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Onorbit Navigation Integrator Results for Typical Shuttle Orbits

Mission Planning and Analysis Division

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SHUTTLE PROGRAM

ONORBIT NAVIGATION INTEGRATOR RESULTS FOR TYPICAL SHUTTLE ORBITS

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CONTENTS

TABLES

FIGURE

1.0 SUMMARY

Three types of navigation onorbit numerical integrators were evaluated, and the following results were obtained:

a. Power integrators with no delta-V incorporation, just coasting; i.e., using Taylor series expansion integrators

   (1) Super G is slightly better than average G for step sizes of 2 and 4 seconds. (A 1200-meter error for delta-T = 4 seconds after 10 revolutions (revs)). Super G is marginal for delta-T = 15 seconds. Neither are adequate for delta-T $\geq$ 30 seconds.

   (2) Spiffy G shows no improvement over super G.

   (3) Super G4 (third order) has slight improvement over super G or spiffy G at steps of 2 and 4 seconds, but is inferior to them for delta-T $\geq$ 15 seconds.

b. Coasting integrators using the Cowell method of special perturbations

   (1) With the exception of Runge-Kutta third order, all third-order (RK or Nystrom) integrators performed rather poorly for delta-T $\geq$ 15 seconds. The RK3 is a remarkable exception, and it competes favorably with fourth-order integrators with delta-T up to 60 seconds. (The RK3 error is less than 1000 meters for delta-T = 60 seconds and 10 revs).

   (2) The fourth-order Nystrom integrators performed as well, or slightly better than the RK's but they are a little slower to execute. The Nystrom 4 with Lear's coefficients ((ref. 1) - NLXD4/4) performed better than all other Nystrom integrators.

   (3) All fourth order integrators at delta-T = 2 seconds had a 0.1-meter or less error when compared to the KS (ref. 2) reference integrator.

   (4) In general, RK4 integrators perform adequately for up to ΔT = 60 seconds and 10 revs with errors less than 1200 meters except for RKL42. Degradation occurs rapidly beyond that ΔT with the exception of RKL41 (ref. 1), which is adequate for up to 120-second time steps.

c. Coasting integrator using the Pines variation of parameter perturbation method.

   (1) The Pines formulation with RKG4 (the standard predictor for onorbit navigation, ref. 3) performs excellent for up to 5-minute (300 seconds) time steps (error less than 200 meters for delta-T = 5 minutes and 10 revs). The 10-minute time steps may be considered.

   (2) The Pines method used more core and executes slower than the Cowell method for a single step. However, for certain applications where longer time steps are permitted, this method is more time efficient.

2.0 INTRODUCTION

This document presents results for using the three onorbit navigation integrators in the onboard software: (a) average G for user parameter propagator (UPP), (b) super G for the onorbit navigation state propagation function, (c) Pines/RKG for the onorbit state prediction function, and (d) other potentially useful Runge-Kutta and Nystrom integrators for onorbit navigation where an analysis task was performed with typical Shuttle orbits (i.e., 100- to 300-n. mi. altitude and small eccentricities).

The acceleration function for a simulated force model (app. E) included a central force field, J2 gravity terms, and a drag perturbation (ref. 4). These were programed into a Hewlett Packard HP9825 desktop calculator (12-digit machine; no double precision). Gravity up to 4x4 (fourth degree-fourth order) was also investigated by CSDL, and its results are included here for completeness.

All cases were run for approximately 10 revs or until position error (RSS) was greater than 100 kilometers.

Integrator step sizes considered were 2, 4, 15, 30, 60, 150, 300, and 600 seconds.

The following integrators were considered in this analysis for total acceleration integration (ref. 1):

a. Series expansion integrators for powered flight:

   (1) Average G - second order

   (2) Super G - second order (app. A)

   (3) Spiffy G - second order (app. A)

   (4) Super G4 - third order (app. A)

b. The RK/NYSTROM integrators using the Cowell method for coasting flight:

   (5) RK3 - standard Runge-Kutta third order

   (6) RKL3 - Lear's coefficient for RK3

   (7) NLXD4/3 - Nystrom-Lear coefficient for NXD4/3

   (8) NXD4/3 - Nystrom third order

   (9) RKG4 - Runge-Kutta-Gill fourth order

   (10) RKL41 - Runge-Kutta-Lear fourth order

   (11) RKL42 - Runge-Kutta-Lear fourth order

   (12) RK4 - standard Runge-Kutta fourth order

(13) NLXD4/4 - Nystrom-Lear coefficient fourth order

(14) NXD4/4 - Nystrom fourth order

c. The following variation of parameters for special perturbation was examined for the coasting flight:

(15) Pines/RKG4 (ref. 3)

Three integrator initial conditions (IC) were used for this analysis and are listed in table I. The positions and velocities (in kilometers and km/sec) listed constitute the state vector at time = zero second. After approximately 10 revolutions (t = 54000 seconds), the state vector propagated by the reference integrator - a KS (Kustannheimo-Stieffel) formulation (ref. 2) with Runge-Kutta 45 integrator (table I) was compared with the tested integrator. The position difference was RSS (root sum squared) to determine an integrator error for the various integrator steps attempted. The position errors are listed in tables II through V.

Integrators (2) and (15) (super G and Pines/RKG) were also tested by Charles Stark Draper Laboratory (CSDL) personnel in the HAL code environment (ref. 5). The CSDL results basically duplicated the results of tables II to V. In addition, reference 5 provides actual execution time for AP101 (Shuttle onboard computer) in extended precision for 1, 5, and 10 revolutions (revs) of propagation for various step sizes. Some of the data in reference 5 are duplicated in this report.

3.0 ANALYSIS RESULTS

3.1 CASE 1 DATA

For case 1, all 15 integrators were initialized with IC#1 and allowed to run for approximately 10 revs (54 000 seconds) with state vectors printed at 10-minute increments. Only J2 gravity perturbation was included in the functional evaluation call to the acceleration function. (Drag was used in case 4 only.) IC#1, which is a 146-n. mi. circular orbit inclined at 30° with the equator, was used as the basic orbit to screen out integrator performance. The integrators that performed well in this environment were further evaluated in cases 2, 3, and 4.

To quickly assess integrator performance, the energy and percent delta energy equations were programed. Although no energy data are presented in this report, it was found that with these parameters, coding errors were detected much sooner than by the normal differencing of state vectors along the reference trajectory. The initial orbit energy ξ_0 was printed at the beginning of the run and subsequently, delta energy $\Delta\xi$ was printed. For conservative orbits; i.e., no drag or self-induced satellite accelerations like uncoupled RCS thrust, the delta energy must be zero along the trajectory: i.e., energy is conserved. (This is a necessary but insufficient condition that indicates to the user that orbit errors are probably not being introduced by the integration scheme selected.) It was found that with 1 digits of delta energy printed, integrator induced errors in the orbit are detected much sooner (in 10 or 20 steps) than they would by differencing the reference state vector with the tested integrator state vector. Therefore, integrator errors could be detected within a few time steps rather than after a rev of data.

The following formulas were used for energy and delta energy:

$$\lambda = \frac{3}{2} \mu K_J R_e^2$$

$$\Lambda = \lambda\left(\frac{X_3^2}{R^5} - \frac{1}{3R^3}\right)$$

$$\xi_i = \frac{\mu}{R} - \frac{V^2}{2} - \Lambda$$

$$\Delta\xi_i = \frac{\xi_i - \xi_o}{\xi_o}$$

where

$\xi_i \stackrel{\Delta}{=}$ orbit energy at time = t_i

$\xi_o \stackrel{\Delta}{=}$ initial orbit energy at time = 0

$\mu \stackrel{\Delta}{=}$ Earth gravitational constant = 398601.0 km^3/sec^2

$R \stackrel{\Delta}{=}$ satellite position vector magnitude, km

$V \stackrel{\Delta}{=}$ satellite velocity vector magnitude, km/sec

$\Lambda \overset{\Delta}{=}$ potential function

$\lambda \overset{\Delta}{=}$ J2 perturbation constant

$X_1, X_2, X_3 \overset{\Delta}{=}$ inertial components of satellite position vector,

X_1 along Earth's equator, X_3 along Earth's North Pole

$K_J \overset{\Delta}{=}$ J2 potential constant = 1.08265×10^{-3}

$R_e \overset{\Delta}{=}$ Earth's radius = 6371.22 km

$\Delta\xi_i \overset{\Delta}{=}$ delta energy from time = 0

Table II shows that the average G integrator performs adequately for steps up to 4 seconds and then degrades rapidly. Super G performs slightly better and is effective up to 15-second step sizes before collapsing. Spiffy G (app. A) performs almost identically to super G in all cases. As mentioned in section 1.0, a 1200 meter error (fig. 1) was noted for super G after 10 revs of propagation for 4-second time steps.

Super G4, developed in appendix A (ref. 6) performed quite well up to 4-second step sizes but degraded quickly for higher steps and was not evaluated further.

The fourth-order Runge-Kuttas used a common fourth-order RK algorithm and only the coefficients were changed for each integrator. Functional flow charts for the third- and fourth-order Runge-Kutta and Nystrom integrators are given in appendixes B and C. All coefficients were obtained from reference 1 and are listed in appendix D.

The functional evaluation subroutine was obtained from reference 7 and is shown in the appendix E flow chart. This flow chart was used by integrators to determine the acceleration vector. Note that only central force field, J2, and drag is used.

All fourth-order Runge Kutta integrators (RKG4, RKL41, RKL42, and RK4) performed quite well with delta steps up to 60 seconds. The RKL41, a Runge-Kutta integrator with Lear's first set of coefficients performed quite well with delta steps up to 150 seconds.

Two third-order Runge-Kutta integrators were tested (a standard RK3 and RKL3 that used the Lear coefficients). The RK3 performed exceptionally well for a third-order integrator and, in fact, performed almost as well or better than the fourth-order Runge-Kutta integrator for delta-T $\leq$ 60 seconds. The RKL3 did not perform well.

Two fourth-order Nystrom integrators were tested: the NLXD4 and the NXD4. The NLXD4, which used the Lear coefficients, performed slightly better than the standard Nystrom NXD4. In general, the Nystrom integrators performed at least as well as the RK's. However, the algorithm took slightly more time to execute because of the extra calculations required in the Nystrom algorithm.

Two third-order Nystrom integrators were tested, and as shown in table II, they performed rather poorly and were quickly discarded (NXD3 and NLXD3).

The last integrator tested was the fourth-order Runge-Kutta-Gill (RKG4) using the Pines variation of parameters formulation technique. The code was obtained directly from the onorbit navigation FSSR (ref. 3). As shown in table II, this formulation and integrator combination performed exceptionally well up to 10-minute steps. However, the Pines code is more complex than Cowell's formulation, and as shown in table VI, it does take about five times longer to execute.

Table VI lists the relative time it took for the various algorithms to perform in the HP9825 environment (a fairly accurate but somewhat slow desktop calculator when compared with a typical powerful and much faster machine such as the UNIVAC 1108). It should be noted that the algorithms coded were general purpose and inefficient, specifically in terms of execution time because if a coefficient of zero were encountered, the algorithm operation was still performed. In any case, this table clearly shows that Pines/RKG is not only the most accurate method but also the slowest method tested for a single step. More or less the same relative execution timing data were observed in reference 5 and are reproduced elsewhere in this report.

3.2 CASE 2 DATA

For case 2, the initial condition IC#2 represented a Skylab reboost rendezvous orbit after the terminal phase finalization (TPF) maneuver. To conserve computer time, some of the small delta step runs were deleted. It was assumed that the trend to higher accuracy for smaller time steps had been established in the case 1 results. Only the average G, super G, spiffy G, RKG4, RKL41, RK3, and the NLXD4 integrators of case 1 were tested for this case.

Results for case 2 were basically the same as for case 1 and are tabulated in table III.

3.3 CASE 3 DATA

Case 3 results are listed in table IV. The IC#3 represented a Skylab insertion orbit of 100 n. mi. circular. Because drag is significant at this altitude, the integrators of case 2 were tested with the drag perturbation of reference 4 (a simplified atmospheric model) in the acceleration function.

The accuracy results for case 3 were very similar to those of cases 1 and 2. This seems to indicate that drag, if modeled properly in the acceleration functional evaluation used by the integrator, will present no difficulty to the integration scheme used. However, it was noted immediately that the orbit energy and delta energy computations were being affected by the slight acceleration produced by the drag. Therefore, if drag or some other nonconservative force are included in the acceleration model, the value of the energy check in evaluating integrator performance is lost.

3.4 CASE 4 DATA

Case 4 results are given in table V. In essence, this case is identical to case 3 except that no drag was included in the evaluation of the acceleration. Some runs with super G4 and Pines/RK6 were added to get more data for this integrator. As with previous cases, the accuracy results were very similar to the other cases. The integrator error differences between cases 3 and 4 (i.e., drag versus no drag) were very minimal. However, the actual position differences after 10 revs were about 30 kilometers (for constant Shuttle surface area) due to the drag.

3.5 CSDL RESULTS

To obtain some performance data for OPS-2 state propagators in HAL code running in AP101 extended precision, CSDL used IC#1 and IC#2 and propagated them for 10 revs for both the super G and Pines/RKG integrators at various integrator step sizes in their AP101 simulation.

Since the onboard flight code was to be used for this analysis, it was decided that an existing ENCKE-Nystrom formulation would be used as a real world reference test case. This formulation consists of a full double precision with a fourth-order/degree gravity model and an integration step size of 30 seconds. (This was later changed to an 8 degree/order gravity model to determine the differences between a 4/4 and 8/8 gravity). Results are given in reference 5 and they can be summarized as follows:

a. For J2 only (gravity = 2 order, zero degree), which is what the HP9825 program was formulated to, the differences between the super G and RKG/Pines were about 1000 meters - basically the same as tables II and V. Reference 5 results can be summarized in table VII. (They can be computed by differencing cases: super G6 and PRKG5; super G6' and PRKG5'.) This same integrator difference is basically observed for cases using higher fidelity gravity models: i.e., super G1 and PRKG1; super G3 and PRKG3; super G5 and PRKG4; and the similar primed cases.

b. Table VII shows that the Pines/RKG, using a 3-minute step size (predictor type operation), performs almost the same as for the 1-minute step size.

4.0 CONCLUSIONS AND RECOMMENDATIONS

a. The super G integrator is a very simple and effective integrator for 2- and 4-second time steps. Since IMU delta-V data can be easily incorporated in the integration scheme, its use as the standard onorbit navigation propagator for the maintenance of the current state has been implemented in the onboard navigation software.

b. The Pines variation-of-parameters formulation method with a Runge-Kutta-Gill (RKG) fourth-order integrator method produces excellent results up to 300-second time steps. Onorbit prediction with this method (3- to 5-minute time steps) has been implemented in the onboard onorbit navigation scheme.

c. The Runge-Kutta third order (ref. 1), using Cowell's method, is an excellent general purpose orbit determination integrator for time steps up to a 60-second duration.

5.0 REFERENCES

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TABLE I.- INTEGRATOR INITIAL CONDITIONS

Param	Unit	Case 1	Case 2	Case 3	Case 4
			t_o = 0.0		
X_o	km	6 649.02	-3 972.220046	-4 192.451762	-4 192.451762
Y_o	km	0	-1 892.121621	4 777.342541	4 777.342541
Z_o	km	0	-5 000.635973	1 636.711626	1 636.711626
$\dot{X}_o$	km	0	4.393527408	-4.85227183	-4.85227183
$\dot{Y}_o$	km/sec	6.705343087	-6.262423238	-2.327477093	-2.327477093
$\dot{Z}_o$	km/sec	3.871331637	-1.112883779	-5.64056083	-5.64056083
H_A	km		284.08	185.05	185.05
H_P	km		269.85	181.38	181.38
i	deg	30°	49.86°	49.93°	49.93°
			Final position @ t_F = 54 000 sec		
		W/O drag	W/O drag	W drag	W/O drag
X_F	km	6 507.6213	-4 068.7385	-4 955.392	-4 968.7873
Y_F	km	1 027.5009	-1 666.3453	- 543.481	- 519.5312
Z_F	km	895.9049	-5 003.6163	-4 263.2106	-4 211.5175

TABLE II.- CASE 1 INTEGRATOR POSITION ERROR AFTER 10 REVS, METERS

[No drag; i = 30°]

	Delta-T							
Integrator	2 sec	4 sec	15 sec	30 sec	60 sec	150 sec	300 sec	600 sec
AVE G	759	3035	42 687	170 682	681 716			
SUPER G	332.2	1 161.1	3 360.8	57 526	228 962			
SPIFFY G	332.5	1 161.2	3 360.8	57 526	228 962			
SUPER G4	22.9	37.2	8 857.3	70 776	563 902			
RKG4		.1	1.1	16.3	227.1	4 558.5	92 327.1	
RKL41		.1	.1	.8	.7	1358.3	50 861	
RKL42	.1	.1	10.5	244.5	6 326	515 726		
RK4	.1	.1	1.8	43.5	1 159.7	100 114		
RK3	.1	.3	9.8	66.9	164.2	84 411.7		
RKL3	21.3	169.6	8 939.8	71 545.3				
NLXD4		.1	.1	1.0	31.7	3 121.6	103 271	
NXD4	.1	.1	2.0	34.8	672.1	39 134.4		
NLXD3	150.7	918.6	17 747.1	82 921.5				
NXD3	390.7	2 406.8	48 173.5	233 569				
PINES-RKG4			.1	.1	.1	1.70	33.4	683.9

TABLE III.- CASE 2 INTEGRATOR POSITION ERROR AFTER 10 REVS, METERS

[No drag; $i = 49.8^{\circ}$
Skylab TPF
H_A = 153.4 n. mi.
H_P = 145.7 n. mi.]

Integrator	Delta-T						
	2 sec	4 sec	15 sec	30 sec	60 sec	150 sec	300 sec
AVE G	752.3	3 009.4	42 312	169 187	675 543		
SUPER G	328.7	1 149.7	3 434.9	55 915	783 477		
SPIFFY G			3 434.9	55 915	783 477		
RKC4		.1	1.0	16.0	223.3	4 500.5	89 065.1
RKL41			.1	.8	2.2	1 228.4	47 179
RK3			.6	10.8	751.0	91 251	
NLXD4			.1	1.0	30.9	3 041.6	100 600

TABLE IV.- CASE 3 INTEGRATOR POSITION ERROR AFTER 10 REVS, METERS

[With drag; $i = 49.93^{\circ}$
Skylab insertion
100-n. mi. circular]

Integrator	2 sec	4 sec	15 sec	30 sec	60 sec	150 sec	300 sec
AVE G	782.9	3 133.4	44 064	176 119			
SUPER G	349.0	1 215.4	3 044.3	64 646	878 152		
SPIFFY G	349.5	1 216.3	3 044.2	64 659	878 232		
RKG4		.1	1.1	15.4	209.0	3 526.5	126 550
RKL41		.1	.1	.8	.7	1 487	54 573
RK3		.1	3.9	14.8	[illegible]37.6	103 727	
NLXD4		.1	.1	1.2	35.6	3 501.4	115 984

TABLE V.- CASE 4 INTEGRATOR POSITION ERROR AFTER 10 REVS, METERS

[No drag; $i = 49.93^\circ$
Skylab insertion
100-n. mi. circular]

Integrator	2 sec	4 sec	15 sec	30 sec	60 sec	150 sec	300 sec	600 sec	1200 sec
AVE G	774.9	3 100.1	43 591	174 299	695 897				
SUPER G	347.3	1 207.9	3 016.8	64 267.8	871 275				
SPIFFY G	347.2	1 208.0	3 016.8	64 267.8	871 275				
SUPER G4	48.1	269.1							
RKG4		.1	1.1	17.1	237.7	4 427.6	116 140		
RKL41		.1	.2	.4	7.3	1 775.3	59 263		
RK3		.1	3.7	14.3	744.7	103 285			
NLXD4		.1	0	1.0	35.2	3 478.7	115 263		
PINES-RKG4			.2	.1	.1	5.4	122.7	1 247.7	42 749

TABLE VI.- INTEGRATOR RELATIVE TIMING DATA FOR hP9825 EXECUTION TIME

Integrator (a)	Order	Approximate time, step/sec (b)
Average G	2	0.293
Super G	2	.320
Spiffy G	2	.327
Super G4	3	.643[c]
RK3	3	.493
RKL3	3	.493
NLXD4/3	3	.527
NXD4/3	4	.527
RKG4	4	.693
RKL41	4	.693
RKL42	4	.693
RK4	4	.693
NLXD4/4	4	.720
NXD4	4	.720
Pines	4	3.375

[a]Nonoptimum code could be reduced significantly for some, especially super G4.
[b]Including F evaluation J2; no drag.
[c]Inefficient coding by programer.

TABLE VII.- CSDL RESULTS

Case no.	Time, step/sec	Gravity deg	Model order	IC no.	Integrator	Total position error, ft 1 rev	5 revs	10 revs
1	4	2	0	1	Super G	2588	13 650	35 358
3	4	2	0	2	Super G	2593	6 064	20 807
2	4	4	4	1	Super G	479	2 182	3 814
4	4	4	4	2	Super G	475	2 156	3 771
2A	8	4	4	1	Super G	1875	7 623	10 836
4A	8	4	4	2	Super G	1856	7 545	10 761
5	60	2	0	1	Pines-RKG	2134	11 634	32 000
8	60	2	0	2	Pines-RKG	2137	4 096	17 714
6	60	4	4	1	Pines-RKG	5	160	556
9	60	4	4	2	Pines-RKG	9.5	161	609
7	180	4	4	1	Pines-RKG	19	185	540
10	180	4	4	2	Pines-RKG	33	415	1 499
6A	60	4	2	1	Pines-RKG	473	1 602	10 204
9A	60	4	2	2	Pines-RKG	2655	8 248	21 888
6B	60	2	2	1	Pines-RKG	390	2 751	13 322
9B	60	2	2	2	Pines-RKG	2126	5 544	16 543
2'	4	4	2	1	Super G	156	3 469	13 383
4'	4	4	2	2	Super G	3117	10 193	25 060

Note: Truth model for above data used an Encke-Nystrom formulation in full double precision with a fourth-order and degree gravity model and an integration step size of 30 seconds.

TABLE VII.- CSDL RESULTS (Concluded)

Case no.	Time, step/sec	Gravity deg	Model order	IC no.	Integrator	Total position error,ft 1 rev	5 revs	10 revs
2"	4	2	2	1	Super G	413	4 698	16 424
4"	4	2	2	2	Super G	2592	7 559	19 780
2A'	8	4	2	1	Super G	1430	8 844	20 402
4A'	8	4	2	2	Super G	4494	15 575	32 043

Note: Truth model for above data used an Encke-Nystrom formulation in full double precision with a fourth-order and degree gravity model and an integration step size of 30 seconds.

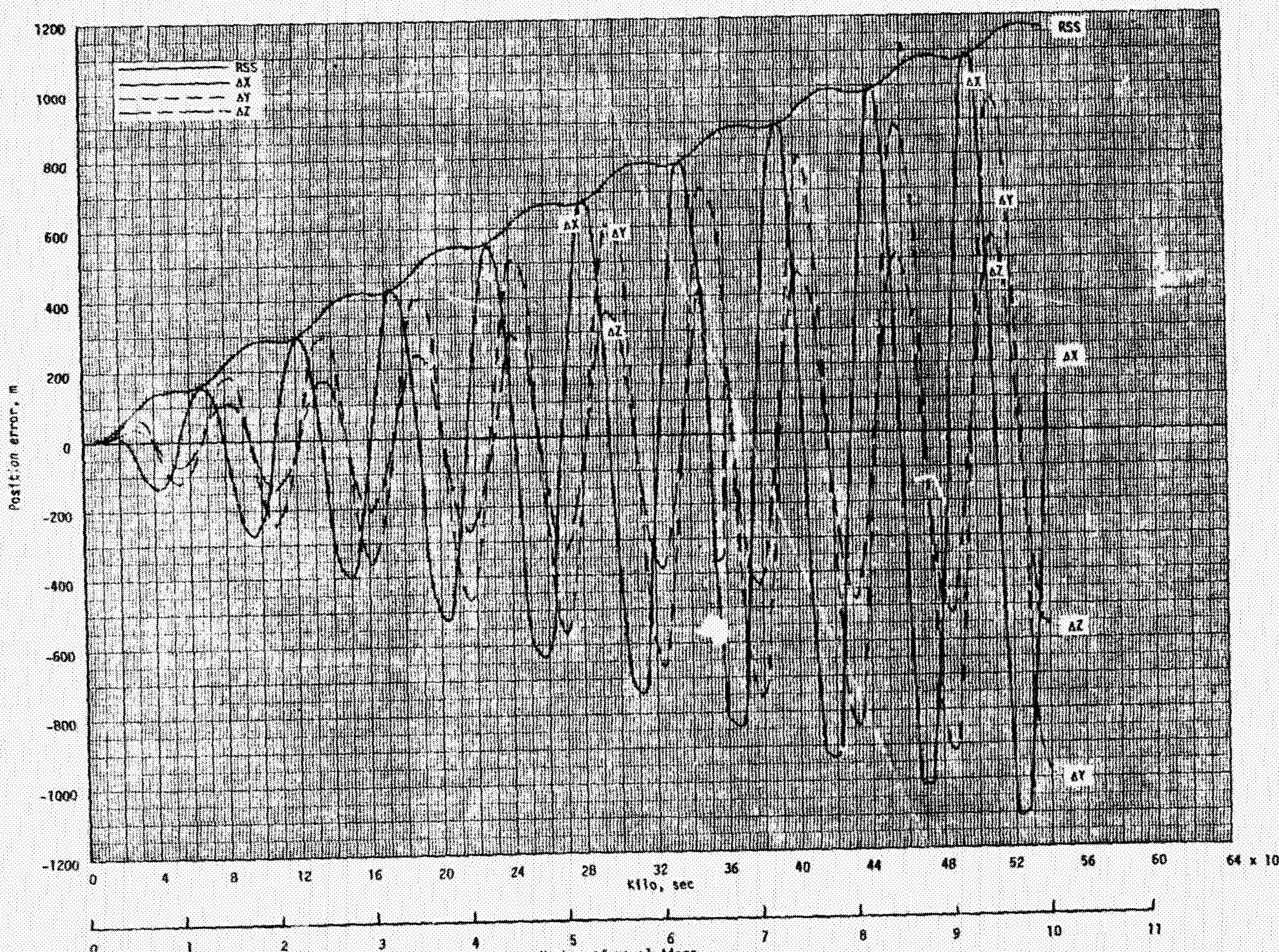


Figure 1.- Typical super G integrator, position error in meters (ΔT = 4 seconds).

APPENDIX A

SUPER G AND SUPER G4 EQUATIONS

Super G and super G4 algorithms:

a. Compute $F_o = F(T_o, R_o, V_o)$

b. Compute R_1, V_1

$$R_1 = R_o + V_o DT + F_o DT^2/2$$

$$V_1 = V_o + F_o DT$$

$$T_1 = T_o + DT$$

c. Evaluate $F_1 = F(T_1, R_1, V_1)$

d. Update R_1, V_1

$$R_1 = R_o + V_o DT + F_o DT^2/2 + A3 \cdot DT^3/6$$

If super G $\rightarrow$ Go to step (e)

$$V_1 = V_o + F_o DT + A3 \cdot DT^2/2$$

Where

$$A3 \equiv (F_1 - F_o)/DT$$

e. If (Spiffy G or Super G) $\rightarrow$ Relabel $R_o \leftarrow R_1$

$V_o \leftarrow V_1$

$T_o \leftarrow T_1$

And go to (a)

f. Evaluate $F_1 = F(T_1, R_1, V_1)$

g. Compute R_2, V_2

$$R_2 = R_1 + V_1 DT + F_1 DT^2/2 + A3 \cdot DT^3/6$$

$$V_2 = V_1 + F_1 DT + A3 \cdot \frac{DT^2}{2}$$

Where

$$A_3 \equiv \frac{F_1 - F_o}{DT}$$

$$T_2 = T_o + 2DT$$

h. Evaluate $F_2 = F\ (T_2, R_2, V_2)$

i. Update R_2, V_2

$$R_2 = R_1 + V_1 DT + F_1 DT^2/2 + A3 \cdot DT^3/6$$

$$V_2 = V_1 + F_1 DT + A3 \cdot DT^2/2$$

Where

$$A3 \equiv (F_2 - F_1)/DT$$

j. Update $F_2 = F\ (T_2, R_2, V_2)$

k. Update R_2, V_2

$$R_2 = R_1 + V_1 DT + F_1 DT^2/2 + A3 \cdot DT^3/6 + A4 \cdot DT^4/24$$

$$V_2 = V_1 + F_1 DT + A3 \cdot DT^2/2 + A4 \cdot DT^3/6$$

Where

$$A3 \equiv \frac{(F_2 - F_o)}{2DT}$$

$$A4 = \frac{F_2 - 2F_1 + F_o}{DT^2}$$

l. Evaluate $F_2 = F\ (T_2, R_2, V_2)$

m. Relabel

$$F_o \leftarrow F_1, \quad T_o \leftarrow T_1$$

$$R_1 \leftarrow R_2, \quad V_1 \leftarrow V_2, \quad F_1 \leftarrow F_2, \quad T_1 \leftarrow T_2$$

GOTO (g)

Derivative a .proximations:

a. Two points (T_o, F_o), (T_1, F_1) Known

$$\dot{F}_1 = \dot{F}_o = \frac{F_1 - F_o}{DT} + 0(DT)$$

b. Three points (T_o, F_o), (T_1, F_1), (T_2, F_2) Known

$$\dot{F}_1 = \frac{F_2 - F_o}{2DT} + 0(DT^2)$$

$$\ddot{F}_1 = \frac{F_2 - 2F_1 + F_o}{DT^2} + 0(DT^2)$$

The errors (DT) and (DT^2) are due to the derivative approximations. Further errors may be introduced due to errors in F_o, F_1, or F_2.

APPENDIX B

FUNCTIONAL FLOW CHART FOR RK3 AND RK4

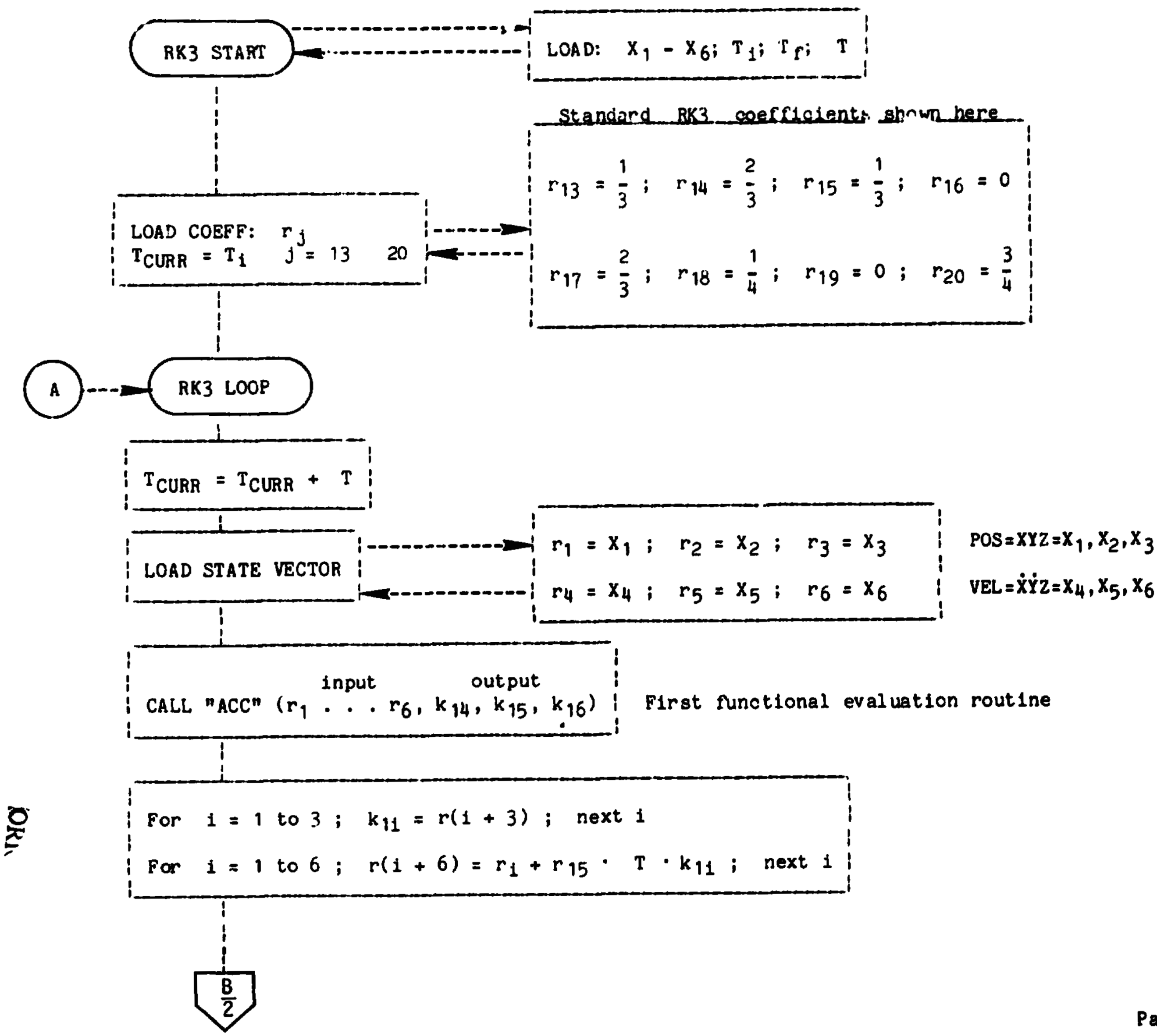


Figure B1.- RK3 functional flow chart.

B

CALL "ACC" (r_7 . . . r_{12}, k_{24}, k_{25}, k_{26}) Second functional evaluation

For i = 1 to 3 ; $k_{2i} = r(i + 9)$; next i

For i = 1 to 6 ; $r(i + 6) = ri + \Delta T(r_{16} \cdot k_{1i} + r_{17} \cdot k_{2i})$; next i

CALL "ACC" (r_7 . . . r_{12}, k_{34}, k_{35}, k_{36} Third functional evaluation

For i = 1 to 3 ; $k_{3i} = r(i + 9)$; next i

For i = 1 to 6 ; $X_i = r_i + \Delta T(r_{18} \cdot k_{1i} + r_{19} \cdot k_{2i} + r_{20} \cdot k_{3i})$; next i

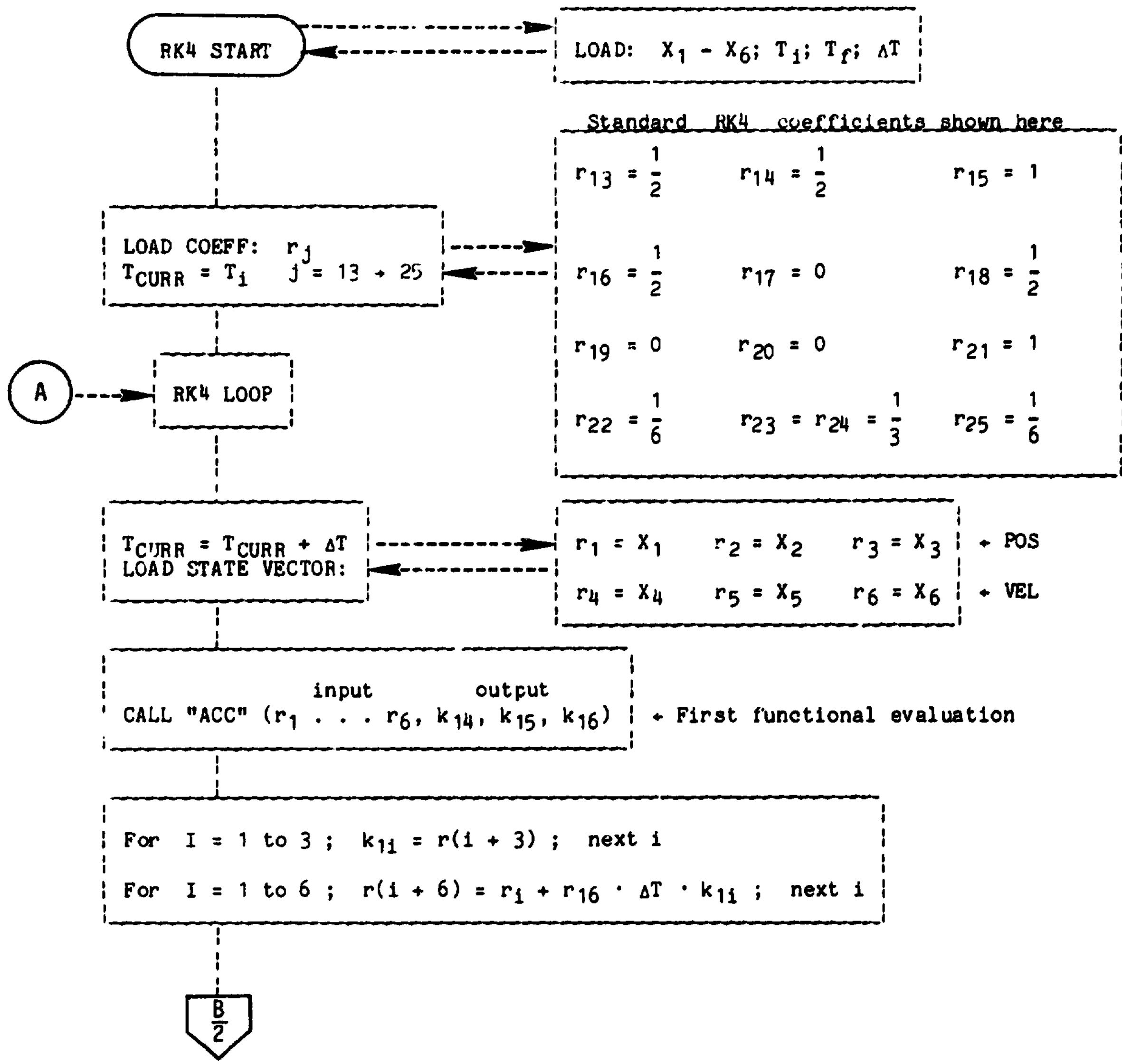
$T_{CURR} = T_f$

No: A T

Yes: PRINT: FINAL STATE AND STOP

Page 2 of 2.

Figure B1.- Concluded.



Page 1 of 2.

Figure B2.- RK4 functional flow chart.

(B)

CALL "ACC" (r_7 . . . r_{12}, k_{24}, k_{25}, k_{26}) ← Second functional evaluation

For i = 1 to 3 ; $k_{2i} = r(i + 9)$; next i

For i = 1 to 6 ; $r(i + 6) = r_i + \Delta T(r_{17} \cdot k_{1i} + r_{18} \cdot k_{2i})$; next i

CALL "ACC" (R_7 . . . r_{12}, k_{34}, k_{35}, k_{36} ← Third functional evaluation

For i = 1 to 3 ; $k_{3i} = r(i + 9)$; next i

For i = 1 to 6 ; $r(i + 6) = r_i + \Delta T(r_{19} \cdot k_{1i} + r_{20} \cdot k_{2i} + r_{21} \cdot k_{3i})$; next i

CALL "ACC" (r_7 . . . r_{12}, k_{44}, k_{45}, k_{46}) ← Fourth functional evaluation

For i = 1 to 3 ; $k_{4i} = r(i + 9)$; next i

For i = 1 to 6 ; $X_i = r_i + \Delta T(r_{22} \cdot k_{1i} + r_{23} \cdot k_{2i} + r_{24} \cdot k_{3i} + r_{25} \cdot k_{4i})$; next i

$T_{CURR} = T_f$?

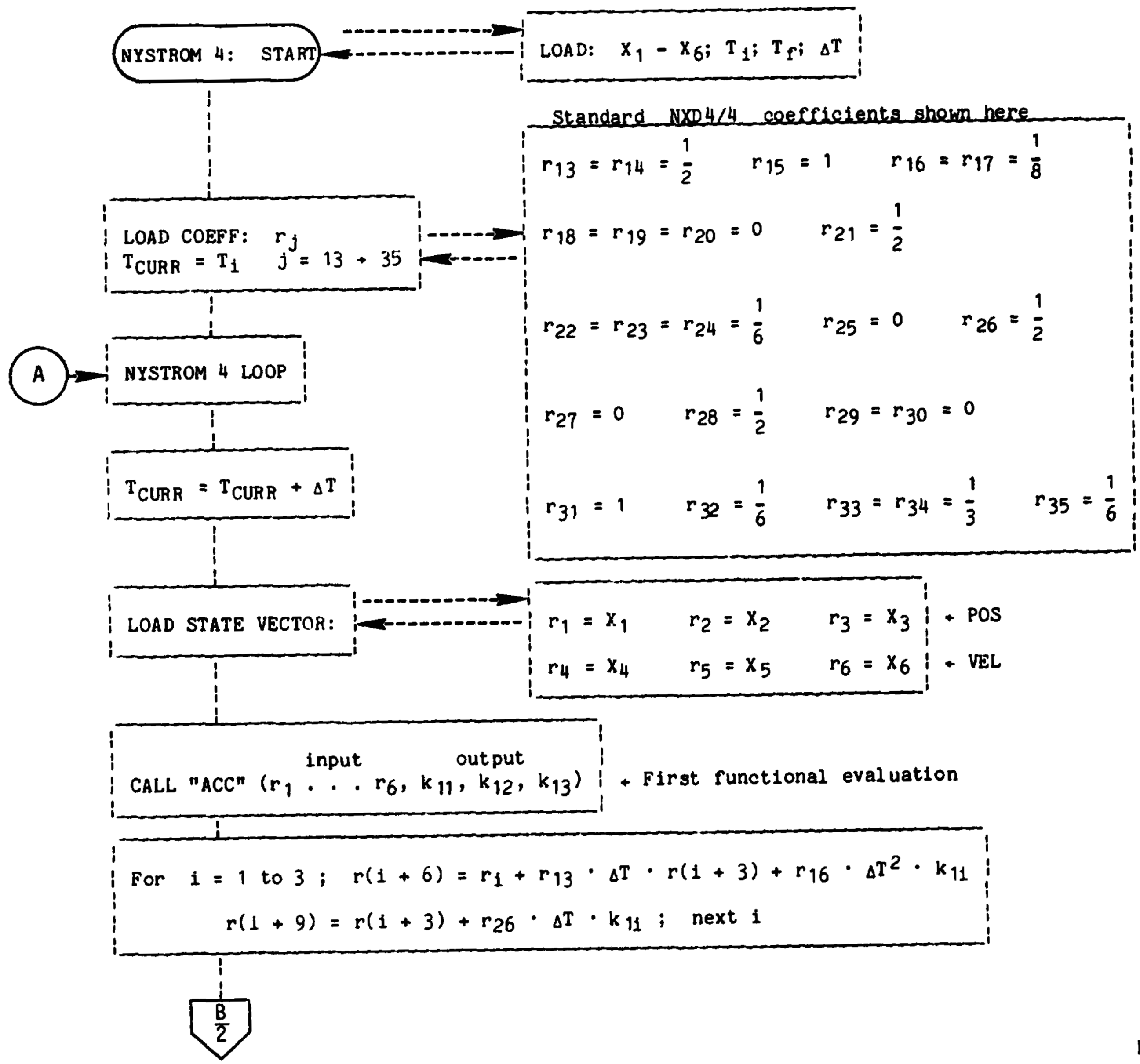
No → (A/T)

Yes → PRINT: FINAL STATE VECTOR AND STOP

Figure B2.- Concluded. Page 2 of 2.

APPENDIX C

FUNCTIONAL FLOWCHART
FOR NYSTROM FOURTH-ORDER INTEGRATOR



Page 1 of 2.

Figure C1.- Nystrom fourth order functional flow chart.

B

CALL "ACC" (r_7 . . . r_{12} [input], k_{21}, k_{22}, k_{23} [output]) ← Second functional evaluation

For i = 1 to 3 ; $r(i+6) = r_i + r_{14} \cdot \Delta T \cdot r(i+3) + \Delta T^2(r_{17} \cdot k_{1i} + r_{18} \cdot k_{2i})$

$r(i+9) = r(i+3) + \Delta T(r_{27} \cdot k_{1i} + r_{28} \cdot k_{2i})$; next i

CALL "ACC" (r_7 . . . r_{12}, k_{31}, k_{32}, k_{33}) ← Third functional evaluation

For i = 1 to 3 ; $r(i+6) = r_i + r_{15} \cdot \Delta T \cdot r(i+3) + \Delta T^2(r_{19} \cdot k_{1i} + r_{20} \cdot k_{2i} + r_{21} \cdot k_{3i})$

$r(i+9) = r(i+3) + \Delta T(r_{29} \cdot k_{1i} + r_{30} \cdot k_{2i} + r_{31} \cdot k_{3i})$; next i

CALL "ACC" (r_7 . . . r_{12}, k_{41}, k_{42}, k_{43}) ← Fourth functional evaluation

For i = 1 to 3 ; $X_i = r_i + \Delta T \cdot r(i+3) + \Delta T^2(r_{22} \cdot k_{1i} + r_{23} \cdot k_{2i} + r_{24} \cdot k_{3i} + r_{25} \cdot k_{4i})$

$X(i+3) = r(i+3) + \Delta T(r_{32} \cdot k_{1i} + r_{33} \cdot k_{2i} + r_{34} \cdot k_{3i} + r_{35} \cdot k_{4i})$; next i

$T_{CURR} = T_f$

No → A T

Yes → PRINT:FINAL STATE VECTOR AND STOP

Figure C1.- Concluded.

Page 2 of 2.

APPENDIX D

INTEGRATOR COEFFICIENTS

TABLE D1.- INTEGRATOR COEFFICIENTS

	RK3	RKL3	NXD3	NLXD3	RK4	RKG4	RKL14	RKL24	NXD4	NLXD4
r_{13}	1/3	$\frac{6-\sqrt{6}}{10}$	1/2	$0.6-\sqrt{.06}$	1/2	1/2	0.15	$(5-\sqrt{5})/10$	1/2	$(5-\sqrt{5})/10$
r_{14}	2/3	$\frac{6+\sqrt{6}}{10}$	1	$.6+\sqrt{.06}$	1/2	1/2	.192	$(5+\sqrt{5})/10$	1/2	$(5+\sqrt{5})/10$
r_{15}	1/3	$\frac{6-\sqrt{6}}{10}$	1/8	$.21-.6\sqrt{.06}$	1	1	1	1	1	1
r_{16}	0	$-(54+19\sqrt{6})/250$	0	$(.15+4\sqrt{.06})/25$	1/2	1/2	.15	$(5-\sqrt{5})/10$	1/8	$(3-\sqrt{5})/20$
r_{17}	2/3	$(102+22\sqrt{6})/125$	1/2	$(5.1+11\sqrt{.06})/25$	0	$(-1+\sqrt{2})/2$	.1536	$-(5+3\sqrt{5})/20$	1/8	0
r_{18}	1/4	1/9	1/6	1/9	1/2	$(2-\sqrt{2})/2$	.0384	$(3+\sqrt{5})/4$	0	$(3+\sqrt{5})/20$
r_{19}	0	$(16+\sqrt{6})/36$	1/3	$(7+20\sqrt{.06})/36$	0	0	6.74526571119	$(-1+5\sqrt{5})/4$	0	$(-1+\sqrt{5})/4$
r_{20}	3/4	$(16-\sqrt{6})/36$	0	$(7-20\sqrt{.06})/36$	0	$-\sqrt{2}/2$	-38.7783195429	$-(5+3\sqrt{5})/4$	0	0
r_{21}			1/2	$(.6-\sqrt{.06})$	1	$(2+\sqrt{2})/2$	33.0330538317	$(5-\sqrt{5})/2$	1/2	$(3-5)/4$
r_{22}			-1	$-(5.4+19\sqrt{.06})/25$	1/6	1/6	1.41435185185	1/12	1/6	1/12
r_{23}			2	$(20.4+44\sqrt{.06})/25$	1/3	$(2-\sqrt{2})/6$	-9.58605664488	5/12	1/6	$(5+\sqrt{5})/24$
r_{24}			1/6	1/9	1/3	$(2+\sqrt{2})/6$	8.95271818848	5/12	1/6	$(5-\sqrt{5})/24$
r_{25}			2/3	$(8+5\sqrt{.06})/18$	1/6	1/6	.218986604542	1/12	0	0
r_{26}			1/6	$(8-5\sqrt{.06})/18$					1/2	$(5-\sqrt{5})/10$
r_{27}									0	$-(5+3\sqrt{5})/20$
r_{28}									1/2	$(3+\sqrt{5})/4$
r_{29}									0	$(-1+5\sqrt{5})/4$
r_{30}									0	$(-5+3\sqrt{5})/4$
r_{31}									1	$(5-\sqrt{5})/2$
r_{32}									1/6	1/12

TABLE D1.- Concluded

	RK3	RKL3	NXD3	NLXD3	RK4	RKG4	RKL14	RKL24	NXD4	NLXD4
r_{33}									1/3	5/12
r_{34}									1/3	5/12
r_{35}									1/6	1/12

Note: In this study, only J2 is considered (except for reference 5). Therefore, for third order integrators, r_{13} and r_{14} constants are not used. Similarly, for fourth order integrators, r_{13}, r_{14}, and r_{15} constants are not used.

APPENDIX E

ACCELERATION FUNCTION SUBROUTINE

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START

LOAD CONSTANTS:

$\mu_E = 3.98601D5$; $J_2 = 1.08265D-3$; $R_B = 6.37122D3$

$\rho_{DBN} = 8.3631D-1$; $C_{AY} = 0.67385D-2$

Intermediate Calculation

$R_{MAG} = \sqrt{r_1^2 + r_2^2 + r_3^2}$; $V_{MAG} = \sqrt{V_1^2 + V_2^2 + V_3^2}$

$R_{5IN} = \dfrac{1}{R_{MAG}^5}$; $R_{7IN} = \dfrac{1}{R_{MAG}^7}$

$X_{LAM} = 1.5 \cdot U_E \cdot J_2 \cdot R_B^2$

$X_{LTB} = X_{LAM} (-5 \cdot r_3^2 \cdot R_{7IN} + R_{5IN})$

COMPUTE CENTRAL FORCE ACCELERATION:

$B_{CF} = -\dfrac{\mu_E}{R_{MAG}^3}$

for $i = 1$ to 3 ; $A_{CFi} = B_{CF} \cdot r_i$; next i

$\frac{A}{2}$

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Page 1 of 2.

Figure E1.- Acceleration function subroutine.

A

COMPUTE J2 (OBLATENESS) ACCELERATION:

for $i = 1$ to 2 ; $A_{J2i} = -X_{LTB} \cdot r_i$; next i

$A_{J23} = -X_{LTB} \cdot r_3 - X_{LAM} \cdot 2 \cdot r_3 \cdot R_{5IN}$

COMPUTE DRAG ACCELERATION:

$$RE = \frac{R_B}{\sqrt{(1.0 + C_{AY} \left(\frac{r_3}{R_{MAG}}\right)^2}}$$

$Z = (R_{MAG} - R_E)/0.3048$

$Z_{FUNCT} = e\ (-24.2 - 0.00289 \cdot Z + 2605/Z)$

$DRAG = -0.5 \cdot \rho_{DBN} \cdot Z_{FUNCT} \cdot V_{MAG}$

for $i = 1$ to 3 ; $A_{Di} = DRAG \cdot V_i$; next i

TOTAL ACCELERATION:

For $i = 1$ to 3 ; $A_{TOTALi} = A_{CFi} + A_{J2i} + A_{Di}$; next i

RETURN

Note 1: State vector units.

$r_1, r_2, r_3 \triangleq$ position in km

$V_1, V_2, V_3 \triangleq$ velocity in km/sec

NOTE 2: $r_3 \triangleq$ position computation along Earth's North Pole.

Figure E1.- Concluded.

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