LIGHT CURVES FOR "BUMP CEPHEIDS" COMPUTED WITH
A DYNAMICALLY ZONED PULSATION CODE

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ABSTRACT

The dynamically zoned pulsation code developed by Castor, Davis, and Davison has been used to recalculate the Goddard model and to calculate three other Cepheid models with the same period (9.8 days). This family of models shows how the bumps and other features of the light and velocity curves change as the mass is varied at constant period. This study, with a code that is capable of producing reliable light curves, shows again that the light and velocity curves for 9.8-day Cepheid models with standard homogeneous compositions do not show bumps like those that are observed unless the mass is significantly lower than the "evolutionary mass." The light and velocity curves for the Goddard model presented here are similar to those computed independently by Fischel, Sparks, and Karp. They should be useful as standards for future investigators.

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I. INTRODUCTION

Cepheids with periods around ten days are known to show secondary bumps in their light curves around the time of maximum light (Cox 1974). Until recently, however, theoretical light curves for Cepheids have been unsatisfactory because of contamination by spurious "zoning bumps" (Keller and Mutschlecner 1971). The new dynamically zoned stellar pulsation code developed by Castor, Davis, and Davison (1977) is capable of producing reliable light curves without these artifacts. We have used a somewhat improved version of this code to recalculate the Goddard model and to calculate three other 9.8-day Cepheid models. We will discuss here the systematic trends that appear in the light and velocity curves for this family of models.

II. THE MODELS

The family of models represents a one-parameter sequence with mass as the independent variable, subject to the constraints that: 1) the period is fixed at 9.8 days, and 2) the models lie along a line in the HR Diagram near the center of and roughly parallel to the instability strip. The characteristics of the models are given in Table 1. The actual masses of the models range from 62 to 99 percent of their respective "evolutionary masses" (corresponding to their luminosities with a standard mass-luminosity relation; see Table 1). The light and velocity curves for the four models are shown in Figures 1 and 2.
<table>
<thead>
<tr>
<th><strong>MODEL PARAMETERS</strong></th>
<th></th>
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<tbody>
<tr>
<td>Mass ($M_\odot$)</td>
<td>4.0</td>
<td>5.0</td>
<td>6.0</td>
<td>7.4</td>
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<td>3849</td>
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<td>5682</td>
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<td>Evolutionary Mass ($M_\odot^*$)</td>
<td>6.46</td>
<td>6.82</td>
<td>7.10</td>
<td>7.44</td>
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<td>Zones in Model (Opt. Thin)</td>
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<td>72(15)</td>
<td>73(12)</td>
<td>74(10)</td>
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<td>Artificial Viscosity Coeff. †</td>
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<td>2.0</td>
<td>2.0</td>
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<tr>
<td>Fractional Core Radius</td>
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<td>0.12</td>
<td>0.13</td>
<td>0.11</td>
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<tr>
<td>Period (days)</td>
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<td>9.78</td>
<td>9.79</td>
<td>9.78</td>
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<td>Peak KE ($10^{42}$ ergs, Expanding)</td>
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<td>5.30</td>
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<td>Periods Calculated ‡</td>
<td>46</td>
<td>52</td>
<td>37</td>
<td>51</td>
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<tr>
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<td>1170</td>
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<td>1230</td>
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<tr>
<td>Periods per Minute (CDC 7600)</td>
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<td>0.67</td>
<td>0.64</td>
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<tr>
<td>Comments</td>
<td>Goddard Evolutionary Model</td>
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</table>

*Log $[M_{\text{evolutionary}}/M_\odot] = 0.287 \log (L/L_\odot) - 0.195$.  
†Stellingwerf (1975) cutoff at 0.1.  
‡Includes periods of amplification.
Fig. 1. Bolometric light curves for four 9.8-day Cepheids. Zero phase is at maximum light, except for the 4.0 M\(_{\odot}\) (Goddard) model, where zero phase was shifted 0.2 units earlier for easier comparison. The model parameters are given in Table 1.
Fig. 2. Velocity curves for four 9.8-day Cepheids. The velocities (positive for radial expansion) are interpolated values that show the motion of the matter at optical depth two-thirds. The phase convention is as in Fig. 1. The model parameters are given in Table 1.
The original version of the code, used by Castor, Davis, and Davison (in Deupree 1976; see also Davis and Davison 1978) to calculate their Goddard model, incorporated an exponential artificial viscosity cutoff. The principal change in the code since then has been to go to the standard Stellingwerf (1975) cutoff. In addition, the value of the cutoff has been increased from one to ten per cent of the isothermal sound speed. These changes account for the higher limiting amplitude and the slightly different appearance of the light and velocity curves for the present Goddard model compared with the earlier one (their model II).

The light and velocity curves for the Goddard model in Figures 1 and 2 should be useful as reference light and velocity curves for the Goddard model. They are quite similar to those presented by Fischel, Sparks, and Karp (in Deupree 1976). It is comforting that there is now reasonable agreement between the light and velocity curves calculated by (at least two) independent investigators for the same model.

III. DISCUSSION OF THE LIGHT AND VELOCITY CURVES

Let us first discuss the features in the light and velocity curves for the Goddard model. Starting at minimum light, we see the phase of rapidly rising light, interrupted by the so-called "artificial viscosity dip." The dip is associated with the compression wave that stops the infall of the envelope. The artificial viscosity plays a role in that it spreads out the compression wave in the mesh. However, the dip itself may not be entirely artificial, since the compression wave undoubtedly raises the effective gravity briefly and can thus cause the light output to drop.
In this model, peak light is preceded by a shoulder in the light curve. There is a broad maximum in the velocity curve that begins at the phase of the shoulder and lasts through the light maximum. The light curve near maximum seems to be governed by changes in the effective gravity at optical depth two-thirds, as there is an increase in the gravity between the shoulder and the peak and a gravity minimum at the peak itself. However, the physical origin of the shoulder is not yet clear.

After maximum light, there is a dip and then the "Christy bump," which is associated with the reflection of a compression wave off the core. The bump in the velocity curve actually agrees in phase with the dip preceding the bump in the light curve. Therefore, in agreement with previous investigators, we identify the feature in the light curve as a "Christy dip," due to the compression that occurs when the reflected wave reaches the surface. Although the light and velocity curves are similar, they differ in detail. This raises some questions about the practice of comparing observed light curves with calculated velocity curves.

We now discuss how these features change as the mass is increased. In general, the Christy bump or dip moves down the descending branch. At the same time, it becomes less prominent, especially in the velocity curve. In fact, there is no bump in the evolutionary mass \((7.4 \, M_\odot)\) model. This confirms the conclusion that, with a standard homogeneous composition, the mass must be less than the evolutionary mass to show bumps like those that are observed.
Another interesting change that occurs as the mass is increased involves the light curve near maximum. In the 5.0 $M_\odot$ model, what was the shoulder in the Goddard model has become the peak, while the peak in the Goddard model has evolved into a bump following maximum light. This bump then moves down the descending branch and becomes less prominent as the mass is further increased.

The ascending branch also changes with increasing mass. It becomes steeper and the "artificial viscosity dip" moves up the ascending branch until it nearly coincides with the peak in the evolutionary mass model. The compression wave associated with the dip increases greatly in strength as this happens. It should be emphasized, though, that this phenomenon is probably amplitude-dependent, and that the limiting amplitude, which also increases with mass (cf. Table 1), may depend on subtle physical or numerical effects in the calculation.

Another interesting phenomenon occurred in the 4.0 and 5.0 $M_\odot$ models, although it is not apparent in the light or velocity curves. In these models, at the phase of rapid expansion, when the hydrogen ionization front is racing inward, a shock forms on the neutral side following the front. This can be understood as a transition in the nature of the ionization front from D to R type (Kahn 1954, Castor 1966). Conditions change too rapidly for a steady-state R-type front to develop, and the front soon returns to its usual D-type behavior. This transition occurs about the time of the Christy dip in the Goddard model, and about the time of the dip just after maximum light in the 5.0 $M_\odot$ model. The front is near optical depth unity at this time, so there should not be any major changes in the emergent spectrum or energy distribution. However,
the following shock may be connected with the weak redshifted Ca II emission features that appear just after maximum light in Beta Doradus (Gratton 1953). The ionization front transition will be discussed in detail elsewhere (Adams and Castor, in preparation).

IV. CONCLUSIONS AND FUTURE PLANS

We have calculated a family of 9.8-day Cepheid models with a code that can produce reliable theoretical light curves. We find that only if the mass is significantly less than the "evolutionary mass" do the bumps in the light and velocity curves occur near maximum light, where they are observed to occur in real ten-day Cepheids (Cox 1974). However, only models with homogeneous population I compositions were calculated. It thus appears that we require either reduced masses, inhomogeneous compositions, or some change in the physics in order to match the observed light curves of ten-day Cepheids.

We plan to continue this program by calculating families of models at other periods to see how the light and velocity curves vary with period. Our goals will be to learn more about the mechanisms that produce bumps in the light and velocity curves and, in general, to see if the Hertzsprung sequence (variation of bump location with period) can be reproduced theoretically. We would also like to calculate some inhomogeneous models to study Art Cox's proposed solution to the "mass anomaly."

Finally, we conclude that there is now a consensus standard Goddard model that can be used as a benchmark for any new pulsation code. It is entirely fitting that this consensus should be reported here at the
Goddard Space Flight Center only two conferences after the idea of a standard Goddard model was originated.

ACKNOWLEDGEMENTS

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REFERENCES


Discussion

J. Wood: Do you have a temperature range for this β Dor Goddard model? From the G-band, I found about 1500° over the cycle, but Rogers and Bell found about 450° from Ha profiles. Sidney Parsons found about 900° from UBV work.

Adams: First of all, I didn't make any attempt to match a particular star. This was simply a theoretical sequence, so the parameters may not be correct for a particular star. The temperature data are available, and could be used to plot loops in the color-color diagram. However, there's a larger question that has to do with limiting amplitudes. The light amplitude shown here was rather large for a Cepheid; therefore, any behavior that depends on the limiting amplitude is questionable. But I don't know whether the temperature range depends on the limiting amplitude.

Wesselink: Have you made any comparison between your light curves and observed light curves?

Adams: I've just taken a cursory look at the observed light curves. I can't claim that I can match any observed light curve. The most important conclusion is that we can calculate good light curves, and they still show the traditional mass anomaly with the bumps. Let me add that even the features seen in the superb light curves produced by Pel and Lub still leave a little freedom for the theoretician to imagine various things happening. It's an extremely difficult problem to get sufficient data.

Connolly: What is causing that sharp dip at rising light and is it a dip, not actually a bump?
Adams: It is a dip. In the theoretical calculations it is due to the fact that when the envelope is falling in, it is stopped by a compression wave or shock wave. The intense compression for a short period of time steals some luminosity and puts it into ionization or compression. Therefore there is a sharp drop in the luminosity as the shock passes through the photosphere at $\tau = 2/3$.

Wesselink: I know one observed light curve -- VZ Her -- which resembles one of yours. It has a dip on the rising light.

Adams: That is interesting. However, I would not go so far as to say that the dip has to exist theoretically, even though it shows up in the models. I think that it is difficult to find any observer who has a well-documented dip in the rising light because that's a very hard thing to see. It looks a lot like a cloud going by, lasting only a very short time.

Pel: But they exist for longer period stars, with 15-30 day periods. There is a "standstill."

Adams: That feature may be a bump, related to the bumps that define the Hertzsprung progression.