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NASA

Technical Memorandum 80656

(NASA-TM-80656) RAIN RATE RANGE PROFILING FROM A SPACEBORNE RADAR (NASA) 28 p HC A03/MF A01 CSCL 04B

N80-25989

Unclas

G3/47 22944

Rain Rate Range Profiling from a Spaceborne Radar

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MARCH 1980

JUN 1980
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NASA STI FACILITY
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National Aeronautics and
Space Administration

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ABSTRACT

At certain frequencies and incidence angles the relative invariance of the surface scattering properties over land can be used to estimate the total attenuation and the integrated rain rate from a spaceborne attenuating-wavelength radar. In this paper the technique is generalized so that rain rate profiles along the radar beam can be estimated, i.e. rain rate determination at each range bin. This is done by modifying the standard algorithm for an attenuating-wavelength radar to include in it the measurement of the total attenuation. Simple error analyses of the estimates show that this type of profiling is possible if the total attenuation can be measured with a modest degree of accuracy.

RAIN RATE RANGE PROFILING FROM A SPACEBORNE RADAR

Robert Meneghini

Introduction

To achieve adequate resolution in remotely sensing precipitation from space, investigators have reconsidered the potential of attenuating wavelength radars. As first shown by Hitschfeld and Bordan [1], the presence of attenuation can severely limit the accuracy in the determination of rain rate. To circumvent this problem, a number of dual measurement techniques have been proposed. Suitable for spaceborne radars are the dual-attenuating wavelength [2,3] and the surface reference techniques [4]. For ground based measurements, additional information is provided by an independent measurement of rainfall rate at some point along the radar beam [1] or by an estimate of attenuation between the radar and a fixed target [5]. One other method, the combined radar-radiometer technique [6], appears to have applications to both ground based and spaceborne measurements.

The fixed target and surface reference methods were originally analyzed from the perspective of providing average or integrated rain rate [4,5]. The observation upon which the techniques rest is that the attenuation factor can be found by forming the ratio of return powers from the target in the presence and absence of precipitation along the beam. The estimate of integrated rain rate then follows upon using an empirical law that relates rain rate to attenuation.

In these techniques, however, no use is made of the information provided by the measured reflectivity factors. The major purpose of the paper is to show that these reflectivity factor measurements along with the total attenuation can be used to give range profiled rain rates, i.e. rain rate determination at each range bin.

To do this, we can begin with either the Hitschfeld-Bordan [1] or the iterative algorithms of Ormsby [7], both of which depend only on the measured reflectivity factors and several empirically derived parameters. Although these algorithms themselves provide bin by bin estimates of rainfall rate, they are susceptible to large errors whenever the total attenuation is significant [1, 7-10]. For

spaceborne radars, the resolution requirement makes it desirable to operate at higher frequencies where the attenuation will be non-negligible for all but very small rain rates. Consequently these techniques in themselves are of limited applicability from space. We can, however, reformulate them so as to account for the fact that the total attenuation has been estimated by means of the surface reference method mentioned above. By incorporating this additional information into the algorithm, we can usually reduce the errors in the estimated rain rate.

The foregoing correction procedure is reminiscent of that proposed by Hitschfeld and Bordan [1]. In their technique an independent point measurement of rain rate along the radar beam (obtained, for example, by means of a rain gage) is used to correct the rain rate estimates up to that point. The difference between the two procedures is that one uses a point measurement of rain rate, the other an estimate of total attenuation. It should be noted that Hitschfeld-Bordan correction technique, like the fixed reference target technique, applies only to ground based radars pointed near the horizon. The surface reference technique which uses the ground as a reference target is suitable for spaceborne or airborne radars.

In this paper we will consider only those estimates which are based in the Hitschfeld-Bordan algorithm since the algorithms derived from it have been shown to be superior to those based on the iterative algorithms. Five estimates will be analyzed: the original Hitschfeld-Bordan (H-B) algorithm, the H-B algorithm that incorporates an estimate of the attenuation, and the H-B algorithm that incorporates a point measurement of rain rate. For each of the last two cases, two distinct estimates are derived depending on whether the additional information (attenuation or point rain rate) is used to correct, the calibration constant or, the parameter a where $k = aZ^{\beta}$, k is the attenuation coefficient in db/km and Z is the reflectivity factor in mm^6/m^3.

Once the five estimates have been written down a simple two-part error analysis is carried out. In the first part we assume that the errors in certain parameters can be characterized by the true value plus a constant offset term. This assumption is helpful in isolating the effects and assessing the relative importance of the various errors. In the second section, the parameters are assumed to be random

with mean values that may be offset from the true value. With these less stringent assumptions, the error analysis should provide an indication of the conditions under which the estimates can be used.

Rain Rate Algorithms

For the rain rate in the jth range bin, the Hitschfeld-Bordan estimate can be written [10],

$$R_j^{H\cdot B} = aZ_{mj}^b \left[1 - K\beta s \sum_{i=1}^{j} \epsilon_i Z_{m_i}^{\beta} \right]^{-b/\beta} \tag{1}$$

where

$$K = 0.2\, a\, \ln(10)$$

$$\epsilon_i = \begin{cases} 1 & i \neq j \\ \frac{1}{2} & i = j \end{cases}$$

The measured reflectivity factor, Z_{mj}, the unattenuated reflectivity factor, Z_j, the radar return power, P_{R_j}, and the calibration constant C are related via the equations,

$$Z_{m_j} = A_j\, Z_j = r_j^2\, P_{R_j}/C \tag{2}$$

where r_j is the distance from the radar to the jth range bin, and where the attenuation factor at the jth range bin, A_j, is defined by

$$A_j = 10^{-0.2\int_0^{r_j} k(x)dx} \tag{3}$$

where k is the attenuation coefficient in db/km. Using the empirical relation $k = aZ^{\beta}$, (3) can be written as

$$A_j = 10^{-0.2\,a\int_0^{r_j} Z^{\beta}(x)dx} \tag{4}$$

Equation (1) follows from the Z-R relationship

$$R_j = aZ_j^b$$

or, on using (2),

$$R_j = aZ_{m_j}^b / A_j^b \tag{5}$$

and the fact that

$$A_j^\beta = \left[1 - K\beta s \sum_{i=1}^{j} \epsilon_i Z_{m_i}^\beta \right] \tag{6}$$

which is a consequence of (2) and (4). The quantity s in (1) and (6) is the radar range resolution which enters through approximating the integral in (4) by a summation, i.e.

$$\int_0^{r_i} Z^\beta(x)dx \sim s \sum_{i=1}^{j} \epsilon_i Z_i^\beta$$

Assume now that a measurement of the attenuation factor out to the nth range bin has been obtained; that is, we have available an estimate A_n for the quantity defined by (3). There are several ways to incorporate this additional information into (1). We discuss two possibilities.

Case I. To account for an offset in the calibration constant C we multiply the set $\{Z_{m_i}\}$ in (1) by a constant Δ. Notice that this is equivalent to multiplying C by Δ^{-1} in (2). The Hitschfeld-Bordan estimate then becomes

$$\hat{R}_j^{H\text{-}B} = a(\Delta Z_{m_j})^b \left[1 - K\beta s \sum_{i=1}^{j} \epsilon_i (\Delta Z_{m_i})^\beta \right]^{-b/\beta} \tag{7}$$

Identifying ΔZ_{m_i} as the corrected form of Z_{m_i}, then from (5) and (6)

$$A_j = \left[1 - K\beta s \sum_{i=1}^{j} \epsilon_i (\Delta Z_{m_i})^\beta \right]^{1/\beta}$$

Letting j go to n in the above formula, using for A_n the measured value $\hat{A}_n$, and solving for Δ gives

$$\Delta = \left[(1 - \hat{A}_n^\beta) \quad K\beta s \sum_{i=1}^{n} \epsilon_i Z_{m_i}^\beta \right]^{1/\beta}$$

This quantity is inserted back into (7) which yields a new estimate of rain rate that we label with a superscript 1:

$$\hat{R}_j^1 = aZ_{m_j}^b \left\{ K\beta s \left[(1 - \hat{A}_n^\beta)^{-1} S_n - S_j \right] \right\}^{-b/\beta} \qquad j \leq n \tag{8}$$

where

$$S_j = \sum_{i=1}^{j} \epsilon_i Z_{m_i}^{\beta} \tag{9}$$

Equation (8) is an alternate form of the Hitschfeld-Bordan algorithm in which the measured quantity $\hat{A}_n$ has been used to correct for errors in the calibration constant.

Case 2. To compensate for offsets in a, we replace K by $K\Delta$ in (1) and identify the corrected form of K by the latter quantity. Using (6) with $K \rightarrow K\Delta$ and proceeding in the same way as Case 1, then

$$\Delta = (1 - \hat{A}_n^{\beta})/K\beta s\, S_n$$

so that the second estimate, labeled with a superscript 2, becomes

$$\hat{R}_j^{(2)} = a\, Z_{m_j}^{b}\, [1 - (1 - \hat{A}_n^{\beta})\, S_j/S_n]^{-b/\beta} \quad j < n \tag{10}$$

As there is only one additional measurement (that of $\hat{A}_n$) we can attempt to eliminate offsets in either a or C but not both. Within the approximations of the error analysis presented later in the paper, the rain rate as determined from (10) is independent of offsets in a but dependent on calibration constant offsets. Exactly the opposite is true of (8).

In their original paper [1], Hitschfeld and Bordan proposed a correction scheme via an independent measurement of rain rate along the radar beam. This may be obtained, for example, by means of a rain gage located under the radar beam. We denote this measured rain rate at or near the nth bin by $\hat{R}_g$. Again we consider two cases. Multiplying the set $\{Z_{m_i}\}$ in (1) by Δ, then

$$R_j = a\,(\Delta Z_{m_j})^b \left[1 - K\beta s \sum_{i=1}^{j} \epsilon_i\, (\Delta Z_{m_i})^{\beta}\right]^{-b/\beta} \tag{11}$$

Letting j increase to n and using R_g for R_n then

$$\hat{R}_g = a\,(\Delta Z_{m_n})^b \left[1 - K\beta s \sum_{i=1}^{n} \epsilon_i\, (\Delta Z_{m_i})^{\beta}\right]^{-b/\beta} \tag{12}$$

Solving (12) for Δ and substituting this into (11) gives (Case 1)

$$\hat{R}_j^{(3)} = a Z_{m_j}^{b}\, \hat{R}_g\, [a^{\beta/b}\, Z_{m_n}^{\beta} + K\beta s\, \hat{R}_g^{\beta/b}\, (S_n - S_j)]^{-b/\beta} \quad j < n \tag{13}$$

Alternatively, K can be replaced by ΔK and an identical procedure carried out, yielding (Case 2)

$$\hat{R}_j^{(4)} = aZ_{m_j}^b \, [1 - \gamma]^{-b/\beta} \quad j < n \tag{14}$$

with

$$\gamma = (1 - (aZ_{m_n}^b / \hat{R}_g)^{\beta/b}) \, [S_j / S_n] \tag{15}$$

Error Analysis

The sources of error in the above estimates arise from random fluctuations or offsets in the reference measurement of rain rate or attenuation, in the parameters appearing in the k-Z, Z-R relations, and in the radar calibration constant. In the empirical k-Z, Z-R relations, the major source of error is caused by the dependence of scattering and attenuation on the temperature and drop size distribution (DSD) of the rain. Atlas and Ulbrich [3] have noted that the DSD exhibits large systematic changes with storm type imposed upon a random component representing the spatial and temporal fluctuations within each class of precipitation. Thus, in general, the assumed parameters in the k-Z, Z-R relations will differ from the true values by an offset plus some random component of zero mean.

Sources of error that affect the accuracy in determining Z_m are the offset in the calibration constant, the sampling errors caused by the random nature of the scatterers, and the receiver noise. In the simple error analysis presented here, sampling errors and finite signal to noise ratios will be neglected; i.e. we assume that the radar has been designed and is operated under conditions for which these errors are small.

Another type of error arises from the uncertainties in the reference measurement. In the fixed target and surface reference methods, the measurement of the attenuation factor will be corrupted by the presence of precipitation in the range bin containing the target. If a rain gage is used for an independent determination of rain rate errors can be caused by a lack of spatial averaging and the existence of too long a temporal average. Additional errors are incurred because of the difference in rain rates between the surface and the radar beam.

The estimates given by (8) and (10) depend on the attenuation factor and the n reflectivity factors. As mentioned above, these quantities can be obtained if either the surface reference or fixed

target method is used. If, however, a radiometric measurement of brightness temperature were available, the attenuation factor could be estimated from this. The radar then would be used only to supply the reflectivity factors.

For spaceborne application over ocean the advantage of this more complicated technique is that such a radar-radiometric sensor could provide rain rate profiling without recourse to the surface reference technique – a technique that is expected to be less accurate over ocean than over land.

The errors in the radiometric measurement of precipitation, like those of the radar, depend on the differences between the actual and assumed values of the temperature and DSD of the scatterers. For spaceborne applications the emission and scattering of radiation from the earth's surface contribute to the observed brightness temperature with the consequence that quantitative measurements of rain rate can be made only over cold backgrounds, e.g. oceans. One disadvantage of the radiometer is its inability to measure the storm height, a quantity needed to obtain the attenuation factor. In a radar-radiometric sensor, the radar would be used to estimate this quantity. Discussions of these and other sources of radiometric error are given in [11-13].

As the attenuation factor may be obtained by several methods, each subject to different sources of error, it would seem necessary to analyze each in order to characterize the errors in A. Previous investigations [4,5] have shown that fairly accurate estimates of A can be made by means of the surface reference or the fixed target techniques. We therefore choose the offset and standard deviation of A to be typical of the values appropriate to a spaceborne radar using the surface reference technique. The results to be presented can also be viewed as determining the accuracy to which A (or R_g) must be known in order that the profiled rain rate be within the given error bounds.

To analyze the estimates it is necessary to characterize the errors in the k-Z, Z-R relations as well as the measurement errors in $\hat{A}$ and $\hat{R}_g$. It is convenient to distinguish two cases where the data set consists of measurements for which: the DSD is relatively constant or the DSD is variable. In the former case, the parameters in, say, the k-Z relation can be written in the following way (where $k = aZ^{\beta}$): fixing β, we define a best fit value a_T that can be determined from the true DSD. The difference between a_T and the assumed value a is a fixed offset error, E_a

$$a_T = a + E_a$$

or

$$a = \delta_a a_T \tag{16}$$

with

$$\delta_a = (a_T - E_a)/a_T \tag{17}$$

When the data consists of measurements over many storms or over a storm in which the DSD changes in time, δ_a should be interpreted as a random variable. For measurements taken over a short period of time (one record) we assume that there is no range variation in a_T. The variability in a_T then arises only from the fluctuations in the mean DSD from record to record. With this assumption we can still use (16) with δ_a taken to be random.

Interpreting the quantities a, $\hat{A}_n$ and $\hat{R}_g$ in a similar manner we can write

$$a = \delta_a a_T \tag{18}$$

$$\hat{A}_n = \delta_A A_n \tag{19}$$

$$\hat{R}_g = \delta_R R_g = \delta_R R_n \tag{20}$$

where R_n, A_n are the true rain rate and the true attenuation factor at the nth range bin. As with δ_a, the quantities δ_a, δ_A, δ_R will be chosen either as constants or random variables depending on the nature of the data set.

The radar calibration constant can be written in the form $C = \delta_C C_T$ where δ_C is a non-random quantity. Using this we can write at the jth range bin

$$Z_{m_j} = Z_{m_j}(T)/\delta_C \tag{21}$$

where $Z_{m_j}(T)$ denotes the true measured reflectivity factor. Notice that as the receiver noise and the sampling errors have been neglected, Z_{m_j} is subject only to errors in the radar calibration constant.

We now use the above equation to write the estimates as function of the five quantities (δ_a, δ_a, δ_A, δ_R, δ_C). To simplify the results, the summations over $Z_{m_j}(T)$ are eliminated by means of the relation

$$A_j^\beta = 1 - 0.2\,(\ln 10)\, a_T \beta s \sum_{i=1}^{j} \epsilon_i\, Z_{m_i}^\beta (T)$$

where A_j is the true attenuation factor up to the jth range bin. This equation can be found from (6), (17) and (19) and by recognizing that the Hitschfeld-Bordon estimate is exact in the absence of errors. We also use the fact that (5)

$$R_j = a_T \, Z^b_{m_j}(T)/A^b_j$$

where all the above quantities are the true values.

Omitting the simple computations and using the same notation for the estimates as above the ratios of the estimated rain rates to the true rain rate at the jth range bin are given by

$$\hat{R}^{H\cdot B}_j/R_j = \frac{\delta_a}{\delta^b_C}\,[1+\epsilon_o]^{-b/\beta} \qquad j < n \tag{22}$$

$$\epsilon_o = A_j^{-\beta}\,(1-A_j^\beta)\,(1-\delta_a/\delta^\beta_C)$$

$$\hat{R}^{(1)}/R_j = \frac{\delta_a}{\delta_a^{b/\beta}}\,[1-\epsilon_1]^{-1} \qquad j < n \tag{23}$$

$$\epsilon_1 = A^\beta_n\, A_j^{-\beta}\,(1-\delta^\beta_A)/(1-\delta^\beta_A\, A^\beta_n)$$

$$\hat{R}^{(2)}/R_j = \frac{\delta_a}{\delta_C}\left[\frac{A_j^\beta\,(1-A^\beta_n)}{A^\beta_n\,(\delta^\beta_A-1)+A^\beta_j\,(1-\delta^\beta_A\,A^\beta_n)}\right]^{b/\beta} \qquad j < n \tag{24}$$

$$\hat{R}^{(3)}_j/R_j = \delta_a\,\delta_R\left[\frac{\epsilon_3+1}{\delta_a^{\beta/b}\,\epsilon_3+\delta_a\,\delta_R^{\beta/b}}\right]^{b/\beta} \qquad j < n$$

$$\hat{R}^{(3)}_n/R_n = \delta_R \tag{25}$$

$$\epsilon_3 = A^\beta_n/(A^\beta_j - A^\beta_n)$$

$$\hat{R}^{(4)}_j/R_j = \frac{\delta_a}{\delta^b_C}\left[\frac{A_j^\beta\,(1-A^\beta_n)}{(A^\beta_j-A^\beta_n)+\epsilon_4\,A^\beta_n\,(1-A^\beta_j)}\right]^{b/\beta} \qquad j < n \tag{26}$$

$$\epsilon_4 = (\delta_a/\delta_C\,\delta_R)^{\beta/b}$$

Certain features of the above estimates are readily deduced. For the Hitschfeld-Bordan estimate (22) we summarize some of the observations made by previous investigators. From the definition of the attenuation factor, it is apparent that the quantity $A_j^{-\beta}$, $\beta > 0$, is an exponentially increasing function of rain rate and range (i.e. of j). As $A_j^{-\beta}$ increases, the absolute value of ϵ_o in (22) will also increase. If $\delta_a/\delta_C^\beta < 1$, the sign of ϵ_o will be positive and $\hat{R}_j^{H\cdot B}$ will tend to be less than the true rain rate, R_j. If $\delta_a/\delta_C^\beta > 1$, ϵ_o will be negative and $R_j^{H\cdot B}$ will generally be larger than R_j. Notice that as $\epsilon_o \to -1$ the estimate fails to exist. In general, the presence of the $A_j^{-\beta}$ factor shows that whenever the total attenuation up to the jth range bin is significant, even modest errors in a or C can result in large errors in $\hat{R}_j^{H\cdot B}$.

For the estimates $\hat{R}_j^{(1)}$, $\hat{R}_j^{(2)}$ nearly the opposite situation exists: for small values of attenuation (out to the nth range bin), the estimates are subject to large errors which then decrease as the attenuation becomes larger. For the $\hat{R}_j^1$ estimate (23) it is easily shown that:

$$1 < [1 - \epsilon_1]^{-1} < (1 - \gamma)^{-1} \quad \text{for} \quad \delta_A < 1 \tag{27}$$

$$(1 + |\gamma|)^{-1} < [1 - \epsilon_1]^{-1} < 1 \quad \text{for} \quad \delta_A > 1 \tag{28}$$

with

$$\gamma = (1 - \delta_A^\beta)/(1 - \delta_A^\beta\, A_n^\beta)$$

We impose the condition $\delta_A^\beta\, A_n^\beta \equiv A_n^\beta < 1$ which is equivalent to stating that only positive values of measured attenuation are admissible.

When the attenuation is large, $A_n^{-\beta} >> 1$, then $\gamma \sim 1 - \delta_A^\beta$ so that $[1 - \epsilon_1]^{-1}$ is bounded between 1 and $\delta_A^{-\beta}$ for $\delta_A < 1$ and between $\delta_A^{-\beta}$ and 1 for $\delta_A > 1$. Thus, for large values of attenuation, the maximum error in the term $[1 - \epsilon_1]^{-1}$ is nearly proportional to the measurement error in the attenuation factor, A_n. If the attenuation is small, however, (27) and (28) show that the error in $[1 - \epsilon_1]^{-1}$ and thus in $\hat{R}_j^{(1)}$ can be large. This same general behavior is true of the $\hat{R}_j^{(2)}$ estimate.

From these considerations and anticipating the numerical results, we conclude that whenever the attenuation is significant the $\hat{R}^{(1)}$, $\hat{R}^{(2)}$ estimates are generally superior to the $\hat{R}^{H\cdot B}$ estimate. For very small values of attenuation, where $\hat{R}^{(1)}$, $\hat{R}^{(2)}$ are subject to large errors, the $\hat{R}^{H\cdot B}$ estimate is preferable and often yields satisfactory results.

The estimates $\hat{R}^{(3)}$, $\hat{R}^{(4)}$ differ from the others in the sense that the rain rate at the nth bin has been controlled rather than the total attenuation. From (25) the following approximation can be deduced,

$$\hat{R}_j^{(3)}/R_j \approx \begin{cases} \delta_a/\delta_a^{b/\beta} & \epsilon_3 << 1 \\ \delta_R & \epsilon_3 >> 1 \end{cases}$$

The quantity ϵ_3 is small when the total attenuation is large and j is not close to n; ϵ_3 is large when the total attenuation is small or j is near n. For other values of ϵ_3, $\hat{R}_j^{(3)}/R_j$ will be intermediate to these extremes. Two characteristics noteworthy of the $\hat{R}^{(3)}$ and $\hat{R}^{(4)}$ are that the errors are bounded both for large and small values of the attenuation and that for $\epsilon_3 >> 1$ the estimate is independent of errors in the quantity 'a', a term which can make a sizeable contribution to the total error.

Results

Figures 1 through 4 illustrate the behavior of the estimates as a function of range for selected values of rain rate and offsets. The radar range resolution is chosen to be equal to .25 km with n = 20 so that the distance from the radar to the nth range bin is 5 km. In all the figures the five ratios $\hat{R}_j/R_j$ of (22) – (26) have been plotted as functions of radar range. Notice that since R_j is the true rain rate, an error less estimate would give a ratio of 1. The wavelength, chosen equal to 0.86 cm, enters primarily in the determination of the attenuation: the results obtained for a 5mm/hr rain rate at $\lambda = 0.86$ cm will be nearly the same as those for R = 8 mm/hr at $\lambda = 1.24$ or R = 65 mm/hr at $\lambda = 3.2$ cm since the attenuation is comparable in all three cases.

In figure 1, the rain rate is fixed at 5 mm/hr with $\epsilon_A = \epsilon_a = \epsilon_R = -0.25$, $\epsilon_a = 0$, $\epsilon_C = .25$ where ϵ denotes the relative error in the quantity appearing in the subscript, e. g. $\epsilon_A = 1 - \delta_A = (A_n - \hat{A}_n)/A_n$. The ratio $\delta_a/\delta_C^{\beta}$ with $\beta = .905$ is significantly greater than 1 and consequently the $\hat{R}^{H\text{-}B}$ becomes much greater than the true rain rate within a short distance from the radar. The estimates $\hat{R}^{(1)}$ through $\hat{R}^{(4)}$ exhibit more gradual changes in range and much smaller errors than in $\hat{R}^{H\text{-}B}$.

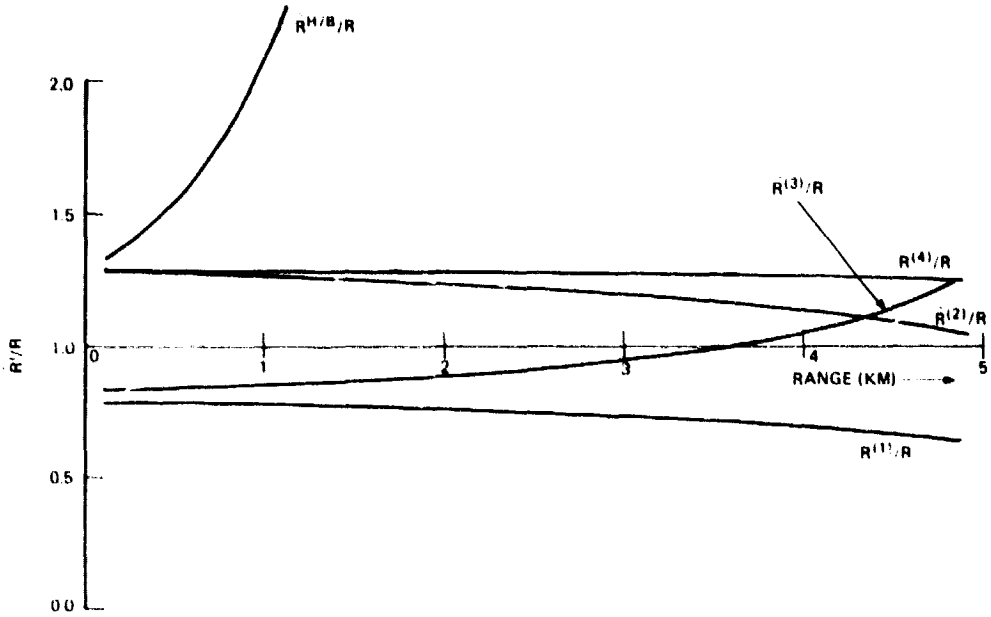

Figure 1. $\hat{R}^i/R$ versus range; (R^i = i^{th} rain rate estimate, R = true rain rate) R = 5 mm/hr; λ = 0.86 cm, $(\delta_a, \delta_A, \delta_\alpha, \delta_C, \delta_R)$ = (1, 1.25, 1.25, .75, 1.25)

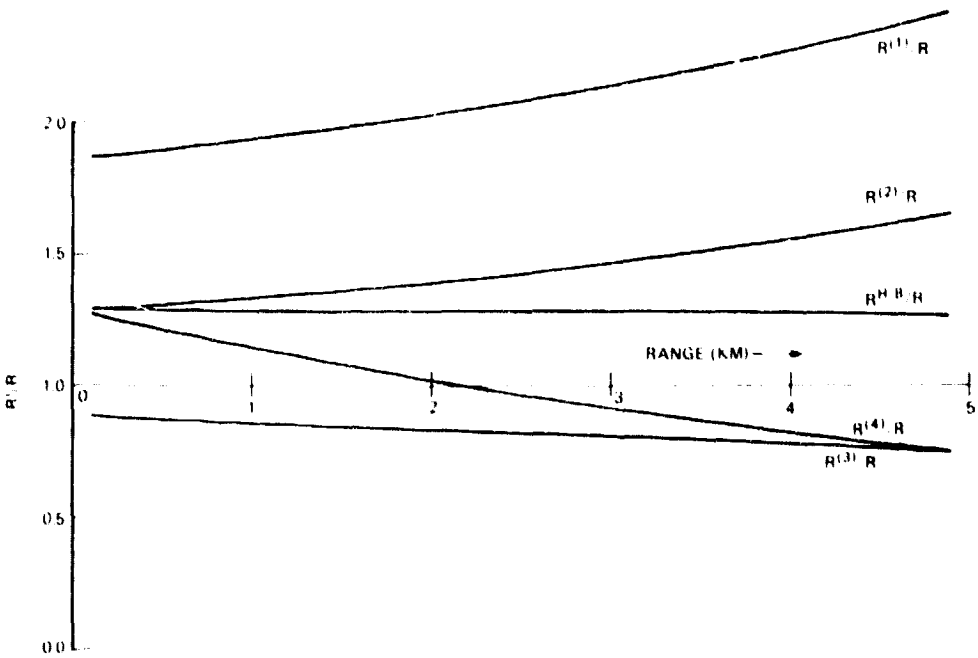

Figure 2. $\hat{R}^i/R$ versus range; (R^i = i^{th} rain rate estimate, R = true rain rate) R = 1 mm/hr; λ = 0.86 cm, $(\delta_a, \delta_A, \delta_\alpha, \delta_C, \delta_R)$ = (1, .75, .75, .75, .75)

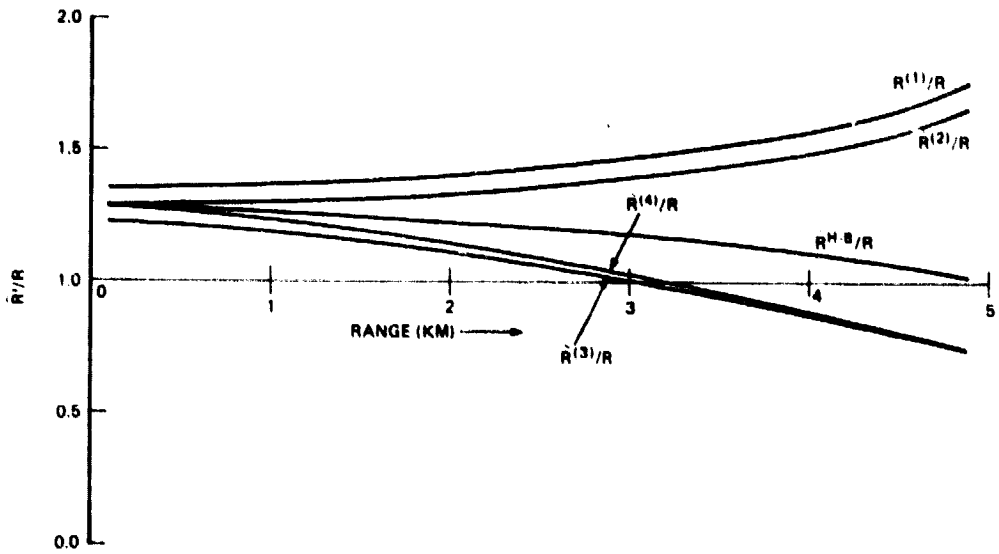

Figure 3. $\hat{R}^i/R$ versus range; (R^i = i^{th} rain rate estimate, R = true rain rate) R = 5 mm/hr; λ= 0.86 cm, $(\delta_a, \delta_A, \delta_\alpha, \delta_C, \delta_R)$ = (1, .75, .75, .75, .75)

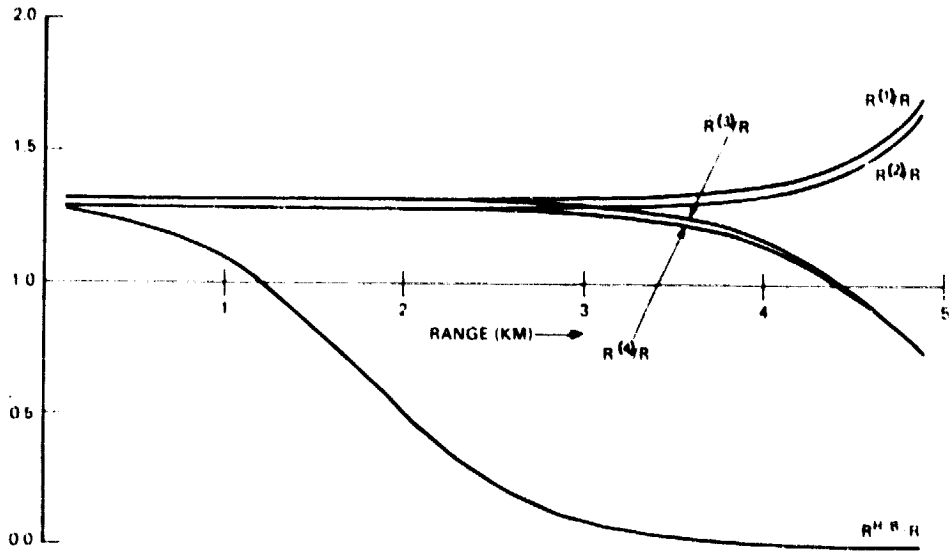

Figure 4. $\hat{R}^i/R$ versus range; (R^i = i^{th} rain rate estimate, R = true rain rate) R = 20 mm/hr; λ = 0.86 cm, $(\delta_a, \delta_A, \delta_\alpha, \delta_C, \delta_R)$ = (1, .75, .75, .75, .75)

In figures 2-4, the relative errors are chosen to be $\epsilon_A = \epsilon_a = \epsilon_R = \epsilon_C = 0.25$, $\epsilon_a = 0$ so that $\delta_a/\delta_C^\beta \sim .97$. In figure 2, assuming R = 1 mm/hr, the errors in $\hat{R}^{(2)}$ and especially in $\hat{R}^{(1)}$ are larger than those of $\hat{R}^{H-B}$, $\hat{R}^{(3)}$ or $\hat{R}^{(4)}$.

In figure (3), with R = 5 mm/hr, $\hat{R}^{H-B}$ decreases more rapidly with range but as δ_a/δ_C^β is close to 1, the overall error remains small. As the rain rate is once more increased to 20 mm/hr (figure 4) the errors in $\hat{R}^{(1)}$ and $\hat{R}^{(2)}$ are considerably reduced whereas $\hat{R}^{H-B}$ is rendered useless because of high attenuation.

The dependence of the estimates on rain rate, as shown in figures 2-4, is general: for low values of attenuation $\hat{R}^{H-B}$ is generally less in error than $\hat{R}^{(1)}$ and $\hat{R}^{(2)}$ while the opposite is true at moderate to high values of attenuation. The transition point, however, is a sensitive function of the ratio δ_a/δ_C^β. In general as the derivation of δ_a/δ_C^β from 1 increases, $\hat{R}^{(1)}$ and $\hat{R}^{(2)}$ will become preferable to $\hat{R}^{H-B}$ at progressively smaller values of rain rate. In the foregoing discussion, the values in α, a, $\hat{R}_g$, $\hat{A}_n$ were assumed to be equal to the best fit value plus a constant offset. As stated previously, this is a reasonable assumption when the DSD is relatively constant over the data set. Of greater interest is the situation in which the DSD varies randomly between the data records. In this case an interpretation of δ_a, δ_α, δ_A, δ_R as random variables is more appropriate. Under this assumption the $\hat{R}_j/R_j$ ratios are themselves random. At $\lambda = 0.86$ cm we can assume without much error that $b = \beta = 1$. Under this assumption, the mean and standard deviation of each of the rain rate estimates can be found by means of straightforward integrations. For simplicity we will present the statistics only for the $\hat{R}^{H-B}$, $\hat{R}^{(1)}$, $\hat{R}^{(2)}$ estimates. The analytic expression for the mean and variance of these quantities are given in the Appendix.

Once the mean and standard deviation of A_n, a and α are chosen, we compute the absolute value of the mean and the standard deviation of the $\hat{R}_i/R_j$ ratios as a function of range for a fixed, uniform rain rate, i.e. $R_j = R$. To simplify the presentation of the results, an additional average is performed over range bins j = 1 to n; that is, we compute the quantities $M_i(R)$, $\bar{\sigma}_i(R)$ where

$$M_i(R) = \frac{1}{n} \sum_{j=1}^{n} \left\langle \left| \frac{\hat{R}_j^i}{R} \right| \right\rangle$$

$$\bar{\sigma}_i(R) = \frac{1}{n}\sum_{j=1}^{n}\left\langle \frac{(\hat{R}_j^i - \langle\hat{R}_j^i\rangle)^2}{R^2}\right\rangle^{1/2}$$

where the angular brackets denote the expectation. The notation $\hat{R}_j^i$ represents the ith rain rate estimate at the jth range bin where R^i denotes either the $\hat{R}^{H-B}$, the $\hat{R}^{(1)}$ or the $\hat{R}^{(2)}$ estimate. The criteria for a good estimate is that $\bar{\sigma}_i$ be small and that $\bar{M}_i$ be close to unity.

We again choose $\lambda = 0.86$ cm, $r_n = 5$ km and $s = 0.25$ km so that $n = 20$. In tables 1 and 2, $(\bar{M}_1^i, \bar{\sigma}_i)$, $(\bar{M}_2, \bar{\sigma}_2)$ and $(\bar{M}_{H-B}, \bar{\sigma}_{H-B})$ are listed as functions of rain rate for the three cases $\delta_C = (1, 1.25, .75)$. These values correspond to calibration errors of 0 db, .97 db and −1.25 db respectively. In table 1, $\sigma_a = \sigma_\alpha = \sigma_A = 0.125$ where σ_a is the standard deviation of σ_a, etc. In table 2, $\sigma_a = \sigma_\alpha = \sigma_A = 0.25$. For both sets of calculations the mean values of δ_a, δ_α and δ_A are assumed to be unbiased. Writing $\langle\delta_a\rangle = 1 - b_a$, etc. The above condition is equivalent to setting $b_a = b_\alpha = b_A = 0$.

Since the $\hat{R}_j^{(1)}$ estimate is independent of calibration errors, $\bar{M}_i$, $\bar{\sigma}_i$ are independent of δ_C. Therefore, in tables 1 and 2 it suffices to list $\bar{M}_i$, $\bar{\sigma}_i$ for the $\delta_C = 1$ case alone. For large rain rates, the mean or standard deviation of the Hitschfeld-Bordon estimate might not exist at one or more range bins. In these instances the range averaged statistics are undefined. We represent this in the tables by writing dashes for the $\bar{M}_{H-B}$, $\bar{\sigma}_{H-B}$.

The results show that $\hat{R}^{H-B}$ is useful only at small values of attenuation, i.e. low rain rates. Of the three estimates $\hat{R}_j^{(2)}$ has, in general, the smallest variability about its mean value. Relative to the $\hat{R}_j^{(1)}$ estimate, its disadvantage is the dependence on the radar calibration error. For example, at $\delta_C = 1.25$, the mean value of $\hat{R}^{(2)}$ is approximately four-fifths of the true rain rate while at $\delta_C = 0.75$ it is approximately one-third greater than the true rain rate. In tables 1 and 2 the $\hat{R}^{(1)}$ estimate is nearly unbiased for the rain rates considered. Notice that the increase in σ_a, σ_α, σ_A from table 1 to table 2 has only a slight effect on the mean values but results in a significant increase in the variance of all three estimates.

In table 3 we have set $\sigma_a = \sigma_\alpha = \sigma_A = 0.25$, $\delta_C = 1$ and $b_a = 0$. As before $(\bar{M}_1, \bar{\sigma}_1)$, $(\bar{M}_2, \bar{\sigma}_2)$ and $(\bar{M}_{H-B}, \bar{\sigma}_{H-B})$ are given as functions of the true rain rate. In the first example, $b_\alpha = b_A = 0.2$ so that

Table 1
Range Averaged Mean and Standard Deviation of $\hat{R}^{(1)}$, $\hat{R}^{(2)}$, $\hat{R}^{H\text{-}B}$
versus the True Rain Rate
$\sigma_a, \sigma_a, \sigma_A = 0.125$
$b_a, b_a, b_A = 0$

Rain Rate (mm/hr)	$\delta_C = 1$		
	$\bar{M}_1, \bar{\sigma}_1$	$\bar{M}_2, \bar{\sigma}_2$	$\bar{M}_{H\text{-}B}, \bar{\sigma}_{H\text{-}B}$
1	1.03, .388	1.0, .144	1.0, .133
2	1.02, .252	1.0, .142	1.01, .168
3	1.02, .238	1.0, .14	1.06, .269
4	1.02, .211	1.0, .138	– –
5	1.02, .206	1.0, .137	– –
10	1.02, .197	1.0, .133	– –
15	1.01, .25	1.0, .141	– –
20	.976, .339	.99, .179	– –
$\delta_C = 1.25$			
1		.804, .115	.756, .098
2		.803, .113	.707, .101
3		.803, .112	.654, .106
4		.803, .111	.599, .112
5		.802, .110	.545, .115
10		.802, .106	.336, .095
15		.8, .113	.229, .067
20		.79, .143	.172, .05
$\delta_C = 0.75$			
1		1.34, .192	1.5, .217
2		1.34, .189	– –
3		1.34, .187	– –
4		1.34, .184	– –
5		1.34, .183	– –
10		1.34, .177	– –
15		1.33, .188	– –
20		1.32, .238	– –

Table 2
Range Averaged Mean and Standard Deviation of $\hat{R}^{(1)}$, $\hat{R}^{(2)}$, $\hat{R}^{H-B}$
versus the True Rain Rate
$\sigma_s, \sigma_a, \sigma_A = 0.25$
$b_s, b_a, b_A = 0$

Rain Rate (mm/hr)	$\bar{M}_1, \bar{\sigma}_1$	$\bar{M}_2, \bar{\sigma}_2$	$\bar{M}_{H-B}, \bar{\sigma}_{H-B}$
$\delta_C = 1$			
1	1.15, .78	1.02, .297	1.01, .269
2	1.11, .544	1.02, .292	1.06, .38
3	1.1, .482	1.02, .287	– –
4	1.1, .456	1.01, .283	– –
5	1.09, .446	1.01, .279	– –
10	1.08, .427	1.01, .269	– –
15	1.08, .436	1.01, .266	– –
20	1.04, .538	.996, .286	– –
$\delta_C = 1.25$			
1		.817, .237	.759, .196
2		.815, .233	.718, .207
3		.813, .229	.682, .236
4		.812, .226	.667, .33
5		.811, .223	– –
10		.807, .215	– –
15		.805, .213	– –
20		.797, .228	– –
$\delta_C = 0.75$			
1		1.36, .395	1.52, .449
2		1.36, .389	–
3		1.36, .382	– –
4		1.35, .377	– –
5		1.35, .372	– –
10		1.34, .359	– –
15		1.34, .355	– –
20		1.33, .381	– –

Table 3
Range Averaged Mean and Standard Deviation of $\hat{R}^{(1)}$, $\hat{R}^{(2)}$, $\hat{R}^{H-B}$
versus the True Rain Rate
$\sigma_s, \sigma_a, \sigma_A = 0.25$
$\delta_C = 1, b_s = 0$

Rain Rate (mm/hr)	$\bar{M}_1, \bar{\sigma}_1$	$\bar{M}_2, \bar{\sigma}_2$	$\bar{M}_{H-B}, \bar{\sigma}_{H-B}$
$b_a = b_A = 0.2$			
1	2.2, 1.5	1.15, .356	.951, .250
2	1.81, 1.0	1.13, .345	.908, .282
3	1.67, .865	1.12, .334	.891, .379
4	1.61, .791	1.11, .325	– –
5	1.57, .765	1.10, .318	– –
10	1.49, .706	1.06, .295	– –
15	1.46, .703	1.05, .286	– –
20	1.40, .787	1.03, .303	– –
$b_a = b_A = -0.2$			
1	.589, .484	.93, .26	1.08, .292
2	.730, .331	.93, .258	– –
3	.777, .304	.94, .257	– –
4	.80, .296	.94, .256	– –
5	.814, .297	.95, .255	– –
10	.842, .298	.966, .253	– –
15	.85, .311	.975, .254	– –
20	.825, .397	.972, .276	– –

$\langle\delta_a\rangle = \langle\delta_A\rangle = 0.8$; in the second example $b_a = b_A = -0.2$ so that $\langle\delta_a\rangle = \langle\delta_A\rangle = 1.2$. In both cases, the $R^{(1)}$ estimate is significantly offset from the true rain rate; moreover, in the case of $b_A = b_a = 0.2$ the standard deviation is very large when the rain rate is small.

For both examples, the $\hat{R}^{(2)}$ estimate is clearly superior to the other two. We conclude that when the radar is well calibrated, $\hat{R}^{(2)}$ yields the most accurate results of the three estimates considered.

Summary

Several recent papers have shown that integrated rain rate estimates can be obtained by means of an attenuating wavelength radar. These estimates are found by means of a measurement of the attenuation factor, A_n. In this paper we have shown that range profiled rain rate estimates also can be deduced by using A_n along with the Hitschfeld-Bordon algorithm and the measured reflectivity factors $\{Z_{mj}\}$. In the derivation it was seen that two estimates, $\hat{R}^{(1)}$, $\hat{R}^{(2)}$ could be obtained from the measured data, the first of which was independent of errors in the calibration constant, C, the latter of which was independent of errors in a, where $k = aZ^{\beta}$. If an independent measurement of rain rate, R_g, is made at some point along the radar beam two additional estimates of rain rate, $\hat{R}^{(3)}$, $\hat{R}^{(4)}$, analogous to $\hat{R}^{(1)}$, $\hat{R}^{(2)}$, can be deduced.

This latter method is essentially the correction scheme proposed by Hitschfeld and Bordan.

For purposes of comparison we added to these four estimates the original range profiled estimate derived by Hitschfeld and Bordan, i.e. $\hat{R}^{H\text{-}B}$. Since $\hat{R}^{H\text{-}B}$ is a function only of $\{Z_{mj}\}$ this estimate is the only one of the five that can be employed if neither A_n nor R_g are available.

To investigate the qualitative behavior of the estimates, a simple error analysis was performed. The major findings were:

1. When the total attenuation is significant, the estimates that incorporate the additional measurement of either A_n or R_n are superior to the $\hat{R}^{H\text{-}B}$ estimate if these quantities can be measured with a modest degree of accuracy.
2. The $\hat{R}^{(3)}$, $\hat{R}^{(4)}$ estimates yield fairly accurate measurements of rain rate for both small and large values of total attenuation. On the other hand, the $\hat{R}^{(1)}$, $\hat{R}^{(2)}$ estimates often suffer

from large errors when the total attenuation is small. The drawback of the $\hat{R}^{(3)}$, $\hat{R}^{(4)}$ estimates is that they can be applied only to ground based radars where an independent measure of point rain rate can be obtained.

3. The $\hat{R}^{(2)}$ estimate is generally more accurate than the $\hat{R}^{(1)}$ estimate when the errors in A_n, C and α are of comparable magnitude. When the errors in C are much greater than those in α, the $\hat{R}^{(1)}$ estimate is superior.

Acknowledgments

I wish to thank Dr. Jerome Eckerman for his assistance and encouragement.

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Appendix
Statistics of the Rain Rate Estimates

To calculate the mean and standard deviation of the normalized rain rate estimates $\hat{R}_j^{H-B}/R_j$, $\hat{R}_j^{(1)}/R_j$, and $\hat{R}_j^{(2)}/R_j$, we must assume some knowledge of the random variables a, α, and A_n. As we are primarily interested in the qualitative behavior of the estimates, we can simplify the computations by making the following assumptions. In the k-Z, Z-R relations, $k = \alpha Z^{\beta}$, $R = aZ^{b}$, we take $b = \beta = 1$. Furthermore, we assume that the true values a_T, α_T and A_n are uniformly distributed random variables so that the probability density function of, for example, δ_a can be written

$$P(\delta_a) = \begin{cases} \dfrac{1}{2\Delta_a}; (1 - b_a) - \Delta_a < \delta_a < (1 - b_a) + \Delta_a \\ 0; \text{otherwise} \end{cases} \tag{A1}$$

with similar expressions for δ_α, δ_A.

We can rewrite (A1) by using the fact that $\Delta_a = \sqrt{3}\,\sigma_a$, where σ_a is the standard deviation of δ_a. The quantity b_a in (A1) is the relative bias in a, i.e.

$$b_a = \left\langle \frac{a_T - a}{a_T} \right\rangle$$

where a_T is the true value, a is the assumed value and the angular brackets denote the expectation operator. Using $\Delta_a = \sqrt{3}\,\sigma_a$, (A1) can be written as

$$P(\delta_a) = \begin{matrix} \dfrac{1}{2\sqrt{3}\,\sigma_a}; [(1 - b_a) - \sqrt{3}\,\sigma_a] < \delta_a < [(1 - b_a) + \sqrt{3}\,\sigma_a] \\ 0; \text{otherwise} \end{matrix} \tag{A2}$$

with

$$\langle \delta_a \rangle = 1 - b_a = 1 - \left\langle \frac{a_T - a}{a_T} \right\rangle \tag{A3}$$

The mean and standard deviation of the estimates can now be computed by means of simple integrations. For the mean and mean square, of the Hitschfeld-Bordan estimate, we obtain

$$\left\langle \frac{\hat{R}_j^{H\text{-}B}}{R_j} \right\rangle = \frac{(1-b_a)}{2\sqrt{3}\,\sigma_a A_j^{-1}(1-A_j)} \ln\left[\frac{1+p_o}{1-p_o}\right] \tag{A4}$$

$$p_o = \frac{\sqrt{3}\,\sigma_a\, A_j^{-1}\,(1-A_j)}{\delta_C + A_j^{-1}(1-A_j)[\delta_C-(1-b_a)]} \tag{A5}$$

$$\left\langle \left(\frac{\hat{R}_j^{H\text{-}B}}{R_j}\right)^2 \right\rangle = \frac{\sigma_a^2 + (1-b_a)^2}{\gamma_1^2 - 3\,\sigma_a^2\,\gamma_2^2} \tag{A6}$$

$$\gamma_1 = 1 + A_j^{-1}\,(\delta_C - 1) + A_j^{-1}\,(1-A_j)\,b_a \tag{A7}$$

$$\gamma_2 = A_j^{-1}\,(1-A_j) \tag{A8}$$

For the mean to exist, the condition $-1 < p_o < 1$ must be satisfied; for the mean square to exist $\sigma_a < \gamma_1^2/3\gamma_2^2$.

For the estimate $\hat{R}_j^{(1)}/R_j$ the mean and mean square are

$$\left\langle \frac{\hat{R}_j^{(1)}}{R_j} \right\rangle = \frac{(1-b_a)}{2\sqrt{3}\,\sigma_a} \ln\left[\frac{1+P_1}{1-P_1}\right] \times \left\{C_o + C_1 \ln\left[\frac{1-q_1}{1+q_1}\right]\right\} \tag{A9}$$

with

$$C_o = (1-A_j^{-1})^{-1}$$

$$P_1 = \sqrt{3}\,\sigma_a/(1-b_a) \tag{A10}$$

$$q_1 = \sqrt{3}\,\sigma_A\, A_n\,(1-A_j^{-1})/[(1-A_n) + b_A\, A_n\,(1-A_j^{-1})] \tag{A11}$$

$$C_1 = A_j^{-1}\,(1-A_n)/2\sqrt{3}\,\sigma_A\, A_n\,(1-A_j^{-1})^2 \tag{A12}$$

$$\left\langle \left(\frac{\hat{R}_j^{(1)}}{R_j}\right)^2 \right\rangle = \frac{\sigma_a^2 + (1-b_a)^2}{(1-b_a)^2 - 3\,\sigma_a^2} \times \left\{C_o^2 + C_2 + C_3 \ln\left[\frac{1-q_1}{1+q_1}\right]\right\} \tag{A13}$$

$$C_2 = \frac{[(1-A_n)/A_j(1-A_j^{-1})]^2}{[(1-A_n) + (1-A_j^{-1})\,b_A\,A_n]^2 - 3\,[\sigma_A\,(1-A_j^{-1})\,A_n]^2} \tag{A14}$$

$$C_3 = (1 - A_n)/\sqrt{3}\,\sigma_A\, A_j\, A_n\, (1 - A_j^{-1})^3 \tag{A15}$$

For both the mean and mean square to exist the following conditions must hold:

$$-1 < q_1 < 1$$

$$-1 < P_1 < 1$$

$$\sigma_a < (1 - b_a)/\sqrt{3}$$

Finally, for the mean and mean square of $\hat{R}_j^{(2)}/R_j$ we have

$$\left\langle \frac{\hat{R}_j^{(2)}}{R_j} \right\rangle = \frac{A_j\,(1 - A_n)\,(1 - b_a)}{2\sqrt{3}\,A_n\,(1 - A_j)\,\sigma_A\,\delta_C}\; \ln\left[\frac{1 + P_2}{1 - P_2}\right] \tag{A16}$$

where

$$P_2 = \frac{\sqrt{3}\,\sigma_A\, A_n\,(1 - A_j)}{A_j\,(1 - A_n) - b_A\, A_n\,(1 - A_j)} \tag{A17}$$

$$\left\langle \left(\frac{\hat{R}_j^{(2)}}{R_j} \right)^2 \right\rangle = \frac{A_j^2\,(1 - A_n)^2\,(\sigma_a^2 + (1 - b_a)^2)}{\delta_C^2\,(\gamma_3^2 - 3\,\sigma_A^2\,\gamma_A^2)} \tag{A18}$$

where

$$\gamma_3 = A_j\,(1 - A_n) - b_A\, A_n\,(1 - A_j) \tag{A19}$$

$$\gamma_4 = A_n\,(1 - A_j) \tag{A20}$$

For the mean and mean square to exist

$$-1 < P_2 < 1$$

$$\sigma_A < \gamma_3/\sqrt{3}\,\gamma_4$$

The standard deviation of the above estimates is given by the square root of the difference between the mean square and square of the mean.