A COMPUTER PROGRAM FOR ESTIMATION FROM INCOMPLETE
MULTINOMIAL DATA

Karen R. Credeur

May 1978

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Date for general release May 1980.

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National Aeronautics and
Space Administration
Langley Research Center
Hampton, Virginia 23665
A COMPUTER PROGRAM FOR ESTIMATION FROM INCOMPLETE MULTINOMIAL DATA

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SUMMARY

This paper presents a computer program for maximum likelihood and Bayesian estimation of the vector $p$ of multinomial cell probabilities from incomplete data. Also included is coding to calculate exact and approximate elements of the posterior mean and covariance matrices. The program is written in FORTRAN IV language for the Control Data CYBER 170 series digital computer system with network operating system (NOS) 1.1. The program requires approximately 44000 octal locations of core storage. A typical case requires from 72 seconds to 92 seconds on CYBER 175 depending on the value of the prior parameter.
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Karen R. Credeur

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This paper describes the main computer program used in reference 1 for estimating the vector $\mathbf{p}$ of multinomial cell probabilities from incomplete data. The data is incomplete in that it contains partially classified observations. Each such partially classified observation is observed to fall in one of two or more selected categories but is not classified further. The estimation criterion is minimization of risk for quadratic loss $L(\mathbf{p}-\hat{\mathbf{p}}) = (\mathbf{p}-\hat{\mathbf{p}})'(\mathbf{p}-\hat{\mathbf{p}})$ for $\hat{\mathbf{p}}$ an estimator of $\mathbf{p}$.

In addition, elements of the posterior mean and covariance matrices are calculated exactly and approximately. A Taylor-series function is used to approximate the posterior covariance matrix. A Taylor-series function, the maximum-likelihood estimate, and the posterior mode are used to approximate the posterior mean.

Monte-Carlo simulation studies are performed for small- and medium-size samples to assess
(1) which of the maximum-likelihood estimate, posterior mode, and Taylor-series approximate posterior mean best minimizes risk for specified values of $\mathbf{p}$;
(2) how well each of these functions approximates the exact posterior mean; and
(3) how well a Taylor-series function approximates elements of the posterior covariance matrix.

Samples are of size 25 and 50, percentage of incomplete data varies around 15 and 40, and probabilities range from the center of the probability simplex $P_2$ to one of its corners. Probabilities equal the means of the prior distributions for varying parameters or are randomly generated from these distributions. An exploratory robustness study is conducted by using the correct prior, a uniform prior, and a perturbed prior in the Bayesian estimators. The iterative algorithm of Dempster, Laird, and Rubin (ref. 2) is used to evaluate all three estimators.
Other discussion, analysis, and results are given in reference 1. Included in the discussion in reference 1 are descriptions of pseudorandom-number generators for the Dirichlet, uniform, and trinomial distributions. Also given are tree diagrams (figs. 5.1 - 5.3 of ref. 1) that illustrate the flow of the computer program.

The computer used is a Control Data Corporation (CDC) CYBER 170 series digital computer system with network operating system (NOS) 1.1. This computer operates with a 60-bit word and single-precision accuracy of about 14.5 significant figures. The programing language is FORTRAN Extended, Version 4.6. The program requires approximately 44000 octal locations of core storage. A typical case requires from 72 seconds to 92 seconds depending on the prior parameter.

A listing of the main computer program is given in the third section. An index precedes the listing. Symbols are defined in the listing. Note that the program is written for DESIGN 2 of reference 1 but can be modified to be DESIGN 1 of reference 1 by deleting the Dirichlet-generative level and changing the dimensioning of the QLMSIJ matrix. DESIGN 1 has a fixed-effects model constituting full factorials (4×2²×3) having four levels* and two replications per cell. DESIGN 2 has a mixed-effects model constituting nested factorials (4-10×2²×3) having four levels and two replications per cell within each of four variations of the prior parameter υ. The ten generations of the Dirichlet probability p in DESIGN 2 are considered random; the remaining factors are considered fixed.

* level 1: four Dirichlet probabilities p
level 2: two sample sizes
level 3: two percentages of incomplete data
level 4: three estimators
To run a case, any necessary changes are made to the following lines in MAIN

\[
\begin{align*}
XNU1 &= \\
XNU2 &= \\
IP &= \\
SEED &=
\end{align*}
\]

In addition, the subroutine GAM, which is a function of the prior parameter \(v\) (NU), and the Hollerith labels

\[
\begin{align*}
\text{DATA ALABEL/10HA. NU} &= (,10H0.1,0.1,9.,10H8). \\
\text{DATA TLABEL/10HTABLE 7.1 /}
\end{align*}
\]

are changed as needed. For values of \(y\) that are less than 10, \(\text{GAM}(y+1)\) is calculated from the relationship \(\text{GAM}(y+1) = y \text{GAM}(y)\) and a starting value. For \(y\) an integer, the starting value is \(\text{GAM}(2) = 1\) and for \(y = x 1/3\), for \(x\) an integer, the starting value is \(\text{GAM}(3 1/3) = 2.7781584804296\). For \(y\) greater than or equal to 10, Stirling's formula is used to approximate the gamma function to 11 significant figures of accuracy.

Because a MODIFY system (ref. 3) is used to maintain the program on a permanent file, a new case is easier to make by changing lines of code rather than reading data cards. Outputs consist of printouts and tapes. Some tapes are directly used as tables. Tapes are also usually input to canned programs for calculating analyses of variance and to a program for summing biases or mean squared error over replication, sample size, percentage of incomplete data, and/or generated Dirichlet probability.

Subroutines URAN, URANV, and MATINV shown in the coding are from the NASA, Langley Research Center, mathematics computer library. They are described in Appendices A, B, and C, respectively. Subroutine URAN
gives a single uniform random number according to the algorithm described in Appendix A. Subroutine URANY gives a vector of uniform random numbers from URAN. Subroutine MATINV solves a system of simultaneous linear equations.

In addition, other computer programs not given in the listing have been written. Among these are programs to test the gamma, Dirichlet, trinomial, and uniform pseudorandom-number generators; to calculate analyses of variance; and to sum mean squared errors and biases over one or more of replication, percentage of incomplete data, sample size, and generated Dirichlet probability. The sums have been used for plots in reference 1. Note that a number of subroutines from IMSL (International Mathematical and Statistical Libraries, Inc.; ref. 4) have been used in calculating the analyses of variance.
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LISTING OF COMPUTER PROGRAM

PROGRAM MAIN(INPUT, OUTPUT, TAPE5, TAPE12)

PROGRAmed by KAREN RACkLEY CREODEUR, SPRING 1977. CDC 6600.
FORTRAN EXTENDED VERSION 4.6, NASA, LANGLEY RESEARCH CENTER

INTEGER COVSKIP
REAL MSE(6,7)
DIMENSION BESTQL(4,2,3),CTSEQOL(3)
DIMENSION R(2,7,2),TUKEY(10),UU(50)
DIMENSION QLMS1(2,2,2),QLMS2(2,2,2),QLMS3(2,2,2),QLMS4(2,2,2)
DIMENSION QLMS5(2,2,2),QLMS6(2,2,2),EBIAS1(2,2,2),EBIAS2(2,2,2)
DIMENSION EBIA3(2,2,2),EMS1(2,2,2),EMS2(2,2,2),EMS3(2,2,2)
DIMENSION QLMS11(10,2,2),QLMS12(10,2,2),QLMS21(10,2,2)
DIMENSION QLMS31(10,2,2),QLMS32(10,2,2),QLMS22(10,2,2)
DIMENSION QLMS41(10,2,2),QLMS42(10,2,2),QLMS51(10,2,2)
DIMENSION QLMS61(10,2,2),QLMS62(10,2,2),QLMS52(10,2,2)
DIMENSION T11(2,2,11),T12(2,2,11),T21(2,2,11),T22(2,2,11)
DIMENSION T31(2,2,11),T32(2,2,11),T41(2,2,11),T42(2,2,11)
DIMENSION T51(2,2,11),T52(2,2,11),T61(2,2,11),T62(2,2,11)
DIMENSION ALABEL(3),TLABEL(1)
COMMON DEP(3,3),QLQL(4,3),E(4,3),IROBUST,NTS,P1,P2,P3,P4TID
COMMON/BEST/BESTEP(3,2),CTDEP,CTDEQL(3),PRDEP(19,3),PRDQL(9,7),SBI
1ASEP(3,3),SBIASQL(3,3)
COMMON/BIASRO/COUNTB(3,8),COUNTRO(8,8)
COMMON/CALEST/APMC11,APMC12,APMC22,CONVCR1,COVSKIP,DMLC1,DMLC2,DML
1C3,DPLD,AAPAC11,EAPAC12,EAPAC22,EAPAC32,EAPAC42,EAPAC52,ISTOP,N12,N13
2NS2,PID,PMLC1,PMLC2,PMLC3,SS,SSX,TIMAP,TIME,P,TIM(2),TIMH1,TIMH2,TIMEP
3NU1,XNU2,XNU3,Z1,Z2,Z3,Z12,Z13,Z23,Z1N,Z2N,Z3N
COMMON/ITKT/AVNUMIT(6),CTNUMIT(6,10)
EQUIVALENCE (E(1,1),PEPM1),(E(1,2),PEPM2),(E(1,3),PEPM3)
EQUIVALENCE (E(2,1),PMLE1),(E(2,2),PMLE2),(E(2,3),PMLE3)
EQUIVALENCE (E(3,1),PPMD1),(E(3,2),PPMD2),(E(3,3),PPMD3)
EQUIVALENCE (E(4,1),PAPM1),(E(4,2),PAPM2),(E(4,3),PAPM3)
EQUIVALENCE (DEP(1,1),EML1),(DEP(1,2),EML2),(DEP(1,3),EML3)
EQUIVALENCE (DEP(2,1),EPM1),(DEP(2,2),EPM2),(DEP(2,3),EPM3)
EQUIVALENCE (DEP(3,1),EAPMN1),(DEP(3,2),EAPMN2),(DEP(3,3),EAPMN3)
EQUIVALENCE (DOQL(1,1),DEPMN1),(DOQL(1,2),DEPMN2),(DOQL(1,3),DEPMN3)
EQUIVALENCE (DOQL(2,1),DML1),(DOQL(2,2),DML2),(DOQL(2,3),DML3)
EQUIVALENCE (DOQL(3,1),DPM1),(DOQL(3,2),DPM2),(DOQL(3,3),DPM3)
EQUIVALENCE (DOQL(4,1),DAPMN1),(DOQL(4,2),DAPMN2),(DOQL(4,3),DAPMN3)

DATA ALABEL/IOHA.NU=(10H0.1,0.1,9.1,10H8) /
DATA TLABEL/IOHTABLE 7.1 /
CONVCRI  RELATIVE-ERROR CONVERGENCE CRITERION (USUALLY 0.0001)
IP    DENOTES, IN THE FOLLOWING ORDER, ONE OF THE EXPECTED P'S
      (.01, .01, .98), (.1, .1, .8), (.2, .3, .5), AND (.3, .3, .3)
NSS   INTEGER SS
NXZ   NUMBER OF TRINOMIAL (X AND Z DATA) SIMULATIONS
PAPMI I-TH T.S. APPROXIMATED POSTERIOR MEAN
PEPMI I-TH EXACT POSTERIOR MEAN
PI    I-TH GENERATED P
PID   PERCENTAGE OF INCOMPLETE DATA
PMLCI I-TH COMPLETE M.L.E.
PMLEI I-TH INCOMPLETE-DATA M.L.E.
PPMDI I-TH POSTERIOR MODE
SS    SAMPLE SIZE
SSN   SS * SUM OF PRIOR PARAMETERS XNU1
XNU   VECTOR OF PRIOR PARAMETERS XNU=(XNU1, XNU2, XNU3)
ZI    NUMBER OF OBSERVATIONS FALLING IN CATEGORY I
ZIJ   NUMBER OF OBSERVATIONS SUCH THAT EACH OBSERVATION IS
      KNOWN TO FALL IN ONE OF CATEGORIES I AND J BUT IS NOT
      FURTHER CLASSIFIED
ZIN   ZI+XNU1

XNU1=0.1
XNU2=0.1
IP=2
SEED=24158739.
GSEED=SEED+100.

INITIALIZE ONE-DIMENSIONAL FORM OF UNIFORM RANDOM-NUMBER
GENERATOR FOR GENERATING GAMMA RANDOM VARIABLES

UN=URAN(GSEED)
PRINT 4, GSEED, UN
XNU3=10.-XNU1-XNU2
XNU=XNU1+XNU2+XNU3

GENERATE A 3-COMPONENT (2-DIM) VECTOR OF DIRICHLET PROBABILITIES

DO 9910 IGEN=1,10
  2  G1=GAMMA(XNU1)
  6  G2=GAMMA(XNU2)
  6  G3=GAMMA(XNU3)
  6  G=G1+G2+G3
P1 = G1/G
P2 = G2/G
P3 = 1 - P1 - P2
IF (P3 < 1.) 7, 7, 5
5 PRINT 3, XNU1, XNU2, XNU3, XNU, IP, P1, P2, P3, PID, NSS, NXZ, CONVCRI
PRINT 6
6 FORMAT(* P3 IS NEGATIVE. REGENERATE DIRICHLET.*////)
GO TO 2
7 PID = 0.15
NSS = 25
NXZ = 200
CONVCRI = 0.0001
KASE = 0
IPID = 1
ISS = 1
IPRINT = 0
1 SEED = SEED + 2.
PRINT 3, XNU1, XNU2, XNU3, XNU, IP, P1, P2, P3, PID, NSS, NXZ, CONVCRI
3 FORMAT(1HM* XNU1*F6.3* XNU2*F6.3* XNU3*F6.3* XNU*F6.2* IP*I2
1* P1*F6.4* P2*F6.4* P3*F6.4* PID*F4.2* NSS*I3* NXZ*I3* CONVC
2RI*F7.5//)
C
C INITIALIZE VECTOR FORM OF UNIFORM RANDOM-NUMBER GENERATOR
C
CALL URANV(SEED, UN)
PRINT 4, SEED, UN
4 FORMAT(* SEED GIVEN URANV IS*E23.14* SEED TRANSFORMED BY URAN FRD
1M THIS SEED IS*E23.14//)
SS = NSS
SSN = SS*XNU
C
DO 95 NREPLIC = 1, 2
C
NUMXZ = NXZR1 = NXZR2 = NCOV = NXZ
ISTOP = 0
COVSKIP = 0
AVTPID = AVDPID = 0.
C
C INITIALIZE COUNTERS FOR % RELATIVE DIFFERENCE AND SIGN OF BIAS
C
DO 28 I = 1, 9
DO 27 J = 1, 7
PRDQL(I, J) = 0.
27 CONTINUE
SBIASEP(I) = 0.
28 CONTINUE
DO 29 I=1,21
SBIASQ(I)=0.
29 CONTINUE
DO 30 I=1,27
PDEP(I)=0.
30 CONTINUE
CTRDEP=CTRQDL(1)=CTRQDL(2)=CTRQDL(3)=0.

C
C INITIALIZE COUNTERS FOR NUMBER OF ITERATIONS FOR CONVERGENCE
C
DO 40 I1=1,6
AVNUMIT(I1)=0.
DO 39 I2=1,7
39 CONTINUE
CTNUMIT(I1,I2)=0.
CTNUMIT(I1,8)=CTNUMIT(I1,9)=CTNUMIT(I1,10)=0.
40 CONTINUE
TIM(1)=TIM(2)=TIMEP=TIMAP=0.

C
C DO 42 J=1,2
BESTEP(1,J)=BESTQ(1,J,1)=BESTQ(1,J,2)=BESTQ(1,J,3)=0.
BESTEP(2,J)=BESTQ(2,J,1)=BESTQ(2,J,2)=BESTQ(2,J,3)=0.
BESTEP(3,J)=BESTQ(3,J,1)=BESTQ(3,J,2)=BESTQ(3,J,3)=0.
BESTQ(4,J,1)=BESTQ(4,J,2)=BESTQ(4,J,3)=0.
42 CONTINUE

C
C INITIALIZE COUNTERS FOR BIAS AND RELATIVE DIFFERENCES
C
DO 45 K2=1,8
DO 44 K1=1,8
COUNTRO(K1,K2)=0.
IF (K1-3) 43,43,44
43 COUNTB(K1,K2)=0.
44 CONTINUE
45 CONTINUE

C
C INITIALIZE AVERAGES AND ERROR SUMMARIES TO ZERO
C
C AVERAGE ESTIMATORS AND THEIR STANDARD ERRORS (S.E.)
C
APMLE1= APEPM1= APAPM1= APPM1= APMCL1= AP1=0.
XAPMLE1=XAPEPM1=XAPAPM1=XAPPM1=XAPMLC1=XAP1=0.
APMLE2= APEPM2= APAPM2= APPM2= APMCL2= AP2=0.
XAPMLE2=XAPEPM2=XAPAPM2=XAPPM2=XAPMLC2=XAP2=0.
AVPMN1=AVPMN2=APPMN1=APPMN2=APMD1=APMD2=0.
FPMN1=FPMN2=APPMN1=APPMN2=PGMD1=PGMD2=0.

AV BIAS, MSE, & VAR(MSE) OF ESTIMATORS FROM EXACT POSTERIOR MEAN

XAPMN1=XAPMN2=XAPMN4=XAPMNSQ=0.
XPMO1 =XPMO2 =XPMO4 =XPMO5Q =0.
XML1 =XML2 =XML4 =XMLSQ =0.

AV BIAS, MSE, & VAR(MSE) OF ESTIMATORS FROM GIVEN OR GENERATED P

YEPMN1 =YAPMN1 =YPMO1 =YML1 =YMLC1 =VAPMN1 =QAPMN1 =QPMO1 =0.
YEPMN2 =YAPMN2 =YPMO2 =YML2 =YMLC2 =VAPMN2 =QAPMN2 =QPMO2 =0.
YEPMN4 =YAPMN4 =YPMO4 =YML4 =VAPMNSQ =QAPMNSQ =QPMO4 =0.
YEPMOSQ =YAPMOSQ =YPMOSQ =YMLSQ =VAPMOSQ =QAPMOSQ =QPMOSQ =0.
REGAPMO=REGPMDO=REGEMPO=REGMLO= REGMLO= YMLC20=0.
REGAPM1=REGPMD2=
REGMCL1=YMLC21=0.
REGAPM2=
REGMCL2=YMLC22=0.

INITIALIZE FOR S.E. OF AVERAGE BIAS

WAPMN1=WPMO1=WML1= WAPMN2=WPMO2=WML2=0.
UMLC1=UAPMN1=UPMN1=UPMD1=UML1=FAPMN1=GAPMN1=GPMO1=0.
UMLC2=UAPMN2=UPMN2=UPMD2=UML2=FAPMN2=GAPMN2=GPMO2=0.

EST AVERAGE, BIAS, S.E., AV % REL DIFF, & MSE FOR COV EST FROM EP

AEPMC11=AAPMC11=BAPMC11=C11MSE=VC11MSE=APRDC11=0.
AEPMC12=AAPMC12=BAPMC12=C12MSE=VC12MSE=APRDC12=0.
AEPMC22=AAPMC22=BAPMC22=C22MSE=VC22MSE=APRDC22=0.
A2EPC11=A2EPC12=A2EPC22=VARMSE=0.
A2APC11=A2APC12=A2APC22= VAR4=0.

GENERATE NUMXZ SETS OF TRINOMIAL DATA, CALCULATE ESTIMATORS, AND
COMPARE THEM AS APPROXIMATIONS FOR THE EXACT POSTERIOR MEAN.
ALSO ASSESS HOW WELL THE T.S. APPROXIMATION DOES FOR THE EXACT
POSTERIOR COVARIANCE MATRIX. COMPARE ESTIMATORS WITH GENERATED
P AND DO EXPLORATORY ROBUSTNESS STUDY

DO 65 NT=1,NXZ
NTE=NT
IROBUST=0

GENERATE MULTINOMIAL COMPLETE DATA X AND INCOMPLETE DATA Z
CALCULATE COMPLETE-DATA MAXIMUM LIKELIHOOD ESTIMATE

CALL GENERATE_DATA(NUU+NSS)
IF (ISTOP-1) 46, 58, 58

CALCULATE EXACT POSTERIOR MEAN

46 CALL EPM

FOR INCOMPLETE DATA Z, CALCULATE MAXIMUM-LIKELIHOOD ESTIMATE,
POSTERIOR MODE, AND TAYLOR-SERIES APPROXIMATED POSTERIOR MEAN
AND TAYLOR-SERIES APPROXIMATED POSTERIOR COVARIANCE MATRICES

CALL METHODS

IF (ISTOP-1) 49, 58, 58

AVERAGE ESTIMATORS

49 APMLE1 = APMLE1 + PMLE1
APMLE2 = APMLE2 + PMLE2
APEPM1 = APEPM1 + PEPM1
APEPM2 = APEPM2 + PEPM2
APAPM1 = APAPM1 + PAPM1
APAPM2 = APAPM2 + PAPM2
APPMD1 = APPMD1 + PPMD1
APPMD2 = APPMD2 + PPMD2
APMLC1 = APMLC1 + PMLC1
APMLC2 = APMLC2 + PMLC2
AP1 = AP1 + P1
AP2 = AP2 + P2

TO CALCULATE S.E. OF AVERAGE ESTIMATORS (WANT S.E. SMALL RELATIVE
TO DIFFERENCE BETWEEN AVERAGE BIAS)

XAPMLE1 = XAPMLE1 + PMLE1
XAPMLE2 = XAPMLE2 + PMLE2
XAPEPM1 = XAPEPM1 + PEPM1
XAPEPM2 = XAPEPM2 + PEPM2
XAPAPM1 = XAPAPM1 + PAPM1
XAPAPM2 = XAPAPM2 + PAPM2
XAPPMD1 = XAPPMD1 + PPMD1
XAPPMD2 = XAPPMD2 + PPMD2

ORIGINAL PAGE IS
OF POOR QUALITY!
DIFFERENCES OF ESTIMATORS FROM EXACT POSTERIOR MEAN

BIAS OF T.S. APPROX POSTERIOR MEAN FROM EXACT POSTERIOR MEAN

XAPMN1 = XAPMN1 + EAPMN1
XAPMN2 = XAPMN2 + EAPMN2
XXAPMN = EAPMN1 * EAPMN1 + EAPMN2 * EAPMN2 + EAPMN3 * EAPMN3

MEAN SQUARE ERROR OF T.S. APM FROM EXACT POSTERIOR MEAN

XAPMNSQ = XAPMNSQ + XXAPMN

FOR VARIANCE OF MEAN SQUARE ERROR

XAPMN4 = XAPMN4 * XXAPMN * XXAPMN

BIAS OF POSTERIOR MODE FROM EXACT POSTERIOR MEAN

XPMD1 = XPMD1 + EPMMD1
XPMD2 = XPMD2 + EPMMD2
XXPMD = EPMMD1 * EPMMD1 + EPMMD2 * EPMMD2 + EPMMD3 * EPMMD3

MEAN SQUARE ERROR OF POSTERIOR MODE FROM EXACT POSTERIOR MEAN

XPMDSQ = XPMDSQ + XPMMD

FOR VARIANCE OF MEAN SQUARE ERROR

XPMD4 = XPMD4 * XPMMD * XPMMD

BIAS OF MLE FROM EXACT POSTERIOR MEAN (INCOMPLETE DATA)

XEM1 = XEM1 + EML1
XEM2 = XEM2 + EML2
XXEM = EML1 * EML1 + EML2 * EML2 + EML3 * EML3

MEAN SQUARE ERROR OF IC-D MLE FROM EXACT POSTERIOR MEAN

XMLESQ = XMLESQ + XXEM
FOR VARIANCE OF MSE

XML4/XML4/XML.XML

DIFFERENCE OF APPROXIMATED POSTERIOR COVARIANCES FROM EXACT POSTERIOR COVARIANCES

IF (COVSKIP=0) 51,51,50
NCOV=NCOV-1
COVSKIP=0
GO TO 53

CUMULATE VALUES TO AVERAGE EXACT POSTERIOR VARIANCES

AEPMC11=AEPMC11+EPMC11
AEPMC12=AEPMC12+EPMC12
AEPMC22=AEPMC22+EPMC22

CUMULATE VALUES TO AVERAGE APPROXIMATE POSTERIOR VARIANCES

AAPMC11=AAPMC11+APMC11
AAPMC12=AAPMC12+APMC12
AAPMC22=AAPMC22+APMC22

FOR S.E. OF COVARIANCE AVERAGES

A2EPC11=A2EPC11+EPMC11*EPMC11
A2EPC12=A2EPC12+EPMC12*EPMC12
A2EPC22=A2EPC22+EPMC22*EPMC22
A2APC11=A2APC11+APMC11*APMC11
A2APC12=A2APC12+APMC12*APMC12
A2APC22=A2APC22+APMC22*APMC22

CUMULATE DIFFERENCES BETWEEN EXACT AND APPROXIMATE POSTERIOR VARIANCES

BAPMC11=BAPMC11+EAPMC11
BAPMC12=BAPMC12+EAPMC12
BAPMC22=BAPMC22+EAPMC22

FOR AVERAGE PERCENT RELATIVE DIFFERENCE

PRDC11=ABS(EAPMC11)/EPMC11
PRDC12=ABS(EAPMC12/EPMC12)
PRDC22=ABS(EAPMC22)/EPMC22
APRC11=APRDC11+PRDC11
APRDC12=APRDC12+PRDC12
APRDC22=APRDC22+PRDC22

FOR MSE OF ELEMENTS OF APPROXIMATED POSTERIOR COVARIANCE MATRIX

EAP2C11=EAPMC11+EAPMC11
EAP2C12=EAPMC12+EAPMC12
EAP2C22=EAPMC22+EAPMC22
C11MSE=C11MSE+EAP2C11
C12MSE=C12MSE+EAP2C12
C22MSE=C22MSE+EAP2C22
VVV=EAP2C11+EAP2C12+EAP2C22
VARMSE=VARMSE+VVV

FOR VARIANCE OF MSE

VC11MSE=VC11MSE+EAP2C11+EAP2C11
VC12MSE=VC12MSE+EAP2C12+EAP2C12
VC22MSE=VC22MSE+EAP2C22+EAP2C22
VAR4=VAR4+VVV+VVV

ADD TO BIAS-SIGN AND %-RELATIVE-DIFFERENCE COUNTS

CALL COUNTS(EAPMC11+PROC11,1)
CALL COUNTS(EAPMC12+PRDC12,2)
CALL COUNTS(EAPMC22+PRDC22,3)

ALSO CHECK DIRECTION OF BIAS OF DEPENDENT COVARIANCES

EC33=EPMC11+EPMC22+EPMC12
EC13=EPMC11-EPMC12
EC23=EPMC22-EPMC12
AC33=APMC11+APMC22+EAPMC12
AC13=APMC11-APMC12
AC23=APMC22-APMC12
BC33=AC33-EC33
BC13=AC13-EC13
BC23=AC23-EC23
PRDC13=ABS(BC13/EC13)
PRDCC3=ABS(BC23/EC23)
PRDC33=ABS(BC33/EC33)
CALL COUNTS(BC13,PRDC13,6)
CALL COUNTS(BC23,PRDC23,7)
CALL COUNTS(BC33,PROC33,8)

DIFFERENCE OF ESTIMATORS FROM GENERATED P

\[ \begin{align*}
\delta \text{YMLC1} &= \text{YMLC1} + \text{DMLC1} \\
\delta \text{YMLC2} &= \text{YMLC2} + \text{DMLC2} \\
\delta \text{YYMLCD} &= \text{DMLC1} \cdot \text{YMLC1} + \text{DMLC2} \cdot \text{YMLC2} + \text{DMLC3} \cdot \text{DMLC3} \\
\delta \text{YYMLCD} &= \text{DMLC1} \cdot \text{YMLC1} + \text{DMLC2} \cdot \text{YMLC2} + \text{DMLC3} \cdot \text{DMLC3} \\
\delta \text{YYMLCD} &= \text{YYMLCD} \\
\delta \text{YYMLCD} &= \text{YYMLCD} \\
\delta \text{REGMLCD} &= \text{REGMLCD} + \text{YYMLCD} \\
\end{align*} \]

NOTE THAT FOR REMAINING COMPARISONS WITH GENERATED P WE ARE
MAINLY INTERESTED IN MSE BECAUSE MAIN CONCERN IS DETERMINING WHICH
ESTIMATOR BEST MINIMIZES QUADRATIC LOSS. HOWEVER, WE WILL ALSO
CALCULATE BIAS FROM GENERATED (OR GIVEN) P.

BIAS OF EXACT POSTERIOR MEAN FROM GENERATED P:

\[ \begin{align*}
\delta \text{YEPMN1} &= \text{YEPMN1} + \text{DEPMN1} \\
\delta \text{YEPMN2} &= \text{YEPMN2} + \text{DEPMN2} \\
\delta \text{YYEPMN} &= \text{DEPMN1} \cdot \text{DEPMN1} + \text{DEPMN2} + \text{DEPMN2} + \text{DEPMN3} \cdot \text{DEPMN3} \\
\end{align*} \]

FOR USUAL MEAN SQUARE ERROR OF EPM FROM GENERATED P

\[ \begin{align*}
\delta \text{YEPMSQ} &= \text{YEPMSQ} + \delta \text{YEPMN} \\
\end{align*} \]

FOR REGRESSION ESTIMATION AND CONTROL VARIATE MSE EPM

\[ \begin{align*}
\delta \text{REGEPMO} &= \text{REGEPMO} + \delta \text{YEPMN} + \delta \text{YYMLCD} \\
\end{align*} \]

FOR VARIANCE OF ALL 3 MEAN SQUARE ERRORS

\[ \begin{align*}
\delta \text{YEPMN4} &= \text{YEPMN4} + \delta \text{YEPMN} + \delta \text{YYEPNN} \\
\end{align*} \]

BIAS OF APPROX POSTERIOR MEAN FROM GENERATED P

\[ \begin{align*}
\delta \text{YAPMN1} &= \text{YAPMN1} + \text{DAPMN1} \\
\delta \text{YAPMN2} &= \text{YAPMN2} + \text{DAPMN2} \\
\delta \text{YYAPMN} &= \text{DAPMN1} + \text{DAPMN1} + \text{DAPMN2} + \text{DAPMN2} + \text{DAPMN3} + \text{DAPMN3} \\
\end{align*} \]

USUAL MEAN SQUARE ERROR FOR APMN FROM GENERATED P

\[ \begin{align*}
\delta \text{YAPMSQ} &= \text{YAPMSQ} + \delta \text{YYAPMN} \\
\end{align*} \]
FOR REGRESSION ESTIMATION AND CONTROL VARIATE MSE
REGAPMO = REGAPMO + YYAPMN * YYMLCD
FOR VARIANCE OF MEAN SQUARE ERRORS
YAPMN4 = YAPMN4 + YYAPMN * YYAPMN
FOR BIAS OF POSTERIOR MODE FROM GENERATED P
YPMD1 = YPMD1 + DPM1
YPMD2 = YPMD2 + DPM2
YPYMD = DPM1 * DPM1 + DPM2 * DPM2 + DPM3 * DPM3
YPMDSQ = YPMDSQ + YPYMD
REGPMDO = REGPMDO + YYPMDO * YYMLCD
YPMD4 = YPMD4 + YYPMDO * YPYMD
BIAS OF MLE FROM GENERATED P
YML1 = YML1 + DML1
YML2 = YML2 + DML2
YML = DML1 * DML1 + DML2 * DML2 + DML3 * DML3
YMLSQ = YMLSQ + YYML
REGMLO = REGMLO + YYML * YYMLCD
YML4 = YML4 * YYML * YYML
BY TWO DIFF CRITERION CALC BEST EST FOR THIS TRINOMIAL TRIAL
CALL BESTEST(BESTOL(11))
FOR S.E. OF AVERAGE BIAS
WAPMN1 = WAPMN1 * EAPMN1 * EAPMN1
WAPMN2 = WAPMN2 * EAPMN2 * EAPMN2
WPM1 = WPM1 + EPM1 * EPM1
WPM2 = WPM2 + EPM2 * EPM2
WML1 = WML1 + EML1 * EML1
WML2 = WML2 + EML2 * EML2
UMLC1 = UMLC1 + DMLC1 * DMLC1
UMLC2 = UMLC2 + DMLC2 * DMLC2
UEPMN1 = UEPMN1 * DEPMN1 * DEPMN1
UEPMN2 = UEPMN2 * DEPMN2 * DEPMN2
UAPMN1 = UAPMN1 * DAPMN1 * DAPMN1
UAPMN2 = UAPMN2 * DAPMN2 * DAPMN2
UPMD1=UPMD1+DPMD1*DPMD1
UPMD2=UPMD2+DPMD2*DPMD2
UML1=UML1+DML1*DML1
UML2=UML2+DML2*DML2

CALCULATE ROBUSTNESS ESTIMATORS (FOR QUADRATIC-LOSS COMPARISON
ONLY). USE WRONG PRIOR IN ESTIMATES.
EXCLUDE EPM BECAUSE OF EXPENSE.

ROBUSTNESS SET 1. UNIFORM PRIOR. RECALCULATE ONLY APPROXIMATE
POSTERIOR MEAN BECAUSE FOR A UNIFORM PRIOR THE POSTERIOR MODE
WILL EQUAL THE ALREADY CALCULATED M.L.E. (FOR INCOMPLETE DATA)

Z1N=Z1+1.
Z2N=Z2+1.
Z3N=Z3+1.
SSN=SS+3.
IROBUST=1
CALL METHODS
IF (ISTOP=1) 55,58,58
55 VAPMN1=VAPMN1+DAPMN1
VAPMN2=VAPMN2+DAPMN2
VAPMN=VAPMN1*DAPMN1+DAPMN2*DAPMN2+DAPMN3*DAPMN3
AVPMN1=AVPMN1+PAPM1
AVPMN2=AVPMN2+PAPM2
FAPMN1=FAPMN1*DAPMN1*DAPMN1
FAPMN2=FAPMN2*DAPMN2*DAPMN2
AFPMN1=AFPMN1*PAPM1*PAPM1
AFPMN2=AFPMN2*PAPM2*PAPM2
VAPMSQ=VAPMSQ+VAPMN
REGAPM1=REGAPM1+VAPMN*YMLCO
VAPMN4=VAPMN4+YVAPMN*VYAPMN
YMLC21=YMLC21+YYMLCD
REGMLC1=REGMLC1+SQMLCD

BY TWO DIFF CRITERION CALC BEST EST FOR THIS TRINOMIAL TRIAL
CALL BESTEST(BESTQL(9))

ROBUSTNESS SET 2. IN BAYESIAN ESTIMATORS APMN AND PMD, USE
PRIOR PARAMETERS 10.(NU/10.+(0.09,0.05,-0.14))

56 Z1N=Z1+XNU1+0.9
Z2N=Z2+XNU2+0.5
Z3N=Z3+XNU3-1.4
SSN=S5*XNU
IROBUST=2
CALL METHODS
IF (ISTOP-1) 57,58,58
57 QAPMN1=QAPMN1+DAPMN1
QAPMN2=QAPMN2+DAPMN2
AQPMN1=QPMMN1+PAPM1
AQPMN2=QPMMN2+PAPM2
QAPMN=QAPMN1+DAPMN1+DAPMN2+DAPMN2+DAPMN3+DAPMN3
QAPMSQ=QPMMSQ+QPMMN
REGAPM2=REGAPM2+QQAPMN*YYMLC2
QAPMN=QAPMN+QQAPMN
YMLC22=YMLC22+YYMLC2
REGMLC2=REGMLC2+QUMLC2
C
QPMD1=QPMD1+DPMD1
QPMD2=QPMD2+DPMD2
AQPMD1=QPMD1+PAPMD1
AQPMD2=QPMD2+PAPMD2
QPMD=QPMD1+DPMD1+DPMD2+DPMD2+DPMD3+DPMD3
QPMDSQ=QPMDSQ+QPMD
REGPMD2=REGPMD2+QQPMD+YYMLC2
QPMD4=QPMD4+QQPMD+QQPMD
C
GAPMN1=GAPMN1+DAPMN1+DAPMN1
GAPMN2=GAPMN2+DAPMN2+DAPMN2
GPMD1=GPMD1+DPMD1+DPMD1
GPMD2=GPMD2+DPMD2+DPMD2
AGPMMN1=AGPMMN1+PAPM1+PAPM1
AGPMMN2=AGPMMN2+PAPM2+PAPM2
PGMD1=PGMD1+PAPMD1+PAPMD1
PGMD2=PGMD2+PAPMD2+PAPMD2
C
BY TWO DIFF CRITERION CALC BEST EST FOR THIS TRINOMIAL TRIAL
CALL BESTEST(BESTQL(17))
C
GO TO 65
58 ISTOP=0
IF (IROBUST-1) 60,61,62
60 NUMXZ=NUMXZ-1
NCOV=NCOV-1
NXZ1=NXZ1-1
NXZ2=NXZ2-1
GO TO 65
C AVERAGE OVER NUMXZ TRINOMIAL RESULTS FOR FIXED XNU=(XNU1,XNU2,XNU3) GENERATED P=(P1,P2,P3), P0D, AND SS

ESTIMATOR MEANS (ESTIMATORS AVERAGED OVER NUMXZ TRIALS)

APMLE1=APMLE1/NUMXZ
APMLE2=APMLE2/NUMXZ
APEPM1=APEPM1/NUMXZ
APEPM2=APEPM2/NUMXZ
APAPM1=APAPM1/NUMXZ
APAPM2=APAPM2/NUMXZ
APPMD1=APPMD1/NUMXZ
APPMD2=APPMD2/NUMXZ
APMLC1=APMLC1/NUMXZ
APMLC2=APMLC2/NUMXZ
AP1=AP1/NUMXZ
AP2=AP2/NUMXZ
AVPMN1=AVPMN1/NXZR1
AVPMN2=AVPMN2/NXZR1
AQPMN1=AQPMN1/NXZR2
AQPMN2=AQPMN2/NXZR2
APPMD1=APPMD1/NXZR2
APPMD2=APPMD2/NXZR2

ND=NUMXZ*(NUMXZ-1)
NC=NCOV*(NCOV-1)
NDR1=NXZR1*(NXZR1-1)
NDR2=NXZR2*(NXZR2-1)

STANDARD ERRORS OF ESTIMATOR MEANS

SE1=SQR((XAPMLE1- NUMXZ*APMLE1*APMLE1)/ND)
SE2=SQR((XAPMLE2- NUMXZ*APMLE2*APMLE2)/ND)
SE3=SQR((XAPEPM1- NUMXZ*APEPM1*APEPM1)/ND)
SE4=SQR((XAPEPM2- NUMXZ*APEPM2*APEPM2)/ND)
SE5=SQR((XAPAPM1- NUMXZ*APAPM1*APAPM1)/ND)
SE6=SQR((XAPAPM2- NUMXZ*APAPM2*APAPM2)/ND)
SE7=SQR((XAPPMD1- NUMXZ*APPMD1*APPMD1)/ND)
SE8=SQR((XAPPMD2- NUMXZ*APPMD2*APPMD2)/ND)
$E9$ = SQRT $\{XAPMLC1$ - NUMXZ * APMLC1 * APMLC1$ / ND$)
$E10$ = SQRT $\{XAPMLC2$ - NUMXZ * APMLC2 * APMLC2$ / ND$)
$E11$ = SQRT $\{XAP1$ - NUMXZ * AP1 * AP1$ / ND$)
$E12$ = SQRT $\{XAP2$ - NUMXZ * AP2 * AP2$ / ND$)
$E13$ = SQRT $\{APMN1$ - NXZR1 * AVPMN1 * AVPMN1$ / NDR1$)
$E14$ = SQRT $\{APMN2$ - NXZR1 * AVPMN2 * AVPMN2$ / NDR1$)
$E15$ = SQRT $\{APMN1$ - NXZR2 * AQPMN1 * AQPMN1$ / NDR2$)
$E16$ = SQRT $\{APMN2$ - NXZR2 * AQPMN2 * AQPMN2$ / NDR2$)
$E17$ = SQRT $\{PGMD1$ - NXZR2 * AOPMD1 * AOPMD1$ / NDR2$)
$E18$ = SQRT $\{PGMD2$ - NXZR2 * AOPMD2 * AOPMD2$ / NDR2$)

PRINT 1079, NUMXZ
PRINT 1080, APMLE1, SE1, APEPM1, SE3, APAPM1, SE5, APPMD1, SE7, APMLC1, SE9
1, AP1, SE11, APMLE2, SE2, APEPM2, SE4, APAPM2, SE6, APPMD2, SE8, APMLC2, SE10, 2AP2, SE12

1079 FORMAT(' AVERAGE ESTIMATORS (MEANS) AND THEIR STANDARD ERRORS ON 1ER*,'14X* TRIALS*')

1080 FORMAT('XAPMN1 * XAPMN1 / NUMXZ
XAPMN2 * XAPMN2 / NUMXZ
XPMD1 * XPMD1 / NUMXZ
XPMD2 * XPMD2 / NUMXZ
XML1 * XML1 / NUMXZ
XML2 * XML2 / NUMXZ
AEPMC11 * AEPMC11 / NCOV
AEPMC12 * AEPMC12 / NCOV
AEPMC21 * AEPMC21 / NCOV
AEPMC22 * AEPMC22 / NCOV
AAPMC11 * AAPMC11 / NCOV
AAPMC12 * AAPMC12 / NCOV
AAPMC21 * AAPMC21 / NCOV
AAPMC22 * AAPMC22 / NCOV
BAPMC11 * BAPMC11 / NCOV
BAPMC12 * BAPMC12 / NCOV
BAPMC21 * BAPMC21 / NCOV
BAPMC22 * BAPMC22 / NCOV
YMLC1 * YMLC1 / NUMXZ
YMLC2 * YMLC2 / NUMXZ
YEPMN1 * YEPMN1 / NUMXZ
YEPMN2 * YEPMN2 / NUMXZ
YAPMN1 * YAPMN1 / NUMXZ
YAPMN2 * YAPMN2 / NUMXZ
YPMD1 * YPMD1 / NUMXZ
YPMD2 * YPMD2 / NUMXZ

C
C AVERAGE BIASES AND COVARIANCE ESTIMATORS
C

XAPMN1 = XAPMN1 / NUMXZ
XAPMN1 = XAPMN1 / NUMXZ
XPMD1 = XPMD1 / NUMXZ
XPMD2 = XPMD2 / NUMXZ
XML1 = XML1 / NUMXZ
XML2 = XML2 / NUMXZ
AEPMC11 = AEPMC11 / NCOV
AEPMC12 = AEPMC12 / NCOV
AEPMC21 = AEPMC21 / NCOV
AEPMC22 = AEPMC22 / NCOV
AAPMC11 = AAPMC11 / NCOV
AAPMC12 = AAPMC12 / NCOV
AAPMC21 = AAPMC21 / NCOV
AAPMC22 = AAPMC22 / NCOV
BAPMC11 = BAPMC11 / NCOV
BAPMC12 = BAPMC12 / NCOV
BAPMC21 = BAPMC21 / NCOV
BAPMC22 = BAPMC22 / NCOV
YMLC1 = YMLC1 / NUMXZ
YMLC2 = YMLC2 / NUMXZ
YEPMN1 = YEPMN1 / NUMXZ
YEPMN2 = YEPMN2 / NUMXZ
YAPMN1 = YAPMN1 / NUMXZ
YAPMN2 = YAPMN2 / NUMXZ
YPMD1 = YPMD1 / NUMXZ
YPMD2 = YPMD2 / NUMXZ
YML1=YML1/NUMXZ
YML2=YML2/NUMXZ
VAPMN1=VAPMN1/NXZ1
VAPMN2=VAPMN2/NXZ1
QAPMN1=QAPMN1/NXZ1
QAPMN2=QAPMN2/NXZ1
QPM01=QPM01/NXZ2
QPM02=QPM02/NXZ2
EBIAS1(NREPLIC,ISS,IPID)=XAPMN1
EBIAS2(NREPLIC,ISS,IPID)=XPM01
EBIAS3(NREPLIC,ISS,IPID)=XML1

CALCULATE S.E. OF BIAS, WANT THIS TO BE SMALL RELATIVE TO THE
DIFFERENCE BETWEEN THE BIASES

SE21=SQRT((WAPMN1-NUMXZ*XAPMN1*XAPMN1)/ND)
SE22=SQRT((WAPMN2-NUMXZ*XAPMN2*XAPMN2)/ND)
SE23=SQRT((WPM01-NUMXZ*XPM01*XPM01)/ND)
SE24=SQRT((WPM02-NUMXZ*XPM02*XPM02)/ND)
SE25=SQRT((WML1-NUMXZ*XML1*XML1)/ND)
SE26=SQRT((WML2-NUMXZ*XML2*XML2)/ND)
SE27=SQRT((A2EPC11-NCOV*AEPMC11*AEPMC11)/NC)
SE28=SQRT((A2EPC12-NCOV*AEPMC12*AEPMC12)/NC)
SE29=SQRT((A2EPC21-NCOV*AEPMC21*AEPMC21)/NC)
SE30=SQRT((A2EPC22-NCOV*AEPMC22*AEPMC22)/NC)
SE31=SQRT((A2APC11-NCOV*AAPMC11*AAPMC11)/NC)
SE32=SQRT((A2APC12-NCOV*AAPMC12*AAPMC12)/NC)
SE33=SQRT((A2APC21-NCOV*AAPMC21*AAPMC21)/NC)
SE34=SQRT((A2APC22-NCOV*AAPMC22*AAPMC22)/NC)
SE35=SQRT((C11MSE-NCOV*BAPMC11*BAPMC11)/NC)
SE36=SQRT((C12MSE-NCOV*BAPMC12*BAPMC12)/NC)
SE37=SQRT((C21MSE-NCOV*BAPMC21*BAPMC21)/NC)
SE38=SQRT((C22MSE-NCOV*BAPMC22*BAPMC22)/NC)
SE39=SQRT((UMLC1-NUMXZ*YMLC1*YMLC1)/ND)
SE40=SQRT((UMLC2-NUMXZ*YMLC2*YMLC2)/ND)
SE41=SQRT((UEPMN1-NUMXZ*YEPMN1*YEPMN1)/ND)
SE42=SQRT((UEPMN2-NUMXZ*YEPMN2*YEPMN2)/ND)
SE43=SQRT((UPMD1-NUMXZ*YPM01*YPM01)/ND)
SE44=SQRT((UPMD2-NUMXZ*YPM02*YPM02)/ND)
SE45=SQRT((UML1-NUMXZ*YML1*YML1)/ND)
SE46=SQRT((UPMN1-NXZ1*VAPMN1*VAPMN1)/ND1)
SE47=SQRT((UPMN2-NXZ1*VAPMN2*VAPMN2)/ND1)
SE48=SQRT((QAPMN1-NXZ2*QAPMN1*QAPMN1)/ND2)
SE49=SQRT((QAPMN2-NXZ2*QAPMN2*QAPMN2)/ND2)
SE50=SQRT((QPM01-NXZ2*QPM01*QPM01)/ND2)
$SE51 = \sqrt{22}$

```c
PRINT 1150, NUMXZ, XAPMN1, SE21, XPM1, SE23, XML1, SE25, XAPMN2, SE22, XPM2, SE36, YEPMN2, SE41, YPMD2, SE43, YML2, SE45, YMLC2, SE37
1150 FORMAT('/* AVERAGE BIASES AND THEIR STANDARD ERRORS OVER */',14,'/* TRI
1ALS */ FROM EPM*15X*XAPM*16X*S.E.*16X*XPMD*16X*S.E. *17X*XML*16X*S.
2E.*10X*P1 *6F20.14/10X*P2, 6F20.14/* FROM P*9X*YEPM*8X*S.
3E.*8X*YAPM*8X*S.E.*8X*YPMD*8X*S.E.*9X*YML*8X*S.E.*8X*YMLC*8X*S.E. *4
4/10X*P1 *10F12.10/10X*P2 *10F12.10/*)
PRINT 1154, APVPMN1, SE13, APMPLE1, SE1, AQPMN1, SE15, AQPMD1, SE17, APVPMN2, SE19, APMPLE2, SE2, AQPMN2, SE16, AQPMD2, SE18
1154 FORMAT('/* ROBUST-ESTIMATOR AVERAGES AND S.E.'MS. WANT S.E. SMALL
1RELATIVE TO DIFFERENCE BETWEEN ESTIMATORS. */9X*APMR1*11X*S.E.*9X*P
2MDR1*MLE*8X*S.E.*11X*APMR2*10X*S.E.*11X*MDR2*10X*S.E. */*/ P1 *4(F1
32.8,F18.14)/* P2 *4(F12.8,F18.14)/*)
PRINT 1155, VAPM1, SE46, YML1, SE44, QAPMN1, SE48, QPMD1, SE50, VAPM2, SE
147, YML2, SE45, QAPMN2, SE49, QPMD2, SE51
1155 FORMAT('/* ROBUST-ESTIMATORS BIASES AND STANDARD ERRORS. */10X, *
1VAPM*11X*S.E.*9X*VPM*9X*S.E.*11X*QAPM*11X*S.E.*11X*QPM*11X*S.E. *1
21X*S.E. */*/ P1 *4(F12.8,F18.14)/* P2 *4(F12.8,F18.14)/*)
```

```c
C MEAN SQUARE ERRORS AND THEIR STANDARD ERRORS
C
XAPMNSQ=XAPMNSQ/NUMXZ
XPMDSQ=XPMDSQ/NUMXZ
XMLSQ=XMLSQ/NUMXZ
EMS1(NREPLIC, ISS, IPID) = XAPMNSQ
EMS2(NREPLIC, ISS, IPID) = XPMDSQ
EMS3(NREPLIC, ISS, IPID) = XMLSQ
YMLC20=YMLC20/NUMXZ
YEPMSQ=YEPMSQ/NUMXZ
YAPMSQ=YAPMSQ/NUMXZ
YPMSQ=YPMSQ/NUMXZ
YMLSQ=YMLSQ/NUMXZ
SD1=SQRT((XAPMN4-NUMXZ*XAPMNSQ*XAPMNSQ)/ND)
SD2=SQRT((XPM4-NUMXZ*XPMDSQ*XPMDSQ)/ND)
SD3=SQRT((XML4-NUMXZ*XMLSQ*XMLSQ)/ND)
SD4=SQRT((REGMLCO-NUMXZ*YMLC20*YMLC20)/ND)
SD5=SQRT((YEPMN4*YEPMN4*YEPMSQ*YEPMSQ)/ND)
SD6=SQRT((YAPMN4*YAPMN4*YAPMSQ*YAPMSQ)/ND)
SD7=SQRT((YPMD4-NUMXZ*YPMSQ*YPMSQ)/ND)
SD8=SQRT((YML4-NUMXZ*YMLSQ*YMLSQ)/ND)
```

C DIFFERENCES BETWEEN MEAN SQUARE ERRORS
23

DM1= XAPMSQ-XPMDSO
DM2= XAPMSQ-YLMSQ
DM3= XPMSQ-XLMSQ
DM4= YMLC20-YAPMSQ
DM5= YMLC20-YAPMSQ
DM6= YMLC20-YPMSQ
DM7= YMLC20-YMLSO
DM8= YPMMSQ-YAPMSQ
DM9= YPMMSQ-YPMSQ
DM10= YPMMSQ-YMLSO
DM11= YAPMSQ-YPMSQ
DM12= YAPMSQ-YMLSO
DM13= YPMDSQ-YMLSO

PRINT 1200, NUMXZ, XAPMSQ, SD1, YPMMSQ, SD5, YMLC20, SD4, XPMSQ, SD2, YAP
MSQ, SD6, YLMSQ, SD8, XLMSQ, SD3, XPMSQ, SD7

1200 FORMAT(/// AVERAGE MEAN SQUARE ERRORS AND THEIR STANDARD ERRORS OVER
PRINT 1300, DM1, DM8, DM12, DM12, DM10, DM10, DM13, DM7

1300 FORMAT(/// DIFFERENCE BETWEEN AVERAGE MSE'S. WANT DIFFERENCE LARGE
1E RELATIVE TO SE(MSE)/// EAPM=EPMD=14.9, 12X*DEPM=DEPM=14.9, 12X
2*DM=DPMD=14.9, 12X*DPMD=DPMD=14.9, 12X*DEPM=DEPM=14.9, 12X
3PM=14.9, 12X*DPMD=DPMD=14.9, 12X
X=NUMXZ
DO 71 I=1,6
IF (I-4) 69, 67, 68
67 X=NXZRI
GO TO 69
68 X=NXZR2
69 AVNUMIT(I)=AVNUMIT(I)/XN
DO 70 J=1,10
70 CTNUMIT(I,J)=CTNUMIT(I,J)/XN
71 CONTINUE
PRINT 2000, IP, PID, SS, NREPLI, NUMXZ, NXZRI, NXZR2, (AVNUMIT(I), I=1,6)

2000 FORMAT(/// NUMBER OF ITER FOR CONVERGENCE AVERAGED OVER NUMBER OF
1 TRINOMIAL SIMULATIONS. IP=I3, PID=F4.2, SS=F3.0, NREPLI=F3.0
212/* NUMXZ=I3, NXZRI=I3, NXZR2=I3, AV NUM ITER FOR MLE=F7
3.3*PHDRO=F7.3*APMRO=F7.3*APMR1=F7.3*PHDRO=F7.3*APMR1=F7.3*APMR2=F7.3
43/1)
PRINT 2010, ((CTNUMIT(I,J), J=1,10), I=1,6), (CTNUMIT(I,J), J=1,10)
1, I=2,5, 1/((CTNUMIT(I,J), J=1,10), I=3,6, 3)
2010 FORMAT(* PROPORTION OF DATA SETS FOR WHICH NUMBER OF ITERATIONS WAS
15 OF SPECIFIED AMOUNTS/10X1 2 3 4 5 6 7
8 8-10 11-15 GT 15*11X1 2 3 4 5 6 7 8
9-10 11-15 GT 15/* MLE *10F6.3* APMR1 *10F6.3* PMDR *10F6.3*
4 PMSR2 *10F6.3* APMR0 *10F6.3* APMR2 *10F6.3//)
YEPMSQ=NUMX*YEPMSQ
YAPMSQ=NUMX*YAPMSQ
YPMDSQ=NUMX*YPMDSQ
YMRSQ=NUMX*YMRSQ
YMLC20=NUMX*YMLC20
T=(1-(P1*P1+P2*P2+P3*P3)/SS)
C
CALL ESTMSE(YEPMSQ,YEPMN4,REGEPMO,YMLC20,REGMLCO,T,NUMX,SE(1))
CALL ESTMSE(YAPMSQ,YAPMN4,REGAPMO,YMLC20,REGMLCO,T,NUMX,SE(7))
CALL ESTMSE(YPMDSQ,YPM4,REGPMO,YMLC20,REGMLCO,T,NUMX,SE(13))
CALL ESTMSE(YMRSQ,YML4,REGML2,YMLC20,REGML2,T,NUMX,SE(19))
CALL ESTMSE(YMLC20,NUMX,YMLC20,T,NUMX,SE(25))
CALL ESTMSE(YAPMSQ,YAPMN4,REGAPM1,YMLC21,REGMLC1,T,NUMX,SE(31))
CALL ESTMSE(YPMDSQ,YPM4,REGPMO,YMLC21,REGMLC2,T,NUMX,SE(37))

C
PRINT 2030, IP,PID,SS,NREPLIC,((MSE(I,J),J=1,7),I=1,6)
2030 FORMAT(* THREE KINDS OF MSE (AND THEIR VARIANCES) FOR QUADRATIC-LO
ISS COMPARISONS. IP=I2, PID=F4.2, SS=F3.0, NREPLIC=I2//18*
2EPM*15X*APM*15X*PMO*15X*MLE*14X*APMR1*13X*APMR2*13X*PMDR2/* REG M
3SE *E18.7* VAR(MSE)*E18.7*/ CV MSE *E18.7* VAR(MSE)*E18.7/*
4 RE MSE *E18.7* VAR(MSE)*E18.7/)"
C
IF (ISS-1) 2035,2035,2040
2035 QLMSI1(IGEN,IPID,NREPLIC)=MSE(5,2)
QLMSI2(IGEN,IPID,NREPLIC)=MSE(5,3)
QLMSI3(IGEN,IPID,NREPLIC)=MSE(5,4)
QLMSI4(IGEN,IPID,NREPLIC)=MSE(5,5)
QLMSI5(IGEN,IPID,NREPLIC)=MSE(5,6)
QLMSI6(IGEN,IPID,NREPLIC)=MSE(5,7)
GO TO 2045
2040 QLMS21(IGEN,IPID,NREPLIC)=MSE(5,2)
QLMS32(IGEN,IPID,NREPLIC)=MSE(5,3)
QLMS42(IGEN,IPID,NREPLIC)=MSE(5,4)
QLMS52(IGEN,IPID,NREPLIC)=MSE(5,5)
QLMS62(IGEN,IPID,NREPLIC)=MSE(5,6)
QLMS62(IGEN,IPID,NREPLIC)=MSE(5,7)
C
C PROPORTIONS FOR BEST ESTIMATOR (BEST IN TERMS OF SMALLEST SUMMED
C SQUARED ERROR AND % REL DIFF FOR SUM BEING OVER THE THREE
C COMPONENTS OF AN ESTIMATOR) AND FOR SIGN OF BIAS
C

2045 CTSEQL(1)=NUMXZ
    CTSEQL(2)=NXZR1
    CTSEQL(3)=NXZR2
    DO 75 I=1,4
    DO 72 IR=1,3
        BESTQL(I,1,IR)=BESTQL(I,1,IR)/CTSEQL(IR)
        BESTQL(I,2,IR)=BESTQL(I,2,IR)/CRTLQ(L(IR)
        SBIASQL(IR,1)=SBIASQL(IR,1)/NUMXZ
    72 CONTINUE
    IF (I-4) 73,75,75
    73 BESTEP(I,1)=BESTEP(I,1)/NUMXZ
        BESTEP(I,2)=BESTEP(I,2)/CRTLDEP
        DO 74 K=1,3
            SBIASEP(K,I)=SBIASEP(K,I)/NUMXZ
        74 CONTINUE
    75 CONTINUE
    DD 76 K=1,3
        SBIASQL(K,5)=SBIASQL(K,5)/NXZR1
        SBIASQL(K,6)=SBIASQL(K,6)/NXZR2
        SBIASQL(K,7)=SBIASQL(K,7)/NXZR2
    76 CONTINUE

C
C CALCULATE % ABS REL DIFF LESS THAN (INSTEAD OF BETWEEN) SPECIFIED
C AMOUNTS
C
    DO 80 I=1,7
    DO 79 II=2,8
        PRDQL(II,I)=PRDQL(II,I)+PRDQL(II-1,I)
        IF (I-4) 78,79,79
    78 PRDEP(II,I)=PRDEP(II,I)+PRDEP(II-1,I)
    79 CONTINUE
    80 CONTINUE
    IR=1
    DO 85 I=1,7
        IF (I-5) 83,81,82
    81 IR=2
        GO TO 83
    82 IR=3
    83 DO 84 II=1,8
        PRDQL(II,I)=PRDQL(II,I)/CRTLQ(L(IR)
    84 CONTINUE
    85 CONTINUE
    DD 88 I=1,3
    DD 88 II=1,8
26 CONTINUE
PRINT 2050, ((BESTEP(I,J),I=1,3), ((BESTOL(I,J),I=2,4), K=1,3), J=1,2)

2050 FORMAT(* PROPORTION OF CASES THAT AN ESTIMATOR IS BEST. FIRST 3 C
1DLNS ARE RESULTS FOR ESTIMATING EPM. REMAINING COLNS, FOR MIN QUA
2D LOSS*/43X*ORIG.-PRIOR ROBUST SET 1UNIFORM-PRIOR ROBUST SET
3 PERTURB-PRIOR ROBUST SET*/20X*MLE PMD APM*11X* *MLE PM
4 D APM*14X*MLE = PMD APM*13X*MLE PMD APM*/ SDQ ERR CRIT
5 *3F6.2,8X3F6.2,3X,2(9X,3F6.2)*/ REL DIFF CRIT *3F6.2,8X3F6.2,3
6X,2(9X,3F6.2)*/
PRINT 2070, ((SBIASEP(I,J)) J=1,3), (SBIASQL(I,K), K=1,7), I=1,3)

2070 FORMAT(* PROPORTION OF CASES IN WHICH DIFFERENCE BETWEEN FIRST COM
1ONENT OF ESTIMATOR AND THAT OF ESTIMATED IS OF A CERTAIN SIGN*/1
25X*EMLE*6X*EPMD*6X*EAPM*16X*QEME*6X*QMLE*5X*QMDR*6X*QAPM*9X*QA
3PMR*9X*QPMR*4X*QAPMR*8X*NEG *3F10.4,10X,4F10.4,5X,F10.4,5X,2F
410.4*5X*IERO *3F10.4,10X,4F10.4,5X,F10.4,5X,2F10.4/6X*POS *3F10.4,
910X,F10.4,5X,F10.4,2F10.4*/
PRINT 2080, ((PRDEP(I,J)) I=1,3), (PROOL(I,K)), K=1,7), I=1,8)

2080 FORMAT(* PROPORTION OF CASES FOR WHICH % ABSOLUTE RELATIVE DIFFERE
1IC FOR EACH OF THE THREE ESTIMATOR COMPONENTS IS LESS THAN SPECIFI
2ED AMOUNTS*/15X*EMLE*6X*EPMD*6X*EAPM*16X*QEME*6X*QMLE*5X*QMDR*6X
3*QAPMRO*9X*QAPMR1*9X*QPMDR2*4X*QAPMR2*8X*NEG *3F10.4,10X,4F10.4,
45X,F10.4,5X,2F10.4,6X*0.1 *3F10.4,10X,4F10.4,5X,F10.4,5X,2F10.4,6X
5*1.0 *3F10.4,10X,4F10.4,5X,F10.4,5X,2F10.4,5X* 5.0*3F10.4,10X,4F
610.4,5X,F10.4,5X,2F10.4,5X*10.0 *3F10.4,10X,4F10.4,5X,F10.4,5X,2F
710.4,5X*15.0 *3F10.4,10X,4F10.4,5X,F10.4,5X,2F10.4,5X*20.0 *3F10.
84,10X,4F10.4,5X,F10.4,5X,2F10.4,5X*25.0 *3F10.4,10X,4F10.4,5X,F10
9.4,5X,2F10.4*/

C PERCENT AVERAGE RELATIVE DIFFERENCE FOR COVARIANCE ESTIMATES
C PARDC11=100.*X*AEPMC11-AAPMC11)/AEPMC11
C PARDC12=100.*X*AEPMC12-AAPMC12)/AEPMC12
C PARDC22=100.*X(AEPMC22-AAPMC22)/AEPMC22
C
C AVERAGE PERCENT RELATIVE DIFFERENCE
C APRDC11=100.*X*APRDC11/NCOV
C APRDC12=100.*X*APRDC12/NCOV
C APRDC22=100.*X*APRDC22/NCOV
C
C SQUARE ROOT MSE DIVIDED BY AVERAGE EPV
C C11MSE=C11MSE/NCOV
C

SE1MSE1/MSE

C

SE11MSE=SORT((VC11MSE- NCov*C11MSE*C11MSE)/NC)
SE12MSE=SORT((VC12MSE- NCov*C12MSE*C12MSE)/NC)
SE22MSE=SORT((VC22MSE- NCov*C22MSE*C22MSE)/NC)
C11RT2=SE11MSE/C11MSE
C12RT2=SE12MSE/C12MSE
C22RT2=SE22MSE/C22MSE
DO 91 J=1,8
DO 90 I=1,8
COUNTRD(I,J)=COUNTRD(I,J)/NC
IF (I-3) 89,89,90
89 COUNTB(I,J)=COUNTB(I,J)/NC
90 CONTINUE
91 CONTINUE

C

PRINT 3000, AEPMC11, SE27, AAPMC11, SE30, PARDC11, APRDC11, C11RT1, C11RT
12, AEPMC12, SE28, AAPMC12, SE31, PARDC12, APRDC12, C12RT1, C12RT2, AEPMC22,
2SE29, AAPMC22, SE32, PARDC22, APRDC22, C22RT1, C22RT2
3000 FORMAT(///12X,*AVERAGE EPV*8X*S.E.*9X*AVERAGE APV*8X*S.E.*7X*Z AV
1 REL DIFF AV % REL DIFF SORT(MSE)/(1) SE(MSE)/MSE///3X*C11
2*8E16, 7/3X*C12 *8E16, 7/3X*C22 *8E16, 7)
PRINT 3005, NCov, IP, PID, SS, NREPL, (COUNTRD(I,1), I=1,6), (COUNTRD(I
1,3), I=1,6), (COUNTRD(I,8), I=1,6), (COUNTRD(I,2), I=1,6), (COUNTRD(I,6
2), I=1,6), (COUNTRD(I,7), I=1,6)
3005 FORMAT(///* PROPORTION DF*I4* CASES IN WHICH PERCENT REL DIFF WAS
1LESS THAN VARYING AMOUNTS. IP=12* PID=4.2* SS=9.0* NREPLI
2C=12//8X*.01 0.1 1.0 5.0 10. 15.*12X*.01 0.1 1.0 35.0 10. 15.*12X*.01 0.1 1.0 5.0 10. 15.*// C11*6F6.
35*6X*C22*6F6. 3,6X*C33*6F6. 3,6X*C13*6F6. 3,6X*C23*6F6. 3
5/)
PRINT 3010, NCov, (COUNTB(I,1), I=1,3), (COUNTB(I,3), I=1,3), (COUNTB(I
1,8), I=1,3), (COUNTB(I,2), I=1,3), (COUNTB(I,6), I=1,3), (COUNTB(I,7), I=
21,3)
3010 FORMAT(///* PROPORTN DF*I4* CASES IN WHICH BIAS IS OF A CERTAIN SIGN*
1//7X*NEG ZERO POS*7X*NEG ZERO POS*7X*NEG ZERO POS*7X*NEG ZER
2 ZERO POS*7X*NEG ZERO POS*7X*NEG ZERO POS*/* C11*3F6.3* C2
32*3F6.3* C33*3F6.3* C12*3F6.3* C13*3F6.3* C23*3F6.3)
C
C AVERAGE TRUE PERCENT (PROPORTION) INCOMPLETE DATA

C AVTPID=AVTPID+PID
AVDPID=AVDPID+DPID
PRINT 3070, PID,NUMXZ,AVTPID,AVDPID
3070 FORMAT(* GIVEN PID IS F4.2* AVERAGE OVER NUMXZ=I3* TRIALS OF TRUE
1 PID IS F6.2* AV DIFF BETWEEN TRUE AND GIVEN PID OVER THESE TRIALS
2 IS F6.2/////) 

C 95 CONTINUE

C KASE=KASE+1.
GO TO (9902,9903,9904,9905) KASE
9902 PID=0.40
IPID=2
IPRINT=1
GO TO 1
9903 PID=0.15
SS=50
ISS=2
GO TO 1
9904 PID=0.40
IPID=2
ISS=2
GO TO 1
9905 CONTINUE

C WRITE(12,8000) TLABEL(1),(ALABEL(I),I=1,3)
8000 FORMAT(63X,A10///7X*REGRESSION-ESTIMATE MSE DATA OVER 200 TRINOMIA
1L SIMULATIONS. TWO REPLICATIONS PER CELL. DESIGN 2. *2A10,A2///)
WRITE(12,8001)
8001 FORMAT(47X*A, ORIGINAL PRIOR IN BAYESIAN ESTIMATORS.***/)
WRITE(12,8010)
8010 FORMAT(12X,27H*************************************************,*SS=25*,27H ************
1*******************************************************************************,*SS=50*,27H ****
2********************************************************************** ESTI- DIR. *,9H**********,* PID=15 *,10H**
3************************************5X,9H**********,* PID=40 *,10H**********,5X,9H***********
4PID=15 *F10H***********,5X,9H***********+ PID=40 *,10H***********/ M
5ATOR GEN. REPLICATION1 REPLICATION2 REPLICATION1 REPLICATI
6ON2 REPLICATION1 REPLICATION2 REPLICATION2 REPLICATION
72*/) 
WRITE(12,8011) (I,((QLMS11(I,J,K),K=1,2),J=1,2),((QLMS12(I,J,K),K
I=1,2),J=1,2),I=1,10)
8011 FORMAT(* APMRO *I2,4(2E15.6,2X)/(9X,4(2E15.6,2X))
WRITE(12,8012) (I, (QLMS21(I,J,K),K=1,2),J=1,2), (QLMS22(I,J,K),K=1,2),J=1,2), (I,1,1), (J,1,1)

8012 FORMAT(* PMDRO *I2,4(2E15.6,2X)/(9X,4(2E15.6,2X))))
WRITE(12,8013) (I, (QLMS31(I,J,K),K=1,2),J=1,2), (QLMS32(I,J,K),K=1,2),J=1,2), (I,1,1), (J,1,1)

8013 FORMAT(* MLE *I2,4(2E15.6,2X)/(9X,4(2E15.6,2X))))
WRITE(12,8000) TLABEL(1), ALABEL(I), I=1,3
WRITE(12,8020) 8020 FORMAT(* UNIFORM AND PERTURBED PRIOR IN BAYESIAN ESTIMATORS. */* */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ */ }
$SOSM=0$

DO 9912 I=1,10
$SUM = SUM+QLMS12(I,IPID,NREPLIC)$
$SOSM=SOSM+QLMS12(I,IPID,NREPLIC)*QLMS12(I,IPID,NREPLIC)$

9912
$T21(IPID,NREPLIC)/10.$
$CALL SUMMARY(TUKEY,10,T21(IPID,NREPLIC,1),T21(IPID,NREPLIC,2),T21(IPID,NREPLIC,3),T21(IPID,NREPLIC,4),T21(IPID,NREPLIC,5))$

$T21(IPID,NREPLIC,6)=SUM/10.$
$T21(IPID,NREPLIC,7)=SORT($(SOSM-10.*T21(IPID,NREPLIC,6)*T21(IPID,NREPLIC,7)/90.)$
$T21(IPID,NREPLIC,8)=T21(IPID,NREPLIC,3)$
$T21(IPID,NREPLIC,9)=T21(IPID,NREPLIC,2)+2.*T21(IPID,NREPLIC,3)+T21(IPID,NREPLIC,4)/4.$
$T21(IPID,NREPLIC,10)=T21(IPID,NREPLIC,4)-T21(IPID,NREPLIC,2)$
$T21(IPID,NREPLIC,11)=T21(IPID,NREPLIC,5)-T21(IPID,NREPLIC,1)$
$SUM=0.$
$SOSM=0.$

DO 9913 I=1,10
$SUM = SUM+QLMS21(I,IPID,NREPLIC)$
$SOSM=SOSM+QLMS21(I,IPID,NREPLIC)*QLMS21(I,IPID,NREPLIC)$

9913
$T22(IPID,NREPLIC)/10.$
$CALL SUMMARY(TUKEY,10,T22(IPID,NREPLIC,1),T22(IPID,NREPLIC,2),T22(IPID,NREPLIC,3),T22(IPID,NREPLIC,4),T22(IPID,NREPLIC,5))$

$T22(IPID,NREPLIC,6)=SUM/10.$
$T22(IPID,NREPLIC,7)=SORT($(SOSM-10.*T22(IPID,NREPLIC,6)*T22(IPID,NREPLIC,7)/90.)$
$T22(IPID,NREPLIC,8)=T22(IPID,NREPLIC,3)$
$T22(IPID,NREPLIC,9)=T22(IPID,NREPLIC,2)+2.*T22(IPID,NREPLIC,3)+T22(IPID,NREPLIC,4)/4.$
$T22(IPID,NREPLIC,10)=T22(IPID,NREPLIC,4)-T22(IPID,NREPLIC,2)$
$T22(IPID,NREPLIC,11)=T22(IPID,NREPLIC,5)-T22(IPID,NREPLIC,1)$
$SUM=0.$
$SOSM=0.$

DO 9914 I=1,10
$SUM = SUM+QLMS22(I,IPID,NREPLIC)$
$SOSM=SOSM+QLMS22(I,IPID,NREPLIC)*QLMS22(I,IPID,NREPLIC)$

9914
$T23(IPID,NREPLIC)/10.$
$CALL SUMMARY(TUKEY,10,T23(IPID,NREPLIC,1),T23(IPID,NREPLIC,2),T23(IPID,NREPLIC,3),T23(IPID,NREPLIC,4),T23(IPID,NREPLIC,5))$

$T23(IPID,NREPLIC,6)=SUM/10.$
$T23(IPID,NREPLIC,7)=SORT($(SOSM-10.*T23(IPID,NREPLIC,6)*T23(IPID,NREPLIC,7)/90.)$
$T23(IPID,NREPLIC,8)=T23(IPID,NREPLIC,3)$
$T23(IPID,NREPLIC,9)=T23(IPID,NREPLIC,2)+2.*T23(IPID,NREPLIC,3)+T23(IPID,NREPLIC,4)/4.$
T22(IPID,NREPLIC,10) = T22(IPID,NREPLIC,4) - T22(IPID,NREPLIC,2)
T22(IPID,NREPLIC,11) = T22(IPID,NREPLIC,5) - T22(IPID,NREPLIC,1)
SUM = 0.
SQSM = 0.
DO 9915 I = 1, 10
SUM = SUM + QLMS31(I, IPID, NREPLIC)
SQSM = SQSM + QLMS31(I, IPID, NREPLIC) * QLMS31(I, IPID, NREPLIC)
9915 TUKEY(I) = QLMS31(I, IPID, NREPLIC)
CALL SUMMARY(TUKEY, 10, T31(IPID, NREPLIC, 1), T31(IPID, NREPLIC, 2), T31(IPID, NREPLIC, 3), T31(IPID, NREPLIC, 4), T31(IPID, NREPLIC, 5))
T31(IPID, NREPLIC, 6) = SUM / 10.
T31(IPID, NREPLIC, 7) = SQRT((SQSM - 10. * T31(IPID, NREPLIC, 6) - T31(IPID, NREPLIC, 7) / 90.)
T31(IPID, NREPLIC, 8) = T31(IPID, NREPLIC, 3)
T31(IPID, NREPLIC, 9) = (T31(IPID, NREPLIC, 2) + 2. * T31(IPID, NREPLIC, 3) + T31(IPID, NREPLIC, 4)) / 4.
T31(IPID, NREPLIC, 10) = T31(IPID, NREPLIC, 4) - T31(IPID, NREPLIC, 2)
T31(IPID, NREPLIC, 11) = T31(IPID, NREPLIC, 5) - T31(IPID, NREPLIC, 1)
SUM = 0.
SQSM = 0.
DO 9916 I = 1, 10
SUM = SUM + QLMS32(I, IPID, NREPLIC)
SQSM = SQSM + QLMS32(I, IPID, NREPLIC) * QLMS32(I, IPID, NREPLIC)
9916 TUKEY(I) = QLMS32(I, IPID, NREPLIC)
CALL SUMMARY(TUKEY, 10, T32(IPID, NREPLIC, 1), T32(IPID, NREPLIC, 2), T32(IPID, NREPLIC, 3), T32(IPID, NREPLIC, 4), T32(IPID, NREPLIC, 5))
T32(IPID, NREPLIC, 6) = SUM / 10.
T32(IPID, NREPLIC, 7) = SQRT((SQSM - 10. * T32(IPID, NREPLIC, 6) * T32(IPID, NREPLIC, 7) / 90.)
T32(IPID, NREPLIC, 8) = T32(IPID, NREPLIC, 3)
T32(IPID, NREPLIC, 9) = (T32(IPID, NREPLIC, 2) + 2. * T32(IPID, NREPLIC, 3) + T32(IPID, NREPLIC, 4)) / 4.
T32(IPID, NREPLIC, 10) = T32(IPID, NREPLIC, 4) - T32(IPID, NREPLIC, 2)
T32(IPID, NREPLIC, 11) = T32(IPID, NREPLIC, 5) - T32(IPID, NREPLIC, 1)
SUM = 0.
SQSM = 0.
DO 9917 I = 1, 10
SUM = SUM + QLMS41(I, IPID, NREPLIC)
SQSM = SQSM + QLMS41(I, IPID, NREPLIC) * QLMS41(I, IPID, NREPLIC)
9917 TUKEY(I) = QLMS41(I, IPID, NREPLIC)
CALL SUMMARY(TUKEY, 10, T41(IPID, NREPLIC, 1), T41(IPID, NREPLIC, 2), T41(IPID, NREPLIC, 3), T41(IPID, NREPLIC, 4), T41(IPID, NREPLIC, 5))
T41(IPID, NREPLIC, 6) = SUM / 10.
T41(IPID, NREPLIC, 7) = SQRT((SQSM - 10. * T41(IPID, NREPLIC, 6) * T41(IPID, NREPLIC, 7) / 90.)
$T_{41}(\text{IPID}, \text{NREPLIC}, 8) = T_{41}(\text{IPID}, \text{NREPLIC}, 3)$

$T_{41}(\text{IPID}, \text{NREPLIC}, 9) = (T_{41}(\text{IPID}, \text{NREPLIC}, 2) + 2 \cdot T_{41}(\text{IPID}, \text{NREPLIC}, 3) + T_{41}(\text{IPID}, \text{NREPLIC}, 5))/4.$

$T_{41}(\text{IPID}, \text{NREPLIC}, 10) = T_{41}(\text{IPID}, \text{NREPLIC}, 4) - T_{41}(\text{IPID}, \text{NREPLIC}, 2)$

$T_{41}(\text{IPID}, \text{NREPLIC}, 11) = T_{41}(\text{IPID}, \text{NREPLIC}, 5) - T_{41}(\text{IPID}, \text{NREPLIC}, 1)$

$\text{SUM} = 0.$

$\text{SOSM} = 0.$

DO 9918 I = 1, 10

SUM = SUM + QLMS42(I, IPID, NREPLIC)

$\text{SOSM} = \text{SOSM} + \text{QLMS52}(I, \text{IPID}, \text{NREPLIC}) \cdot \text{QLMS42}(I, \text{IPID}, \text{NREPLIC})$

9918

TUKEY(I) = QLMS42(I, IPID, NREPLIC)

CALL SUMMARY(TUKEY, 10, T42(\text{IPID}, \text{NREPLIC}, 1), T42(\text{IPID}, \text{NREPLIC}, 2), T42(\text{IPID}, \text{NREPLIC}, 3), T42(\text{IPID}, \text{NREPLIC}, 4), T42(\text{IPID}, \text{NREPLIC}, 5))

$T_{42}(\text{IPID}, \text{NREPLIC}, 6) = \text{SUM}/10.$

$T_{42}(\text{IPID}, \text{NREPLIC}, 7) = \text{SQRT}((\text{SQSM} - 10 \cdot T_{42}(\text{IPID}, \text{NREPLIC}, 6)) / 90.)$

$T_{42}(\text{IPID}, \text{NREPLIC}, 8) = T_{42}(\text{IPID}, \text{NREPLIC}, 3)$

$T_{42}(\text{IPID}, \text{NREPLIC}, 9) = (T_{42}(\text{IPID}, \text{NREPLIC}, 2) + 2 \cdot T_{42}(\text{IPID}, \text{NREPLIC}, 3) + T_{42}(\text{IPID}, \text{NREPLIC}, 4))/4.$

$T_{42}(\text{IPID}, \text{NREPLIC}, 10) = T_{42}(\text{IPID}, \text{NREPLIC}, 4) - T_{42}(\text{IPID}, \text{NREPLIC}, 2)$

$T_{42}(\text{IPID}, \text{NREPLIC}, 11) = T_{42}(\text{IPID}, \text{NREPLIC}, 5) - T_{42}(\text{IPID}, \text{NREPLIC}, 1)$

$\text{SUM} = 0.$

$\text{SOSM} = 0.$

DO 9919 I = 1, 10

SUM = SUM + QLMS51(I, IPID, NREPLIC)

$\text{SOSM} = \text{SOSM} + \text{QLMS51}(I, \text{IPID}, \text{NREPLIC}) \cdot \text{QLMS51}(I, \text{IPID}, \text{NREPLIC})$

9919

TUKEY(I) = QLMS51(I, IPID, NREPLIC)

CALL SUMMARY(TUKEY, 10, T51(\text{IPID}, \text{NREPLIC}, 1), T51(\text{IPID}, \text{NREPLIC}, 2), T51(\text{IPID}, \text{NREPLIC}, 3), T51(\text{IPID}, \text{NREPLIC}, 4), T51(\text{IPID}, \text{NREPLIC}, 5))

$T_{51}(\text{IPID}, \text{NREPLIC}, 6) = \text{SUM}/10.$

$T_{51}(\text{IPID}, \text{NREPLIC}, 7) = \text{SQRT}((\text{SQSM} - 10 \cdot T_{51}(\text{IPID}, \text{NREPLIC}, 6)) / 90.)$

$T_{51}(\text{IPID}, \text{NREPLIC}, 8) = T_{51}(\text{IPID}, \text{NREPLIC}, 3)$

$T_{51}(\text{IPID}, \text{NREPLIC}, 9) = (T_{51}(\text{IPID}, \text{NREPLIC}, 2) + 2 \cdot T_{51}(\text{IPID}, \text{NREPLIC}, 3) + T_{51}(\text{IPID}, \text{NREPLIC}, 4))/4.$

$T_{51}(\text{IPID}, \text{NREPLIC}, 10) = T_{51}(\text{IPID}, \text{NREPLIC}, 4) - T_{51}(\text{IPID}, \text{NREPLIC}, 2)$

$T_{51}(\text{IPID}, \text{NREPLIC}, 11) = T_{51}(\text{IPID}, \text{NREPLIC}, 5) - T_{51}(\text{IPID}, \text{NREPLIC}, 1)$

$\text{SUM} = 0.$

$\text{SOSM} = 0.$

DO 9920 I = 1, 10

SUM = SUM + QLMS52(I, IPID, NREPLIC)

$\text{SOSM} = \text{SOSM} + \text{QLMS52}(I, \text{IPID}, \text{NREPLIC}) \cdot \text{QLMS52}(I, \text{IPID}, \text{NREPLIC})$

9920

TUKEY(I) = QLMS52(I, IPID, NREPLIC)

CALL SUMMARY(TUKEY, 10, T52(\text{IPID}, \text{NREPLIC}, 1), T52(\text{IPID}, \text{NREPLIC}, 2), T52(\text{IPID}, \text{NREPLIC}, 3), T52(\text{IPID}, \text{NREPLIC}, 4), T52(\text{IPID}, \text{NREPLIC}, 5))
T52(IPID,NREPLIC,6) = SUM/10.
T52(IPID,NREPLIC,7) = SQRT((SQSM-10.*T52(IPID,NREPLIC,6)*T52(IPID,NREPLIC,6))/90.)
T52(IPID,NREPLIC,8) = T52(IPID,NREPLIC,3)
T52(IPID,NREPLIC,9) = (T52(IPID,NREPLIC,2)+2.*T52(IPID,NREPLIC,3)+T5
12(IPID,NREPLIC,4))/4.
T52(IPID,NREPLIC,10) = T52(IPID,NREPLIC,4) - T52(IPID,NREPLIC,2)
T52(IPID,NREPLIC,11) = T52(IPID,NREPLIC,5) - T52(IPID,NREPLIC,1)
SUM=0.
SOSM=0.
DO 9921 I=1,10
SUM = SUM+QLMS61(I,IPID,NREPLIC)
SOSM=SOSM+QLMS61(I,IPID,NREPLIC)*QLMS61(I,IPID,NREPLIC)
9921 TUKEY(I) = QLMS61(I,IPID,NREPLIC)
CALL SUMMARY(TUKEY,10,T61(IPID,NREPLIC,1),T61(IPID,NREPLIC,2),T61(IPID,NREPLIC,3),T61(IPID,NREPLIC,4),T61(IPID,NREPLIC,5))
T61(IPID,NREPLIC,6) = SUM/10.
T61(IPID,NREPLIC,7) = SQRT((SQSM-10.*T61(IPID,NREPLIC,6)*T61(IPID,NREPLIC,6))/90.)
T61(IPID,NREPLIC,8) = T61(IPID,NREPLIC,3)
T61(IPID,NREPLIC,9) = (T61(IPID,NREPLIC,2)+2.*T61(IPID,NREPLIC,3)+T6
12(IPID,NREPLIC,4))/4.
T61(IPID,NREPLIC,10) = T61(IPID,NREPLIC,4) - T61(IPID,NREPLIC,2)
T61(IPID,NREPLIC,11) = T61(IPID,NREPLIC,5) - T61(IPID,NREPLIC,1)
SUM=0.
SOSM=0.
DO 9922 I=1,10
SUM = SUM+QLMS62(I,IPID,NREPLIC)
SOSM=SOSM+QLMS62(I,IPID,NREPLIC)*QLMS62(I,IPID,NREPLIC)
9922 TUKEY(I) = QLMS62(I,IPID,NREPLIC)
CALL SUMMARY(TUKEY,10,T62(IPID,NREPLIC,1),T62(IPID,NREPLIC,2),T62(IPID,NREPLIC,3),T62(IPID,NREPLIC,4),T62(IPID,NREPLIC,5))
T62(IPID,NREPLIC,6) = SUM/10.
T62(IPID,NREPLIC,7) = SQRT((SQSM-10.*T62(IPID,NREPLIC,6)*T62(IPID,NREPLIC,6))/90.)
T62(IPID,NREPLIC,8) = T62(IPID,NREPLIC,3)
T62(IPID,NREPLIC,9) = (T62(IPID,NREPLIC,2)+2.*T62(IPID,NREPLIC,3)+T6
12(IPID,NREPLIC,4))/4.
T62(IPID,NREPLIC,10) = T62(IPID,NREPLIC,4) - T62(IPID,NREPLIC,2)
T62(IPID,NREPLIC,11) = T62(IPID,NREPLIC,5) - T62(IPID,NREPLIC,1)
9925 CONTINUE
NREPLIC=1
WRITE(3,5100) (ALABEL(I),I=1,3)
5100 FORMAT(63X*TABLE 7.5*///* DATA SUMMARIES, CENTRAL VALUES, AND SPRE
1ADS, MULT. BY 10, OVER 10 DIRICHLET SIMULATIONS FOR Q.L. REG-EST M
2SE. FIRST REPLIC. DESIGN 2.///57X,3A10//////

4990 NSS=25
PID=0.15
ITABLE=0
WRITE(5,5105)

5105 FORMAT(19X,53H****************************DATA SUMMARY***************************
15X,36H****************CENTRAL VALUES****************4X17H********SPREADS*******/
2* ESTIMATOR SS PID L EXTREME L HINGE MEDIAN U HINGE U
3EXTREME*7X*MEAN (S.E.) MEDIAN TRIMEAN MIDSPREAD RANGE*
4/) 4991 WRITE(5,5510) NSS,PID,T11(IPID,NREPLIC,K),K=1,11,T21(IPID,NREP
1LIC,K),K=1,11,T31(IPID,NREPLIC,K),K=1,11,T41(IPID,NREPLIC,K),K
2=1,11,T51(IPID,NREPLIC,K),K=1,11,T61(IPID,NREPLIC,K),K=1,11
1X*APMR2*9X*5F11.7,3XF9.6*(F9.7*)*2F9.6,9X*2F9.6,3X*2F9.6/3X*MLE*11X*5F11.
27,3X*F9.6*(F9.7*)*2F9.6,3X*2F9.6/3X*APMR1*9X,5F11.7,3XF9.6*(F9
3.7*)*2F9.6,3X*2F9.6/3X*APMR2*9X,5F11.7,3XF9.6*(F9.7*)*2F9.6,3X
42F9.6,3X*APMR2*9X,5F11.7,3XF9.6*(F9.7*)*2F9.6,3X,2F9.6/) 4992 PID=0.40
GO TO (4992,4993,4994,4995) ITABLE
4993 NSS=50
PID=0.15
GO TO 4991
4994 PID=0.40
GO TO 4991
4995 WRITE(5,5112)
5112 FORMAT(/3X* NOTE THAT A ZERO BEFORE A DECIMAL DENOTES AN EXACT Z
1ERO. OTHERWISE, THE ZERO IS ROUNDED.*)
IF (NREPLIC-1)* 4996,4996,4998
4996 NREPLIC=2
WRITE(5,4997)
4997 FORMAT(/) WRITE(5,5115) (ALABEL(I),I=1,3)
5115 FORMAT(63X*TABLE 7.6///* DATA SUMMARIES, CENTRAL VALUES, AND SPRE
1ADS MULT. BY 10, OVER 10 DIRICHLET SIMULATIONS FOR Q.L. REG-EST M
2SE. SECOND REPLIC. DESIGN 2.///57X,3A10//////
GO TO 4990
4998 CONTINUE
END
FUNCTION GAMMA(GG)

C GENERATE A GAMMA RANDOM VARIABLE
C TO DO SO USE ALGORITHM GT FROM AHRENS AND DIETER (1974) "COMPUTER
C METHODS FOR SAMPLING FROM GAMMA, BETA, POISSON, AND BINOMIAL
C DISTRIBUTIONS", P229, COMPUTING. VOL. 12

C NOTE THAT FOR 1/77 SIMULATION STUDY, GG RANGES FROM 0.1 TO 9.8
C
C OBTAIN INTEGER PART OF GG
C
K=GG

C OBTAIN FRACTIONAL PART OF GG
C
F=GG-K

C OBTAIN INTEGER PART OF GAMMA
C
GI=0.
IF (K=0) 14,14,8
8 I=0.
GP=1.
10 I=I+1
UU=URAN(0.)
GP=GP*UU
IF (I-K) 10,12,12
12 GI=-ALOG(GP).

C OBTAIN FRACTIONAL PART OF GAMMA
C
14 GF=0.
IF (F=0.) 40,40,15
15 B=(2.7182818284590+F)/2.7182818284590
DF=1./F
FMN1=F-1.
16 UU=URAN(0.)
GP=B*UU

C GENERATE NEW UNIFORM RANDOM NUMBER FOR TESTS IN FOLLOWING STEPS
C 18 AND 30
C
UU=URAN(0.)
IF (GP=1.) 18,18,30
C
C GF IS LESS THAN OR EQUAL TO 1.
C
10 GF=GP**OF
   TEST=EXP(-GF)
   IF (UU-TEST) 40,40,16
C GF IS GREATER THAN 1.
C
30 GF=-ALOG((B-GP)**OF)
   TEST=GF**FMIN1
   IF (UU-TEST) 40,40,16
40 GAMMA=G1+GF
RETURN
END
SUBROUTINE GENXZ(UU, NSS)

GENERATE MULTINOMIAL COMPLETE (X) AND INCOMPLETE (Z) DATA
ALSO CALCULATE MAXIMUM-LIKELIHOOD-ESTIMATES FROM COMPLETE DATA

COMMON DEP(3,3),DOL(4,3),E(4,3),IROBUST,NTS,P1,P2,P3,TPID
COMMON/CALEST/APMC11,APMC12,APMC21,APMC22,CONVCR,COVSKIP,DMC1,DMC2,DM
IC3,DP10,EAPMC11,EAPMC12,EAPMC21,EAPMC22,EPMC11,EPMC12,EPMC21,EPMC22,ISTOP,N12,N13
2,N23,PID,PMLC1,PMLC2,PMLC3,SS,SN,TIMAP,TIMEP,TIM(2),TIM21,TIM31,X
3NU1,XNU2,XNU3,Z1,Z2,Z3,Z12,Z13,Z23,Z1N,Z2N,Z3N
COMMON/DATA/XDATA(2),ZDATA(6)
DIMENSION UU(NSS)
INTEGER Y1,Y2,W1,W3,V2,V3,COVSKIP
EQUIVALENCE (E(1,1),PEPM1), (E(1,2),PEPM2), (E(1,3),PEPM3)
EQUIVALENCE (E(2,1),PML1), (E(2,2),PML2), (E(2,3),PML3)
EQUIVALENCE (E(3,1),PPML1), (E(3,2),PPML2), (E(3,3),PPML3)
EQUIVALENCE (P1,1),PAPM1), (P1,2),PAPM2), (P1,3),PAPM3)
EQUIVALENCE (EP(1,1),EML1), (EP(1,2),EML2), (EP(1,3),EML3)
EQUIVALENCE (EP(2,1),EPMD1), (EP(2,2),EPMD2), (EP(2,3),EPMD3)
EQUIVALENCE (EP(3,1),EAPMN1), (EP(3,2),EAPMN2), (EP(3,3),EAPMN3)
EQUIVALENCE (DOL(1,1),DEPM1), (DOL(1,2),DEPM2), (DOL(1,3),DEPM3)
EQUIVALENCE (DOL(2,1),DML1), (DOL(2,2),DML2), (DOL(2,3),DML3)
EQUIVALENCE (DOL(3,1),DPM1), (DOL(3,2),DPM2), (DOL(3,3),DPM3)
EQUIVALENCE (DOL(4,1),DAPMN1), (DOL(4,2),DAPMN2), (DOL(4,3),DAPMN3)

RECALL THAT NSS IS THE INTEGER FORM OF THE SAMPLE SIZE SS

CALCULATE APPROX AMOUNT OF DATA GOING INTO EACH OF THE 4 GROUPS

H1=4*PID/2
H2=H4
H11=1.-3.*H2
E3=H1+H2

SET END POINTS

P12=P1+P2
E1=P1*E3
E2=P12*E3
E4=E3+P1*H2
E5=E3+P1*H2
E6=H1+2.*H2
E7=E6+P1*H2
E8=E6+P12*H2

C C INITIALIZE Z AND DUMMY VARIABLES Y1, W1, AND V1
C
Z1=0.
Z2=0.
Z3=0.
Y1=0
Y2=0
W1=0
W3=0
V2=0
V3=0

C C GENERATE X, Z DATA
C C CALL UNIFORM PSEUDO RANDOM-NUMBER GENERATOR
X(N+1) = (43490275647445.0*X(N)) MOD(2EXP(48))
SPECTRAL NUMBERS C(2)=2.839, C(3)=2.095, C(4)=1.819, C(5)=0.978
C C CALL URANV(0.,NSS,UU)
C
DO 85 I=1,NSS
U=UU(I)
IF (E1-U) 2,2,40
2 IF (E2-U) 4,4,45
4 IF (E3-U) 6,6,50
6 IF (E4-U) 8,8,55
8 IF (E5-U) 10,10,60
10 IF (E6-U) 12,12,65
12 IF (E7-U) 14,14,70
14 IF (E8-U) 80,80,75
40 Z1=Z1+1.
GO TO 85
45 Z2=Z2+1.
GO TO 85
50 Z3=Z3+1.
GO TO 85
55 Y1=Y1+1
GO TO 85
60 Y2=Y2+1
GO TO 85
65 W3=W3+1
GO TO 85
70 W1=W1+1
GO TO 85
75 V2=V2+1
GO TO 85
80 V3=V3+1
85 CONTINUE
N12=Y1+Y2
N13=W1+W3
N23=V2+V3
C OBTAIN REAL FORM OF SHARED INCOMPLETE DATA
Z12=N12
Z13=N13
Z23=N23
C OBTAIN COMPLETE DATA X
X1=Z1+Y1+W1
X2=Z2+Y2+V2
X3=Z3+W3+V3
C CALCULATE COMPLETE-DATA MAXIMUM-LIKELIHOOD ESTIMATES
PMLC1=X1/SS
PMLC2=X2/SS
PMLC3=1.-PMLC1-PMLC2
IF (PMLC3=0.) 90,95,95
90 ISTOP=1
PRINT 92, XNU1,XNU2,XNU3,P1,P2,P3,PI0,SS,X1,X2,X3,Z1,Z2,Z3,Z12,Z13
1,Z23,PMLC1,PMLC2,PMLC3,NSS,NTS
92 FORMAT(//10 GENXZ. COMPLETE-DATA MLE NEGATIVE P. XNU=3F10.4, GE
1NERATED P=3F10.6, P ID=F6.2, SS=F4.0/X=3F6.0/Z=6F6.0/P 2ML=3F10.6/NSS=13,I3*13* TH TRINOMIAL SIMUL*)
RETURN
95 DMLC1=PMLC1-P1
DMLC2=PMLC2-P2
DMLC3=PMLC3-P3
Z1N=Z1+XNU1
Z2N=Z2+XNU2
Z3N=Z3+XNU3
C PUT X AND Z DATA INTO SEPARATE BLOCK COMMON BECAUSE PROGRAM WON'T
C CORRECTLY RUN IF BLANK COMMON X,Z DATA IS PUT INTO KTITER, COUNTS,
AND BESTEST (PERHAPS PROBLEMS WITH E EQUIVALENCE STATEMENTS)

XDATA(1)=X1
XDATA(2)=X2
ZDATA(1)=Z1
ZDATA(2)=Z2
ZDATA(3)=Z3
ZDATA(4)=Z12
ZDATA(5)=Z13
ZDATA(6)=Z23

TRUE PERCENT (PROPORTION) INCOMPLETE DATA

TPID=(Z12+Z13+Z23)/SS
DPIO=PIO-TPID
RETURN
END
SUBROUTINE EPM

C PROGRAM TO CALCULATE EXACT POSTERIOR MEAN AND COVARIANCE MATRICES

COMMON DEP(3,3),DQL(4,3),E(4,3),IROBUST,NTS,P1,P2,P3,TPID
COMMON/CALEST/APMC11,APMC12,APMC22,CONVCR1,COVSkip,OML1,OML2,OML3
IC3,OPID,EPMC11,EPMC12,EPMC22,EPMC31,EPMC32,ISTOP,N12,N13
Z1,N23,PIP,PMC11,PMC12,PMC31,SS,SSN,TImAP,TImEP,TIm2,TIm31,X

5 NNU1,NNU2,NNU3,Z1,Z2,Z3,Z12,Z13,Z23,Z1N,Z2N,Z3N

INTEGER COVSkip

EQUIVALENCE (E(1,1),PEPM1),(E(1,2),PEPM2),(E(1,3),PEPM3)
EQUIVALENCE (E(2,1),PMLE1),(E(2,2),PMLE2),(E(2,3),PMLE3)
EQUIVALENCE (E(3,1),PPMD1),(E(3,2),PPMD2),(E(3,3),PPMD3)
EQUIVALENCE (E(4,1),PAPM1),(E(4,2),PAPM2),(E(4,3),PAPM3)
EQUIVALENCE (DEP(1,1),EM11),(DEP(1,2),EM21),(DEP(1,3),EM31)
EQUIVALENCE (DEP(2,1),EMP11),(DEP(2,2),EMP12),(DEP(2,3),EMP13)
EQUIVALENCE (DEP(3,1),EAPM11),(DEP(3,2),EAPM12),(DEP(3,3),EAPM13)
EQUIVALENCE (DQL(1,1),DEPM11),(DQL(1,2),DEPM12),(DQL(1,3),DEPM13)
EQUIVALENCE (DQL(2,1),DM11),(DQL(2,2),DM12),(DQL(2,3),DM13)
EQUIVALENCE (DQL(3,1),DPMD1),(DQL(3,2),DPMD2),(DQL(3,3),DPMD3)
EQUIVALENCE (DQL(4,1),DAPMN1),(DQL(4,2),DAPMN2),(DQL(4,3),DAPMN3)

C

N121=N12+1
N131=N13+1
N231=N23+1

3 Y1=Z1N

YY1A=GAM(Y1)

IF (Z12-0.) 4,4,8

4 IF (Z13-0.) 5,5,8

5 IF (Z23-0.) 6,6,8

C COMPLETE-DATA CASE, ALL ZIJ=0.

C

6 SUM12=YY1A*GAM(Z2N)*GAM(Z3N)

S12P1=Z1N*SUM12
S12P1S=(Z1N+1.)*S12P1
S12P2=Z2N*SUM12
S12P2S=(Z2N+1.)*S12P2
S12P1P2=Z1N*Z2N*SUM12
GO TO 55

8 Z2N12=Z2N+Z12

Z3N13Z3N+Z13+Z23

C SIJKP DENOTES ZIJ SUM FOR POSTERIOR MEAN P(K) CALCULATIONS.

C SIMILARLY, IJKP DENOTES A TERM OF THIS SUM.
C

35 CONTINUE

C

40 CONTINUE

C
$S_{12P2} = S_{12P1} + T_{12P2} $

$S_{12P1SO} = S_{12P1SO} + T_{12P1SO} $

$S_{12P2SQ} = S_{12P2SQ} + T_{12P2SQ} $

$S_{12P1P2} = S_{12P1P2} + T_{12P1P2} $

90 CONTINUE

C ELEMENTS OF POSTERIOR MEAN OF P GIVEN Z

59 D1 = SUM12 * SSN

D2 = D1 * (SSN + 1.)

PEPM1 = S12P1  /D1

PEPM2 = S12P2  /D1

PEPM3 = 1. - PEPM1 - PEPM2

IF (PEPM3 < 0.) 230, 201, 201

201 CALL SECOND TIM2

C ELEMENTS OF POSTERIOR COVARIANCE MATRIX OF P GIVEN Z

X1X2 = S12P1P2 /D2

EPMC12 = X1X2 - PEPM1 * PEPM2

X1SQ = S12P1SO /D2

EPMC11 = X1SO - PEPM1 * PEPM1

X2SQ = S12P2SQ /D2

EPMC22 = X2SO - PEPM2 * PEPM2

DEPMN1 = PEPM1 - P1

DEPMN2 = PEPM2 - P2

DEPMN3 = PEPM3 - P3

RETURN

230 PRINT 231

231 FORMAT(/// EXACT POSTERIOR MEAN. NEGATIVE P.///)

PRINT 240, PEPM1, EPMC11, X1SO, S12P1, S12P1SO, PEPM2, EPMC22, X2SO, S12P2,

S12P2SQ, PEPM3, EPMC12, XIX2, SUM12, S12P1P2, NTS

240 FORMAT(/// 7H PEPM1=E15.8, 4X, 8H EPMC11=E15.8, 4X, 8H X1SO=E15.8, 4X, 7H

S12P1=E15.8, 4X, 9H S12P1SO=E15.8, 4X, 7H PEPM2=E15.8, 4X, 8H EPMC22=E15.8, 4X,

24X, 6H X2SO=E15.8, 4X, 7H S12P2=E15.8, 4X, 9H S12P2SQ=E15.8, 7H PEPM3=E1

35.8, 4X, 8H EPMC12=E15.8, 4X, 6H XIX2=E14.7, 4X, 7H SUM12=E14.7, 4X, 9H S1

42P1P2=E14.7, 4X, 7H NTS=E13/1

ISTOP=1

RETURN

END
FUNCTION GAM(X)

EVALUATE GAM(X) FOR CASES FROM PRIOR (.1,.1,.9.8)

IF (X-9.2) 1,35,35

ARGUMENT IS LESS THAN OR EQUAL TO 9.1. TO INSURE 11 SIGNIFICANT PLACES OF ACCURACY IN GAMMA, USE 11 SIGNIFICANT-FIGURE VALUE CALCULATED FROM LOG(GAM(X)) FROM ABRAMOWITZ AND STEGUN OR DAVIS.

CALCULATE GAM(0.1), GAM(1.1), AND GAM(2.1) BY HAND FROM GAM(3.1) AND RELATION GAM(X+1) = X GAM(X)

1 IF (ABS(X-9.1)-1.E-13) 2,2,4
    GAM(9.1)
    2 GAM=49973.708949629
    RETURN

4 IF (ABS(X-8.1)-1.E-13) 6,6,8
    GAM(8.1)
    6 GAM=6169.5936974051
    RETURN

8 IF (ABS(X-7.1)-1.E-13) 10,10,12
    GAM(7.1)
    10 GAM=868.9568588072
    RETURN

12 IF (ABS(X-6.1)-1.E-13) 14,14,16
    GAM(6.1)
    14 GAM=142.45194406569
    RETURN

16 IF (ABS(X-5.1)-1.E-13) 18,18,20
    GAM(5.1)
    18 GAM=27.9317537538371
    RETURN

20 IF (ABS(X-4.1)-1.E-13) 22,22,24
    GAM(4.1)
    22 GAM=6.8126228630175
    RETURN

24 IF (ABS(X-3.1)-1.E-13) 26,26,28
    GAM(3.1)
    26 GAM=2.1976202783927
    RETURN

28 IF (ABS(X-2.1)-1.E-13) 30,30,32
    GAM(2.1)
    30 GAM=1.046485846854
    RETURN

RETURN
32 IF (ABS(X-1.1)-1.E-13) 34,34,36
C GAM(1.1)
34 GAM=0.95135076987
RETURN
C GAM(9.1)
36 GAM=9.5135076987
RETURN
C ARGUMENT X IS GREATER THAN OR EQUAL TO 10.
C USE STIRLING'S FORMULA TO OBTAIN AN APPROXIMATION TO GAMMA
C THAT IS ACCURATE TO 11 SIGNIFICANT FIGURES
C
35 XSQ=X*X
XCU=XSQ*X
XFIFTH=XSQ*XCU
C Y IS THE APPROXIMATED NATURAL LOG(BASE E) OF GAMMA(X)
C
Y=(X-0.5)*ALOG(X)-X+0.91893853320467+1./(12.+X)-1./(360.*XCU)+1./(11260.*XFIFTH)
 IF (X-22.) 40,45,45
40 Y=Y-1./(1660.*XFIFTH*XSQ)
45 GAM=EXP(Y)
RETURN
END
SUBROUTINE METHODS

FOR INCOMPLETE DATA CALCULATE MY APPROXIMATION, POSTERIOR MODE,
AND MAXIMUM-LIKELIHOOD ESTIMATE

INTEGER COVSKIP
DIMENSION A(3,3),B(3,1),IPIVOT(3),IWK(6)
COMMON DEP(3,3),DOL(4,3),E(4,3),IROBUST,NTS,P1,P2,P3,TPID
COMMON/CALEST/APMC11,APMC12,APMC22,CONVCRI,COVSKIP,DMLC1,DMLC2,DML
1C3,DPID,EAPMC11,EAPMC12,EAPMC22,EPMC11,EPMC12,EPMC22,ISTOP,N12,N13
2,N23,PIID,PMLE1,PMLE2,PMLE3,SS,SSN,TIMAP,TIMEP,TIMC2,TIM21,TIM31,X
3NU1,NU2,NU3,Z1,Z2,Z3,Z12,Z13,Z23,Z1N,Z2N,Z3N
COMMON/ITKT/AVNUMIT(6),CTNUMIT(6,7)
EQUIVALENCE (E(1,1),PEPM1),(E(1,2),PEPM2),(E(1,3),PEPM3)
EQUIVALENCE (E(2,1),PMLE1),(E(2,2),PMLE2),(E(2,3),PMLE3)
EQUIVALENCE (E(3,1),PPMD1),(E(3,2),PPMD2),(E(3,3),PPMD3)
EQUIVALENCE (E(4,1),PAPM1),(E(4,2),PAPM2),(E(4,3),PAPM3)
EQUIVALENCE (DEP(1,1),EML1),(DEP(1,2),EML2),(DEP(1,3),EML3)
EQUIVALENCE (DEP(2,1),EAPMN1),(DEP(2,2),EAPMN2),(DEP(2,3),EAPMN3)
EQUIVALENCE (DOL(1,1),DEPM1),(DOL(1,2),DEPM2),(DOL(1,3),DEPM3)
EQUIVALENCE (DOL(2,1),DML1),(DOL(2,2),DML2),(DOL(2,3),DML3)
EQUIVALENCE (DOL(3,1),DPMD1),(DOL(3,2),DPMD2),(DOL(3,3),DPMD3)
EQUIVALENCE (DOL(4,1),DAPMN1),(DOL(4,2),DAPMN2),(DOL(4,3),DAPMN3)

CHECK TO INSURE THAT CONVERGENCE CRITERION IS NOT TOO STRICT

IF (IROBUST-1) .GT.100,.50

INCOMPLETE-DATA MAXIMUM-LIKELIHOOD ESTIMATE

1 PMLE1=PEPM1
PMLE2=PEPM2
PMLE3=PEPM3
K=1
5 PL1=PMLE1
PL2=PMLE2
PMLE12=PMLE1+PMLE2
PMLE13=PMLE1+PMLE3
PMLE23=PMLE2+PMLE3
IF (PMLE12-1.E-14) 7,7,8
7 TEMP=0.,
GO TO 9
8 TEMP=Z12/PMLE12
9 PMLE1=(Z1+PMLE1*(TEMP+Z13/PMLE13))/SS
PMLE2 = (Z2 + PMLE2 * (TEMP * Z23 / PMLE23)) / SS
PMLE3 = 1.0 - PMLE1 - PMLE2
IF (PMLE3 = 0.0) 21, 14, 14

C
14 IF (PMLE1 = 0.1) 15, 15, 16
15 IF (ABS(PMLE1 - PL1) < 0.00001) 17, 20, 20
16 IF (ABS(PMLE1 - PL1) / PMLE1 - CONVCR1) 17, 20, 20
17 IF (PMLE2 = 0.1) 18, 18, 19
18 IF (ABS(PMLE2 - PL2) < 0.00001) 25, 20, 20
19 IF (ABS(PMLE2 - PL2) / PMLE2 - CONVCR2) 25, 20, 20
20 K = K + 1
IF (K = 1000) 5, 5, 21

21 PRINT 22, K, XNU1, P1, PL1, PL2, CONVCR1, IROBUST, NTS, TPID, PMLE3, XNU2, P2
1, PMLE1, PMLE2

22 FORMAT /// * EXCESS NUMBER OF ITERATIONS FOR INCOMPLETE-DATA M.L.E.
2(RD)*F7.5 * IROBUST=I2, NTS=I3, TPD=I6, PMLE3=I6, XNU2=I6
39.4* P2=*F9.6 * PMLE1=I1.8 * PMLE2=I1.8
IF (K = 1030) 25, 25, 250

C
CONVERGENCE FOR MAXIMUM-LIKELIHOOD ESTIMATE INCOMPLETE DATA

25 EML1 = PMLE1 - PEPM1
EML2 = PMLE2 - PEPM2
EML3 = PMLE3 - PEPM3
DML1 = PMLE1 - P1
DML2 = PMLE2 - P2
DML3 = PMLE3 - P3

C
INCREMENT COUNTER FOR NUMBER OF ITERATIONS FOR CONVERGENCE

CALL KTITER(K, 1)

C
POSTERIOR MODE

50 T1 = Z1N - 1.
T2 = Z2N - 1.
K = 1
D = SSM - 3.
PPMD1 = PEPM1
PPMD2 = PEPM2
PPMD3 = PEPM3
55 PL1 = PPMD1
PL2 = PPMD2
PPMD12 = PPMD1 + PPMD2
PPMD13 = PPMD1 + PPMD3
PPMD23 = PPMD2 + PPMD3
IF (PPMD12 - 1. E-14) N7, 57, 58
57 TEMP = 0.
GO TO 59
58 TEMP = Z12/PPMD12
59 PPMD1 = (T1 + PPMD1 * (TEMP + Z13/PPMD13)) / 0
IF (PPMD1.LT.0.) PPM01 = 0.
PPMD2 = (T2 + PPMD2 * (TEMP + Z23/PPMD23)) / 0
IF (PPMD2.LT.0.) PPM02 = 0.
PPMD3 = 1. - PPMD1 - PPMD2
IF (PPMD3.LT.0.) 71, 64, 64
64 IF (PPMD1.LT.0.1) 65, 65, 66
65 IF (ABS(IPPM1 - PL1) < 0.00001) 67, 70, 70
66 IF (ABS(IPPM1 - PL1) / PPMD1 - CONVCR1) 67, 70, 70
67 IF (PPMD2.LT.0.1) 68, 68, 69
68 IF (ABS(IPPM2 - PL2) < 0.00001) 75, 70, 70
69 IF (ABS(IPPM2 - PL2) / PPMD2 - CONVCR1) 75, 70, 70
70 K = K + 1
IF (K.LT.1001) 55, 55, 71
71 PRINT 72, K, XNU1, PL1, PL2, CONVCR1, IROBUST, NTS, TPIO, PPM03, XNU2, P2
1, PPM01, PPMD2

CONVERGENCE FOR POSTERIOR MODE INCOMPLETE DATA

75 EPMD1 = PPMD1 - PEPM1
EPMD2 = PPMD2 - PEPM2
EPMD3 = PPMD3 - PEPM3
DPMD1 = PPMD1 - P1
DPMD2 = PPMD2 - P2
DPMD3 = PPMD3 - P3

INCREMENT COUNTER FOR NUMBER OF ITERATIONS FOR CONVERGENCE

J = 2
IF (IROBUST.EQ.2) J = 5
CALL KTITER(K, J)
MY TAYLOR-SERIES APPROXIMATED POSTERIOR MEAN AND COVARIANCE MATRICES

100 PAPM1=PEPM1
   PAPM2=PEPM2
   PAPM3=PEPM3
   K=1
105 PL1=PAPM1
   PL2=PAPM2
   PL3=PAPM3
   PAPM12=PAPM1+PAPM2
   PAPM13=PAPM1+PAPM3
   PAPM23=PAPM2+PAPM3
   IF (PAPM12<1.E-14) 107,107,108
107 TEMP=0.
   GO TO 109
108 TEMP=Z12/PAPM12
109 PAPM1=(Z1H+PAPM1*(TEMP+Z13/PAPM13))/SSN
   PAPM2=(Z2H+PAPM2*(TEMP+Z23/PAPM23))/SSN
   PAPM3=1.-PAPM1-PAPM2
   IF (PAPM3<0.) 121,114,114
114 IF (PAPM1<0.1) 115,115,116
115 IF (ABS(PAPM1-PL1)<0.00001) 117,120,120
117 IF (PAPM2<0.1) 118,118,119
118 IF (ABS(PAPM2-PL2)<0.00001) 125,120,120
119 IF (ABS(PAPM3<PL3)<0.00001) 125,120,120
120 K=K+1
   IF (K>1000) 105,105,121
121 PRINT 122, K,XNU1,P1,PL1,PL2,CONVCR1,IROBUSTR,NTS,TPI0,PAPM3,XNU2,P
   IF (K>1030) 125,125,250
C CONVERGENCE FOR T.S. APPROX. POSTERIOR MEAN INCOMPLETE DATA
C
125 EAPMN1=PAPM1-PEPM1
   EAPMN2=PAPM2-PEPM2
   EAPMN3=PAPM3-PEPM3
   DAPMN1=PAPM1-P1
   DAPMN2=PAPM2-P2
DAPM3 = PAPM3 - P3

INCREMENT COUNTER FOR NUMBER OF ITERATIONS FOR CONVERGENCE

IF (IROBUST = 1) 127, 128, 129
127 J = 3
GO TO 130
128 J = 4
GO TO 130
129 J = 6
130 CALL KTITER(K, J)
IF (IROBUST .GT. 0) RETURN

APPROXIMATED POSTERIOR VAR/COV MATRIX. NONITERATIVE METHOD.

P12 = PAPM1 + PAPM2
P13 = PAPM1 + PAPM3
P23 = PAPM2 + PAPM3

CAUTION. INSURE THAT P12, P13, AND P23 ARE NOT IN COMMON FROM
GENERATED P1, P2, AND P3

R12 = PAPM1/P12
R13 = PAPM1/P13
R21 = PAPM2/P12
R23 = PAPM2/P23

SSN = SUM OF DATA PLUS SUM OF PRIOR PARAMETERS XNUI

T = SSN * (SSN + 1.)
P12SQ = P12 * P12
P13SQ = P13 * P13
P23SQ = P23 * P23
ZRPA21 = Z12 * R21/P12
ZRPA13 = Z13 * R13/P13
ZRPA12 = Z12 * R12/P12
ZRPA23 = Z23 * R23/P23

C CALCULATE A(1,1)
A112 = (ZRPA21 + Z13/P13)**2/T
A113 = (ZRPA21 * R21)/(P12 * T)
A114 = Z13/(P13SQ * T)
A(1,1) = -1. + A112 - A113 - A114

C CALCULATE A(1,2)
A121 = (ZRPA13 - ZRP12) * (ZRPA21 + Z13/P13)
A122 = ZRP12 * R21/P12
\[ A_{123} = \frac{ZRP_{13}/P_{13}}{1} \]
\[ A(1,2) = 2 \cdot \left( A_{121} + A_{122} - A_{123} \right) / T \]
\[ TEMP = Z_{12}/P_{12}^{2} \]

**C CALCULATE A(1,3)**
\[ A_{131} = (ZRP_{12} - ZRP_{13})^{2} \]
\[ A_{132} = ZRP_{12} \cdot R_{12}/P_{12} + ZRP_{13} \cdot R_{13}/P_{13} \]
\[ A(1,3) = (A_{131} - A_{132}) / T \]

**C CALCULATE B(1,1)**
\[ B(1,1) = -(SSN \cdot PAPM_{1} \cdot P_{23} + ZRP_{12} \cdot PAPM_{2} + ZRP_{13} \cdot PAPM_{3}) / T \]

**C CALCULATE A(2,1)**
\[ A_{211} = (ZRP_{21} + Z_{13}/P_{13}) \cdot (ZRP_{23} - ZRP_{21}) \]
\[ A_{212} = TEMP \cdot R_{21}^{2} \]
\[ A(2,1) = (A_{211} + A_{212}) / T \]

**C CALCULATE A(2,2)**
\[ A_{221} = TEMP \cdot PAPM_{1} \cdot Z_{23} \cdot (1 - 2 \cdot PAPM_{1}) / P_{23}^{2} \]
\[ A_{222} = TEMP \cdot PAPM_{2} \cdot Z_{13}^{2} \cdot (1 - 2 \cdot PAPM_{2}) / P_{13}^{2} \]
\[ A_{223} = TEMP \cdot Z_{12} \cdot (P_{13} - 2 \cdot PAPM_{1}) \]
\[ A(2,2) = -T \cdot A_{221} + A_{222} + A_{223} / T \]

**C CALCULATE A(2,3)**
\[ A_{231} = (ZRP_{12} + Z_{23}/P_{23}) \cdot (ZRP_{13} - ZRP_{12}) \]
\[ A_{232} = TEMP \cdot R_{12}^{2} \]
\[ A(2,3) = (A_{231} - A_{232}) / T \]

**C CALCULATE B(2,1)**
\[ B(2,1) = PAPM_{1} \cdot PAPM_{2} \cdot (SSN \cdot TEMP) / T \]

**C CALCULATE A(3,1)**
\[ A_{311} = (ZRP_{21} - ZRP_{23})^{2} \]
\[ A_{312} = ZRP_{21} \cdot R_{21}/P_{12} \]
\[ A_{313} = ZRP_{23} \cdot R_{23}/P_{23} \]
\[ A(3,1) = (A_{311} - A_{312} - A_{313}) / T \]

**C CALCULATE A(3,2)**
\[ A_{321} = (-ZRP_{12} - Z_{23}/P_{23}) \cdot (ZRP_{21} - ZRP_{23}) \]
\[ A_{322} = ZRP_{12} \cdot R_{12}/P_{12} \]
\[ A_{323} = ZRP_{23}/P_{23} \]
\[ A(3,2) = 2 \cdot (A_{321} + A_{322} - A_{323}) / T \]

**C CALCULATE A(3,3)**
\[ A_{331} = (ZRP_{12} + Z_{23}/P_{23})^{2} \]
\[ A_{332} = ZRP_{12} \cdot R_{12}/P_{12} \]
\[ A_{333} = ZRP_{23}/P_{23}^{2} \]
\[ A(3,3) = -T + A_{332} - A_{333} - A_{334} / T \]

**C CALCULATE B(3,1)**
\[ B(3,1) = -(SSN \cdot PAPM_{2} + P_{13} \cdot ZRP_{21} \cdot PAPM_{1} + ZRP_{23} \cdot PAPM_{3}) / T \]
SOLVE SYSTEM \( A \times X = B \) FOR \( X \). \( X \) IS VECTOR OF COVARIANCES \( C_{11}, C_{12}, C_{22} \)

CALL MATINV(3,3,A,1,B,1,DETERM,ISCALE,IPIVOT,IWK)

IF (ABS(DETERM) < 5.0E-14) 212,212,220

212 PRINT 214, DETERM,K,XNU1,XNU2,P1,P2,PAPM1,PAPM2,((A(I,J)),J=1,3),B(I,1),I=1,3)


320 FORMAT(/"DIFFERENCE OF APPROXIMATED POSTERIOR COVARIANCES FROM EXACT POSTERIOR COVARIANCES. C11, C12, AND C22"

220 APMC11=B(1,1)
APMC12=B(2,1)
APMC22=B(3,1)

IF (APMC11 .EQ. 0.) 225,225,221

221 IF (APMC22 .EQ. 0.) 225,225,222

222 EAPMC11=APMC11-EPMC11
EAPMC12=APMC12-EPMC12
EAPMC22=APMC22-EPMC22
RETURN

225 PRINT 226, APMC11,APMC12,APMC22,EPMC11,EPMC12,EPMC22,XNU1,XNU2,PAPM1,PAPM2,NTS

226 FORMAT(/"APPROXIMATED VARIANCE IS NEGATIVE. APMC11=*E21.14,* AP"

1HC12=*E18.11,* APMC22=*E18.11,* EPMC11=*E18.11,* EPMC12=*E18.11,*

2EPMC22=*E20.13,* XNU1=*F4.0,* XNU2=*F4.0,* PAPM1=*F5.3,* PAPM2=*F5

4.3,* NTS=*I3/"

230 COVSKIP=1
RETURN

250 ISTOP=1
RETURN

END
SUBROUTINE COUNTS(BIAS, RELDIFF, J)

C COUNT NUMBER OF NUMxZ UNTERMINATED TRIALS THAT HAVE NEGETAIVE,
ZERO, AND POSITIVE BIAS AND THAT HAVE ABSOLUTE RELATIVE
DIFFERENCES LESS THAN CERTAIN PERCENTAGES.

BIAS = (APPROX-EXACT) OR (APPROX-GENERATED P)
RELDIFF = ABS(BIAS/EXACT) OR ABS(BIAS/GENERATED P)
(RECALL THAT COV IS NEG SO WANT DENOMINATOR INCLUDED IN ABS VALUE)

J DENOTES, IN SUBSEQUENT ORDER, ONE OF EAPMC11, EAPMC12, EAPMC22,
(BIAS OF APPROX T.S. EXPANSION FOR EXACT POSTERIOR COV)
DMLC1 AND DMLC2 (COMPLETE-DATA MLE BIAS FROM GENERATED OR GIVEN P)
(THUS, J=3 REFERS TO EAPMC22)

COMMON DEP(3,3), DQL(4,3), E(4,3), IROBUST, NTS, P1, P2, P3, TPID
COMMON/BIASRD/COUNTB(3,8), COUNTRD(8,8)
AB = ABS(BIAS)
IF (AB > 1.0E-15) 3, 3, 1
1 IF (BIAS > 0.) 2, 2, 4
C NEGATIVE BIAS
2 COUNTB(1,J) = COUNTB(1,J) + 1.
GO TO 5
C ZERO BIAS (CDC 6600 COMPUTER ACCURACY IS 14 SIGN FIGURES BUT
C CONSIDER ONLY 15 DECIMAL PLACES FOR ZERO BIAS
3 COUNTB(2,J) = COUNTB(2,J) + 1.
GO TO 5
C POSITIVE BIAS
4 COUNTB(3,J) = COUNTB(3,J) + 1.
5 IF (RELDIFF > 0.25) 8, 8, 30
6 COUNTRD(8,J) = COUNTRD(8,J) + 1.
C 25 PERCENT RELATIVE DIFFERENCE
7 IF (RELDIFF > 0.20) 10, 10, 30
9 COUNTRD(10,J) = COUNTRD(10,J) + 1.
C 20 PERCENT RELATIVE DIFFERENCE
10 IF (RELDIFF > 0.15) 12, 12, 30
11 COUNTRD(12,J) = COUNTRD(12,J) + 1.
C 15 PERCENT RELATIVE DIFFERENCE
12 IF (RELDIFF > 0.10) 14, 14, 30
13 COUNTRD(14,J) = COUNTRD(14,J) + 1.
C 10 PERCENT RELATIVE DIFFERENCE
14 IF (RELDIFF > 0.05) 16, 16, 30
15 COUNTRD(16,J) = COUNTRD(16,J) + 1.
C 1 PERCENT RELATIVE DIFFERENCE
   IF (RELDIFF-0.01) 18,18,30
18 COUNTRD(3,J)=COUNTRD(3,J)+1.
C .1 PERCENT RELATIVE DIFFERENCE
   IF (RELDIFF-0.001) 20,20,30
20 COUNTRD(2,J)=COUNTRD(2,J)+1.
C .01 PERCENT RELATIVE DIFFERENCE
   IF (RELDIFF-0.0001) 22,22,30
22 COUNTRD(1,J)=COUNTRD(1,J)+1.
30 CONTINUE
   IF (RELDIFF-0.15) 40,31,31
31 IF (J-4) 33,40,32
32 IF (J-5) 40,40,33
33 PRINT 35 J,NTS,BIAS,RELDIFF,TPID,IROBUST,E(1,1),E(1,2),E(4,1),E(4,2)
35 FORMAT(* SUBR COUNTS. J=*I2* NTS=*I3* BIAS=*F9.7* RELDIFF=*F5.12* TPID=*F6.2* IROBUST=*I2* PEPM1,2=*2F7.4* PAPM1,2=*2F7.4)
40 CONTINUE
   RETURN
END
SUBROUTINE ESTMSE(Y, Y2, XY, X, X2, TXMSE, MSEC)

CALCULATE ESTIMATES OF MSE AND SAMPLE VARIANCE OF THESE ESTIMATES.

REAL MSE(6), MSECV

FOR TERM = (PE1-P1)**2 + (PE2-P2)**2 + (PE3-P3)**2 AND CONTROL-VARIATE
TERM CV = (PMLECD1-P1)**2 + (PMLECD2-P2)**2 + (PMLECD3-P3)**2
FOR PE- DENOTING ONE OF ESTIMATORS EPM, APM, PMO, AND MLE AND
PMLECD- DENOTING COMPLETE-DATA MLE

Y = SUM OF N TERM
Y2 = SUM OF N TERM**TERM
XY = SUM OF N TERM*TERMCV
X = SUM OF N TERMCV
X2 = SUM OF N TERMCV**TERMCV
N = NUMBER OF TERMS
TXMSE = TRUE MEAN SQ ERROR OF CONTROL VARIATE
MSECV = USUAL SAMPLE MSE FOR THE CONTROL VARIATE

MSE(1) IS USUAL MSE (B=0 IN MSE REGRESSION ESTIMATE)
MSE(2) IS VAR OF MSE(1)
MSE(3) IS USUAL CONTROL-VARIATE MSE (B=1 IN MSE REGRESSION EST)
MSE(4) IS VAR OF MSE(3)
MSE(5) IS LEAST-SQUARES REGRESSION ESTIMATE MSE (B=LEAST-SQS EST)
MSE(6) IS VAR OF MSE(5)

NOTE THAT MSE(5) SHOULD HAVE SMALLEST VARIANCE. HOWEVER, IT WILL
BE A BIASED ESTIMATE. HENCE, USE IT IN ANALYSES ONLY IF IT
DIFFERS FROM EITHER ONE OF TWO UNBIASED ESTIMATES BY NO MORE
THAN 1%. (IE, IT CAN DIFFER BY MORE THAN 1% FROM EITHER MSE(1) OR
MSE(3) BUT NOT BOTH.)

GENERAL FORM OF ESTIMATED MSE IS

MSE = MSE(1)*B*(TXMSE-MSECV)

USUAL MSE (B=0)

XN = N*(N-1)
MSE(1) = Y/N
MSE(2) = (Y2 - N*MSE(1)*MSE(1))/XN

USUAL CONTROL-VARIATE MSE (B=1)
XN=N*(N-2)
T0=2.*XY
MSECV=X/N
T1=N*MSECV
T2=2.*N*TXMSE
T3=T2*TXMSE/2.
D=TXMSE-MSECV
MSE(3)=MSE(1)+D
MSE(4)=Y2-T0*X2+T2*(MSE(1)-MSECV)+T3-N*MSE(3)*MSE(3)/XN

C
C LEAST-SQUARES REGRESSION-ESTIMATE MSE (B=LEAST-SQUARES ESTIMATE)
C
B=(XY-T1*MSE(1))/(X2-T1*MSECV)
B2=B*B
MSE(5)=MSE(1)+B*D
MSE(6)=(Y2-B*T0+B2*X2+B*T2*(MSE(1)-B*MSECV)+B2*T3-N*MSE(5)*MSE(5))/XN
RETURN
END
SUBROUTINE KTITER(K, J)
C
INCREMENT COUNTERS FOR (1) AVERAGING NUMBER OF ITERATIONS AN
ESTIMATOR REQUIRED AND (2) DETERMINING HOW MANY CASES IN A
REPLICATION TOOK A SPECIFIED NUMBER OF ITERATIONS

K IS NUMBER OF ITERATIONS REQUIRED TO MEET CONVERGENCE CRITERION
J DENOTES, IN SUBSEQUENT ORDER, ONE OF ESTIMATORS MLE, PMDRO,
APMRO, APMR1, PMDR2, APMR2 (THUS, J=4 REFERS TO APMR1)

COMMON DEP(3,3),DQLU(4,3),E(4,3),IROBST,NTS,P1,P2,P3,TPID
COMMON/DATA/XDATA(2),IDATA(6)
COMMON/ITKT/AVNUMIT(6),CTNUMIT(6,10)

FOR AVERAGING NUMBER OF ITERATIONS FOR JTH ESTIMATOR

AVNUMIT(J)=AVNUMIT(J)+K

INCREMENT COUNTER FOR NUMBER OF ITERATIONS

I=1
IF (K=1) 20,20,2
2 I=2
IF (K=2) 20,20,3
3 I=3
IF (K=3) 20,20,4
4 I=4
IF (K=4) 20,20,5
5 I=5
IF (K=5) 20,20,6
6 I=6
IF (K=6) 20,20,7
7 I=7
IF (K=7) 20,20,8
8 I=8
IF (K=8) 20,20,9
9 I=9
IF (K=9) 20,20,10
10 I=10
20 CTNUMIT(J,I)=CTNUMIT(J,I)+1
IF (K=25) 30,25,25
25 PRINT 27, NTS,K,J,TPID,XDATA(1),XDATA(2),IROBST,(E(II,JJ),JJ=1,2
2)II=1,4,(IDATA(II),II=1,6)
27 FORMAT(* SUBR KTITER, NTS=I3* NUMBER OF ITER IS*I4* FOR METHOD
1 J=II* (MLE,PMDRO,APMRO,APMRO,PMR2,APM2), TPID=Ef4.2* X(C.D.)
}
CONTINUE
RETURN
END
SUBROUTINE SUMMARY(X,N,LE,LH,M,UH,UE)

C TUKEY'S FIVE-POINT DATA SUMMARY. ROUTINE SORTS INPUT VECTOR X OF
C LENGTH N AND THEN CALCULATES LOWER EXTREME LE, LOWER HINGE LH,
C MEDIAN M, UPPER HINGE UH, AND UPPER EXTREME UE.
C REFERENCE "EXPLORATORY DATA ANALYSIS" BY JOHN W. TUKEY
C FORTRAN EXTENDED VERSION 4.6, CDC 6600 COMPUTER (14 SIGN DIGIT S.P.)
C PROGRAMER IS KAREN R CREDEUR, NASA, LANGLEY RESEARCH CENTER

DIMENSION X(N)
REAL LE,LH,M

C SORT DATA IN ASCENDING ORDER

CALL ASORT(X,1,N)

C MEDIAN

XN=(N+1.)/2.
K=XN*1.E-12
M=X(K)
IF (ABS(XN-K)-1.E-8) 5,5,1
1 M=(M+X(K+1))/2.

C HINGES

5 XN=(K+1)/2.
K=XN*1.E-12
LH=X(K)
UH=X(N+1-K)
IF (ABS(XN-K)-1.E-8) 15,15,10
10 LH=(LH+X(K+1))/2.
UH=(X(N-K)+UH)/2.

C EXTREMES

15 LE=X(1)
UE=X(N)
RETURN
END
SUBROUTINE BESTEST(BESTQL)

BY TWO DIFFERENT CRITERION (SUMMED ABSOLUTE RELATIVE DIFFERENCE
RELO— AND SUMMED SQUARED ERROR SE— FOR SUM BEING OVER THE THREE
COMPONENTS OF AN ESTIMATOR) DETERMINE WHICH ESTIMATOR IS BEST FOR
A GIVEN ONE OF THE TRINOMIAL SIMULATION TRIALS

TIES SCORE AS BEST EACH ESTIMATOR THAT TIES FOR BEST.

THE FOUR ESTIMATORS IN E SHOULD BE IN THE FOLLOWING ORDER PEPM,
PMLE, PPMD, PAPM. BIASES SHOULD BE IN CORRESPONDING ORDER.

DIMENSION BESTQL(4,2),RDEP(3),RELQL(4),RELDQL(4,3)
DIMENSION SEEP(3),SEQL(4),V(4),W(4),X(3),Y(3)
COMMON DEP(3,3),DEQL(4,3),E(4,3),IROBUST,NTP,P1,P2,P3,TPID
COMMON/BEST/BESTEP(3,2),CTRQEP,CTRLQL(3),PRDEP(9,3),PRQL(9,7),SBI
IASEP(3,3),SBIASQL(3,7)
COMMON/DATA/XDATA(2),ZDATA(6)

IR=IROBUST+1
IF (IROBUST-1) 1,100,150

FOR EPM COMPARISONS

1 DO 10 I=1,3
SEEP(I)=0.
  DO 2 J=1,3
    X(I)=SEEP(I)=SEEP(I)*DEP(I,J)*DEP(I,J).
2 CONTINUE

INCORPORATE SUBROUTINE COUNTS TWICE (ONCE FOR EACH OF EP AND QL
COMPARISONS) IN THIS SUBROUTINE TO SAVE PROGRAM EXECUTION COST OF
MANY SUBROUTINE CALLS AND INDEX RESETTING.

DETERMINE SIGN OF FIRST COMPONENT OF ESTIMATOR

IF (ABS(DEP(I,1))=1.E-13) 5,5,3
  IF (DEP(I,1)-0.) 4,4,6
C NEGATIVE BIAS
  4 SBIASEP(1,I)=SBIASEP(1,I)+1.
     GO TO 10
C ZERO BIAS (CONSIDER ONLY 13 DECIMAL PLACES)
  5 SBIASEP(2,I)=SBIASEP(2,I)+1.
     GO TO 10
C POSITIVE BIAS
6 SBIASEP(3,1)=SBIASEP(3,1)+1.
10 CONTINUE
   CALL ASORT(X,1,3)
   DO 20 I=1,3
      IF (SSEP(I).EQ.X(I)) BESTEP(I,1)=BESTEP(I,1)+1.
   20 CONTINUE
C DETERMINE WHETHER ANY PEPH COMPONENT IS ZERO
C
ICK=0
   IF (E(1,1)-1.E-10) 30,25,25
25 IF (E(1,2)-1.E-10) 30,26,26
26 IF (E(1,3)-1.E-10) 30,32,32
30 ICK=1
32 IF (ICK=0) 33,33,56
33 CTRDEP=CTRDEP+1.
   DO 35 I=1,3
      RDEP(I)=0.
      DO 34 J=1,3
         RELDEP(I,J)=ABS(DEP(I,J))/E(I,J)
         Y(I)=RDEP(I)+RELDEP(I,J)
      34 CONTINUE
   35 CONTINUE
   CALL ASORT(Y,1,3)
   DO 40 I=1,3
      IF (RDEP(I).EQ.Y(I)) BESTEP(I,2)=BESTEP(I,2)+1.
40 CONTINUE
C FOR DETERMINING PROPORTION OF CASES FOR WHICH % ABSOLUTE RELATIVE DIFFERENCE FOR EACH OF ALL THREE ESTIMATOR COMPONENTS IS LESS THAN SPECIFIED AMOUNTS (INCORPORATED IN PART FROM SUBROUTINE COUNTS)
C
   DO 55 I=1,3
      II=1
      DO 53 J=1,3
         GO TO (41,43,45,47,49,51,520,528,53) II
41 IF (RELDEP(I,J)-0.0001) 53,42,42
42 II=2
43 IF (RELDEP(I,J)-0.001) 53,44,44
44 II=3
45 IF (RELDEP(I,J)-0.01) 53,46,46
46 II=4
47 IF (RELDEP(I,J)-0.05) 53,48,48
48 II=5
49 IF (RELDEP(I,J)-0.10) 53,50,50
50 II=6
51 IF (RELDEP(I,J)-0.15) 53,52,52
52 II=7
520 IF (RELDEP(I,J)-0.20) 53,525,525
525 II=8
528 IF (RELDEP(I,J)-0.25) 53,530,530
530 II=9
53 CONTINUE
PRDEP(I,J)=PRDEP(I,J)+1.
55 CONTINUE
IF (RELDEP(3,1)-0.15) 541,543,543
541 IF (RELDEP(3,2)-0.15) 543,545,545
543 PRINT 544, NTS, IROBUST, TPID, RELDEP(3,1), RELDEP(3,2), (E(I,J), J=1,2)
545 I=1,4,3, (E(I,J), J=1,2), I=2,3, (DEP(I,J), J=1,2), I=3, XDATA(1)
546 ,XDATA(2), (ZDATA(I), I=1,6)
547 FORMAT(15X*SUBR BESTEST, NTS=*I3*, IROBUST=*I1*, TPID=*F4.2*, RELDE
548 P(3,1-2)=*2F5.2*, PAPM1,2=*2F6.4*, PAPM1,2=*2F6.4*, PMLE1,2=*2F5.3
549 Z=15X*PPM01,2=*2F7.4*, DEP=*3(2F7.4,3X1,X,Z)=2F4.0,2X,6F4.0)
56 II=1
57 L=2
58 ICK=0
59 IF (P1-1.E-10) 59,57,57
60 IF (P2-1.E-10) 59,58,58
61 IF (P3-1.E-10) 59,61,61
62 ICK=1
63 DO 75 I=II,4
64 SEQL(I)=0.
65 DO 610 J=1,3
66 V(I)=SEQL(I)+SEQL(I)+DQL(I,J)*DQL(I,J)
67 610 CONTINUE
68 IF (IR-2) 62,615,64
69 IF (I-3) 75,75,63
70 K=1
71 GO TO 65
63  K=5
64  G0 TO 65
65  IF (ABS(DQL(I,1))<1.E-13) 69, 69, 67
67  IF (DQL(I,1)<0.) 68, 68, 70
C NEARATIVE BIAS
68  SBIASQL(1,K)=SBIASQL(1,K)+1.
69  G0 TO 75
C ZERO BIAS (CONSIDER ONLY 13 DECIMAL PLACES)
70  SBIASQL(2,K)=SBIASQL(2,K)+1.
75  CONTINUE
C POSITIVE BIAS
70  SBIASQL(3,K)=SBIASQL(3,K)+1.
75  CONTINUE
CALL ASORT(V,L)
DO 76 I=L
76  IF (SEQL(I).EQ.V(L)) BESTOL(1,1)=BESTOL(I,1)+1.
C
C UNSORT V FOR ROBUSTNESS SETS
C DO 770 I=L
770  V(I)=SEQL(I)
C
C IF ANY ESTIMATOR IS ZERO, SKIP DIV FOR RELATIVE DIFF FOR ALL EST.
C IF (ICK.GT.0) RETURN
C TRDQL(IR)=CTRDQL(IR)+1.
DO 78 I=1
78  RELDQL(I,1)=ABS(DQL(I,1))/P1
RELDQL(I,2)=ABS(DQL(I,2))/P2
RELDQL(I,3)=ABS(DQL(I,3))/P3
RDQL(I)=0.
DO 77 J=1,3
77  W(I)=RDQL(I)=RDQL(I)+RELDQL(I,J)
77  CONTINUE
78  CONTINUE
CALL ASORT(W,L)
DO 79 I=L
79  IF (RDQL(I).EQ.W(L)) BESTOL(1,2)=BESTOL(I,2)+1.
C
C UNSORT W FOR ROBUSTNESS SETS
CONTINUE
   IF (IROBUST.EQ.1) II=4
   DO 98 I=II+4
      II=1
      IF (IR-2) 80,81,82
50  K=I
      GO TO 83
80  K=5
      GO TO 83
81  K=I+3
82  DO 96 J=1,3
     GO TO (84,86,88,90,92,94,950,958,96) II
84  IF (RELDQL(I,J)-0.0001) 96,85,85
85  II=2
86  IF (RELDQL(I,J)-0.001) 96,87,87
87  II=3
88  IF (RELDQL(I,J)-0.01) 96,89,89
89  II=4
90  IF (RELDQL(I,J)-0.05) 96,91,91
91  II=5
92  IF (RELDQL(I,J)-0.10) 96,93,93
93  II=6
94  IF (RELDQL(I,J)-0.15) 96,95,95
95  II=7
960  CONTINUE
     PRODQL(II,K)=PRODQL(II,K)+1.
96  CONTINUE
     RETURN

C ROBUSTNESS SET 1. UNIFORM PRIOR.

C

100  II=3
    L=2

C SET POSTERIOR MODE EQUAL TO M.L.E.

C

DO 102 I=1,3
   DQL(3,I)=DQL(2,I)
102  CONTINUE
GO TO 61

C ROBUSTNESS SET 2. PERTURBED PRIOR

150 I1=3
   GO TO 61
   END
APPENDIX A

LANDEY LIBRARY FUNCTION URAN

Language: COMPASS

Purpose: URAN generates uniformly distributed random numbers over the interval (0,1).

Use: $Y = URAN(X)$

- $X$ An input real number on which three conditions exist:
  - $X = 0$, The next random number is generated and returned. If no previous call was made, a default seed of 17171274321477413155B is provided.
  - $X < 0$, A random number is not generated but the last previously generated random number (or the seed if no random number has been generated) is returned.
  - $X > 0$, The exponent part of $X$ is set to 1717B and the low order bit is set to one. This result is returned as the seed of a new sequence, and any additional calls to URAN will be based on a sequence using this seed.

Method: This pseudorandom-number generator is multiplicative with algorithm

$$X_{i+1} = 43490275647445 X_i \mod(2^{48}).$$

Each random number is generated from the previous one by taking the lower order 48 bits of the 96 bit product produced by $X_{i+1} = 43490275647445 X_i$. The exponent of the product is such that $X_{i+1}$ is constrained to lie between 0 and 1.

Accuracy: The generator has a full period of $2^{46}$. Extensive statistical testing for randomness and distribution were performed to establish its validity as a reliable random number generator.

SPECTRAL NUMBERS:

<table>
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<tr>
<th></th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2.839</td>
<td>2.095</td>
<td>1.819</td>
<td>0.978</td>
</tr>
</tbody>
</table>


Storage: 13 octal locations

Subroutine date: March 1, 1977
APPENDIX B

LANGLEY LIBRARY SUBROUTINE URANV

Language: COMPASS

Purpose: URANV generates uniformly distributed random numbers over the interval (0,1).

Use: CALL URANV(X,N,V)

X An input real number on which three conditions exist:

X = 0, A vector of random numbers is generated using the last random number generated on the previous call as a seed. If no previous call was made, a default seed of 17171274321477413155B is provided.

X < 0, The last random number calculated by the routine, or the default seed if no previous call was made, is returned in V(1). V(2), ..., V(N) are not altered.

X > 0, The first random number is found by packing an exponent of 1717B and the coefficient part of X into V(1), and setting the low order bit to one. Random numbers V(2), ..., V(N) are then calculated using the algorithm given under METHOD.

N Input integer specifying the number of random numbers to be returned in V.

N ≤ 1, V(1) is calculated and returned.

N > 1, V(1), ..., V(N) are calculated and returned.

V An output one-dimensional real array dimensioned at least N. On output, V will contain the N calculated random numbers.

Method: This pseudorandom-number generator is multiplicative with algorithm

\[ X_{i+1} = 43490275647445 \times X_i \mod(2^{48}). \]

Each random number is generated from the previous one by taking the lower order 48 bits of the 96 bit product produced by \( X_{i+1} = 43490275647445 \times X_i \). The exponent of the product is such that \( X_{i+1} \) is constrained to lie between 0 and 1.
Accuracy: The generator has a full period of $2^{46}$. Extensive statistical testing for randomness and distribution were performed to establish its validity as a reliable random number generator.

SPECTRAL NUMBERS:

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.839</td>
<td>2.095</td>
<td>1.819</td>
<td>0.978</td>
</tr>
</tbody>
</table>


Storage: 25 octal locations

Subroutine date: March 1, 1977
APPENDIX C

LANGLEY LIBRARY SUBROUTINE MATINV

Language: FORTRAN

Purpose: MATINV solves the matrix equation \( AX = B \), where \( A \) is a square coefficient matrix and \( B \) is a matrix of constant vectors. The solution to a set of simultaneous equations, the matrix inverse, and the determinant may be obtained.

Use: CALL MATINV(MAX,N,A,M,B,IOP,DETERM,ISCALE,IPIVOT,IWK)

MAX The maximum order of \( A \) as stated in the DIMENSION statement of the calling program

N The order of \( A \); \( 1 \leq N \leq MAX \)

A A two-dimensional array of coefficients. On return to the calling program, \( A^{-1} \) is stored in \( A \).

M The number of column vectors in \( B \). On return to the calling program, \( X \) is stored in \( B \) if \( M > 0 \); for \( M = 0 \), the subroutine is used only for inversion.

B A two-dimensional array of the constant vectors \( B \). On return to the calling program, \( X \) is stored in \( B \).

IOP Option to compute the determinant:
\( 0 \) Compute the determinant.
\( 1 \) Do not compute the determinant.

DETERM Gives the value of the determinant by the formula
\[
\text{Det}(a) = 10^{(100 \times \text{ISCALE})}(\text{DETERM}) \quad \text{when} \quad \text{IOP} = 0.
\]
For \( \text{IOP} = 1 \), the determinant is set to 1. For a singular matrix and \( \text{IOP} = 0 \) or \( \text{IOP} = 1 \), the determinant is set to zero.

ISCALE A scale factor computed by the subroutine to keep the results of computation within the floating-point word size of the computer

IPIVOT A one-dimensional array of temporary storage used by the subroutine

IWK A two-dimensional array of temporary storage used by the subroutine

Restrictions: Arrays \( A, B, \) IPIVOT, and INDEX have variable dimensions in the subroutine. The maximum size of these arrays must be specified in a DIMENSION statement of the calling program as \( A(\text{MAX},\text{MAX}) \), \( B(\text{MAX},M) \), \( \text{IPIVOT}(\text{MAX}) \), and \( \text{IWK}(\text{MAX},2) \). The original matrices \( A \) and \( B \) are destroyed. They must be saved
APPENDIX C

by the user if there is further need for them. The determinant is set to zero for a singular matrix.

Method: Jordan's method is used to reduce a matrix $A$ to the identity matrix $I$ through a succession of elementary transformations $\ell_n$, $\ell_{n-1}$, ..., $\ell_1$. If these transformations are simultaneously applied to $I$ and to a matrix $B$ of constant vectors, the results are $A^{-1}$ and $X$ where $AX = B$. Each transformation is selected so that the largest element is used in the pivotal position. (See ref. (a).)

Accuracy: Total pivotal strategy is used to minimize the rounding errors; however, the accuracy of the final results depends upon how well-conditioned the original matrix is. A return with $\text{DETERM} \neq 0$ does not guarantee accuracy in the solutions of inverse.


Storage: 516 octal locations

Subroutine date: January 1, 1975
REFERENCES


A Computer Program For Estimation From Incomplete Multinomial Data

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Hampton, Va. 23665

National Aeronautics and Space Administration
Washington, D.C. 20546

This paper describes and lists the main computer program written for results given in NASA TM 78703. Coding is given for maximum likelihood and Bayesian estimation of the vector \( p \) of multinomial cell probabilities from incomplete data. Also included is coding to calculate and approximate elements of the posterior mean and covariance matrices. The program is written in FORTRAN IV language for the Control Data CYBER 170 series digital computer system with network operating system (NOS) 1.1. The program requires approximately 44000 octal locations of core storage. A typical case requires from 72 seconds to 92 seconds on CYBER 175 depending on the value of the prior parameter.

computer program, estimation, multinomial, incomplete data, simulation