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A COMPARATIVE EVALUATION OF SNOWMELT RUNOFF MODELS

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CHAPTER I

INTRODUCTION

Snowmelt runoff is a primary source of water supply in many mountainous areas of the world, including much of the western U.S. Water is stored all winter in the form of snowpack, then released by melting during the spring and summer months. Because most of these areas experience very little summer precipitation, almost all the available water is derived from melted snow.

In order to maximize the economic benefits of the water supply, forecasts of runoff volume are required. These forecasts are used for a variety of purposes, including planning for agriculture, municipal water supply, power generation, pollution control, recreation, navigation, and flood control. Such a variety of uses requires a variety of forecast lengths, from one day to seasonal. The accuracy of these forecasts has a significant effect on the economic benefits of managing the water resource.

In recent years, a variety of forecast procedures have been developed and used to predict snowmelt runoff. Where previously most of the operational models were empirical, such as the regression equation (SCS, 1970), a number of conceptual models have recently been developed (Martinec, 1975; Anderson, 1976; Corps of Engineers, 1975). Some of these models have been developed to utilize new data sources, most notably the measurements of snow covered area made from Landsat satellite imagery.

The primary purpose of this study is to test and compare the accuracy of a representative sample of the available snowmelt models. Most of the newer models have only been used on a few watersheds to date and direct comparisons of accuracy can only be made if all models are tested with a data base from the same watershed and the same years of record. Testing all of the available models would be extremely expensive and time consuming, so only a representative sampling is used.

Some of the techniques used in various models for increasing accuracy of prediction are also to be tested. Spatial separation of the watershed into smaller, more homogeneous areas is thought to improve accuracy of forecast. This seems reasonable because the watersheds on which the models are used are usually mountainous, encompassing a wide range of slopes and elevations. Some of the models use snow covered area data for improving the estimate of snowpack storage; this is another of the techniques which will be tested.

Another objective of this study is to determine whether accurate snowmelt runoff forecasts can be made using only snow covered area data. If this is possible, data collection for the snowmelt models would be greatly simplified. Many of the currently operational models are based on snow water equivalent measurements of the snowpack. These measurements must be made in the field, either by automated data collection stations or by man. Because the measurement sites are often remote, collection of this data is expensive. If the Landsat derived snow covered area data can be substituted for the snow water equivalents, data collection will be less expensive and easier.

Some data collection systems, notably the Landsat system of deriving snow cover area, are subject to delays between data observation and data collection. The effect of these delays on model accuracy should be evaluated; some models may only provide acceptable accuracy when used with real-time data.

CHAPTER II

LITERATURE REVIEW

Many different models have been developed and used for predicting snowmelt runoff (Leaf, 1977; Baker and Carder, 1977; Zuzel and Cox, 1978). These models vary considerably in complexity; the simplest models are based solely on statistical techniques, while the most complex methods attempt to model the individual processes involved in the melting of a snowpack. Some models are designed to predict streamflow for any given day or series of days, (Leaf, 1977; Martinec, 1975; Tangborn, 1977) while other models give only seasonal predictions (Zuzel and Cox, 1978). Generally, snowmelt models may be categorized on the basis of complexity and length of forecast period.

Empirical models are based on statistical correlations between predictor variables and the criterion variable, volume of snowmelt runoff. This type of model is most often used for seasonal predictions. Snow water equivalent measurements, previous runoff volumes, and precipitation totals are the most common predictor variables (SCS, 1970; USACE, 1956). Theory is not very important in formulating empirical models; the objective is to explain as much of the variation in the criterion values as possible using whatever data are available. It is quite common for these models to include two predictors expressed in different units, such as snow water equivalent (in inches) and previous winter runoff (in volumetric units).

Water balance models are more conceptual than the simple empirical models. The water balance is an accounting of all the water entering and leaving the basin. The volume of water stored in the snowpack is estimated from precipitation or water equivalent data; allowances are made for losses due to evaporation, groundwater storage, and transpiration; the remaining volume is the seasonal snowmelt runoff prediction (Zuzel and Cox, 1978). Loss rates may be estimated either empirically or conceptually, as may the snowpack storage. Most water balance models are somewhat empirical.

Short-term runoff predictions usually require models of greater complexity than the models used for seasonal runoff. Not only must the total volume of water stored in the snowpack be estimated, but also the proportion of that volume that will melt and leave the watershed as streamflow in a given time period must be estimated. The amount of water generated by melting snow is a function of the energy available for this purpose. Therefore, the most complex snowmelt models are generally based on an energy balance (Zuzel and Cox, 1978).

Energy balance procedures attempt to model the physical processes involved in snowmelt runoff. The amount of available energy is commonly estimated by the air temperature, although some models include such factors as incoming solar radiation, cloud cover, albedo, and net long-wave radiation (Anderson, 1976). These models often require that the watershed be subdivided into small, homogeneous areas so that the available energy for each location can be estimated more accurately (Leaf, 1977). Since snowmelt models are generally used in mountainous areas, slope and aspect can result in large differences in incident energy from one area to another. Evaporation, transpiration and groundwater losses are also estimated conceptually in some energy budget models (Leaf, 1977).

Model Selection

To test the study objectives, models having significant differences in important characteristics had to be selected. Criteria for model selection include the frequency of current usage, input data requirements and whether or not these data are typically available, the degree of model complexity, and the length of forecast period. Additionally, because snow covered area (SCA) is more readily available than in previous decades, models that either included SCA or were capable of being modified to include it were given more consideration.

Three models were selected for comparison, with several methods of evaluation for each model. The model types studied were the regression model, the Tangborn model, and the Martinec model.

The Regression Models

The most common form of empirical model is the linear regression. These models are widely used for snowmelt runoff predictions in the western U.S. (USACE, 1956; SCS, 1970). They are easily calibrated and can use many different hydrologic variables as predictor variables. These models are used for making seasonal runoff forecasts, but due to the empirical nature of the method, they may also be used to give predictions for shorter time periods.

Linear regression models are based on the assumption that there is a linear relationship between the predictor variables and the criterion variable. This assumption implies that as the value of the predictor variable increases, the value of the criterion variable changes at a constant rate. The equation that relates the value of the criterion to the value of the predictor is of the form:

$$Y = a + bX \qquad (2\text{-}1)$$

in which Y is the criterion variable, X is the predictor variable, and a and b are the regression coefficients (Miller and Freund, 1977).

Many hydrologic variables have approximately linear relationships with the volume of snowmelt runoff. A few of these variables are snow water equivalent, winter precipitation, and snow covered area. The linearity of the relationships is due to the fact that these variables are indicators of the volume of water stored in the snowpack. Because the relationships between these predictor variables and the volume of runoff are only approximately linear, many different lines may be drawn which appear to fit the data. Some of the lines pass through a number of the data points, but due to deviations from linearity, a straight line that will pass through all of the data points can not be drawn.

The method of selecting the best regression line for a set of data points is based on minimizing the sum of squares of the errors. For each observed value of the predictor, two values of the criterion variable appear; the first is the corresponding observed value and the second is the value predicted by the regression equation. The difference between these two values is termed the error of prediction. The regression line is defined as the line that results in the minimum value of the sum of the squares of the errors. The coefficients of the regression line can be derived using the equations:

$$b = \frac{\Sigma XY - (\Sigma X \Sigma Y)/n}{\Sigma X^2 - (\Sigma X)^2/2} \qquad (2\text{-}2)$$

and

$$a = (\Sigma Y)/n - b(\Sigma X)/n \qquad (2\text{-}3)$$

in which X and Y are the predictor and criterion variables, respectively, and n is the number of observations (Hays, 1965). By using these equations, the line of best fit can be determined.

In natural systems the value of the criterion variable is often a function of more than one predictor. The relationships between the criterion variable and the predictors may be assumed to be linear, resulting in a prediction equation of the form:

$$Y = a + \sum_{i=1}^{p} b_i X_i \qquad (2\text{-}4)$$

in which Y is the criterion variable, X_i is the i^{th} predictor variable, and a and b_i are the regression coefficients. Models of this type are called multiple linear regressions. The regression coefficients are unique and may be calculated from equations similar to Eqs. 2-2 and 2-3. In many cases, the inclusion of more than one predictor variable results in a more accurate model (Davis, 1973).

The Tangborn Model

The Tangborn equation is a water balance model (Tangborn and Rasmussen, 1976). The structure of the model was established conceptually, but calibration is accomplished using regression methods. The model may be used for any length of forecast period from one day to the entire snowmelt season. The only data required are daily precipitation and runoff values, although daily temperature may be included for short forecast periods.

The basic form of the model is:

$$R_s^* = a\ P_w + b - R_w \qquad (2\text{-}5)$$

in which R_s^* is the predicted runoff volume, P_w is the total depth of precipitation observed during the preceding winter, R_w is the winter runoff, and a and b are regression coefficients. The structure of the model is based on the assumption that the volume of water stored on the watershed is equal to the amount of winter precipitation minus the winter runoff. The regression coefficients represent losses and modifications such as transpiration, groundwater storage, and evaporation.

An important feature of the Tangborn model is the test season modification. In using this method, a short test season prediction model with the structure of Eq. 2-5 is developed. At the end of the test season, the error of the test season prediction is evaluated and used to modify the prediction for the forecast season. The form of the forecast model becomes:

$$R_s^{**} = R_s^{*} - ce_t = a(P_w+P_t) + b - (R_w+R_t) - ce_t \qquad (2\text{-}6)$$

in which R_s^{**} is the revised runoff prediction; R_s^{*} is the original prediction; P_w and P_t are the winter and test season precipitation, respectively; R_w and R_t are the winter and test season runoff volumes, respectively; a, b, and c are coefficients; and e_t is the error of the test season prediction. The reasoning behind this modification is that the test season error is a result of the inaccuracy of estimating basin storage by subtracting winter runoff from winter precipitation. Because the forecast season prediction is based on the same estimate, the test season error should be related to the prediction season error.

Figure 1 shows the relationship of the various seasons. In order to use the test season approach, data from the present and a number of previous years are compiled. For each year, precipitation and runoff totals are computed for the winter and test seasons; runoff totals are also computed for the prediction season of each year, except for the current year (the value for

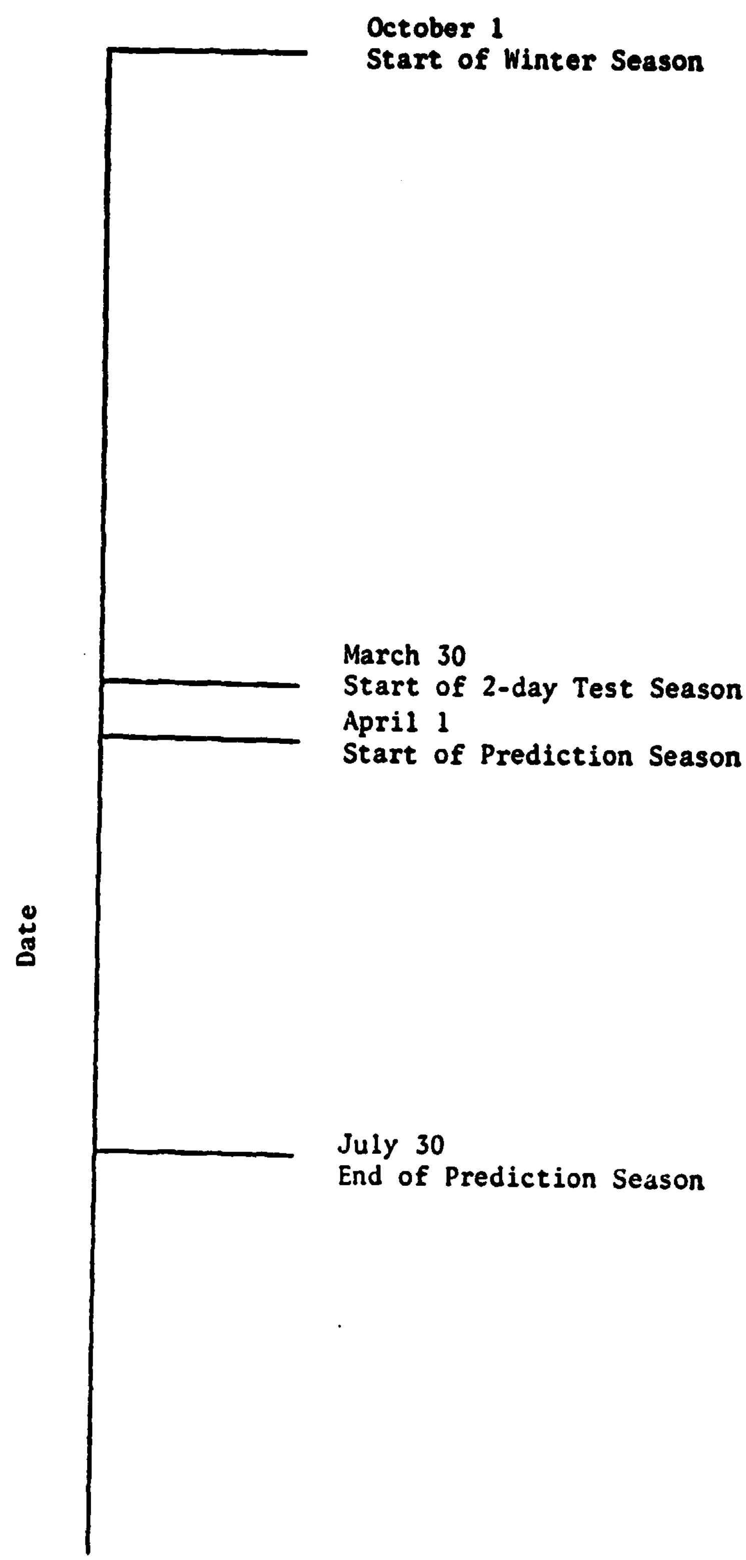

FIGURE 1. Relationship of the Winter, Test, and Prediction Seasons for the Tangborn Model

the current year is not yet known). Note that the prediction date, April 1, is at the end of the test season; therefore, observed values of runoff and precipitation during the test season are available for the current year. Once all the data has been obtained, the observed test season runoff volumes are regressed onto the winter precipitation values, resulting in a calibrated equation of the form:

$$R_t^* = a\ P_w + b - R_w \tag{2-7}$$

in which R_t^* is the predicted test season runoff. The test season error in each year is then computed by the equation:

$$e_t = R_t^* - R_t \tag{2-8}$$

Next, a model for estimating the prediction season runoff is formed by regressing the prediction season runoff on the sum of the winter and test season precipitation depths for each of the previous years:

$$R_s^* = a(P_w+P_t) + b - (R_w+R_t) \tag{2-9}$$

The errors are then calculated in a manner similar to that used for the test season:

$$e_s = R_s^* - R_s \tag{2-10}$$

in which e_s is the prediction season error. The coefficient of the test season error, c in Eq. (2-6), can then be determined.

The coefficient is computed using the test season and prediction season errors from previous years, according to the equation:

$$c = \frac{\Sigma(e_t * e_s)}{\Sigma(e_t)^2} \tag{2-11}$$

The original runoff season prediction, R_s^*, which was calculated for the current year in Eq. (2-9), is adjusted by the product of c and the current year test season error; the final prediction is:

$$R_s^{**} = R_s^* - ce_t = a(P_w+P_t) + b - (R_w+R_t) - ce_t \tag{2-12}$$

in which R_s^{**} is the final prediction.

When using the Tangborn model for prediction periods of a few days, accuracy may be increased by including temperature in the model (Tangborn, 1978). Tangborn suggested the following composite temperature variable, A_t:

$$A_t = \alpha\overline{T} + (1-\alpha)\Delta T \tag{2-13}$$

in which $\overline{T}$ is the daily mean temperature, ΔT is the daily range of temperature, and α is a coefficient. The daily mean temperature is computed from the observed maximum and minimum temperatures for the day; the range of temperature is the difference between the maximum and minimum observed values. The reasoning behind this equation is that the average daily mean temperature is an estimator of the amount of convective energy available for melting snow, and that the difference between maximum and minimum temperatures can be used to estimate the amount of radiant energy available for this purpose. Large differences between the daily maximum and minimum are indicative of clear skies, while a small daily range of temperature indicates cloud cover and, therefore, less radiant energy. The relative importance of the two components (radiative and convective) is controlled by the coefficient α. When the temperature term is included in the Tangborn model, the equation becomes:

$$R_s^{***} = a(P_w+P_t) + b - (R_w+R_t) - ce_t - e_s^* \qquad (2\text{-}14)$$

in which e_s^* is the prediction season error estimated from the temperature function, and R_s^{***} is the revised runoff prediction. The value of the prediction season error is estimated from the temperature function A_t using the equation:

$$e_s^* = dA_t + e \qquad (2\text{-}15)$$

in which d and e are coefficients determined by regression. Tangborn reports a minimum reduction in standard error of estimate of nine percent due to inclusion of this temperature term (Tangborn, 1978).

The Martinec Model

The Martinec model is conceptually derived and may be used for prediction periods of one day or longer (Martinec,1975).

The amount of energy available for anowmelt runoff is estimated by a daily temperature index. Data requirements include daily temperature, precipitation, and snow covered area. The form of the model is:

$$Q_n^* = c(dTSCA+P)\ A(1-K) + KQ_{n-1} \qquad (2\text{-}16)$$

in which Q_n^* is the predicted volume of runoff for day n, c is a dimensionless runoff coefficient, d is a degree-day factor, T is the value of the daily temperature index on day n, A is the total area of the watershed, SCA is the percentage of the area that is covered by snow on day n, K is a dimensionless recession coefficient, and Q_{n-1} is the volume of runoff observed on the previous day. The value of the daily temperature index is computed using hourly data if available; otherwise, the daily maximum and minimum temperatures are used. The daily index is a measure of the average number of degrees above freezing for the temperature on that day. The values are expressed in degree-days celsius.

The first term of Eq. (2-16) represents the amount of water that is generated by precipitation and melting snow on day n and that is expected to leave the watershed on that day. The value of the degree-day factor, d, is expressed in inches of water per degree Celsium; therefore, when the temperature index is multiplied by this factor, an estimate of the depth of water generated by snowmelt is obtained. This depth is multiplied by the total area of the watershed, A, and by the percentage of the total area that is covered by snow (SCA) to get an estimate of the volume of water produced by melting snow on day n. The precipitation, P, is assumed to be a constant depth over the entire watershed; therefore, the product of P and A is an estimate of the volume of rainfall on day n. The sum of the volume of melted snow and the volume of precipitation is referred to as the generated runoff.

Not all of the generated runoff leaves the watershed on the day of generation. Some is lost to groundwater storage and evapotranspiration; this proportion is represented by c, the runoff coefficient. Furthermore, on large watersheds the outlet of the basin is quite a distance from the source of much of the generated melt; therefore, much of the water is in transit to the outlet for several days. The proportion of water that does not reach the outlet on the day that it is generated is represented by K, the recession coefficient. Thus, only the proportion (1-K) of the runoff generated on day n actually reaches the outlet on day n.

The second term in the equation, $K \cdot Q_{n-1}$, is called the recession term. It represents the amount of water generated on previous days that is expected to appear as runoff on day n. Because K is nearly equal to 1 on large watersheds, this recession term is often considerably larger than the generated runoff term.

CHAPTER III

DATA BASE DEVELOPMENT

The selection of models was subject to four important constraints. First, the models selected should be representative of those in use and reflect variation in levels of conceptual development. Second, the models selected should be designed for use over a range of forecast periods, from one day to the entire snowmelt season. Third, the input requirements of the model should be similar to the input data that is usually available for forecasting. Fourth, a data base that includes all input requirements for all models must be available for a single watershed. The three models described in the previous chapter satisfy these requirements.

Data requirements vary significantly for the three models. Conceptual models generally require a more extensive data base than simple empirical models. Also, short-term models require that the data be collected more frequently than long-term models. While some of the data requirements of the models are the same, these data requirements can be most easily discussed by considering separately the required data base for each model.

Input Data Requirements of the Models

Regression Models

Regression models, which are used for long-term forecasting, can include almost any hydrologic variable as a predictor; the worth of any variable depends on its correlation with the forecast criterion variable, which is the amount of runoff observed during the forecast period. Frequently used predictor variables include snow water equivalent measurements, winter precipitation, and winter runoff. The snow water equivalent is measured at many sites in the mountains of the western U.S., commonly on the first days of February, March, April, and May. Precipitation and runoff are also measured at many locations, usually on a daily basis; these daily values are summed up to derive the seasonal totals, which are used in the regression equations. In this study, snow covered area data were tested for use in regression equations; the percentages of the total watershed area covered by snow on April and May 1 were used as predictor variables.

Tangborn Model

The Tangborn model can be used for both short-term and long-term prediction. The model requires daily values of precipitation and runoff during the snowmelt season; also, the total precipitation and runoff observed during the preceding winter is needed. No other data are required, although a modification to the short-term model has been proposed by Tangborn (1978); this modified model requires daily maximum and minimum temperatures, in addition to the precipitation and runoff data.

Martinec Model

The Martinec model is used for predicting runoff for short time periods (up to 15 days in this study). Data required on a daily basis are the hourly or maximum and minimum temperature, precipitation, snow covered area, and runoff. The accuracy of the model may be improved by subdividing the watershed into elevation zones. If separation by elevation is required, the daily snow covered area data must be separated into elevation zones; the zonal temperature and precipitation data can be extrapolated from base station readings.

Selection of a Test Watershed

In order to allow direct comparison of the results of testing the various models, all of the models should be tested on the same watershed using data from the same years. Data required for this testing program are daily average temperature, daily precipitation, daily snow covered area (divided into elevation zones), daily runoff, and monthly snow water equivalents. This data must be available for a number of years to ensure a representative sample. Furthermore, the models must be tested for years other than those for which they are calibrated in order to simulate a true prediction situation.

The Kings River watershed, in the Sierra Nevada mountains of California, was selected as the test site. The watershed is large with a total area of 1545 square miles. The elevation ranges from less than 1000 feet to nearly 13,000 feet. The general orientation of the basin is east-west, as shown in Fig. 2. All of the data required for calibrating and testing the models are available for this watershed, except daily snow covered area, which must be interpolated from a few observations

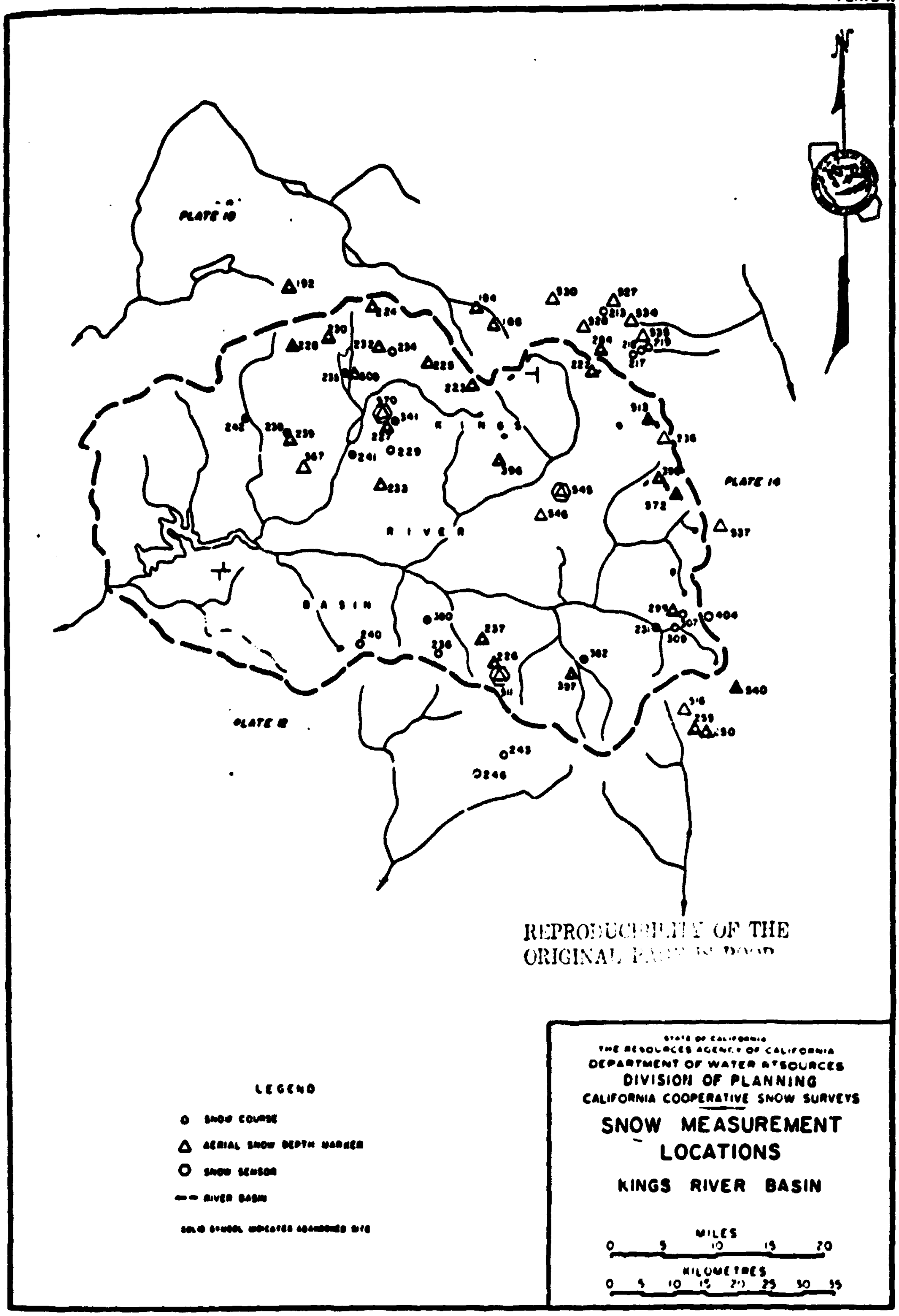

FIGURE 2. Map of Kings River Watershed Showing Location of Snow Courses

each year.

Until 1973, observations of snow covered area were made by low altitude aerial mapping performed by the U.S. Army Corps of Engineers. This process is expensive, and snow covered area data were only collected for a few major snow basins prior to the Landsat satellite program. For most basins, then, snow covered area records only go back to 1973, which is an insufficient length of record for this study. The Kings River basin is one for which snow covered area data are available from before the Landsat program. All other required data are available from a number of stations in the basin, with at least 25 years of record. The Corps of Engineers began mapping the snow covered area on the Kings River basin four times per year in 1952, and continued to do so until 1973. Since that year, Landsat imagery has been used to derive snow covered area data as often as possible. The Landsat satellites provided imagery of the Kings River watershed every 18 days in the period 1973-1977. Unfortunately, cloud cover often obscured the basin; the actual measurement interval for snow covered area is as great as 36 days.

Derivation of Daily Snow Covered Area Data

The ideal data base for testing these models would include daily observations of snow covered area, divided into elevation zones, for at least 15 years. This data was not available for any watershed, so the missing data were generated from the observations that were available. Earlier investigators (Moravec, 1977) have suggested that a good estimation of the snow cover depletion curve could be derived from four or five data points. The observed snow covered area is plotted versus date of observation, and a smooth S-shaped curve is drawn through the data points. When this method was applied to the Kings River data collected for 1973 through 1977, the smooth depletion curve required could not be drawn for one of the years without gross inaccuracies. The graphs are shown in Figure 3. Note that in 1977 the snow cover falls off in April, then increases later in the season by 25 percentage points. Clearly, an S-shaped depletion curve cannot be drawn for this year. Judgments as to the suitability of the S-curve method for the Kings River basin are based only on the years 1973-1977, because the data provided for the other years (1952-1972) consists of only four observations per year, which were made on May 1, May 15, June 1, and June 15 of each year; in most years this is the period of maximum ablation, but the data give little indication of the snow covered area or ablation rate early in the melt season.

Estimates of the snow covered area on April 1 of each year were required for use as a predictor variable in the April regression models. Observations of snow covered area for dates prior to April 1 were available only from the Landsat data. Values of April 1 snow cover were derived from the S-curves in Fig.3 for the years from 1973 to 1977. In order to estimate the April 1 values for the remaining years, a regression model was formed and calibrated using the data from 1973-1977. The data available for making these estimates consisted of previous winter precipitation, daily temperature, and the April snow water equivalents.

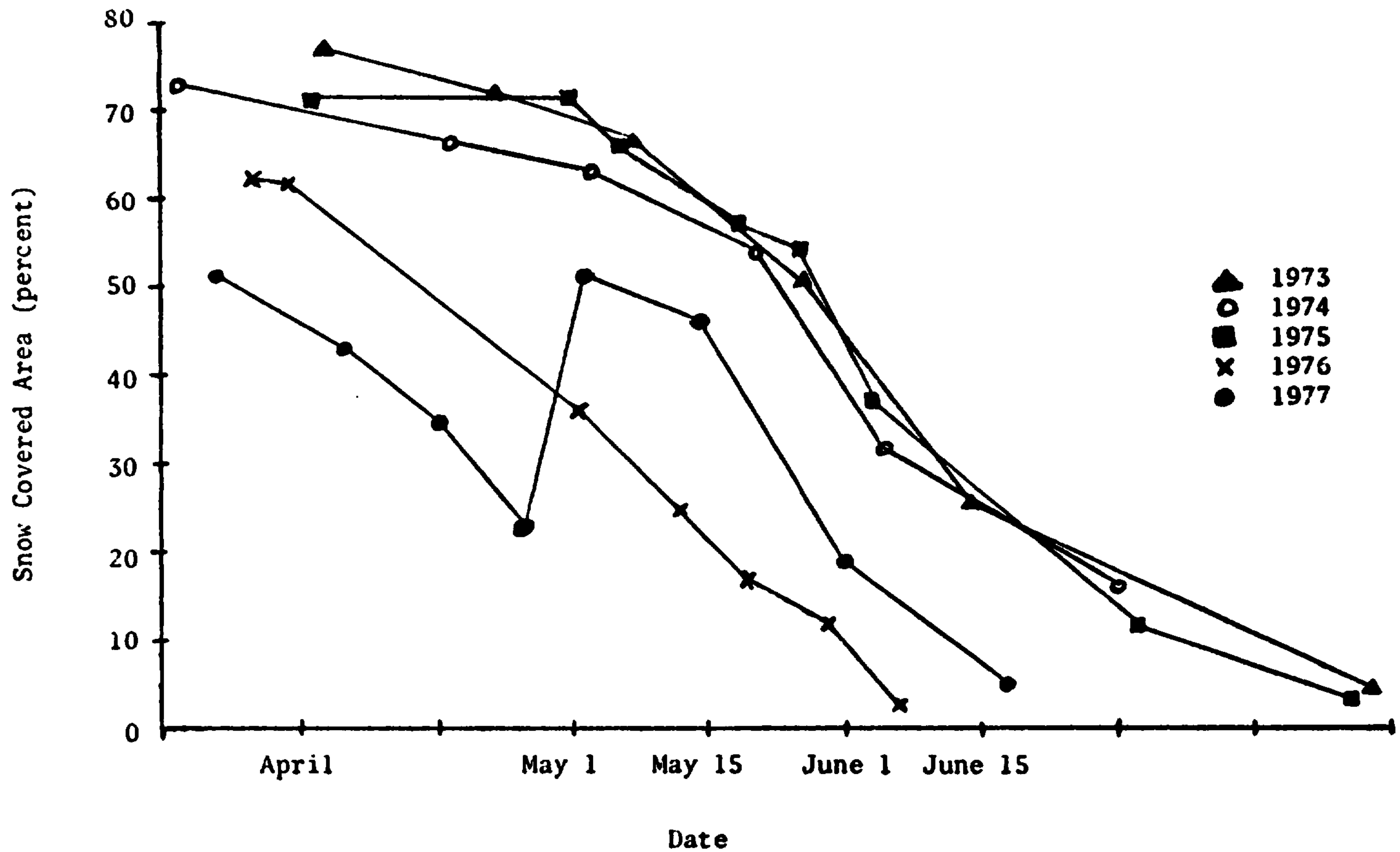

FIGURE 3. Snow Cover Depletion Curves for Years 1973 to 1977 Inclusive

Investigation showed that the best regression model that could be formulated used the precipitation total from October 1 to April 1 for estimating April 1 snow covered area. Although only five years of data (1973-77) were available for calibrating these models, the correlation coefficients showed that a significant relationship existed. Due to the amount of error inherent in this method of estimating snow cover before May 1, it was decided that daily snow covered area values would not be derived for this time period. Daily values of snow covered area were derived only for the period May 1-June 15, during which observed data were available every 15 days for the years prior to 1973.

Daily snow covered area values were generated simply by straight line interpolation between the four observed values for each year. While this method certainly smooths out the day-to-day variation in snow covered area, no data were available that could be used to calibrate a model that would reflect the daily variation more accurately. Straight line interpolation would probably be much less accurate in other time periods, but generally the period from May 1 to June 15 is one in which the snow cover is melting quickly; the beginning and end of the depletion curves were not observed during this time period (see Fig. 3). This suggests that May 1 - June 15 is the time of maximum snowmelt-derived runoff, a suggestion that is confirmed by the runoff hydrographs. Since runoff from snowmelt is maximum during this time period, it makes sense to test the models for short-term runoff prediction on the data from this time period. Prediction of runoff during August, for instance, should not be based on snow data, because there is very little snow being melted during August. Just how late in the summer the runoff can be successfully predicted from snow data is a factor that must be determined.

Once daily values of snow covered area were generated, the data had to be divided into elevation zones. Data from the Landsat imagery of Kings River basin for 1973-1977 has been compiled for elevation zones at intervals of 500 feet. Graphing the total snow covered area versus the percent in each zone for these years showed that there was generally a high correlation between the values. These graphs are shown in Fig. 4. Regression equations were developed for predicting the percentage of snow cover in each zone from the total snow covered area. These equations were then used with the daily snow covered area values previously generated to derive daily values for each elevation zone for the time period May 1 - June 15. Thus, an estimate of the required daily zonal snow covered area data was developed from the available data base. Almost certainly, the estimated data exhibits less daily variation than would the true values; but in the absence of measured data, these estimates must be used for testing the runoff prediction models.

Prediction of Input Data for Use During the Forecast Period

When actually making a forecast of runoff for any given time period, the values of temperature, precipitation, and snow covered

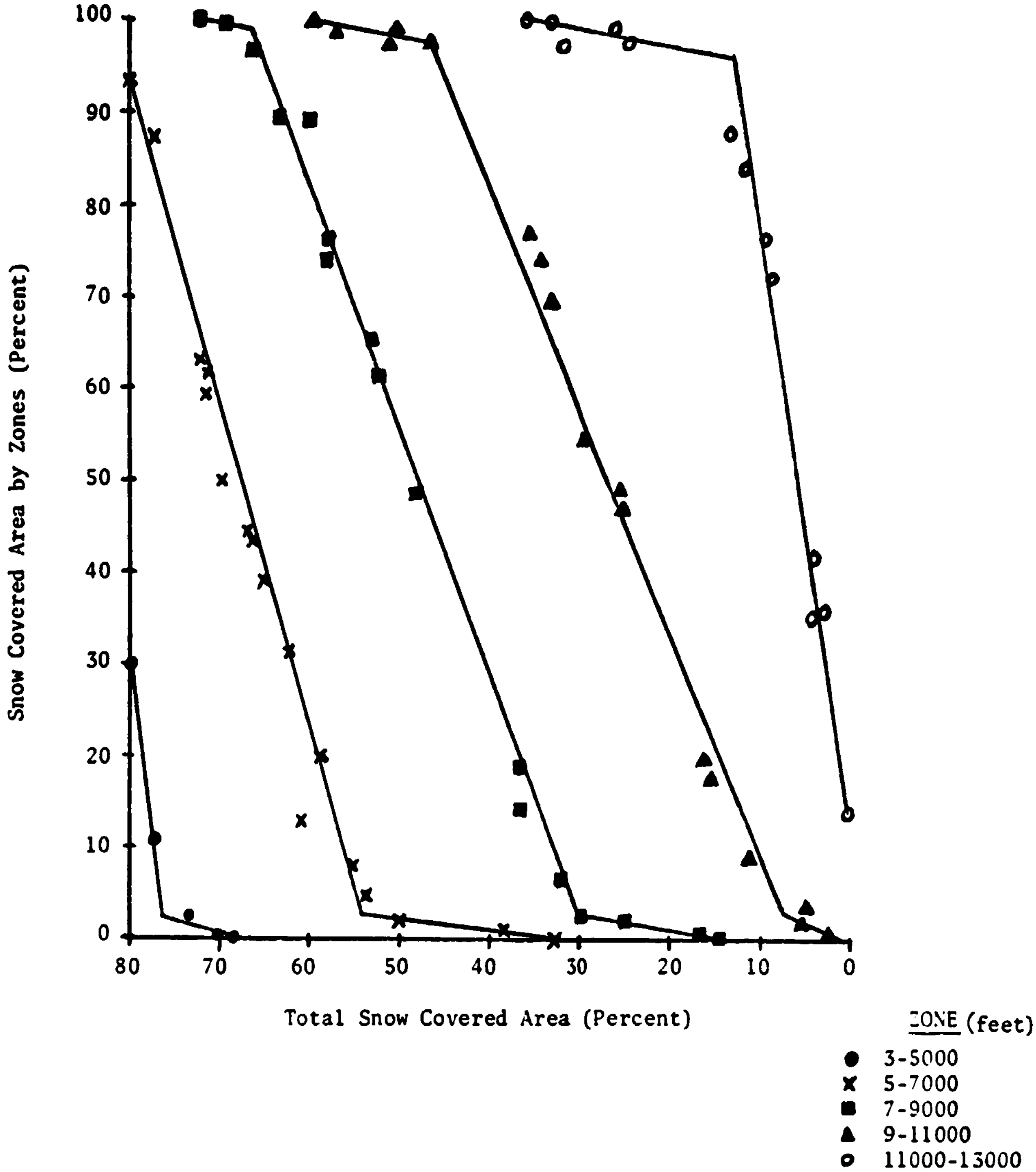

FIGURE 4. Relationship of Zonal vs. Total Snow Covered Area Values

area are not available for the days during the forecast period; data are only available for the days prior to the forecast date. In an actual prediction situation, the Martinec model would require predictions of the temperature, precipitation, and snow cover for each day of the forecast period. Many methods of predicting these values are available. An investigation was conducted to determine the most accurate method of prediction for each variable and the best methods were incorporated into the Martinec model. The actual measured daily values are used in model calibration, and the estimates are needed only for testing the model.

Temperature Prediction

Temperature can be predicted for a few days at a time using a model based on the normal temperature for each day and the deviation from normality observed on the preceding days. First, the normal average daily temperatures are calculated for each date by averaging the daily temperatures observed on that date during the previous 24 years. These 24-year normal daily temperatures define a smooth temperature curve, as shown in Fig. 5. When a prediction is made, the difference between the normal temperature and the actual temperature for each of the previous few days is calculated; the average of these differences is the deviation from normality expected for the next few days. Thus, if the temperature on the previous few days was lower than normal by an average of five degrees, the predicted

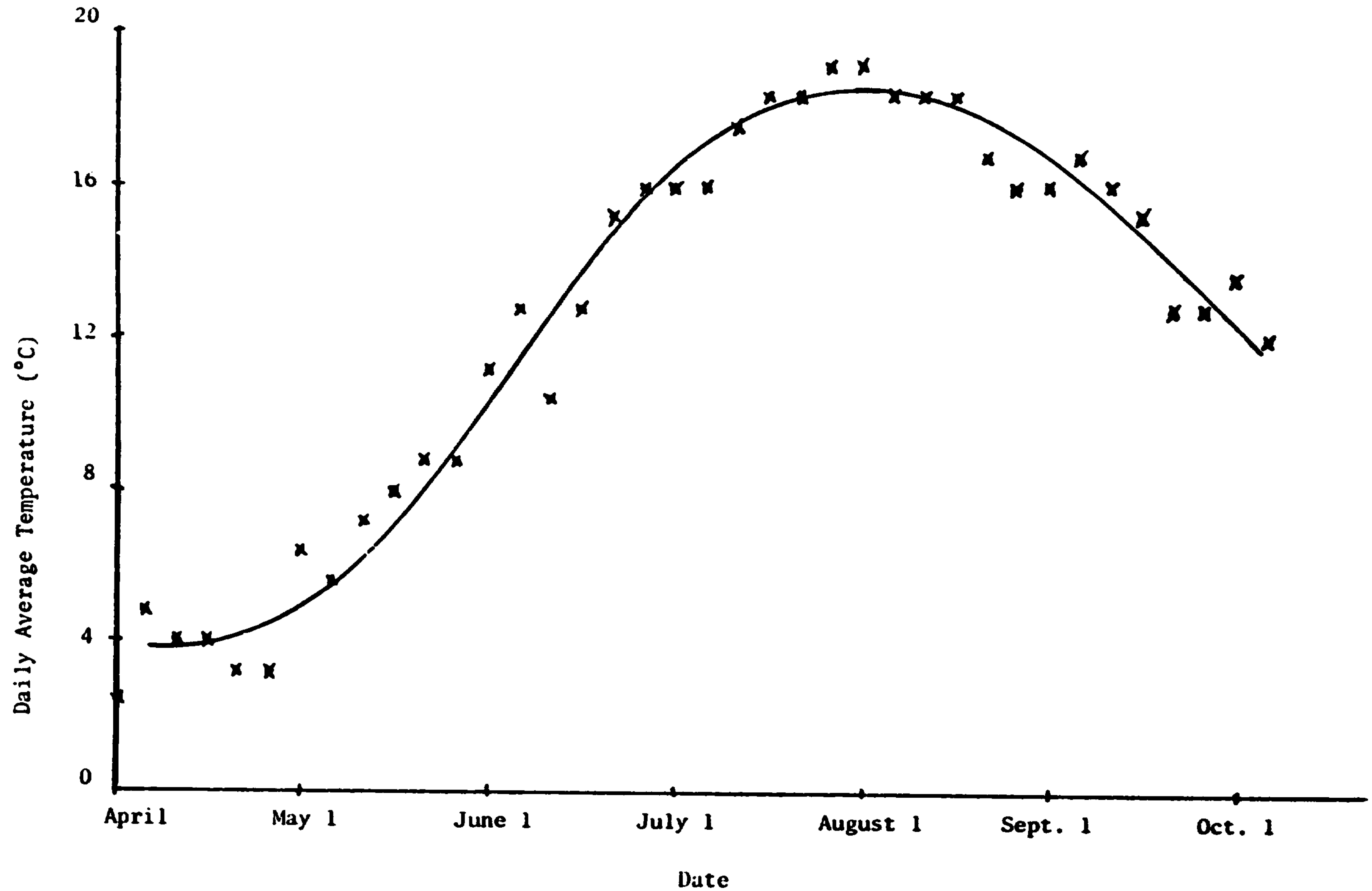

FIGURE 6. The Daily Normal Temperature Curve

temperatures for the next few days would be five degrees below the normal temperatures for those days. The accuracy of this method of predicting temperature is dependent on the number of previous days used in computing the average deviation from normal. For one day predictions, the correlation coefficient between predicted and observed values is 0.861 when only the previous day deviation is used, but if the deviation for three previous days is used the correlation is only 0.690. For three day predictions, the corresponding correlations are 0.669 and 0.566. The temperature can be predicted most accurately by using the deviation from normal for the previous day alone, rather than the average deviation for the previous few days.

The alternative method of temperature prediction is to assume that the actual temperature on each day will be equal to the normal temperature for that date. Tests showed that for prediction periods of more than eight days the normal temperature provides a better estimate of the observed value than does the method using the previous day's deviation, while for shorter time periods the previous deviation method was more accurate. Comparison of the accuracy of the two methods for various time periods is shown in Fig. 6.

Prediction of Precipitation

Precipitation is much more difficult to predict than temperature, due to the intermittent nature of the phenomenon. In an actual short-term prediction situation, a good weather forecast

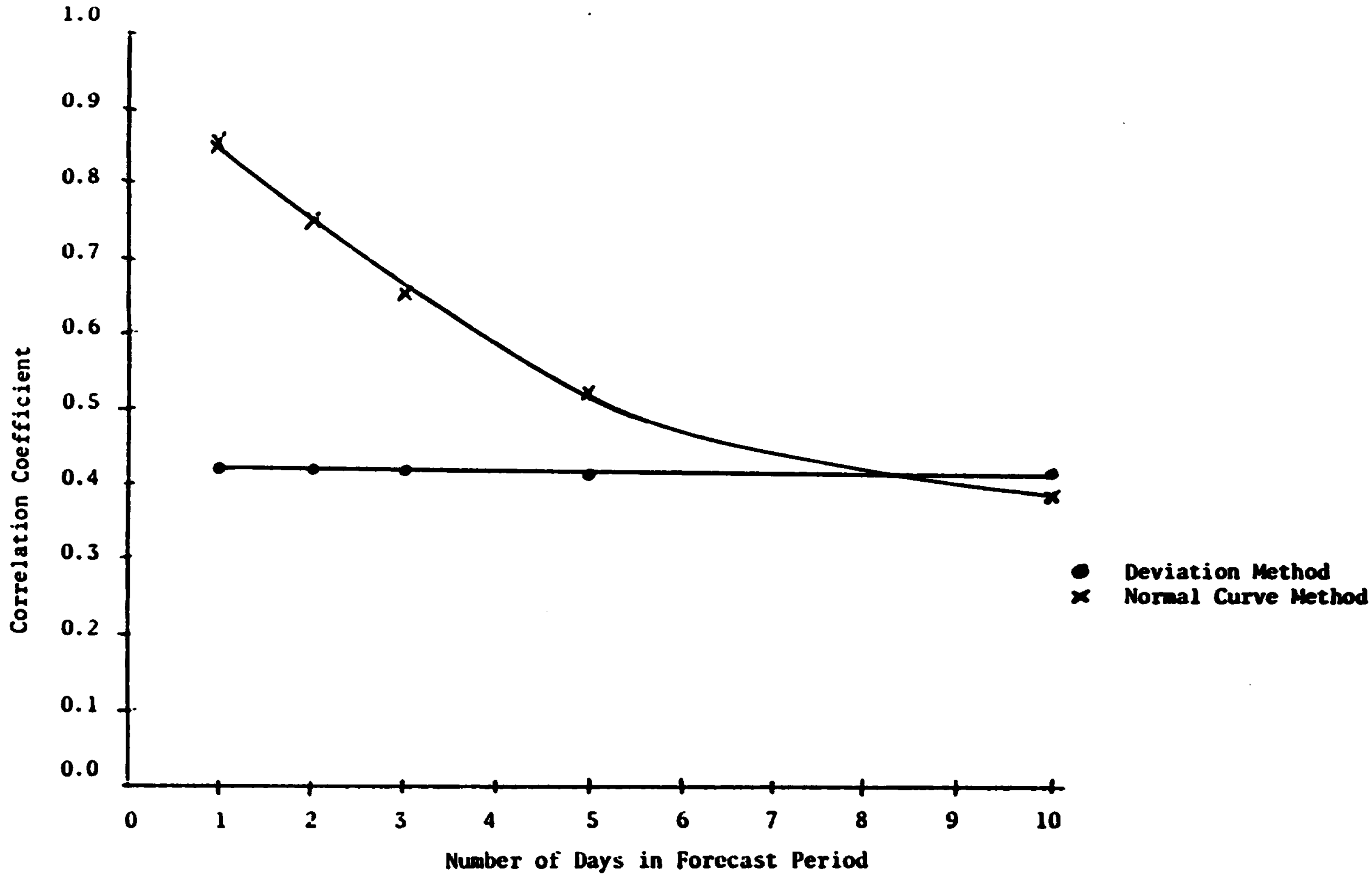

FIGURE 6. Comparison of the Accuracy of Two Methods of Temperature Prediction

would probably be available; but for this study, some method of predicting precipitation from the previously collected data was required. Two possible methods are: 1) use the 24-year normal precipitation for any particular time period, or 2) assume that there will be no precipitation, because very little precipitation occurs during the months of May, June, and July.

The correlation coefficient cannot be used for comparing the accuracy of these two methods because all of the predictions made by the second method are zero; the correlation between a constant and any variable must be zero, because the constant does not vary. Therefore, the standard error of estimate is used for comparing these two methods of predicting precipitation. Using the normal precipitation for the time period gave standard errors ranging from 0.207 for a one day forecast to 0.212 for a ten day forecast. When the precipitation was always predicted as zero, the standard error was 0.217 for all time periods. These standard errors of estimate must be compared with the standard deviation of the observed rainfall, which was 0.213. The conclusion is that neither method is very accurate, but using the normal precipitation for the period is preferable to assuming zero precipitation.

Prediction of Snow Covered Area During Period of Forecast

The other statistic for which daily predictions are necessary is snow covered area. One method of predicting these values is to assume that the rate of decline in snow covered area is a function of both the temperature and the present area of the snowpack.

Since the day-to-day change in predicted temperature is gradual for short forecast periods, the predicted rate of melt should also vary gradually. For prediction periods of five days or less, the rate of snow cover depletion was assumed to be constant and equal to the rate observed on the last day before the forecast date. Care must be taken to insure that the predicted value of snow covered area does not go below zero when using this method for periods of more than 5 days. When the estimated value of SCA is less than zero, the depletion rate is derived from the average rate of melt observed during the specified length of time for the initial value of snow covered area. Thus, if the initial value (on the prediction date) is 18 percent, and if analysis of past records shows that when the snow covered area equals 18 percent, the average value observed six days later is 14 percent, then the snow cover depletion during a six day prediction period can be assumed to be four percent. Assessing the accuracy of these methods of predicting snow covered area is impossible because measured data are not available; but these methods do accurately predict the interpolated daily values discussed above.

Assembling the Data Base

The data required for testing on the Kings River watershed were assembled from a variety of sources. The California Department of Water Resources provided the snow water equivalent data, which consisted of observations made on the first of April every year at 14 snow courses in the King River basin. Locations of these snow courses are shown in Fig.2. The temperature and precipitation records were collected by the National Weather Service at five sites in the basin, although data from only one site (Grant Grove) were used in this study. The temperature was reported as the maximum and minimum values observed on each date. The daily temperature index was calculated using these two values. The snow covered area figures for 1952 to 1973 were reported by the U.S. Army Corps of Engineers; the snow cover was mapped by observers from low altitude aircraft, four times per year. Due to weather conditions, the flights could not always be made on the required dates (May 1, May 15, June 1, and June 15) so the values were adjusted where necessary. The Landsat snow covered area statistics, divided into 500-feet elevation zones, were provided by NASA.

The runoff data used as the criterion variable in the testing program was provided by the Kings River Water Association. During the period under consideration (1953-1977), a number of significant water storage and diversion structures were built in the Kings River basin. The Kings River Water Association has developed a method of estimating the unimpaired runoff from the whole basin from data collected by the U.S. Geological Survey at a number of streamflow gages within the basin. Note that the data used in testing are these estimates of unimpaired daily runoff, just as though the basin were still in its natural state. All runoff volumes were supplied as an average streamflow rate for each day, expressed in cubic feet per second; in order to allow these statistics to be equated with the estimates of snowmelt and precipitation volumes generated in the Martinec model, the values were converted to volumes expressed in thousands of acre-feet.

CHAPTER IV

CALIBRATION OF SNOWMELT MODELS

The models chosen for this study use predictor variables that have a cause-and-effect relationship to runoff volume. Before the models can be used, though, the various model parameters that help to define these relationships must be calibrated for the particular forecast date. Thus, model calibration must be performed before the models can be tested for accuracy. This chapter describes the process of model calibration and reports the goodness-of-fit statistics that result from calibration.

Split-Sample Analysis

In order to evaluate the effectiveness of the various models, the available data must be split into two subsets. One subset of the data base is used to calibrate the models, while the remainder of the data is reserved for testing. In this way, the models can be tested on data that are independent of the data used in calibration. With only 24 years of data available for both calibration and testing, the accuracy of the models when used with the test data is likely to be dependent on the way in which the sample is split. Splitting the sample in half gives 12 years for calibration and 12 years for testing; this seems to be the best division available for a 24-year data set. It was expected that the accuracy of the test results would be directly dependent on the criterion used in splitting the sample. Maximum accuracy was expected when the sample was split by ranking the observations in order of decreasing observed runoff during the time period for which predictions were to be made. For instance, if the models were to be tested for accuracy of 3-day predictions from May 1, the years would be ranked according to the amount of runoff observed during that 30-day period; then, the data from years with an even number in rank could be used for calibration, and the data from years with an odd number in rank could be reserved for testing. Since the calibration and test sets would have similar means, standard deviations, and ranges, good predictions would be expected. Conversely, the lowest levels of accuracy would be expected when the 12 years with lowest values were used for calibration and the 12 years with highest values were used for testing. In this case, all of the test data would lie outside the range of values for which the model had been calibrated; therefore, the accuracy of prediction was expected to be comparatively low. Due to the anticipated effects on accuracy of various methods of splitting the samples, each model was calibrated and tested with a variety of subsets of the data base, each of which included 12 of the data years. The subsets were arrived at by the following method. First, the observations were ranked on each of three different criteria, forming three separate lists; then, each list was split into odd-versus-even and high-versus-low data sets. Criteria for the three lists were the total runoff volumes for the periods May 1-May 31, April 1-June 30, and April 1-September 30. The ranking lists and the resulting data sets are shown in Table 1. Note that the result of using the high-versus-low split on the April 1-June 30 list is the same as that obtained by using the same method

TABLE 1

Ranking of The Data Years and The Resulting Data Sets

Year	May 1-May 31 Runoff (thousands of acre-feet)	Rank	April 1-June 30 Runoff (thousands of acre-feet)	Rank	April 1-Sept. 30 Runoff (thousands of acre-feet)	Rank
1954	477.4	8	980.3	11	2162.1	11
1955	330.2	14	782.1	15	1796.0	15
1956	508.2	7	1305.5	6	3319.7	5
1957	315.3	15	897.6	12	2059.6	12
1958	755.3	2	1745.6	2	4245.3	3
1959	192.0	21	498.0	21	1127.9	20
1960	230.5	20	521.5	19	1125.7	21
1961	161.3	22	381.7	22	871.8	22
1962	418.9	10	1299.5	7	3071.8	7
1963	460.3	9	1146.8	9	2989.9	8
1964	263.1	17	579.2	17	1317.8	18
1965	415.3	11	1097.3	10	2911.9	9
1966	371.5	13	790.2	13	1737.9	16
1967	611.6	5	1685.0	3	5088.2	2
1968	246.4	18	524.3	18	1909.3	13
1969	1122.7	1	2552.2	1	6668.9	1
1970	398.1	12	787.0	14	1844.8	14
1971	272.8	16	721.5	16	1713.0	17
1972	235.4	19	509.4	20	1168.3	19
1973	750.2	3	1518.5	4	3448.4	4
1974	620.7	4	1384.4	5	3200.7	6
1975	523.9	6	1149.4	8	2627.7	10
1976	159.9	23	282.3	23	720.5	23
1977	83.3	24	260.4	24	589.5	24

TABLE 1 cont.

Calibration Data Sets					Test Data Sets				
#1	#2	#3	#4	#5	#1	#2	#3	#4	#5
1954	1955	1956	1955	1957	1956	1954	1954	1954	1954
1955	1957	1957	1959	1959	1957	1956	1955	1956	1955
1958	1959	1958	1960	1961	1959	1958	1959	1957	1956
1960	1960	1961	1961	1963	1963	1962	1960	1958	1958
1961	1961	1965	1964	1964	1964	1963	1962	1962	1960
1962	1964	1968	1966	1966	1965	1965	1963	1963	1962
1968	1966	1970	1968	1967	1966	1967	1964	1965	1965
1970	1968	1971	1970	1970	1967	1969	1966	1967	1968
1971	1971	1972	1971	1973	1969	1970	1967	1969	1969
1974	1972	1973	1972	1974	1972	1973	1969	1973	1971
1975	1976	1975	1976	1975	1973	1974	1974	1974	1972
1977	1977	1977	1977	1977	1976	1975	1976	1975	1976

with the April 1-September 30 list; for this reason, there are only five distinct ways of dividing the 24 data years.

Calibration of the Regression Models

A stepwise regression program was used to calibrate multivariate linear models. In selecting the first predictor variable, stepwise regression selects the predictor variable that explains the highest percentage of the variation of the criterion variable and develops a simple linear prediction model. The correlation between the predicted and observed values of the criterion variable is computed, along with the standard error of estimate. Then the predictor variable that will result in the greatest increase in explained variation is selected to enter the model and a two-predictor model is formed. An F-test is used to measure the significance of increases in explained variance. The correlation coefficient and standard error for this two-predictor model are computed. A third predictor variable is then selected to enter the model and another model is formed. This process continues until all the available predictors have been included, or until the introduction of the remaining predictors will not result in any significant increase in explained variation. Inclusion of predictors is determined only by statistical relationships, with no conceptual judgment being exercised once the original set of eligible predictors has been chosen.

In this way, a set of models is produced, each accompanied by the correlation coefficient and standard error of estimate. The best of these models is then selected by the researcher on the basis of the goodness-of-fit statistics and the rationality of the model. Rationality is judged by examining the signs of the regression coefficients and the magnitudes of the standardized partial regression coefficients. For instance, if the sign of the regression coefficient for the snow water equivalent is negative, a larger value of the snow water equivalent will result in a smaller predicted volume of runoff. Clearly, this is not reasonable. The irrationality of coefficients in some equations is caused by high levels of correlation between the predictor variables. Irrational models can sometimes be used effectively in cases where all the input data lie within the range of values for which the model was calibrated; generally, rational models should be selected for use.

The data base for the regression models consisted of 24 observations of each of the variables listed in Table 2. Regression models were developed both with and without snow covered area data so that comparisons could be made to assess the value of this data as a predictor of snowmelt runoff. Regression equations were considered to be a long-term prediction method, so predictions were made for periods of 15, 30, 45, 60, 90, 120, and 150 days starting on both April 1 and May 1.

The snowpack index referred to in Table 2 is calculated from snow water equivalent measurements made at 14 snow courses in the Kings River basin. The locations of these snow courses are shown in Fig. 2. The snowpack index is formed by dividing the snow water equivalent at each snow course by the mean value from previous years and then averaging the quotients.

The winter precipitation used for this study is the total

TABLE 2

Predictor Variables for the Regression Models

Variable	Mean	Standard Deviation
Snowpack Index (percent)	95.1	56.5
October-March Precipitation Total (inches)	32.8	15.3
October-April Precipitation Total (inches)	37.2	17.1
April 1 Snow Covered Area (percent)	67.3	9.8
May 1 Snow Covered Area (percent)	54.5	14.7
Product of Snowpack Index and April 1 Snow Covered Area (percent^2)	6903.5	5420.1
Product of Snowpack Index and May 1 Snow Covered Area (percent^2)	5717.5	4489.0

amount of precipitation measured from October 1 of the preceding year up to the forecast date. Thus, predictions made on April 1 used October-March precipitation, and predictions made on May 1 used the October-April precipitation total. Data from the Grant Grove station were used in this study because this station seems to be most representative of the entire watershed (Tangborn, 1978).

The snow covered area data is expressed as a percentage of total watershed area measured on May 1 and predicted for April 1, as described previously. The product of the snow covered area and the snowpack index was used as a predictor variable because it is a rational way of estimating the volume of water stored on the basin (Rango, et al., 1975).

The data base was used to generate eight different regression models for each forecast date. Four of these models used only one of the predictor variables listed in Table 2 and the other four used two of the predictors. The two-predictor models used the combinations: snowpack index and winter precipitation, snowpack index and snow covered area, winter precipitation and snow covered area, and winter precipitation and the product of snowpack index and snow covered area. Due to the high intercorrelations among the four predictor variables, the two-predictor models were only slightly more accurate than the single predictor models. The goodness-of-fit statistics for the calibration of all of the regression models are shown in Table 3.

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variable: Snowpack Index

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.789	.670	.632	.722	.342
	Se	26.9	24.1	28.2	22.5	33.8
	Sy	41.8	30.9	34.8	31.0	34.3
30	R	.840	.623	.756	.653	.520
	Se	55.1	44.5	52.1	42.9	58.3
	Sy	96.8	54.2	75.9	54.0	65.1
45	R	.899	.656	.883	.679	.729
	Se	82.7	69.6	71.5	67.8	98.1
	Sy	180.2	88.0	145.4	88.1	136.7
60	R	.933	.812	.947	.814	.905
	Se	99.6	74.4	90.8	77.9	107.7
	Sy	263.6	121.4	269.4	127.7	241.3
90	R	.950	.890	.953	.897	.901
	Se	145.2	96.0	146.2	86.3	204.8
	Sy	442.6	200.5	460.3	186.4	450.8
120	R	.947	.891	.938	.898	.833
	Se	174.0	108.1	198.4	97.9	346.5
	Sy	516.9	227.4	544.7	212.1	596.3
150	R	.943	.894	.933	.899	.818
	Se	188.1	109.7	216.4	100.2	383.9
	Sy	539.2	233.2	572.3	218.1	636.1

R = Correlation Coefficient
Se = Standard Error of Estimate
Sy = Standard Deviation of Observed Values

TABLE 3

Summary Statistics for Calibration of Regression Models
Predictor Variable: October-March Precipitation Total
Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.834	.547	.727	.552	.517
	Se	24.2	27.2	25.0	27.1	30.7
	Sy	41.8	30.9	34.8	31.0	34.3
30	R	.851	.534	.834	.490	.648
	Se	53.4	48.1	44.0	49.4	52.0
	Sy	96.8	54.2	75.9	54.0	65.1
45	R	.912	.621	.915	.598	.749
	Se	77.6	72.3	61.7	74.1	95.0
	Sy	180.2	88.0	145.4	88.1	136.7
60	R	.954	.800	.950	.824	.943
	Se	82.8	76.4	88.5	75.9	84.2
	Sy	263.6	121.4	269.4	127.7	241.3
90	R	.954	.803	.947	.909	.945
	Se	139.5	125.4	155.6	81.5	155.2
	Sy	442.6	200.5	460.3	186.4	450.8
120	R	.951	.795	.950	.909	.928
	Se	167.3	144.6	178.7	92.7	232.4
	Sy	516.9	227.4	544.7	212.1	596.3
150	R	.949	.799	.948	.912	.924
	Se	178.8	147.0	190.3	94.1	256.0
	Sy	539.2	233.2	572.3	218.1	636.1

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variable: April Snow Covered Area

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.840	.564	.731	.567	.531
	Se	23.8	26.8	24.9	26.8	30.4
	Sy	41.8	30.9	34.8	31.0	34.3
30	R	.852	.553	.841	.509	.661
	Se	53.2	47.4	43.1	48.8	51.2
	Sy	96.8	54.2	75.9	54.0	65.1
45	R	.914	.642	.918	.618	.759
	Se	76.7	70.8	60.6	72.6	93.3
	Sy	180.2	88.0	145.4	88.1	136.7
60	R	.958	.813	.948	.837	.945
	Se	79.6	74.2	89.6	73.4	82.9
	Sy	263.6	121.4	269.4	127.7	241.3
90	R	.955	.797	.943	.912	.939
	Se	138.2	126.9	160.5	80.4	162.9
	Sy	442.6	200.5	460.3	186.4	450.8
120	R	.952	.789	.947	.911	.921
	Se	165.6	146.5	182.9	91.7	243.4
	Sy	516.9	227.4	554.7	212.1	596.3
150	R	.950	.793	.947	.913	.916
	Se	176.6	148.9	193.4	93.1	267.4
	Sy	539.2	233.2	572.3	218.1	636.1

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variable: Product of Snowpack Index and April Snow Covered Area

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.820	.663	.656	.705	.347
	Se	25.1	24.3	27.5	23.0	33.7
	Sy	41.8	30.9	34.8	31.0	34.3
30	R	.866	.619	.786	.634	.535
	Se	50.7	44.7	49.3	43.8	57.7
	Sy	96.8	54.2	75.9	54.0	65.1
45	R	.918	.657	.909	.671	.725
	Se	75.1	69.6	63.7	68.5	98.7
	Sy	180.2	88.0	145.4	88.1	136.7
60	R	.943	.820	.961	.826	.906
	Se	92.1	73.0	78.1	75.6	107.1
	Sy	263.6	121.4	269.4	127.7	241.3
90	R	.957	.893	.957	.919	.904
	Se	131.5	94.8	139.8	77.0	450.8
	Sy	442.6	200.5	460.3	186.4	450.8
120	R	.956	.895	.943	.922	.847
	Se	159.6	106.6	189.7	86.3	332.5
	Sy	516.9	227.4	544.7	212.1	596.3
150	R	.953	.898	.938	.923	.835
	Se	172.0	107.8	207.5	87.9	367.5
	Sy	539.2	233.2	572.3	218.1	636.1

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variables: Snowpack Index and October-March Precipitation Total

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.835	.670	.731	.730	.630
	Se	25.4	24.1	26.2	23.4	29.4
	Sy	41.8	30.9	34.8	31.0	34.4
30	R	.855	.623	.834	.633	.681
	Se	55.5	44.5	44.0	44.7	52.7
	Sy	96.8	54.2	75.9	54.0	65.1
45	R	.916	.669	.923	.681	.755
	Se	79.9	72.2	62.1	71.4	99.1
	Sy	180.2	88.0	145.4	88.1	136.7
60	R	.956	.841	.970	.853	.947
	Se	85.2	72.6	72.2	73.7	85.8
	Sy	263.6	121.4	269.4	127.7	241.3
90	R	.962	.896	.972	.941	.947
	Se	133.6	98.4	120.0	69.8	159.6
	Sy	442.6	200.5	460.3	186.4	450.8
120	R	.959	.896	.966	.941	.931
	Se	161.4	111.6	155.8	79.2	240.6
	Sy	516.9	227.4	544.7	212.1	596.3
150	R	.956	.899	.963	.943	.929
	Se	174.6	113.0	170.7	80.1	261.2
	Sy	539.2	233.2	572.3	218.1	636.1

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variables: Snowpack Index and April Snow Covered Area

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.841	.670	.735	.727	.654
	Se	25.0	24.1	26.0	23.5	28.6
	Sy	41.8	30.9	34.8	31.0	34.3
30	R	.856	.626	.641	.659	.701
	Se	55.3	46.7	43.1	44.9	51.3
	Sy	96.8	54.2	75.9	54.0	65.1
45	R	.918	.678	.926	.684	.763
	Se	79.0	71.5	60.9	71.1	97.7
	Sy	180.2	88.0	145.4	88.1	136.7
60	R	.960	.848	.971	.860	.949
	Se	82.1	71.1	71.8	72.1	84.2
	Sy	263.6	121.4	269.4	127.7	241.3
90	R	.963	.895	.971	.942	.943
	Se	131.8	98.8	121.3	69.5	165.5
	Sy	442.6	200.5	460.3	186.4	450.8
120	R	.961	.895	.966	.941	.922
	Se	159.1	112.1	156.7	79.1	254.9
	Sy	516.9	227.4	544.7	212.1	596.3
150	R	.957	.898	.963	.943	.919
	Se	172.1	113.5	170.8	80.0	277.6
	Sy	539.2	233.2	572.3	218.1	636.1

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variables: October-March Precipitation Total and April Snow Covered Area

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.840	.624	.731	.623	.531
	Se	23.8	26.7	24.9	26.8	30.4
	Sy	41.8	30.9	34.8	31.0	34.3
30	R	.852	.628	.841	.597	.661
	Se	53.2	46.6	43.1	47.9	51.2
	Sy	96.8	54.2	75.9	54.0	65.1
45	R	.914	.717	.918	.710	.759
	Se	76.7	67.8	60.6	68.6	93.3
	Sy	180.2	88.0	145.4	88.1	136.7
60	R	.958	.834	.950	.861	.945
	Se	79.6	74.2	88.5	71.9	82.9
	Sy	263.6	121.4	269.4	127.7	241.3
90	R	.955	.805	.947	.912	.945
	Se	138.2	131.4	155.6	84.6	155.2
	Sy	442.6	200.5	460.3	186.4	450.8
120	R	.952	.799	.950	.911	.928
	Se	165.6	151.3	178.7	91.7	232.4
	Sy	516.9	227.4	544.7	212.1	596.3
150	R	.950	.803	.948	.913	.924
	Se	176.6	153.7	190.3	93.1	256.0
	Sy	539.2	233.2	572.3	218.1	636.1

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variables: October-March Precipitation Total and Product of Snowpack Index and April Snow Covered Area

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.837	.665	.729	.718	.662
	Se	25.3	25.5	26.3	23.9	28.4
	Sy	41.8	30.9	34.8	31.0	34.3
30	R	.869	.619	.834	.647	.685
	Se	53.0	44.7	46.3	45.5	52.4
	Sy	96.8	54.2	75.9	54.0	65.1
45	R	.924	.665	.928	.671	.752
	Se	76.3	72.6	59.8	68.5	99.7
	Sy	180.2	88.0	145.4	88.1	136.7
60	R	.959	.839	.973	.852	.945
	Se	83.0	73.0	68.6	73.9	87.5
	Sy	263.6	121.4	269.4	127.7	241.3
90	R	.964	.895	.970	.945	.946
	Se	129.3	99.0	124.7	67.7	162.2
	Sy	442.6	200.5	460.3	186.4	450.8
120	R	963	.896	.964	.946	.932
	Se	154.4	111.8	160.5	76.1	239.3
	Sy	516.9	227.4	544.7	212.1	596.3
150	R	.960	.899	.961	.948	.929
	Se	166.9	113.0	175.3	76.7	260.0
	Sy	539.2	233.2	572.3	218.1	636.1

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variable: Snowpack Index

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.905	.639	.930	.648	.803
	Se	39.8	30.2	29.5	30.3	51.4
	Sy	89.1	37.5	76.5	37.9	82.1
30	R	.876	.824	.962	.779	.942
	Se	95.2	46.8	58.6	57.9	69.0
	Sy	188.0	78.7	205.5	88.1	196.0
45	R	.885	.809	.962	.798	.946
	Se	141.3	82.6	89.1	80.8	105.5
	Sy	289.3	134.1	312.7	127.9	310.7
60	R	.911	.816	.956	.830	.898
	Se	160.6	107.4	122.1	93.0	191.8
	Sy	372.0	177.2	398.7	159.0	414.9
90	R	.917	.820	.943	.830	.825
	Se	186.7	123.8	168.5	109.5	332.9
	Sy	445.4	206.1	481.0	186.9	561.0
120	R	.914	.824	.938	.832	.810
	Se	199.3	126.1	184.1	112.5	369.6
	Sy	467.5	212.0	507.8	193.3	600.7

TABLE 3

Summary Statistics for Calibration of Regression Equations

Predictor Variable: October-April Precipitation Total

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.893	.644	.892	.653	.623
	Se	42.0	30.1	36.2	30.1	67.4
	Sy	89.1	37.5	76.5	37.9	82.1
30	R	.937	.846	.932	.854	.904
	Se	68.8	44.0	78.1	48.0	87.7
	Sy	188.0	78.7	205.5	88.1	196.0
45	R	.927	.796	.932	.878	.884
	Se	114.0	85.1	118.6	64.2	152.6
	Sy	289.3	134.1	312.7	127.9	310.7
60	R	.947	.758	.950	.867	.944
	Se	125.9	121.3	131.1	83.2	143.3
	Sy	372.0	177.2	398.7	159.0	414.9
90	R	.950	.747	.966	.853	.968
	Se	146.6	143.8	130.4	102.5	148.9
	Sy	445.4	206.1	481.0	186.9	561.0
120	R	.949	.751	.970	.854	.969
	Se	154.8	146.8	130.1	105.4	156.7
	Sy	467.5	212.0	507.8	193.3	600.7

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variable: May 1 Snow Covered Area

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.498	.187	.613	.091	.364
	Se	81.1	38.6	63.3	39.6	80.2
	Sy	89.1	37.5	76.5	37.9	82.1
30	R	.699	.205	.736	.380	.719
	Se	141.0	80.7	145.8	85.5	142.8
	Sy	188.0	78.7	205.5	88.1	196.0
45	R	.761	.459	.762	.544	.775
	Se	196.0	124.0	212.3	112.6	206.0
	Sy	289.3	134.1	312.7	127.9	310.7
60	R	.753	.498	.752	.571	.822
	Se	256.9	161.1	275.6	136.8	247.9
	Sy	372.0	177.2	398.7	159.0	414.9
90	R	.750	.510	.741	.588	.831
	Se	309.1	186.0	338.7	158.6	327.7
	Sy	445.4	206.1	481.0	186.9	561.0
120	R	.744	.507	.735	.585	.825
	Se	327.8	191.7	361.0	164.4	355.7
	Sy	467.5	212.0	507.8	193.3	600.7

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variable: Product of Snowpack Index and May 1 Snow Covered Area

Forecast Date: May 1

Length of Forecast (Days)		Data Set #				
		1	2	3	4	5
15	R	.874	.367	.920	.380	.685
	Se	45.5	36.7	31.5	36.7	62.8
	Sy	89.1	37.5	76.5	37.9	82.1
30	R	.920	.707	.965	.755	.927
	Se	77.1	58.4	56.7	60.6	77.2
	Sy	188.0	78.7	205.5	88.1	196.0
45	R	.946	.838	.962	.866	.946
	Se	98.8	76.7	90.1	67.2	105.5
	Sy	289.3	134.1	312.7	127.9	310.7
60	R	.967	.866	.950	.904	.935
	Se	98.9	92.9	130.5	71.4	154.1
	Sy	372.0	177.2	398.7	159.0	414.9
90	R	.972	.876	.931	.915	.895
	Se	110.2	104.2	184.3	79.3	262.6
	Sy	445.4	206.1	481.0	186.9	561.0
120	R	.969	.879	.925	.916	.884
	Se	120.7	106.2	202.7	81.5	294.3
	Sy	467.5	212.0	507.8	193.3	600.7

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variables: Snowpack Index and October-April Precipitation Total

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.914	.678	.937	.686	.811
	Se	40.1	30.5	29.5	30.5	53.2
	Sy	89.1	37.5	76.5	37.9	82.1
30	R	.937	.882	.973	.870	.962
	Se	72.5	40.9	52.5	48.1	59.2
	Sy	188.0	78.7	205.5	88.1	196.0
45	R	.928	.848	.973	.893	.958
	Se	119.4	78.5	79.7	63.6	98.8
	Sy	289.3	134.1	312.7	127.9	310.7
60	R	.949	.836	.976	.897	.961
	Se	129.9	107.5	95.2	77.7	126.8
	Sy	372.0	177.2	398.7	159.0	414.9
90	R	.952	.835	.979	.888	.968
	Se	150.3	125.4	108.1	94.9	148.9
	Sy	445.4	206.1	481.0	186.9	561.0
120	R	.951	.839	.980	.890	.969
	Se	159.6	127.4	111.7	97.3	164.2
	Sy	467.5	212.0	507.8	193.3	600.7

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variables: Snowpack Index and May Snow Covered Area

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.905	.744	.933	.708	.831
	Se	39.8	27.7	30.5	29.6	50.5
	Sy	89.1	37.5	76.5	37.9	82.1
30	R	.914	.824	.965	.797	.952
	Se	84.5	49.3	59.7	58.9	66.6
	Sy	188.0	78.7	205.5	88.1	196.0
45	R	.944	.845	.968	.865	.968
	Se	105.5	79.2	86.3	71.0	86.3
	Sy	289.3	134.1	312.7	127.9	310.7
60	R	.960	.863	.961	.901	.949
	Se	115.2	98.8	121.3	76.1	144.3
	Sy	372.0	177.2	398.7	159.0	414.9
90	R	.963	.871	.948	.908	.910
	Se	132.9	112.1	170.0	86.6	257.8
	Sy	445.4	206.1	481.0	186.9	561.0
120	R	.958	.873	.943	.909	.899
	Se	147.5	114.3	187.0	89.3	291.2
	Sy	467.5	212.0	507.0	193.3	600.7

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variables: October-April Precipitation Total, May Snow Covered Area

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.895	.696	.897	.701	.636
	Se	43.9	29.7	37.4	29.9	70.1
	Sy	89.1	37.5	76.5	37.9	82.1
30	R	.951	.852	.957	.873	.909
	Se	64.5	45.5	66.2	47.5	90.3
	Sy	188.0	78.7	205.5	88.1	196.0
45	R	.959	.877	.965	.942	.905
	Se	90.8	71.2	91.1	47.3	146.3
	Sy	289.3	134.1	312.7	127.9	310.7
60	R	.972	.862	.975	.944	.965
	Se	96.9	99.2	97.9	58.2	120.3
	Sy	372.0	177.2	398.7	159.0	414.9
90	R	.973	.859	.986	.939	.985
	Se	112.9	116.7	90.0	70.9	105.5
	Sy	445.4	206.1	481.0	186.9	561.0
120	R	.971	.861	.987	.939	.985
	Se	123.0	119.2	89.7	73.3	115.0
	Sy	467.5	212.0	507.8	193.3	600.7

TABLE 3

Summary Statistics for Calibration of Regression Models

Predictor Variables: October-April Precipitation Total and Product of Snowpack Index and May Snow Covered Area

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.898	.655	.939	.667	.685
	Se	43.4	31.3	29.2	31.2	66.1
	Sy	89.1	37.5	76.5	37.9	82.1
30	R	.943	.863	.983	.876	.943
	Se	69.0	43.9	41.6	47.0	72.0
	Sy	188.0	78.7	205.5	88.1	196.0
45	R	.951	.891	.981	.938	.951
	Se	98.9	67.3	66.7	49.0	106.7
	Sy	289.3	134.1	312.7	127.9	310.7
60	R	.972	.894	.983	.954	.967
	Se	95.9	87.9	82.1	53.0	117.0
	Sy	372.0	177.2	398.7	159.0	414.9
90	R	.977	.898	.984	.954	.970
	Se	106.2	100.2	96.2	61.7	150.0
	Sy	445.4	206.1	481.0	186.9	561.0
120	R	.975	.901	.984	.956	.970
	Se	116.0	101.5	100.9	63.0	162.0
	Sy	467.5	212.0	507.8	193.3	600.7

The correlation coefficients in Table 3 range from 0.091 to 0.987. The values vary considerably between the data sets, with the highest values resulting from calibration with sets #1, 3, and 5 and the lowest from sets #2 and 4. The correlations tend to be higher for forecast periods of 60 days or more than for shorter periods; with the exception of the May snow covered area model, all regressions resulted in correlations greater than 0.7 for forecasts of 60 days or longer. The correlations for the four single predictor models (based on snowpack index, October-March precipitation, snow covered area, and the product of snowpack index and snow covered area) are all nearly equal for periods of 60 days or more from April 1, but for May 1 forecasts, the snow covered area models generally show much lower correlations than the other single predictor models.

Calibration of the Tangborn Models

The Tangborn models are calibrated using regression; however, they differ from the regression approach in that the structure of the model was established conceptually. The predictor variables are established through a conceptual interpretation of the processes. For long-term predictions (15 days or more), the variables used are the total precipitation and total runoff measured during the winter preceding the prediction period. The equation has the form

$$R^* = aP_w + b - R_w \tag{4-1}$$

where R^* is the predicted runoff, P_w is the total depth of winter precipitation, R_w is the previously observed winter runoff volume, and a and b are the regression coefficients that require calibration.

In addition to evaluating optimum values of the regression coefficients, calibration of this long-term model involves determining the optimum starting date for the winter season. The winter precipitation and runoff volumes are dependent on the date selected to define the start of winter; October 1 is commonly used, but it is not necessarily the optimum starting date. In order to determine the optimum date, total values of winter runoff and precipitation were compiled for each date from September 1 to October 30. Prediction equations were formed, and the correlation coefficients and standard errors of estimate were calculated. Comparison of these statistics showed that, for the Kings River watershed, prediction accuracy was not sensitive to the winter starting date, with almost no change in the goodness-of-fit statistics observed for the various start dates. Therefore, it seemed reasonable to use October 1 as the starting date for the winter period.

In calibrating these equations, the accuracy of the results is also dependent on the length of the test season. The optimum test season length was determined by using all 24 years of data to make seasonal predictions; test seasons of one to five days were tried, and it was concluded from the results that a test season of one day gave the greatest accuracy of prediction.

Now that the optimum winter start date and test season length had been determined, calibration of the coefficients was performed. For the long-term models, coefficients were derived for prediction periods of 15, 30, 45, 60, 90, 120, and 150 days starting from both April 1 and May 1. Short-term models were calibrated for prediction periods of 1, 2, 3, 5, and 10 days starting on May 1, May 15, June 1, and June 15. Calibration was accomplished by regression on the 12 years of data selected for the purpose. Since the sample was split in a number of different ways, as explained previously, separate regressions were performed for each set of calibration data. The resulting equations, along with the goodness-of-fit statistics, appear in Table 4.

The correlation coefficients in Table 4 range from 0.521 to 0.961. The correlations are generally higher for data sets #1, 3, and 5 than for sets #2 and 4. Models for the long term prediction periods are more accurate than those for forecasts for periods of ten days or less; for forecasts of 60 days or more, all the data sets give correlations of at least 0.74. The short term predictions seem to be more accurate for forecast dates of May 1 and June 15 than for May 15 and June 1; this may be due to the fact that the peak flows occur most often in late May or early June.

Calibration of the Martinec Model

The basic form of the Martinec Model is:

$$Q^*_n = c\cdot(d\cdot T\cdot SCA+P)\cdot A\cdot(1-K) + K\cdot Q_{n-1} \qquad (4\text{-}2)$$

TABLE 4

Summary Statistics for Calibration of Tangborn Models

Forecast Date: April 1

Length of Forecast (Days)		Data Set #				
		1	2	3	4	5
15	R	.761	.680	.629	.617	.738
	Se	28.4	23.8	28.3	25.6	24.2
	Sy	41.8	30.9	34.8	31.0	34.3
30	R	.710	.689	.742	.547	.709
	Se	71.5	41.2	53.4	47.4	48.1
	Sy	96.8	54.2	75.9	54.0	65.1
45	R	.806	.779	.825	.673	.665
	Se	111.7	57.9	86.3	68.3	107.1
	Sy	180.2	88.0	145.4	88.1	136.7
60	R	.920	.869	.912	.879	.895
	Se	108.4	63.1	116.2	63.9	112.7
	Sy	263.6	121.4	269.4	127.7	241.3
90	R	.954	.811	.937	.931	.935
	Se	139.8	122.9	169.1	71.6	167.3
	Sy	442.6	200.5	460.3	186.4	450.8
120	R	.952	.803	.934	.933	.955
	Se	165.6	142.2	204.8	80.1	186.4
	Sy	516.9	227.4	544.7	212.1	596.3
150	R	.949	.806	.930	.934	.956
	Se	178.2	144.7	221.1	81.7	196.3
	Sy	539.2	233.2	572.3	218.1	636.1

TABLE 4

Summary Statistics for Calibration of Tangborn Models

Forecast Date: Ma. 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.943	.911	.844	.902	.615
	Se	31.1	16.2	43.1	17.1	68.0
	Sy	89.1	37.5	76.5	37.9	82.1
30	R	.941	.922	.922	.949	.880
	Se	66.8	31.9	83.3	29.0	97.7
	Sy	188.0	78.7	205.5	88.1	196.0
45	R	.939	.801	.951	.909	.867
	Se	104.7	84.1	101.4	55.8	162.2
	Sy	289.3	134.1	312.7	127.9	310.7
60	R	.957	.759	.951	.892	.925
	Se	112.7	121.1	129.2	75.5	165.5
	Sy	372.0	177.2	398.7	159.0	414.9
90	R	.961	.747	.957	.877	.951
	Se	128.8	143.7	146.2	94.2	181.5
	Sy	445.4	206.1	481.0	186.9	561.0
120	R	.959	.751	.958	.878	.952
	Se	138.4	146.7	153.5	97.2	192.0
	Sy	467.5	212.0	507.8	193.3	600.7
150	R	.959	.760	.959	.888	.954
	Se	140.8	142.9	153.0	92.2	194.1
	Sy	473.8	209.8	514.1	191.0	614.6

TABLE 4

Summary Statistics for Calibration of Short-Term Tangborn Models

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
1	R	.845	.911	.659	.740	.521
	Se	1.83	1.18	3.13	1.99	3.07
	Sy	3.26	2.71	3.97	2.82	3.43
2	R	.870	.910	.703	.779	.563
	Se	3.79	2.47	5.76	3.82	6.33
	Sy	7.32	5.69	7.72	5.81	7.31
3	R	.895	.895	.769	.800	.569
	Se	5.68	4.06	7.58	5.51	9.93
	Sy	12.1	8.68	1.3	8.76	11.5
5	R	.908	.847	.860	.827	.591
	Se	10.9	8.38	10.8	8.78	16.8
	Sy	24.8	15.0	20.2	14.9	19.9
10	R	.933	.821	.808	.851	.664
	Se	23.7	17.7	29.1	25.9	38.0
	Sy	63.0	29.5	47.2	28.8	48.4
15	R	.943	.911	.844	.902	.615
	Se	31.1	16.2	43.1	17.1	68.0
	Sy	89.1	37.5	76.5	37.9	82.1

TABLE 4

Summary Statistics for Calibration of Short-Term Tangborn Models

Forecast Date: May 15

Length of Forecast (Days)		Data Set # 1	2	3	4	5
1	R	.745	.752	.774	.826	.645
	Se	5.07	2.27	5.33	2.19	6.63
	Sy	7.24	3.29	8.02	3.70	8.27
2	R	.766	.741	.787	.803	.690
	Se	9.83	4.76	10.7	5.00	12.3
	Sy	14.6	6.76	16.5	7.99	16.2
3	R	.788	.777	.801	.809	.739
	Se	14.5	6.33	16.2	7.53	17.3
	Sy	22.4	9.59	25.9	12.2	24.4
5	R	.823	.846	.841	.850	.803
	Se	23.2	9.20	25.8	11.2	25.3
	Sy	39.0	16.4	45.4	20.2	40.5
10	R	.852	.914	.892	.877	.927
	Se	41.4	14.1	41.8	20.6	31.9
	Sy	75.5	33.1	88.4	40.8	81.1
15	R	.855	.904	.924	.873	.954
	Se	62.1	22.0	52.2	30.7	40.6
	Sy	114.1	49.3	130.4	60.1	129.9

TABLE 4

Summary Statistics for Calibration of Short-Term Tangborn Models

Forecast Date: June 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
1	R	.836	.608	.910	.571	.757
	Se	5.21	3.57	4.04	3.02	6.23
	Sy	9.06	4.29	9.31	3.51	9.10
2	R	.837	.574	.904	.548	.719
	Se	10.0	8.02	7.98	5.98	12.9
	Sy	17.4	9.34	17.8	6.82	17.7
3	R	.848	.573	.902	.546	.715
	Se	14.1	12.8	11.9	8.68	19.2
	Sy	25.4	14.9	26.2	9.88	26.1
5	R	.854	.638	.900	.608	.722
	Se	22.1	21.5	19.9	13.1	31.7
	Sy	40.6	26.6	43.5	15.7	43.7
10	R	.835	.787	.921	.752	.767
	Se	46.1	34.7	33.7	25.6	58.5
	Sy	79.9	53.7	82.5	37.0	86.9
15	R	.843	.808	.941	.785	.827
	Se	64.3	44.7	41.3	33.6	72.8
	Sy	113.9	72.4	116.0	51.8	123.4

TABLE 4

Summary Statistics for Calibration of Short-Term Tangborn Models

Forecast Date: June 15

Length of Forecast (Days)		Data Set # 1	2	3	4	5
1	R	.877	.813	.943	.708	.915
	Se	3.52	2.33	2.35	2.50	3.22
	Sy	6.98	3.81	6.71	3.37	7.59
2	R	.883	.787	.937	.680	.907
	Se	6.77	4.77	4.87	5.09	6.94
	Sy	13.8	7.37	13.3	6.62	15.8
3	R	.900	.785	.936	.685	.897
	Se	9.27	7.02	7.28	7.29	11.1
	Sy	20.2	10.8	19.7	9.53	24.0
5	R	.923	.794	.924	.716	.879
	Se	13.5	11.4	13.5	10.9	20.7
	Sy	33.6	17.8	33.5	14.9	41.3
10	R	.933	.795	.915	.765	.879
	Se	25.3	21.6	28.3	18.4	43.4
	Sy	66.7	34.0	66.7	27.3	86.7
15	R	.930	.782	.923	.778	.877
	Se	35.3	31.5	38.1	24.2	67.3
	Sy	91.7	48.1	94.6	36.7	133.5

in which Q_n^* is the predicted volume of runoff for day n, c is a dimensionless runoff coefficient, d is a degree-day factor, T is the temperature index on day n, SCA is the percentage of total area covered by snow on day n, P is the precipitation on day n, A is the total area of the watershed, K is a dimensionless recession coefficient, and Q_{n-1} is the observed volume of runoff for the previous day. The basis for this model was explained previously.

Calibration of the Martinec model requires the optimization of the model parameters. The optimum values of the parameters are expected to vary from one watershed to another, so model calibration is required anytime the model is to be used on a new watershed. The parameters to be optimized are the runoff and recession coefficients and the degree-day factor (c, K, and d, respectively, in eq. 4-2).

Because the Kings River watershed is large and encloses a wide range of elevations, meteorological conditions are not likely to be uniform over the whole area. Therefore, dividing the watershed into elevation zones should improve the accuracy of prediction . Effective temperatures and precipitation were calculated separately for each zone; also, separate snow covered area data were compiled. The zonal temperature values were determined by finding the difference between the temperature station elevation and the mean elevation of each zone, then multiplying by a temperature lapse rate and adding the product to the station temperature. In this way an estimate of the average daily temperature in each zone was obtained. If the zonal temperature value was below zero, any precipitation in that zone was assumed to have fallen as snow; in this case, the precipitation was not added to the generated runoff total. The modified form of the Martinec equation is given by:

$$Q_n^* = c(1-K) \sum_{i=1}^{m} (d_i T_i SCA_i + P_i)\ A_i + KQ_{n-1} \qquad (4\text{-}3)$$

in which m is the number of elevation zones. This equation reflects the varying meteorological conditions over the watershed.

Both the original form (Eq.4-2) and the modified from (Eq.4-3) of the Martinec model were calibrated. Values of K were estimated by three different methods. The modified form of the model was tested both with the degree-day factor being the same for each zone and with the factor being allowed to vary from one zone to the next. In both cases, the runoff coefficient was the same for all zones.

Three different optimization techniques (analytical, numerical, and subjective) were used in calibrating the Martinec model. The objective of each of these methods is to explain the maximum possible variation in the criterion variable.

The subjective optimization technique is generally used to optimize one parameter at a time. Using the investigator's knowledge of the model and the system, the value of each parameter is adjusted by trial and error until there is no significant improvement in the error function.

The other two optimization techniques require that all parameters be optimized simultaneously. The criterion by which accuracy of prediction is measured is the sum of squares of the errors. This sum of squares, called the criterion function, is an indicator of the goodness-of-fit between the observed and predicted values. Model accuracy is best when the sum of squares is lowest. Thus, the model can be calibrated by adjusting the parameter values to give the minimum sum of squares of the errors. Two methods of doing this are: (1) Determine the parameters analytically, using the partial derivatives of the criterion function; or (2) determine the values numerically using the pattern search method.

Subjective Optimization of the Recession Coefficient

In theory, the recession coefficient K has a different value for each day; these values can be calculated from the equation (Martinec, 1975):

$$K_n = \frac{Q_{n+1}}{Q_n} \tag{4-4}$$

As mentioned above, an initial investigation indicated that the recession term is far more important than the generated runoff term. Therefore, accuracy of the model is very sensitive to the values of K. In a predictive situation, the values of Q_{n+1} and Q_n are not available, so K must be estimated. This can be done in a number of ways.

The first method, which was suggested by Martinec (1975), estimates K using the equation:

$$K_n = aQ_{n-1}^b \tag{4-5}$$

in which a and b are empirical coefficients. Values of K that are considered true values are derived with Eq. 4-4 and regressed on daily values of Q_{n-1} using the logarithmic form of the linear regression equation:

$$\log K_n = \log a + b \log Q_{n-1} \tag{4-6}$$

The optimum values of a and b for each of the 24 years of record are shown in Table 5.

When making forecasts, the optimum values of a and b given in Table 5 can not be used. Two methods for estimating a and b are feasible. First, the average values for the 24 years of record were recommended by Martinec (1975). For the Kings River watershed the mean values of a and b are 1.02484 and -0.00351, respectively. Use of the average values implies that the annual variation in a and b is random. Second, if the annual variation is not random but results from variation in other factors, then better estimates can be obtained by relating the optimum values of a and b (Table 5) to the factor or factors responsible for the variation. An analysis showed that a and b were related to the October-April precipitation. Regression analysis provided linear prediction equations of the form:

$$\hat{a} = e_1 \text{ (October-April precipitation)} + e_2 \tag{4-7}$$

and

$$\hat{b} = e_3 \text{ (October-April precipitation)} + e_4 \tag{4-8}$$

TABLE 5

Values of The Optimum Recession Term Parameters for The Martinec Model

Recession Term - $a\,Q_{n-1}^{b}$

Year	a	b
1954	0.9294	-0.00140
1955	0.9766	-0.00005
1956	0.9281	-0.00229
1957	0.9313	-0.00650
1958	0.9797	0.00097
1959	0.9875	-0.00475
1960	0.9523	-0.00148
1961	1.0228	-0.00394
1962	1.0154	-0.01327
1963	0.9620	-0.00005
1964	1.0160	-0.00875
1965	0.9422	-0.00788
1966	1.0000	-0.00013
1967	1.0000	-0.00019
1968	1.0000	-0.00013
1969	1.0000	-0.00013
1970	1.0000	-0.00013
1971	1.9775	-0.00184
1972	1.0476	-0.01238
1973	1.0000	-0.00013
1974	0.9951	-0.00091
1975	1.0000	-0.00027
1976	0.9500	-0.00291
1977	0.9826	-0.01563

in which $\hat{a}$ and $\hat{b}$ are the predicted values of a and b, and e_1, e_2, e_3, and e_4 are regression coefficients. For the 24 years of record these equations provided correlation coefficients of -0.544 and 0.505 for Eqs. 4-7 and 4-8, respectively. For making forecasts, separate equations were derived for each of the five 12-year data sets. The use of constant values of a and b for any one year does not imply that K is constant because K depends on Q_{n-1}.

The second method of estimating K is to assume that there is good serial correlation in the runoff values and use the runoff values from the two previous days. In this case, the predicted value of K is set equal to the most recent observed value available, which is calculated by the equation:

$$K_n^* = K_{n-2} = \frac{Q_{n-1}}{Q_{n-2}} \tag{4-9}$$

The accuracy when applying Eq. 4-9 is better than that obtained when the average values of a and b are applied with Eq. 4-5. However, the accuracy when applying Eq. 4-9 is similar to that obtained when equations such as Eqs. 4-7 and 4-8 are used with Eq. 4-5; in this case, the accuracy varies from year to year.

The Relative Importance of the Recession Term

Eq. 4-9 was used to derive estimates of K for the years from 1954 to 1958, inclusive. Predictions of runoff were made for each day of the period from May 1 to June 15 for each of the five years using the model:

$$Q_n^* = KQ_{n-1} \tag{4-10}$$

A different value of K was used for each day. Eq. 4-10 is the recession term of the Martinec model, Eq. 4-2. The importance of the recession term on this watershed is such that more than 81 percent of the total variance was explained with this term only. The relative importance of the recession term when compared to the generated runoff term indicates that the model is probably not sensitive to the values of the c and d coefficients of Eqs. (4-2) and (4-3).

Subjective Optimization of the Runoff and Degree-day Coefficients

As noted above, the Martinec model is not sensitive to the values of c and d in Eq. 4-3. It is known that insensitive coefficients quite often fail to approach the population values and are sometimes irrational (Dawdy and O'Donnell, 1965). Also, it is difficult to relate irrational coefficients to other characteristics of the system.

Martinec (1975) has evaluated some field estimated values of d, the degree-day factor. This parameter represents the amount of snow that melts for each degree-day. Since the rate of snowmelt depends on the available energy, the wind speed, and the vapor pressure deficit (among other factors), the temperature alone cannot be expected to completely determine the amount of snowmelt. Martinec suggested a value in the range of 0.35-0.60 centimeters per degree (Celsius)-day. This corresponds to a range of 0.138-0.236 inches per degree-day. Martinec also suggested that the value varies systematically throughout the snowmelt season and also varies from one elevation zone to the next. These values derive from the experimental approach to calibration of the model.

The runoff coefficient, c, can be expected to vary considerably from watershed to watershed. This parameter is an indicator of the proportion of the water incident on the basin that leaves the basin as streamflow. Because groundwater storage and evapotranspiration losses vary considerably between watersheds, the value of c can be expected to vary as well.

Once values of K and d have been selected, c can be determined subjectively by making forecasts with an initial estimated value of c, examining the errors of the predictions, and then adjusting the value of c in such a way as to lessen the errors. Using values of K derived from (Eq.4-9) and the values of d suggested by Martinec, subjective optimization gave an optimum c value of about 0.15.

Analytical Calibration of the Martinec Model

Analytical calibration is performed by taking the partial derivatives of the criterion function, which is the sum of the squares of the errors, with respect to each of the coefficients to be optimized. These partial derivatives are then set equal to zero, because the derivatives equal zero at the minimum value of the criterion function. The partial derivatives form a set of simultaneous equations that can be solved for the optimum values of the coefficients. The advantage of analytical calibration is that it leads to a unique solution and is reproduceable;

two different researchers calibrating the same equation with the same data will arrive at the same optimum values. The disadvantages are that the partial derivatives do not always form an independent set of simultaneous equations and that the evaluation of the equations and the substitutions involved in solving them are cumbersome.

When optimizing the coefficients, it is desirable to optimize all of the coefficients simultaneously. Analytical calibration of the Martinec model was attempted using the estimates of K given by Eq. 4-5. The equation is:

$$Q_n^* = c(dTSCA+P)\ A\ (1-aQ_{n-1}{}^{b}) + (aQ_{n-1}{}^{(b+1)}) \qquad (4\text{-}11)$$

When the partial derivatives are calculated, no independent equation involving b results. Therefore, this form of the model with K estimated by Eq. 4-5 cannot be optimized analytically. Another method of estimating K must be chosen.

Assuming that K is estimated by Eq. (4-9), the criterion function for the Martinec equation can be written:

$$F = \sum_{n=1}^{m} (Q_n^* - Q_n)^2 = \sum_{n=1}^{m} [(dTSCA+P)\ Ac\ (1-K) + KQ_{n-1} - Q_n]^2 \qquad (4\text{-}12)$$

in which m is the number of days for which predictions are made. The partial derivative with respect to c is:

$$\frac{\partial F}{\partial c} = 2 \sum_{n=1}^{m} [(dTSCA+P)\ cA\ (1-K) + KQ_{n-1} - Q_n][(dTSCA+P)(1-K)A] \qquad (4\text{-}13)$$

which equals zero at the minimum value of Eq. 4-11:

$$0 = \sum_{n=1}^{m} [(dTSCA+P)^2(1-K)^2cA^2 + A(1-K)(dTSCA+P)\ KQ_{n-1}$$

$$- A(dTSCA+P)(1-K)\ Q_n] \qquad (4\text{-}14)$$

Assuming that the degree-day factor does not vary from zone to zone, Eq. (4-13) can be rearranged to:

$$0 = cAd^2 \sum_{n=1}^{m} [T\ SCA\ (1-K)^2] + 2cAd \sum_{n=1}^{m} [TPSCA(1-K)^2]$$

$$+ cA \sum_{n=1}^{m} [P^2(1-K)^2] + d \sum_{n=1}^{m} [TSCAK(1-K)Q_{n-1}]$$

$$+ \sum_{n=1}^{m} [K(1-K)PQ_{n-1}] - d \sum_{n=1}^{m} [TSCA(1-K)Q_n] - \sum_{n=1}^{m} [P(1-K)Q_n]$$

(4-15)

The partial derivative of Eq. (4-11) with respect to d is given by:

$$\frac{\partial F}{\partial d} = 2 \sum_{n=1}^{m} [cA(1-K)(dTSCA+P) + KQ_{n-1} - Q_n]\ (cA(1-K)TSCA)$$

(4-16)

which, when set equal to zero, becomes:

$$0 = cAd \sum_{n=1}^{m} [T^2SCA^2(1-K)^2] + cA \sum_{n=1}^{m} [TSCAP(1-K)^2]$$

$$+ \sum_{n=1}^{m} [TSCAK(1-K)Q_{n-1}] - \sum_{n=1}^{m} [TSCA(1-K)Q_n] \qquad (4\text{-}17)$$

Eqs. (4-15) and (4-17) are simultaneous and independent; they can be solved for the optimum values of c and d once the summations are calculated. The resulting values are different for each year. In testing the Martinec model, the average values of c and d were used because the yearly values were not highly correlated with any of the data available on May 1. The accuracy of the model when tested with the averages of the analytically derived values of c and d was not greater than the accuracy achieved with values derived using numerical optimization.

An alternative way of analytically calibrating the Martinec model assumes that K is constant for each year, rather than a function of the previous day runoff. With this assumption, optimum values of K, c, and d can be derived from the three simultaneous and independent equations obtained from the partial derivatives. The use of a constant K for each year simplifies the prediction model significantly, but also results in a decrease in accuracy when compared with the methods in which K is a function of the runoff from the previous day.

Numerical Calibration of the Martinec Model

The numerical calibration method is an iterative process that requires a computer. The program used in this study is referred to as pattern search. This program starts with initial estimates of the coefficients to be optimized (supplied by the programmer) and calculates the predictions and errors for each case using these initial estimates. The sum of squares of the errors is then calculated. Next, each coefficient is sequentially decremented by a given amount, ususlly about 5 to 10 percent of the initial estimates. The sum of squares of the errors is calculated for each new set of parameters and compared with that produced by the original set. The best new set of parameters (i.e., the set with the lowest sum of squares) becomes the set of base values, and these values are then incremented. The process continues until variation of the parameters does not result in a significnatly lower sum of squares than does the base set. At this point, the parameters have converged on the optimum values, that is, the set of values that minimizes the sum of squares of the errors. If the initial parameter estimates are properly selected, numerical optimization should provide final parameter estimates that are similar to those that would be obtained if an analytical solution were possible.

In using the pattern search method, K was assumed to be a function of Q_{n-1}. The optimum values of c and d were to be determined, so the following model was used in the pattern search program:

$$Q_n^* = c(1-aQ_{n-1}{}^b)\sum_{i=1}^{m}[(d_iT_iSCA_i+P_i)A_i] + aQ_{n-1}{}^{(b+1)} \qquad (4\text{-}13)$$

The degree-day factor K is different for each zone, and six elevation zones were used. There was no snow cover in the lowest elevation zone during the forecast period, so this zone did not need a degree-day factor; therefore, only five values of degree-day factor were required. The pattern search model was used to determine the optimum set of values for the parameters a, b, c, d, d_2,

d_3, d_4, and d_5. The resulting values of d were negative for some zones, which is irrational. All values of the degree-day factor must be greater than zero. Therefore, the pattern search program was modified in order to constrain all the values of d to be greater than zero. The resulting values of d were nearly zero for some zones and did not vary systematically with elevation.

The model was also optimized with the degree-day factor being constant from zone to zone. The equation for this model is:

$$Q_n^* = c(1-aQ_{n-1}{}^b)\sum_{i=1}^{m}[(dT_i SCA_i + P_i)A_i] + aQ_{n-1}{}^{(b+1)} \qquad (4\text{-}19)$$

With this equation there were only four parameters to be optimized because there is only one value of d.

The accuracies of these three versions of the modified Martinec model were compared on the basis of the sum of squares of the errors for each year. The model in which the degree-day factor varied from zone to zone and was not constrained to be greater than zero was most accurate, but the d values were irrational. Constraining the values of d to be greater than zero increased the sum of squares of the errors by two to four percent for most years when compared with the unconstrained model. When the value of the degree-day factor was assumed to be constant from one zone to the next, the sum of squares increased by less than two percent when compared with the constrained form of the spatially distributed model.

These results show that the values of c and d are not very important to the accuracy of prediction; that is, the model is not sensitive to these parameters. In order to determine just how important the values of c and d are, the program was used with only the recession term of the Martinec model:

$$Q_n^* = aQ_{n-1}^{(b+1)} \tag{4-20}$$

A comparison of the results of this investigation with the results of using the full model is shown in Table 6. It is obvious that this model which is based only on the recession term, requires far less data to operate than does the original Martinec form; the only data required for the recession model is daily runoff, while the original version also required daily temperature, precipitation, and snow covered area. Whether the improved accuracy obtained with the original model justifies the added expense of collecting all this data is a question that will depend on both the watershed and project objectives. It should be emphasized that this result will not necessarily be valid for all watersheds; for watersheds that are characterized by low serial correlation, one would expect the first term to be more important.

TABLE 6

Comparison of The Accuracy of the Entire Martinec Model Vs. Accuracy of the Recession Term, 1954-1958 Calibration

Year	Recession Term Model Correlation Coefficient	Recession Term Model Sum of Squares of the Errors	Martinec Model Correlation Coefficient	Martinec Model Sum of Squares of the Errors
1954	.950	1.1376×10^8	.969	8.1712×10^7
1955	.908	2.3632×10^8	.942	1.6749×10^8
1956	.906	2.7011×10^8	.944	1.5675×10^8
1957	.834	5.7226×10^8	.871	4.6871×10^8
1958	.927	2.4326×10^8	.947	1.8302×10^8

CHAPTER V

TESTING THE SNOWMELT MODELS

Each model was tested for accuracy of prediction using each of the five 12-year test data sets. Data sets #1, 3, and 5 were used to evaluate the accuracy of the model when used with data from within the range of calibration data; data sets #2 and 4 were used to test the model with data from outside the calibration range. The accuracy of prediction for each model and each data set was measured by the correlation between predicted and observed runoff volumes and by the standard error of estimate.

Significance of the Correlation Coefficient

In evaluating the results of the testing program, it was necessary to compare the correlations and standard errors of the various models. These goodness-of-fit statistics are shown for the regression models in Table 7, for the Tangborn models in Table 8, and for the Martinec models in Table 9. Conclusions as to which models are most accurate are based on these comparisons. Because the models were tested on only 12 years of data, the resulting correlations and standard errors are only approximations of the values that would result from a more extensive testing program. Therefore, the fact that one model resulted in a higher correlation coefficient than another model does not necessarily mean that the first model is superior;

TABLE 7

Summary Statistics for Testing of Regression Models
Predictor Variable: Snowpack Index
Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.709	.646	.807	.674	.911
	Se	31.3	32.9	27.3	33.1	20.7
	Sy	42.3	41.2	44.1	42.8	47.9
30	R	.852	.751	.894	.751	.948
	Se	49.3	62.9	47.0	63.7	36.5
	Sy	89.9	91.0	99.8	92.0	109.3
45	R	.927	.858	.931	.858	.974
	Se	70.3	92.1	75.2	93.1	48.9
	Sy	179.0	171.0	196.1	173.0	206.1
60	R	.972	.921	.963	.924	.976
	Se	83.9	106.8	93.9	109.3	81.1
	Sy	340.6	261.8	332.4	272.1	356.7
90	R	.954	.886	.941	.881	.972
	Se	198.0	220.8	220.9	219.6	154.8
	Sy	626.8	453.9	662.0	422.2	631.6
120	R	.921	.809	.907	.802	.967
	Se	335.5	372.3	362.0	370.8	211.8
	Sy	822.6	604.3	821.0	592.5	791.8
150	R	.916	.792	.900	.786	.964
	Se	370.6	417.1	401.5	415.6	235.6
	Sy	880.1	651.9	878.9	640.4	842.0

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variable: October-March Precipitation Total

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.743	.720	.762	.744	.860
	Se	29.7	29.9	30.0	30.0	25.6
	Sy	42.3	41.2	44.1	42.8	47.9
30	R	.860	.759	.820	.775	.888
	Se	48.2	62.1	59.9	61.0	52.7
	Sy	89.9	91.0	99.8	92.0	109.3
45	R	.901	.810	.858	.822	.928
	Se	81.4	105.8	105.8	103.3	80.3
	Sy	179.0	171.0	196.1	173.0	206.1
60	R	.967	.930	.962	.936	.966
	Se	91.6	101.2	94.8	100.7	96.6
	Sy	340.6	261.8	332.4	272.4	356.7
90	R	.968	.941	.973	.943	.981
	Se	165.3	160.9	149.4	154.7	129.6
	Sy	626.8	453.9	622.0	442.2	631.6
120	R	.957	.921	.962	.921	.982
	Se	250.5	247.5	236.5	242.3	154.9
	Sy	822.6	604.3	821.0	592.5	791.8
150	R	.954	.912	.958	.913	.980
	Se	276.8	280.2	263.3	274.7	174.0
	Sy	880.1	651.9	878.9	640.4	842.0

TABLE 7

Summary Statistics for Testing of Regression Models
Predictor Variables: April 1 Snow Covered Area
Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.744	.716	.764	.740	.855
	Se	29.6	30.1	29.9	30.2	26.0
	Sy	42.3	41.2	44.1	42.8	47.9
30	R	.864	.758	.820	.775	.884
	Se	47.5	62.2	59.9	61.0	53.6
	Sy	89.9	91.0	99.8	92.0	109.3
45	R	.904	.809	.859	.822	.925
	Se	80.2	105.4	105.2	103.4	82.0
	Sy	179.0	171.0	196.1	173.0	206.1
60	R	.966	.930	.964	.936	.966
	Se	91.8	101.0	92.7	100.3	96.2
	Sy	340.6	261.8	332.4	272.1	356.7
90	R	.965	.940	.972	.942	.982
	Se	172.3	163.1	153.1	155.7	126.1
	Sy	626.8	453.9	622.0	442.2	631.6
120	R	.953	.919	.959	.920	.984
	Se	260.6	250.3	242.5	243.9	146.9
	Sy	822.6	604.3	821.0	592.5	791.8
150	R	.950	.911	.956	.912	.983
	Se	286 9	282.2	269.3	275.6	164.2
	Sy	880.1	651.9	878.9	640.4	842.0

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variable: Product of Snowpack Index and April 1 Snow Covered Area

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.723	.676	.776	.695	.878
	Se	30.7	31.8	29.2	32.2	24.1
	Sy	42.3	41.2	44.1	42.8	47.9
30	R	.854	.753	.850	.753	.900
	Se	49.1	62.8	55.1	63.4	49.9
	Sy	89.9	91.0	99.8	92.0	109.3
45	R	.932	.854	.899	.854	.948
	Se	67.9	93.2	90.3	94.4	68.5
	Sy	179.0	171.0	196.1	173.0	206.1
60	R	.968	.947	.958	.943	.971
	Se	89.1	88.2	99.6	94.6	89.1
	Sy	349.6	261.8	332.4	272.1	356.7
90	R	.945	.923	.940	.924	.971
	Se	214.9	182.9	223.0	177.5	159.5
	Sy	626.8	453.9	622.0	442.2	631.6
120	R	.915	.862	.911	.861	.971
	Se	347.7	320.8	355.0	316.0	199.3
	Sy	822.6	604.3	821.0	592.5	791.8
150	R	.910	.848	.906	.847	.969
	Se	382.5	361.9	390.9	357.3	218.8
	Sy	880.1	651.9	878.9	640.4	842.0

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variables: Snowpack Index and October-March Precipitation Total

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.743	.646	.746	.674	.613
	Se	31.3	34.7	32.5	34.9	41.8
	Sy	42.3	41.2	44.1	42.8	47.9
30	R	.860	.751	.820	.751	.888
	Se	50.8	66.4	63.2	67.1	55.5
	Sy	89.9	91.0	99.8	92.0	109.3
45	R	.901	.860	.885	.858	.928
	Se	85.8	96.5	101.0	98.2	84.6
	Sy	179.0	171.0	196.1	173.0	206.1
60	R	.967	.946	.968	.950	.966
	Se	96.6	93.8	92.3	94.0	101.8
	Sy	340.6	261.8	332.4	272.1	356.7
90	R	.970	.909	.961	.931	.981
	Se	168.2	209.0	189.7	178.2	136.6
	Sy	626.8	453.9	622.0	442.2	631.6
120	R	.950	.837	.943	.879	.982
	Se	284.3	365.1	301.7	311.8	163.3
	Sy	822.6	604.3	821.0	592.5	791.8
150	R	.946	.823	.939	.867	.980
	Se	314.1	409.6	333.2	352.8	183.4
	Sy	880.1	651.9	878.9	640.4	842.0

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variables: Snowpack Index and April 1 Snow Covered Area

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.744	.646	.748	.674	.593
	Se	31.2	34.7	32.4	34.9	42.6
	Sy	42.3	41.2	44.1	42.8	47.9
30	R	.864	.761	.820	.751	.767
	Se	50.0	65.2	63.2	67.1	77.6
	Sy	89.9	91.0	99.8	92.0	109.3
45	R	.904	.858	.885	.858	.925
	Se	84.5	97.2	100.9	98.2	86.4
	Sy	179.0	171.0	196.1	173.0	206.1
60	R	.966	.947	.969	.951	.966
	Se	96.8	92.6	90.9	93.1	101.4
	Sy	340.6	261.8	332.4	272.1	356.7
90	R	.968	.907	.960	.932	.982
	Se	173.0	211.0	192.3	177.7	133.0
	Sy	626.8	453.9	622.0	442.2	631.6
120	R	.948	.834	.941	.880	.984
	Se	289.7	368.1	306.3	311.3	154.8
	Sy	822.6	604.3	821.0	592.5	791.8
150	R	.945	.820	.938	.868	.983
	Se	319.5	412.8	337.8	351.9	173.1
	Sy	880.1	651.9	878.9	640.4	842.0

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variables: October-March Precipitation Total and April Snow Covered Area

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.744	.660	.764	.684	.855
	Se	31.2	34.2	31.5	34.5	27.4
	Sy	42.3	41.2	44.1	42.8	47.9
30	R	.864	.717	.820	.729	.884
	Se	50.0	70.1	63.2	69.6	56.5
	Sy	89.9	91.0	99.8	92.0	109.3
45	R	.904	.772	.859	.781	.925
	Se	84.5	120.2	110.9	119.3	86.4
	Sy	179.0	171.0	196.1	173.0	206.1
60	R	.966	.924	.962	.930	..966
	Se	96.8	110.8	99.9	110.5	101.4
	Sy	340.6	261.8	332.4	272.1	356.7
90	R	.965	.942	.973	.942	.981
	Se	181.6	168.3	157.5	164.1	136.6
	Sy	626.8	453.9	662.0	422.2	631.6
120	R	.953	.922	.962	.920	.982
	Se	274.7	258.8	249.3	257.1	163.3
	Sy	822.6	604.3	821.0	592.5	791.8
150	R	.950	.913	.958	.912	.980
	Se	302.4	294.6	277.6	290.5	183.4
	Sy	880.1	651.9	878.9	640.4	842.0

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TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variables: October-March Precipitation Total and Product of Snowpack Index and April Snow Covered Area

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.743	.669	.754	.678	.311
	Se	31.3	33.8	32.0	34.7	50.3
	Sy	42.3	41.2	44.1	42.8	47.9
30	R	.854	.753	.825	.753	.888
	Se	51.8	66.2	62.4	66.9	55.5
	Sy	89.9	91.0	99.8	92.0	109.3
45	R	.932	.852	.882	.854	.928
	Se	71.5	98.9	102.1	99.5	84.6
	Sy	179.0	171.0	196.1	173.0	206.1
60	R	.967	.952	.964	.952	.966
	Se	96.6	88.9	98.2	92.2	101.8
	Sy	340.6	261.8	332.4	272.1	356.7
90	R	.962	.928	.956	.940	.981
	Se	188.1	187.0	201.3	166.5	136.6
	Sy	626.8	453.9	662.0	422.2	631.6
120	R	.939	.868	.940	.889	.982
	Se	311.7	332.0	309.4	299.9	163.3
	Sy	822.6	604.3	821.0	592.5	791.8
150	R	.936	.854	.937	.877	.980
	Se	343.7	374.5	339.1	340.6	183.4
	Sy	880.1	651.9	878.9	640.4	842.0

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variable: Snowpack Index

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.927	.861	.907	.864	.971
	Se	37.9	48.6	45.5	48.5	25.2
	Sy	96.4	91.0	102.8	91.8	100.0
30	R	.977	.874	.925	.885	.946
	Se	58.6	101.0	99.5	100.4	87.7
	Sy	260.6	197.9	249.6	205.9	258.5
45	R	.976	.886	.931	.883	.952
	Se	95.8	149.3	156.3	149.6	131.3
	Sy	417.3	307.1	407.4	303.3	409.0
60	R	.944	.846	.909	.836	.955
	Se	190.7	220.7	238.1	220.4	165.9
	Sy	551.9	394.6	545.5	383.2	534.5
90	R	.909	.768	.875	.758	.951
	Se	327.5	368.3	379.7	367.1	225.6
	Sy	749.3	548.0	749.0	536.3	696.1
120	R	.904	.752	.869	.742	.948
	Se	362.1	412.3	419.5	410.9	250.2
	Sy	807.0	596.2	807.9	584.7	746.9

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variable: October-April Precipitation Total

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.785	.627	.751	.639	.923
	Se	62.6	74.3	71.3	74.1	40.5
	Sy	96.4	91.0	102.8	91.8	100.0
30	R	.940	.858	.946	.871	.967
	Se	93.4	106.5	85.1	106.1	69.6
	Sy	260.6	197.9	249.6	205.9	258.5
45	R	.946	.870	.957	.861	.984
	Se	142.0	158.7	124.1	162.0	75.5
	Sy	417.3	307.1	407.4	303.3	409.0
60	R	.970	.943	.982	.930	.987
	Se	140.2	137.4	107.3	148.0	89.9
	Sy	551.9	394.6	545.5	383.2	534.5
90	R	.981	.966	.987	.954	.986
	Se	151.5	148.6	125.5	169.3	119.6
	Sy	749.3	548.0	749.0	536.3	696.1
120	R	.982	.965	.986	.953	.985
	Se	158.8	163.3	140.5	184.7	136.1
	Sy	807.0	596.2	807.9	584.7	746.9

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variable: May 1 Snow Covered Area

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.531	.076	.476	.216	.706
	Se	85.6	95.1	94.9	94.0	74.3
	Sy	96.4	91.0	102.8	91.8	100.0
30	R	.785	.496	.730	.602	.785
	Se	169.2	180.2	179.0	172.4	167.8
	Sy	260.6	197.9	249.6	205.9	258.5
45	R	.819	.518	.770	.619	.813
	Se	251.0	275.4	272.5	249.8	249.6
	Sy	417.3	307.1	407.4	303.3	409.0
60	R	.878	.604	.832	.714	.825
	Se	277.2	329.7	317.6	281.2	316.9
	Sy	551.9	394.6	545.5	383.2	534.5
90	R	.896	.653	.852	.756	.812
	Se	349.1	435.6	411.8	367.9	426.0
	Sy	749.3	548.0	749.0	536.3	696.1
120	R	.897	.654	.851	.757	.811
	Se	374.6	473.2	445.4	400.9	458.2
	Sy	807.0	596.2	807.9	584.7	746.9

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variable: Product of May 1 Snow Covered Area and April Snowpack Index

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.876	.778	.839	.783	.965
	Se	48.7	59.9	58.8	59.9	27.6
	Sy	96.4	91.0	102.8	91.8	100.0
30	R	.974	.913	.945	.919	.966
	Se	61.4	84.5	85.7	84.9	70.6
	Sy	260.6	197.9	249.6	205.9	258.5
45	R	.976	.928	.960	.925	.972
	Se	95.4	119.7	120.2	120.7	101.5
	Sy	417.3	307.1	407.4	303.3	409.0
60	R	.965	.917	.960	.913	.973
	Se	151.3	165.2	160.4	163.9	129.3
	Sy	551.9	394.6	545.5	383.2	534.5
90	R	.945	.862	.945	.858	.969
	Se	256.5	291.1	257.3	288.5	179.8
	Sy	749.3	548.0	749.0	536.3	696.1
120	R	.942	.849	.941	.846	.967
	Se	284.9	330.6	286.4	327.4	199.8
	Sy	807.0	596.2	807.9	584.7	746.9

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variables: Snowpack Index and October-April Precipitation Total

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.927	.794	.877	.801	.971
	Se	39.9	61.2	54.5	60.8	26.5
	Sy	96.4	91.0	102.8	91.8	1000.0
30	R	.940	.908	.948	.915	.965
	Se	98.4	91.6	88.2	91.7	75.3
	Sy	260.6	197.9	249.6	205.9	258.5
45	R	.946	.921	.955	.907	.970
	Se	149.7	132.6	133.3	141.5	109.5
	Sy	417.5	307.1	407.4	303.3	409.0
60	R	.970	.907	.960	.927	.987
	Se	147.8	184.0	168.9	158.5	93.4
		551.9	394.6	545.5	383.2	534.5
90	R	.981	.844	.961	.894	.986
	Se	159.2	324.8	229.9	266.2	126.1
	Sy	749.3	548.0	749.0	536.3	696.1
120	R	.982	.832	.962	.885	.985
	Se	167.4	365.5	244.9	301.3	143.4
	Sy	807.0	596.2	807.9	584.7	746.9

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variables: Snowpack Index and May Snow Covered Area

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.927	.885	.917	.882	.978
	Se	39.9	46.8	45.3	47.9	23.0
	Sy	96.4	91.0	102.8	91.8	100.0
30	R	.969	.872	.933	.893	.947
	Se	71.2	107.1	99.2	102.4	91.7
	Sy	260.6	197.9	249.6	205.9	258.5
45	R	.974	.900	.947	.898	.956
	Se	103.9	147.7	145.1	147.5	133.3
	Sy	417.3	307.1	407.4	303.3	409.0
60	R	.979	.872	.935	.865	.959
	Se	123.9	213.6	214.5	212.5	167.7
	Sy	551.9	394.6	545.5	383.2	534.5
90	R	.962	.803	.907	.798	.946
	Se	226.7	361.5	349.5	357.4	248.8
	Sy	749.3	548.0	749.0	536.3	696.1
120	R	.958	.787	.899	.783	.943
	Se	255.1	406.9	390.6	402.1	275.9
	Sy	807.0	596.2	807.9	584.7	746.9

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variables: October-April Precipitation Total and May Snow-covered Area

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.785	.659	.730	.662	.923
	Se	66.0	75.7	77.7	76.1	42.7
	Sy	96.4	91.0	102.8	91.8	100.0
30	R	.931	.859	.919	.872	.967
	Se	104.9	111.8	108.8	111.6	73.3
	Sy	260.6	197.9	249.6	205.9	258.5
45	R	.938	.872	.934	.863	.982
	Se	160.2	166.3	160.6	169.5	86.1
	Sy	417.3	307.1	407.4	303.3	409.0
60	R	.975	.951	.974	.938	.987
	Se	134.9	134.5	137.0	146.4	96.2
	Sy	551.9	394.6	545.5	383.2	534.5
90	R	.989	.980	.985	.966	.984
	Se	123.0	119.7	145.0	152.7	135.1
	Sy	749.3	548.0	749.0	536.3	696.1
120	R	.990	.980	.984	.966	.983
	Se	126.4	131.9	158.7	166.7	150.6
	Sy	807.0	596.2	807.9	584.7	746.9

TABLE 7

Summary Statistics for Testing of Regression Models

Predictor Variables: October-April Precipitation Total and Product of Snowpack Index and May Snow Covered Area

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.785	.497	.810	.489	.965
	Se	66.0	87.2	66.6	88.6	29.1
	Sy	96.4	91.0	102.8	91.8	100.0
30	R	.940	.905	.950	.919	.976
	Se	98.4	93.1	86.1	89.9	61.8
	Sy	260.6	197.9	249.6	205.9	258.5
45	R	.978	.936	.964	.927	.981
	Se	95.2	119.1	120.4	125.7	87.2
	Sy	417.3	307.1	407.4	303.3	409.0
60	R	.978	.945	.976	.945	.992
	Se	127.8	143.1	131.5	138.1	76.4
	Sy	551.9	394.6	545.5	383.2	534.5
90	R	.966	.902	.976	.910	.990
	Se	214.7	261.8	180.9	245.3	109.5
	Sy	749.3	548.0	749.0	536.3	696.1
120	R	.965	.891	.975	.901	.985
	Se	235.5	298.8	197.3	280.2	143.4
	Sy	807.0	596.2	807.9	584.7	746.9

TABLE 8

Summary Statistics for Testing of Tangborn Models

Forecast Date: April 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.747	.511	.753	.583	.410
	Se	29.5	37.1	30.5	36.4	45.8
	Sy	42.3	41.2	44.1	42.8	47.9
30	R	.856	.441	.781	.558	.541
	Se	48.7	85.6	65.5	80.1	96.4
	Sy	89.9	91.0	99.8	92.0	109.3
45	R	.841	.436	.828	.546	.746
	Se	101.4	161.4	115.4	152.0	143.9
	Sy	179.0	171.0	196.1	173.0	206.1
60	R	.912	.739	.938	.784	.901
	Se	146.2	185.0	121.1	177.3	162.1
	Sy	340.6	261.8	332.4	272.1	356.7
90	R	.937	.895	.951	.891	.940
	Se	229.4	212.6	201.0	210.9	225.2
	Sy	626.8	453.9	662.0	442.2	631.6
120	R	.942	.899	.955	.901	.929
	Se	290.2	277.2	254.6	269.0	307.7
	Sy	882.6	604.3	821.0	592.5	791.8
150	R	.941	.894	.956	.899	.926
	Se	331.4	306.7	271.3	293.9	332.9
	Sy	880.1	651.9	878.9	640.4	842.0

TABLE 8

Summary Statistics for Testing of Tangborn Models

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
15	R	.740	.494	.689	.525	.828
	Se	68.0	82.9	78.2	82.0	58.9
	Sy	96.4	91.0	102.8	91.8	100.0
30	R	.899	.770	.903	.778	.943
	Se	119.8	132.5	112.6	135.5	90.0
	Sy	260.6	197.9	249.6	205.9	258.5
45	R	.915	.838	.908	.818	.979
	Se	177.1	175.6	179.0	182.8	86.6
	Sy	417.3	307.1	407.4	303.3	409.0
60	R	.936	.910	.955	.890	.980
	Se	203.4	171.6	169.2	183.2	110.6
	Sy	551.9	394.6	545.5	383.2	534.5
90	R	.948	.934	.976	.923	.980
	Se	249.8	205.8	169.4	215.9	147.1
	Sy	749.4	548.0	749.0	536.3	696.1
120	R	.949	.931	.978	.925	.978
	Se	266.2	228.5	178.1	233.3	163.3
	Sy	807.0	596.2	807.9	584.7	746.9
150	R	.950	.930	.978	.925	.978
	Se	269.4	236.0	181.5	239.7	167.0
	Sy	819.5	612.0	822.8	600.6	756.8

TABLE 8

Summary Statistics for Testing of Short-Term Tangborn Models

Forecast Date: May 1

Length of Forecast (Days)		Date Set # 1	2	3	4	5
1	R	.665	.619	.760	.644	.813
	Se	3.89	4.20	3.38	3.90	3.14
	Sy	4.96	5.1	4.96	4.86	5.15
2	R	.694	.634	.761	.664	.834
	Se	7.43	8.08	7.02	7.53	5.86
	Sy	9.85	9.97	10.31	9.61	10.13
3	R	.718	.630	.739	.667	.858
	Se	10.43	11.56	11.11	10.69	8.00
	Sy	14.29	14.18	15.74	13.69	14.83
5	R	.758	.587	.684	.632	.830
	Se	15.45	19.18	20.82	17.66	15.12
	Sy	22.57	22.60	27.21	21.74	25.82
10	R	.786	.515	.706	.539	.756
	Se	35.08	49.87	49.25	47.95	44.42
	Sy	54.06	55.46	66.28	54.28	64.68
15	R	.740	.494	.689	.525	.828
	Se	68.0	82.94	78.18	81.99	58.88
	Sy	96.38	90.97	102.84	91.83	100.05

TABLE 8

Summary Statistics for Testing of Short-Term Tangborn Models

Forecast Date: May 15

Length of Forecast (Days)		Data Set # 1	2	3	4	5
1	R	.694	.405	.397	.506	.782
	Se	6.79	8.21	7.96	8.34	5.28
	Sy	9.00	8.56	8.27	9.22	8.09
2	R	.734	.438	.514	.547	.808
	Se	12.83	15.41	14.74	15.67	10.30
	Sy	18.01	16.34	16.38	17.85	16.68
3	R	.769	.466	.627	.575	.825
	Se	18.81	22.60	20.58	22.98	15.75
	Sy	28.03	24.35	25.18	26.78	26.61
5	R	.805	.525	.734	.551	.850
	Se	29.64	36.18	30.58	36.59	26.26
	Sy	47.61	40.53	42.92	41.82	47.56
10	R	.929	.763	.892	.790	.908
	Se	40.27	55.57	41.81	55.36	45.10
	Sy	103.7	81.92	88.36	86.09	102.86
15	R	.951	.811	.920	.830	.926
	Se	54.53	79.67	66.09	79.41	64.08
	Sy	168.14	129.68	160.79	135.90	161.37

TABLE 8

Summary Statistics for Testing Short-Term Tangborn Models

Forecast Date: June 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
1	R	.893	.640	.766	.753	.931
	Se	6.55	8.96	9.39	7.56	5.37
	Sy	13.86	11.12	13.93	10.96	14.06
2	R	.872	.601	.735	.727	.924
	Se	14.28	18.71	20.01	15.75	11.28
	Sy	27.85	22.31	28.14	21.87	28.20
3	R	.864	.601	.727	.718	.920
	Se	21.89	27.69	30.07	23.52	17.12
	Sy	41.50	33.04	41.79	32.25	41.79
5	R	.865	.599	.729	.705	.928
	Se	35.26	44.25	47.79	37.87	25.90
	Sy	66.99	52.67	66.58	50.90	66.42
10	R	.898	.617	.775	.657	.950
	Se	54.92	75.71	78.91	68.62	37.89
	Sy	118.84	91.72	119.04	86.79	115.79
15	R	.929	.703	.828	.764	.957
	Se	61.47	89.08	94.09	81.83	46.92
	Sy	158.74	119.41	160.12	109.89	154.42

TABLE 8

Summary Statistics for Testing of Short-Term Tangborn Models

Forecast Date: June 15

Length of Forecast (Days)		Data Set # 1	2	3	4	5
1	R	.957	.877	.821	.832	.950
	Se	2.81	3.86	5.74	3.98	2.91
	Sy	9.22	7.66	9.58	6.84	8.91
2	R	.938	.863	.822	.831	.946
	Se	6.91	8.30	11.72	8.27	6.04
	Sy	19.00	15.65	19.64	14.17	17.79
3	R	.915	.836	.838	.809	.947
	Se	11.90	13.07	16.54	12.65	8.62
	Sy	28.08	22.69	28.89	20.52	25.62
5	R	.881	.799	.883	.773	.946
	Se	23.57	23.85	23.88	22.89	14.43
	Sy	47.54	37.83	48.55	34.44	42.37
10	R	.893	.830	.932	.805	.962
	Se	48.62	47.24	39.96	47.17	25.90
	Sy	103.01	80.74	105.28	75.76	90.44
15	R	.883	.828	.918	.793	.971
	Se	76.38	72.08	65.80	74.13	32.17
	Sy	155.37	122.48	158.42	116.03	129.10

TABLE 9

Summary Statistics for Testing of Martinec Models

Forecast Date: May 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
1	R	.971	.965	.977	.961	.990
	Se	1.15	1.28	1.01	1.28	.688
	Sy	4.96	5.10	4.96	4.86	5.15
2	R	.960	.943	.953	.939	.980
	Se	2.64	4.16	3.00	3.17	1.92
	Sy	9.85	9.97	10.3	9.61	10.1
3	R	.940	.902	.922	.895	.958
	Se	4.66	5.85	5.82	5.84	4.08
	Sy	14.29	14.2	15.7	13.7	14.8
5	R	.862	.733	.834	.717	.841
	Se	11.0	14.7	14.4	14.5	13.4
	Sy	22.6	22.6	27.2	21.7	25.8
10	R	.866	.652	.806	.642	.792
	Se	25.9	40.2	37.6	39.8	37.8
	Sy	54.1	55.5	66.3	54.3	64.7
15	R	.921	.721	.860	.709	.867
	Se	36.0	60.4	50.3	62.0	47.8
	Sy	96.4	91.0	102.8	91.8	100.0

TABLE 9

Summary Statistics for Testing of Martinec Models

Forecast Date: May 15

Length of Forecast (Days)		Data Set # 1	2	3	4	5
1	R	.913	.878	.891	.967	.878
	Se	3.52	3.92	3.59	2.26	3.70
	Sy	9.0	8.56	8.27	9.22	8.09
2	R	.778	.790	.725	.940	.698
	Se	10.8	9.60	10.8	5.85	11.4
	Sy	18.0	16.3	16.4	17.9	16.7
3	R	.627	.731	.584	.907	.542
	Se	20.9	15.9	19.6	10.8	21.4
	Sy	28.0	24.3	25.2	26.8	26.6
5	R	.320	.657	.377	.828	.312
	Se	43.2	29.3	38.1	22.5	43.3
	Sy	47.6	40.5	42.9	41.8	47.6
10	R	.792	.599	.640	.754	.686
	Se	60.6	62.8	71.1	54.2	71.7
	Sy	103.7	81.9	96.6	86.1	102.9
15	R	.763	.729	.657	.793	.690
	Se	104.1	85.0	116.0	79.3	111.8
	Sy	168.1	129.7	160.8	135.9	161.4

TABLE 9

Summary Statistics for Testing of Martinec Models

Forecast Date: June 1

Length of Forecast (Days)		Data Set # 1	2	3	4	5
1	R	.992	.949	.988	.964	.990
	Se	1.63	3.34	2.09	2.80	1.88
	Sy	13.9	11.1	13.9	11.0	14.1
2	R	.986	.940	.980	.948	.982
	Se	4.37	7.30	5.42	6.64	5.15
	Sy	27.8	22.3	28.1	21.9	28.2
3	R	.982	.933	.976	.942	.971
	Se	7.45	11.4	8.67	10.4	9.49
	Sy	41.5	33.0	41.8	32.2	41.8
5	R	.973	.916	.984	.936	.964
	Se	14.8	20.3	11.5	17.2	16.9
	Sy	67.0	52.7	66.6	50.9	66.4
10	R	.962	.645	.991	.802	.972
	Se	31.2	67.1	15.3	49.7	26.1
	Sy	118.8	91.7	119.0	86.8	115.8
15	R	.920	.727	.982	.797	.925
	Se	59.5	78.5	29.0	63.6	56.2
	Sy	158.7	119.4	160.1	109.9	154.4

TABLE 9

Summary Statistics for Testing of Martinec Models

Forecast Date: June 15

Length of Forecast (Days)		Data Set # 1	2	3	4	5
1	R	.952	.920	.919	.917	.974
	Se	2.70	2.87	3.61	2.62	1.92
	Sy	9.22	7166	9.58	6.84	8.91
2	R	9.13	.868	.844	.869	.957
	Se	7.43	7.45	10.1	6.71	4.92
	Sy	19.0	15.6	19.6	14.2	17.8
3	R	.869	.803	.769	.821	.941
	Se	13.3	12.9	17.7	11.2	8.33
	Sy	28.1	22.7	28.9	20.5	25.6
5	R	.793	.699	.627	.745	.898
	Se	27.7	25.9	36.2	22.0	17.8
	Sy	47.5	37.8	48.5	34.4	42.4
10	R	.816	.777	.948	.843	.892
	Se	57.0	48.7	32.2	39.1	39.1
	Sy	103.0	80.7	105.3	75.8	90.4
15	R	.759	.848	.951	.842	.869
	Se	96.9	62.2	46.7	60.0	61.1
	Sy	155.4	122.5	158.4	116.0	129.1

it may be that if both models were tested with many more years of data, the second model would yield a correlation equal to or higher than that of the first model. For this reason, some measure of the significance of the differences in correlation is required.

A test is needed to determine the truth of the hypothesis that the two correlations being compared are significantly different. Fisher (1942) has constructed a test of this hypothesis that can be used with samples of moderate size drawn from bivariate normally distributed populations. The test is based on a function known as the Fisher R to Z transformation:

$$Z = \frac{1}{2} \log_e \frac{1+R}{1-R} \qquad (5\text{-}1)$$

in which R is the sample correlation coefficient. The Z values of this function that correspond to various values of the correlation coefficient have been calculated and are presented in Table 10. Determination of the significance of the difference between two correlation coefficients is judged using the test statistic:

$$z = \frac{Z_1 - Z_2}{\left[\left(\frac{1}{N_1-3}\right)+\left(\frac{1}{N_2-3}\right)\right]^{1/2}} \qquad (5\text{-}2)$$

in which Z_1 and Z_2 are the values of the variable given by Eq. (5-1) and corresponding to the two correlation coefficients, N_1 and N_2 are the numbers of observations on which each correlation coefficient is based, and z is the value of a random variable having a standard normal distribution. The test statistic is

TABLE 10

Values of the Fisher R-to-Z Transformation

$$Z = \tfrac{1}{2} \log_e \frac{(1 + R)}{(1 - R)}$$

R	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.5	.549	.563	.576	.590	.604	.618	.633	.648	.662	.678
0.6	.693	.709	.725	.741	.758	.775	.793	.811	.829	.848
0.7	.867	.887	.908	.929	.950	.973	.996	1.020	1.045	1.071
0.8	1.099	1.127	1.157	1.188	1.221	1.256	1.293	1.333	1.376	1.422
0.9	1.472	1.528	1.589	1.658	1.738	1.832	1.946	2.092	2.298	2.646

compared with a critical value obtained from tables for the normal distribution function to determine the significance of the difference in correlation.

In order to conclude that the accuracy of two models is significantly different, a level of significance must be chosen. This level of significance will determine the magnitude of differences in correlation that will be considered significant. The level of significance should be chosen by examining the consequences of accepting an incorrect hypothesis and weighing these consequences against the results of rejecting a correct hypothesis. Such a choice can be made only by those with the knowledge necessary to evaluate the consequences; choice of the proper level of significance is beyond the scope of this investigation. Significance of the results of the testing program must therefore be judged by the reader. Once a level of significance has been selected, the statistical tools presented here may be used to compare the accuracies of the models.

Table 11 shows the values of the test statistic corresponding to various levels of significance. In order to determine whether one model is significantly more accurate than another at the chosen level of significance, the test statistic is computed according to Eq. (5-2); if the resulting value is greater than the value given in Table 11, the difference in model accuracy is deemed significant. For example, the accuracy of the short-term Tangborn model can be compared with the accuracy of the Martinec model for making 3-day predictions with a forecast

TABLE 11

Values of the Test Statistic Corresponding to Various Levels of Significance

$$Z = \frac{Z_1 - Z_2}{\left[\left(\frac{1}{n_1 - 3}\right) + \left(\frac{1}{n_2 - 3}\right)\right]^{\frac{1}{2}}}$$

Level of Significance α =	Minimum Value of Z for Significant Difference
0.01	2.575
0.05	1.960
0.10	1.645
0.20	1.280

date of May 15 using data set #3. The correlation coefficient for the Tangborn model is 0.627; the correlation coefficient for the Martinec model is 0.855. The corresponding Z-values, from Table 10, are $Z_{0.855}=1.275$ and $Z_{0.627}=0.7364$; the test statistic calculated from Eq. (5-2) is:

$$z = \frac{Z_1 - Z_2}{\left[\frac{1}{(N_1-3)} + \frac{1}{(N_2-3)}\right]^{1/2}} = \frac{1.275 - .7364}{[2/9]^{1/2}} = 1.1425 \quad (5\text{-}3)$$

Comparing the test value with the values in Table 11 shows that the difference in correlation is not significant even at the 20 percent level. It must be noted that a difference in correlation of 0.228, such as was tested here, is not always insignificant; significance is a function of the values of the correlations, not just the difference between them. If the first correlation was 0.980, and the difference between the correlations was 0.228, the second correlation would be 0.752. The Z-values would be $Z_{0.980}=2.298$ and $Z_{0.752}=0.978$; the test statistic would equal 2.8, and the difference in correlation would be judged to be significant at the one percent level.

Significance of the Standard Error

A hydrologic basis for comparison of the models would simplify interpretation of the test results. If the prediction given by model A is nearly the same as that given by model B in each of a series of test years, these two models may be considered hydrologically similar. It is possible to determine which pairs of models are hydrologically similar by examining the predicted values for each test year for each model, but this would be very time consuming due to the volume of data. Instead, the standard error of estimate was tested for use as an indicator of hydrologic similarity.

In order to determine whether similar standard errors were indicative of similarity of predictions, the predictions given by a representative sample of the models were compared. Table 12 shows a sample comparison; predictions that were made with the Tangborn model for a 90-day forecast period starting on April 1 are compared with predictions that were made with regression equations for the same data set. The comparison demonstrates that even though the standard errors of estimate of two models are similar, the predictions for individual years are not necessarily similar. Due to the high correlations among the predictor variables for the various regression models, the predictions given by some of these models were similar. However, there was little similarity between predictions made with the Tangborn model and those made with any of the regression models. The Tangborn predictions for short forecast periods were generally not similar to the Martinec results, even when the standard errors were nearly equal. The conclusion is that the standard errors of estimate can not be used to judge hydrologic similarity of individual predictions among the models.

Testing the Correlations

The correlation coefficients were compared several ways.

TABLE 12

Example of Comparison of Predictions of Models With Similar Standard Errors

	Tangborn (Se = 212.6)		Regression With Snowpack Index (Se = 220.8)		Regression With October-March Precipitation (Se = 160.9)	
Observed	Predicted	Error (%)	Predicted	Error (%)	Predicted	Error (%)
975.4	969.0	-0.7	1247.3	+27.9	1018.1	+4.4
1289.7	1572.0	+21.9	1075.3	-16.6	1340.9	+4.0
1731.7	1315.2	-24.0	1652.0	-4.6	1341.8	-22.5
1288.8	1223.8	-5.0	1631.8	+26.6	1270.7	-1.4
1134.9	821.9	-27.6	670.6	-40.9	752.0	-33.7
1085.8	1049.6	-3.3	1105.7	+1.8	944.1	-13.1
1644.9	1580.3	-3.9	1338.4	-18.6	1445.5	-12.1
2527.7	2379.8	-5.9	2684.0	+6.2	2317.2	-8.3
782.7	833.0	+6.4	660.5	-15.6	778.7	-0.5
1506.6	1358.9	-9.8	1793.6	+19.0	1391.8	-7.6
1375.4	966.3	-29.7	1297.9	-5.6	1106.5	-19.6
1142.3	1215.2	+6.4	1287.8	+12.7	892.3	-21.9

In order to judge the significance of differences in correlation, the level of significance was arbitrarily set at 20 percent. Table 13 is arranged in such a way that, given a level of significance and one correlation coefficient, the nearest significantly different correlation value can be read from the table.

Correlations for the various data sets were compared with each other; it was expected that the accuracy of the models would be significantly better when used with data sets #1, 3, and 5, rather than with sets #2 and 4. The correlations for the various lengths of forecast were also compared for each model type, as were the figures for the different forecast dates. These comparisons should indicate the conditions under which each type of model is most accurate. Once the optimum conditions are determined for each type of model, the models may be compared to determine which type is most accurate for each forecast period.

Effect of Data Sets on Model Accuracy

Data sets #1, 3, and 5 were formed by ranking the data years in order of decreasing runoff for various time periods, then splitting the 24-year sample by taking the even-ranked years for calibration and the odd-ranked years for testing. Data sets #2 and 4 were formed by using the low runoff years for calibration and the high runoff years for testing. Thus, when the models were tested with sets #1, 3, and 5, the test data was within the range of data for which the model had been calibrated. With data sets #2 and 4, however, the models were tested with data from outside the range of calibration. In

TABLE 13

Values of Significantly Different Correlation Coefficients for Various Levels of Significance

R_1	R_2 (α = 0.01)	R_2 (α = 0.05)	R_2 (α = 0.10)	R_2 (α = 0.20)
.995	.945	.969	.977	.984
.990	.892	.938	.954	.967
.985	.843	.908	.932	.951
.980	.795	.880	.909	.935
.975	.750	.852	.888	.919
.970	.706	.824	.866	.903
.965	.664	.798	.845	.888
.960	.624	.771	.825	.872
.955	.586	.746	.805	.858
.950	.550	.720	.784	.842
.940	.481	.672	.746	.813
.930	.417	.625	.708	.784
.920	.358	.582	.672	.756
.910	.305	.540	.637	.728
.900	.252	.499	.603	.701
.890	.206	.461	.570	.674
.880	.160	.424	.538	.649
.870	.119	.388	.506	.623
.860	.080	.354	.476	.598
.850	.040	.320	.447	.574
.840	.007	.288	.419	.550
.830	-.026	.258	.391	.526
.820	-.057	.229	.364	.504
.810	.087	.200	.330	.480
.800	-.115	.174	.313	.459

many cases the differences in the resulting correlation coefficients are not significant; but the pattern of the significant differences indicates that the regression models generally were more accurate with sets #1, 3, and 5 than with sets #2 and 4 for forecasts of 90 days or more. The Tangborn model was more accurate with data sets #1, 3, and 5 for all lengths of forecast, especially during June. The Martinec model was more accurate with sets #1, 3, and 5 for all prediction periods during June.

In general, the models were found to be significantly more accurate when tested with data from within the range of data used in calibration. Thus, if the data used for calibration is representative of the watershed hydrology that the basin will experience in future years, then the goodness-of-fit statistics are probably representative of the forecast accuracy. The conclusion is that accuracy of the models is determined by the degree to which the calibration data represents the data base as a whole. If a model is calibrated with only five years of data that is representative of the entire range of data,the resulting equation may be more accurate than a model calibrated with 20 years of non-representative data.

Variation of Accuracy with Forecast Length

A comparison of the correlation coefficients for various lengths of forecast was performed for each model and data set. Results of the regression equations were significantly more accurate for forecast lengths of 60, 90, and 120 days than for 15 and 30 day periods; the same was true of the long-term Tangborn results. When the Tangborn model was used for short-term predictions, the accuracy for the 15 day period was significantly greater than for periods of 1, 2, or 3 days. The Martinec model was significantly more accurate for 1 and 2 day predictions than for longer periods.

Variation of Accuracy With Forecast Dates

The regression models based on the snowpack index were significantly more accurate for a forecast date of April 1 than for May 1 forecasts. The other regressions generally performed equally well for both dates. The short-term models, both Tangborn and Martinec, were less accurate for forecasts made on May 15 than for forecasts made on the other dates. This is due to the fact that the peak flows occur in late May in most years; the forecasts are less accurate at this time because there is greater random variation in the criterion variable. Results of the long-term Tangborn testing did not vary significantly with forecast date.

Differences in Accuracy of the Models

Comparison of the correlations showed that the regression based on snow covered area was usually the least accurate long-term prediction model. It must be remembered that for 19 of the 24 data years, the true values of snow covered area for April 1 were not available, so estimates were generated by a regression equation that was calibrated with only five years of data. This may be the reason for the poor performance of the regression models based on snow covered area.

The regression models that did not include snow covered area generally gave more accurate results than the Tangborn model, especially for forecast periods of 45 and 60 days. The Martinec model was significantly better than the Tangborn for prediction periods of 1, 2, and 3 days except in late June, when the differences in correlation were not significant.

Analysis of Objectives

The testing program was designed to provide answers to the four specific investigations described in Chapter I. Some of these investigations were limited by the type of data available. The results of the investigations are discussed in this section.

The Value of Spatial Separation of a Watershed

In conceptual models, the coefficients and variables that define components of the methods are used to reflect variation in snowmelt and runoff. For example, the first term of the Martinec model represents the runoff generation processes (precipitation and snowmelt) and also the loss processes (evapotranspiration and groundwater storage). Coefficients are included in this term in order to approximate the rates of these processes. On a large watershed with a wide range of elevation, such as the Kings River basin, the physical conditions that control the rates of these processes show significant variation both spatially and temporally. Therefore, it is reasonable to expect that dividing the watershed into smaller, more homogeneous areas and determining separate parameter values for each subarea should improve the accuracy of the predictions. Some of the factors that control the rate of the snowmelt process are air temperature, wind velocity, and vapor pressure deficit. Each of these climatic variables is a function of elevation; therefore, a rational way of subdividing the watershed into smaller, more homogeneous areas is by using elevation zones.

In order to test the hypothesis that accuracy of prediction can be increased by dividing the watershed into smaller areas, the Martinec model was calibrated and tested twice. The first analysis considered the entire watershed as one area, with the temperature index, T, the degree-day factor, d, and the runoff coefficient, c, assumed constant over the watershed. The pattern search method of optimization was used to calibrate the parameter values, and the resulting model, which had the form of Eq. (4-11), was tested using each of the five test data sets. In the second analysis, the area of the watershed was divided into six elevation zones. By May 1 there was no snow cover below an elevation of 3000 feet in any of the years of record. Therefore, the first zone consisted of all points at an elevation below 3000 feet. The rest of the watershed ranges in elevation from 3000 to nearly 13,000 feet and was divided into five zones using an elevation interval of 2000 feet. Different values of temperature were developed for each zone by using a constant lapse rate and the difference between the temperature station elevation and the median elevation of the zone. Both the degree-day factors and the runoff coefficients were allowed to vary from zone to zone and were calibrated using the pattern search method with model:

$$Q_n^* = (1-aQ_{n-1}{}^b)\sum_{i=1}^{6} c_i(d_iT_iSCA_i+P_i)A_i \ + aQ_{n-1}{}^{(b+1)} \tag{5-4}$$

The analyses of Chapter 4 showed that irrational parameter values resulted from the model of Eq. (4-10). When both the runoff coefficient and the degree-day factor were zonal dependent, the resulting values were highly irrational. The runoff coefficient represents the proportion of generated melt that leaves the watershed as streamflow. Due to the longer travel distance of water generated in the upper elevation zones, it was expected that the higher elevation zones should have lower runoff coefficients. The zonal runoff coefficients derived by the pattern search did not vary systematically with elevation, contrary to expectations. Due to this irrationality, the model was re-formulated with a constant runoff coefficient but varying degree-day factors. The resulting model has the form of Eq. (4-25). Pattern search was used to calibrate the model, but the resulting parameter values were still irrational. Some of the degree-day factors were negative, which is not physically possible. Again, the values did not vary systematically with elevation, which indicates that the values are irrational. This irrationality is the result of the insensitivity of the model to these parameters.

Because the more complex forms of the model could not be successfully calibrated, the degree-day factor and runoff coefficient were assumed to be constant from one elevation zone to the next. The resulting model is given by Eq. (4-26). In this model, the temperature and effective precipitation vary from zone to zone, but the degree-day factor and runoff coefficient are the same for all zones. This model was calibrated and then tested using each of the five 12-year test data sets. Comparing the calibration results of this model with the results of calibrating the other zoned models, Eqs. (4-25) and (5-4), indicates that very little statistical accuracy is lost by assuming that the runoff coefficient and degree-day factor are constant over the entire watershed. The differences in the sum of the squares of the errors for the calibration data averaged less than two percent between the lumped and spatially distributed models.

The lumped parameter and spatially distributed models were compared using the 5 test data sets; statistics for the lumped parameter model are given in Table 14, and the statistics for the distributed form of the Martinec model appear in Table 9. The test statistic of Eq. (5-2) is used to compare the correlation coefficients. For the spatially distributed model, the rational coefficients derived from the pattern search calibration of Eq. (4-25) were used. A 20 percent level of significance was used for decision-making. For most cases there were no significant differences. However, for forecast periods of 1, 2, and 3 days that start on May 15, the lumped model results

TABLE 14

Correlation Coefficients for Testing of Lumped Parameter Martinec Model

Forecast Date	Length of Forecast (Days)	Data Set # 1	2	3	4	5
May 1	1	.968	.962	.979	.957	.986
	2	.953	.940	.957	.936	.973
	3	.929	.908	.935	.901	.948
	5	.842	.772	.872	.754	.840
	10	.789	.645	.854	.625	.747
	15	.821	.689	.885	.683	.752
May 15	1	.994	.988	.987	.990	.977
	2	.984	.970	.974	.974	.953
	3	.968	.941	.956	.948	.935
	5	.918	.870	.905	.878	.901
	10	.888	.713	.818	.735	.908
	15	.891	.749	.848	.765	.922
June 1	1	.983	.975	.988	.966	.994
	2	.970	.961	.980	.944	.989
	3	.964	.955	.979	.932	.986
	5	.967	.969	.988	.938	.992
	10	.979	.969	.986	.945	.987
	15	.988	.960	.985	.954	.980
June 15	1	.955	.916	.968	.885	.972
	2	.926	.857	.943	.810	.956
	3	.903	.804	.920	.742	.949
	5	.873	.711	.884	.632	.938
	10	.914	.772	.921	.716	.955
	15	.924	.792	.925	.749	.961

were significantly better than the distributed model results. The lumped model was also superior for 15-day predictions from June 1 and June 15 with data sets 1, 3, and 5. The conclusion is that spatial separation of the model input does not improve the accuracy on this watershed, at least not with the Martinec model as applied in this study; if the models were modified to include a routing term, spatial separation might prove to be more helpful.

The Value of Real-Time Data

Another objective was to evaluate the effect on accuracy of delays in daily data collection. The Martinec model was used to evaluate the effect of delays in the availability of data. This was accomplished by assuming that data from previous days were not available; lag times of 1, 3, and 5 days were tested. The delayed data were the daily snow covered area, temperature, precipitation, and runoff.

In order to forecast runoff, the Martinec model requires predictions of the precipitation, temperature, and snow covered area for the forecast period. The precipitation predictions are based on observations from previous years, not on observations from the previous few days, so a time lag in collection of precipitation data will not affect the accuracy of the model. For prediction periods of greater than five days, the temperature predictions are also derived from past years; but for predictions of less than five days, the temperature prediction is based on the value observed on the previous day. If there is a time lag in collection of the temperature data, the accuracy of the predictions may be lessened. Snow covered area is also predicted from the values observed on the previous days, and a time lag in collection of this data may also affect accuracy.

The Martinec model was tested with delays in data collection of one, three, and five days. All five test data sets were used; the resulting goodness-of-fit statistics are shown in Table 15. In order to determine the effect on accuracy of delays in data collection, the values in Table 15 are compared with each other and with the statistics presented in Table 9 for the Martinec model with real-time data.

Comparison of the correlation coefficients in Tables 15 and 9 indicate that at a significance level of 10 percent there is no effect on accuracy for predictions of 3, 5, 10, or 15 days. For forecast periods of one or two days, the accuracy achieved by using real-time data is significantly higher than the accuracy achieved when there is a three day or five day lag between data measurement and availability. When the time lag is one day, the results are significantly more accurate than with a five day time lag for forecast periods of one or two days.

Accuracy of the Snow Covered Area Model

One of the objectives of this study was to determine whether snow covered area data can be used alone to accurately predict snowmelt runoff. No other data is to be included in the model, so an empirical equation must be developed. Linear regression models relating snow covered area to seasonal runoff were calibrated for both April 1 and May 1, as described in Chapter 4.

TABLE 15

Correlation Coefficients for Testing of Martinec Model with Time Lags of One, Three, and Five Days - Data Set #1

Forecast Date	Length of Forecast (Days)	Time Lag (Days) 1	3	5
May 1	1	.938	.838	.932
	2	.933	.858	.857
	3	.922	.878	.878
	5	.856	.872	.894
	10	.861	.860	.880
	15	.920	.905	.884
May 15	1	.905	.952	.843
	2	.777	.918	.834
	3	.640	.907	.821
	5	.878	.883	.824
	10	.781	.796	.785
	15	.755	.769	.763
June 1	1	.975	.940	.896
	2	.963	.927	.895
	3	.954	.938	.904
	5	.970	.955	.919
	10	.960	.964	.927
	15	.925	.940	.912
June 15	1	.871	.787	.909
	2	.821	.718	.887
	3	.788	.857	.869
	5	.831	.852	.835
	10	.824	.855	.814
	15	.780	.815	.756

TABLE 15

Correlation Coefficients for Testing of Martinec Model with Time Lags of One, Three, and Five Days - Data Set #2

Forecast Date	Length of Forecast (Days)	Time Lag (Days) 1	3	5
May 1	1	.925	.748	.663
	2	.908	.762	.701
	3	.880	.784	.754
	5	.740	.761	.798
	10	.662	.686	.713
	15	.724	.726	.722
May 15	1	.871	.797	.373
	2	.821	.768	.291
	3	.787	.388	.277
	5	.367	.360	.291
	10	.601	.604	.571
	15	.731	.732	.710
June 1	1	.933	.727	.641
	2	.920	.651	.554
	3	.914	.528	.519
	5	.586	.510	.502
	10	.634	.586	.581
	15	.723	.703	.701
June 15	1	.844	.738	.786
	2	.815	.690	.739
	3	.790	.657	.711
	5	.663	.658	.693
	10	.755	.750	.781
	15	.842	.841	.845

TABLE 15

Correlation Coefficients for Testing of Martinec Model with Time Lags of One, Three, and Five Days - Data Set #3

Forecast Date	Length of Forecast (Days)	Time Lag (Days) 1	3	5
May 1	1	.928	.725	.661
	2	.911	.745	.694
	3	.895	.781	.748
	5	.841	.838	.840
	10	.821	.855	.868
	15	.861	.861	.864
May 15	1	.829	.783	.820
	2	.605	.599	.791
	3	.413	.812	.747
	5	.704	.706	.662
	10	.607	.611	.587
	15	.626	.630	.603
June 1	1	.915	.793	.898
	2	.868	.715	.884
	3	.832	.868	.881
	5	.936	.885	.897
	10	.981	.955	.956
	15	.983	.981	.982
June 15	1	.860	.793	.932
	2	.774	.712	.911
	3	.702	.863	.900
	5	.907	.878	.891
	10	.936	.915	.926
	15	.954	.943	.939

TABLE 15

Correlation Coefficients for Testing of Martinec Model with Time Lags of One, Three, and Five Days - Data Set #4

Forecast Date	Length of Forecast (Days)	Time Lag (Days) 1	3	5
May 1	1	.918	.730	.632
	2	.901	.751	.682
	3	.869	.777	.741
	5	.718	.746	.778
	10	.648	.672	.694
	15	.711	.715	.711
May 15	1	.951	.882	.642
	2	.924	.877	.601
	3	.892	.760	.561
	5	.688	.671	.500
	10	.747	.763	.703
	15	.790	.799	.783
June 1	1	.928	.817	.752
	2	.904	.784	.702
	3	.894	.709	.682
	5	.795	.700	.672
	10	.789	.740	.723
	15	.795	.785	.780
June 15	1	.771	.742	.835
	2	.731	.729	.805
	3	.707	.690	.781
	5	.701	.699	.753
	10	.813	.805	.832
	15	.869	.881	.827

TABLE 15

Correlation Coefficients for Testing of Martinec Model with Time Lags of One, Three, and Five Days - Data Set #5

Forecast Date	Length of Forecast (Days)	Time Lag (Days) 1	3	5
May 1	1	.969	.813	.677
	2	.964	.830	.712
	3	.957	.876	.791
	5	.873	.921	.906
	10	.831	.908	.934
	15	.891	.934	.949
May 15	1	.837	.790	.840
	2	.641	.622	.835
	3	.471	.865	.820
	5	.714	.808	.784
	10	.666	.760	.746
	15	.672	.741	.718
June 1	1	.922	.862	.958
	2	.874	.782	.956
	3	.829	.933	.954
	5	.955	.940	.958
	10	.965	.953	.960
	15	.925	.919	.921
June 15	1	.899	.770	.921
	2	.873	.692	.905
	3	.861	.821	.895
	5	.839	.830	.872
	10	.844	.846	.873
	15	.844	.857	.861

Of all the long-term prediction models tested, the least accurate was that based on snow covered area alone. For a forecast date of April 1, the snow covered area model was nearly as accurate as the other regression models; but for May 1 forecasts, the snow covered area model was significantly less accurate than the others. This is true for all data sets and all forecast lengths tested, from 15 to 150 days. The reason for this lack of accuracy is simply that the May 1 snow covered area statistics do not correlate as highly with the runoff volumes as do the predictor variables in the other regression equations.

Length of Record for Calibration

The fourth specific objective of this study was to determine the length of record required for model calibration. The models used in this study could all be calibrated with as little as one year of record, but generally the accuracy of a model can be expected to increase as the number of years used for calibration increases. At some number of calibration years, perhaps eight or ten, the increase in accuracy resulting from adding an additional year is no longer significant. It must be noted, however, that the characteristics of the calibration data years are at least as important as the length of record.

The correlation coefficients resulting from calibration of the regression models and the Tangborn models with each data set appear in Tables 3 and 4. Comparison of the values for the various data sets shows that the higher correlations are usually the result of using data sets #1, 3 and 5 for calibration, rather than sets #2 and 4. This implies that the choice of years used for calibration may be more important than the number of years used.

Each of the models was also calibrated using all 24 data years; generally, the results of this calibration were not significantly better (at the 20 percent level) than the results of calibrating with sets #1, 3, and 5. Therefore, 12 years seems to be an adequate length of record for calibrating the models as long as the 12 years are representative of the entire range of data. Any set of data years that constitutes a representative sample of the population of all possible data years should be sufficient for calibration; the number of years required is dependent on the characteristics of the population.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions result from the comparisons discussed above:

1. Accuracy of the Tangborn model and the regression models is greater if the test data falls within the range of calibration data than if the test data lies outside the range of calibration data.
2. The regression models are significantly more accurate for forecasts of 60 days or more than for shorter prediction periods.
3. The Tangborn model is more accurate for forecasts of 90 days or more than for shorter prediction periods.
4. The Martinec model is more accurate for forecasts of one or two days than for periods of 3,5,10, or 15 days.
5. Accuracy of the long-term models seems to be independent of forecast date; exceptions are the snowpack index regression model and the snow covered area regression model, both of which are more accurate for April 1 forecasts than for May 1 forecasts.
6. The short-term models are least accurate for forecast periods in late May and early June; this coincides with the period of peak flow for most years.
7. The Martinec model is the best of those tested for one and two day forecasts.

8. With the exception of the snow covered area model, the regression models are all roughly equal in accuracy; these regressions are the most accurate of all models tested for forecasts of 60 days or more.
9. Spatial separation of the watershed by elevation zones does not improve the accuracy of the Martinec model on the Kings River Basin.
10. Delays in data collection of more than one day may significantly lessen the accuracy of the Martinec model; real-time data is desirable.
11. The regression model using only snow covered area as input data is not as accurate as the other regression models for May 1 forecasts.
12. The sufficiency of a calibration data base is a function not only of the number of years of record but also of the accuracy with which the calibration years represent the total population of data years. Twelve years appears to be a sufficient length of record for each of the models considered here, as long as the twelve years are representative of the population.

Recommendations

All of the conclusions listed above were drawn from the results of testing the models on just one watershed. The Kings River basin is very large, and many of the conclusions may not be true for smaller watersheds. Therefore, testing of the models on at least one small watershed is recommended.

The Kings River basin was chosen for this study mainly because snow covered area data had been collected for many years. This data may have been collected for many other watersheds in the western U.S. as well, but there doesn't appear to be any one agency that can supply pre-Landsat snow covered area data for a variety of watersheds. Apparently, most of the snow cover data from before 1973 was not published. It would be much easier to perform studies such as this one if some control agency were established that would compile and supply all the hydrologic data that has been collected over the years. If all of these statistics were assembled in one spot, the task of organizing a data base for a study such as this one would be greatly facilitated.

As discussed in Chapter 5, there seems to be no accepted method for judging the hydrologic similarity of two models. A basis of comparison would be very useful for interpreting the results of studies such as this one. If two models give comparable errors in each of a series of test years, only one of the models must be studied because both models represent the same relationships of cause and effect between the input values and the predictions. Discriminant analysis may prove to be the best way of comparing the test results of various models. It would be useful to perform such an analysis of the results of this study.

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