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COMPUTER PREDICTION OF LARGE
REFLECTOR ANTENNA RADIATION PROPERTIES

by

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ABSTRACT

BOTULA, ALAN BERNARD. Computer Prediction of Large Reflector Antenna Radiation Properties. (Under the direction of JAMES FRANK KAUFFMAN.)

A program for calculating reflector antenna radiation patterns has been rewritten and extended to include a new class of reflector surfaces. The original program, developed jointly by North Carolina State University and NASA, was capable of computing patterns for reflectors with smooth analytic surfaces. The revised version allows the reflector to be composed of a number of panels, removing the restriction that the surface must be smooth. These individual panels, however, are subject to the restrictions originally imposed upon the entire surface; that is, they must be analytic surfaces.

The theoretical foundation for the program is as follows: Geometrical optics techniques are used to trace rays from a feed antenna to the reflector surface and back to a mathematical plane just in front of the reflector. The resulting tangential electric field distribution, assumed to be the only source of forward radiation, is integrated numerically to calculate the radiation pattern for a desired set of angles. When the reflector is composed of more than one panel, each panel is treated as a separate antenna, the ray-tracing procedure and integration being repeated for each panel. The results of the individual aperture plane integrations are stored and summed to yield the relative electric field strength over the angles of interest.

An example and several test cases are included to demonstrate the use of the program and verify the new method of computation.
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1. INTRODUCTION

This thesis reports the work done to extend the capabilities of the computer algorithm, REFLECTR, a Fortran program developed jointly by North Carolina State University and the National Aeronautics and Space Administration to compute reflector antenna radiation patterns.

The original program was published in the IEEE Transactions on Antennas and Propagation in 1976 [1]. At this stage, it was capable only of computing patterns for paraboloidal reflectors. It was called PREFPAT, an acronym for Paraboloidal REFlector PATterns. The geometrical formulation was later changed to provide the calculated co- and cross-polarized fields which would actually be measured on an antenna range. This revised version appeared later that year in a North Carolina State University report [2]. Between 1976 and 1978, the program was extended to include ellipsoidal and spherical surfaces. It also received its present name, REFLECTR. A report on these changes was published by NASA in May of 1978 by Dr. Pradeep K. Agrawal [3]. Dr. Agrawal continued to improve the program, adding the ability to analyze planar reflectors in addition to the other surfaces [4].

The objective of the work reported herein is to generalize REFLECTR to make it capable of computing patterns for more complex surfaces; in particular, reflectors constructed from several panels or sections. The strategy for the algorithm is to represent each panel as a separate antenna and superpose their radiation patterns to find the total secondary pattern for the configuration. Among the consequences of this work were the automation of several operations formerly
done manually, the addition of equations for parabolic cylinders to allow analysis of deployable gored reflectors, a numerical technique for aperture boundary definition, and improved output formatting with more control over output operations.

This latest version of REFLECTR is written in standard Fortran-G using no compiler-specific advantages. With the exception of the plotting package, it should run on any Fortran compiler with little change. A complete description of how to organize an input file for the program for several reflector configurations is provided in Section 6.

The increase in the generality of the algorithm has not caused a substantial increase in computer time. In fact, the revised version requires only about six percent more time than the old program when making a pattern computation for a given single-panel reflector.
2. THEORETICAL DEVELOPMENT

2.1 Review of Theory

The new and old versions of REFLECTR share a common theoretical core. Before either program is discussed individually, this basic theory will be reviewed.

The approach taken in this algorithm is to assume that the reflector is in the far field of the feed antenna and to use geometrical optics ray tracing to compute the electric field over a planar aperture just in front of the reflector surface. The tangential field over this aperture is assumed to be the only source of forward radiation and is integrated to yield the composite radiation pattern of the antenna. Edge diffraction effects are neglected.

A reflector antenna is described in terms of two cartesian coordinate systems, the reference (unprimed) system and the feed (primed) system. These are shown in relation to a typical reflector antenna in Figure 1. The feed system is used to describe the feed antenna radiation pattern. The reflector geometry, the location of the aperture plane, and the overall radiation pattern are described in the reference system. In general, these coordinate systems are related by a rotation and a translation, each with three orthogonal components.

The majority of the operations in the algorithm are performed in the reference coordinate system. The reflector surface is described by a function of the reference coordinates x, y, and z. Five types of surfaces are presently available to the program: planes, spheres, ellipsoids, paraboloids, and parabolic cylinders. Other second-order
FIG. 1. COORDINATE SYSTEMS
polynomial functions may be added with little difficulty. The composite radiation pattern is expressed in a spherical coordinate system which is based on the reference cartesian coordinates. Convention is followed in that the theta angle is the polar angle measured from the positive z-axis and phi is the angle in the xy plane increasing from the positive x-axis to the positive y-axis.

The feed system allows the feed antenna radiation pattern to be described independently of its location or orientation. The phase center of the feed antenna is assumed to be at the origin of its coordinate system and the antenna points in the negative x' direction. Its radiation pattern is described in a spherical coordinate system, which is based on the feed cartesian coordinates using the same convention as the reference spherical system. The feed coordinate angles are denoted $\theta'$ and $\phi'$ to differentiate between these and the reference system angles. Field strength and phase information for a given set of ray angles is obtained by evaluating a known function, which represents an assumed feed pattern, or by interpolating from measured data. Because the feed antenna points in the negative x' direction, the main beam will be oriented at $\phi' = 180^\circ$ instead of $\phi = 0^\circ$. Hence, the radiation pattern must be shifted in the phi coordinate by $180^\circ$ before it is used in the program.

Once a ray is generated in the feed system, it must be expressed in the reference system. The transformation is performed by a three-dimensional rotation matrix $[A]$. A derivation of the rotation matrix appears in [2].
The rotation matrix is computed from the three angles used to define the rotations required to make the feed system parallel to the reference system. These angles will be called ALPHA, BETA, and GAMMA. Figure 2, taken from Agrawal [3], shows these angles as they relate to the feed coordinate axes. ALPHA is a rotation about the z'-axis. It could be considered the azimuthal aiming angle for the feed antenna. BETA is a rotation about the x'-axis, providing a means for effectively rotating the reflector. GAMMA is a rotation about the y'-axis. This angle is commonly used to define the feed pointing angle for offset reflectors. All angles are defined to be positive if the rotation is counter-clockwise when looking in the negative direction along the axis of rotation.

The reflector is illuminated by a raster scan of rays over the angles it subtends. As computer storage is presently allocated, as many as 2750 rays, defined by their angles theta and phi, may be created for this purpose. This limitation may be removed by increasing the dimensions of the P array (See Appendix D). For reasons explained later, the outermost rays are intended to miss the reflector.

For purposes of illustration, a single ray will be traced through the series of operations resulting in the calculation of a point on the aperture plane. The ray begins as a set of angles θ' and φ' in the feed system. The appropriate θ'- and φ'-polarized electric field strengths and the initial phase are taken from the feed antenna pattern. This constitutes five pieces of information associated with a given ray. A unit vector, \( \mathbf{s}_i \), in the direction of the ray is computed using the familiar relations
FIG. 2. FEED ROTATION ANGLES
\[ s_1 = s_x \hat{x} + s_y \hat{y} + s_z \hat{z} \]

\[ s_x = \sin \theta \cos \phi \]

\[ s_y = \sin \theta \sin \phi \]

\[ s_z = \cos \theta \]

The \( \theta \) and \( \phi \) components of the field are similarly resolved to their rectangular components. Recall that the feed system is translated with respect to the reference system. The reference origin is known as a point in the feed system and this coordinate triplet may be thought of as the translation vector \( \overrightarrow{0'} \overrightarrow{0} \) expressed in the feed system (See Figure 1). The point where the ray strikes the reflector surface is defined by the vector \( \overrightarrow{V} \) as shown in Figure 3. Thus,

\[ \overrightarrow{V} = r \hat{s}_1 - \overrightarrow{0'} \overrightarrow{0} \]

where \( r \) is the distance to the intersection point from the feed origin.

To find \( \overrightarrow{V} \), the vectors \( \hat{s}_1 \) and \( \overrightarrow{0'} \overrightarrow{0} \) must be expressed in the reference coordinate system. The transformation into the reference system is accomplished by forming a 3 by 2 matrix \([BB]\) whose columns are the ray unit vector \( \hat{s}_1 \) and the translation vector, and premultiplying it by the rotation matrix \([A]\). \([A]\) operates on each column of \([BB]\) independently, causing the ray unit vector and the translation vector to be rotated into the reference system.

\[ [A][BB] = [B] = [b_{ij}] \]
FIG. 3. VECTOR OPERATION
The resulting matrix \([B]\) has the transformed ray unit vector as its first column and the transformed translation vector as its second. All points in the path of the ray are now described in the reference system by the parametric equations

\[
\begin{align*}
x &= b_{11}r - b_{12} \\
y &= b_{21}r - b_{22} \\
z &= b_{31}r - b_{32}
\end{align*}
\]

where \(r\) is the distance travelled from the feed origin along the ray. Simultaneous solution of the above parametric equations with the equation of the reflector surface will yield the point of intersection \((x_0, y_0, z_0)\) and the distance from the feed origin \(R\). The unit normal to the surface is evaluated at this point and used to compute a vector in the direction of the reflected ray,

\[
\hat{s}_r = \hat{s}_i - 2(\hat{n} \cdot \hat{s}_i)\hat{n}.
\]

Because the reflector is assumed to be in the far field of the feed antenna, the electric field incident on the reflector surface, denoted \(\hat{E}_i\), is attenuated by a factor \(1/R\). The reflected field, assuming a perfect conductor, is given by

\[
\hat{E}_r = 2(\hat{n} \cdot \hat{E}_i)\hat{n} - \hat{E}_i.
\]

To find the intersection of the reflected ray with the aperture plane, another set of parametric equations is formulated,
\[ x = x_0 + d \cos (\alpha_x) \]
\[ y = y_0 + d \cos (\alpha_y) \]
\[ z = z_0 + d \cos (\alpha_z) \]

where \( d \) is the distance travelled along the ray. The direction cosines are easily computed by

\[ \cos \alpha_x = \frac{(\hat{x} \cdot \hat{r}_r)}{|\hat{r}_r|} \]
\[ \cos \alpha_y = \frac{(\hat{y} \cdot \hat{r}_r)}{|\hat{r}_r|} \]
\[ \cos \alpha_z = \frac{(\hat{z} \cdot \hat{r}_r)}{|\hat{r}_r|} . \]

The parametric equations are solved simultaneously with the equation of the aperture plane \((x = x_c)\) to give the point of intersection \((x_a, y_a, z_a)\) and the distance \( d \). The phase of the field upon reaching the aperture plane is computed using

\[ \phi = \frac{2\pi (R + d)}{\lambda} + \phi_0 \]

where \( \phi_0 \) is the phase of the feed antenna radiation pattern at the angles defining that ray. There are five quantities computed at the point at which the ray intersects the aperture plane: the \( y \) and \( z \) coordinates, the \( y \) and \( z \) components of the electric field, and the phase of the field. A more detailed discussion of the above operations is provided by Kauffman [2].
The field strength radiated by a reflector antenna at a particular point in space is computed by evaluating two integrals over the aperture plane. It is necessary to integrate only those points which result from reflections from the actual surface and not from its mathematical extension. To do this, the reflector boundary must be defined on the aperture plane allowing the extraneous points to be identified. Methods for finding and describing the boundary are discussed later. A series of edge points is interpolated on the boundary using information from points which lie outside of the aperture. The reason for over-illuminating the reflector as mentioned earlier is to insure that there will be points external to the aperture with which to interpolate edge points. The points external to the aperture are then discarded. The next processing step is to quantize the aperture plane points in their y-coordinate. A series of equispaced constant-y lines, called grid bars, is created across the bounded aperture. If the field strength and phase vary slowly across the aperture compared to the grid bar spacing, little error is incurred if the point is simply assigned a new y-coordinate corresponding to the nearest grid bar.

The integrals to be evaluated are:

\[ E_0 = \iint_{\text{Aperture}} ER_z \cos \phi \exp\{j[ky \sin \theta \sin \phi + kz \cos \theta - \phi]\} dydz \]

\[ E_\phi = \iint_{\text{Aperture}} (ER_y \sin \theta + ER_z \cos \theta \sin \phi) \exp\{j[ky \sin \theta \sin \phi + kz \cos \theta - \phi]\} dydz. \]
The development of these integrals may be found in Appendix A. They are evaluated in two steps. First, the z-dependent part of each integral is computed as a line integral along each individual grid bar. The results of the grid bar integrations, weighted by the grid spacing, are then summed to give the total integrals. This procedure is repeated for each pair of angles in the desired pattern. When the pattern is complete, the field value of largest magnitude in each polarization is assigned the value of zero decibels. The remaining field strength values are tabulated and plotted in dB relative to these maximum values. The above operations form the core of both algorithms.

2.2 Review of the Original Program

The latest version of REFLECTR prior to this work was organized as shown in Figure 4. The subroutines in the double-lined boxes will be briefly discussed to describe how the various tasks were distributed in the program and by what means they were accomplished. The main procedure (MAIN) was little more than a calling program to the five principal subroutines shown below it and will receive no further attention.

NPUT was the first routine called by the main program. Its primary function was to read the input data for the reflector under computation. In addition to the basic geometrical data, the old version required the programmer to supply parameters for an aperture plane boundary ellipse; the grid bar location and spacing; and the minimum, maximum, and incremental values of $\theta'$ and $\phi'$ for the illumination pattern. All of these had to be calculated prior to running the program. NPUT passed the illumination pattern specifications to the subroutine called FILL.
FIG. 4. STRUCTURE OF ORIGINAL VERSION
The function of FILL was to load the main working array of the program, hereafter referred to as the P array, with illumination data. FILL had no established form. It was simply a name assigned to any subroutine which would accept minimum, maximum, and incremental values of theta and phi and load the P array with ray data. If some type of information was needed for this purpose, such as a table of field strengths for the feed antenna, it was read in at this time. FILL would load the P array and return to NPUT, which would then compute some trigonometric constants and return control to the main procedure.

The next subroutine called was APERTUR. APERTUR was responsible for tracing the rays to the aperture plane, determining whether or not they resided within the aperture boundary ellipse, and interpolating a set of edge points. The routine began by computing the rotation matrix, then proceeded to trace each ray to the aperture plane. When the location of a point on the aperture plane was found, it was tested for residence by evaluating the aperture plane ellipse equation using its y- and z-coordinates. Aperture blockage was tested similarly. If a point was inside the ellipse, it was stored at the beginning of the P array. If a point was on the edge of the ellipse, it was stored at the end of the array. If a point was found to be outside, it was discarded. When two successive points were on opposite sides of the ellipse, an edge point was interpolated between them and stored with the naturally occurring edge points.

QUANTIZ prepared the aperture plane for integration by quantizing the locations of the aperture plane points in the y-coordinate. The edge points found by APERTUR were used in a spline interpolation
routine to compute a new set of edge points at the grid bar locations. The interior points were quantized separately and concatenated in the array with the new edge points. Finally, the array was sorted; first with respect to y, then with respect to z.

APRPLT made a plot of the locations of the computed points on the aperture plane both before and after quantizing. Different symbols were used for every three dB variation in electric field strength across the aperture. A modified version of APRPLT is used in the revised program.

A feature of the old program was that in the radiation pattern request, one could specify a pattern referenced to the beam maximum. The computer estimated the beam maximum by finding the ray corresponding to $\theta = 90^\circ; \phi = 180^\circ$ in the feed system and stored the angles of its reflection from the reflector surface in the reference system. This feature was not retained in the revised program.

2.3 Numerical Techniques for the Revised Program

In order to accommodate multi-panelled surfaces, several new numerical techniques were added to the algorithm. These additions are now discussed in detail.

Each panel of the reflector is described by a surface code, surface parameters, and a list of edge points. The surface code and parameters allow the computer to select the correct surface equations with which to compute the ray reflections. The edge points, hereafter referred to as perimeter points, are the vertices of a convex polygon which approximates the boundary of the panel in question. The perimeter points are expressed in the reflector cartesian coordinate system and
must be input to the computer in order as one follows the boundary of
the reflector panel either clockwise or counterclockwise. These need
not lie in a single plane as they will be projected onto the aperture
plane in order to estimate the aperture boundary.

The large number of panels possible in a reflector antenna moti-
vated the development of an algorithm to delineate the angular extent
and incrementation of the feed pattern to be used for the illumination
of a given panel. By performing the reverse of the ray-tracing proce-
dure, the perimeter points can be expressed as pairs of angles in the
feed spherical coordinate system. The details of this operation may
be found in Appendix B. With this information, the angles subtended by
the panel may be found and the illumination array created. The angles
defined by each perimeter point are stored in the illumination pattern
with the generated angles and projected to the aperture plane. These
projected perimeter points define the aperture boundary for the particu-
lar panel.

The illumination pattern is composed under three constraints. The
first constraint is that a certain number of rays are to be generated.
This number, NPTPPL, is supplied by the programmer for each panel. It
is the approximate number of rays which he desires to use in illuminat-
ing the particular panel. The algorithm will generate a pattern with
equal theta and phi increments which contains a number of rays as close
to NPTPPL as possible. Second, the algorithm will generate an addi-
tional set of rays one angular increment beyond the angles actually
subtended by the reflector in order to slightly over-illuminate the
panel. These extra rays are reflected from the mathematical extension
of the surface and used to interpolate edge point data later in the program. The third constraint involves quantizing considerations. If, in the feed system, phi is held constant and theta is varied, a constant-phi contour will be traced on the aperture plane. The program attempts to make the grid bars fall on these constant-phi contours. When this is done, the distance the points must be moved during quantizing is minimized. Another consideration concerning the grid bars is that the outermost bars come close to the edge of the reflector. If this is not done, the panel will be represented in quantized form as being narrower than is actually the case. If both of these conditions are to be met, some forethought must be exercised when creating the illumination array. The illumination pattern is constructed such that the outermost phi angles exceed the angular extents of the reflector panel (as determined by the perimeter points) by 4/5 of an angular increment. The angles just within the angles subtended by the reflector are then 1/5 of an angular increment inside the boundary angles. The resulting point distribution on the aperture plane will then exhibit qualities favorable to quantizing. The creation of the illumination pattern is thus a more complex procedure than it was in the old program, but it is almost completely automatic and less haphazard. It should be noted here that this system is hardly infallible and tends toward greater deviation as the rays become less parallel with the x-axis of the reference system; that is, for panels which lie near the edges of large reflectors. In these regions, the constant-phi contours bend into arcs, and a grid bar may cross several contour lines.
The field strength is usually low in these regions, however, and this deviation has not been found to cause problems.

A major difference exists between the old and revised programs regarding the method by which the aperture plane points are prepared for integration, both in the handling of the edge points and in the determination of the aperture plane boundary. No attempt is made to distinguish between interior and exterior points in the APRTUR routine; instead, all of the rays are traced to the aperture plane and the array of points is delivered to the quantizing routine. The quantizing routine computes the position and spacing of the grid bars in accordance with the criteria stated earlier and assigns each point, except for those which are the projections of the perimeter points, a new y-coordinate corresponding to that of the nearest grid bar.

The aperture boundary is convex and closed; thus, each grid bar will intersect it at two points. The intersection points may be considered the endpoints of the grid bar, since any aperture plane point not lying on the resulting line segment does not reside in the aperture. Once the z-coordinate of each endpoint is known, residence of a data point in the aperture may be determined by comparing the z-coordinate of the data point with those of the endpoints for that particular grid bar.

To find the endpoints, the computer works its way around the polygonal boundary, formulating the line equations given by adjacent perimeter points. When the line equation for a particular pair of points has been found, any grid bars lying between the generating points are intersected with the line and are assigned the resulting z-coordinate.
as one of their endpoints. After the entire boundary has been processed, each grid bar will have two endpoints and the extraneous points may be identified. This operation is pictured in Figure 5.

One of the major changes in the program is the procedure of integrating the aperture field panel by panel instead of all at once. This change is numerically justified by the fact that the phase of the fields for various panels is measured in absolute angle from the same phase reference and the integrations are all performed over the same plane. It is justified from a theoretical standpoint in that the linearity of the integrals and Maxwell's equations allow the superposition principle to be applied.

The program has the ability to compute single-panel antennas using an elliptical rather than a polygonal boundary. This has the advantage that a smooth edge will be created and only four perimeter points are required in the input data. The points supplied to the program are those constituting the maximum and minimum y- and z-coordinates; that is, the highest, lowest, leftmost, and rightmost points on the reflector as seen by the feed antenna. These points are projected to the aperture plane in the same way as those used to define a polygonal edge, and an ellipse is formulated using the distance between the high and low z-coordinate as the minor axis and the distance between the high and low y-coordinates as the major axis (See Figure 6). The grid bar endpoints are computed by solving the ellipse equation for z using the y-values of the grid bars. The solution is multi-valued, and the two solutions for each grid bar are its endpoints. This option will cause the edge to be misrepresented in configurations where the illuminating
Fig. 5. Finding the Boundary
FIG. 6. SINGLE-PANEL TECHNIQUE
rays take on extreme values of the angular coordinates, but it does serve as a valuable simplification in the use of the program in many instances.
3. THE STRUCTURE OF REFLECTR

3.1 Overview

A block diagram of the revised version of REFLECTR is shown in Figure 7. The primary subroutines, pictured in the double-lined boxes, will be examined in detail in this section. The main procedure is last since a knowledge of the other primary routines is prerequisite to its discussion. The remaining subroutines function mainly in a support capacity and will be discussed in Section 5.

With the exception of the plotting package (APRPLT and PLTSET), the entire program is written using double-precision constants and functions. IBM double precision employs eight bytes of storage for each real variable and sixteen bytes for each complex variable. This corresponds to fifteen significant figures in real variables and in both the real and imaginary parts of the complex variables. The program has been tried with four-byte variables with a significant degradation in the results. In its present form, REFLECTR requires 258K of memory.

As a means of introducing some of the variables in the program, the eight common storage blocks will be listed and explained:

BLOCKG/YCBL, ZCBL, HFMABL, HFMIBL

(Aperture plane blockage information.)

YCBL, ZCBL -

y and z center coordinates of the aperture plane blockage ellipse.
FIG. 7. STRUCTURE OF REVISED VERSION
HFMABL, HFMIBL -

Half-major and half-minor axes of the aperture plane blockage ellipse (along y and z, respectively).

CONTRL/NOPT(3), NLIST, IOPT, ILIST(100)

(Program control variables.)

NOPT(3) -

Three numbers specifying options regarding printer, plotter, and aperture plane data output, respectively (See Section 4).

NLIST -

The number of panels for which the algorithm will print complete illumination and quantizing data.

IOPT -

A variable which is zero when the program is to run normally and one when the single-panel option is in effect.

ILIST(100) -

The specific panels for which the algorithm is to provide complete illumination and quantizing data (See Section 4).

DIMENS/YDIM, ZDIM, YCT, ZCT

(Dimensional data for the bounded aperture.)

YDIM, ZDIM -

The maximum y- and z-dimensions for the aperture to be plotted by APRPLT or APRMAP.

YCT, ZCT -

The y and z center coordinates of the aperture to be plotted.

EXTENT/YMIN, YMAX, ZMIN, ZMAX

(Dimensional data for the bounded aperture.)

YMIN, YMAX -

The lower and upper limits of the bounded aperture in the y-coordinate.
ZMAX, ZMIN -

The lower and upper limits of the bounded aperture in the z-coordinate.

(This is actually the same information as is contained in the DIMENS block but in a form which is more convenient for operations other than plotting.)

FEED/EP(91), ET(91), NP, NT.

(Feed antenna parameters.)

EP(91), ET(91) -

Arrays containing the electric field strengths of the feed antenna in one degree increments off-axis in the $\theta = 90^\circ$ and $\phi = 180^\circ$ planes, respectively.

NP, NT -

The number of increments of phi and theta used in the illumination pattern, respectively.

(The contents of this block will change depending on how the feed antenna is described. The above block was used in the first test case for which the feed pattern was interpolated from E- and H-plane tables.)

MATH/PI, PI2, PID2, DTOR, RTOD

(Mathematical constants.)

PI, PI2, PID2 -

Pi, two times pi, and pi divided by two.

DTOR, RTOD -

Constants for degree-to-radian conversion and radian-to-degree conversion.

PARAMS/AORORF, BELLP, PSI, ?LPNPNT(3), PLNORM(3), FEED(3), ALPHA, BETA, GAMMA, XLAM, SURFCE, NPNL, NPOINT

(Antenna system parameters.)
AORORF -

The focal length of a paraboloidal reflector, the focal length of a parabolic cylindrical reflector, the radius of a spherical reflector, or the semi-major axis of an ellipsoidal reflector (along x), depending on which surface is intended to represent the reflector.

BELLP -

The semi-minor axes (along y and z) of an ellipsoidal reflector surface. Note that this does not define a completely arbitrary ellipsoid since the axes along y and z must be equal.

PSI -

The angle of rotation of a parabolic cylinder whose focal line is in the yz plane as measured from the positive y-axis when looking in the negative-x direction in the reflector coordinate system (See Appendix C).

PLNPNT(3) -

The coordinates of a point on a planar reflector surface (x, y, z).

PLNORM(3) -

The components of a unit normal vector to a planar reflector surface (x, y, z).

FEED(3) -

The reference coordinate system origin as expressed in the feed coordinate system (x', y', z').

ALPHA, BETA, GAMMA -

The angles of rotation between the two coordinate systems as described in Section 2.1.

XLAM -

The wavelength of the radiation emitted by the feed antenna.

SURFCE -

An integer variable containing the value of the surface code.  
1 - surface is a plane.  
2 - surface is an ellipsoid.  
3 - surface is a sphere.
4 - surface is a paraboloid.
5 - surface is a parabolic cylinder.

NPNL -

The number of panels comprising the reflector. If the value supplied to the computer is one or greater, the program expects a list of perimeter points and other surface parameters for each panel and the aperture boundary is approximated by a polygon. Entering the value of zero causes the program to use an ellipse to represent the boundary—the single-panel option.

NPOINT -

The number of points or rays which are being stored for processing in the P array at any given time.

PATTN/ ETOT(2,400), AMINOR (3,5), AMAJOR(5), MINOR(5), MAJOR(5), NANGLE(5)

(Information concerning the overall radiation pattern.)

ETOT(2,400) -

A complex array which contains the summation of the aperture plane integration results from the individual reflector panels. The first subscript is one or two, denoting the Θ- or φ-directed electric field, respectively. The pattern cuts are stacked end to end in the array. Presently, 400 angles may be stored.

AMINOR(3,5) -

A pattern request array. The first subscript is one, two, or three, denoting the lower, upper, or incremental values of the minor angle of the pattern. The second subscript denotes the particular request.

AMAJOR(5) -

A pattern request array which contains the angle defining the principle plane of the pattern cut. The subscript denotes the particular request.

MINOR(5) -

A pattern request array containing the angular coordinate (theta or phi) to which AMINOR applies. The subscript denotes the particular request.
MAJOR(5) -

A pattern request array containing the angular coordinate (theta or phi) of the principle plane of the pattern cut. The subscript denotes the particular request.

NANGLE(5) -

Contains the total number of angles in each requested pattern cut as determined by AMINOR. The subscript denotes the particular request.

The remaining variables will be introduced as the subroutines are examined. The reader is referred to Appendix D for a listing of the code while reading this section.

3.2 NPUT

NPUT is primarily an input/output routine. As it begins, IOPT, the single-panel option variable, is set to zero, indicating that the program is to run in the multi-panelled reflector option. The first four cards are read into the TITLE array as character data. NPUT then reads values for FEED, ALPHA, BETA, GAMMA, XLAM, SURFCE, NPNL, AORORF, BELLP, PSI, PLNPNT, PLNORM, XC, YCBL, ZCBL, HFMA, AND HFMI. These numbers are all read regardless of whether or not they are applicable to the reflector under study. If a value is not needed, it is simply not used. Following this, any data required for the feed antenna pattern is read by appropriate statements. This section of NPUT is often changed to accommodate differences in the information required by a particular FILL routine. Any data read here is passed to FILL through the FEED common block. If NPNL was entered as zero, the four extreme points on the reflector are read into the P array. This is followed by the output option data.
Statements 77 to 98 are the pattern request input loop. AMINOR, AMAJOR, MINOR, and MAJOR are read for consecutive requests during which the total number of pattern cuts and angles are monitored. If the number of cuts is greater than five, if the total number of angles involved is greater than 400, or if any one pattern contains more than 75 angles, an error message is printed and the program terminates. These maximum numbers are storage constraints which may be changed, if desired, and if sufficient memory is available in the computer. The number of pattern cuts is limited by the dimensioning of AMINOR, AMAJOR, MAJOR, MINOR, and NANGLE. The total number of angles in all patterns is limited by the dimensioning of ETOT, FIELDY, and FIELDZ. The number of angles in any given pattern is restricted by the dimensions of the arrays YFLD, ZFLD, and PWER in the main procedure.

The DO-loop to statement 108 clears as many locations in the ETOT array as are required to store all of the angles in the patterns. Following this step, the information received by the computer is printed and the mathematical constants in the MATH common block are calculated.

3.3 APRTUR and APRIN

In the first call to APRTUR, the rotation matrix is computed from the rotation angles ALPHA, BETA, and GAMMA. Because it is a rotation matrix, \([A]\) belongs to the class of Orthogonal Matrices. An Orthogonal matrix is non-singular and its inverse is equal to its transpose. AINV is thus obtained simply. The NPTPPL variable is set to 2250 to signify that in the case of the single-panel option, 2250 rays are to be generated to illuminate the panel. NPERIM is set to four in order to accommodate the four extreme points which are entered when the single-panel
option is used. If the program is to run normally; that is, using a polygonal aperture boundary, the APRIN routine will be called and NPERIM and NPTPPL will be updated to the appropriate values. In subsequent calls to APRTUR, the above steps are skipped.

If the single-panel option is not in effect, the APRIN subroutine is called to read data for the reflector panel. APRIN reads NPERIM, SURFCE, and NPTPPL on the first card of the panel data and screens these values for easily detected errors. If any are found, an error message is printed and the program stops execution. The perimeter points are then read into the P array with the x-, y-, and z-coordinates occupying the first, second, and third locations, respectively. These are followed by any other geometrical information needed for that surface. For example, were the panel an elliptical surface, values would be expected for AORORF and BELLP. If the output control variables dictate that data is to be printed for this panel, APRIN will do so. If not, it returns to APRTUR. After a call to APRIN, the values of SURFCE and whatever other parameters are required for the panel have been updated. For example, if the overall reflector was not an ellipsoid, an ellipsoidal panel would cause AORORF, BELLP, and SURFCE to be changed.

The statements in APRTUR between 50 and 65 are concerned with finding the angles subtended by the reflector or the reflector panel. The perimeter points are expressed as angles in the feed system and monitored to record the maximum and minimum θ' and ϕ' angles represented by the set of points. This procedure is applied to both the perimeter points and the four extreme points in the single-panel
I. option. Appendix B details the geometrical formulation for this operation.

The statements between 65 and 95 create the illumination array from the information found above. DELP and DELT are the angular dimensions of the panel in phi and theta, respectively. It is required that

\[ NP \times NT = NPTPPL. \]

An estimate of an appropriate NP is found by

\[ NP \times [NP \times (DELT/DELP)] = NPTPPL \]

\[ NP = [(DELP/DELT) \times NPTPPL]^{0.5} + 1. \]

One is added to round NP to the next larger integer. The statement following this insures that the illumination pattern will have a set of angles generated at the phi angular center of the reflector by forcing NP to be odd.

If the reflector subtends DELP radians, it is desirable to have

\[ ANGINC \times (NP - 1) = DELP + (0.8 \times ANGINC)^2. \]

(See Section 2.3 concerning the illumination pattern.) Simplifying the above, the angular increment is found to be

\[ ANGINC = DELP/(NP - 2.6). \]

NT is computed such that a set of rays exist at the 0° angular center of the reflector and the rays corresponding to the upper and lower limits of 0° miss the reflector. The resulting maxima, minima, and the
total number of angles generated are computed and, if desired, printed. The DO-loop to statement 95 loads the complete set of illumination angles into the P array just after the angle pairs corresponding to the perimeter points. The FILL routine is then called to supply the angle pairs in the P array with field strength and phase information.

3.4 FILL

Many FILL subroutines have been written for REFLECTR. One of the most useful is described in this section. It may be used for any antenna for which the E- and H-plane patterns are available as long as these patterns are symmetrical with respect to the negative $x'$-axis and may be adequately represented when sampled at one-degree intervals. The subroutine must be provided with two tables of relative field strengths in one-degree increments—one in the $\theta' = 90^\circ$ plane and one in the $\phi' = 180^\circ$ plane.

Consider the feed coordinate system and a typical ray as shown in Figure 8. The ray and the $x'$-axis define a plane. Let $\delta$ be the angle formed between the ray and the negative $x'$-axis in this plane. $\delta$ is given by

$$\delta = \arccos(\sin \theta' \cos \phi').$$

Because it is the angle between the ray and the axis of radiation symmetry, $\delta$ determines the values for the electric field to be taken from both the $\theta' = 90^\circ$ and $\phi' = 180^\circ$ patterns. Interpolation from the field strength tables is linear. Let $\epsilon$ be the angle between the $x'y'$ plane and the plane containing the ray and the $x'$-axis. Measured from the fourth quadrant of the $x'y'$ plane, $\epsilon$ is given by
FIG. 8. DEFINITION OF ANGLES FOR FILL

FIG. 9. INTERPOLATION ELLIPSE
\[ \theta = \arctan \left( \frac{\cos \theta \cdot \sin \phi}{\sin \theta \cdot \sin \phi} \right). \]

This angle determines the proportion of each pattern used in computing the final electric field strength for the ray. The interpolation scheme is based on an ellipse as shown in Figure 9. The half-major and half-minor axes are the \( \theta = 90^\circ \) plane and \( \phi = 180^\circ \) plane field strengths, respectively. The ellipse equation is written in polar form to express the radius in terms of the angle \( \theta \).

\[
\left( \frac{u}{E_{\theta=90}} \right)^2 + \left( \frac{v}{E_{\phi=180}} \right)^2 = 1
\]

\[ u = r \cos \theta \]
\[ v = r \sin \theta \]

\[ r = \frac{E_{\theta=90} \cdot E_{\phi=180}}{E_{\theta=90} \cdot \sin^2 \theta + E_{\phi=180} \cdot \cos^2 \theta} \]

The code for this subroutine may be found in Appendix C. PROJX is the direction cosine of the ray with the negative \( x' \)-axis; that is, the cosine of the angle \( \delta \). PROJY, the cosine of the angle \( \epsilon \), is set to zero. \( \epsilon \) is undefined when \( \delta = 0 \); thus, it is not computed in this case. ANGLX is the measure of \( \delta \) in degrees. LO and IHI are the indices of the array locations for the integral angles just below and just above \( \delta \), respectively. These are used to linearly interpolate the electric field in each principal plane. The elliptical interpolation is then carried out and the result is loaded into the proper location.
P array. The subroutine is presently written so that the antenna is \( \phi^- \)-polarized and the initial phase is zero.

### 3.5 QUANTZ

It is assumed that the rays representing the extreme values of \( \phi^- \) miss the reflector; hence, there are \( NP-2 \) constant-\( \phi^- \) contours on the reflector. \( NBARS \), the number of grid bars, is set to this value. An appropriate number of locations in the array of grid bar endpoints, \( Z \), are set to \( 10^{20} \). This number is used as a flag later in the program to determine whether the particular location has received an endpoint. After initializing the \( Z \) array, a loop finds the maximum and minimum \( y^- \) and \( z^- \)-coordinates represented in all of the perimeter point projections and writes these points into the \( PR5 \) array. This array must be 5 by \( n \) where \( n \) is one greater than the number of perimeter points expected for any particular panel. The array is presently 5 by 41. The information found in this loop is used to compute plotting parameters (\( YCT \), \( ZCT \), \( YDIM \), and \( ZDIM \)) and the grid bar locations. Note that the grid bars are placed in such a way that the outermost grid bars are \( 1/5 \) of a grid spacing inside the aperture. Lines 31 through 41 constitute the quantizing loop. A point is first examined to determine whether the \( y^- \)-coordinate lies within the established \( y^- \)-range of the aperture. If it does, it is assigned the \( y^- \)-coordinate of the nearest grid bar. If not, it is discarded. \( NPOINT \) is adjusted to account for the points which are deleted in this operation. The remaining points are sorted; first with respect to \( y^- \), then with respect to \( z^- \). The sorting process may be thought of as arranging the grid bars end to end in the array.
The next processing step is to compute the grid bar endpoints. The method using perimeter points is described first. As stated before, the basic strategy is to formulate line equations with adjacent perimeter points and intersect these with any grid bars which might lie between them. The variables involved are defined below:

**KDEX**

- The index of the perimeter point currently under consideration.

**Y1, Z1**

- The y- and z-coordinates of the first point used to formulate the line equation.

**Y2, Z2**

- The y- and z-coordinates of the second point used to formulate the line equation.

**SLOPE**

- The slope of the line.

**B**

- The z-intercept of the line.

**YLO, YHI**

- The lesser and greater values of the y-coordinate of the two perimeter points under consideration.

**Z(2,J)**

- The array of grid bar endpoints. Z(1,J) and Z(2,J) are the lower and upper endpoints of the Jth grid bar, respectively.

**INDEX**

- The index of the location in the Z array in which the computed endpoint is stored.
YQ -

The y-coordinate of the grid bar which is to be intersected with the line.

ZEE -

The z-coordinate of the computed endpoint.

ILOAD -

A loading index controlling which of two possible locations in the Z array will receive the computed endpoint once the second subscript is fixed.

A flowchart of the process used to load the Z array is shown in Figure 10. To begin, the first point in the PR5 array is copied into the location following the last perimeter point. KDEX is set to 2 to start the loop at the second point in the array. Y2 and Z2 are assigned the y- and z-coordinates of the first point. Y1 and Z1 are taken from the second point in the array and the difference between Y1 and Y2 is checked to insure that the line will not have infinite slope. Since the boundary is assumed to be convex, a vertical line must have one of the two outermost y-coordinates of the reflector. The grid bars were placed inside of these extremes; thus, a vertical line will intersect no grid bars. If Y1 and Y2 differ by too small an amount, the algorithm rejects the line and goes to the next point pair. Otherwise, the slope and z intercept are computed and YLO and YHI are determined. The first grid bar beyond YLO is the first candidate for intersection with the line. The y-coordinate and its index in the Z array are computed. If the grid bar is beyond YHI, the line is abandoned and the next line is formulated. If it is within the range of the line, the intersection is found and stored in the Z array. The index and y-coordinate are incremented by 1 and GRID, respectively, until YHI has
COPY FIRST POINT AT END OF ARRAY

KDEX = 2

Y2 = PR5(1,1)
Z2 = PR5(2,1)

Y1 = PR5(1, KDEX)
Z1 = PR5(2, KDEX)

? Y1 - Y2 ~ 0

yes

no

COMPUTE: SLOPE, B

Y1 - Y2

< 0

YLO = Y1

YHI = Y2

> 0

YLO = Y2

YHI = Y1

INDEX = (YLO - GRIDLO)/GRID + 0.5

INDEX = INDEX + 1

compute YQ

? YQ > YHI

yes

no

ILOAD = 1
ZEE = m(YQ) + B

Z(ILOAD, INDEX) = ZEE

? Z(I, INDEX) < 10^0

no

yes

ILOAD = 2

Z(ILOAD, INDEX) = ZEE

EXIT

FIG. 10. FLOWCHART FOR LOADING Z ARRAY
been exceeded. The next line is then formulated and the process continues until the boundary has been closed.

After the Z array is loaded, a short loop following the loading procedure insures that Z(1,J) is less than Z(2,J) and that all locations of the array have been loaded with intersection coordinates.

If the single-panel option is in effect, the endpoints are found by solving the boundary ellipse equation at the grid bar locations. HFMAEX and HFMIE are the half-major and half-minor axes of this ellipse, respectively.

The next section of QUANTZ imposes the boundary on the aperture. The approach taken is to step through the data points in each grid bar in ascending order, discarding those which are outside the aperture and retaining those which are inside or on the edge. When a grid bar endpoint is crossed, a new data point is interpolated with that Z-coordinate. Any point found to reside in the aperture plane blockage ellipse is assigned zero field strength and phase. Upon completion of a grid bar, the number of retained and interpolated points is checked and the grid bar is deleted if this number is less than three. This is a requirement of the integration routine. A flowchart for this section of QUANTZ appears in Figure 11. The variables are defined below in order of their appearance.

N -

One greater than the total number of points retained in the aperture (including blocked points) at any given time.

L -

The total of points retained on the present grid bar (including blocked points) at any given time during the processing of that grid bar.
L=0 N=0
PBLK=0
YQ=P(1,1)

COMPUTE: IDEX

DO 900 I=1,NPOINT

? P(1,I)=YQ

? L>2

delete line:
N=N-L

L=0
YQ=P(1,I)
COMPUTE: IDEX

PBLK(I)=P(1,I)
PBLK(2)=P(2,I)
COMPUTE:
TEST
TESTBL

TEST
<0

TESTBL

<0

not blocked:
LOAD_PBLK

N=N+1
L=L+1

TEST

=0

(edge)

L

=0

(first point)

TEST*

TESTO

>0

(intermediate)

INTERPOLATE

EDGE POINT (PINT)

NCHG = 0

FIG. 11a. PARTIAL FLOWCHART FOR QUANTZ
Move last point up one location in the P array.

$NCHG = 1$

$PBLK(1) = PINT(1)$
$PBLK(2) = PINT(2)$

COMPUTE $TESTBL$ for $PINT$

$TESTBL$

LOAD:
$P(I, N-NCHG)$ with $PBLK(J)$

LOAD:
$P(I, N-NCHG)$ with $PINT(J)$

$N = N + 1$
$L = L + 1$

FIG. IIb. PARTIAL FLOWCHART FOR QUANTZ
PBLK -

An array representing a blocked point. The field and phase values are zero and the first two locations contain the y- and z-coordinates of the point under consideration.

YQ -

The y-coordinate of the present grid bar.

IDEX -

The second subscript of the location of the Z array which contains the endpoints of the grid bar at YQ.

TEST -

A variable which is negative, zero, or positive if the data point is outside, on, or between the grid bar endpoints, respectively.

TESTBL -

A variable which is negative, zero, or positive if the data point is outside, on, or inside the blockage ellipse, respectively.

TESTO -

The value of TEST for the previous data point.

POLD -

The previous data point.

NCHG -

A variable controlling where a new point is loaded into the P array. NCHG = 0 means that the new point is to be loaded normally, while NCHG = 1 means that the point is to be loaded one location before the location indicated by N.

The only complication in the process occurs when the algorithm encounters an interior point after an exterior point. A point is interpolated on the boundary after the interior point has already been stored in the array. The interior point should follow the interpolated
point in the array because of its higher z-coordinate; thus, the points
would be out of order if the interpolated point were loaded normally.
When this situation arises, the interior point is moved up one space in
the array and the interpolated point is loaded into the gap by subtract-
ing NCHG from the loading index N. If the interpolation is a result of
moving from an interior point to an exterior point, NCHG is zero and
the interpolated point is loaded in normal order.

The end of QUANTZ sets NPOINT to the proper value and returns the
perimeter points to the P array, storing them after the aperture points.
Quantizing data is then printed and the P array is written to a disk
file (excluding the perimeter points at the end) if the output control
code indicates.

3.6 INTGRT

INTGRT is essentially the section of the old RADPAT subroutine
which computed the radiation pattern integrals. The sections which
performed other tasks have been deleted or moved to other parts of the
program. INTGRT accepts the quantized P array and the pattern request
from the main procedure and returns the integrated results for each
angle of the pattern in the FIELDY and FIELDZ arrays in complex form.
The integrals evaluated by INTGRT are:

\[ E_0 = \iint ER_z \cos \theta \exp \left\{ j[ky \sin \theta \sin \phi + kz \cos \theta - \phi]\right\} \, dydz \]
Aperture

\[ E_\phi = \iint (ER_y \sin \theta + ER_z \cos \theta \sin \phi) \exp \left\{ j[ky \sin \theta \sin \phi + kz \cos \theta - \phi]\right\} \, dydz \]
Aperture
where \( E_{Ry}, E_{Rz}, \) and \( \phi \) are the \( y \)- and \( z \)-components and phase of the electric field at the point \((y,z)\) in the aperture. The derivation of these integrals appears in Appendix D. Each of these integrals is divided into a set of integrations along each grid bar and a transverse integration of the results from the grid bars. The grid bar integrals are:

\[
ZI = \int ER_z \cos \phi \exp[j(kz \cos \theta - \phi)] \, dz
\quad y = \text{const.}
\]

\[
YI = \int (ER_y \sin \theta + ER_z \cos \theta \sin \phi) \exp[j(kz \cos \theta - \phi)] \, dz.
\quad y = \text{const.}
\]

The total integrals are the transverse integrals.

\[
E_\theta = \text{FLDZ} = \int ZI \exp[j(ky \sin \theta \sin \phi)] \, dy
\quad \text{All bars}
\]

\[
E_\phi = \text{FLDY} = \int YI \exp[j(ky \sin \theta \sin \phi)] \, dy
\quad \text{All bars}
\]

Integration is by the trapezoidal rule with the accuracy being enhanced by linearly interpolating a series of equidistant intermediate points between the existing data points. The number of subdivisions, called NPARTS, is presently seven.

A partial flowchart for INTGRT is shown in Figure 12. The variable SEN signifies the beginning and end of the P array. The first time it is encountered indicates that the first grid bar has just been integrated. The second time indicates that the last grid bar has been integrated. The subroutine begins by determining the first \( \theta \) and \( \phi \).
FIG. 12. PARTIAL FLOWCHART FOR INTGRT
angles in the pattern and computes their sines and cosines as required in the integrals. Although the integrals are evaluated simultaneously, only the \( E_\theta \) integral will be discussed for the sake of clarity. The first two grid bars are integrated and these results form the first trapezoid of the transverse integral. This trapezoid is evaluated and stored in FLDZ. The next grid bar closes the second trapezoid, which is evaluated and summed into FLDZ. Each successive grid bar integral closes another trapezoid for the transverse integral such that the final grid bar closes the last trapezoid and FLDZ contains the result of the aperture integration for the theta polarization of the field. FLDZ is stored in FIELDZ(1) and the computation continues with the next set of angles. When the integrals have been evaluated for all angles in the pattern, the subroutine returns to the main procedure.

### 3.7 Main Procedure

The main procedure orchestrates the principal subroutines described earlier in the section to make a radiation pattern calculation. The variables introduced are defined below:

**ETOT(2,400)** -

A summing array for the complex values returned by INTGRT. FIELDY and FIELDZ are summed into ETOT(1,:) and ETOT(2,:), respectively.

**YFLD(75), ZFLD(75)** -

Arrays containing the magnitudes of the unnormalized \( \phi \) and \( \theta \)-polarized electric field strengths, respectively, in a final antenna pattern.
POWER(T5) -

An array containing the unnormalized power for angles in a final antenna pattern.

PR(2,500) -

An array containing the perimeter points of all panels of the reflector for use by the APRMAP subroutine. The y- and z-coordinates are denoted by the first subscript being one or two, respectively. Panels are punctuated in the array by a very large number in the y-coordinate location.

MLVL -

The stack pointer for the PR array. It denotes the next location pair [PR(1,•), PR(2,•)] to receive a perimeter point.

YLO, YHI -

The lowest and highest y-coordinate possessed by any panel yet processed.

ZLO, ZHI -

The lowest and highest z-coordinate possessed by any panel yet processed.

ISUM -

The index of the last pair of locations in the ETOT array to receive or supply pattern data.

NANG -

The number of angles in the present pattern.

FMAXY, FMAXZ -

The maximum φ- and ω-polarized electric field magnitudes, respectively, in a particular pattern.

PWR -

The total power corresponding to FMAXY and FMAXZ.

D -

The current value of the incremented angle in a particular pattern.
FMYDB, FMZDB, PWRMDB -

The decibel values of FMAXY, FMAXZ, and PWR, respectively.

DBZZ, DBZY -

The relative power in dB of the $\theta$-polarized electric field with respect to FMZDB and FMYDB, respectively.

DBYZ, DBYY -

The relative power in dB of the $\phi$-polarized electric field with respect to FMZDB and FMYDB, respectively.

The procedure begins by defining the decibel function and calling NPUT to read the reflector antenna parameters and computational requests. A loop is then begun which is performed once for each panel. APRTUR is called to create the illumination array and trace the rays to the aperture plane; then QUANTZ is called to prepare the aperture for integration. APRPLT is called if the output control code dictates.

The dimensions of the aperture are checked with the current maximum and minimum values (YLO, YHI, ZLO, and ZHI), after which the perimeter points are stored in the PR array in locations indicated by the stack pointer MLVL. MLVL is updated and punctuation for that panel is stored after its perimeter points. The punctuation is used by APRMAP to distinguish between separate panels.

Patterns are stacked in the ETOT array as shown in Figure 13. INTGRT is called once for each pattern request. Each time a pattern is returned by INTGRT, FIELDY and FIELDZ are summed into the portion of the stack containing that pattern. ISUM is the pointer indicating the
FIG. 13. THE ETOT ARRAY
last pair of locations in ETOT to receive values. It is updated after every pattern and reset to zero after every panel.

After these steps have been performed, APRTUR is called to begin computation of the next panel. The process is repeated until all panels are finished, after which an APRMAP plot is generated, if specified by the output option code.

The rest of the main procedure is concerned with printing tables of electric field strengths for each pattern request. The first pattern is taken from the ETOT array and the magnitudes of the $\phi$- and $\theta$-polarized fields are placed in the YFLD and ZFLD arrays, respectively. The maximum and minimum of each of these arrays is found and assigned to FMAXY and FMAXZ, respectively. If these values are greater than $10^{-10}$, they are converted to dB and the pattern from which they are taken is computed, tabulated, and plotted. If one or both of them are less than $10^{-10}$, the entire pattern is set to $-60$ dB. This is an indication to the programmer that the magnitude of that polarization of the field is insignificantly small or that some problem exists in the calculation. Once the pattern maximum has been deemed large enough to output the pattern, the lowest value converted to dB for an individual angle is $10^{-15}$. Any value smaller than this is assigned the value of $-100$ dB. This provides 100 dB of dynamic range for the weakest pattern qualifying for computation. The above precautions are exercised to prevent problems with the logarithm function in the computer.

The relative field values (DBZZ, DBZY, DRYZ, and DBYY) are computed and the table is printed one line at a time. PLOT4 is
called to make a printer plot of the table entries. The above steps are repeated until all requested patterns have been printed.
4. OUTPUT CONTROL AND PROGRAM OPTIONS

4.1 Introduction

In order to better monitor the calculation, a large quantity of output is available from the program. Selection of specific output is accomplished by means of a three-digit output option code which is supplied to the program along with the antenna parameters in the input file. This section describes the printing, plotting, and permanent storage abilities of the program and how the code may be used to exercise them.

The output option specification requires between one and eight cards, depending on which values are supplied for the output option code. These cards appear just before the pattern request cards in the input file. The option code, known to the computer as NOPT(3), is punched in the first three columns of the first card. Another field on the first card is for the variable NLIST. The remaining seven cards, if required, are for the dimensioned variable ILIST(100). These variables will be explained later in the section.

4.2 Printing Options

An example of the printed output from REFLECTR may be found in Appendix E. The portion of the output prior to the illumination data is not under control of the option code. The panel illumination and quantizing data (also in Appendix E) are under the control of the first digit of the code. If it is desired to print panel data for all panels, NOPT(1) is specified as two. It may be desirable to print this information only for several specific panels. In this case, NOPT(1) is
specified as one and the total number of panels for which data is requested is entered for the NLIST parameter. NLIST is right-justified in column 10 of the first card. If NOPT(1) is one, the program will look for the numbers of NLIST specific panels on the following cards according to a 16I5 format. Panels are numbered by the program in the order in which they are received. If it were required to print panel data for panels 6, 12, and 19, for example, NLIST would be given the value 3 and the first three I5 fields on the next card would contain the numbers 6, 12, and 19. These need not be supplied in any specific order and no more option cards would be required. If no panel data is to be printed, NOPT(1) is specified as zero. Printed output following the panel quantizing data; that is, the computed radiation pattern of the antenna, is not under control of the code.

4.3 Plotting Options

Two types of plotted output are available from REFLECTR: a plot of data points in the aperture plane (APRPLT) and a plot of the panel aperture boundaries on the aperture plane (APRMAP). APRPLT plots the aperture data points for a single panel (an example appears in Appendix E). APRMAP, which also appears in Appendix E, plots the boundaries of all of the apertures in one plot. Plotting is controlled by the second digit of the output option code, NOPT(2). If all of the panel apertures are to be plotted, NOPT(2) is specified as two. If only specific panel apertures are to be plotted, NOPT(2) is specified as one. In this case, the procedure followed is identical to that for the printing option. If both the printing and plotting options are specified as one, the panels to be printed and plotted must be the same. That is, only one
set of cards for ILIST follow the option code card; thus, there is no
provision for printing and plotting different sets of panels. APRMAP
is called automatically if any panels are plotted. If no plotting is
to occur, NOPT(2) is specified as zero. Note that NLIST and ILIST only
need to be provided if NOPT(1) or NOPT(2) equals one.

4.4 Disk Storage Options

REFLECTR has the ability to write the post-QUANTZ P array to a
disk file for later examination or to be retrieved at the end of the
program. This option is controlled by the third digit of the code,
NOPT(3). If NOPT(3) is specified as two, the P array will be stored for
all panels, then retrieved and printed at the end of the program. If
NOPT(3) is one, the P array will be stored for all panels. Finally,
specifying NOPT(3) as zero will cause the program not to store any
aperture data. The data may be examined at a later time by submitting
a job which calls the PTLIST subroutine described in Section 5.4.
5. THE SUPPORT CODE

5.1 Introduction

The support subroutines are subroutines which do not participate directly in the radiation pattern calculation. Instead, they aid the calculation in some way--either by preparing output or by performing simple data processing tasks. These subroutines do not appear in Appendix C with the principal subroutines. Instead, a listing of each is provided along with its description in this section.

The plotting package does not appear in this section. Plotting subroutines tend to be extremely installation-dependent and it was felt that a plotting package using local software would not be useful in general application. Agrawal's version of APRPLT [3] makes a good example on which to base a new version. There is enough information in Section 3.7 and the test case plots in Appendix E with which to synthesize a new version of APRMAP.

5.2 PLOT4

PLOT4 is the printer plot subroutine which produces the output shown in Figure 14. It plots the table of relative field strengths comprising the radiation pattern over a 60 dB dynamic range with one angular increment to a line and one dB to a print column. If a value to be plotted is more than 60 dB below the maximum value, a "<" character is printed at -60 dB. The angular axis is labelled every five angular increments and the lower plot border is always extended to the next even multiple of five increments. A listing of the subroutine appears in Table 1.
FIG. 14. PLOT4 OUTPUT
TABLE I. PLOT4

SUBROUTINE PLOT4(NAME,FMAX,F,NT,MINOR,AMINOR)
IMPLICIT REAL*8 (A-H,0-Z)
REAL*8 NAME(8),MINOR
DIMENSION LINE(71)
DIMENSION F(NT),AMINOR(3)
DATA IBLANK,LESS,IDOT,IDASH,I/1H ,I/1H,1H,-,1H/,
PRINT 10,NAME,MINOR
10 FORMAT(///25X,8A8,///7X,A5,DEG) -60',7X,'-50',7X,'-40',7X,
'-30',7X,'-20',7X,'-10',7X,'0',7X,'+10',7X,
:9X,9X,**,9X,**,9X,**,9X,**,9X,**,9X,**,9X,**,
:DO 20 J=1,70
20 LINE(J)=IDASH
DO 22 K=1,70,5
22 LINE(K)
DO 29 N=1,70
29 LINE(N)=ISLANK
TEST=DFLOAT(J-1)/5.0
IF (TEST-DINT(TEST)).EQ.0.0) LINE(2)=IDASH
IF (LINE(2).EQ.IDASH) LINE(70)=IDASH
41 PLACE=F(J)-FMAX+60.0
IF (PLACE) 90,80,70
70 LINE(1)=I
K=PLACE+1.5
LINE(K)=IDOT
GO TO 99
80 LINE(1)=IDOT
GO TO 98
90 LINE(1)=LESS
98 LINE(2)=IBLANK
99 PRINT 105,L,ANG,LINE
105 FORMAT(20X,71A1)
170 CONTINUE
DO 515 L=2,70
515 LINE(L)=IBLANK
LINE(1)=I
II=NT+1
JJ=NT+6
DO 520 L=II,JJ
TEST=DFLOAT(L-1)/5.0
IF (TEST-DINT(TEST)).EQ.0.0) GO TO 524
520 PRINT 105,L,ANG,LINE
524 DO 525 M=1,70
525 LINE(M)=IDASH
DO 545 N=1,71,5
545 LINE(N)=I
PRINT 105,LINE
RETURN
END
The lines of the plot are composed one at a time in the character array \texttt{LINE(71)}. The array begins and ends with the character "I". On every fifth multiple of the angular increment, \texttt{LINE(2)} and \texttt{LINE(70)} are loaded with the "-" character and the value of the angle is printed beside the line. The dot is placed by using the value of the relative field strength to compute the index of \texttt{LINE} which will locate it properly with respect to the dB axis. The upper and lower plot borders are generated by loading all 71 locations with dashes, then writing an "I" over every fifth dash.

The calling arguments are \texttt{NAME(.), FMAX, F(-), NT, MINOR, and AMINOR(.).} \texttt{NAME(.)} is a 1-by-8 array of eight-byte variables containing the title of the plot. The title must be 64 characters long or padded with blanks to make 64 characters. \texttt{FMAX} is the maximum unnormalized field strength magnitude for the entire pattern in dB. \texttt{F(-)} is an array containing the unnormalized magnitudes of the pattern field strengths in dB. \texttt{NT} is the number of angles in the pattern. Finally, \texttt{MINOR} is the name of the angle to be varied in the pattern, and \texttt{AMINOR(i)} is for the low, high, and incremental values of this angle for \(i\) equal to one, two, and three, respectively.

5.3 IO

\texttt{IO} is the output control function subroutine. A referral to the output control variables is made by supplying \texttt{IO} with an integer code for the particular output operation and the number of the panel currently under computation. \texttt{IO} returns a 1 if the operation is permitted and a 0 if it is not.
A listing of IO is presented in Table 2. INTENT is the output operation code variable. The codes are:

1. Print panel illumination and quantizing data.
2. Plot the P array after quantizing (APRPLT).
3. Create a map of panel projections with the plotter (APRMAP).
4. Write the P array to a disk file after quantizing.
5. Read the P array from the disk file and print it at the end of the program.

ITER, or iteration, is the number of the present panel.

5.4 Utility Routines

Utility routines are those subroutines which perform standard data processing tasks. Included in this section are SETM, MOVEM, INTPL, MULT32, PTSORT, and PTLIST. All of these routines are relatively simple; thus, only a brief description of each will be provided.

SETM is listed in Table 3. It loads N locations of a singly-dimensioned array (A) with the constant X. Because of the way arrays are indexed in the computer, SETM may also be used to load X into rows of doubly-dimensioned arrays—such as the P array. For example, the Nth row of the P array could be loaded with X using the statement

\[
\text{CALL SETM (X,P(1,N),5).}
\]

SETM is used primarily in this capacity.

MOVEM appears in Table 4. Its function is to copy N values from one singly-dimensioned array (AOLD) into another array of the same type (ANEW). Like SETM, MOVEM may also be used to operate on rows of doubly-dimensioned arrays. The main use of MOVEM in this program is to
TABLE 2. 10

```fortran
FUNCTION IO(INTENT, ITER)
IMPLICIT REAL*8 (A-H,O-Z)
INTEGER SURFACE
COMMON/CTRL/NOPT(3), MLIST, IOPT, ILIST(100)
GO TO (20, 30, 40, 50, 60), INTENT
20 IF (NOPT(1).EQ.0) GO TO 90
   IF (NOPT(1).EQ.2) GO TO 91
22 DO 25 I=1, MLIST
   IF (ILIST(I).EQ. ITER) GO TO 91
25 CONTINUE
   GO TO 90
30 IF (NOPT(2).GT.0) GO TO 91
   GO TO 90
40 IF (NOPT(2).EQ.0) GO TO 90
   IF (NOPT(2).EQ.2) GO TO 91
   GO TO 22
50 IF (NOPT(3).GE.1) GO TO 91
   GO TO 90
60 IF (NOPT(3).EQ.2) GO TO 91
90 IO=0
   RETURN
91 IO=1
   RETURN
END
```

TABLE 3. SETM

```fortran
SUBROUTINE SETM(X, ANEW, N)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION ANEW(N)
IF(N.LE.0) RETURN
DO 100 I=1, N
   ANEW(I)=X
100 CONTINUE
   RETURN
END
```

TABLE 4. MOVEM

```fortran
SUBROUTINE MOVEM(AOLD, ANEW, N)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION AOLD(N), ANEW(N)
IF(N.LE.0) RETURN
DO 100 I=1, N
   ANEW(I)=AOLD(I)
100 CONTINUE
   RETURN
END
```
copy 1-by-5 vectors such as PBLK(5) into rows of the P array, as shown below:

```
CALL MOVEM (PBLK, P(1,N),5).
```

INTPL is a linear interpolation subroutine used for the edge point interpolations in the aperture plane. It is listed in Table 5. PLO( ) and PHI( ) are arrays containing the data for the two known points with the lower and higher z-coordinates, respectively. Z( ) is an array containing the z-coordinates of the endpoints for the grid bar on which the interpolation is to be performed. The z-coordinate of the endpoint between the known points is assigned to the interpolated point PINT( ). The phase and both components of the electric field are interpolated based on the relationship between the z-coordinate of the endpoint and those of the known points.

MULT32 premultiplies a 3-by-2 matrix (OPB) with a 3-by-3 matrix (OPA) and stores the result in a 3-by-2 array (ROP). It is listed in Table 6.

PTSORT sorts a 5-by-n array A(5,n) in ascending order, first with respect to the numbers stored in the first column A(1,n), then with respect to the second column, A(2,n). The first and second columns correspond to the y- and z-coordinates of the data points in the P array, respectively. A listing of PTSORT is presented in Table 7.

NROW is the length of the array in the second dimension. SWAP( ) is an intermediate array used to store information during a row exchange. The operation of the subroutine is simple. The first row is compared to every other row in the array. If any row is found which should appear before the first row, the information in the rows is
TABLE 5. INTPL

SUBROUTINE INTPL(PLO, PHI, PINT, Z)
IMPLICIT REAL*8 (A-N, O-Z)
DIMENSION PLO(5), PHI(5), PINT(5), Z(2)
ZC=2(Z)
IF (PLO(2).GT.Z(1)) ZC=Z(2)
FAC=(ZC-PLO(1))/PHI(1)-PLO(1))
DO 10 I=1,5
10 PINT(I)=PLO(I)+FAC*PHI(I)-PLO(I))
RETURN
END

TABLE 6. MULT32

SUBROUTINE MULT32(ROP, OPA, OPB)
IMPLICIT REAL*8 (A-N, O-Z)
DIMENSION ROP(3,2), OPA(3,3), OPB(3,2)
ROP(1,2)=OPA(1,1)*OPB(1,2)+OPA(1,2)*OPB(2,2)+OPA(1,3)*OPB(3,2)
ROP(2,2)=OPA(2,1)*OPB(1,2)+OPA(2,2)*OPB(2,2)+OPA(2,3)*OPB(3,2)
ROP(3,2)=OPA(3,1)*OPB(1,2)+OPA(3,2)*OPB(2,2)+OPA(3,3)*OPB(3,2)
ROP(1,1)=OPA(1,1)*OPB(1,1)+OPA(1,2)*OPB(2,1)+OPA(1,3)*OPB(3,1)
ROP(2,1)=OPA(2,1)*OPB(1,1)+OPA(2,2)*OPB(2,1)+OPA(2,3)*OPB(3,1)
ROP(3,1)=OPA(3,1)*OPB(1,1)+OPA(3,2)*OPB(2,1)+OPA(3,3)*OPB(3,1)
RETURN
END

TABLE 7. PTSORT

SUBROUTINE PTSORT(A, NCOL, NROW)
IMPLICIT REAL*8 (A-N, O-Z)
DIMENSION A(NCOL, NROW), SWAP(5)
DO 900 I=2, NROW
DO 900 J=1, NROW
IF ( A(I,J)-A(I,1-1) ) 100, 200, 900
100 DO 150 K=1, NCOL
SWAP(K)-A(K,1-1)
A(K,1-1)-A(K,1)
A(K,J)-SWAP(K)
150 GO TO 900
200 IF ( A(2,J)-A(2,1-1) ) 220, 900, 900
220 DO 250 K=1, NCOL
SWAP(K)=A(K,1-1)
A(K,1-1)=A(K,J)
250 A(K,J)-SWAP(K)
900 CONTINUE
RETURN
END
interchanged. After scanning the entire array, the first row will be that which has the lowest numbers in the first and second columns. The second row is then compared to the third and greater rows, and so forth, until the last row is reached. At this point, the array is sorted.

PTLIST reads the aperture data points which have been stored on disk earlier in the program and prints them in tables, one for each panel. NLIST is the number of points in the aperture and IPNL is the number of the panel corresponding to the aperture. A negative number read in the NLIST field indicates the end of the file. A listing of PTLIST appears in Table 8.
**TABLE 8. PTLIST**

```plaintext
SUBROUTINE PTLIST
REAL*8 Q(5)
READ(7,18) NLIST,IPNL
IF (NLIST.LE.0) GO TO 8
PRINT 20, NLIST, IPNL
DO 4 I=1,NLIST
READ(7,21) Q
PRINT 22, Q
CONTINUE
GO TO 2
8 PRINT 19
STOP
18 FORMAT(2I8)
19 FORMAT(//10X,'********** END OF RECORD ON DATA SET 7 **********')
20 FORMAT('1',//10X,'LISTING OF APERTURE PLANE POINTS AS SUPPLIED TO',
        'RADPAT',//10X,'POINTS INTEGRATED FOR PANEL NUMBER',I4,
        '//14X,'Y-COORDINATE',8X,'Z-COORDINATE',10X,'EY (V/M)',12X,
        'EZ (V/M)',10X,'PHASE (RADIANS)//')
21 FORMAT(5D16.10)
22 FORMAT(10X,F15.8,6X,F15.8,6X,D16.10,4X,D16.10,4X,F15.8)
RETURN
END
```
6. EXAMPLE AND FIRST TEST CASE

6.1 Introduction

The use of the program will now be demonstrated. In addition, a card-by-card description of the general input file will be provided before considering the specific antenna configurations. The example is taken from work reported by Rusch [5] and was used in an earlier report on REFLECTR prior to the work reported herein. The reflector will be described in general, then analyzed by the program in three roughly equivalent configurations. In Case I, the single-panel option will be used to represent the reflector as a single piece of material with an elliptical edge. In Case II, the reflector will be represented as a single panel with the edge defined by 20 perimeter points. This will closely approximate the elliptical edge. Finally, the reflector will be described as three panels such that these panels together form the surface of Case III. The object of the procedure is to compute nearly identical patterns in all three cases and to demonstrate agreement with the calculations in [5]. The first case verifies that the point processing changes made in QUANTZ will not adversely affect the performance of the program. The second case shows that a reflector edge may be adequately represented by a polygon. The third representation shows that it is numerically justified to superpose the patterns of the component panels to obtain the total pattern and that the algorithm is working correctly. In each case, the complete input file will be developed and listed.
6.2 Description of the Problem

The antenna, shown in Figure 15, is an offset-fed parabolic dish with a focal length of 31.5 inches. Measured in the focus-centered reference coordinates, the physical reflector lies between 21.8 and 84.1 degrees in the theta coordinate. The aperture projects onto the yz plane as a one-meter circle. The phase center of the feed antenna is located at

\[
x = -1.854 \text{ in.}
\]
\[
y = 0.0
\]
\[
z = -3.966 \text{ in.}
\]

in the reference system with the horn tilted upward 40 degrees.

The feed system origin is placed at the phase center of the feed antenna. The system is inclined 40 degrees in order to make \( \theta = 90^\circ \), \( \phi = 180^\circ \) correspond to the boresight of the antenna. This coordinate system rotation is given by \( \text{ALPHA} = 0^\circ \), \( \text{BETA} = 0^\circ \), and \( \text{GAMMA} = -40^\circ \). The translation vector must be expressed in feed system coordinates. This is found by

\[
x' = 1.854 \sin 40^\circ - 3.966 \sin 40^\circ = -1.129 \text{ in.}
\]
\[
y' = 0.0
\]
\[
z' = 3.966 \cos 40^\circ + 1.854 \sin 40^\circ = 4.230 \text{ in.}
\]

The aperture plane should be placed just forward of the reflector surface. Some clearance should be allowed to insure that the over-illumination rays encounter the aperture plane before striking the reflector surface. The maximum x-coordinate of the surface in this example may be found by using the polar form of the parabola equation.
FEED - SCALAR HORN
15 DB AMPLITUDE TAPER
AT 34°
Y POLARIZED
4.99 GHZ

FIG. 15. OFFSET-FED PARABOLA
\[ r = \frac{2f}{1 + \cos \theta_{\text{max}}} = \frac{2(31.5)}{1 + \cos 68.2^\circ} = 45.94 \text{ in.} \]
\[ x = -r \cos \theta_{\text{max}} = -45.94 \cos 68.2^\circ = -17.06 \text{ in.} \]

The aperture plane will be placed at \( XC = -16.0 \text{ in.} \) to provide the necessary clearance.

The feed antenna for this reflector is a scalar horn with a 15 dB amplitude taper at 34.1 degrees. Rusch [5] computed field strengths for this antenna in one-degree increments off boresight in both the E- and H-planes. These values are used by the FILL routine described in Section 3.4 to assign field strengths to the illumination array. The radiation is \( \phi \)-polarized with a wavelength of 2.367 inches [4.99 GHz].

6.3 General Input File

To help clarify the organization of the input file for REFLECTR, the general input file is explained below. The three examples in the following sections will provide files for actual computations.

<table>
<thead>
<tr>
<th>Card(s)</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>Title cards</td>
</tr>
<tr>
<td>5</td>
<td>FEED(1-3), ALPHA, BETA, GAMMA, XLAM</td>
</tr>
<tr>
<td>6</td>
<td>SURFCE, NPNL, AORORF, BELLP, PSI</td>
</tr>
<tr>
<td>7</td>
<td>PLNPNT(1-3), PLNORM(1-3)</td>
</tr>
<tr>
<td>8</td>
<td>XC, YCBL, ZCBL, HFMABL, HFMIBL</td>
</tr>
<tr>
<td>9-N</td>
<td>Any data required by the FILL subroutine.</td>
</tr>
<tr>
<td>(N+1)-(N+4)</td>
<td>Four extreme points (X, Y, Z) on the edge of the reflector. One point goes on each card (single panel option only).</td>
</tr>
</tbody>
</table>
N+5

NOPT, NLIST
(If NOPT specifies that only certain panels are to be printed or plotted, cards containing the list of these panels follow this card.)

N+6

MAJOR, AMAJOR, MINOR, AMINOR(1-3)
(Pattern request cards. There will be between one and five of these cards in this location—the order of the requests is not important.)

N+7

DONE typed in the first four columns of the card.

These cards are followed by the panel data unless the single panel option is exercised. The organization of the panel data appears below.

Card(s) Information
1 NPERIM, SURFCE, NPTPPL
2-(NPERIM+1) (X, Y, Z) Perimeter points.
(One perimeter point is put on each card.)
NPERIM+2 AORORF or AORORF, BELLP or AORORF, PSI or PLNPNT(1-3), PLNORM(1-3) depending on which parameters are needed to describe the surface specified by SURFCE.

The files for the individual panels are concatenated in the main input file after the DONE card. The order in which the panels appear is unimportant; however, they will be numbered by the program in this order.

6.4 First Configuration

Case I uses the single-panel option. In addition to the general information computed in the previous section, four points defining the aperture boundary ellipse must be found. Since these points are used to find the maximum and minimum $\theta^*$ and $\phi^*$ values for the illumination array, these four points must represent the extreme points on the dish
in each angular coordinate. The $\theta$-extrema are the uppermost and lowermost points on the dish and may be found as follows:

**Upper point** -

\[
\begin{align*}
    r &= \frac{2f}{1 + \cos \theta_{\text{max}}} = \frac{2(31.5)}{1 + \cos 68.2^\circ} = 45.94 \text{ in.} \\
    x &= -r \cos \theta_{\text{max}} = -17.060 \text{ in.} \\
    y &= 0.0 \\
    z &= r \sin \theta_{\text{max}} = 42.654 \text{ in.}
\end{align*}
\]

**Lower point** -

\[
\begin{align*}
    r &= \frac{2f}{1 + \cos \theta_{\text{min}}} = \frac{2(31.5)}{1 + \cos 5.9^\circ} = 31.58 \text{ in.} \\
    x &= -r \cos \theta_{\text{min}} = -31.416 \text{ in.} \\
    y &= 0.0 \\
    z &= r \sin \theta_{\text{min}} = 3.246 \text{ in.}
\end{align*}
\]

The two points representing the $y$-extrema of the dish are almost exactly the $\phi$-extrema as well. The $z$-coordinates of these points are identical.

\[
    z = \frac{z_{\text{min}} + z_{\text{max}}}{2} = \frac{42.654 + 3.246}{2} = 22.95 \text{ in.}
\]

The dish is one meter wide and symmetrical with respect to the $xz$ plane.

\[
y = \pm \frac{39.37}{2} = \pm 19.685 \text{ in.}
\]
Finally, the paraboloid equation provides the x-coordinate.

\[ x = \frac{y + z}{4f} \quad -f = -24.244 \text{ in.} \]

Table 9 shows the input file for Case I. The first four cards contain title information which is reproduced at the beginning of the printout. The coordinate system information (FEED, ALPHA, BETA, and GAMMA) and the wavelength (XLAM) appear on Card 5. Cards 6 and 7 contain information about the surface of the reflector. Card 6 is for SURFCE, NPNL, AORORF, BELLP, and PSI. Card 7 is for PLNPNT and PLNORM. For this reflector, SURFCE, NPNL, and AORORF are given values of 4, 0, and 31.5, respectively. None of the other parameters are required for this surface, so all are given the value zero. Card 8 is for aperture plane information—XC, YCBL, ZCBL, HFMABL, and HFMIBL. In this case, there is no aperture blockage, so HFMABL and HFMIBL are both set to zero. Cards 9 through 46 are the tables of electric field strengths for the particular feed antenna in one-degree increments. The values begin at 0 degrees and progress to 90 degrees as one reads text—line by line from left to right. The \( \phi = 180^\circ \) plane follows the \( \theta = 90^\circ \) plane. The card order that follows is peculiar to the single-panel option. Cards 47 through 50 are for the extreme points on the dish. Unlike the perimeter points, they may be entered in any order. Card 51 is for the output option code. Here the computer is instructed to exercise the print and plot options for all panels and to write the aperture plane point array onto a disk file at the end of the QUANTZ subroutine. Since it is not requested that output options be exercised for any particular panels, NLIST has been left blank and there are no panel number cards.
### TABLE 9. CASE I INPUT

<table>
<thead>
<tr>
<th></th>
<th>OFFSET-FED PARABOLIC DISH</th>
<th>RUSCH/AEROJET</th>
<th>CASE ONE</th>
</tr>
</thead>
<tbody>
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<td></td>
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for ILIST following Card 51. Cards 52 and 53 are the radiation pattern requests. Patterns are requested in the $\phi = 0^\circ$ plane for $\theta$ from $68.0^\circ$ to $98.0^\circ$ by $0.5^\circ$, and in the $\theta = 83^\circ$ plane for $\phi$ from $-15.0^\circ$ to $15.0^\circ$ by $0.5^\circ$. The next card has DONE typed in the first four columns, which signifies that no more pattern requests follow. All of the above cards are read by the NPUT subroutine. This completes the input file for Case I. The output appears in Appendix E.

6.5 Second Configuration

In Case II, the program will run normally with 20 points on the dish edge selected as perimeter points. It was previously noted that the dish edge projects to a one-meter circle on the $yz$ plane. The perimeter points are calculated by choosing twenty equispaced points on this circle and solving the paraboloid equation for the appropriate $x$-coordinate. The input file for this test case is shown in Table 10. The only changes in the file before Card 47 are the title cards and the NPNL parameter. NPNL is changed from 0 to 1 to indicate that the panel is to be treated normally and perimeter points are to be expected. The four extreme points used in Case I are deleted so that the option code card becomes Card 47 and the pattern requests become Cards 48 and 49. Card 50 is the DONE card. The panel data begins on Card 51 with NPERIM, SURFCE, and NPTPPL. NPTPPL is entered as 2250 to match the default value used in the single-panel option. The next NPERIM cards each contain the coordinates of a perimeter point $(x, y, z)$ in its order of appearance as one traces around the dish edge. The final card in the panel data file is reserved for whatever parameters are required for the panel surface. In this case, the focal length of the paraboloid is
TABLE 10. CASE II INPUT

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needed (AORORF). Were the panel part of an ellipsoid, both AORORF and BELLP would be expected on this card. A parameter received in a panel data file replaces any value supplied previously. The output received from this configuration is shown in Appendix E.

6.6 Third Configuration

Case III requires the synthesis of three separate panels from the single parabolic dish. The division of the surface and the location of the perimeter points are shown in Figure 16. The perimeter points on the edge of the dish are the same as those used in Case II. The points on the face of the dish were found by relating the y- and z-coordinates with line equations and using the paraboloid equation to find the corresponding x-coordinates. The density of illumination is kept close to that of the first two cases by requesting that each panel be illuminated by 800 rays.

The input file is shown in Table 11. It is identical to the file for Case II through Card 50 except for the title cards, the NPNL specifications, and the NPTPPL parameter. The illustrative purposes, the panels have been input in the order shown in Figure 16, although the panels may be input in any desired sequence. Organization of the panel input files follows from Case II. The output appears in Appendix E.

6.7 Results

The results of these calculations are shown in Figures 17(a,b). The solid line represents Rusch's calculations. The circles, triangles, and squares represent the results of Cases I, II, and III, respectively. Whenever the Case II or III points fall on those computed from Case I,
FIG. 16. PANELS AND PERIMETER POINTS FOR CASE III
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**NOTE:** The values in the table represent offset-fed parabolic dish data for BUSCH/AEROJET. The illumination is interpolated from tables, and no blockage is present. The reflector is one panel defined by 20 perimeter points. The values are for May 22, 1980, and are presented in NC State University (NCSU) POMR: BOTULA FCLTY: KILL38000. The table includes offset values from 1.000000 to 0.000000, with corresponding illumination values at each point.
TABLE II. (continued).

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**FIG. 17a. PARABOLA H-PLANE PATTERN**
FIG. 17b. PARABOLA E-PLANE PATTERN
only the circles are plotted. In all cases, the H-plane results agree almost perfectly with Rusch's calculations. The major deviation in the E-plane pattern occurs where Rusch predicts a beam shoulder and REFLECTR predicts a shallow null. This is probably due to the fact that such points on the beam are extremely dependent on the placement of the feed phase center. Rusch accounted for the difference in phase center between the E- and H-plane feed patterns, while we did not. Still, the E-plane patterns agree well with Rusch's computed pattern.
7. SEGMENTED SPHERICAL REFLECTOR TEST CASE

7.1 Description of the Antenna

In order to test the algorithm with measured data, a segmented spherical reflector antenna was constructed and tested by NASA at the Langley Research Center. The reflector surface consisted of 54 triangular-shaped planar facets, the vertices of which were located on a sphere with a 24-inch radius. The center of the surface was elevated 21 degrees above the negative x-axis. The boundary of the reflector was hexagonal in shape and measured 11.6 inches from point to point. The surface was fabricated from fiberglass molded onto a machined metal negative and sprayed with conducting paint. The feed antenna was a dual mode horn with a 16 dB amplitude taper at 23 degrees. Illuminating radiation was y-polarized at a frequency of 35 GHz. The center of the horn aperture was located at -12.52 inches on the negative x-axis and the feed antenna was elevated 40 degrees above horizontal. A diagram of the antenna appears in Figure 18.

7.2 Computation and Results

For computational purposes, the phase center of the feed horn was assumed to be 0.5(λ) or 0.169 inches behind the aperture. The aperture plane was placed at x = -18.7 inches. The triangular panels were represented by three perimeter points corresponding to their three vertex points as computed for the mold surface. Each panel was illuminated with 150 rays. Patterns were computed in the θ = 91° plane with φ varied from 0 to 16 degrees and in the φ = 0° plane with θ varied.
54 PANELS

DUAL MODE HORN  
16 DB AMPLITUDE  
TAPER AT 23°  
Y POLARIZED  
36 GHZ

FIG. 18. FACETED SPHERICAL REFLECTOR
from 81 to 101 degrees. The incremental angle was 0.5 degrees in both cuts. The algorithm performed the computation in 6.3 minutes.

The results of the computation are compared with the measured patterns in Figures 19(a,b). The H-plane pattern compares extremely well with the measured results. The E-plane pattern exhibits a slightly wider beamwidth and a higher amplitude of the fourth sidelobe. The fourth sidelobe is a grating lobe caused by the periodicity of the reflector surface in the y-coordinate. It is present regardless of the polarization of the illuminating radiation. It should be noted that the program exactly predicts the location of the lobe as well as the fact that it is more steeply sloped on its inward edge. The level of the lobe, however, is about 6 dB lower than predicted.

One possible explanation for these discrepancies is that the vertex points used in the program were computed for the mold and not measured from the actual surface. The reflector may have sprung out somewhat in the xy plane after being removed from the mold, which would cause the reflector surface analyzed by the program to be different from that used in the pattern measurements. The exact location of the vertex points on the fiberglass reflector will be measured in the future and the computation will be repeated in hopes of obtaining better agreement in the E-plane.
FIG. 19a. SPHERE H-PLANE PATTERN
FIG. 19b. SPHERE E-PLANE PATTERN
8. CONCLUSIONS

An algorithm has been developed to compute radiation patterns for large segmented reflector antennas by extending an existing algorithm for smooth-edged, single-panel reflectors. Included in this algorithm are new techniques for aperture boundary definition, feed antenna description, output formatting, and reflector illumination. Equations for parabolic cylinders have been included so that the program can be used to analyze deployable umbrella reflector antennas.

The algorithm has been tested by comparing calculations with calculations made by Rusch [5], and with measured data, with the results presented in Sections 6 and 7.

Because of the discrepancies between the calculated and measured E-plane patterns of the faceted spherical reflector, it is recommended that the reflector surface be measured and the program rerun to see if the pattern might be better predicted. If the spherical test case does not achieve better results, a return to the theory underlying the program would be required to determine what electromagnetic effects are contributing to the error and how they might be included in the program.


APPENDIX A

DEVELOPMENT OF THE APERTURE PLANE INTEGRALS

In a source-free region, the electric field at a point is given by

\[ \mathbf{E} = - \mathbf{\nabla} \mathbf{v} - j \omega \mathbf{A} - \frac{1}{c} \mathbf{\nabla} \times \mathbf{\Phi} \]  

(1)

where sinusoidal time variation is assumed.

Supplying the Lorentz gauge condition, this can be rewritten as

\[ \mathbf{E} = \frac{\mathbf{\nabla} \cdot \mathbf{A}}{j \omega \epsilon} - j \omega \mathbf{A} - \frac{1}{c} \mathbf{\nabla} \times \mathbf{F}. \]  

(2)

By the equivalence principle [6], the tangential aperture electric field becomes a magnetic current source of strength \( \mathbf{\dot{M}} = -2(\mathbf{n} \times \mathbf{E}) \). The radiated electric field strength is thus

\[ \mathbf{\dot{E}} = \frac{1}{c} \mathbf{\nabla} \times \mathbf{\dot{F}} \]  

(3)

where

\[ \mathbf{\dot{F}} = \frac{1}{4\pi} \int_{V'} \frac{\epsilon \mathbf{\Phi} e^{jkr}}{r} dV'. \]  

(4)

The integration over the source volume is independent of the observation coordinates acted upon by the \( \mathbf{\nabla} \) operator. (3) and (4) are combined and the curl is taken of the integrand.

\[ \mathbf{\dot{E}} = \frac{1}{4\pi} \int_{V'} \mathbf{\nabla} \times \left( \frac{\epsilon \mathbf{\Phi} e^{-jkr}}{r} \right) dV'. \]  

(5)

To expand the integrand, we make use of the vector identity

\[ \mathbf{\nabla} \times (\phi \mathbf{\dot{M}}) = (\mathbf{\nabla} \phi) \times \mathbf{\dot{M}} + \phi (\mathbf{\nabla} \times \mathbf{\dot{M}}) \]  

(6)

and note that \( \mathbf{\dot{M}} \) is a function only of source coordinates, thus...
\[ \phi(\nabla \times \mathbf{M}) = 0 \]

\[ \nabla \phi = \nabla \left( \frac{e^{-jkr}}{r} \right) \]

\[ = -\left\{ \hat{x} \left( \frac{x-x'}{r} \right) + \hat{y} \left( \frac{y-y'}{r} \right) + \hat{z} \left( \frac{z-z'}{r} \right) \right\} \frac{e^{-jkr}}{r} (jk + \frac{1}{r}). \quad (7) \]

If \( \mathbf{M} \) is represented as

\[ \mathbf{M} = \hat{x} M_x + \hat{y} M_y + \hat{z} M_z, \quad (8) \]

the cross product is

\[ \nabla \times \frac{e^{-jkr}}{r} = \left\{ \begin{array}{c}
- \left[ \frac{M_z}{r} \left( \frac{y-y'}{r} \right) + \frac{M_y}{r} \left( \frac{z-z'}{r} \right) \right] \hat{x} \\
+ \left[ \frac{M_z}{r} \left( \frac{x-x'}{r} \right) + \frac{M_y}{r} \left( \frac{x-x'}{r} \right) \right] \hat{y} \\
+ \left[ - \frac{M_x}{r} \left( \frac{x-x'}{r} \right) + \frac{M_x}{r} \left( \frac{y-y'}{r} \right) \right] \hat{z}
\end{array} \right\} \frac{e^{-jkr}}{r} \left( \frac{jk + \frac{1}{r}}{r} \right). \quad (9) \]

We now make the far field approximations:

(A) \[ \left[ \frac{jk + \frac{1}{r}}{r} \right] \approx \left[ \frac{jk}{r} \right] \]

(B) \[ r \approx r_0 - \left[ x' \sin \phi \cos \phi + y' \sin \phi \sin \phi + z' \cos \theta \right] \]

(See Figure A1)

(C) \[ x-x' \approx x \]
\[ y-y' \approx y \]
\[ z-z' \approx z \]

(D) \[ \frac{x}{r} \approx \sin \cos \phi \]
\[ \frac{y}{r} \approx \sin \sin \phi \]
\[ \frac{z}{r} \approx \cos \phi \]

Using these approximations in the integrand (9), we obtain
FIG. A1. FAR FIELD APPROXIMATION (B)
The electric field integral becomes
\[
\mathbf{E} = \frac{-j k r_0}{4 \pi r} \int_{\mathbf{v}'} G \, d\mathbf{v}'
\] (11)

where
\[
G = \begin{bmatrix}
-M_z \sin \phi \sin \theta + M_y \cos \phi \\
-M_x \cos \theta + M_z \sin \cos \phi \\
-M_y \sin \cos \phi + M_x \sin \sin \phi
\end{bmatrix} \hat{x}
\] (12)

\[
\cdot \exp \{ j k [ x' \sin \cos \phi + y' \sin \sin \phi + z' \cos \theta] \}.
\]

The source is an aperture in the y'z' plane. \( \mathbf{M} \) is given by
\[
\mathbf{M} = -2(\hat{n} \times \mathbf{E})
\]
\[
= 2[-\hat{x} \times (E_x \hat{x} + E_y \hat{y} + E_z \hat{z})]
\]
\[
= -2 \hat{z} E_y + 2 \hat{y} E_z.
\]

Thus
\[
M_x = 0
\]
\[
M_y = 2 E_z
\]
\[
M_z = -2 E_y
\] (13)

and the electric field in the far field is given by
\[ E_x = -\frac{jkr_0}{4\pi r_0} \int_{s'} \left[ E_y \sin \theta \sin \phi + E_z \cos \theta \right] e^{jkr(y \sin \theta \sin \phi + z' \cos \theta)} \, dy' \, dz' \] (14)

\[ E_y = -\frac{jkr_0}{4\pi r_0} \int_{s'} \left[ -E_y \sin \theta \cos \phi \right] e^{jkr(y \sin \theta \sin \phi + z' \cos \theta)} \, dy' \, dz' \] (15)

\[ E_z = -\frac{jkr_0}{4\pi r_0} \int_{s'} \left[ -E_z \sin \phi \right] e^{jkr(y \sin \theta \sin \phi + z' \cos \theta)} \, dy' \, dz' \] (16)

The vectors \( \hat{\phi} \) and \( \hat{\theta} \) in spherical coordinates are given by
\[ \hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi \] (17)
\[ \hat{\theta} = \hat{x} \cos \phi \cos \theta + \hat{y} \sin \phi \cos \phi - \hat{z} \sin \theta \] (18)

\[ E_\theta = -\frac{jkr_0}{4\pi r_0} \int_{s'} U \exp \left\{ jkr(y \sin \theta \sin \phi + z' \cos \theta) \right\} \, dy' \, dz' \] (19)

where
\[
U = \cos \phi \cos \theta \left[ E_y \sin \theta \sin \phi + E_z \cos \theta \right] + \sin \theta \left[ E_z \sin \phi \cos \phi \right] \\
+ \sin \phi \left[ E_y \sin \phi \cos \theta - \sin \theta \sin \phi \cos \phi \right] \\
= E_y \left( \sin \phi \sin \cos \phi \cos \theta - \sin \theta \sin \phi \cos \phi \cos \theta \right) \\
+ E_z \left( \cos ^2 \phi \cos \phi + \sin ^2 \phi \cos \phi \right) \\
U = E_z \cos \phi \ . \] (20)

\[ E_\phi = -\frac{jkr_0}{4\pi r_0} \int_{s'} V \exp \left\{ jkr(y \sin \theta \sin \phi + z' \cos \theta) \right\} \, dy' \, dz' \] (21)

where
\[
V = -\sin \phi \left[ E_y \sin \phi \sin \theta + E_z \cos \theta \right] + \cos \phi \left[ -E_y \sin \theta \sin \phi \right] \\
= E_y \left( -\sin ^2 \phi \sin \theta - \cos ^2 \phi \sin \theta \right) - E_z \sin \phi \cos \phi \\
V = -E_y \sin \phi - E_z \sin \phi \cos \theta \ . \] (22)
We are interested in the relative magnitudes and phases of these field components. Furthermore, the relative phase of each component needs only to be preserved with respect to itself. Hence, the constant premultiplier may be dropped. The initial phase of the fields on the aperture is accounted for as \( \phi \) in the phase term. We have

\[
E_0 = \iint E_z \cos \phi \exp\{i[ky \sin \theta \sin \phi + kz \cos \theta - \phi]\} \, dy' \, dz' \tag{23}
\]

Aperture

\[
E_\phi = \iint [E_y \sin \theta + E_z \cos \sin \phi] \exp\{i[ky \sin \theta \sin \phi + kz \cos \theta - \phi]\} \, dy' \, dz' \tag{24}
\]

Aperture

These integrals are evaluated numerically in the program.
APPENDIX B
DEVELOPMENT OF RAY BACKTRACING EQUATIONS

The object of tracing a ray back to the feed antenna is to express a point on the reflector surface by its angles $\theta'$ and $\phi'$ in the feed spherical coordinate system.

Let us define a new coordinate system which is parallel to the feed system but centered on the reference origin. These coordinates are denoted $x''$, $y''$, and $z''$. The point on the reflector is expressed as a vector and rotated into the double-prime system by the inverse of the original rotation matrix.

$$
\begin{bmatrix}
A^{-1} & x \\
y & y'' \\
z & z''
\end{bmatrix}
= 
\begin{bmatrix}
x'' \\
y'' \\
z''
\end{bmatrix}
= \vec{v}'
$$

The translation to the feed system is accomplished by adding the translation vector $O'O$. The $x$, $y$, and $z$ components of this vector are called FEED(1), FEED(2), and FEED(3), respectively.

The distance $R$ of the point from the feed origin is found by taking the magnitude of the vector $O'O + \vec{v}'' = \vec{v}'$.

$$R = \sqrt{(x'')^2 + (y'')^2 + (z'')^2}$$

The angle $\theta'$ is found by

$$z' = R \cos \theta'$$

$$\theta' = \cos^{-1} \left( \frac{z'}{R} \right)$$

The angle $\phi'$ is found by
y' = R \sin \theta' \sin \phi' \\
\phi' = \sin^{-1} \left( \frac{y'}{R \sin \theta'} \right).

R and \sin \theta' will always be positive. If \( y' \) is positive, the desired \( \phi' \) is less than \( \pi \) radians. The arcsine function will return an angle between 0 and \( \pi/2 \). If \( y' \) is zero, the desired \( \phi' \) is \( \pi \), but the arcsine returns 0. Finally, if \( y' \) is negative, the desired \( \phi' \) is greater than \( \pi \), and the arcsine returns a negative angle. It is concluded that if the returned angle is subtracted from \( \pi \), the desired angle is obtained.

\phi' = \pi - \sin^{-1} \left( \frac{y'}{R \sin \theta'} \right)
APPENDIX C

ADDITION OF PARABOLIC CYLINDERS

Standard form of the parabolic cylinder equation shall be with the directrix in the xz plane, opening in the positive x direction. The focal point of the directrix lies on the coordinate origin. This surface is shown in Figure C1.

\[
z^2 = 4f(f + x)
\]  

where \(f\) is the focal length of the directrix. We now allow the surface to be rotated about the x-axis. Rotation is considered as positive when counter-clockwise if the observer looks in the negative-x direction. The angle of rotation is denoted by \(\psi\). The coordinate system rotation is pictured in Figure C2.
An arbitrary point $P$ may be rotated from the unprimed to the primed coordinates by the relations

$$z' = z \cos \psi - y \sin \psi$$  \hspace{1cm} (2)
$$y' = z \sin \psi + y \cos \psi$$  \hspace{1cm} (3)

The parametric equations for a ray are

$$x = RB_{11} - B_{12}$$
$$y = RB_{21} - B_{22}$$
$$z = RB_{31} - B_{32}$$  \hspace{1cm} (4)

First account for the rotation of the surface by substitution of $z'$ in (2) for $z$ in (1).

$$(z')^2 = 4f(f + x)$$
$$(z \cos \psi - y \sin \psi)^2 = 4f(f + x)$$
\[ z^2 \cos^2 \psi - 2yz \sin \psi \cos \psi + y^2 \sin^2 \psi = 4f(f + x). \]  \hspace{1cm} (5)

Then substitute the parametric expressions for \( x, y, \) and \( z \).

\[
\begin{align*}
(RB_{31} - R_{32}) \cos \psi & - 2(RB_{21} - B_{22})(RB_{31} - B_{32}) \cos \psi \sin \psi \\
+ (RB_{21} - B_{22})^2 \sin^2 \psi & = 4f(f + RB_{11} - B_{12}) \\
(R^2 B_{31}^2 - 2R B_{31} B_{32} + B_{32}^2) \cos^2 \psi & - 2(R^2 B_{21} B_{31} - R B_{31} B_{22} - R B_{21} B_{32} + B_{22} B_{32}) \cos \psi \sin \psi \\
+ (R^2 B_{21}^2 - 2R B_{21} B_{32} + B_{22}^2) \sin^2 \psi & = 4f^2 - 4f R B_{11} \\
+ 4f B_{12} & = 0.
\end{align*}
\]  \hspace{1cm} (6)

The last equation (6) is of the form

\[ AR^2 + BR + C = 0 \]  \hspace{1cm} (7)

where

\[ A = B_{31}^2 \cos^2 \psi - 2B_{21} B_{31} \cos \psi \sin \psi + B_{21}^2 \sin^2 \psi \]  \hspace{1cm} (8)

\[ B = -2B_{31} B_{32} \cos^2 \psi + 2(B_{31} B_{22} + B_{21} B_{32}) \cos \psi \sin \psi \\
- 2B_{21} B_{22} \sin^2 \psi - 4f B_{11} \]  \hspace{1cm} (9)

\[ C = B_{32}^2 \cos^2 \psi - 2B_{22} B_{32} \cos \psi \sin \psi + B_{22}^2 \sin^2 \psi - 4f^2 + 4f B_{12} \]  \hspace{1cm} (10)

Expressions (8), (9), and (10) are evaluated by the program and (7) is solved to find the intersection point of the ray with the surface.

Using (5), the inside normal of the surface may be found at any point by taking the negative of the gradient.

\[ \nabla \phi(x, y, z) = \hat{n}(x, y, z) \]  \hspace{1cm} (11)

where
\[ \phi(x,y,z) = z^2 \cos^2 \psi - 2yz \sin \psi \cos \psi + y^2 \sin^2 \psi - 4f(f + x) = 0 \]

\[-\nu_{\phi_x} = 4f \]  
\[-\nu_{\phi_y} = 2z \sin \psi \cos \psi - 2y \sin^2 \psi \]  
\[-\nu_{\phi_z} = 2y \sin \psi \cos \psi - 2z \cos^2 \psi . \]  

A unit vector is obtained by normalizing this vector. The common factor of 2 is removed from each component prior to normalization such that

\[ \hat{n} = \frac{\hat{x}}{2f/NMAG} + \frac{\hat{y}(z \sin \psi \cos \psi - y \sin^2 \psi)}{NMAG} \]
\[ + \frac{\hat{z}(y \sin \psi \cos \psi - z \cos^2 \psi)}{NMAG} \]

where

\[ NMAG = [4f^2 + (z \sin \psi \cos \psi - y \sin^2 \psi)^2 + (y \sin \psi \cos \psi - z \cos^2 \psi)^2]^{1/2} \]
APPENDIX D
LISTING OF THE CODE FOR REFLECTR
PROGRAM REFLECTR

IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 MAJOR(5),MINOR(5)
INTEGER SURFACE
COMPLEX*16 ETOT(2,400),FIELDY(400),FIELDZ(400)
COMMON/APPRPM/NPTbPL,NbERIM
COMMON/CONTRL/NOPT(3),NLIST,IOPT,ILIST(100)
COMMON/DIMENS/YMIN,YMAX,ZMIN,ZMAX
COMMON/EXTENT/YMIN,YMAX,ZMIN,ZMAX
COMMON/MATH/PI,PI2,PID2,DTOR,RTOD
COMMON/PATTRN/ETOT,AMINOR(3,5),AMAJOR(5),MINOR,MAJOR,NANGLE(5)
COMMON/PARAMS/ALPHA,BETA,GAMMA,XLAM,XC,SURFACE,NPNL,NPOINT

COMMON/DIM/750,75,75,500
DATA DONE/5HDONE /
DATA MLVL,NPARTS/0,7/
DATA YLO,YHI,ZLO,ZHI/1.OD+10,-1.OD+10,1.OD+10,-1.OD+10/
FDB(X)=20.0*DLOG10(X)
PDB(X)=10.0*DLOG10(X)

CALL NPUT(P,NPAT)
CALL APRTUR(P,1)
PRINT 777
CALL QUANTZ(P,NPERIM,1)
IF (IO(3,1).EQ.0) GO TO 80
ISW=1
IF (IOPT.EQ.1) ISW=-1
CALL APRPLT(P,1,NPAT)
PRINT 780

DO 150 L=1,NPERIM
PR(1,L+MLVL)=P(1,L+NPAT)
150 PR(2,L+MLVL)=P(2,L+NPAT)
MLVL=MLVL+1
95 DO 770 I = I+1,NPAT

NANG=NANGLE(I)
DO 200 K=1,NPAT
CALL INTGRT(P,MAJOR(K),AMAJOR(K),AMINOR(1,K),FIELDY,FIELDZ)
PRINT 779
NANG=NANG+1
200 DO 770 I = I+1,NPAT

ETOT(1,L+ISUM)=ETOT(1,L+ISUM)+FIELDY(L)
ETOT(2,L+ISUM)=ETOT(2,L+ISUM)+FIELDZ(L)
150 DO 770 I = I+1,NPAT

ISUM=ISUM+1
400 CONTINUE
PRINT 781
IF (IOPT.EQ.1) GO TO 420
IF (IO(2,1).EQ.0) GO TO 420
PRINT 782

420 CONTINUE
PRINT 782

END
FMAXY = -1.00 + 40  
FMAXZ = -1.00 + 40  
DO 450 J=1,NANG  
YFLD(J) = CDABS(ETOT(1,J+ISUM))  
ZFLD(J) = CDABS(ETOT(2,J+ISUM))  
FMAXY = DMAX1(FMAXY, YFLD(J))  
450   FMAXZ = DMAX1(FMAXZ, ZFLD(J))  
ISUM = ISUM + NANG  
DAMINOR(1,I) = YFLD(J) - C * DABS(ETOT(1, J + ISUM))  
FMYDB = -60.000  
FMZDB = -60.000  
PWRMDB = -60.000  
PWR = FMAXZ * FMAXZ + FMAXY * FMAXY  
IF (FMAXY.GT.1.0D-10) FMYDB = FDB(FMAXY)  
IF (FMAXZ.GT.1.0D-10) FMZDB = FDB(FMAXZ)  
IF (PWR.GT.1.0D-10) PWRMDB = PDB(PWR)  
PRINT 600, MAJOR(I), AMAJOR(I), MINOR(I), (AMINOR(J,I), J=1, I)  
600   FORMAT(1H1, 24X,  
  ' TABLE OF ELECTRIC FIELD STRENGTHS (DB)', 8H +', 24X,  
  ' PRINCIPAL PLANE OF CUT IS ', AS, ' ', F8.3, ' DEG'  
PRINT 666, MINOR(I)  
666   FORMAT(/13X, AS, 4X, 'DB(Z/Z)', 4X, 'DB(Y/Z)', 4X, 'DB(Z/Y)', 5X,  
  '+DB(Y/Y)', 5X, 'PWRDB', 10X)  
DO 700 K=1,NANG  
PWR(K) = PWRMDB - 100.00  
DBY = FMYDB - 100.00  
DBZ = FMZDB - 100.00  
PWR = 2 * FLD(K) + YFLD(K) * YFLD(K)  
IF (YFLD(K).GT.1.0D-15) DBY = FDB(YFLD(K))  
IF (ZFLD(K).GT.1.0D-15) DBZ = FDB(ZFLD(K))  
IF (PWR.GT.1.0D-10) PWRMDB = PDB(PWR)  
IF (FMYDB.EQ.-60.000) DBY = -100.000  
IF (FMZDB.EQ.-60.000) DBZ = -100.000  
DBZ = DBZ - DBY  
D3BY = DBY - DBZ  
PWRMDB = PWR(K) - PWRMDB  
PRINT 690, D, D3Z, DBZ, D3BY, DBY, PWRDB  
690   FORMAT(10X, F9.3, 5F11.5)  
DO = DAMINOR(3, I)  
YFLD(K) = DBY  
ZFLD(K) = DBZ  
700   CONTINUE  
PRINT 750, FMAXX, FMINDB, FMAY, FMYDB  
750   FORMAT(/15X, 'MAXIMUM FIELD VALUES -'/15X,  
  '20LOG(MAX(FIELD-Z))=',1PE15.7, ', 20LOG(MAX(FIELD-Y))=',1PE15.7)  
PRINT 755, NPARTS  
755   FORMAT(/14X, 'INTERPOLATION NUMBER USED FOR INTEGRATION IS .........', 5X)  
PRINT 765, MAJOR(I), AMAJOR(I)  
765   FORMAT(1H1, 20X, 'PRINCIPAL PLANE = ', AS, F7.3, ' DEGREES')  
CALL PLOT4(64H NORMALIZED Z-COMPONENT OF SECONDARY PATTERN (DB)  
  , FMZDB, ZFLD, NANG, MINOR(1, I), AMINOR(J, I))  
PRINT 765, MAJOR(I), AMAJOR(I)  
CALL PLOT4(64H NORMALIZED Y-COMPONENT OF SECONDARY PATTERN (DB)  
  , FMYDB, YFLD, NANG, MINOR(I), AMINOR(I, K))  
PRINT 765, MAJOR(I), AMAJOR(I)  
CALL PLOT4(54H NORMALIZED POWER PATTERN (DB)  
  , PWRMDB, PWR, NANG, MINOR(I), AMINOR(I, I))
SUBROUTINE NPUT(P,NPAT)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 MAJOR(5),MINOR(5)
COMPLEX*16 ETOT(2,400)
INTEGER SURFCE
COMMON/BLOCKG/YCBL,ZCBL,HFMABL,HFMIBL
COMMON/CONTRL/NOPT(3),NLIST,IOPT,ILIST(100)
COMMON/FEED/EP(91),ET(91),NP,MT
COMMON/MATH/PI,PI2,PID2,DTOR,RTOD
COMMON/PATNR/AOROF,BELL,P,PLNPNT(3),PLNORM(3),FEED(3),
ALPHA,BETA,GAMMA,XLAM,SC,SURFCE,NPNL,NPOINT
COMMON/PATNR/ETOT,AMINOR(3,5),AMAJOR(5),MINOR,MAJOR,NANGLE(5)
DIMENSION P(5,2750),TITLE(40)
DATA DONE/5HDONE
IOPT=0
READ 200, TITLE
READ(1,10) FEED,ALPHA,BETA,GAMMA,XLAM
10 FORMAT(7F10.4)
READ(1,20) SURFCE,NPNL,AOROF,BELL,P,PLNPNT,PLNORM
20 FORMAT(I1,14X,I5,3F10.4/6F10.4)
READ(1,30) XC,YCBL,ZCBL,HFMABL,HFMIBL
30 FORMAT(5F10.4)
READ(1,40) EP
40 FORMAT(5F15.6)
IF (NPNL.LE.0) READ(1,50) ((P(I,J),I-1,3),J=1,4)
50 FORMAT(3F10.4)
READ(1,60) NOPT,NLIST
60 FORMAT(3I1,2X,15)
IF (NOPT(1).EQ.0) READ(1,70) (ILIST(I),I-1,NLIST)
70 FORMAT(16I5)
ISUM=0
NPAT=1
77 READ(1,80) MAJOR(NPAT),AMAJOR(NPAT),MINOR(NPAT),AMINOR(1,NPAT),
I=1,3)
80 FORMAT(A5,5X,F10.4,A5,5X,F10.4)
IF (MAJOR(NPAT).EQ.DONE) GO TO 98
NANGLE(NPAT)=AMINOR(2,NPAT)-AMINOR(1,NPAT)/AMINOR(3,NPAT)+1.5
IF (NANGLE(NPAT).GT.75) GO TO 88
ISUM=ISUM+NANGLE(NPAT)
NPAT=NPAT+1
IF (NPAT.LT.6) GO TO 77
PRINT 573
STOP
83  PRINT 574
STOP
98  IF (ISUM.LE.400) GO TO 105
PRINT 575,ISUM
STOP
105  NPAT=NPAT-1
DO 108 L=1,ISUM
108  ETOT(1,L)=(0.0D0,0.0D0)
GO TO 120,130,140,150,160,SURFCE
120  PRINT 579, TITLE, XLAM, FEED, ALPHA, BETA, GAMMA, AORORF, BELLP
GO TO 170
130  PRINT 580, TITLE, XLAM, FEED, ALPHA, BETA, GAMMA, AORORF
GO TO 170
140  PRINT 581, TITLE, XLAM, FEED, ALPHA, BETA, GAMMA, AORORP
GO TO 170
150  PRINT 582, TITLE, XLAM, FEED, ALPHA, BETA, GAMMA, AORORF, PSI
170  IF (NPNL.GE.1) GO TO 180
IOPT=1
NPNL=1
NPAT=1
PRINT 583, ((P(I,J),I=1,3),J=1,4)
180  PRINT 584, XC,YCBL,ZCBL,HFMABL,HFMIBL,NPNL
PRINT 586
PRINT 587
PRINT 590,EP
PRINT 586
PRINT 588
PRINT 590,ET
PRINT 576, NPAT
DO 190 M=1,NPAT
190  PRINT 577, MAJOR(M), MAJOR(M), MINOR(M), (AMINOR(KK,M), KK=1,3)
200  FORMAT(10A8)
573  FORMAT(' ****** ERROR - MORE THAN 5 PATTERN ')
574  FORMAT(' ****** ERROR - MORE THAN 15 PATTERN REQUEST ')
575  FORMAT(' ****** ERROR - REQUESTED 15 ANGLES TO BE ',
576  FORMAT(' ****** ERROR - REQUESTED ANGLES TO BE ',
577  FORMAT(' ****** ERROR - REQUESTED ANGLES TO BE ',
578  FORMAT(' PLANAR REFLECTOR FAR FIELD RADIATION ')
579  FORMAT(' ELLIPTICAL REFLECTOR FAR FIELD RADIATION ')
SUBROUTINE APTRUR (P, ICALL)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 WHAT(3),NMAG
INTEGER SURFCE
COMMON/APRFRM,NPTPPL,NPERIM
COMMON/CONTRL,NOPT(3),NLIST,IOPT,ILIST(100)
COMMON/FEED,EOP(91),ET(91),NP,NT
COMMON/MATH,PI,P1,P2,PID2,DTOR,RTOD
COMMON/PARAMS/AORORF,BELLP,PSI,PLNPNT(3),NPNL,NPOINT
  REAL*8 ALPHA,BETA,GAMMA, XLAM,XC,SURFCE
  PI=3.141592653589793D0
  P1=PI+PI
  P2=0.5*PI
  RTOD=180./PI
RETURN
END
DIMENSION A(3,3),AINV(3,3),B(3,2),BB(3,2),C(3),EI(3),ER(3),
P(5,2750),SR(3),X(3)

IF (ICALL.GT.1) GO TO 50
ALPHAR=ALPHA*DTOR
BETAR=BETA*DTOR
GAMMAR=GAMMA*DTOR
A(1,1)=DCOS(ALPHAR)*DCOS(GAMMAR)-DSIN(ALPHAR)*DSIN(BETAR)*
   DSIN(GAMMAR)
A(1,2)=DSIN(ALPHAR)*DCOS(GAMMAR)+DCOS(ALPHAR)*DSIN(BETAR)*
   DSIN(GAMMAR)
A(1,3)=DCOS(BETAR)*DSIN(GAMMAR)
A(2,1)=DSIN(ALPHAR)*DCOS(GAMMAR)
A(2,2)=DCOS(ALPHAR)*DCOS(BETAR)
A(2,3)=DSIN(BETAR)
A(3,1)=DSIN(ALPHAR)*DSIN(GAMMAR)+DCOS(ALPHAR)*DSIN(BETAR)*
   DCOS(GAMMAR)
A(3,2)=DCOS(ALPHAR)*DSIN(GAMMAR)-DCOS(ALPHAR)*DSIN(BETAR)*
   DCOS(GAMMAR)
A(3,3)=DCOS(BETAR)*DCOS(GAMMAR)

DO 40 I=1,3
   DO 40 J=1,3
   40 AINV(I,J)=A(I,J)

NPERIM=4
NPTPPL=2250

50 IF (IOPT.EQ.0) CALL APRIN (P,ICALL)
   TMAX=0.OD0
   TMIN=PI
   PMIN=PI+PID2
   PMAX=PID2
   DO 65 I=1,NPERIM
      DO 60 J=1,3
      60 X(J)=AINV(J,1)*P(1,1)+AINV(J,2)*P(2,1)+AINV(3,3)*P(3,1)
      R=DSQRT((X(1)+FEED(1))**2+(X(2)+FEED(2))**2+(X(3)+FEED(3))**2)
      P(1,I)=DARCOS((X(3)+FEED(3))/R)
      SINTHT=DSIN(P(1,I))
      IF (SINTHT.LT.1.D-10) SINTHT=1.D-10
      P(2,I)=PI-DARSIN((X(2)+FEED(2))/(R*SINTHT))
      IF (P(2,I).GT.TMAX) TMAX=P(2,I)
      IF (P(2,I).LT.TMIN) TMIN=P(2,I)
      IF (P(2,I).GT.PMAX) PMAX=P(2,I)
      IF (P(2,I).LT.PMIN) PMIN=P(2,I)
   65 CONTINUE
   DELP=PMAX-PMIN
   DEXP=MAX-TMIN
   NP=DSQRT(DELP*DFLOAT(NPTPPL)/DELT)+1.0
   NP=((NP-1)/2)*2+1
   ANGINC=DELP/(DFLOAT(NP)-2.6)
   NTD2=DELT/(2.0*ANGINC)+1.0
   NT=2*NTD2+1
   PMIN=PMIN-0.8*ANGINC
   PMAX=PMAX+0.8*ANGINC
   TCT=(TMAX+TMIN)/2.0
   TMIN=TCT-DFLOAT(NTD2)*ANGINC
   TMAX=TCT+DFLOAT(NTD2)*ANGINC
   DO 95 J=1,NT
      DO 95 K=1,NP
      P(I,NPERIM+(J-1)*NP+K)=TMIN+(J-1)*ANGINC
      95 P(I,NPERIM+(J-1)*NP+K)=TMIN+(J-1)*ANGINC
   NTNP=NT*NP
   NPOINT=NPERIM+NTNP
   PMIN=PMIN*RTOD
   PMAX=PMAX*RTOD
   PMIN=PMIN+RTOD
   PMAX=PMAX+RTOD
Illumination Data

ILLUMINATION DATA

\( \Theta \) ILLUMINATION FROM ...................... \( F_9.3 \) TO \( F_9.3 \)

\( \Phi \) ILLUMINATION FROM ...................... \( F_9.3 \) TO \( F_9.3 \)

INCREMENTAL ANGLE (DEG) .................................. \( F_7.4 \)

THEREFORE TOTAL NUMBER OF GENERATED RAYS .................. \( I_7 \)

TOTAL NUMBER OF APERTURE PLANE POINTS .................. \( I_7 \)

SURFACE

AR=0.0

BR=B(1,1)*PLNORM(1)+B(2,1)*PLNORM(2)+B(3,1)*PLNORM(3)

CR=-B(1,2)*PLNORM(1)

=-(B(2,2)+B(3,2)+B(3,1))*PLNORM(3)

GO TO 180

AR=B(1,1)**2/AORORF**2+B(2,1)**2+B(3,1)**2)/BELLP**2

BR=-2.0*(B(1,1)*B(1,2)/AORORF**2+B(2,1)*B(2,2)+B(3,1)*B(3,2))/BELLP**2

CR=B(1,2)**2/AORORF**2+B(2,2)**2+B(3,2)**2)/BELLP**2-1.0

GO TO 180

AR=B(1,1)*B(1,1)+B(2,1)*B(2,1)+B(3,1)*B(3,1)

BR=-2.0*(B(2,1)*B(2,2)+B(3,1)*B(3,2)+2.0*AORORF*B(1,1))

CR=B(2,2)*B(2,2)+B(3,2)*B(3,2)+4.0*AORORF**2+B(1,2)-4.0*AORORF**2

GO TO 180

AR=B(1,1)*B(3,1)*CPSI+CPSI-2.0*B(2,1)+B(3,1)*CPSI+CPSI

=+B(2,1)*B(2,1)*SNPSI+SNPSI

BR=-2.0*B(3,1)*B(3,2)*CPSI+2.0*AORORF*B(1,1)

CR=B(3,2)*B(3,2)*CPSI+CPSI-2.0*B(2,2)*B(3,2)*CPSI+CPSI

GO TO 180

R=(-BR+DSQRT(BR*BR-4.*AR*CR))/(AR+AR)

CONTINUE

X0=B(1,1)*R-B(1,2)

Y0=B(2,1)*R-B(2,2)

Z0=B(3,1)*R-B(3,2)

GO TO (220,230,240,250,260),SURFACE
220 NHAT(1) = PLNORM(1)
NHAT(2) = PLNORM(2)
NHAT(3) = PLNORM(3)
GO TO 288

230 NHAT(1) = XO * BELL**2 / DSQRT(XO**2 * BELL**4 + (YO**2 + ZO**2) * AORORF**4)
NHAT(2) = YO * AORORF**2 / DSQRT(XO**2 * BELL**4 + (YO**2 + ZO**2) * AORORF**4)
NHAT(3) = ZO * AORORF**2 / DSQRT(XO**2 * BELL**4 + (YO**2 + ZO**2) * AORORF**4)
GO TO 288

240 NHAT(1) = -XO / AORORF
NHAT(2) = -YO / AORORF
NHAT(3) = -ZO / AORORF
GO TO 288

250 NHAT(1) = 2.0 * AORORF / DSQRT(4.0 * AORORF**2 + YO**2 + ZO**2)
NHAT(2) = -YO / DSQRT(4.0 * AORORF**2 + YO**2 + ZO**2)
NHAT(3) = -ZO / DSQRT(4.0 * AORORF**2 + YO**2 + ZO**2)
GO TO 288

260 NMAG = DSQRT(4.0 * AORORF**2 + (YO * SNPSI + ZO * CSPSI)**2 + (YO * SNPSI - ZO * CSPSI)**2)
NHAT(1) = 2.0 * AORORF / NMAG
NHAT(2) = SNPSI * (ZO * CSPSI - YO * SNPSI) / NMAG
NHAT(3) = CSPSI * (YO * SNPSI - ZO * CSPSI) / NMAG

288 SCALAR = 2.0 * (B(1,1) * NHAT(1) + B(2,1) * NHAT(2) + B(3,1) * NHAT(3))
DO 295 L = 1, 3
SR(L) = B(L,1) - SCALAR * NHAT(L)
ETI = P(3,1) / R
EPI = P(4,1) / R
C(1) = COST * COSP * ETI - SINP * EPI
C(2) = COST * SINP * ETI + COSP * EPI
C(3) = - SINT * ETI
DO 400 N = 1, 3
EI(N) = E(N,1) * A(N,M) * C(M)
SCALAR = 2.0 * (EI(1) * NHAT(1) + EI(2) * NHAT(2) + EI(3) * NHAT(3))
DO 500 K = 1, 3
ER(K) = SCALAR * NHAT(K) - EI(K)
Y = Y0 + (XC - XO) / SR(2) / SR(1)
Z = Z0 + (XC - XO) / SR(3) / SR(1)
D = DSQRT((XC - XO) + (Y - Y0) + (Z - Z0) + (Y - Y0) + (Z - Z0))
PHASE = PI2 * (R + D) / XLAM + P(5, I)
P(1, I) = Y
P(2, I) = Z
P(3, I) = ER(2)
P(4, I) = ER(3)
P(5, I) = PHASE

400 EI(N) = E(N,1) * A(N,M) * C(M)
500 ER(K) = SCALAR * NHAT(K) - EI(K)
DO 600 K = 1, 3
600 CONTINUE
RETURN
END

SUBROUTINE APRIN(P, ICALL)
IMPLICIT REAL*R(A-H2O-Z)
INTEGER SURFCE
COMMON/APPRPM/NPTPPL, NPERIM
COMMON/CONTRL/NOPT(3), NLIST, IOPT, ILIST(100)
COMMON/PARAMS/AORORF, BELL, PSI, PLNPNT(3), PLNORM(3), FEED(3),
* ALPHA, BETA, GAMMA, XLAM, INC, SURFCE, NPNL, NPOINT
DIMENSION P(5, 2750)
READ(1, 10) NPERIM, SURFCE, NPTPPL
10 FORMAT(3I5)
   IF (NPERIM.GT.2) GO TO 250
   IF (SURFCE.GT.5) GO TO 250
   IF (NPTPL.GT.2500) GO TO 270
   IF (NPERIM*SURFCE.LE.0) GO TO 250
   READ(1,20) ((P(I,J),I=1,NPERIM),J=1,NPERIM)
20 FORMAT(3I10.4)
28 GO TO (30,40,50,50,60),SURFCE
30 READ(1,35) PLNPMT,PLNORM
35 FORMAT(6PI0.4)
40 READ(1,45) AORORF,BELLP
45 FORMAT(2FI0.4)
100 CONTINUE
199 IF (10(1,ICALL).GT.20.0) RETURN
200 FORMAT(1X,'REFLECTOR PANEL NUMBER',I4,
   , ' EXECUTION TERMINATING ************')
   PRINT 200, ICALL
250 PRINT 252,ICALL
252 FORMAT(///' ffffffffff INPUT ERROR ON CARD ONE FOR PANEL NUMBER',
   , ' 14,' EXECUTION TERMINATING ************)
   STOP
260 PRINT 262,ICALL
262 FORMAT(///' STORAGE DOES NOT EXIST FOR NUMBER OF'
   , ' PERIMETER POINTS SPECIFIED - PANEL',I4,' ************')
   STOP
270 PRINT 272,ICALL
272 FORMAT(///' MAXIMUM ILLUMINATION REQUEST IS 2500',
   , ' RAYS - PANEL',I4,' ************')
   NPTPL=2500
28 GO TO 28
320 PRINT 401,PLNPMT,PLNORM,NPERIM
330 PRINT 402,AORORF,BELLP,NPERIM
340 PRINT 403,AORORF,NPERIM
350 PRINT 404,AORORF,NPERIM
360 PRINT 405,AORORF,PSI,NPERIM
401 FORMAT(///10X,'PANEL IS A PLANAR SURFACE',///
   , ' A POINT ON THE REFLECTOR SURFACE (X,Y,Z).............',F7.2
   , ' COMPONENTS OF UNIT NORMAL TO SURFACE (X,Y,Z).............',F7.2
   , ' NUMBER OF USER-SUPPLIED EDGE POINTS.................',I7)
402 FORMAT(///10X,'PANEL IS AN ELLIPTICAL SECTION'///
   , ' MAJOR AXIS OF ELLIPTICAL REFLECTOR...............',F7.2/
   , ' MINOR AXIS OF ELLIPTICAL REFLECTOR...............',F7.2/
   , ' NUMBER OF USER-SUPPLIED EDGE POINTS..............',I7)
403 FORMAT(///10X,'PANEL IS A SPHERICAL SECTION'///
   , ' RADIUS OF REFLECTOR SPHERE........................',F7.2/
   , ' NUMBER OF USER-SUPPLIED EDGE POINTS..............',I7)
404 FORMAT(///10X,'PANEL IS A PARABOLIC SECTION',///
   , ' FOCAL LENGTH OF THE PARABOLA....................',F7.2/
   , ' NUMBER OF USER-SUPPLIED EDGE POINTS..............',I7)
405 FORMAT(///10X,'PANEL IS SECTION OF A PARABOLIC CYLINDER'///
FOCAL LENGTH OF THE PARABOLA.

FOCAL LINE ROTATION FROM Y-AXIS (PSI).

NUMBER OF USER-SUPPLIED EDGE POINTS.

END

SUBROUTINE FILL(P,NPT)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/FEED/EP(91),ET(91),NP,NT
COMMON/MATH/PI,PI2,PI2,DTOR,RTOD
DIMENSION P(5,NPT)
DO 100 I=1,NPT
PSX=DSIN(P(1,I))*DCOS(P(2,I))
PSY=0.0D0
IF (DABS(P(2,I)-PI).GT.1.0D-5)
PSY=DCOS(DATAN(PSX/DSIN(P(1,I))/DSIN(P(2,I))))
SINLX=DCOS(PSX)*RTOD
LO=ANGLX+1.0D0
HI=LO+1
PPFLD=(ANGLX-DFLOAT(LO-1))*(EP(LO)-ET(LO))+ET(LO)
TPFLD=(ANGLX-DFLOAT(LO-1))*(ET(LO)-ET(LO))+ET(LO)
CSE2=PSY*PSY
SINE2=1.0D0-CSE2
P(3,I)=0.0D0
P(4,I)=PPFLD*TPFLD/(DSQRT(TPFLD*TPFLD*CSE2+)
PPFLD*PPFLD*SINE2))
P(5,I)=0.0D0
100 CONTINUE
RETURN
END

SUBROUTINE QUNTZ(P,NPERIM,ICALL)
IMPLICIT REAL *8 (A-H,O-Z)
INTEGER SURFCE
COMMON/BLOCK/VCBL,2CBL,HFMBBL,HFMBBL
COMMON/CTRL/NOPT(1),NLIST,IOPT,ILIST(100)
COMMON/DIMENS/YDIM,ZDIM,YCT,ZCT
COMMON/EXTENT/YMIN,YMAX,ZMIN,ZMAX
COMMON/FEED/EP(91),ET(91),NP,NT
COMMON/PARAMS/ALPH,BETA,GAMMA,XLAX,XC,SURFCE,NPNT,POINT
DIMENSION P(5,2750),PINT(5),POLD(5),PRLK(5),PR5(5,41),E(2,101)
NBARS=NP-2
YMIN=1.0D+10
YMAX=-1.0D+10
ZMIN=1.0D+10
ZMAX=-1.0D+10
NOS=2*NBARS
CALL SETM(1.0D+20,2,NOS)
DO 20 I=1,NPERIM
IF (P(1,I).GT.YMAX) YMAX=P(1,I)
IF (P(1,I).LT.YMIN) YMIN=P(1,I)
IF (P(2,I).GT.ZMAX) ZMAX=P(2,I)
IF (P(2,I).LT.ZMIN) ZMIN=P(2,I)
CALL MOVEM(P(1,I),PR5(1,I),5)
YDIM=YMAX-YMIN
YCT=(YMAX+YMIN)/2.
ZDIM=ZMAX-ZMIN
ECT = (EMAX-E_MIN)/2.
GRID = (YMAX-Y_MIN)/(DFLOAT(MBARS)-0.6)
GRIDLO = Y_MIN+GRID/5.0D0
GRIDHI = Y_MAX-GRID/5.0D0

31 IBGM=IPERIM+1
INDEX=IPERIM
DO 100 I=IBGM,NPOINT
IF (P(1,I).GT.YMAX) GO TO 98
IF (P(1,I).LT.YMIN) GO TO 99
NGRID = (P(1,I)-GRIDLO)/GRID+0.5
P(1,I) = GRIDLO+DFLOAT(MGRID)*GRID
CALL MOVEM(P(1,I),P(1,I-INDEX),5)
GO TO 100

98 INDEX=INDEX+1
100 CONTINUE
NPOINT=NPOINT-INDEX
CALL PTSORT(P,5,NPOINT)
IF (IOPT.EQ.1) GO TO 422
CALL MOVEM(PR5(1,1),PR5(1,INDEX+1),5)
KDEX=2
Y2=PR5(1,1)
Z2=PR5(2,1)
200 Y1=PR5(1,KDEX)
Z1=PR5(2,KDEX)
IF (DMA(Y1-Y2).LT.1.0D-5) GO TO 400
SLOPE = (Z1-Z2)/(Y1-Y2)
4 = Z2*SLOPE*Y2
F(Y1-Y2) = 220, 230, 230
220 YHI=Y2
YLO=Y1
GO TO 240
230 YHI=Y1
YLO=Y2
240 INDEX=(YLO-GRIDLO)/GRID+1.0
INDEX=INDEX+1
Y=GRIDLO+DFLOAT(INDEX-1)*GRID
IF (Y,YT,GTH) GO TO 400
ILOAD=1
ZEE=SLOPE*Y+B
IF ((Z1,INDEX).LT.1.0D+10) ILOAD=2
Y(ILOAD,INDEX)=ZEE
GO TO 250
400 Y2=Y1
Z2=Z1
KDEX=KDEX+1
IF (KDEX.LE.NPERIM+1) GO TO 200
DO 420 I=1,MBARS
IF ((Z(1,I)+Z(2,I)).GT.1.0D+10) GO TO 1005
110 Z(2,I)-Z(1,I) 419,420,420
240 CONTINUE
Z(2,I)=Z1
422 HFMAX=YMIN/2.0D0
HFMIN=YMAX/2.0D0
DO 430 I=1,MBARS
Y=GRIDLO+DFLOAT(I-1)*GRID
ZZ=HFMAX*DSQRT((1.0D0-(Y-YCT))/HFMAEX)**2
Z(1,I)=ZZ+ECT
Z(2,I)=ZZ+ECT
430 CONTINUE
444 L=0
N=1
CALL SETM(0.0,PBLK,5)
YQ=P(1,1)
IDEX=INT((YQ-GRIDLO)/GRID+1.001)
DO 900 I=1,NPOINT
IF (P(1,I).EQ.YQ) GO TO 480
IF (L.GT.2) GO TO 470
N=N-1
470 L=0
YQ=P(1,1)
IDEX=INT((YQ-GRIDLO)/GRID+1.001)
480 PBLK(1)=P(1,1)
PBLK(2)=P(2,1)
TEST=-1.0
IF (P(2,1).EQ.Z(1,IDEX).OR.P(2,1).EQ.Z(2,IDEX)) TEST=0.0
IF (P(2,1).LT.Z(1,IDEX).AND.P(2,1).LT.Z(2,IDEX)) TEST=1.0
TESTBL=HFMABL*HFMABL*HFMIBL*HFMIBL
  -HFMABL*HFMABL*(P(2,1)-ZCBL)*(P(2,1)-ZCBL)
  -HFMIBL*HFMIBL*(P(1,1)-YCBL)*(P(1,1)-YCBL)
IF (TEST) 701,501,501
501 IF (TESTBL.LE.0.0) GO TO 510
CALL MOVEM(PBLK,P(1,N),5)
GO TO 515
510 CALL MOVEM(P(1,I),P(1,N),5)
515 N=N+1
L=L+1
IF (TEST.EQ.0.0) GO TO 800
701 IF (L.EQ.0) GO TO 800
IF (TEST*TESTO) 704,800,800
704 CALL INTPL(POLD,P(1,I),PINT,1,IDEX)
NCHG=0
IF (TEST.LT.0.0) GO TO 711
CALL MOVEM(P(1,N-1),P(1,N),5)
NCHG=1
711 CALL MOVEM(PINT,PBLK,2)
TESTBL=HFMABL*HFMABL*HFMIBL*HFMIBL
  -HFMABL*HFMABL*(PINT(2)-ZCBL)*(PINT(2)-ZCBL)
  -HFMIBL*HFMIBL*(PINT(1)-YCBL)*(PINT(1)-YCBL)
IF (TESTBL.LE.0.0) GO TO 720
CALL MOVEM(PBLK,P(1,N-NCHG),5)
GO TO 725
720 CALL MOVEM(PINT,P(1,N-NCHG),5)
725 N=N+1
L=L+1
800 CALL MOVEM(P(1,I),POLD,5)
TESTO=TEST
CONTINUE
900 NPOINT=N-1
NLOC=5*NPERIM
CALL MOVEM(PR5,P(1,N),NLOC)
IF (I0(4,1).EQ.1) WRITE(7,9) NPOINT,ICALL
TEST=-1.0
IF (I0(1,ICALL).EQ.0) RETURN
PRINT 950, YMIN,YMAX,ZMIN,ZMAX,CMIDLO,CRIDNI,GRID,NBARS,NPOINT
950 FORMAT(//" QUANTIZING DATA-"//
  ."  POINT PATTERN EXTENTS ON APERTURE PLANE.......YMIN=",F7.2/
  .........YMAX=",F7.2/
  ........ZMIN=",F7.2/
  ........ZMAX=",F7.2/
  ...."GRID RANGES FROM..................",F8.3," TO","F8.3/
  ....SPACING BETWEEN GRID BARS IS................",F8.4/
  .... THEREFORE NUMBER OF GRID BARS........................",I4/)
SUBROUTINE INTGRT(P, MAJOR, MINOR, FLDY, FLDZ)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 MAJOR, MINOR
COMPLEX*16 CTE14P, CZI, C22, CY1, CY2, TS1, DS1, DY1, ZIOLD, YIOLD,
YI, FLDY, FLDZ, FIELDZ(200), FIELDY(200)
INTEGER SURFCE
COMMON/MATH/PI, PI2, PID2, OTOR, RTOD
COMMON/PARAMS/AORORF, BELLP, PHI, PLNPMT(3), PLNORM(3), FEED(3),
ALPHA, BETA, GAMMA, XLM, XC, SURFCE, NPML, NPOINT
DIMENSION AMINOR(3), P(S, 2750)
DATA NPHI, NTHETA/5NPHE/, SMTMETA/
SEH=999.0
NPART=7
NPAR=1/NPARTS
ZLAM=PIZ/XLM
CALL SETM(SEN, P(I, NPOINT+1), S)
DEG=MAJOR
DEGR=DEG*DTOR
DLOR=AMINOR(1)*DTOR
DICR=AMINOR(3)*DTOR
DSTOP=AMINOR(2)*DTOR DICR+0.5
MTH=0
D=DLOR
IF (MAJOR .EQ. NPHI) GO TO 3400
400 COSP=DCOS(DEGR)
SINP=DSIN(DEGR)
COST=DCOS(D)
SINT=DSIN(D)
GO TO 3425
1400 COST=DCOS(D)
SINT=DSIN(D)
COST=DCOS(DEGR)
SINT=DSIN(DEGR)
1425 MTH=MTH+1
CTSP=COST*SINP
ZK=ZLM***COST
YK=ZLM*SINP*SINT
IOLD=1
INEX=2
FLDY=(0.0, 0.0, 0.0)
FLDZ=(0.0, 0.0, 0.0)
YOLD=SEN
Y=(0.0, 0.0, 0.0)
ZI=(0.0, 0.0, 0.0)
1450 CONTINUE
IF (P(1, IOLD) .NE. P(1, INEX)) GO TO 4000
X=P(2, IOLD)
ERY=P(3, IOLD)
ERZ=P(4, IOLD)

PH = P(5, IOLD)
DZ = (P(2, INEW) - Z) * RPART
DERZ = (P(3, INEW) - ERY) * RPART
DPH = (P(5, INEW) - PH) * RPART
CTEMP = CDEXP(DCMPLX(0.0D0, ZK*Z-PH))
CZ1 = ERZ*COSP*CTEMP
CY1 = (ERY*SINT+ERZ*CTSP)*CTEMP
TSY = (0.0, 0.0)
TSZ = (0.0, 0.0)
DO 3700 N=1, NPARTS
Z = Z + DZ
ERY = ERY + DERY
ERZ = ERZ + DERZ
PH = PH + DPH
CTEMP = CDEXP(DCMPLX(0.0D0, ZK*Z-PH))
CZ2 = ERZ*COSP*CTEMP
CY2 = (ERY*SINT+ERZ*CTSP)*CTEMP
TSZ = TSz + CZ1 + CZ2
TSY = TSY + CY1 + CY2
CZ1 = CZ2
CY1 = CY2
3700 CONTINUE
ZI = ZI + TSZ*(.5*DZ)
YI = YI + TSY*(.5*DZ)
3900 IOLD = IOLD + 1
INEW = INEW + 1
GO TO 3450
4000 CONTINUE
YNEW = P(1, IOLD)
IF(YOLD .EQ. SEN) GO TO 4400
4200 DZI = (ZI - ZIOLD) * RPART
DYI = (YI - YIOLD) * RPART
DY = (YNEW - YOLD) * RPART
CTEMP = CDEXP(DCMPLX(0.0D0, YK*YOLD))
CZ1 = ZIOLD*CTEMP
CY1 = YIOLD*CTEMP
TSY = (0.0, 0.0)
TSZ = (0.0, 0.0)
DO 4300 N=1, NPARTS
YOLD = YOLD + DY
ZIOLD = ZIOLD + DZI
YIOLD = YIOLD + DYI
CTEMP = CDEXP(DCMPLX(0.0D0, YK*YOLD))
CZ2 = ZIOLD*CTEMP
CY2 = YIOLD*CTEMP
TSZ = TSz + CZ1 + CZ2
TSY = TSY + CY1 + CY2
CZ1 = CZ2
CY1 = CY2
4300 CONTINUE
FLDZ = FLDZ + TSZ*(.5*DY)
FLDY = FLDY + TSY*(.5*DY)
4400 CONTINUE
YOLD = YNEW
ZIOLD = ZI
YIOLD = YI
YI = (0.0, 0.0)
ZI = (0.0, 0.0)
IF(P(1, INEW) .NE. SEN) GO TO 3900
FIELDY(NTH) = FLDY
FIELDZ(NTH) = FLDZ
D = D + DICK
IF (D.GT.DSTOPR) GO TO 5000
IF (MAJOR.EQ.HPHI) GO TO 400
GO TO 3400
5000 CONTINUE
RETURN
END
APPENDIX E

OUTPUT FOR THE EXAMPLE
**OFFSET-FED PARABOLIC DISH RUSCH/AEROJET**  
**ILLUMINATION INTERPOLATED FROM TABLES; NO BLOCKAGE; SINGLE-PANEL OPTION**  
**USED IN THIS RUN.**

**INPUT PARAMETERS—**

<table>
<thead>
<tr>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>WAVELENGTH OF ELECTRIC FIELD</td>
<td>2.3670</td>
</tr>
<tr>
<td>LOCATION OF COORDINATE ORIGIN WRT FEED ( (x,y,z) )</td>
<td>(-1.129 ), 0.0, 0.0</td>
</tr>
<tr>
<td>FEED ROTATION ANGLES ( (\alpha, \beta, \gamma) )</td>
<td>0.0, 0.0, (-40.300 )</td>
</tr>
<tr>
<td>FOCAL LENGTH OF THE REFLECTOR</td>
<td>31.500</td>
</tr>
<tr>
<td><strong>PROGRAM IN SINGLE PANEL MODE</strong></td>
<td></td>
</tr>
<tr>
<td>MINIMUM-Y POINT ON THE REFLECTOR ( (x,y,z) )</td>
<td>(-17.060 ), 0.0, 42.654</td>
</tr>
<tr>
<td>MAXIMUM-Y POINT ON THE REFLECTOR ( (x,y,z) )</td>
<td>(-31.416 ), 0.0, 3.247</td>
</tr>
<tr>
<td>MINIMUM-Z POINT ON THE REFLECTOR ( (x,y,z) )</td>
<td>(-24.244 ), 19.685, 22.950</td>
</tr>
<tr>
<td>MAXIMUM-Z POINT ON THE REFLECTOR ( (x,y,z) )</td>
<td>(-24.244 ), 19.685, 22.950</td>
</tr>
<tr>
<td>APERTURE PLANE LOCATION ( (x,c) )</td>
<td>(-16.00 )</td>
</tr>
<tr>
<td>FEED SHADOW CENTER COORDINATES IN APERTURE PL ( (x,c) )</td>
<td>0.0, 0.0</td>
</tr>
<tr>
<td>HALF MAJOR AXIS OF FEED SHADOW ( (c) )</td>
<td>0.0</td>
</tr>
<tr>
<td>HALF MINOR AXIS OF FEED SHADOW ( (c) )</td>
<td>0.0</td>
</tr>
<tr>
<td>NUMBER OF PANELS IN REFLECTOR</td>
<td>1</td>
</tr>
</tbody>
</table>
NUMBER OF PATTERN GROUPS REQUESTED: 

PHI = 0.0
THETA = 1/2,000

THETA FROM 68.0000 TO 98.0000 BY 0.5000
PHI FROM -15.0000 TO 15.0000 BY 0.5000

ILLUMINATION DATA

THETA ILLUMINATION FROM 57.704 TO 116.651
PHI ILLUMINATION FROM 149.186 TO 210.814
INCREMENTAL ANGLE (DEG) 1.3397
THEORETICAL TOTAL NUMBER OF GENERATED RAYS 2115
TOTAL NUMBER OF APERTURE PLANE POINTS 2119

------------ FINISHED APRTUP --------------

QUANTIZING DATA

POINT PATTERN EXTENTS ON APERTURE PLANE
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MAX= 19.73
ZMIN= 5.36
ZMAX= 42.72

GRID RANGES FROM -19.550 TO 19.550
SPACING BETWEEN GRID BARS 0.8887

THEORETICAL NUMBER OF GRID BARS 1643
NUMEBER OF POINTS SUPPLIED TO RADPAT 48

------------ FINISHED QUANTZ --------------
**** EXECUTED APRPLT ****
------------ FINISHED INTGRT --------------
------------ FINISHED INTGRT --------------

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**Table of Electric Field Strengths**

**Principal Plane of Cut is PHI = 0.0 Deg**

Angle Theta from 68.000 to 98.000 by 0.500 Deg
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\[ 20 \log (\text{MAX(FIELD-2)}) = 20 \log (\text{MAX(FIELD-1)}) \]

\[ \text{INTERPOLATION NUMBER USED FOR INTEGRATION} \]
PARABOLIC REFLECTOR FAR FIELD RADIATION PATTERN CALCULATION

OFFSET-FEED PARABOLIC DISH
RUSCH/TAEROJET
ILLUMINATION INTERPOLATED FROM TABLES; NO BLOCKAGE
REFLECTOR IS ONE PANEL DEFINED BY 20 PERIMETER POINTS
MAY 22, 1980
NGC
PGM: KOJULA
FCTY: HILLSBRO

INPUT PARAMETERS:

WAVELENGTH OF ELECTRIC FIELD: *********** 2.747
LOCATION OF COORDINATE ORIGIN WRT FEED (X,Y,Z)... -1.126 0.0 0.229
FEED ROTATION ANGLES (ALPHA,BETA,GAMMA)........... 0.0 0.0 -40.0
FOCAL LENGTH OF THE REFLECTOR........................ 71.500
APERTURE PLANE LOCATION (Xc)............................ -16.00
FEED SHADOW CENTER COORDINATES IN APERTURE PLAT... 0.0 0.0 0.0
HALF MAJOR AXIS OF FEED SHADOW.......................... 0.0
HALF MINOR AXIS OF FEED SHADOW.......................... 0.0
NUMBER OF PANELS IN REFLECTOR.......................... 1

PATTERN OF FEED IN ONE DEG INCREMENTS OFFSET-

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## PATTERN OF FEED IN ONE DEG INCREMENTS OFF-AXIS -

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<td>THETA = 83.0000</td>
<td>PHI FROM -15.0000 TO 15.0000 BY 0.5000</td>
</tr>
</tbody>
</table>
REFLECTOR PANEL NUMBER 1

PANEL IS A PARABOLIC SECTION

FOCAL LENGTH OF THE PARABOLA: 31.50
NUMBER OF USER-SUPPLIED EDGE POINTS: 20

ILLUMINATION DATA:

THETA ILLUMINATION FROM: 57.280 TO 117.056
PHI ILLUMINATION FROM: 148.817 TO 211.183
INCREMENTAL ANGLE (DEG): 1.2997
THEORETICAL TOTAL NUMBER OF GENERATED RAYS: 2303
TOTAL NUMBER OF APERTURE PLANE POINTS: 2323

------------------------- FINISHED APERTUR ---------------

QUANTIZING DATA:

POINT PATTERN EXTENTS ON APERTURE PLANE:

\[ Y_{MIN} = -19.73 \]
\[ Y_{MAX} = 19.73 \]
\[ Z_{MIN} = 5.39 \]
\[ Z_{MAX} = 42.71 \]

GRID RANGES FROM: -19.558 TO 19.558

STAMPING BETWEEN GRID BAPS IS: 0.8503
THEORETICAL NUMBER OF GRID BAPS: 47
NUMBER OF POINTS SUPPLIED TO RACPA: 1729

------------------------- FINISHED QUANTY -------------------

**** EXECUTED APFRT ****

------------------------- FINISHED INTEGR -------------------

- PATTERN COMPUTATIONS COMPLETE ---------------

**** EXECUTED APRMAP ****
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Note: The table continues with more data points for different values of Theta.
### Table of Electric Field Strength (100)

Principal Plane of Cut is Theta = 93,000 Deg

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<th>Angle Phi from -15,000 to 15,000 by 0.500 Deg</th>
<th>D(V/Z)</th>
<th>D(V/Y)</th>
<th>D(V/Y)</th>
<th>D(V/Y)</th>
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</thead>
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Note: The table values are approximate and may require further calculation or correction for precision.
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</table>

**MAXIMUM FIELD VALUES:**

20LOG(MAX(FIELD-Z)) = 20LOG( 1.0273426D 00 ) = 0.2343062

20LOG(MAX(FIELD+Y)) = 20LOG( 1.678170D 05 ) = 0.8516667

**INTERPOLATION NUMBER USED FOR INTEGRATION IS:**

7
### Parabolic Reflector Far Field Radiation Pattern Calculation

**Case Three**

**Illumination Interpolated From Tables: No Blockage; Three Panels**
- Panel One: 20 Points
- Panel Two: 21 Points
- Panel Three: 21 Points

**Input Parameters**:
- Wavelength of Electric Field: 2.1 GHz
- Location of Coordinate Origin WRT Feed (X,Y,Z): -1.129, 0.0, 0.228
- Feed Rotation Angles (Alpha, Beta, Gamma): 0.0, 0.0, -40.300
- Focal Length of the Reflector: 31.500
- Aperture Plane Location Xc: -15.00

**Feed Shadow Center Coordinates in Aperture Plane**
- Half Major Axis of Feed Shadow: 0.0
- Half Minor Axis of Feed Shadow: 0.0

**Pattern of Feed in One Deg Increments Off-Axis**

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### Pattern of Feed in One Deg Increments Off-Axis

**Theta-Plane**

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**Number of Pattern Groups Requested**

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<th>Theta from 08.0000 to 08.0000 by 0.5000</th>
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**Theta = 83.0600** | Phi from -15.0000 to 15.0000 by 0.5000
**Reflector Panel Number 1**

Panel is a Parabolic Section

**Focal Length of the Parabola:**

| Number of User-Supplied Edge Points | 11.50 |

**Illumination Data:**

- **Theta Illumination From:** 61.131 to 108.416
- **Phi Illumination From:** 178.818 to 211.326
- Therefore total number of generated rays: 1,4776
- Total number of aperture plane points: 773

---

**Quantizing Data:**

- **Point Pattern Extents on Aperture Plane:**
  - **XMIN** = 19.73, **YMIN** = 0.00
  - **XMAX** = 0.00, **YMAX** = -8.99
  - **XMIN** = 36.35
- **Grid Ranges From:**
  - -19.535 to -9.193
- **Spacing Between Grid Raps is:**
  - 1.0471
- Therefore number of grid raps: 21
- Number of Points Supplied to RAPDAT: 405

---

**Finished Quant?**

- Executed Apfel 8888
- Finished Integ 8888
REFLECTOR PANEL NUMBER 2

PARAMETERS:
- PANEL IS A PARABOLIC SECTION
- FOCAL LENGTH OF THE PARABOLA: 31.50
- NUMBER OF USER-SUPPLIED EDGE POINTS: 21

ILLUMINATION DATA:
- THETA ILLUMINATION FROM: 56.916 TO 84.887
- PHI ILLUMINATION FROM: 140.252 TO 197.047
- INCREMENTAL ANGLE (DEG): 1.2574
- THEREFORE TOTAL NUMBER OF GENERATED RAYS: 897
- TOTAL NUMBER OF APERTURE PLANE POINTS: 918

FINISHED AERTUP

QUANTIZING DATA:
- POINT PATTERN EXTENTS ON APERTURE PLANE:
  - YMIN = -11.56
  - YMAX = 19.73
  - ZMIN = 23.82
  - ZMAX = 42.71
- GRID RANGES FROM:
  - 11.391 TO 19.886
- THEREFORE NUMBER OF GRID RAYS: 37
- NUMBER OF POINTS SUPPLIED TO PADDAT: 560

FINISHED QUANTZ

FINISHED INTGRT

FINISHED INTGRT
REFLECTOR PANEL NUMBER 3

PANEL IS A PARABOLIC SECTION

FOCAL LENGTH OF THE PARABOLA .......................... 31.50
NUMBER OF USER-SUPPLIED EDGE POINTS ....................

ILLUMINATION DATA:

THETA ILLUMINATION FROM .................................. 80.971 TO 116.367
PHI ILLUMINATION FROM .................................. 148.569 TO 203.272
INCREMENTAL ANGLE (DEG) .................................. 1.6099
TOTAL NUMBER OF GENERATED RAYS ......................... 624

----------------- FINISHED APRPLD -----------------

QUANTIZING DATA:

POINT PATTERN EXTENTS ON APERTURE PLANE ............... YMIN = -11.70
y................... YMAX = 19.73
................... ZMIN = 5.38
................... ZMAX = 24.24

GRID RANGES FROM .................................... 11.593 TO 25.178
SPACING BETWEEN GRID PANS TO ......................... 0.5751
THEORETICAL NUMBER OF GRID POINTS .................. 23
NUMBER OF POINTS SUPPLIED TO RADMAT ............ 520

----------------- FINISHED PLANTZ -----------------

**** EXECUTED APRPLT ****

----------------- FINISHED INTGRT -----------------

----------------- PATTERN COMPUTATIONS COMPLETE -----------------

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**MAXIMUM FIELD VALUES:**

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\[ \text{20LOG}(\text{MAX(FIELD-Y)})=20LOG(1.65813930-03)=16.036256 \]

**INTERPOLATION NUMBER USED FOR INTEGRATION IS:** 7
### TABLE OF ELECTRIC FIELD STRENGTHS (129)

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**Angle Phi** from -15.000 to 15.000 by 0.500 Deg

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CASE II

MAP OF PANEL PROJECTIONS

ORIGINAl PAGE IS
OF POOR QUALITY
APERTURE PLANE AFTER QUANTIZING
APERTURE PLANE AFTER QUANTIZING

CASE III
CASE III

MAP OF PANEL PROJECTIONS

X-AXIS

Y-AXIS

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0.0

45.0 40.0 35.0 30.0 25.0 20.0 15.0 10.0 5.0