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PRELIMINARY RESULTS FROM A FOUR-WORKING SPACE, DOUBLE-ACTING PISTON, STIRLING ENGINE CONTROLS MODEL

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Work performed for
U.S. DEPARTMENT OF ENERGY
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Prepared for
Fifteenth Intersociety Energy Conversion Engineering Conference
Seattle, Washington, August 18-22, 1980
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Washington, D.C. 20545
Under Interagency Agreement EF-77-A-31-1040

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SUMMARY

A four-working space, double-acting piston, Stirling engine simulation is being developed for controls studies. The development method is to construct two simulations, one for detailed fluid behavior, and a second model with simple fluid behavior but containing the four working space aspects and engine inertias, validate these models separately, then upgrade the four working space model by incorporating the detailed fluid behavior model for all four working spaces.

The single working space (SWS) model contains the detailed fluid dynamics. It has seven control volumes in which continuity, energy, and pressure loss effects are simulated. Comparison of the SWS model with experimental data shows reasonable agreement in net power versus speed characteristics for various mean pressure levels in the working space.

The four working space (FWS) model was built to observe the behavior of the whole engine. The drive dynamics and vehicle inertia effects are simulated. To reduce calculation time, only three volumes are used in each working space and the gas temperatures are fixed (no energy equation). Comparison of the FWS model predicted power with experimental data shows reasonable agreement. Since all four working spaces are simulated, the unique capabilities of the model are exercised to look at working fluid supply transients, short circuit transients, and piston ring leakage effects. The FWS model has been upgraded by using the detailed SWS model for each of the four working spaces. Currently the detailed FWS model is being reworked to reduce the amount of calculation time per cycle.

NUMERICAL

A area, m^2
C_p specific heat at constant pressure, J/(kg-K)
C_v specific heat at constant volume, J/(kg-K)
F force, N
f() function of
G gear ratio
h heat transfer coefficient, J/(sec-m^2-K)
I inertia, N-m-sec^2
J mechanical equivalent of heat, 1.0 (N-m)/J

A four-working space, double-acting piston, Stirling engine simulation is being developed at NASA Lewis Research Center for controls studies. The development method is to construct two simulations which together form a complete model. The models are: a single working space model, with detailed fluid behaviour, and a second model, with simple fluid behaviour but containing the four working space aspects and the engine mechanical inertias. The approach is to then combine them into a single detailed controls model.
model. The single-working space (SWS) model was developed to determine the number of control volumes (lumps) and equation types needed to adequately represent the thermodynamics of the engine. The SWS model has been written to approximate a detailed performance model (Ref. 1) developed by Tew. The SWS model differs from the performance model in that the SWS model has fewer control volumes, includes integrated flow pressure drop effects and uses average values of heat transfer coefficients and flow resistances determined from the performance model results. Also the SWS model does not contain shuttle or conduction losses since these only influence efficiency but do not impact control behaviour.

A second model was constructed to study the behaviour of the total engine system. The model consists of four working spaces (FWS) between four pistons. To reduce the calculation time on the digital computer, only three control volumes are used in each working space and gas temperatures are fixed within each volume (no energy equation). The drive dynamics are also included and the FWS model can be run by either forcing the piston motion or as an engine.

Included in this paper are the results obtained from both models compared with some experimental data. Also, since all four working spaces are simulated, the unique capability of the FWS model is exercised to study various phenomena not easily duplicated by a single working space model. To demonstrate this capability, working fluid supply transients, short circuit (braking) effects, and piston ring leakage performance are presented.

Finally, the capabilities of both models have been combined into a single detailed controls simulation which will eventually be used with a detailed engine control system simulation to explore various controls concepts.

**SINGLE WORKING SPACE (SWS) MODEL**

A single working space model of a Stirling engine was developed to determine an adequate representation for the fluid dynamics and thermodynamics of the engine. Seven control volumes were selected one each for the expansion space and heater, three for the regenerator to give a reasonable temperature distribution, and one each for the cooler and compression spaces. A schematic of the model is shown in Fig. 1.

**SWS Model Equations**

The equations used to model the fluid dynamics and thermodynamics are simplifications to the complex partial differential flow equations. Mass flow is assumed to occur due to pressure differential between gas modes. From Fig. 1, some representative equations are, for mass flow:

\[ \dot{w}_i = \frac{1}{\Delta x_{i+1}} (P_i - P_{i+1}) \quad i = 1, 2, \ldots, 6 \]  

(1)

The change in stored mass in a volume is:

\[ \dot{w}_{s_i} = \dot{w}_{i-1} - \dot{w}_i \quad i = 1, 2, \ldots, 7 \]

\[ \dot{w}_0 = \dot{w}_7 = \text{Constant} \]  

(2)
The energy equation used to calculate the change in temperature in a volume is complex due to the oscillatory nature of the Stirling cycle. The form of the equation (Ref. 2) is:

\[ w_s T_i = \omega_{i-1} (\gamma_{i-1} - T_i) - \omega_i (\gamma_i - T_i) + \frac{\Sigma Q}{C_v} + \frac{\Sigma \text{Work}}{C_v} \quad i = 1, 2 \ldots 7 \]

(3)

where the heat flows to and from the gas are modeled as:

\[ \dot{Q} = nA(\Delta T) \]

(4)

The primed variables are interface volume temperatures and are determined by upwind differencing (Ref. 1). The method assumes that the gas temperatures have the same temperature profile as the regenerator matrix temperature. A representative equation is:

\[
\begin{align*}
\dot{\omega}_i > 0 & \quad T'_i = T_i - \frac{(T_{m3} - T_{m5})}{4.0} + \frac{\dot{\omega}_i \Delta T (T_{m3} - T_{m5})}{4.0 w_{s1}} \\
\dot{\omega}_i < 0 & \quad T'_i = T_{i+1} - \frac{(T_{m3} - T_{m5})}{4.0} + \frac{\dot{\omega}_i \Delta T (T_{m3} - T_{m5})}{4.0 w_{s1+1}} \\
\dot{\omega}_i > 0 & \quad T'_i = T_i \\
\dot{\omega}_i < 0 & \quad T'_i = T_{i+1}
\end{align*}
\]

\[ i = 1, 2 \]

(5)

The regenerators are modeled as thermal lags:

\[ T_{m_i} = \frac{h_i A_i}{c_{p_m} w_{s_m}} (T_i - T_{m_i}) \quad i = 3, 4, 5 \]

(6)

Pressure is calculated using the ideal gas law based on volume temperatures. Variable volumes are calculated using the drive geometry to calculate piston position as a function of crank angle. These equations are described later.

Equations of the forms of (2), (3), and (6) for the seven volumes result in a 17th order model. The integration technique used is a backward difference method which utilizes a multivariable Newton-Rhapson iteration (Ref. 3). This technique was chosen because of the large difference in time constants which results when flow resistance is considered.

Comparison of the SWS model with the Tew performance model shows agreement of pressures within 2 percent and temperatures within 8 percent over a cycle at steady state. This is considered adequate for controls analysis where performance detail is not so important.
The second model generated was a four working space model. Since the purpose of the model was to investigate the four working space aspects of the whole engine, the fluid dynamic model of each working space was simplified. Only three control volumes are used in each working space. One volume for the heater (and expansion) space, one volume for the regenerator, and one volume for the cooler (and compression) space. All volumes are assumed isothermal—with the expansion and compression space gas temperatures being the average steady state temperatures from the SWS model. The regenerator gas and mesh temperatures are assumed to be the same. While the model is quite simple it will be shown that it gives a fairly good representation of the engine. The model also has the drive dynamics included as well as a simple model for vehicle load effects. A schematic of the model is shown in Fig. 2.

**FWS Model Equations**

The fluid dynamic equations used in the FWS model are simplified from the SWS model. From Fig. 2, the pistons are numbered in the order in which they reach top stroke as is indicated by the arrows in the piston heads. Double subscripted variables indicate: first subscript, a volume position within a working space; second subscript, the working space. Some representative equations are:

\[\dot{w}_{i,J} = \frac{(P_{i,J} - P_{i+1,J})}{K_{i,J}} \]  
\[\dot{w}_{s_{i+1,J}} = \dot{w}_{i,J} - \dot{w}_{i+1,J} \]

\[\dot{w}_{0,J} = \dot{w}_{3,J} = 0,0 \]

and

\[P_{i,J} = \frac{w_{s_{i,J}}RT_{i,J}}{V_{i,J}} \]

Variable volumes are calculated as a function of piston position:

\[V_{1,1} = V_{1,1_u} + (\frac{L}{2} - x_1)A_p \]

Piston position is a function of crank angle:

\[x_i = (\frac{L}{2}) \cos \theta_i + \sqrt{R_L^2 - \left(\frac{L}{2}\right)^2 \sin^2 \theta_i} - \sqrt{R_L^2 - \left(\frac{L}{2}\right)^2} \]  
\[i = 1,2 \ldots 4 \]
Torque is calculated as a function of differential forces on the pistons which are summed through the drive geometry:

\[ T_i = \left( \frac{\delta}{2} \right) [P_{1i}A_p - P_{3i}A_p(A_p - A_r)\sin \theta_1] \left( 1 + \frac{\left( \frac{\delta}{2} \right) \cos \theta_1}{\sqrt{K_i^2 - \left( \frac{\delta}{2} \right)^2 \sin^2 \theta_1}} \right) \]  

(12)

Also included in the FWS model are simplified vehicle load effects and engine power losses due to mechanical friction and auxiliaries. Figure 3 shows a schematic of how these losses are calculated. Torques from the four pistons are summed to form indicated torque. Torque due to engine friction \( \mathcal{T}_F \) is subtracted to form brake torque. This is then available for auxiliaries \( \mathcal{T}_a \) and also for vehicle load effects such as rolling resistance \( \mathcal{T}_r \) and drag \( \mathcal{T}_d \). The vehicle inertia is brought into the torque equation through the year ratio to give an effective engine inertia. The summation of torques is integrated to give engine speed and integrated again to give crank angle, \( \theta_1 \), which in turn is used to generate piston position (Eq. (11)). The torque equations for the losses are linear and nonlinear functions of \( \theta \) with the torque equation:

\[ \text{I}_{\text{eff}} \dot{\theta} = \mathcal{T}_1 + \mathcal{T}_2 + \mathcal{T}_3 + \mathcal{T}_4 - \mathcal{T}_f(\dot{\theta}) - \mathcal{T}_d(\dot{\theta}) - \mathcal{T}_r(\theta) \]  

(13)

COMPARISON OF FWS AND SWS MODELS WITH EXPERIMENTAL DATA

Both the SWS and FWS models were run at various engine speeds from 1000 to 4000 rpm and three different mean pressure levels ranging from 5 to 15 MPa. Power was calculated for each cycle until steady state was reached. Power calculations were made by integrating the area of the pressure-volume curves in the expansion and compression spaces; and then adding the results at the end of the cycle. The power output predicted by the SWS model was multiplied by four to give gross engine power. The FWS model sums up the predicted power (indicated) from all four working spaces. From these values, mechanical and auxiliary power losses are subtracted to get net power. The results are shown in Fig. 4. Both models show reasonable agreement over the whole map with the SWS model showing slightly better agreement than the FWS model at the higher mean pressures and speeds, although neither model predicted the fall in power at 4000 rpm and 15 MPa.

UNIQUE CAPABILITIES OF THE FWS MODEL

The purpose of the models discussed in this report is for control studies. The FWS model is simplified but can be used to study overall engine performance in response to various control schemes and inputs. Also, since all four working spaces are simulated, the FWS model can be used to study phenomena not easily duplicated by a single working space model. Some applications of these will now be shown.
Working Fluid Supply Transient

The mean pressure control system for Stirling engines modulates engine power by changing the amount of working fluid in the cycle. In order to increase engine speed, working fluid is supplied to the engine. From Fig. 2, fluid is supplied through the piston rods; when the pistons are near bottom (min) stroke. The fluid is supplied to the compression space through a timing slot in the piston rod. A block diagram of the supply system is shown in Fig. 5. The check valves are open as long as the supply pressure is greater than the corresponding compression space pressure. The timing slot area varies as a function of piston position. Flow is modeled as a linear function of pressure drop:

\[ \dot{V}_{\text{sup}} = \frac{A}{k} (P_{\text{sup}} - P_{3,i}) \quad i = 1, 2, \ldots, 4 \quad (14) \]

A typical supply transient is shown in Fig. 6. The starting engine speed is 2000 rpm. After the simulation settled out, working fluid is added. This is shown as a function of crank angle in the top graph. Note that the amount of working fluid added decreases with each cycle since the working space pressure level continues to rise. Only the supply for the second working space is shown. Supplies for the other working spaces are similar but phase shifted by 90° with some overlap. Note the increase in torque amplitude when injection begins. This is due to the timing effect of the injections. Also, as fluid is added, the compression space pressure rises. The mean pressure shown is an average value for the sinusoidal compression space pressure. The pressure rise is 3 MPa in 0.24 seconds.

The torque wave shape shows four cycles for every engine cycle before the supply transient starts. This occurs due to the summation of the torques from all four pistons. When the working fluid is added to the cycle, the torque starts to rise. There is no transient torque drop seen in the engine. This is probably due to the absence of the energy equation for the incoming fluid in the compression space. The addition of cold working fluid to the compression space in the actual engine has a quenching effect on the temperature causing pressure to drop which results in a transient drop in torque. Note that the torque continues to show the effect of the four working spaces during the supply transient by the changes in wave shape during the rise and fall of the torque wave shape. These changes occur when the working fluid is added to the various working spaces. The rather slow speed response can be attributed to the assumed vehicle weight causing the effective inertia to be quite large. For the figure shown, engine speed increased 30 rpm in 0.24 seconds (8 cycles) which resulted in a vehicle speed change from 31.86 to 32.5 km/hr.

Piston Ring Leakage

The next effect studied is piston ring leakage. This allows working fluid to pass from one working space to an adjacent one. This analysis would be difficult to duplicate on a single working space model since the flow can redistribute through all four working spaces. It is assumed in this analysis that all four piston rings have the same leakage area. The leakage terms are designated as \( \dot{V}_L \) in Fig. 2. The leakage flow is modeled using an orifice equation:
The results are shown in Fig. 7. The nominal leakage area chosen was 0.002 cm\(^2\) which corresponds to a gap of about 0.000025 cm. The resultant effective leakage flow resistance at 15 MPa mean pressure and 4000 rpm is about 30 times greater than the flow resistance in the working spaces. This results in a maximum flow rate 130 times larger in the working spaces than through the leakage area. The maximum flow rate ratio drops with speed to 30 to 1000 rpm and 15 MPa; and with leakage area increase, to 34 at 1000 rpm and 0.008 cm leakage area. The simulation was run at four different engine speeds from 1000 to 4000 rpm and six different leakage areas from 0 to 0.01 cm\(^2\). The leakage flow area increased, the resultant net power of the engine dropped. For a leakage area of 0.004 cm\(^2\), a power reduction of 12 percent at 4000 rpm and almost 34 percent at 1000 rpm is shown. This analysis shows the value of a four working space model to determine power losses that can occur due to faulty piston rings in a Stirling engine. The effects of a failure of a single ring can also be easily investigated with a four working space model of this type.

Short Circuit Analysis

Rapid engine braking of the Stirling engine is accomplished by short circuiting of the working spaces. The analysis of a short circuit control mode is most effectively done with a four working space engine model. Figure 8 shows a schematic of the short circuit system. The compression space of each working space is connected to two different plenums one containing high pressure (PSHTMAX) and one containing a low pressure (PSHTMIN). Check valves allow flow to the high pressure plenum when the corresponding compression space pressure is greater than PSHTMAX. A second set of check valves let flow to the compression space when the corresponding compression space pressure is lower than PSHTMIN. Thus working fluid is removed from the compression space when the pressure is at a maximum thereby lowering the pressure; and added to the compression space when the pressure is at a minimum thereby increasing the pressure. The system thus decreases the pressure swing in the working spaces and thus lowers the power output of the engine. A valve connects the two plenums and determines the amount of engine braking that is accomplished. With the valve shut, the plenum pressures reach their respective maximum and minimum values. With the valve open, pressure in the high pressure plenum falls and pressure in the low pressure plenum rises. Note that the system requires working fluid to be shuttled to and from two different plenums and four working spaces. This would very difficult to simulate using only a single working space model.

Figure 9 shows the net steady state power as a function of speed and valve opening ratio. Note the large drop in power as the valve is open for any constant speed. For example, if the valve is half open, the power drops from 34.5 to 1.25 kW at 4000 rpm. If the valve is open sufficiently, the power goes negative which indicates the engine is absorbing power from the
load. The data for these curves was obtained by forcing piston position as a function of time (drive dynamics not active) and letting the simulation settle out. At the highest valve openings (0.5 to 1.0) the power starts to increase as speed drops.

The torque curves corresponding to the power curves in Fig. 9 are shown in Fig. 10. Note that the torque also drops significantly with valve opening at constant speed. At 4000 rpm, it drops from 86 to 0 Nm at 0.5 area ratio. As speed decreases, torque continues to drop off for the higher valve area settings. Figures 9 and 10 indicate that short circuiting is a very effective means of supplying engine braking torque.

SUMMARY

Presented in this report are preliminary results from a simplified four working space Stirling engine simulation. The model was constructed to simulate overall engine behaviour, rather than precise performance, for controls studies. While the model is quite simple it exhibits reasonable results when compared to steady-state engine test results, and gives both steady-state and transient results which are representative of Stirling engine behaviour. Also presented is a single working space model which has a detailed model of the fluid dynamic and thermodynamic aspects of a single working space. This model also shows reasonable results when compared to experimental data. Currently, the FWS model has been upgraded by incorporating the detailed fluid behaviour model for all four working spaces. This detailed model will be a reference model for Stirling engine controls studies. The simplified models will be normally used when they yield valid results. But, in some cases, such as short circuiting, a detailed four working space model is required. Results from the detailed model should compare even more favorably with the Stirling engine behaviour since the heat transfer characteristics of the different phenomena simulated can be incorporated into the model. For example, the quenching effect of the cool supply fluid on the compression space temperature can be included. This could result in a better prediction of the torque wave shape during a supply transient. Future efforts will include conceptual controls studies such as variable stroke control and detailed modeling of the current mean pressure control.

REFERENCES


**Figure 1.** Schematic of a single working space (SWS) model.

**Figure 2.** Schematic of four working space (FWS) model.
Figure 3. - Stirling engine drive dynamics.

Figure 4. - Comparison of net power versus speed for the SWS and FWS models and engine data.
Figure 5. Schematic of working fluid supply system.

Figure 6. Working fluid supply transient.
Figure 7. - Net power versus speed for various piston ring leakage areas, $P$ - 15 MPa.

Figure 8. - Schematic of short circuit system.
# Title and Subtitle
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# Supplementary Notes

# Abstract
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# Key Words (Suggested by Author(s))
Stirling engine
Controls
Modeling

# Distribution Statement
Unclassified - unlimited
STAR Category 34
DOE Category UC-96

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