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HANDLING QUALITIES OF LARGE FLEXIBLE CONTROL-CONFIGURED AIRCRAFT

Grant No. NSG 4018

Final Technical Report

for Period

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Introduction

This is a final report on Grant No. NSG 4018. The project began on January 1, 1979 and was to have terminated on December 31, 1979. A no-cost time extension was requested and approved extending the termination date to June 30, 1980.

Dr. Robert L. Swaim, Principal Investigator, has devoted ten percent time throughout the project. One Ph.D. student, Mr. Supat Poopaka, has been employed fifty percent time throughout the project. Mr. Poopaka is completing his Ph.D. thesis on the project and expects to complete it and take the final examination, where he defends the thesis, in September 1980. Three copies of the thesis will be submitted to the Technical Officer, Mr. Glenn Gilyard, and two copies to the NASA Scientific and Technical Information Facility in late September.

As the details of the research will be documented in the thesis, this final report on the project will be just a brief summary.

Problem Statement

The effects on handling qualities of low frequency symmetric elastic mode interaction with the rigid body dynamics of a large flexible aircraft was analyzed by use of a mathematical pilot modeling computer simulation. An extension of the optimal control model for a human pilot was made so that the mode interaction effects on the pilot's control task could be assessed. Pilot ratings were determined for a longitudinal tracking task with parametric variations in the undamped natural frequencies of the two lowest frequency symmetric elastic modes made to induce varying amounts of mode interaction.

A summary of our mathematical formulation of the problem is contained in Attachment 1, which is a paper presented by Mr. Poopaka at the Eleventh
Results

Our approach of relating numerical performance index values associated with the frequency variations used in several dynamic cases to a numerical Cooper-Harper pilot rating has proved successful in discriminating when the mathematical pilot can or cannot separate rigid from elastic response in the tracking task. The detailed presentation of these results will be contained in the forthcoming Ph.D. thesis by Mr. Poopaka. An outline of the thesis content is presented below.

Thesis Outline

I. Introduction
   A. Handling Qualities and Pilot Ratings
   B. Pilot Rating Assessment Techniques
   C. Background on the Effects of Dynamic Aeroelasticity on Handling Qualities
   D. Objectives and Scope of Study
   E. Plan of Presentation

II. Literature Survey
   A. Past Results on Elastic Airplane Research

III. Equations of Motion
   A. Longitudinal Equations of Motion for a Flexible Airplane
   B. Turbulence Model
   C. Attitude Director Equations
   D. B-1 Flight Condition

IV. Pilot Modeling
   A. Manual Control and Pilot Models
   B. Optimal Control Model
   C. Internal Model
   D. Closed-Loop Performance Equations
   E. Hess's Pilot Rating Method
Thesis Outline (cont.)

V. Effects of Elastic Mode Interaction on Handling Qualities
   A. Define Controllable Boundary
   B. Controllable Boundary Based on Open-Loop Parameters

VI. Conclusions and Recommendations

VII. Selected Bibliography

VIII. Appendix A. Numerical Values of Stability Derivatives and Equations of Motion

IX. Appendix B. Derivations Related to Singular Perturbation Theory

X. Appendix C. Computational Algorithms

Project Reports and Presentations

A list of the reports and presentations generated on the project is given below.


4. Supat Poopaka, "Handling Qualities of Large Flexible Aircraft," presented at Eleventh Southwestern Graduate Research Conference in Applied Mechanics, Oklahoma State University, April 11-12, 1980.


HANDLING QUALITIES OF LARGE FLEXIBLE AIRCRAFT

Supat Poopaka
School of Mechanical and Aerospace Engineering
Oklahoma State University
April 1980

Summary

The effects on handling qualities of elastic mode interaction with the rigid-body dynamics of a large flexible aircraft are discussed. An extension of the optimal control model for the human operator is made so that the mode interaction effects on the pilot's control task can be assessed.

Introduction

The handling qualities are the flying qualities of a piloted aircraft. They are the characteristics of an aircraft that govern the ease and precision with which a pilot is able to perform the control task required in support of the aircraft mission flight phase. The pilot's subjective opinion on the handling qualities is called a pilot rating and usually based on the Cooper-Haper pilot rating scale [1].

The effects of elastic mode interaction on the handling qualities have been reported recently in Reference 2 based on a ground-based pilot-flown simulation. In this report we develop an extension of the optimal control model for the human operator so that the mode interaction effects on handling qualities can be easily assessed.

Problem Statement

The dynamics of an aircraft to be controlled by the human pilot are described by a set of small perturbation equations of motion:

\[
\dot{x}_a(t) = A_a x_a(t) + B_a u(t) + E_a w(t)
\]

\[
y_a(t) = C_a x_a(t) + D u(t)
\]

(1)
where $x_a = \text{col.} \{x_{ag}, x_{ar}, x_{ae}\}$, $x_{hy}$ is a turbulence state vector, $x_{ar}$ is a rigid body states, $x_{ae}$ is an elastic body states, $u$ is a control input, $y_a$ is a displayed output, and $w$ is a zero-mean, gaussian, white noise process with autocovariance.

$$E\{w(t_1)w(t_2)\} = \mathcal{W} \delta(t_1 - t_2)$$

For a cruise, level flight condition, eqn. (1) represents the longitudinal equations of motion and $y_a(t)$ is an attitude director equation.

$$y_a(t) = \theta(t) - \sum_{j=1}^{n} \phi_j(x_p) \xi_j(t) = \theta(t) - \theta_e(t)$$

where $x_p$ indicates pilot fuselage station, $\phi_j(x_p)$ the slope of the $j^{th}$ symmetric elastic mode at that station, $\xi_j(t)$ the generalized displacement, $\theta(t)$ the rigid body pitch angle, and $\theta_e(t)$ the elastic contribution to total pitch angle (see Fig. 1).

The task of the pilot is to keep the rigid pitch response following the command director. Since only the total pitch response is shown to the pilot, therefore he has to distinguish rigid body motion from the total motion. We want to establish the boundary between when the pilot can visually separate the rigid body motion from the total motion in terms of undamped natural frequencies of the elastic modes. The rigid body
mode parameters will be maintained at the values known to give good
handling qualities by the pole-placement technique while the elastic
mode frequencies will be parametrically varied to introduce the mode in-
teraction. The pilot model developed for computer simulation of this
task is presented in the next section.

The Pilot Model

The optimal control model for the human operator developed here is
a modified version of the standard optimal control model[3,4]. The model
structure is illustrated in Fig. 2.

\[
\begin{align*}
\text{Aircraft Dynamics} & \quad \xrightarrow{\text{Display}} \quad \text{Human Operator Model} \\
& \quad \xrightarrow{\text{Delay}} \quad \text{Neuromotor} \quad \xrightarrow{\text{Control Gain}} \quad \text{Estimation} \\
& \quad \xrightarrow{\text{Log \ (TA^2 + I) }^{-1}} \quad \text{Control Output} \\
& \quad \xrightarrow{\text{Estimation}} \quad \text{Perceived Information} \\
& \quad \xrightarrow{\text{Display}} \quad \text{Output} \\
& \quad \xrightarrow{\text{Human Operator Model}} \quad \text{Error} \\
\end{align*}
\]

Fig. 2 Optimal Control Model for Human Operator

The relation \( u(t) = r(t-\tau) \) is approximated by a first-order Padé
approximation which can be expressed in the state variable form as

\[
\begin{align*}
\dot{x}(t) &= z(t) - r(t) \\
\dot{z}(t) &= -\frac{2}{\tau} I z(t) + \frac{\tau}{c} I r(t) \\
\end{align*}
\] (4)

The perceived information \( y_p(t) \) is a noisy version of \( y_a(t) \), i.e.,

\[
y_p(t) = y_a(t) + v_y(t)
\] (5)
where \( v_y(t) \) is a zero-mean, gaussian, white noise with autocovariance

\[
E\{v_y(t_1)v_y'(t_2)\} = \gamma_y \delta(t_1-t_2)
\]  

(6)

and \( \gamma_y \) is known to scale with mean square of \( y \):

\[
\gamma_{y^2} = e_{y^2} E\{y(t)^2\}
\]  

(7)

where the observation noise/signal ratio \( e_{y^2} = 0.01\pi \).

The pilot generates a command control input \( m(t) \) which is then transformed to \( r(t) \) via the relation

\[
T_n \dot{r}(t) + r(t) = m(t) + \nu_m(t)
\]  

(8)

where \( T_n \) is the neuromotor lag matrix and \( \nu_m(t) \) is a zero-mean, gaussian, white noise, with autocovariance

\[
E\{\nu_m(t_1)\nu_m'(t_2)\} = \gamma_m \delta(t_1-t_2)
\]  

(9)

and \( \gamma_m \) is known to scale with \( E(m_i(t)^2) \), i.e.,

\[
\gamma_{m_i} = e_{m_i} E\{m_i(t)^2\}
\]  

(10)

where the motor noise/signal ratio \( e_{m_i} = 0.003\pi \).

Equations (4) and (8) may be augmented to (1) to define an augmented system of equations

\[
\begin{align*}
\dot{x}_c(t) &= A_c x_c(t) + B_c m(t) + E_c \omega_c(t) \\
y_a(t) &= C_c x_c(t)
\end{align*}
\]  

(11)
The aircraft dynamics of eqn. (1) can be written in the form

\[ \dot{x}_1(t) = A_1 x_1(t) + A_{12} x_2(t) + B_1 u(t) + E_1 w(t) \]

\[ \dot{x}_2(t) = A_{21} x_1(t) + A_{22} x_2(t) + B_2 u(t) + E_2 w(t) \]

\[ y_d(t) = C_1 x_1(t) + C_2 x_2(t) + D u(t) \]  

(12)

where \( x_1 \) is a vector of the rigid body state variables, \( x_2 \) is a vector of the elastic mode state variables, and \( \mu \) is a small positive scalar constant which can be unknown in this study.

In performing estimation and control command generation, it is necessary for the pilot to have a knowledge of the aircraft dynamics, the human limitation parameters \( (\tau, \tau_n, V_y, V_m) \) and weighting coefficients of the performance index. This knowledge is called the internal model. In our aircraft dynamics model, the display consists of a slowly varying part due mainly to the rigid body dynamics and a high frequency oscillation part from the elastic mode. From the past experiment [2] it is evident that the pilot ignores the low amplitude high frequency oscillation part of the display. Therefore, in the pilot model, we will use the slowly varying dynamics subsystem as the internal model.

By the singular perturbation technique [5, 6], we can decompose the slowly varying part from the system (12) by letting \( \mu \to 0^+ \), from which we get

\[ \dot{x}_d(t) = A_d x_d(t) + B_d u(t) + E_d w(t) \]

\[ y_d(t) = C_d x_d(t) + D_d u(t) \]  

(13)
where
\[A_d = A_{II} - A_{II} A_{II}^{-1} A_{II} B_{II}\]
\[B_d = B_1 - A_{II} A_{II}^{-1} B_{II}\]
\[C_d = C_1 - C_{II} A_{II}^{-1} A_{II}\]
\[D_d = D_1 - C_{II} A_{II}^{-1} B_{II}\]
\[E_d = E_1 - A_{II} A_{II}^{-1} E_{II}\]

The pilot control task is assumed to be adequately reflected in the choice of a control \(r(t)\) that minimizes the performance index
\[
J(r) = \lim_{T \to \infty} E\left\{\int_0^T \left[ y_d(t) Q_y y_d(t) + \dot{r}(t) Q_r \dot{r}(t) \right] dt \right\}
\]
conditioned on the perceived information \(y_p(t)\). The command control \(m(t)\) of eqn. (8) is then given by
\[
m(t) = -L_{opt} \hat{x}_e(t) = -L \hat{x}_e(t)\]
where
\[
\hat{x}_s = [\hat{x}_d, z]^{\prime}, \quad \hat{x}_e = [\hat{x}_e, z, y]^{\prime}, \quad T_n = P_{zz} P_{zz}^{-1}
\]
\[
L = [L_{opt}, 0], \quad L_{opt} = P_{zz}^{-1} \hat{x}_s, \quad P = \begin{bmatrix} P_{II} & P_{IE} \ P_{EI} & P_{II} \end{bmatrix}
\]
\[
A_0 P + P A_0 + C_0 Q_y C_0 - P B_0 Q_r^{-1} P = 0
\]
\[
A_0 = \begin{bmatrix} A_d & B_d & -B_d \\ 0 & -\frac{1}{N^2} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C_0 = [C_{II} D_{II}]^{-1} D_{II}
\]

The weighting \(Q_r\) is chosen such that the resulting \(T_n\) is equal to 0.1 second, which is a typical value for the human neuromotor lag.

The state \(\hat{x}_e(t)\) is the best estimate of \(x_e(t)\) generated by a Kalman filter,
\[
\hat{x}_e(t) = A_e \hat{x}_e(t) + B_e m(t) + \Sigma C_e V_e^{-1} [y_p(t) - C_e \hat{x}_e(t)]
\]
where $\Sigma$ satisfies the equation

$$\begin{align*}
0 &= A_{\xi}\Sigma + \Sigma A_{\xi} + E_0\bar{W}_k E_0' + \Sigma C_0' V_0^{-1} C_0 \Sigma \\
A_{\xi} &= \begin{bmatrix}
A_k & B_k & -B_k \\
0 & 0 & 4/\varpi^2 \\
0 & 0 & -1/\varpi^2
\end{bmatrix},
B_k = \begin{bmatrix}
0 \\
0 \\
T \varpi
\end{bmatrix},
E_0 = \begin{bmatrix}
E_d & 0 \\
0 & 0
\end{bmatrix}
\end{align*}$$

(18)

$$W_k = \begin{bmatrix}
W_k' \\
0 \\
V_0
\end{bmatrix}$$

Combining eqns. (5) with (11), (15) and (17) yields the closed loop system

$$\begin{align*}
\dot{x}_c(t) &= A_c x_c(t) - B_c L \ddot{x}_c(t) + E_c u_c(t) \\
\ddot{x}_c(t) &= (A_k - B_k L^2) \ddot{x}_c(t) + \Sigma C_k' V_0^{-1} \left[ C_k \dot{x}_c - C_0 \ddot{x}_c + u(t) \right]
\end{align*}$$

(19)

The covariances $E(y_a(t)^2)$ and $E(m_i(t)^2)$ for eqns. (7) and (10) can be obtained by solving eqns. (19) for cov.($x_c, \ddot{x}_c$).

**Conclusion**

The technique developed here is being applied to determine the handling qualities of a large flexible aircraft of Reference 2 in a longitudinal tracking task with parametric variations in the undamped natural frequencies of the two lowest frequency, symmetric elastic modes made to induce varying amounts of mode interaction. The pilot rating is found by using the technique of Reference 7.

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References


