I. INTRODUCTION

For the past two years my collaborators and I at the University of Texas at Austin have worked toward establishing an independent distance scale for Cepheid variables. The basis of our approach is to make use of the apparent magnitude and the visual surface brightness, inferred from an appropriate color index, to determine the angular diameter variation of the Cepheid. When combined with the linear displacement curve, obtained from the integrated radial velocity curve, the distance and linear radius are determined. The great attractiveness of the method is its complete independence of all other stellar distance scales. Even though a number of practical difficulties currently exist in implementing the technique, our preliminary results may nonetheless be useful by virtue of this independence.

II. VISUAL SURFACE BRIGHTNESSES

The critical component of this technique is the relation between a color index and the visual surface brightness parameter $F_V$, defined by

$$F_V = 4.2207 - 0.1V_o - 0.5\log \varphi,$$

where $V_o$ is the apparent visual magnitude corrected for interstellar extinction and $\varphi$ is the stellar angular diameter in milliseconds of arc. Measured values of the stellar angular diameter are now available for nearly one hundred stars (Barnes, Evans, and Moffett 1978). Of the readily available photometric indices for these stars, we have found the Johnson (1966) V-R index to be the most tightly correlated with $F_V$ (Barnes and Evans 1976; Barnes, Evans, and Parsons 1976; Barnes et al. 1978). Knowledge of $(V-R)_0$ alone permits $F_V$ to be inferred from the mean relation with a typical uncertainty of $\pm 0.033$ over the spectral type range $04 - M8$. This uncertainty is entirely attributable to the observational errors. No luminosity class effects are apparent in the relation.

Although no Cepheid variable has an observed angular diameter, the presence of several other kinds of variables on the relation, even though observed at random times in their cycles, led Barnes, Evans and Parsons (1976) to suggest that the $F_V - (V-R)_0$ calibration would apply to Cepheids throughout their pulsations. Barnes et al. (1977) examined this
assumption by the strategem of exploring its consequences when applied to Cepheids for which the requisite photometry and radial velocities had been published. They found the assumption to be verified within the rather loose constraints permitted by the quality of the data (see also Evans 1977).

In the present discussion I will explore the validity of this assumption by determining the \( F_V - (V-R)_0 \) relation from Cepheids themselves and comparing the result to the relation for stars of known angular diameter. (This work will be published elsewhere in its entirety by T.G. Barnes and B.J. Beardsley.)

III. CEPHEID SURFACE BRIGHTNESSES: THE SLOPE

Thompson (1975) has devised a method for determining the visual surface brightness of a Cepheid, to within an unknown additive constant, from photometry and radial velocities. His method requires a priori knowledge of the star's linear radius, but not of the distance or luminosity. To obtain the linear radii, we have adopted Balona's (1977) recent results for the Cepheid period-radius relation:

\[
\log R/R_0 = 1.213 + 0.6021 \log P. \tag{2}
\]

This relation assumes a conversion factor between radial velocity and pulsational velocity of \( p=1.31 \).

The photometric and radial velocity data were essentially the same as used by Barnes et al. (1977) and referenced therein. Color excesses were taken from Parsons and Bell (1975). We adopted \( R = 3.3 \) and \( E(V-R)/E(B-V) = 0.84 \).

To ensure that the problem of phase-matching the photometric and radial velocity data was minimized, we used only those eleven Cepheids (with BVRI data) for which simultaneous photometry and radial velocities have established the relative phases (Breger 1967, Evans 1976). We simply shifted the radial velocity phases until minimum radial velocity occurred at the appropriate phase relative to the \( V \) light curve. (The mean shift was \(-0.005\) in phase.)

For each Cepheid, Thompson's method yields the variation in \( F_V \) with color index to within an unknown constant. Figure 1 shows a representative result. In all eleven cases the data are consistent with a linear relation between \( F_V \) and \( (V-R)_0 \) and with no distinction between rising and falling branches of the light curve. This confirms the observational and theoretical arguments for linearity given by Barnes et al. (1977).

Figure 2 shows that the slopes are independent of period and have a scatter about their mean value in accordance with the observational uncertainties. The mean slope is \(-0.360 \pm 0.010\) (s.e.m.). This slope is somewhat less negative than the best fit to the A-F-G stars of known angular diameter given by Barnes et al. (1977), in agreement with their, and Evans' (1977), suspicions.

Both the linearity and common slope are also found if the surface brightness is correlated with \( (B-V)_0 \). In this case we obtain \(-0.211 \pm 0.003\) (s.e.m.), in comparison with Thompson's value of \(-0.211 \pm 0.003\) and Balona's value of \(-0.215 \pm 0.002\).
Figure 1: The variation in $F_V$ with $(V-R)_0$ for X Cygni. Dots represent phases of falling light and crosses, phases of rising light. The line is a least-squares fit to the data.

Figure 2: The individual Cepheid slopes plotted against the logarithm of the period. The error bars are one sigma values based on the uncertainties in the photometry, the phase-matching, and the adopted linear radii. The mean slope and its uncertainty are shown.
IV. CEPHEID SURFACE BRIGHTNESSES: THE ZERO POINT

Until a Cepheid angular diameter is actually measured, the zero point must be acquired either by assuming that Cepheids have surface brightnesses similar to non-variable F supergiants or by using model atmosphere results. Happily, both choices give the same result for the short period Cepheids.

Parsons (1969, 1970a, 1971) has demonstrated that his model atmosphere fluxes for F and G supergiants accurately match the observed fluxes in the blackbody six-color system for a large selection of variables and non-variables. One parameter obtained in the fitting procedure is the stellar angular diameter, tabulated by Parsons (1970b) and Parsons and Bouw (1971). The uncertainties in the angular diameters are +0.03 dex from the fitting procedure (Parsons 1971), +0.02 dex from the model physics (Parsons 1978, private communication), and +0.02 dex from the interstellar extinction (our estimate). The last uncertainty cancels out when eq. (1) is used to compute the visual surface brightness parameter.

Only one star in Parsons' lists has an observed angular diameter, the F8Ia star 8 CMa. Parsons (1970b) gives log $\phi = 0.56 \pm 0.04$, whereas Hanbury Brown et al. (1974) measured $0.56 \pm 0.06$. This gives us considerable confidence that his mean angular diameters for Cepheids are accurate.

There are eighteen Cepheids with both angular diameters and BVRI photometry. We have computed visual surface brightnesses for these using eq. (1). Recall that the interstellar extinction term in the computation of log $\phi$ cancels the same term in eq. (1) leaving $F_V$ independent of interstellar extinction. (Interstellar reddening still enters to the extent that incorrect elimination of it affects the fit of the model fluxes to the observed fluxes.)

Figure 3 compares the visual surface brightnesses of the Cepheids to those for similar temperature stars of known angular diameter. For Cepheids bluer than $(V-R)_0 = 0.6$ the agreement is excellent. In particular note the agreement with 8 CMa at $(V-R)_0 = 0.47$. Because of the paucity of observed angular diameters in this color range, the mean curve from Barnes et al. (1978) is very poorly determined. It could easily be altered to fit simultaneously the short period Cepheids and the few observed angular diameter stars.

The long period Cepheids are clearly discordant. After examining the uncertainties involved, we are convinced that the long period Cepheids cannot lie on the same relation as the non-variable stars of measured angular diameter.

Ignoring the Cepheids to the red of $(V-R)_0 = 0.6$, we find the model atmosphere values of $F_V$ to be represented by the regression line

$$F_V = 3.945 - 0.338 (V-R)_0 \pm 0.025 \pm 0.048$$

The slope of this relation is in agreement with the value found in the previous section by a total different approach. However, the uncertainty in the slope in eq. (3) is considerably larger than found earlier. Hence we have adopted the previous value of the slope and used the model atmosphere results to establish the zero point:

$$F_V = 3.957 - 0.360 (V-R)_0 \pm 0.006 \pm 0.010$$
Figure 3: The relation between visual surface brightness parameter and \( (V-R)_o \) for Cepheids (plus signs) and for stars of measured angular diameter (all other symbols). Uncertain values are enclosed in parentheses. The smooth curve is the fit adopted by Barnes et al. (1978).

where both uncertainties are standard errors of the mean. This line is shown in Figure 3.

V. A PRELIMINARY DISTANCE SCALE

With the relation between \( F_v \) and \( (V-R)_o \) now established, we return to the determination of the Cepheid distances. We have used eq. (4) and the technique described by Barnes et al. (1977) to determine distances for the seven short-period Cepheids for which the requisite data exist and for which Evans (1976) has established phase matching (\( \eta \) Aql, RT Aur, \( \delta \) Cep, \( W \) Gem, \( \zeta \) Gem, \( S\) Sge, and \( T\) Vul). A weighted mean of the seven yields a distance scale \((14.2 \pm 6.8\%)\) larger than the Fernie and Hube (1968) scale and \((18.2 \pm 6.3\%)\) larger than the Sandage and Tammann (1969) scale.

The above uncertainties are the random uncertainties only, whereas more realistic values should include the contributions of systematic errors. There are three of these to consider:

1) The uncertainty in the conversion factor from radial velocity to pulsational velocity enters directly into the distance scale. With a conservative estimate of \( p = 1.31 \pm 0.06 \), this contributes \( \pm 4.6\% \).

2) The uncertainty in the slope in eq. (4) enters in a manner dependent upon the mean color and amplitude of the Cepheid. The uncertainty in the distance due to this ranges from \( \pm 4.2\% \) to \( \pm 13.5\% \), with a mean of \( \pm 7.2\% \) which we will adopt.

3) The uncertainty in the zero point in eq. (4) enters exponentially, giving \( \pm 2.8\% \). However, this represents only the observational and theoretical scatter in the zero point and does not include any systematic
uncertainty in the model physics. As mentioned earlier, the uncertainty in model physics leads to +0.02 dex in log φ, or another 4.7% in distance.

Altogether the systematic errors are ±10.2% in distance.

Combining the variances, we find a preliminary distance scale (14.2 ±12.2)% larger than Fernie and Hube, and (18.2 ±12.0)% larger than Sandage and Tammann. Averaging these and expressing the result as a change in absolute magnitude, we find the short-period Cepheids 0.33 ± 0.23 mag brighter than on the old distance scale. This is quite consistent with the currently recommended change in the Hyades distance modulus.

In conclusion, we have shown that the surface brightnesses of Cepheid variables may be determined from the Cepheids themselves, that for short-period Cepheids the resultant values are in good agreement with non-variables of the same color, and that the preliminary distance scale to which these values lead supports full effect of the new Hyades distance scale upon Cepheid luminosities.

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REFERENCES

Discussion

Schmidt: Could you tell us how the radii are affected by the zero point?

Barnes: I haven't done that for this group of Cepheids. It will increase the radii somewhat over what I have obtained previously.

Wesselink: What does the value -0.33 mean?

Barnes: The -0.33 magnitude is the shift in the mean distance modulus or the mean absolute magnitude that is implied by this result on seven Cepheids.

Evans: In your graph for X Cyg, in the surface brightness-(V-R) relation, do you think you would get a somewhat steeper slope if you confined yourself to the descending branch of the light curve? You'd definitely get different slopes from different branches.

Barnes: There are typically only a few points on the rising branch; I think that there are only four for X Cyg. I don't think that it is statistically significant that three lie above and only one below the curve. Looking at the other 10 Cepheids, there seems to be a random scatter of the points about the rising and falling branches.

A. Cox: You've already published data from seven Cepheids. Are these the same stars, and is this the same answer? If not, how different is it?

Barnes: The 1977 paper used these seven Cepheids plus two others. The present result is different by about 1.5 $\sigma$ -- about 0.3 mag brighter than the previous result in absolute magnitude. The reason for that is threefold.
In our early work, we didn't know about Nancy Evans' work on relative phasing, so we just shifted the curves until they looked right. Secondly, we have an improved technique for interpolating in the displacement curve at the phases for which photometry exists. Finally, the 1977 computations were based on a preliminary surface brightness -- color relation from stars of known angular diameter. The present results are based on a relation inferred from the Cepheids themselves.