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PRELIMINARY DESIGN OF
LARGE REFLECTORS WITH FLAT FACETS

By
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ABSTRACT

A concept for approximating curved antenna surfaces using flat facets is discussed. A preliminary design technique for determining the size of the reflector surface facets necessary to meet antenna surface accuracy requirements is presented. A proposed large Microwave Radiometer Satellite is selected as an application, and the far-field electromagnetic response of a faceted reflector surface is compared with that from a spherical reflector surface.

INTRODUCTION

Recent research studies [1], [2] indicate increased interest in the feasibility of assembling and operating large-aperture antennas in space. A challenging potential application of a large space antenna system is the Microwave Radiometer Satellite (MRS) [3], [4]. When orbiting at an altitude of 650 km and operating at a frequency of 1 GHz, this antenna system is required to produce simultaneously 200 contiguous beams each having a footprint diameter of 1 km on earth. This resolution requirement plus the requirement for 90 percent beam efficiency yields a reflector with a 1150m spherical radius and an aperture diameter of 660m.

A typical structural concept for such a system is shown in Fig. 1. The concept consists of a large structural truss network with a mesh or membrane reflector surface. The triangular arrangement of nodes shown in Fig. 1 is typical of many proposed concepts for large space structures because of desirable structural characteristics. Of interest in the present paper is how the structural nodal pattern might be used to support flat

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membrane facets that would satisfactorily approximate the desired reflector surface. One attractive approach for erecting a large reflecting antenna in space is to assemble an array of deployable truss modules each of which supports a reflecting facet as depicted in Fig. 2.* Near-optical flatness of these facets in this reflector concept can be achieved by tensioning a membrane with as few connections to the truss as possible [5]. A nearly triangular membrane facet is shown in Fig. 3 in which mirror-quality flatness has been achieved using only three tensioned-cable attachments.

Antenna accuracy requirements are often specified by the root-mean-square deviation ($\delta_{\text{rms}}$) of the actual surface from the desired surface. The purpose of the present paper is to present a preliminary design technique for determining the size of the reflector surface facets required to meet specified antenna surface accuracy requirements. Only the regular but known deviation of the flat facet from the desired surface is considered and errors from other sources are not included. Equations and generalized design curves are presented in the paper for both triangular and hexagonal surface facets which approximate spherical or paraboloidal surfaces. The proposed Microwave Radiometer Satellite reflector is chosen as an illustrative example, and the far-field electromagnetic response of reflector surfaces with either triangular or hexagonal facets is compared with that from a spherical reflector surface.

REFLECTOR SURFACE ERRORS

Faceted Surface Geometry

To subdivide a surface of revolution the geometric scheme proposed in [6] was adopted. The mapping procedure is illustrated in Fig. 4. A surface of revolution is divided at the aperture circle into $N$ equal segments. A line connecting these points to the center of the reflector forms the $N$ sided pyramid shown in the upper part of Fig. 4. Each side of the pyramid is then subdivided into $M$ equal parts (bays) to form subelements. Finally, the points of intersection of these triangles are projected or mapped on the surface using a suitable origin of coordinates to obtain the final nodal coordinates of the members. It was found [6] that for shallow reflectors six aperture divisions ($N=6$) and a center of projection at the center of curvature for the point at the center of the reflector resulted in nearly uniform triangular facets that were close to having equal sides. This method of subdivision was used in all the studies of this report.

*A patent disclosure on this "Deployable Module for Constructing Large Surfaces" has been filed at the Langley Research Center by H. Bush, M. Mikulas, Jr., and R. Wallsom.
Root-Mean-Square Surface Deviation

To determine the acceptable size of a facet for preliminary antenna reflector designs, the root-mean-square deviation ($\delta_{\text{rms}}$) of the flat triangular surfaces from the ideal reflector surface is employed. Equations for determining $\delta_{\text{rms}}$ for a general triangle with vertices located on a sphere or paraboloid are given in the Appendix.

An illustrative preliminary design result for the Microwave Radiometer Satellite reflector is shown in Fig. 5. In this example, a spherical reflector was investigated with six circumferential subdivisions ($N=6$) to obtain nearly equal-sided facets. The surface deviation is shown in Fig. 5 for the number of bays $M$ ranging from 13 to 25. The vertical bar for each $M$ indicates the range of discrete member lengths and is plotted at the $\delta_{\text{rms}}$ for that configuration. The geometry of the surface with $M=16$ is shown in Fig. 6.

Simplified Facet Sizing Equations

If the reflector under consideration is shallow, the facets tend toward equilateral triangles and the principal curvatures are nearly equal for any surface of revolution. Thus the $\delta_{\text{rms}}$ calculation for an equilateral triangle on a spherical surface should be a good approximation for the actual geometry. In this case the equations of the Appendix are greatly simplified. The maximum deviation $H$ of a sphere of radius $R$ from the plane of an equilateral triangle with vertices on the sphere is

$$H = \frac{1}{6} \frac{L^2}{R}$$

where $L$ is the length of one side of the triangle. The effective sphere is defined as that which minimizes the $\delta_{\text{rms}}$ with respect to the triangular facet. As shown in the appendix the distance between the sphere containing the vertices and the effective sphere is $3/4 \, H$. The corresponding $\delta_{\text{rms}}$ is

$$\delta_{\text{rms}} = \frac{1}{8\sqrt{15}} \frac{L^2}{R}$$

Thus, the member length required to satisfy a certain $\delta_{\text{rms}}$ requirement is then
where \( F \) is the focal length of the reflector taken as one-half the radius of curvature at the center of the reflector and \( D \) is the aperture diameter. The general design curve contained in Eq. (3) is plotted in Fig. 7(a). Assuming the reflector is shallow \((F/D > 0.5)\) the number of bays \( M \) (assuming \( N=6 \)) is given approximately by

\[
M = \frac{1}{\frac{F}{D}}
\]

(4)

The total number of members \( N_m \) in the reflector is given in [6] by

\[
N_m = 9M^2 + 3M
\]

(5)

and the total number of triangular facets \( N_f \) is

\[
N_f = 6M^2
\]

(6)

Figure 7(b) presents generalized curves for the number of members and facets based on Eqs. (5) and (6).

For the Microwave Radiometer Satellite, the accuracy of the design formula (Eq. (3)) is indicated in Fig. 5, where results from Eq. (3) are seen to be in the middle of the range of actual member lengths for each value of number of bays \( M \). Thus, the accuracy of estimated facet size presented in Eq. (3) is adequate for preliminary design.
Hexagonal and Square Facets

Even though identical regular polygons cannot in general be mapped onto a curved surface, the results of Fig. 5 show for the triangle the assumption of equal sides provide the basis for a good estimate of required member length to achieve a given $\delta_{\text{rms}}$. Making a similar assumption for the square and hexagon results in an equation similar to Eq. (3) as summarized in the following table.

<table>
<thead>
<tr>
<th>FACET TYPE</th>
<th>$C$</th>
<th>FORMULA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hexagonal</td>
<td>4.046</td>
<td>$\frac{L}{D} = C \sqrt{\frac{\delta_{\text{rms}}}{F}}$</td>
</tr>
<tr>
<td>Square</td>
<td>6.160</td>
<td></td>
</tr>
<tr>
<td>Triangular</td>
<td>7.872</td>
<td></td>
</tr>
</tbody>
</table>

For equal-sided hexagonal facets, it can be shown also that

$$M = \frac{1}{3} \left( \frac{1}{\frac{L}{D}} + 1 \right)$$

$$N_m = 3M^2 - 3M$$

and

$$N_f = 3M^2 - 3M + 1$$

The required number of members and facets for hexagonal and triangular facets are compared in Fig. 7(b). For the same reflector accuracy requirements, a larger number of hexagonal members is required and each member is shorter than its triangular facet counterpart (Eq. (7)).

ELECTROMAGNETIC PERFORMANCE

Ideal Reflector

The 200 beams mentioned in the Introduction are obtained by stacking 200 identical feed horns along a circular arc that is concentric to a spherical reflector (Fig. 8(a)). Each feed horn is pointed along the radius
of curvature and thus illuminates a part of the reflector. Due to the spherical symmetry, each of these illuminated spots on the reflector is essentially identical. Dimensions corresponding to one illuminated spot on the reflector are shown in Fig. 8(b). The feed is a z-polarized circular corrugated horn. Its far-field radiation pattern is essentially axisymmetric and has approximately a cosine-squared behavior as shown in Fig. 9. The edge illumination for a spot on the reflector turns out to be about -27 dB with respect to the peak illumination at the center of the spot.

To compute the secondary far-field radiation pattern [7], first, the rays are traced from the feed horn to the reflector and then to an aperture plane located in front of the reflector. Next, using equations of geometric optics for each ray, the electric field is found at many points in the aperture plane and by numerically performing a double integration over the aperture plane the secondary far-field radiation pattern is computed.

Assuming that the reflector surface is perfectly spherical, the computed secondary far-field pattern for the configuration of Fig. 8(b) is shown in Fig. 10 for the xz-plane. The secondary pattern is also axisymmetric. The beam efficiency of the secondary pattern at two and a half beam widths away from the main beam is about 94 percent which upon including the effect of spillover energy from the feed corresponds to an overall beam efficiency of about 91 percent.

Reflector with Triangular Facets

Now let the reflector surface be generated from flat equilateral triangular facets. If the permissible surface error is assumed to be 0.012λ (λ/25 for a frequency of 1 GHz), then the use of the geometry of Fig. 8 and Eq. (3) yields a facet member length of 20.7m. The secondary far-field pattern for the faceted configuration is shown in Fig. 11. The results from the ideal spherical reflector are also shown. Note from the insert in Fig. 11 that the geometric pattern is different in the y and z directions which results in different far-field patterns in the two directions. The surface deviation has a three-fold symmetry along the axis, and the average period of its variation along the xz-plane (E-plane) is smaller than the average period of variation along the xy-plane (H-plane). In other words, the surface deviation in the xz-plane has a smaller effective period than in the xy-plane. Apparently, as a result of this, as shown in Fig. 11, not only do the far-field patterns in the E- and the H-plane differ, but the pattern in the xz-plane (E-plane) has higher side lobes than in the xy-plane (H-plane). It appears therefore, that it is not the magnitude of the RMS surface deviation alone, but also the way in which this deviation varies over the reflector surface that determines its effect on the secondary far-field pattern.
Comparison of Reflectors Made From
Square, Triangular, and Hexagonal Facets

The points made at the end of the preceding paragraph are further
illustrated in Fig. 12 where the computed far-field radiation patterns in
the H-plane (xy-plane) are plotted for the same reflector when it is made
out of planar hexagonal, triangular, and square facets such that the RMS
surface deviation in each case is the same (=λ/25). In this case, as
shown in Fig. 7(b), there will be fewer hexagons than triangles. Thus,
the average period of surface deviation variation in the hexagonal case is
slightly larger than in the triangular case. Or, equivalently, the surface
deivation is more slowly varying over the reflector approximated by planar
hexagons than over the reflector approximated by planar triangles.
Apparently as a result, the pattern for the reflector made out of hexagonal
facets is slightly better than that for the reflector made out of
triangular facets. However, for the reflector surface made out of square
facets, the pattern has quite high side lobes. This is attributed to the
regularity of the surface deviation. Calculations made at angles from the
reflectors axis greater than shown in Fig. 12 indicated presence of
grating side lobes which would be of significance to the MRS application and
would require further investigation.

CONCLUSIONS

A concept for approximating curved reflector surfaces using flat surface
facets has been discussed. A preliminary design technique for determining
the size of the reflector surface facets required to meet antenna surface
accuracy requirements has been presented. Results suggest that practical-
sized facets which meet stringent accuracy requirements are feasible.

Electromagnetic analysis of far-field patterns of a large spherical
radiometer reflector has been performed. Results for square, hexagonal and
triangular faceted reflector surfaces suggest that the far-field patterns
are governed by both the magnitude of the root-mean-square surface
deivation and the variation of the surface deviation over the reflector.
For the same root-mean-square deviation, it appears that more slowly
varying periodic deviation cause less deterioration of the far-field than
those with shorter periods. Also, slightly irregular facet geometries seem
to result in better side lobe behavior than more regular patterns. For the
three types of facets considered herein, the main beam and first side lobe
of a perfect spherical reflector were closely approximated by all three
faceted reflectors. However, for side-lobes beyond the first lobe,
deterioration occurs with hexagonal facets showing the best performance and
triangular facets a close second. The presence of grating side lobes was
noted but the investigation of their significance is beyond the scope of
this paper.
For large space antennas, triangular facets, which are desirable structurally, lead to the fewest number of structural members to meet accuracy requirements, and have acceptable electromagnetic performance. Considering all their factors, they appear to be the best configuration investigated for approximating curved reflector surfaces.
APPENDIX

SURFACE ERROR OF A GENERAL TRIANGLE WITH VERTICES ON A SURFACE OF REVOLUTION

In this appendix exact equations are derived to determine the root mean square deviations of a general triangle from a parabolic or spherical surface. These equations can be used to estimate surface errors for deep reflectors ($F/D < 1/2$) and for shallow shells can be shown to reduce to the approximate equations contained in the main text.

Consider the general triangle shown in Fig. 13 located on a surface of revolution with vertices located at $x_i, y_i, z_i, (i = 1, 3)$. A vertical plane $P$ containing the $x$-axis and the centroid of the triangle forms and angle $\theta$ with respect to the $y$-axis. Now consider a two-dimensional coordinate system $(\xi, \eta)$ in the plane of the triangle with origin at $x_1, y_1, z_1$, and the $\xi$ axis parallel to the intersection of plane $P$ with the plane of the triangle: The centroid of the triangle is at

$$y_c = \frac{1}{3}(y_1 + y_2 + y_3)$$

$$z_c = \frac{1}{3}(z_1 + z_2 + z_3)$$

The angle between the $x$-axis and the $\xi$-axis is

$$\chi = \cos^{-1} \frac{B y_c + C z_c}{\sqrt{(B y_c + C z_c)^2 + y_c^2 + z_c^2}}$$

where $B$ and $C$ are constants defining the plane of the triangle

$$x = A + By + Cz$$
The angle \( \theta \) is given by

\[
\theta = \tan^{-1} \left( \frac{z_i}{y_c} \right)
\]

The coordinates \( \xi_i \) can now be determined as

\[
\xi_i = \left[ (y_i - y_1) \cos\theta + (z_i - z_1) \sin\theta \right] \frac{1}{\sin\gamma}
\]

From this it follows (Fig. 13) that

\[
\eta_i = \pm \sqrt{\xi_i^2 - \xi_1^2}
\]

The planar coordinates of all the vertices are now known for the general triangle. Using a shallow shell approximation the equation of the surface of revolution can be written for the deviation from the \( \xi,\eta \) plane as

\[
w = a + b\xi + c\eta + \frac{1}{2R\xi} \xi^2 + \frac{1}{2R\eta} \eta^2
\]

where \( a \) is the normal distance from the points \( \xi_i, \eta_i \) to the surface representing the effective reflector surface. In order to minimize the r.m.s. error, \( a \) is varied and \( w(\xi_i, \eta_i) = a \). The remaining constants are then

\[1\]
\[ b = \frac{1}{4S} (n_3 d_1 - n_2 d_3) \]

\[ c = \frac{1}{4S} (\xi_2 d_3 - \xi_3 d_2) \]

\[ S = \frac{\xi_2 n_3 - \xi_3 n_2}{2} \quad \text{area of triangle} \]

\[ d_i = \frac{\xi_i^2}{R_{\xi}} + \frac{n_i^2}{R_{\eta}} \quad \text{(A8)} \]

\[ R_\xi \quad \text{radius of curvature in meridian direction} \]

\[ R_\eta \quad \text{radius of curvature in circumferential direction} \]

The rms error is found from the following integral:

\[ \phi = \iint w^2 d\xi d\eta \]

\[ = S [a^2 \frac{ef}{\xi} + \frac{f^2}{120} - \frac{S^2}{90R_\xi R_\eta}] \quad \text{(A9)} \]
where \( r = \frac{\xi_2^2 - \xi_2 \xi_3 + \xi_3^2}{R_\xi} + \frac{\eta_2^2 - \eta_2 \eta_3 + \eta_3^2}{R_\eta} \)

then

\[
\delta_{\text{rms}} = \sqrt{\frac{\mathcal{Q}}{s}}
\]  

(A10)

Application to a Sphere

For a sphere the following equations apply:

\[ R_\xi = R_\eta = R = \text{constant} \]

A = \delta = \text{constant}  

(A11)

Though it is impossible to define a faceted surface with equilateral triangles that have vertices lying on a sphere (except for small numbers of facets) the geometry resulting from the subdivision used in Ref. 7 results in very nearly equilateral triangles for practical reflectors. For the case of an equilateral triangle:

\[
\xi_2 = \xi_3 = L \sqrt{\frac{3}{2}}
\]

(A12)

\[-\eta_2 = \eta_3 = L/2\]
This expression can be minimized with respect to \( a \) to yield

\[
\phi = \frac{\sqrt{3} L^2}{4} \left[ a^2 - \frac{aL^2}{4R} + \frac{1}{60} \frac{L^4}{R^2} \right]
\]

The minimum occurs at \( a = \frac{1}{3} \frac{L^2}{R} \) which is \( 3/4 \) of the maximum distance the sphere and the plane of the triangle.

Application to Parabolic Surface

The equation of the parabolic surface containing the vertices of the triangular facets is

\[
y^2 + z^2 = 4F (x_0 - x)
\]

The equation defining the effective paraboloid is

\[
y^2 + z^2 = 4F' (x_0 - x - \delta)
\]

where \( F' \) and \( \delta \) are varied to minimize \( \delta_{\text{rms}} \).
The relation between \( \delta \), \( F' \) and the parameter \( a \) that appears in \( \delta_{\text{rms}} \) can be obtained by passing a line through the centroid of the triangle normal to its plane. The equation of that line in a plane passing through the centroid in terms of \( r (r^2 = x^2 + y^2) \) and \( x \) is

\[
x = x_c + \frac{(r - r_c)}{\sqrt{B^2 + c^2}}
\]  

(A16)

Solving this equation and equation (A14) yields the solution for the radius \( r_q \) at which the line pierces the parabolic surface as

\[
r_q = -2F\frac{F}{\sqrt{B^2 + c^2}} + 2\sqrt{\frac{F^2}{B^2 + c^2} + F\left(x_0 - x_c + \frac{r_c}{\sqrt{B^2 + c^2}}\right)}
\]  

(A17)

Similarly the point that pierces the effective parabolic is

\[
r_p = -2F'\frac{F'}{\sqrt{B^2 + c^2}} + 2\sqrt{\frac{F'^2}{B^2 + c^2} + F\left(x_0 - x_c - \delta + \frac{r_c}{\sqrt{B^2 + c^2}}\right)}
\]  

(A18)
then

\[ x_q = x_0 - \frac{r_q}{4F} \]

\[ x_p = x_0 - \frac{r_p}{4F} \]

\[ a = \sqrt{(x_p - x_q)^2 + (r_p - r_q)^2} \]  

(A19)

The radii of curvature for the parabola are

\[ R_\xi = 2F \left[ 1 + \left( \frac{r_c}{2F} \right)^2 \right]^{3/2} \]

and

\[ R_\eta = r_c \sqrt{1 + \left( \frac{2F}{r_c} \right)^2} \]
Procedure For Numerical Calculations

It is necessary to determine the value of $\delta$ and for a parabola, $F'$, that will minimize $\delta_{\text{rms}}$ in order to establish the effective reflector surface. For a real surface all lengths are slightly different and the general procedure must be used. A starting value of $\delta$ is taken as $3/4$ of the maximum deviation found between any triangular facet and the desired surface. The general expression for $\phi$ (Eq. A9) is evaluated for each facet and summed over all facets. The results divided by total area is $\delta_{\text{rms}}^2$. Variations of $\delta$ generally show the initial value chosen yields minimum $\delta_{\text{rms}}$ except for very deep reflectors. For parabolic surfaces the initial $F'$ is taken as $F$ and for practical reflectors this was found to yield a minimum $\delta_{\text{rms}}$.
REFERENCES


(a) Modular element with membrane surface facet

Fig. 2 Modular Antenna
Fig. 3  Tensioned triangular membrane facet
Fig. 4  Geometry scheme for subdividing reflector surface
Fig. 5  Surface errors and facet sizes for various subdivisions of MRS reflector
(a) Member lengths

Fig. 7 Generalized design curve for reflector surface with flat facets
(b) Members and Facets

Fig. 7 Concluded
Fig. 8
Geometry of MRS spherical reflector antenna

(a) Overall Dimensions

Center of Curvature

θ = 0.088°

8.8°

Feed Arc

305 m

15°

Feed Horns

Spherical Reflector

1150 m

577 m

660 m
(b) Detail of one illuminated spot on reflector

Fig. 8 Concluded
Fig. 9  Radiation pattern of circular corrugated feed horn
Fig. 10  Secondary far-field pattern of ideal spherical reflector
Fig. 11  Secondary far-field pattern of reflector having flat triangular facets
Fig. 12  
Comparison of secondary far-field patterns for reflectors having flat square, triangular or hexagonal facets
Fig. 13  Geometry of general triangle located on surface of revolution
A concept for approximating curved antenna surfaces using flat facets is discussed. A preliminary design technique for determining the size of the reflector surface facets necessary to meet antenna surface accuracy requirements is presented. A proposed large Microwave Radiometer Satellite is selected as an application, and the far-field electromagnetic response of a faceted reflector surface is compared with that from a spherical reflector surface.