ORBIT/ATTITUDE ESTIMATION FOR THE GOES SPACECRAFT USING VAS LANDMARK DATA

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ABSTRACT

A software system is described which provides for batch least-squares estimation of spacecraft orbit, attitude, and camera bias parameters using image data from the Geostationary Operational Environmental Satellites (GOES). The image data are obtained by the Visible and Infrared Spin Scan Radiometer (VISSR) Atmospheric Sounder (VAS). The resulting estimated parameters are used for absolute image registration. Operating on the Digital Equipment Corporation (DEC) PDP-11/70 computer, the FORTRAN system also includes the capabilities of image display and manipulations. An overview of the system is presented as well as some numerical results obtained from observations taken by the SMS-2 satellite over a 3-day interval in August 1975.
SECTION 1 - INTRODUCTION

A variety of spacecraft (S/C) exist which transmit images to the ground to provide meteorological and Earth resource information. Several studies have been concerned with the use of this imaging data for the estimation of the S/C orbit and attitude. Such an estimation procedure can be used for several purposes. The one with which this report is concerned is the use of the estimated S/C orbit and attitude (O/A) parameters for absolute image registration. The estimated O/A parameters are used to predict the geodetic latitude and longitude (\( \phi, \lambda \)) which correspond to a specified location on an Earth picture. This allows accurate geodetic coordinate determination for temporal phenomena, such as clouds or sea swells.

There are two categories of image data; those from three axis stabilized S/C and those from spin-stabilized S/C.

The Landsat and Earth Resource Technology Satellites (ERTS) are examples of three axis stabilized S/C. These produce image data from high inclination (polar) close Earth (900 km altitude) orbits. The use of this data is discussed in Reference 1 which describes a software system for the display and manipulation of image data as well as the use of an extended Kalman filter estimator for the O/A parameter determination.

The geosynchronous Geostationary Operational Environmental satellites (GOES) are examples of spin-stabilized S/C which produce image data. An overview of O/A estimation using this type of data is given in Reference 2, where sample numerical results are presented for the first geostationary Synchronous Meteorological Satellite (SMS-1).
This paper describes a software system developed to provide Bayesian weighted least-squares estimation of spacecraft orbit and attitude parameters using picture data obtained from the VAS (VISSR Atmospheric Sounder) instrument to be flown on the GOES-D. The data consist of ground control points of known geodetic coordinates located on pictures of the Earth taken by the GOES spacecraft. The VAS/NAVPAK (VISSR Atmospheric Sounder Navigation Package) system operates on the Digital Equipment Corporation PDP 11/70 computer.
As shown in Figure 1, the VAS/NAVPAK system can be divided into four functions. First, the Data Base Management (DBM) portion controls file and data manipulation. Second, the picture display and cursor navigation portion controls: (1) picture display on the I2S, such as image zooming; (2) cursor navigation, including the extraction of picture coordinates \((\ell, e)\) and the automatic moving of the cursor to the picture coordinates corresponding to a specified longitude and latitude; (3) automatic grey scale correlation between a prestored chip (16 x 16 pixel reference landmark) and a search area about the cursor; (4) the creation of landmark observations. The third VAS/NAVPAK function is the O/A and camera bias estimation. This portion of the system provides for weighted least-squares (DC) estimation of the satellite orbit, attitude, and camera biases. The fourth VAS/NAVPAK function produces the specific navigation parameters which are required over a specified prediction interval (usually 2 days). The navigation parameters are used to annotate the picture data.

2.1 Picture Display and Cursor Navigation

Cursor navigation is the prediction of picture coordinates \((\ell, e)\) corresponding to a specified geodetic latitude and longitude, given the estimated satellite orbit and attitude and the camera biases for some epoch time.

This is the method by which a prestored video reference area (taken from a VAS picture) is correlated with an area surrounding the cursor on the image displayed by the operator.

2.2 Orbit/Attitude Estimation

The S/C O/A estimation is done with the classical Bayesian weighted least-squares technique. The estimator can use either landmark data, radar tracking data, or both. Only the capability for using landmark data will be
Figure 1. VAS/NAVPAK Overview
presented. It is assumed that the working observation files of landmark data have been created before beginning the O/A estimation.

The computational procedure followed for the O/A proceeds in the following steps:

1. **An a priori estimate is provided of the solve-for parameters.** These parameters will be a subset of:

   \[ \begin{align*}
   \overline{\mathbf{r}}_0, \overline{\mathbf{v}}_0 & \quad \text{the S/C position and velocity} \\
   \chi_i, i=1,5 & \quad \text{S/C attitude model} \\
   \psi_i, i=1,5 & \quad \text{coefficients} \\
   \xi & \quad \text{camera bias} \\
   \rho & \quad \text{camera bias} \\
   \Delta \gamma_0 & \quad \text{camera bias}
   \end{align*} \]

2. **For each observation, the S/C position and velocity are found by integrating the equations of motion to the observation time, } t_{\text{obs}}.** For the VAS/NAVPAK system, the integration is performed with a 12th order Cowell method, as described in Reference 3. The force model is selectable by the user and can include a spherical harmonic geopotential expansion terms up to 21 x 21, lunar/solar third body perturbations, and solar radiation pressure.

3. **For each observation time, an observation (\ell, e) pair and partial derivatives are computed corresponding to the geodetic coordinates (\phi, \lambda) of the landmark using the S/C position, velocity, attitude, and camera biases.**

4. **The computed observation pair is used to calculate the observation residuals.** The residual is examined to see if it meets the editing criteria. If it does not, it is not used in the solution.
5. After steps 2, 3, and 4 have been performed for all the observations, the new estimate of the epoch S/C state, the attitude, and camera biases, and their covariance matrix, is computed.

6. The new estimate of the solve-for parameters are compared with the previous to see if the least squares process has converged. If the solution is judged to have not converged, the new estimate replaces the a priori in step 1, and the process is repeated.

2.3 Navigation Parameter Output

Spacecraft parameters can be generated for a sequence of overlapping time intervals covering a specified output span. These parameters include spacecraft ephemerides, attitude information, camera biases, eclipse times, and Chebyshev coefficients for position, beta angle, and retransmission correction.
SECTION 3 - THE OBSERVATION MODEL

The observational model in VAS/NAVPAK is a modification of that used in the SMS NAVPAK (Reference 4). The camera bias and attitude representations for the VAS/NAVPAK observational model were reformulated, consulting the VAS working group (Reference 5) and with the assistance of R. Pajerski (GSFC).

The SMS and GOES are geosynchronous spinning spacecraft designed for taking pictures of the Earth in several wavelengths. A camera, or VISSR (Visible and Infrared Spin Scan Radiometer), transmits data to a ground station where a complete picture of the Earth is assembled. The data consist of a grid or matrix of intensity measurements. A line number and an element number specify the location of the intensity measurement within the grid. The line number ℓ, corresponds roughly to longitude. These are shown schematically in Figure 2. For the visible wavelength observations, each picture element (pixel) intensity measurement corresponds nominally to an area on Earth of dimension 1/2 mile by 1/2 mile square. Of course, near the edge of the Earth, foreshortening will enlarge and distort this square. Options exist to handle data whose dimensions are integer multiples of this unit (i.e., 2-mile by 4-mile data). Associated with each line of the picture is a time and angular quantity which relates the starting position of the line to the direction of the Sun in inertial space.

At the ground station preprocessing is performed and full resolution picture segments of 1024 x 1024 pixels are generated. In order to create a landmark observation, the operator first displays a picture or subset of a picture on the 12S. Then, an identification is made of a particular location on the picture (ℓ, e) pair which corresponds to a known geodetic latitude and longitude on Earth. The geodetic coordinates and the picture coordinates with associated quantities such as time and Sun angle are transferred to an observation file. This constitutes a single landmark observation pair.
Figure 2. Schematic VAS/GOES Picture
Figure 3 shows the GOES satellite relative to the earth at an instant of time. Except for specific camera constants, the SMS is almost identical to the GOES. Both satellites are cylindrical spinning objects with the longitudinal symmetry axis nearly aligned with the spin axis. The spin axis in turn is nearly aligned with the polar axis of the Earth pointed southward. As the satellite spins, the camera scans across the face of the Earth's disk, from west to east measuring the light intensity for each pixel along a line. The relation between the \((\ell, e)\) coordinates of each picture and the camera orientation can be shown by comparing the image in Figure 2 with Figure 3. The element, \(e\), is related to the azimuthal camera angle, \(q\). This angle is measured in the satellite spin plane and is the angle between the line of sight (LOS) vector to the landmark and the LOS vector to the left (west) edge of the Earth. The conversion to line element is

\[
e = \frac{q}{\text{RPE}}
\]

where RPE is the number of radians per line element. The satellite spin plane in Figure 4, perpendicular to the spin axis \(z'\), is shown coincident with the spacecraft (S/C) symmetry plane, perpendicular to the S/C longitudinal symmetry axis \(z_{S/C}\). In the actual development of the observation equations the general case of a misaligned spin axis is considered.

The line number related to the camera elevation angle, \(\alpha\), as

\[
\ell = \frac{\alpha}{\text{RPL}} + \ell_0
\]

where RPL is the number of radians per line and \(\ell_0\) is the line number which corresponds to a zero elevation setting of the camera.

The relation of the picture coordinates \((\ell, e)\) to coordinates of a location on the Earth \((\phi, \lambda)\) depends upon the spacecraft position and attitude, and the camera constants and biases. Several coordinate system transformations are required to express this relation.

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Figure 3. VAS/OES Configuration
The satellite spin plane coordinate system, the \((x', y', z')\) system in Figure 3, must be related to the Earth inertial coordinate system in which the satellite position is computed. Figure 4 shows a spacecraft spin plane coordinate system relative to true-of-date coordinates. The \(x'\) axis lies in the true-of-date \((xz)\) plane at an angle of \(\chi\) with respect to the true-of-date \((-x)\) axis. The \(y'\) axis forms a right hand orthogonal system.

The transformation matrix \(S\) from the \((x, y, z)\) system into the \((x', y', z')\) system is

\[
\begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix} = S \cdot \begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
-cos\chi & 0 & sin\chi \\
-sin\psi sin\chi & cos\psi & cos\chi sin\psi \\
-cos\psi sin\chi & sin\psi & -cos\psi cos\chi
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]

(3)

Since the positive spin axis \(z'\) is nearly aligned with the negative \(z\) axis of the true-of-date system, the angles \(\chi\) and \(\psi\) will always be relatively small. Also shown on Figure 4 are the right ascension and declination angles \((\alpha, \delta)\) which are conventionally used to represent the location of the \(z'\) axis. The declination angle is near -90 degrees. The relation of \((\chi, \psi)\) to \((\alpha, \delta)\) is

\[
\tan \chi = \frac{\cos \alpha}{\tan \delta}
\]

\[
\sin \psi = \sin \alpha \cos \delta.
\]

The location of the \(z'\) axis in \((\alpha, \delta)\) is expressed as a time varying function as

\[
\delta = \delta_0 + \delta_1 t + \delta_2 \sin(\delta_3 t + \delta_4)
\]

(5)

and

\[
\alpha = \alpha_0 + \alpha_1 t + \alpha_2 \sin(\alpha_3 t + \alpha_4)
\]

(6)

The model represented by equations (5) and (6) is a symmetric one. Because of the spin stability of the \(S/C\) axis, perturbations to \((\alpha, \delta)\) or \((\chi, \psi)\) are expected to be small.
NOTES: (1) \((x, y, z)\) is the true-of-date system. The \((xy)\) plane is the true-of-date Earth's equatorial plane, while \(z\) points along the Earth's axis of rotation.

(2) \((x', y', z')\) is the spacecraft spin system. The positive \(z'\) axis lies roughly in the direction of the \(-z\) true-of-date axis.

(3) The \(x'\) axis lies in the \(xz\) plane at an angle \(X\) above the \(-x\) axis. Hence, \(X\) is measured in the \((x, z)\) (or, equivalently, \((-x, z)\)) plane from the \(-x\) axis to the \(+x'\) axis. The angle \(X\) is measured positive towards the \(+z\) axis.

Figure 4. True-of-Date and Spin Plane Coordinates
Since the spin axis is nearly aligned with the negative z axis of the true-of-date system, the right ascension angle, $\alpha$, is sensitive to the precision with which it is computed. For example, if the magnitude of the xy plane projection of $z'$ (line OA in Figure 4) is nearly zero, then a change in sign would cause $\alpha$ to change by 180 degrees. Such a change can occur on successive iterations in the estimation process. The result would be to create divergent oscillations in the attitude correction vector $(\alpha, \delta)$. Therefore, it is advantageous to use the $(X, \psi)$ coordinates for the spin axis location.

The angle $\psi$ is analogous to declination and is the angle between the xz true-of-date plane and the spin axis. It is measured from the xz plane (perpendicular to the xy plane) to the $z'$ axis. The angle $\chi$ is analogous to right ascension and is the angle between the z axis and the projection of the spin axis onto the xz plane.

The model for the $(X, \psi)$ coordinates of the spin axis can be written in a form similar to those of equations (5) and (6)

$$\psi = \psi_0 + \psi_1 t + \psi_2 \sin(\psi_3 t + \psi_4) \quad (7)$$

and

$$\chi = \chi_0 + \chi_1 t + \chi_2 \sin(\chi_3 t + \chi_4). \quad (8)$$

Figure 5 shows the S/C symmetry coordinate system. The $z_{S/C}$ axis is parallel to the S/C longitudinal symmetry axis and $x_{S/C}$ points to the zero elevation angle in the actual VISSR plane. The S/C symmetry plane is perpendicular to the S/C symmetry axes and is the reference plane from which the true camera elevation is measured. Two VISSR planes are shown; the actual VISSR plane is the plane swept out in elevation as the camera is moved from one spin cycle to another, while the nominal VISSR plane is the plane in which the camera motion is supposed to occur. The angle (measured in the symmetry plane) between the sun sensor plane and the nominal VISSR plane is $\gamma_0$. 
Figure 5. Spacecraft Symmetry Coordinate System
The angle measured in the S/C symmetry plane between the actual and nominal VISSR planes, ΔYo, is the first camera bias. A second camera bias is ι, which is an elevation (or line) bias angle. When the camera is set at a zero elevation setting, represented by line xS/C0, its true elevation angle is ι.

Angle a is the elevation of a landmark above (or below) the nominal zero elevation point and Δa is the amount by which the elevation is incremented each spin cycle. At the beginning of a picture, a is set to a negative value corresponding to the northern part of the earth and then incremented to positive values towards the southern portion of the earth.

Figure 6 shows the spin plane (or attitude) coordinate system first introduced in Figure 4. Because of the inertial motion of the spin axis (equations (7) and (8)) and the rotation of the earth, the location of a landmark with respect to the spin frame is changing. Moreover, the daily motion of the sun and the spin axis inertial motion causes the solar positions to change with respect to the spin coordinates. However, at the time of a landmark observation, t_S, the azimuth of a landmark, γ^2_S, and the azimuth of the sun, γ^1, can be determined with respect to the spin system.

Figure 7 shows the spin coordinate system relative to the S/C symmetry frame. The symmetry frame is rotating with respect to the spin frame but Figure 7 depicts the instant that the VISSR plane intersects with x' axis of the spin frame. Notice that the xS/C axis is shown coincident with the x' axis at this instant. This choice is tantamount to forcing the z' axis to lie in the yS/Cz'S/C plane. This choice is allowable because the bias ι can absorb the elevation difference (between x' and xS/C) which would occur if z' did not lie in the yS/Cz'S/C plane at this moment.

An angle ρ is defined as the angle, measured in the spin plane, between the nominal VISSR and actual VISSR planes. This represents an azimuthal bias which allows the modeling of error in the azimuthal location of the VISSR plane.
Ib Z

*SPIN AXIS

...l

DI-electro

x ° (x', y', z') IS THE SPIN COORDINATE SYSTEM; Z' POINTS ALONG THE SPIN AXIS, Y' IS DEFINED SO THAT IT IS NORMAL TO THE SPIN AXIS AND LIES IN THE TRUE-OF-DATE XY PLANE, X' FORMS A RIGHT HANDED SYSTEM WITH Y' AND Z'.

γ₁ THE AZIMUTH OF THE SUN IN THE SPIN PLANE

γ₂ THE AZIMUTH OF THE LANDMARK IN THE SPIN PLANE.

θ' IS THE ELEVATION ANGLE OF A LANDMARK ABOVE (OR BELOW) THE SPIN PLANE.

Figure 6. Spin Plane Coordinate System
Z' lies in the Z_{S/C} Y_{S/C} plane

α' is the elevation angle of the landmark in the spin system

α + δ is the corresponding elevation of the landmark measured in the actual VISSR plane

δ is the camera elevation misalignment bias.

Figure 7. Camera and Spin Biases
The angle $\gamma$ is the angle in the spin plane between the actual VISSR plane and
the sensor plane, and $a$ is the elevation of the camera at the time a landmark
was observed in the S/C symmetry coordinate system.

Since $\hat{X}_{\text{sun}}, \hat{X}_{\text{sun}}$ and the sun direction form a right spherical triangle,

$$\xi = -\tan^{-1}(\tan A \sin \delta_\odot) .$$

(9)

The picture coordinate, $l$, is the line number or elevation coordinate and is
given by,

$$l = \frac{a - e}{RPL} + \lambda_0$$

(10)

where RPL is the radians/line conversion constant and $\lambda_0$ is the line corre-
spending to $a = 0$. In practice the elevation angle $a'$ is found in the spin plane
and then converted to $a$.

The second picture coordinate, $e$, corresponds to an azimuthal angle (measured
in the spin plane) between the left (west) edge of the earth and the landmark.

The situation is shown in Figure 8 which depicts the spin plane as viewed from
the north. The satellite is spinning clockwise. The picture coordinate, $e$, is
thus,

$$e = \frac{\gamma_2 - \gamma_1 - \beta + \gamma - \xi}{RPE} \mod 2\pi$$

(11)

where RPE is the number of radians per element and $\beta$ is the angle through which
the satellite has turned from the instant of sun observation by the sun sensor
to the observation of the left edge of the earth by the VISSR. The angle $\gamma_1$ is
the azimuth of the sun and $\gamma_2$ the azimuth of the landmark. The angle $\beta$
determined by finding, for each line of the picture, the first or leftmost pixel
of that line. Each revolution, a body-mounted sun sensor on the satellite detects
the sun and produces a sun pulse. For each revolution, a time interval called
the $\beta$-time ($T_\beta$), is computed. This time, which should elapse between the
sighting of the sun by the sun sensor and the alignment of the camera with the
Figure 8. Spin Plane
desired left edge of the earth picture, is used to detect the first element of each line. For each line, the values $T_\beta$ and $t_0$ (time of the average sun pulse) are available as recorded data. Since there are 3144960 counts per half spin

$$\beta = \frac{\pi T_\beta}{3144960} \quad \text{(radians)} \quad (12)$$

or

$$\beta = \frac{\pi (T_\beta - 8 \cdot 165)}{3144960} + \pi \quad (13)$$

when the sun pulse is 180 degrees out of phase.
SECTION 4 - NUMERICAL RESULTS

The sample results shown below (Figure 9) are for a three day span of SMS-2 data obtained from three images taken twenty-four hours apart. Additional preliminary results taken from NAVPAK runs using a longer data span supplied by NOAA indicates that sub-pixel accuracy is possible by using a suitable set of solve-for parameters and a longer, denser data set. The full results of these and other evaluations of VAS/NAVPAK (e.g., force and attitude model evaluations, propagation/prediction capability evaluation, etc.) will be published in a future paper.
ITERATION REPORT FOR ITERATION 3

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END = 750901 150600.00
EPCH = 750630 150600.00

--- OBSERVATION SUMMARY BY TYPE ---

| TYPE / TOTAL NO. / ACCEPTED / WEIGHTED RMS / MEAN RESIDUAL / STANDARD DEV |
|-----------------------------|-------------------------------|---------------------------------|
| ELEM                        | 29                            | 0.6566D+01                      | -0.2211D+01                     | 0.3238D+01                       |
| LINE                        | 24                            | 0.7913D+01                      | -0.1195D-02                     | 0.3956D+01                       |
| RANC                        | 0                             | 0.0000D+00                      | 0.0000D+00                      | 0.0000D+00                       |
| RDIS                        | 0                             | 0.0000D+00                      | 0.0000D+00                      | 0.0000D+00                       |

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Figure 9. Sample Numerical Results
REFERENCES


