FINAL REPORT
High Pressure Laser Plasma Studies
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I. INTRODUCTION

A nuclear pumped laser, operating at a wavelength of 1.79 \( \mu \text{m} \) on the 3d(1/2)-4p(3/2) transition in argon with \(^3\text{He}\) as the majority gas has been produced by Jalufka et. al. at NASA-Langley Research center.\(^1\) A model, proposed by Harries and Wilson, to explain the operation of this laser uses collisional radiative recombination of \(\text{Ar}^+\) as the population mechanism of the 3d(1/2) state.\(^2\) This model also proposes dissociative recombination of \(\text{Ar}_2^+\) as a population mechanism for the 4p(3/2) state.

Work done by Biondi et. al. at the University of Pittsburgh confirms that dissociative recombination populates the 4p level but indicates that dissociative recombination dominates over collisional radiative recombination in pure argon at low pressures.\(^3\) As the laser developed at NASA-Langley operates in the high-pressure regime, calculations have been performed by DePaola et. al. (see Appendix 1) extrapolating well-known low-pressure results to the high-pressure regime. From the results of these calculations there is some question in predicting laser operation at 1.79 \( \mu \text{m} \) in a \(^3\text{He}-\text{Ar}\) plasma.\(^9\)

The purpose of this study is the investigation of the
energy pathways in He-Ar gas mixtures at low pressure is accomplished by observing the effects of varying partial pressures on the emissions of levels lying above the 4p level in argon during a pulsed afterglow. An attempt was made to investigate the population mechanisms of the 3d level in pure argon by observing emission from the 3d(1/2)-4p(3/2) transition in a high-pressure plasma excited by a high-energy electron beam.
II. THEORY

A nuclear pumped, $^3$He-Ar laser, excited by the $^3$He(n,p)$^3$H reaction, was created by Jalufka et. al. in 1976.\(^1\) The laser operates at a wavelength of 1.79 μm. on the 3d(1/2)-4p(3/2) transition in atomic argon. A model for the laser, proposed by Harries and Wilson, assumes that the population mechanism of the 3d(1/2) state is collisional radiative recombination of Ar\(^+\) and that dissociative recombination of Ar\(_2\) populates the 4p(3/2) state.\(^2\)

A flow diagram, illustrating the energy pathways proposed, is shown in figure 1. The greater part of the fission energy produces helium ions, He\(^+\), and excited helium atoms in a metastable state, He(2\(^3\)S). Some of the He(2\(^3\)S) is converted to a molecular helium metastable, He\(_2\)(2\(^3\)Σ). Argon ions are created from both atomic and molecular helium metastables by Penning ionization. The helium ions are converted to molecular helium ions, He\(_2^+\), which also serve to ionize the argon by charge transfer collisions. The ionic argon may recombine by collisional radiative recombination or be converted to Ar\(_2^+\). The products of collisional radiative recombination cascade through excited states and may produce a population inversion between the 3d(1/2) and 4p(3/2) states. Molecular argon ions undergo dissociative recombination which
Figure 1. Flow Diagram of Processes Important to the $^3$He-Ar Laser.

Dissociative recombination is assumed to populate the lower laser level.
He\textsuperscript{+} + e\textsuperscript{-} \rightarrow He \textsuperscript{+} + 2He \rightarrow He\textsubscript{2}\textsuperscript{+}

He\textsuperscript{+} \rightarrow He\textsuperscript{*}

He\textsuperscript{+} + e\textsuperscript{-} \rightarrow He\textsuperscript{*} + He \textsuperscript{+}

He\textsuperscript{*} \rightarrow 1.79 \mu m Laser

Ar\textsuperscript{+} + e\textsuperscript{-} \rightarrow Ar\textsuperscript{*} + Ar \textsuperscript{+}

Ar\textsuperscript{+} + e\textsuperscript{-} \rightarrow Ar\textsuperscript{*} + Ar \textsuperscript{+}

Ar\textsuperscript{*} \rightarrow Ar\textsubscript{2}\textsuperscript{+} + 2Ar

Ar\textsuperscript{+} \rightarrow Ar\textsubscript{2} + e\textsuperscript{-}
is presumed to populate the 4p(3/2) state thus becoming a source of population for the lower laser level.

The assumption that dissociative recombination populates the lower laser level is supported by the work of Biondi and Shiu which shows that the 4p states are the most strongly populated. Experiments performed by Frommhold and Biondi, in 1969, indicate that dissociative recombination dominates over collisional radiative recombination in pure argon at low pressure.

Calculations performed by DePaola et al. (see Appendix 1) provide a characterization of high-pressure 3He-Ar plasmas based on extrapolation of the knowledge of similar low-pressure plasmas. Results of these calculations for a 3He-Ar plasma at a total pressure of 2 atmospheres with a 1% concentration of argon and a thermal neutron flux of $10^{18}$ neutrons cm$^{-2}$ sec$^{-1}$ are quoted here:

- Gas Temperature $T_o = 300$ K
- Electron Temperature $T_e = 536$ K
- Electron Density $N_e = 2.08 \times 10^{14}$ cm$^{-3}$
- $\text{Ar}^+$ Density $\{\text{Ar}^+\} = 7.57 \times 10^{13}$ cm$^{-3}$
- $\text{Ar}_2^+$ Density $\{\text{Ar}_2^+\} = 1.08 \times 10^{14}$ cm$^{-3}$

Collisional radiative rate coefficients for the minority gases in various helium-dominated gas mixtures have been calculated by Whitten et al. For a He-Ar mixture at 2 atmospheres total pressure the rate coefficient is:

$$\alpha_{\text{CRR}} = 2.55 \times 10^{-9} \text{ cm}^3 \text{ sec}^{-1}$$

and the rate coefficient for dissociative recombination of $\text{Ar}_2^+$ is:

$$\alpha_D = 9.1 \times 10^{-7} (T_e / T_o)^{-0.61} \text{ cm}^3 \text{ sec}^{-1}$$
The assumption may be made that all of the products of collisional radiative recombination pass through the 3d level and are distributed according to the densities of the individual states within the level. The same assumption is made with respect to the products of dissociative recombination cascading through the 4p level. The rates of population of the two states are thus:

\[
\frac{d\{Ar(3d(1/2))\}}{dt} = \frac{2}{20} \alpha_{CRR} \{Ar^+\} N_e = 4.02 \times 10^{18} \text{ cm}^{-3} \text{ sec}^{-1}
\]

\[
\frac{d\{Ar(4p(3/2))\}}{dt} = \frac{4}{12} \alpha_{DR} \{Ar^+\} N_e = 4.78 \times 10^{21} \text{ cm}^{-3} \text{ sec}^{-1}
\]

The rate of population of the lower laser level is thus seen to be three orders of magnitude greater than that of the upper laser level and laser action is not predicted in this model.

The characteristics of an existing 3He-Ar laser may be used to predict the value of the collisional radiative recombination rate coefficient necessary to produce lasing. A laser described by DeYoung et al. operates at 2 atmospheres total pressure with a 1% argon concentration. The peak power output of the laser is 3.4 Watts for a thermal neutron flux of \(10^{17}\) neutrons-cm\(^{-2}\)-sec\(^{-1}\). The efficiency of the laser is reported to be 0.09%. The power deposited in a laser of this type is:

\[
\xi = \sigma \phi (3\text{He}) E_F V
\]

where \(\sigma = 5 \times 10^{-21}\) cm\(^2\) is the fission cross-section of \(^3\text{He}\), \(\phi\) is the thermal neutron flux, \(E_F = 0.79\) Mev is the energy released per fission event and \(V\) is the active volume. The
power output is given by:
\[
\xi_0 = \varepsilon \xi_i = \varepsilon \phi \{^3\text{He}\} E_F V
\]
where \( \varepsilon \) is the efficiency of the laser. Thus the active volume is given by:
\[
V = \frac{\xi_0}{\varepsilon \phi \{^3\text{He}\} E_F} = 1.162 \text{ cm}^3
\]
The peak power output of 3.4 Watts corresponds to \( 3.06 \times 10^{19} \) photons-sec\(^{-1} \) at 1.79 \( \mu \text{m} \). As the \(^3\text{He}-\text{Ar} \) laser is a steady-state laser the source term for the population of the 3d(1/2) state must be equal to the loss term. Using the model of Harries and Wilson this requirement becomes:
\[
\alpha_{\text{CRR}} N_e \{\text{Ar}^+\} V = 3.06 \times 10^{19}
\]
or:
\[
\alpha_{\text{CRR}} = \frac{3.06 \times 10^{19}}{N_e \{\text{Ar}^+\} V}
\]
The computer program used by DePaola et. al. was modified slightly (see Appendix 2) to calculate the electron and ion densities of a \(^3\text{He}-\text{Ar} \) plasma at 2 atmospheres with a thermal neutron flux of \( 10^{17} \) neutrons-cm\(^{-2}\)-sec\(^{-1} \). The results are:
\[
\text{Electron Density } N_e = 5.53 \times 10^{13} \text{ cm}^{-3}
\]
\[
\text{Ar}^+ \text{ Density } \{\text{Ar}^+\} = 1.81 \times 10^{13} \text{ cm}^{-3}
\]
And the required collisional radiative rate coefficient is:
\[
\alpha_{\text{CRR}} = 2.63 \times 10^{-8}
\]
a full order of magnitude greater than that calculated by Whitten et. al.\(^5\)
III. EXPERIMENTAL PROCEDURES AND EQUIPMENT

The gas mixtures of interest in this study were excited by a pulsed electrical discharge. Pulse duration was 4.5 ±0.05 μsec with a peak current of 40±5 amps (see figure 7). Electrode spacing was 7.5 cm in a glass discharge tube with quartz windows. The discharge tube was 2.5 cm in diameter and 1 m long.

Figure 2 shows a block diagram of the vacuum and gas-handling system. The pumping system consists of a 4-inch oil diffusion pump with a liquid nitrogen cold trap and auxiliary roughing pump. The diffusion pump was capable of bringing the entire gas-handling system to a pressure of $10^{-7}$ Torr and the discharge tube was brought to a pressure of $10^{-5}$ Torr or better prior to each gas fill.

The gas inlet system consisted of reservoirs of research grade helium and argon with controlled-flow valves connected to the remainder of the system through a length of 1/4-inch copper tubing. Gas pressure was monitored by an absolute
Figure 2. Block Diagram of Vacuum and Gas-Handling System.
pressure gauge calibrated from 0.1 to 20 Torr in 0.1 Torr gradations and a differential gauge calibrated from 0 to 800 Torr in 10 Torr gradations attached to the gas inlet system. Under vacuum conditions pressure could be monitored by the mass spectrometer or by an ionization gauge (Varian model 563) calibrated from $10^{-4}$ to $10^{-8}$ and attached to the gas chamber.

Preparation of Gas Mixtures

In preparing the gas mixtures to be studied the entire system was first flushed with the minority gas to reduce impurity levels. The minority gas was used in flushing as insurance against possible incomplete pumping due to the low conductance of the 1/4-inch tubing used in the gas inlet system. Following this the discharge tube was overpressured with the desired gas mixture and subsequently pumped to 10 Torr. Before each run the discharge tube was isolated from the vacuum and gas-handling system.

Data Acquisition System

The data acquisition system is shown schematically in figure 3. Data was accumulated by a photomultiplier tube (RCA model 7625) attached to a 0.5-meter Ebert scanning spectrometer (Jarrel-Ash model 82-000). The spectrometer was calibrated to within 5 angstroms with a mercury arc lamp and location of individual lines was verified by maximizing the signal
The signal from the PMT was routed through a photon Discriminator/Amplifier before going to the photon counter.
from the photomultiplier as viewed on an oscilloscope. The photomultiplier signal was then routed to a photon discriminator/amplifier (Mech-Tronics model 511) to a photon counting system.

The photon counting system consisted of a photon counter (Ortec model 9315) with a sampling/control unit (Ortec model 9320). The photon counting system was operated in a 30-step boxcar mode allowing counting to take place during a pre-selected time period at 30 different temporal positions in the active discharge and afterglow (see figure 4). A pre-trigger was supplied to the sampling/control unit by the discharge power supply trigger 1 msec before the initiation of the current pulse. A variable delay line in the sampling/control unit served to initiate counting approximately 20 μsec in advance of the current pulse for each cycle. During step 1 the photon counter is enabled by a 40 μsec gate signal supplied by the sampling/control unit. This is repeated for each trigger pulse received by the sampling/control unit for the duration of an integration interval of 5000 current pulses. At the end of the integration interval the data accumulated by the photon counter is transferred to a buffer where it is available for recording. The delay to the gate signal is then incremented by twice the selected gate width and step 2 begins. Operation during step 2 is identical to that of step 1 except that counting begins at a later time in the discharge/afterglow. A gate width of 40 μsec gave samples as far as 2.4 msec into the afterglow. An integration interval of 200 sec yielded over 5000 current pulses.
Figure 4  Schematic of Timing for the Photon Counting System.

The gate signal in step 1 began 20µsec before the current pulse.
from a pulse rate of 25 sec\(^{-1}\).

**Calibration of the Optical System**

The spectral response of the spectrometer and photomultiplier tube was calibrated with a tungsten filament lamp (ITT model T-10) placed behind the discharge tube. The temperature of the filament was 1500 K. The spectrum of the lamp, corrected for the self-absorption of the filament, is shown in figure 5, and the spectral response of the optical system is shown in figure 6. The response curve shows the signal from the photomultiplier tube, as a function of wavelength, relative to the response at 3600 angstroms.

**Electron Beam System**

The system used for high-pressure measurements is shown schematically in figure 7. It consists of an electron beam machine supplied by Maxwell Research Co. attached to a stainless steel plasma cell 28.5 inches long and 12 inches in diameter. Two CaF\(_2\) windows, 2 inches in diameter, were attached to the cell at its midpoint. The electron beam delivered approximately 7.5 amps-cm\(^{-2}\) at 60 kV in 900 nsec pulses. The gas-handling system and preparation procedures were similar to those described above with the exception of the total pressures. A pressure of 300 Torr was used for the electron beam measurements. Data was accumulated by a liquid nitrogen-cooled InAs infrared detector (Barnes Engineering model 0211) attached to the same spectrometer described above. The spectrometer was equipped with an infrared grating (Jarrel-
Figure 5  Spectral Response of Calibration Lamp.
Blackbody curve of a tungsten filament at 1500°C corrected for self-absorption. Intensity is in arbitrary units.
SPECTRAL RESPONSE OF CALIBRATION LAMP
Figure 6 Spectral Response of the Optical System.

Response relative to the response at 3600 ångströms is shown.
SPECTRAL RESPONSE OF OPTICAL SYSTEM

RELATIVE RESPONSE vs WAVELENGTH (Å)
Figure 7. Block Diagram Showing High-Pressure Apparatus.
Ash #1347D8A9). The signal from the detector was amplified by a 60-db gain low-noise preamplifier (Ortec model 7506) and viewed directly on an oscilloscope (Tektronix model 466) in storage mode.
IV. EXPERIMENTAL RESULTS

The results of this study are presented in three sections. First, the intensities of observed emission lines are presented. A quantitative discussion of the results is presented in the second section. This is followed by a summary of the conclusions drawn from this work.

Intensities of Some emission Lines in Argon

Emissions of lines emanating from the 7s, 6p, 6s, 5d, 5p, 4d and 4p levels in argon were monitored during the active discharge and afterglow in three different gas mixtures. The gas mixtures studied were pure argon, 99% helium with 1% argon and a 50-50 mixture of helium and argon. Plots of photons accumulated vs. time are shown for several representative lines in figures 8 - 10. The relative intensities of each of the lines are tabulated in tables 1-3. Relative intensities are reported for the active discharge and 5 times during the afterglow. Transitions chosen for analysis are the 4p'(1/2)-4s'(1/2) transition at 7503 angstroms and the 5p'(1/2)-4s'(1/2) transition at 4259 angstroms. The 7503 angstrom line was chosen because the 4p' level lies well below the ground state of Ar$^+_2$ and is one of the levels of interest in the $^3$He-$^3$Ar laser. The 4259 angstrom line was chosen because...
Figure 8. Plot of Intensity Versus Time at 7503 Angstroms.

Uncertainty is $\pm 0.14X(\text{Intensity})^{1/2}$

Symbols are:  
+- pure argon  
$x$-1% argon  
*-50% argon

The current pulse occurs at 20 $\mu$sec.
INTENSITY VS. TIME FOR 7503 ANGSTROMS

INTENSITY (PHOTONS)

TIME (MS)
Figure 9. Plot of Intensity Versus Time for 5888 Angstroms.

Uncertainty is $\pm 0.14 \times \text{Intensity}^{1/2}$

Symbols are:  
+--pure argon  
X--1% argon  
*--50% argon

The current pulse occurs at 20 μsec.
Figure 10. Plot of Intensity Versus Time for 4259 Angstroms.

Uncertainty is ±0.14X(Intensity)\(^{1/2}\)

Symbols are: +--pure argon

X--1% argon

*--50% argon

The current pulse occurs at 20 µsec.
INTENSITY VS. TIME AT 4259 ANGSTROMS

TIME (NS)

INTENSITY (PHOTONS)

$10^7$ $10^6$ $10^5$ $10^4$ $10^3$ $10^2$ $10^1$
<table>
<thead>
<tr>
<th>Wavelength (angstroms)</th>
<th>Time after trigger (μsec)</th>
</tr>
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<tbody>
<tr>
<td>7503</td>
<td>23391 83147 71844 58626 2739 6268</td>
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<tr>
<td>7030</td>
<td>12352 28019 13232 3709 432</td>
</tr>
<tr>
<td>6937</td>
<td>3712 15929 6003 2386 761 188</td>
</tr>
<tr>
<td>5888</td>
<td>4431 5710 2386 662 123 18</td>
</tr>
<tr>
<td>5650</td>
<td>449 3869 761 395 97 17</td>
</tr>
<tr>
<td>4259</td>
<td>1382 2348 188 365 72 11</td>
</tr>
<tr>
<td>3649</td>
<td>698 93 11 3 1</td>
</tr>
</tbody>
</table>

Table 1. Relative Intensities in Pure Argon.

Trigger occurs 20 μsec before current pulse.
<table>
<thead>
<tr>
<th>Wavelength (angstroms)</th>
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<th>160</th>
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<td>100</td>
<td>10</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2. Relative Intensities in 99% Helium With 1% Argon.

Trigger occurs 20 μsec before current pulse.
<table>
<thead>
<tr>
<th>Wavelength (angstroms)</th>
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<th>80</th>
<th>160</th>
<th>240</th>
<th>320</th>
<th>400</th>
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<tr>
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<td>10</td>
</tr>
<tr>
<td>3649</td>
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<td>142</td>
<td>41</td>
<td>12</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3. Relative Intensities in 50% Helium with 50% Argon. Trigger occurs 20 μsec before current pulse.
level lies near the 3d level.

The evolution of the chosen transitions may be examined through the ratios of their emissions. The ratio of emission at 4259 angstroms to that at 7503 angstroms in pure argon falls off from 0.059 during the discharge to 0.028 50 µsec after the discharge and 0.006 130 µsec after the discharge. A similar situation is encountered in the 1% and 50% mixtures. This is to be expected.

As the partial pressures of the gas mixtures are altered a marked effect is observed. The ratio of emission at 4259 angstroms to that at 7503 angstroms rises from 0.059 in pure argon to 0.090 in helium with 1% argon then falls off to 0.056 in the 50% mixture, during the active discharge. Early in the afterglow the ratio remains nearly constant as the partial pressures change but by 130 µsec into the afterglow the behavior is similar to that of the active discharge.

The high-pressure experiment yielded indeterminant results. Infrared radiation from an electron beam-generated plasma of pure argon at 300 Torr was scanned over the range of 1.5 µm to 1.84 µm. No signal was observed above the radio-frequency noise generated by the electron beam machine indicating that the sensitivity of the IR detection apparatus was insufficient.

Discussion

The plasma is assumed to be in a quasi-steady state, thus the intensity of emission from a given state, as expressed in number of photons is equal to the rate of population of
that state. In the model of Harries and Nilson dissociative recombination is assumed to be a population mechanism for the 4p level but not for the 5p level\(^2\). Collisional radiative recombination is assumed to be a population mechanism for both levels.

The collisional radiative recombination rate coefficient is typically expressed as the sum of a neutral part and an electronic part. In the low-pressure regime it may be assumed that the total rate coefficient is approximately equal to the electronic part. The collisional radiative rate coefficient is expressed as:

\[
\alpha_{\text{CRR}} = A_{\text{CRR}} N_e T_e^{-x}
\]

where \(A_{\text{CRR}}\) is a constant and \(x\) is unknown. The dissociative recombination rate coefficient is

\[
\alpha_d = A_d T_e^{-0.61}
\]

Thus the ratio, \(R\), of emission at 4259 angstroms to that at 7503 angstroms may be expressed as:

\[
R = \frac{A_{\text{CRR}} N_e T_e^{-x} N_e \{\text{Ar}^+\}}{A_{\text{CRR}} N_e T_e^{-x} N_e \{\text{Ar}^+\} + A_d T_e^{-0.61} N_e \{\text{Ar}_2^+\}}
\]

The quasi-steady state assumption imposes the requirement that:

\[
d\{\text{Ar}_2^+\}/dt = k\{\text{He}\}\{\text{Ar}\}\{\text{Ar}^+\} - A_d T_e^{-0.61} N_e \{\text{Ar}_2^+\} = 0
\]

where \(k\{\text{He}\}\{\text{Ar}\}\{\text{Ar}^+\}\) is the rate of formation of \(\text{Ar}_2^+\). Or:

\[
\{\text{Ar}^+\} = \frac{k\{\text{He}\}\{\text{Ar}\}\{\text{Ar}^+\}}{A_d T_e^{-0.61} N_e}
\]

Thus:

\[
R = \frac{A_{\text{CRR}} T_e^{-x} N_e^2 \{\text{Ar}^+\}}{A_{\text{CRR}} T_e^{-x} N_e^2 \{\text{Ar}^+\} + k\{\text{He}\}\{\text{Ar}\}\{\text{Ar}^+\}}
\]
In this expression \( k, A_{CRRT}, \{\text{He}\} \) and \( \{\text{Ar}\} \) are constant with respect to time. Thus the ratio of the ratios at two different times during the afterglow may be written:

\[
\frac{R_1}{R_2} = \frac{N_{e1}/N_{e2}}{(T_{e1}/T_{e2})^{2-x}}
\]

and:

\[
-x = \ln\left(\frac{R_1}{R_2}\frac{N_{e2}/N_{e1}}{(T_{e1}/T_{e2})^2}\right)/\ln(T_{e1}/T_{e2})
\]

The time evolution of electron density and electron temperature in a plasma under conditions similar to those in this study were measured by Monchicourt et al. Their results are shown in Table 4 along with the other data pertinent to this discussion. The calculated values of \(-x\) are found to be 4.05 for pure argon, 0.255 for 1% argon and 6.43 for 50% argon.

Conclusions

Results of the high-pressure observations are inconclusive, indicating a need for further work in this area. The electron temperature dependences calculated from the results of the low-pressure observations, on the basis of the model of Harries and Wilson, are incompatible with generally accepted values. The model of Harries and Wilson and the results of this study are seen to be inconsistent with each other.
<table>
<thead>
<tr>
<th>Time after discharge</th>
<th>50 µsec</th>
<th>130 µsec</th>
</tr>
</thead>
<tbody>
<tr>
<td>R in pure argon</td>
<td>0.028</td>
<td>0.014</td>
</tr>
<tr>
<td>R in 99% He with 1% Ar</td>
<td>0.028</td>
<td>0.020</td>
</tr>
<tr>
<td>R in 50% He with 50% Ar</td>
<td>0.030</td>
<td>0.012</td>
</tr>
<tr>
<td>Electron Temperature</td>
<td>780 K</td>
<td>710 K</td>
</tr>
<tr>
<td>Electron density</td>
<td>$9.0 \times 10^{11}$ cm$^{-3}$</td>
<td>$7.7 \times 10^{11}$ cm$^{-3}$</td>
</tr>
</tbody>
</table>

Table 4. Parameters Used in Calculation of Temperature Dependence of Collisional Radiative Recombination Rate Coefficient.
APPENDIX I

ESTIMATIONS OF ELECTRON DENSITIES AND TEMPERATURES
IN $^3\text{He}$-DOMINATED PLASMAS

by

B. D. DePaola, S. D. Marcum, H. K. Wrench, B. L. Whitten

and W. E. Wells
ESTIMATIONS OF ELECTRON DENSITIES AND TEMPERATURES
IN \(^3\text{He}\)-DOMINATED PLASMAS

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B. D. De Paola, S. D. Marcum, H. K. Wrench, E. L. Whitten
and H. E. Walls

INTRODUCTION

It is well recognized that in order to make efficient use of the large energies derived from a fission reactor with a Nuclear Pumped Laser (NPL), a medium that will lase at high pressure must be employed. This is only one of the many features of future NPL's that will make them markedly different from common lasers of today. With this in mind we have made an attempt to characterize a high-pressure, \(^3\text{He}\)-dominated NPL plasma, based on what is known of similar low pressure plasmas.

Many processes important to NPL's are strongly dependent upon electron temperature and/or electron density. Also, due to the nature of nuclear pumping, performing diagnostics on NPL plasmas is a very difficult, expensive and hazardous task. These facts provide the impetus for performing the calculations described here. These calculations are straightforward extrapolations of low pressure results into the high pressure regime. Several of the uncertainties inherent in this approach are also discussed.

THE PURE \(^3\text{He}\) PLASMA

Taking helium as the sole species in the plasma, the following reactions were considered:

1) \( S = \sigma \phi n_3 \varepsilon_f \frac{\text{He}^+}{\text{He}} \)

Equation 1 describes the production of \( \text{He}^+ \) where \( \sigma \) is the fission cross-section for \(^3\text{He} \), \( \phi \) is the thermal neutron flux (assumed to be \( 10^{18} \text{ n-cm}^2\text{-sec}^{-1} \)), \( n_3 \) is the density of \(^3\text{He} \), \( \varepsilon_f \) is the energy released per fission of \(^3\text{He} \) and \( \text{He}^+ \) is the
Energy invested per electron-ion pair produced \( = (7.5e7) \).

2) \( S_x = S / .55 \).

This equation accounts for production of \( \text{He}(2^3S) \) metastables which proceeds at roughly twice the rate of \( \text{He}^+ \) production.

3) \( \text{He}^+ + 2\text{He} \rightarrow \text{He}_2^+ + \text{He} \).

3-body conversion to \( \text{He}_2^+ \). This process varies as \( 70^2 \text{He} \) (average of several measurements).

4) \( \text{He}(2^3S) + \text{He}(2^3S) \rightarrow \text{He}^+ + \text{He} + e. \)

Metastable-metastable ionization which has the rate coefficient

\[ \beta = 1.8 \times 10^{-9} \text{ cm}^3\text{-sec}^{-1}. \]

5) \( \text{He}(2^3S) + 2\text{He} \rightarrow \text{He}_2(2^3S) + \text{He}. \)

3-body conversion of metastables which scales as \( \varepsilon \text{He}^2 \).

6) \( \text{He}_2^+ + e \rightarrow \text{He} + e. \)

This represents collisional-radiative recombination which includes a very large number of possible processes.

7) \( \text{He}(2^3S) + e \rightarrow \text{He} + e. \)

The last process considered is superelastic relaxation. The rate for this reaction is temperature dependent \( (T_e)^{1/2} \) and was derived from a detailed balancing calculation based on the forward process. (Reactions involving the helium singlet metastable were not considered because being only .8eV away from the \( \text{He}(2^3S) \) state it is very quickly converted to the triplet metastable).

Using the above processes the following rate equations were developed:

\[ \frac{d[\text{He}^+]}{dt} = \frac{5}{2} \beta \cdot [\text{He}(2^3S)] - 78 \cdot \varepsilon_{\text{He}} \cdot [\text{He}^+] \]

Here the first source term is due to ionization by high energy electrons produced by the fission products of \( ^3\text{He} \); the second is due to metastable-metastable ionization. The loss term is due to 3-body conversion. For \( \text{He}_2^+ \) we have

\[ \frac{d[\text{He}_2^+]}{dt} = 78 \cdot \varepsilon_{\text{He}} \cdot [\text{He}^+] - \alpha \cdot [\text{He}_2^+] \cdot n_e, \]

where

\[ \alpha = 4.5 \times 10^{-20} (T_e/T_o)^{-4.5} n_e + 5 \times 10^{-27} (T_e/T_o)^{-1} n_e, \]

\( T_e, T_o, n_e, \) no.
are electron and gas temperatures and the electron and neutral densities, respectively. The source term is due to 3-body conversion whereas the loss is due to collisional-radiative recombination. Note the electron temperature and density dependencies. For helium metastables the rate equation is

\[
\frac{d[He(2^3S)]}{dt} = S_x + 0.7 [He^+] n_e - 0.05 [He(2^3S)]^2 n_e - 0.6 \frac{P_T}{n_e} [He(2^3S)]
\]

where \( \gamma = 7 \times 10^{-11} T_e^{-1/2} \). The source terms here are due to excitation by high energy electrons produced by the fission products of \(^3\)He and collisional-radiative recombination of \( He^+ \). Losses are due to metastable-metastable ionization, superelastic relaxation and 3-body conversion of metastables, respectively. For electron density we have

\[
\frac{d n_e}{dt} = S + 1/2 \beta [He(2^3S)]^2 - \alpha [He^+] n_e
\]

Here, of course, we have the same source terms as with \( He^+ \); the loss is assumed to be due to collisional-radiative recombination. Also considered was the following energy balance equation:

\[
\frac{d(3/2 n k T_e)}{dt} = S \Delta e_n-e + 1/2 \beta [He(2^3S)]^2 \Delta e^*-e + \gamma n_e [He(2^3S)] \Delta e^*-e
\]

\[
- \frac{20}{11} \frac{3/2 k}{I_e} (T_e - T_0) \nu_{eq}
\]

The source terms are due to ionization by high energy electrons produced by the \(^3\)He fission products, metastable-metastable ionization and superelastic relaxation. \( \Delta e_n-e \), \( \Delta e^*-e \) and \( \Delta e^*-e \) are defined as the energies transferred to the maxwellian part of the electron energy distribution by a non-maxwellian electron at 10, 15 and 20 eV respectively. The loss term is due to electron-neutral collisions with \( \nu_{eq} \) being the electron-neutral collision frequency.

We have an electron energy distribution like that of Figure 1. High energy electrons produced in REGION III by the \(^3\)He fission products cascade down in energy toward a primarily maxwellian region of the distribution (REGION I).
Beyond the first excitation potential of He, inelastic collisions dominate. At and below the first excitation potential of helium production of He$^+$ and He($2^3S$) occurs. Wells et. al. have shown that the energy transferred to the Maxwellian part of the electron energy distribution by a non-Maxwellian electron is a function of electron density and pressure (Figure 2). All values of $\Delta \varepsilon$ used in this calculation were extrapolated from the work of Wells.

Using the above rate equations in steady-state form we calculated electron temperature and density as a function of pressure from 100 to 2300 torr. With these temperatures and densities we were also able to calculate the densities of He$^+$, He$_2^+$ and the density of the He($2^3S$) metastable, all as functions of pressure. However, pure helium is not expected to make a very good lasing medium, so we next considered a mixture of helium and argon. Results for the pure helium plasma will be shown later in comparison to those for the helium-argon mixture.

**THE He-Ar (1% Ar) Plasma**

Upon adding 1% argon to the plasma, all reactions considered earlier with helium alone are still operative. However, several other reactions with argon must now be taken into account. In Figure 3, equations 1-7 are for a pure helium plasma. Equations 8-15 include possible He-Ar reactions as well as Ar-Ar reactions. Eq. 8 - charge exchange with He$^+$; 9 - charge exchange with He$_2^+$; 10 - 3-body charge exchange; 11 - Penning ionization of argon; 12 - dissociative recombination; 13 - 3-body conversion; 14 - collisional-radiative recombination; and 15 - associative ionization to Ar$_2^+$. In the case of collisional-radiative recombination we assumed this reaction to be identical to collisional-radiative recombination of He.

The rate equations listed earlier must now be modified to include these reactions. Figure 4 lists the updated set of rate equations; additions to the
rate equations for the pure helium plasma are underlined. For example, He⁺ can now be lost through charge exchange with Ar. He₂ can also be lost through charge exchange as well as 3-body conversion, etc. However, considering steady state conditions, those equations can be solved for electron temperature and density as functions of pressure, as well as the densities of all other species in the plasma.

RESULTS

Figure 5 is a plot that compares electron temperature with and without argon. Both are plotted against pressure from 100 to 2300 torr. Note that the electron temperature is over 1000K at low pressures, but tends toward room temperature at higher pressures. This is important because of the dependence of the various coefficients on electron temperature. Also note that the addition of 1% Ar to the plasma does not significantly affect the electron temperature.

Figure 6 compares electron densities with and without argon. Here the variation in electron density with pressure is not so great as was the temperature change, but again note the dependence of the rate coefficients on electron density. Figure 7 is a plot of He⁺ density versus pressure with 0% and 1% Ar. Upon adding argon to the plasma the He⁺ density is lower at all pressures. This is reasonable considering that charge exchange between He⁺ and Ar can now occur. Similarly, Figure 8 shows He₂⁺ density with and without argon plotted against pressure. The large decrease in He₂⁺ concentration at all pressures is due to 2- and 3-body charge exchange with argon. Figure 9 plots the helium metastable (He(2S)) density versus pressure also with and without argon. The decrease in the helium metastable density with 1% Ar at all pressures is due to Penning ionization of argon. Also, densities of Ar⁺, Ar₂⁺ and Ar were calculated as functions of pressure and are shown in Figure 10. The argon excited state density represents an accumulation of all argon excited states that exist between the ionization limits of the atomic
and molecular ions into a single state. This was done so that associative ionization of argon could be considered.

**CONCLUSIONS**

The above results are reasonable based on extrapolation of the knowledge of low-pressure $^3$He-dominated plasmas. However:

1. What is the collisional-radiative recombination rate coefficient at high pressures? We have employed a low pressure coefficient in our calculations that may or may not be valid at higher pressures.

2. We have assumed that recombination of $\text{Ar}^+$ is in the collisional radiative form, which seems reasonable, but we've assumed collisional radiative recombination of argon to be identical to that of helium. This later assumption may well be a dubious one.

3. What other processes are important at higher pressures that we could not consider? It is possible that $\text{He}_2^+$, $\text{Ar}_3^+$ and/or $\text{He}-r^+$ achieve large densities at higher pressures such that they become important species in the plasmas. These could not be considered in our analysis.

These are examples of the questions that this type of calculation raises. While it is true that there are more sophisticated methods of calculating electron temperatures and densities, given the uncertainties in cross sections and other data we feel that the approach outlined here is sufficient for the present.

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Figure 1: Electron Energy Distribution and Energy Balance Equation Used.
Heating of a maxwellian distribution due to a non maxwellian electron

Figure 2:
REACTIONS CONSIDERED

1. \( S = \sigma \phi \frac{n_3}{(\text{He})} \frac{E_f}{W} \). (He\(^+\) source)

2. \( S_e = S/56 \). (He(2\(^3\)S) source)

3. \( \text{He}^+ + 2\text{He} \rightarrow \text{He}_2^+ + \text{He} \)

4. \( \text{He}(2\(^3\)S) + \text{He}(2\(^3\)S) \rightarrow \text{He}^+ + \text{He} + e \)

5. \( \text{He}(2\(^3\)S) + 2\text{He} \rightarrow \text{He}_2(2\(^3\)S) + \text{He} \)

6. \( \text{He}_2^+ + e + X \xrightarrow{\text{CRR}} \)

7. \( \text{He}(2\(^3\)S) + e \rightarrow \text{He} + e \)

\( = 78p^2_{\text{He}} \)

\( \beta = 1.8 \times 10^{-9} \text{ cm}^3\cdot\text{sec}^{-1} \)

\( = 0.6p^2_{\text{He}} \)

\( = 7 \times 10^{-11} \frac{1}{T_e}^{1/2} \)

OTHER REACTIONS WITH ARGON (1%)

8. \( \text{He}^+ + \text{Ar} \rightarrow \text{Ar}^+ + \text{He} \)

9. \( \text{He}_2^+ + \text{Ar} \rightarrow \text{Ar}^+ + 2\text{He} \)

10. \( \text{He}_2^+ + \text{Ar} + \text{He} \rightarrow \text{Ar}^+ + 3\text{He} \)

11. \( \text{He}(2\(^3\)S) + \text{Ar} \rightarrow \text{Ar}^+ + \text{He} + e \)

12. \( \text{Ar}^+ + e \rightarrow \text{Ar}^+ + \text{Ar} \)

13. \( \text{Ar}^+ + \text{Ar} + \text{He} \rightarrow \text{Ar}_2^+ + \text{He} \)

14. \( \text{Ar}^+ + e + X \xrightarrow{\text{CRR}} \)

15. \( \text{Ar}^+ + \text{Ar} \rightarrow \text{Ar}_2^+ \)

\( \sigma = 10^{-17} \text{ cm}^2 \)

\( \beta_1 = 2.2 \times 10^{-10} \text{ cm}^3\cdot\text{sec}^{-1} \)

\( \gamma_1 = 2.4 \times 10^{-29} \text{ cm}^6\cdot\text{sec}^{-2} \)

\( \beta_2 = 7 \times 10^{-16} \text{ cm}^2 \)

\( \sigma_0 = 9.1 \times 10^7 \frac{(300/T_e)^{61/3}}{\text{ cm}^3\cdot\text{sec}^{-1}} \)

\( \sigma_0 = 375p^0_{\text{He}}p^0_{\text{Ar}} \)

\( 2 \times 10^{-9} \text{ cm}^3\cdot\text{sec}^{-1} \)

Figure 3: Reactions Considered in this Study.
RATE EQUATIONS

\[
\frac{d[\text{He}^+]}{dt} = S + \frac{1}{2} \beta \left( \text{He}(2^3S) \right) - 78 \frac{p_{\text{He}}}{p_{\text{Ar}}} \left[ \text{He}^+ \right] - \sigma \left[ \text{He}^+ \right] \left[ \text{Ar} \right]
\]

\[
\frac{d[\text{He}_2^+]}{dt} = 78 \frac{p_{\text{He}}}{p_{\text{Ar}}} \left[ \text{He}^+ \right] - \alpha \left[ \text{He}_2^+ \right] n_e - \beta_1 \left[ \text{He}_2^+ \right] \left[ \text{Ar} \right] - \gamma_1 \left[ \text{He}_2^+ \right] \left[ \text{Ar} \right] \left[ \text{He} \right]
\]

\[
\frac{d[\text{He}(2^3S)]}{dt} = S_x + \frac{7}{2} \alpha \left[ \text{He}_2^+ \right] n_e - \beta \left[ \text{He}(2^3S) \right]^2 - \gamma \left[ \text{He}(2^3S) \right] n_e - \frac{2}{3} \frac{T_{\text{He}}}{T_{\text{Ar}}} \left[ \text{He}(2^3S) \right]
\]

\[
\frac{d n_e}{dt} = S + \frac{1}{2} \beta \left[ \text{He}(2^3S) \right]^2 - \alpha \left[ \text{He}_2^+ \right] n_e + \beta_3 \left[ \text{Ar} \right] \left[ \text{He}(2^3S) \right] - \epsilon \left[ \text{Ar}_2 \right] n_e
\]

\[
\frac{d[\text{Ar}^*]}{dt} = \sigma \left[ \text{Ar}^+ \right] \left[ \text{Ar} \right] + \beta_1 \left[ \text{He}_2^+ \right] \left[ \text{Ar} \right] + \gamma_1 \left[ \text{He}_2^+ \right] \left[ \text{Ar} \right] \left[ \text{He} \right] + \beta_3 \left[ \text{He}(2^3S) \right] \left[ \text{Ar} \right]
\]

\[
\frac{d[\text{Ar}^+]}{dt} = 2 \times 10^{-9} \left[ \text{Ar}^+ \right] \left[ \text{Ar} \right] + 376 \frac{p_{\text{He}}}{p_{\text{Ar}}} \left[ \text{Ar}^* \right] - \epsilon \left[ \text{Ar}_2 \right] n_e
\]

\[
\frac{d[\text{Ar}_2^+]}{dt} = \alpha \left[ \text{Ar}^+ \right] n_e - 2 \times 10^{-9} \left[ \text{Ar}^* \right] \left[ \text{Ar} \right] \left[ \text{Ar}^* \right] \left[ \text{Ar}_2 \right] n_e
\]

Where:

\[
S = \sigma \phi \frac{n_e}{\left( \text{He} \right) \frac{c}{2}}
\]

\[
S_x = \frac{S}{2}, 55
\]

\[
\gamma = 7 \times 10^{-11} \left( \frac{c}{T_{\text{Ar}}} \right), 1/2
\]

\[
\alpha = 5 \times 10^{-20} \left( \frac{c}{T_{\text{Ar}}} \right)^{-4 \pm 5} n_e + 5 \times 10^{-27} \left( \frac{c}{T_{\text{Ar}}} \right)^{-12} n_o
\]

\[
\sigma \phi = 5 \times 10^{-13}
\]

\[
\beta_1 = 2.2 \times 10^{-10}
\]

\[
\beta_3 = 5.2 \times 10^{-11}
\]

\[
\gamma = 9.1 \times 10^{-7} \left( \frac{c}{T_{\text{Ar}}} \right)^{1 \pm 6}
\]

**Figure 4**: Rate Equations Employed in this Study.
Figure 6: Plot of Electron Density versus Pressure.
Figure 7: Plot of He⁺ Density versus Pressure.
Figure 9: Plot of He(2\textsuperscript{3}S) Density versus Pressure.
APPENDIX II

Program to Calculate Electron Densities and Temperatures
THIS PROGRAM SOLVES A SYSTEM OF NONLINEAR EQUATIONS BY
SUCCESSIVE ITERATIONS. A SYSTEM OF SEVEN SIMULTANEOUS
EQUATIONS HAS BEEN REDUCED ANALYTICALLY TO TWO NONLINEAR,
SIMULTANEOUS EQUATIONS IN ELECTRON DENSITY AND ELECTRON
TEMPERATURE. IN EACH ITERATION STEP THE SSP SUBROUTINE
RTMI, SUPPLIED BY IBM IS USED TO SOLVE A NONLINEAR EQUATION.
THE SUBROUTINE RTMI SOLVES FOR A ROOT OF AN EXTERNAL FUNCTION.
SET EQUAL TO ZERO.

COMMON DE(3), XNEO, TEO, EXAB(3), PH, PA, P, V, SIG13, FLX, FF, SIG1, B11, B2, GA
- M, SIG4.
REAL YZ(610, 9)
EXTERNAL FCT
EXTERNAL FCN

FORMAT PRINTED OUTPUT
550 FORMAT( ' ', METASTABLE DENSITY = ', E14.7)
570 FORMAT( ' ', ELECTRON DENSITY = ', E14.7)
590 FORMAT( ' ', ELECTRON TEMPERATURE=',E14.7)
610 FORMAT( ' ', AR2=',E14.7)
611 FORMAT( ' ', AR=',E14.7)
612 FORMAT( ' ', RECOMBINATION=',E14.7)
613 FORMAT( ' ', HE2=',E14.7)
614 FORMAT( ' ', HE=',E14.7)
620 FORMAT( ' ', ** NO CONVERGENCE *****)
640 FORMAT( ' ', ** NO CONVERGENCE *****)
670 FORMAT( ' ', '4, GARbage GARbage GARbage GARbage')
680 FORMAT( ' ', '4, ')

DEFINE PARAMETERS AND INITIALIZE VARIABLES
N=1
N=2
XNEO=1.0E12
TEO=305.
PH=1504.8
X=.01
PA=X*PH
P=PA+PH
PA1=PA
PH1=PH
P1=P
V=6.0E4
SIG13=5.0E-21
FLX=1.0E17
FF=2.3E4
SIG1=1E-17
B11=1.8E-9
B2=2.2E-10
GA1=2.4E-29
SIG4=7.0E-16

THE FOLLOWING SOLVES AN ENERGY PARTITION EQUATION FOR THE
ENERGY TRANSFERRED TO THE NEAP-MAXWELLIAN PART OF A NON-
MAXWELLIAN ELECTRON ENERGY DISTRIBUTION FUNCTION
200 AA=6.93E-5*XNEO
P=PH+PA
\[
\begin{align*}
\beta &= 1.34 \times 10^3 E_1^{19} P \\
\gamma &= 2.36 \times 10^3 E_1^{19} P \\
AA &= \sqrt{AA/BB} \\
XX &= (AA/CC)^{1/3} \\
AB &= \sqrt{3/AA} - \sqrt{3.55E19/P} \\
A &= \sqrt{A/BB} \\
Y &= \frac{(AA/CC)^{1/3}}{} \\
ASC &= \frac{AAA \times (\arctan(3/AAA) - \arctan(1.29E-4 \times TEO/AAA))}{1.73 \times \arctan((2 \times \sqrt{FLOAT(N)}) - XK)/(XK^1.73))} \\
N &= N+1 \\
DE &= ABC + XAB(NN) - ABB \\
IFLAG &= 0 \\
GO TO 295 \\
290 \quad \text{DO 295 N=10,20,5} \\
295 \quad \text{CONTINUE} \\
C \quad \text{CALCULATE ELECTRON TEMPERATURE} \\
S &= FC(T(TEO)) \\
\text{IF (S) 1011,1012,1013} \\
C \quad \text{CALL SUBROUTINE TO SEARCH FOR RIGHT-HAND BOUNDARY USED IN RTMI} \\
1011 \quad \text{CALL TPOS(S,SI,XRI,XLI)} \\
\quad \text{GO TO 36} \\
1012 \quad \text{TEN=TEO} \\
\quad \text{GO TO 37} \\
C \quad \text{CALL SUBROUTINE TO SEARCH FOR LEFT-HAND BOUNDARY USED IN RTMI} \\
1013 \quad \text{CALL TNEG(S,SI,XRI,XLI)} \\
C \quad \text{CALL SUBROUTINE TO SOLVE ENERGY BALANCE EQUATION} \\
36 \quad \text{CALL RTMI(TEN,F,FC(T,XLI,XRI,1,,1000,IER)} \\
\quad \text{FN=F} \\
\quad \text{IERN=IER} \\
\text{IF (.IER-2)37,1999,37} \\
C \quad \text{CHECK CONVERGENCE OF ENERGY BALANCE EQUATION} \\
37 \quad \text{IF((ABS(TEO-TEN)/TEO)-0.0001)710,710,700} \\
700 \quad \text{TEO=TEN} \\
\quad \text{GO TO 200} \\
C \quad \text{CALCULATE ELECTRON DENSITY} \\
710 \quad S=FCN(XNEO) \\
\text{IF(S)1001,1002,1003} \\
C \quad \text{CALL SUBROUTINE TO SEARCH FOR RIGHT-HAND BOUNDARY USED IN RTMI} \\
1001 \quad \text{CALL NPOS(S,SI,XRI,XLI)} \\
\quad \text{GO TO 56} \\
1002 \quad XNEN=XNEO \\
\quad \text{GO TO 57} \\
C \quad \text{CALL SUBROUTINE TO SEARCH FOR LEFT-HAND BOUNDARY USED IN RTMI} \\
1003 \quad \text{CALL NNEG(S,SI,XRI,XLI)} \\
C \quad \text{CALL SUBROUTINE TO SOLVE ELECTRON DENSITY CONTINUITY EQUATION} \\
56 \quad \text{CALL RTNI(XNEN,F,FCN,XLI,XRI,1,,1000,IER)}
\end{align*}
\]
IF (IERN-2)57, 1999, 57
C CHECK CONVERGENCE OF ELECTRON DENSITY CONTINUITY EQUATION
57 IF((ABS(XNEO-XNEN)/XNEO)-0.0001)465, 465, 460
460 XNEO=XNEN
GO TO 200
1999 WRITE(6, 620)
465 IF (IFLAG)480, 480, 470
470 WRITE(6, 640)
C CALCULATE ION AND METASTABLE DENSITIES FROM SOLUTIONS FOR ELECTRON TEMPERATURE AND ELECTRON DENSITY
480 ALF9=4.0E-20*(300./TEN)**4*XNEN+1.8E-10*(300./TEN)**6
ALFD=9.1E-7*(300./TEN)**.61
ALF12=7.0E-11*SQRT(TEN)
C1=SIG1*FLX*3.55E16*PH*FF/(78.*PH**2+SIG1*V**3.55E16*PA)
C2=5*B11/(78.*PH**2+SIG1*V**3.55E16*PA)
C3=78.*PH**2/(B2*3.55E16*PA+GAM*1.26025E33*PA*PH+ALF9*XNEN)
C4=(SIG1*V**3.55E16*PA+(B2+GAM*3.55E16*PH)*3.55E16*PA*C3)/(ALF9*XNE-376.*PH*PA)
C5=SIG4*V**3.55E16*PA/(ALF9*XNEN+376.*PA*PH)
C60=B11-(27.*ALF9*PH*PH*B11*XNEN/(78.*PH**2+3.55E-1*V*PA))/(ALF9*XNEN+2.2E-10*3.55E16*PA)
BBI0=1.49E6*PA+ALF12*XNEN+.6*PH*PH
CC10=-SIG1*FLX*3.55E16*PH*FF/.56-(54.6*ALF9*PH*PH*SIG1*FLX*3.55E16*PH*FF*XNEN/(78.*PH**2+3.55E-1*V*PA))/(ALF9*XNEN+2.2E-10*3.55E16*PA)
C10=(-BB10+SQR(BB10**2-4*AA10*CC10))/2/AA10
490 HE=C1+C2*C10*C10
HE2=C3*HE
AR=C4*HE+C5*C10
FINIS=ALF9*AR*XNEN/(2.E-9*3.55E16*PA+ALF9*XNEN))
AR2=(2.E-9*FINIS*3.55E16*PA+376.*PH*PA*AR)/(ALFD*XNEN)
XNE=AR+AR2+HE+HE2
IF(ABS(XNE-XNEN)/XNEN-.01 )15, 15, 10
10 PH=PH-(HE+HE2)/3.55E16
PA=PA1-(AR+AR2)/3.55E16
XNE=XNEN
GO TO 200
C PRINT RESULTS
15 WRITE(6, 680).
WRITE(6, 500)PH1, PA1, P1
WRITE(6, 550)C10
WRITE(6, 570)XNEN
WRITE(6, 590)TEN
WRITE(6, 611) AR
WRITE(6, 610) AR2
WRITE(6, 613)HE2
WRITE(6, 614) HE
WRITE(6, 570)XNE
STOP
END

SUBROUTINE TP0S(S, S1, XRI, XLI)
C THIS SUBROUTINE SEARCHES FOR THE RIGHT-HAND BOUNDARY USED
C BY RTNI IN SOLVING THE ENERGY BALANCE EQUATION.
C THE SEARCH IS PERFORMED BY INCREASING THE ARGUMENT OF
C THE EXTERNAL FUNCTION FCT IN THE DIRECTION OF INCREASING VALUE
C OF THE FUNCTION UNTIL A POSITIVE VALUE IS ENCOUNTERED. CONTROL
C IS THEN RETURNED TO THE MAIN PROGRAM.

COMMON DE(3),XNEO,TEO,XAB(3),PH,PA,P,V,SIG13,FLX,FF,SIG1,B11,B2,GA
-N,SIG4
I1=0
J1=0
TE=1.01*TEO
TE1=TEO
S=FCT(TEO)
S1=FCT(TE)
IF (S)1004,1004,1028
1004 IF (S-S1)1007,1005,1005
997 TE1=TEO
1005 TE=TE1/1.01
S=FCT(TE)
IF(S)1001,1001,1028
1001 I1=I1+1
IF (I1=800)1006,1006,1003
1003 IF (J1=800)999,999,1028
1006 TE1=TE
GO TO 1005
999 TE1=TEO
1007 TE=TE1/1.01.
S=FCT(TE)
IF(S)1002,1002,1028
1002 J1=J1+1
IF(J1=800)1008,1008,1009
1009 IF (J1=800)997,997,1028
1008 TE1=TE
GO TO 1007
1028 XI=TE
XLI=TEO
RETURN
END

SUBROUTINE TNEG(S,S1,XRI,XLI)
C THIS SUBROUTINE SEARCHES FOR THE LEFT-HAND BOUNDARY USED
C BY RTNI IN SOLVING THE ENERGY BALANCE EQUATION.
C THE SEARCH IS PERFORMED BY DECREASING THE ARGUMENT OF
C THE EXTERNAL FUNCTION FCT IN THE DIRECTION OF DECREASING VALUE
C OF THE FUNCTION UNTIL A NEGATIVE VALUE IS ENCOUNTERED. CONTROL
C IS THEN RETURNED TO THE MAIN PROGRAM.

COMMON DE(3),XNEO,TEO,XAB(3),PH,PA,P,V,SIG13,FLX,FF,SIG1,B11,B2,GA
-N,SIG4
J1=0
II=0
TE1=TEO
TE=1.01*TEO
S=FCT(TEO)
S1=FCT(TE)
SUBROUTINE NPOS(S,S1,XRI,XLI)
C. THIS SUBROUTINE SEARCHES FOR THE RIGHT-HAND BOUNDARY USED
C BY RTNI IN SOLVING THE ELECTRON DENSITY CONTINUITY EQUATION.
C. THE SEARCH IS PERFORMED BY INCRENENTING THE ARGUMENT
C THE EXTERNAL FUNCTION FCN IN THE DIRECTION OF INCREASING VALUE
C OF THE FUNCTION UNTIL A POSITIVE VALUE IS ENCOUNTERED. CONTROL
C. IS THEN RETURNED TO THE MAIN PROGRAI.
COMMON DE(3),XNEO,TEO,XAB(3),PH,PA,P,V,SIG13,FLX,FF,SIG1,B1i,B2
J1=0
II=0
XNE=1.01+XNEO
XNE1=XNEO
S=FCN(XNEO)
S1=FCN(XNE)
IF (S1)11,11,28
11 IF (S-S1)17,12,12
12 XNE=XNE1/1.01
S=FCN(XNE)
IF(S)10,10,28
10 II=II+1
IF (II-1000)13,13,8
8 IF(J1-1000)6,6,28
13 XNE1=XNE
GO TO 12
6 XNE1=XNEO
17 XNE=XNE1*1.01
SUBROUTINE NNEG(S,SI,XRI,XLI)

C THIS SUBROUTINE SEARCHES FOR THE LEFT-HAND BOUNDARY USED
C BY RTMI IN SOLVING THE ELECTRON DENSITY CONTINUITY EQUATION.
C THE SEARCH IS PERFORMED BY INCREMENTING THE ARGUMENT OF
C THE EXTERNAL FUNCTION FCT IN THE DIRECTION OF DECREASING VALUE
C OF THE FUNCTION UNTIL A NEGATIVE VALUE IS ENCOUNTERED. CONTROL
C IS THEN RETURNED TO THE MAIN PROGRAM.

COMMON DE(3),XNEO,TEO,XAB(3),PH,PA,P,V,SIG13,FLX,FF,SIG1,B11,B2,GA
-H,SIG4
J1=0
II=0
XNE=1.01*XNEO
XNE1=XNEO
S=FCN(XNEO).
S1=FCN(XNE)
IF (S-S1)10,11,12
7 XNE1=XNEO
10 XNE=XNE1/1.01
S=FCN(XNE)
IF (S)27,14,18
14 II=II+1
IF (II-1000)11,11,13
13 IF (J1-1000)8,8,28
11 XNE1=XNE
12 GO TO 10
8 XNE1=XNEO
15 XNE=XNE1*1.01
S=FCN(XNE)
IF (S)27,18,18
18 J1=J1+1
IF (J1-1000)16,16,17
17 IF (II-1000)7,7,28
16 XNE1=XNE
GO TO 15
28 XRI=XNE
XLI=XNEO
RETURN
END
FUNCTION FCT(TEN)
C EXTERNAL FUNCTION USED BY RTI IN SOLVING ENERGY BALANCE
C EQUATION
COMMON DE(3),XNEO,TEO,XAB(3),PH,PA,P,V,SIG13,FLX,FF,SIG1,B11,B2,CA
-M,SIG4
ALFD=9.1E-7*(300./TEN)**.61
ALF9=4.0E-20*(300./TEN)**4*XNEO+1.8E-10*(300./TEN)*P
ALF12=7.0E-11*SQRT(TEN)
AA10=B11-(27.3*ALF9*PH*PH*B11*XNEO/(78*PH*PH+3.55E-1*V*PA))/(ALF9*XNEO+2.2E-10*3.55E16*P
-NEO+2.2E-10*3.55E16*PA+2.4E-29*1.26025E33*PH*PA))
BB10=1.491E6*PA+ALF12*XNEO+6.6*PH*PH
CC10=-SIG13*FLX*3.55E16*PH*FF/.56-(54.6*ALF9*PH*PH*SIG13*XAB(3)*FLX*3.55E
-16*PH*FF*XNEO/(78*PH*PH+3.55E-1*V*PA))/(ALF9*XNEO+2.2E-10*3.55E16*P
CC10=-SIG13*FLX*3.55E16*PH/FF/.56-(54.6*ALF9*PH*PH*SIG13*XAB(3)*FLX*3.55E
-A+2.4E-29*1.26025E33*PH*PA))
C10=-(BB10+SQRT(BB10**2-4*AA10*CC10))/2/AA10
XXX=SIG13*FLX*FF*3.55E16*PH*DE(1)-1.74E-17*ABS(TEN-300.)*SQRT(TEN)-3.55E16*PA
C10**2*DE(2)+7E-11k*DE(3)*C10*XNEO*SQRT(TEN)
FCT=XXX/I.E15
RETURN
END

FUNCTION FCN(XNE)
C EXTERNAL FUNCTION USED BY RTI IN SOLVING ELECTRON DENSITY
C CONTINUITY EQUATION
COMMON DE(3),XNEO,TEO,XAB(3),PH,PA,P,V,SIG13,FLX,FF,SIG1,B11,B2,CA
-M,SIG4
ALFD=9.1E-7*(300./TEO)**.61
ALF9=4.0E-20*(300./TEO)**4*XNE+1.8E-10*(300./TEO)*P
ALF12=7.0E-11*SQRT(TEO)
C1=SIG13*FLX*3.55E16*PH*FF/(78.*PH**2+SIG13*V*3.55E16*PA)
C2=.5*B11/(78.*PH**2+SIG13*V*3.55E16*PA)
C3=78.*PH**2/(B2*3.55E16*PA+CAN*1.26025E33*PA*PH+ALF9*XNE)
C4=(SIG13*V*3.55E16*PA+(B2+GA*3.55E16*PH)*3.55E16*PA*C3)/(ALF9*XNE
-N+376.*PH*PA)
C5=SIG4*V*3.55E16*PA/(ALF9*XNE+376.*PA*PH)
AA10=B11-(27.3*ALF9*PH*PH*B11*XNE/(78*PH*PH+3.55E-1*V*PA))/(ALF9*X
-NEO+2.2E-10*3.55E16*PA+2.4E-29*1.26025E33*PH*PA))
BB10=1.491E6*PA+ALF12*XNE+6.6*PH*PH
CC10=-SIG13*FLX*3.55E16*PH*FF/.56-(54.6*ALF9*PH*PH*SIG13*XAB(3)*FLX*3.55E
-16*PH*FF*XNE/(78*PH*PH+3.55E-1*V*PA))/(ALF9*XNE+2.2E-10*3.55E16*P
-NEO+2.4E-29*1.26025E33*PH*PA))
C10=-(BB10+SQRT(BB10**2-4*AA10*CC10))/2/AA10
-4*AA10
-4*AA10
HEI=C1+C2*C10
HEII=C3*HEI
ARI=C4*HEI+C5*C10
ARX=ALF9*ARI*XNE/(2.E-9*3.55E16*PA+(ALF9*XNE))
ARI1=(2.E-9*ARX*3.55E16*PA+376.*PH*PA*ARI)/(ALFD*XNE)
YYY=HEI+HEII+ARI+ARI1-XNE
FCT=YYY/I.E15
RETURN
END
REFERENCES


7. Ortec Publication No. 2687 (ML581): O3C 0976

