M_X$	24
M_Z^2	11	$M_Y M_Z$	25
M_X^2	12	$M_Y M_X$	26
$F_N F_Y$	13	$M_Z M_X$	27
$F_N F_A$	14		

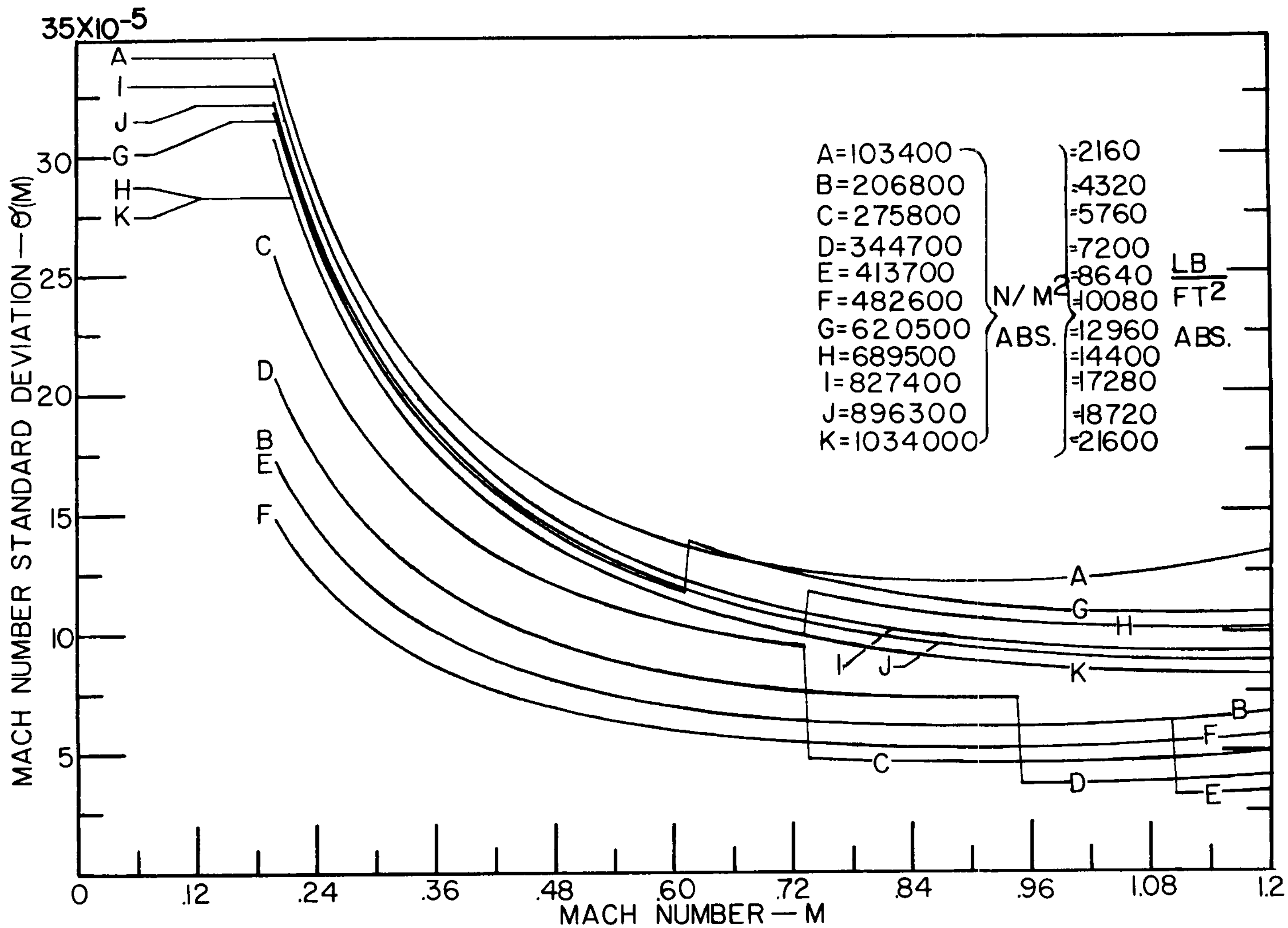


Figure 1.- σ(M) versus Mach number for various stagnation pressures using different types of pressure transducers.

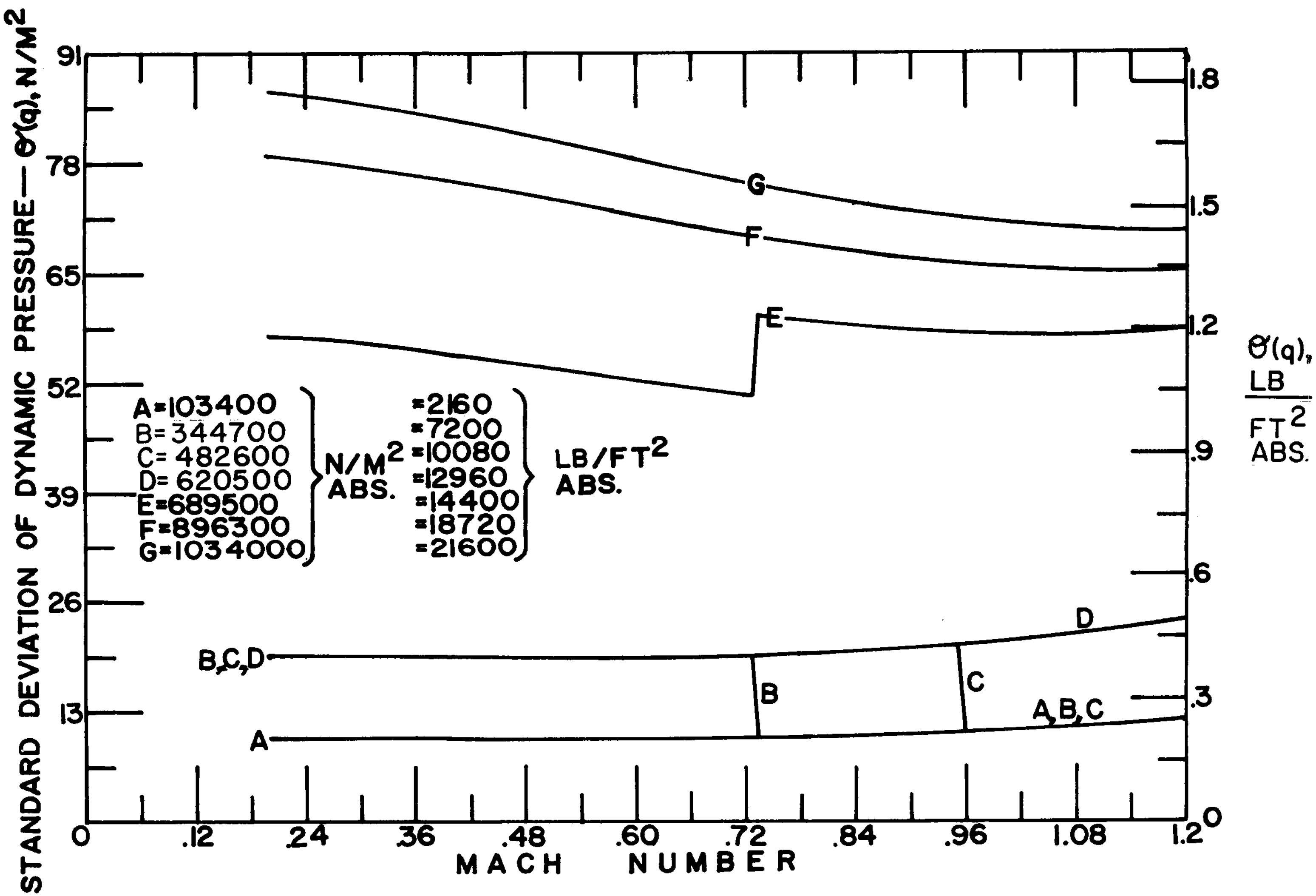


Figure 2.- σ(q) versus Mach number for various stagnation pressures using different types of pressure transducers.

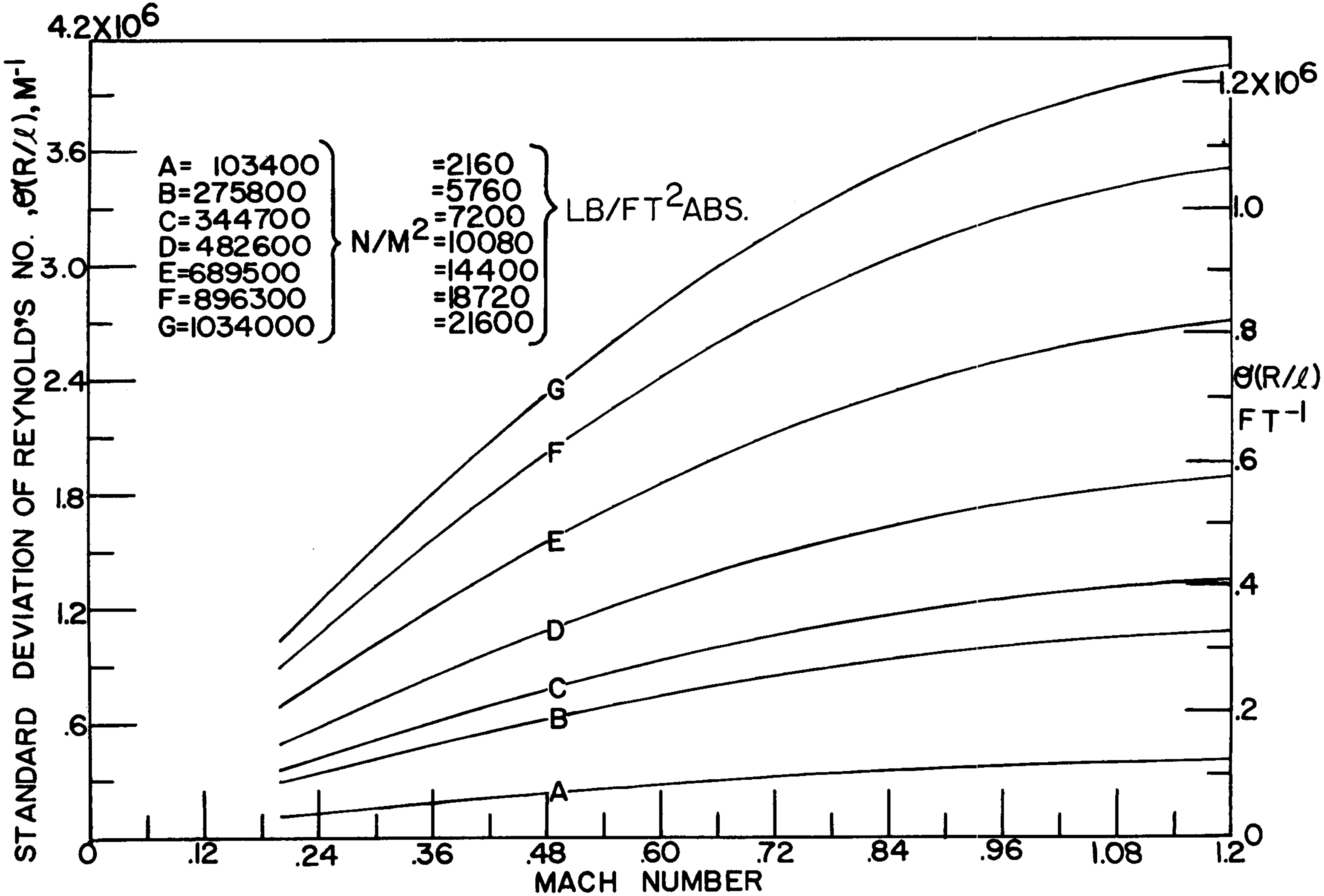
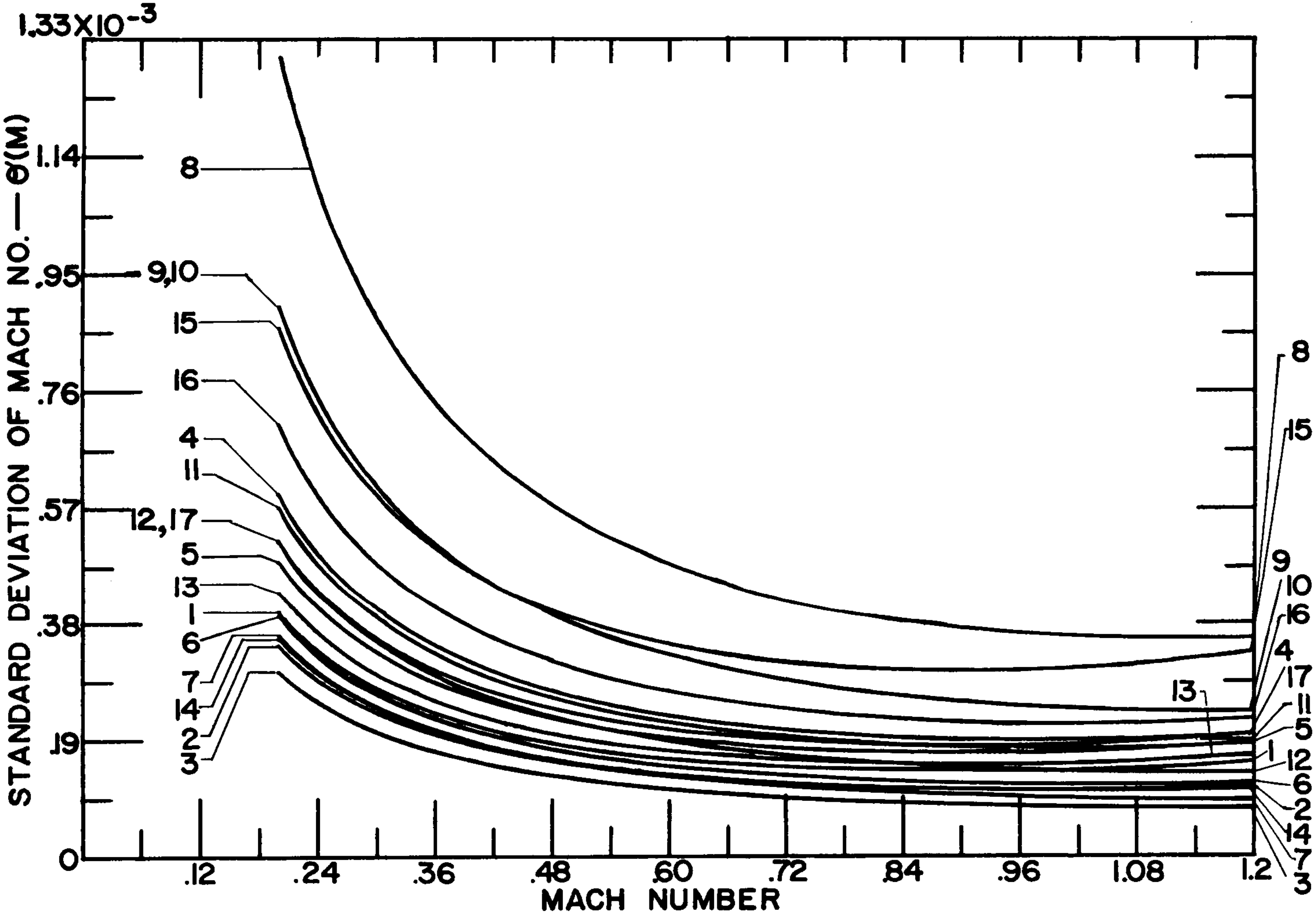


Figure 3.- $\sigma(R/\ell)$ versus Mach number at $T = 77.78$ K (140° R) for various stagnation pressures using different types of pressure transducers.



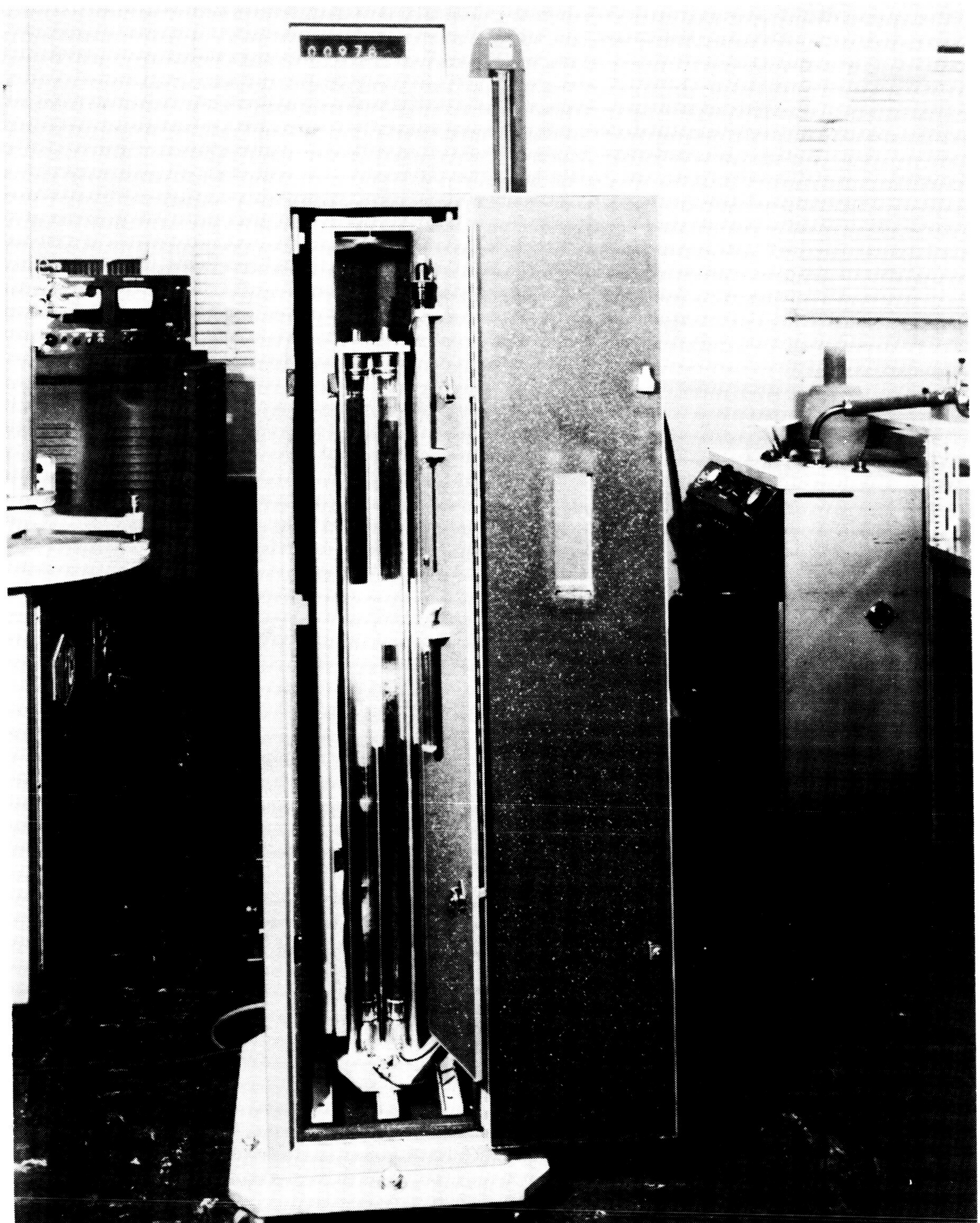
(a) σ(M) versus Mach number for various stagnation pressures.

Figure 4.- σ(M) versus Mach number for various stagnation pressures using only fused-quartz Bourdon tube pressure gauges with different full scales.

Curve	Wind-tunnel operating p_t		Instrument full scale used for stagnation pressure		Instrument full scale used for static pressure	
	N/m^2	lb/ft^2	N/m^2	lb/ft^2	N/m^2	lb/ft^2
1	8.963×10^5	18 720	10.34×10^5	21 600	10.34×10^5	21 600
2	8.963	18 720	10.34	21 600	6.895	14 400
3	8.963	18 720	10.34	21 600	3.447	7 200
4	5.171	10 800	10.34	21 600	6.895	14 400
5	5.171	10 800	6.895	14 400	6.895	14 400
6	5.171	10 800	6.895	14 400	3.447	7 200
7	5.171	10 800	6.895	14 400	2.068	4 320
8	2.068	4 320	10.34	21 600	3.447	7 200
9	2.068	4 320	6.895	14 400	2.068	4 320
10	2.068	4 320	6.895	14 400	1.034	2 160
11	2.068	4 320	3.447	7 200	2.068	4 320
12	2.068	4 320	3.447	7 200	1.034	2 160
13	2.068	4 320	2.068	4 320	2.068	4 320
14	2.068	4 320	2.068	4 320	1.034	2 160
15	1.034	2 160	2.068	4 320	2.068	4 320
16	1.034	2 160	2.068	4 320	1.034	2 160
17	1.034	2 160	1.034	2 160	1.034	2 160

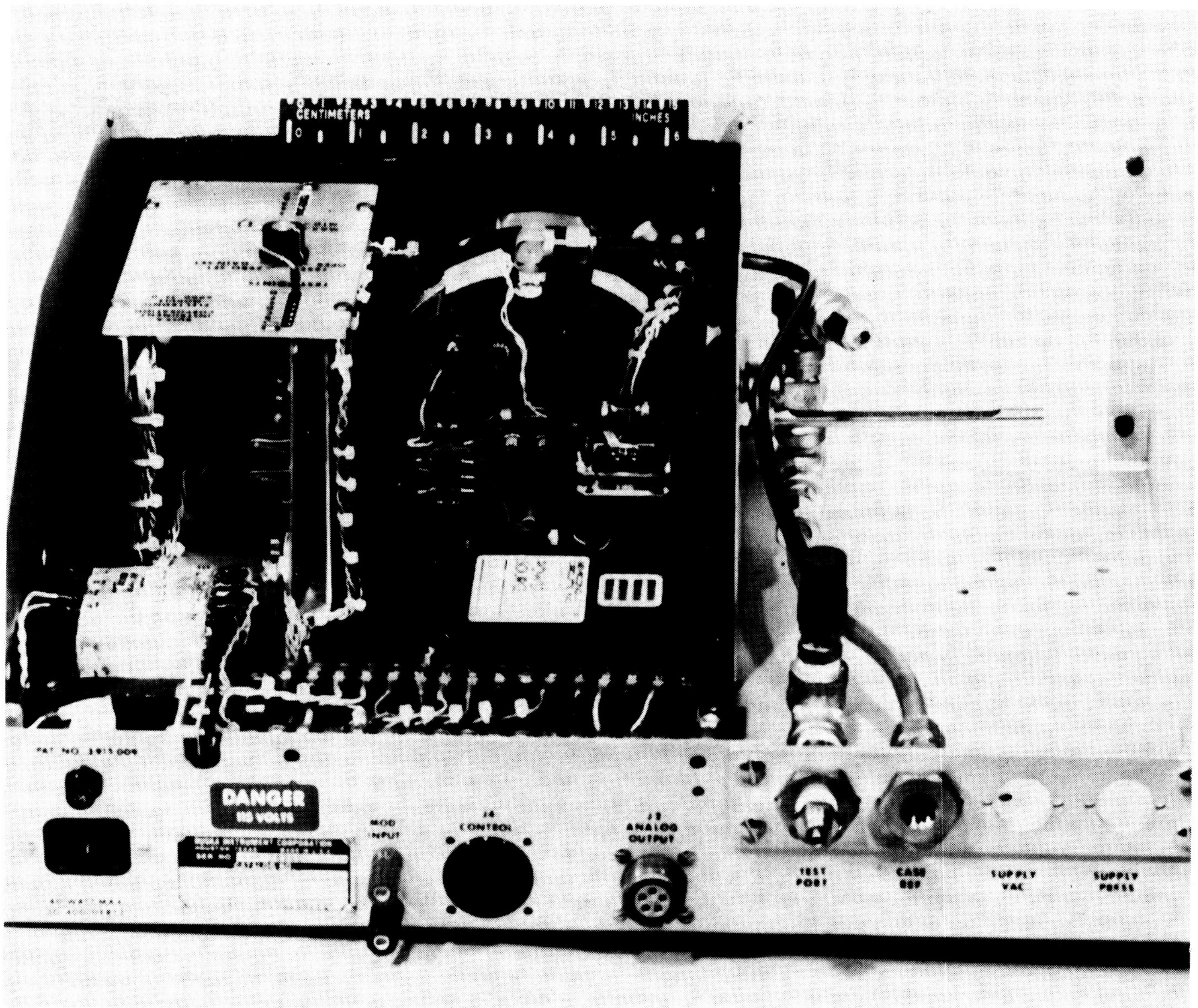
(b) Key to curves.

Figure 4.- Concluded.



L-79-335

Figure 5.- Acoustic U-tube mercury pressure manometer.



L-79-336

Figure 6.- Fused-quartz Bourdon tube pressure gauge.

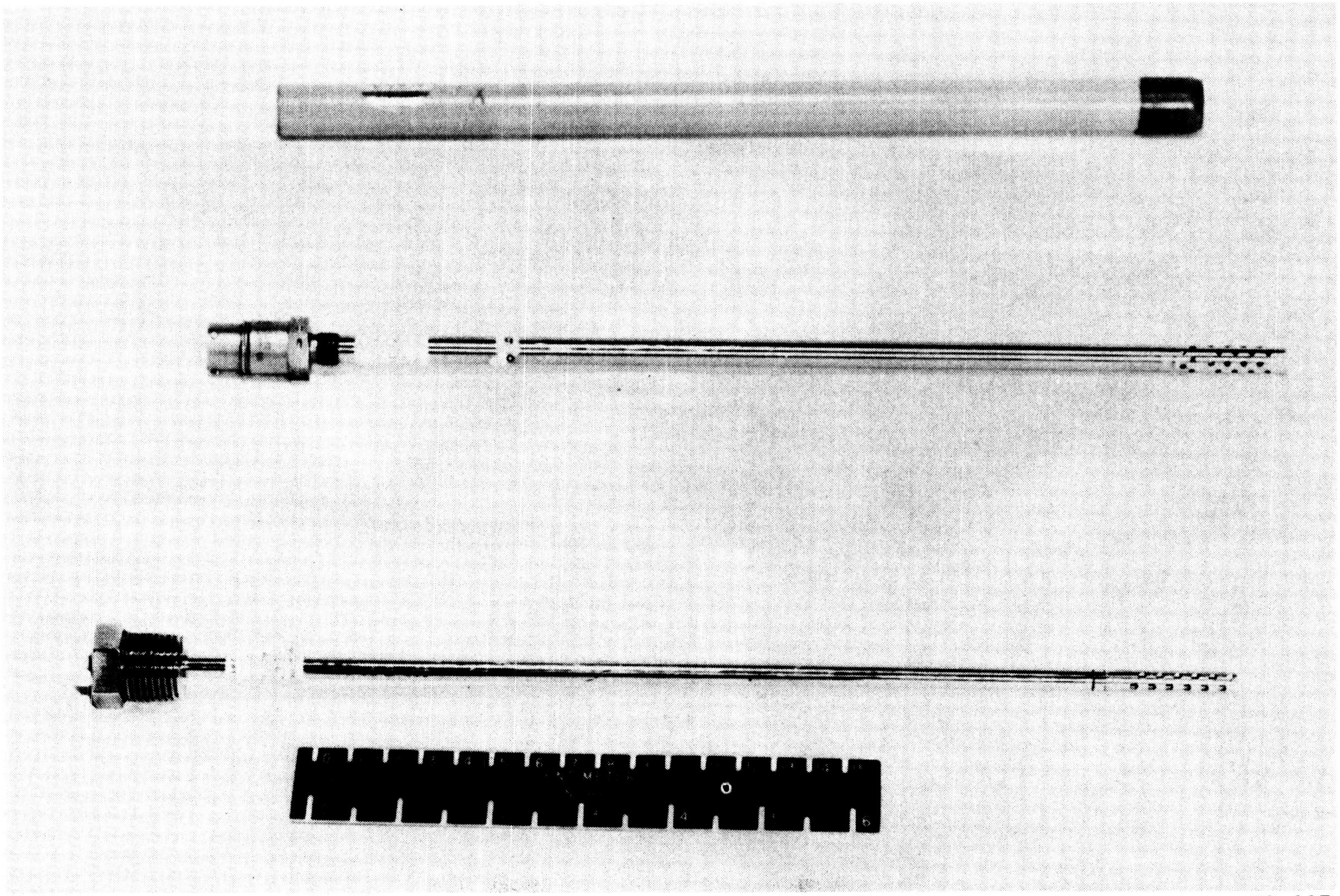


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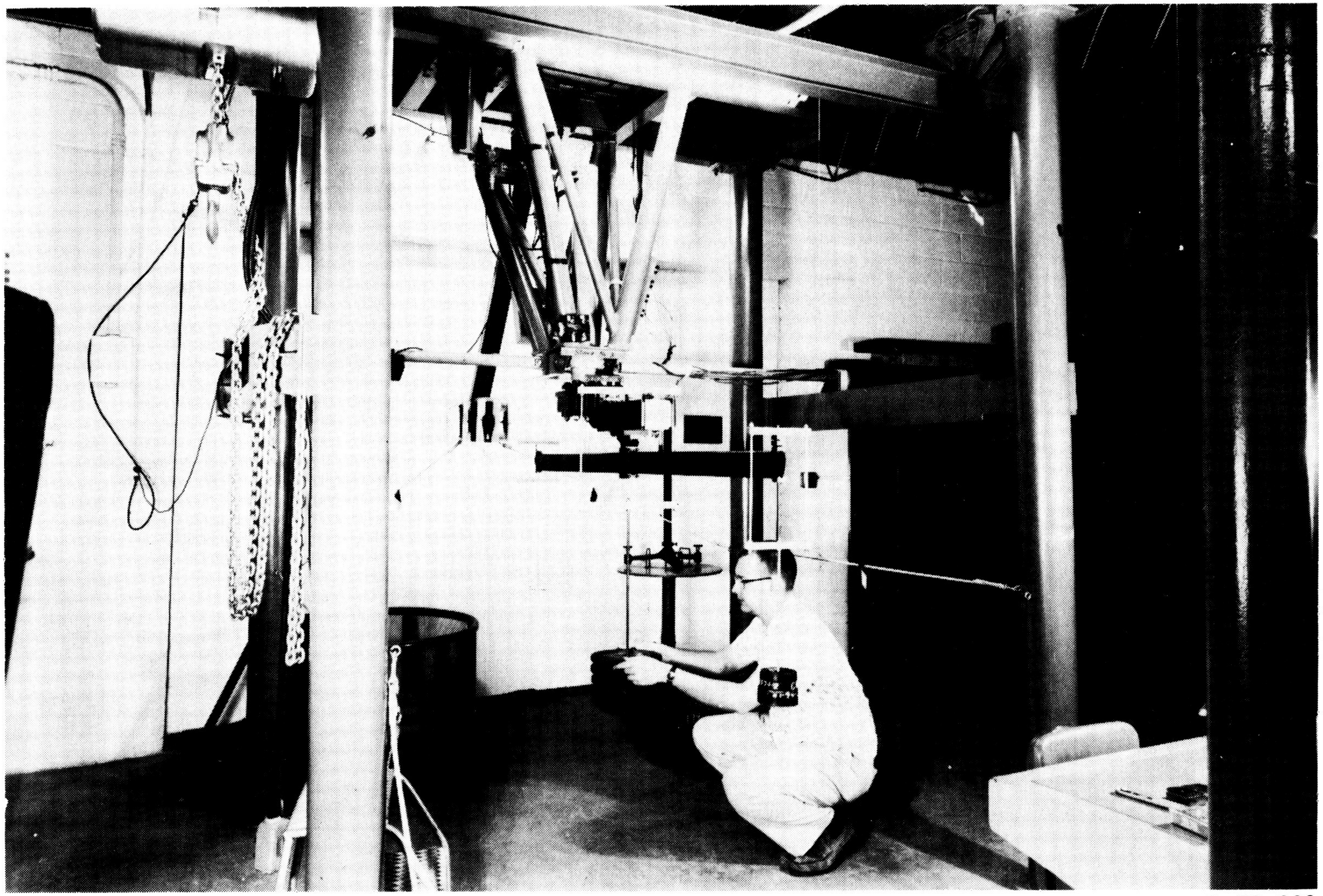
Figure 7.- Platinum resistance thermometer with sheath.



L-79-338

Figure 8.- Six-component strain-gauge-balance fixture being loaded.

1. Report No. NASA TP-1572	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle INSTRUMENT ERROR ANALYSIS AS IT APPLIES TO WIND-TUNNEL TESTING		5. Report Date December 1979
		6. Performing Organization Code
7. Author(s) Emanuel Rind		8. Performing Organization Report No. L-13222
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15. Supplementary Notes		

16. Abstract

This paper encompasses that part of error analysis which deals with errors resulting from the instrumentation used in measuring pertinent wind-tunnel parameters. The parameters, for this discussion and analysis, are limited to those required for the wind-tunnel model measurements. The pertinent parameters, their standard deviations, and the theoretical derivation of them, are given. BASIC programs and plots for the standard deviations of dynamic pressure, Mach number, and Reynolds number are included for the National Transonic Facility. A literature search was made back to 1934 and it was found that information of this kind is almost nonexistent.

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