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AN INVESTIGATION OF THE MOTION OF SMALL PARTICLES AS RELATED TO THE FORMULATION OF ZERO GRAVITY EXPERIMENTS

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FINAL REPORT
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INTRODUCTION

With the advent of a manned orbiting laboratory, many historically fundamental experiments can potentially be performed where gravitational effects are absent or minimal. Study of Motion of Small Particles is one example of such an experiment. The motion of small particles in various media is referred to as Brownian motion. Though Robert Brown did not discover Brownian motion, his contribution was to establish Brownian motion as an important phenomenon and to refute by experiment facile mechanical explanations of the phenomenon [1]. Albert Einstein developed the full mathematical theory of the movements of the particles, neglecting body forces [2]. He deduced a formula which makes it possible to calculate the magnitude of the Brownian displacements. Harvey Fletcher proposed modifications to this formula and suggested a new method for studying Brownian movements [3,4]. Leopard Infeld conducted theoretical investigation of Brownian motion [5].

The theoretical formulations have, for the most part, neglected gravitational body forces while the gravitational effects are unavoidably included in the experimental results due to terrestrial testing. Performing small particle experiments in an orbiting laboratory would make it possible to eliminate gravitational influence.

Laser Doppler Anemometer Techniques as described in Reference 6, provide an excellent opportunity to conduct such experiments with greater accuracy than ever before. The purpose of this project was to investigate the feasibility and evaluate the scientific worth of experiments in space by conducting earth based experiments.

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\[a\] The numbers in the brackets refer to the list of references given at the end.
THEORY OF BROWNIAN MOVEMENTS

Some basic equations of interest are given here [3].

The average absolute value of displacement of a small particle in a gaseous medium with no outside forces is given by

\[ \bar{x} = \sqrt{\frac{2}{\pi}} \left( \bar{x}^2 \right)^{\frac{1}{2}} \tag{1} \]

where \( \bar{x}^2 \) is the group average of the squares of the individual displacements.

The magnitude of Brownian displacement for a spherical particle moving uniformly through an incompressible gaseous medium can be written as

\[ \bar{x} = \frac{2}{3\pi} \left( \frac{Et}{\mu \alpha} \right)^{\frac{1}{2}} \tag{2} \]

where

- \( E \) = average energy agitation of the gas molecule
- \( t \) = time interval being considered
- \( \mu \) = coefficient of viscosity
- \( \alpha \) = particle radius
- \( \kappa \) = a function of \( \frac{z}{\alpha} \)
- \( z \) = mean free path

If the particle is moving under the influence of constant outside force, assume that the particle displacement is composed of two components:

\[ x_{\text{gravity}} = Vt \]

and

\[ x_{\text{Brownian}} = U \sqrt{t} \]

where, \( U \) = displacement in one second due to Brownian movements

The total displacement is given by

\[ b = Vt + U \sqrt{t} \]

\[ \therefore U = bt^{\frac{1}{2}} - Vt^{\frac{1}{2}} \]
The number $n$ out of $n$ events that will have the time of fall, through a fixed distance, between $t_1$ and $t_2$ can be expressed as:

$$n = -n\left(\frac{h^2}{\pi}\right)^{1/2} \int_{t_1}^{t_2} (bt^{-3/2} + Vt^{-1/2}) \exp\left(-\frac{h}{t} (b-Vt)^2\right) dt$$ \hspace{1cm} (3)

To obtain an expression of the distribution of frequencies, one utilizes the equation for frequency shift, i.e.,

$$\omega = \Delta f = \frac{c}{t}$$

substituting $t = \frac{c}{\omega}$ in equation 3,

$$n = \frac{n}{2\pi} \left(\frac{h^2}{\pi}\right)^{1/2} \int_{\omega_g}^{\omega_1} \left(\frac{bc}{\omega^{3/2}} + Vc \omega^{-3/2}\right) \exp\left(-\frac{h}{c\omega} (\omega b - Vc)^2\right) d\omega$$ \hspace{1cm} (4)

If the frequency due to gravity is $\omega_g$, then the average of the average value of all frequencies below $\omega_g$ and of all frequencies above $\omega_g$ is given by

$$\bar{\omega} = \frac{\omega_g + \omega_c}{2b} + \frac{c}{8b^2h}$$

The first term on the right hand side is due to gravity and the second term is due to Brownian Motion.
EXPERIMENTAL SYSTEM

Laser Doppler Method

The Doppler effect is the fact that there is a change in frequency with which energy reaches a receiver when the receiver and the energy source are in motion relative to one another [6]. In the Laser Doppler System, energy is transmitted to a moving scatterer (tracer) which then becomes a source and the energy is transmitted to a receiver. This technique is based on the fact that radiation passing through a fluid is scattered by tracers in the fluid. In Laser Doppler Systems small particles are suspended in the fluid and act as tracers. The particles are assumed to move with the same velocity as the fluid surrounding the particle (no-slip boundary condition).

The scattered radiation gives information regarding the velocity of the particles. This information manifests itself by frequency shifting the radiation striking the particles.

The amount that the source frequency is shifted, $\Delta f$, upon striking the tracer and returning to a receiver is called the Doppler Shift or Doppler Frequency.

DESCRIPTION OF THE SYSTEM

The experimental system is shown in Figures 1 and 2. It is of a dual beam-single channel type utilizing forward scatter. It is easier to obtain strong signals with forward scatter than with back scatter. The main components of the system are described below:

Helium-Neon Gas Laser - 15 mW power and provides focused and coherent light having a wavelength of $6.328 \times 10^{-7}$ m.

Polarization Rotator - orients the polarity of the laser beam for optimum efficiency.
Transmitting Optics - consists of beam splitter and focussing lens. The beam splitter splits the beam into two parallel beams, each displaced 25 mm from the original beam.

Particle Container - contains the tracer particles suspended in water.

Receiving Optics - consists of lenses to collect scattered light.

Photomultiplier - converts light energy received into proportional electrical signal with a quantum efficiency of 13%.

Particle Doppler Analysis Processor - measures the time period of burst signals generated by particles passing through the test volume.

Data Sorter - forms histograms of signal time and prints the data.

The Laser and the optics were supplied by Thermo-Systems, Inc. and the electronics were supplied by Spectron Development Laboratories, Inc. The small diameter particles used as tracers were Dow Uniform Latex Particles supplied by Dow Diagnostics.

**TSI SYSTEM**

The laser beam is separated into two beams of equal intensity and these beams are focussed at the measuring location as shown in Figure 3. Where the two beams cross, the wave fronts interfere to form alternate regions of high and low intensity light which causes variations in the intensity of light scattered by the particles crossing the measuring location.
The scattered light is picked up by the photomultiplier and is converted to an electrical signal whose frequency is proportional to the rate at which the particle is crossing the interference fringes.

The following equations are applicable:

\[ \frac{d_f}{f} = \frac{\lambda}{2 \sin \frac{\phi}{2}} \]

\[ V = f \cdot d_f \]

where:
- \( \lambda \) = wave length of incident light = 6.328 \times 10^{-7} \text{ m} 
- \( f \) = frequency detected at Photomultiplier, Hz
- \( V \) = velocity of flow perpendicular to fringes, m/s
- \( d_f \) = distance between fringes (=1.6 mm for 120 mm lens)
- \( \phi \) = angle of the beam intersection (=25° for 120 mm lens)

The system is equipped with 3 sets of lenses of focal lengths, 120 mm, 250 mm and 600 mm. For lower velocities and lower frequencies, as in the present application, shorter focal length lenses are preferable. Therefore, 120 mm lenses were used.

The relationship between the velocity of particle and the frequency detected is shown in Figure 4.
DATA

The latex particles were suspended in water in a plastic container. The particles were stirred well to make all the particles to be suspended. Sufficient time was allowed for the fluid to become quiet so that the particles can be considered to be moving freely. The collection of data was started at this point. A typical printout from the Data Sorta is shown in Figure 5. The summary of data collected is shown in the table below.

<table>
<thead>
<tr>
<th>RUN #</th>
<th>PARTICLE SIZE ( \mu \text{m} )</th>
<th>AVERAGE VELOCITY ( \text{m/s} )</th>
<th>STANDARD DEVIATION ( \text{m/s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0.085</td>
<td>0.2241</td>
<td>0.01568</td>
</tr>
<tr>
<td>15</td>
<td>0.085</td>
<td>0.01415</td>
<td>0.1015</td>
</tr>
<tr>
<td>16</td>
<td>0.085</td>
<td>0.02342</td>
<td>0.183</td>
</tr>
<tr>
<td>17</td>
<td>0.091</td>
<td>0.03965</td>
<td>0.2713</td>
</tr>
<tr>
<td>20</td>
<td>0.091</td>
<td>0.00624</td>
<td>0.1027</td>
</tr>
<tr>
<td>21</td>
<td>0.091</td>
<td>0.0205</td>
<td>0.1916</td>
</tr>
<tr>
<td>23</td>
<td>0.091</td>
<td>0.0707</td>
<td>0.351</td>
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<tr>
<td>30</td>
<td>0.176</td>
<td>0.06867</td>
<td>0.113</td>
</tr>
<tr>
<td>32</td>
<td>0.176</td>
<td>0.00289</td>
<td>0.01296</td>
</tr>
<tr>
<td>35</td>
<td>0.176</td>
<td>0.05023</td>
<td>0.01773</td>
</tr>
<tr>
<td>36</td>
<td>0.176</td>
<td>0.09242</td>
<td>0.1892</td>
</tr>
<tr>
<td>38</td>
<td>0.176</td>
<td>0.04258</td>
<td>0.04211</td>
</tr>
</tbody>
</table>
CONCLUSIONS

The values of the standard deviation obtained in most cases are significantly large compared to the corresponding velocity. Therefore, their accuracy is suspect and no attempt is made to draw meaningful conclusions from these results. The failure to obtain useful data might have been due to the difficulty in keeping sufficient number of particles suspended in water without settling down. It is also possible that the instrumentation employed was not sophisticated enough for the type of measurements.

ACKNOWLEDGEMENT

The author is grateful to National Aeronautics and Space Administration for supporting this project.
REFERENCES


2. Albert Einstein, Annalen der Physik, 17 (1905), p 549

3. Harvey Fletcher, "A Verification of the Theory of Brownian Movements and a Direct Determination of the Value of NE for Gaseous Ionization", The Physical Review, V 33, No. 2 (1911), p 81


Figure 2. Details of Experimental Set-up
Figure 4. Velocity vs Frequency for 120 mm Lenses
<table>
<thead>
<tr>
<th>RANGE (METERS/SEC)</th>
<th>NUMBER (FRACTION OF TOTAL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OVER 1.600-02</td>
<td>9.149-01</td>
</tr>
<tr>
<td>1.600-02 to .441-02</td>
<td>0.000-00</td>
</tr>
<tr>
<td>1.441-02 to 1.280-02</td>
<td>0.000-00</td>
</tr>
<tr>
<td>1.280-02 to .120-02</td>
<td>0.000-00</td>
</tr>
<tr>
<td>.120-02 to 9.609-01</td>
<td>0.000-00</td>
</tr>
<tr>
<td>9.609-01 to .000-01</td>
<td>0.000-00</td>
</tr>
<tr>
<td>8.000-01 to 6.400-01</td>
<td>0.000-00</td>
</tr>
<tr>
<td>6.400-01 to 4.800-01</td>
<td>0.000-00</td>
</tr>
<tr>
<td>4.800-01 to 3.200-01</td>
<td>0.000-00</td>
</tr>
<tr>
<td>3.200-01 to 1.600-01</td>
<td>0.000-00</td>
</tr>
<tr>
<td>OVER 1.599-01</td>
<td>0.000-00</td>
</tr>
<tr>
<td>1.599-01 to 1.441-01</td>
<td>0.000-00</td>
</tr>
<tr>
<td>1.441-01 to 1.279-01</td>
<td>0.000-00</td>
</tr>
<tr>
<td>1.279-01 to 1.120-01</td>
<td>0.000-00</td>
</tr>
<tr>
<td>1.120-01 to 9.609-02</td>
<td>0.000-00</td>
</tr>
<tr>
<td>9.609-02 to 7.999-02</td>
<td>2.951-04</td>
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<tr>
<td>7.999-02 to .399-02</td>
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<tr>
<td>.399-02 to 4.800-02</td>
<td>0.000-00</td>
</tr>
<tr>
<td>4.800-02 to 3.199-02</td>
<td>0.000-00</td>
</tr>
<tr>
<td>3.199-02 to 1.599-02</td>
<td>8.470-02</td>
</tr>
</tbody>
</table>

AVERAGE VELOCITY = \( 6.867 \times 10^4 \) METERS/SEC

STANDARD DEVIATION = 1.131 \( \times 10^2 \)

Figure 5. Sample Printout of Data