SUBSTRUCTURE PROCEDURE FOR INCLUDING
TILE FLEXIBILITY IN STRESS ANALYSIS OF
SHUTTLE THERMAL PROTECTION SYSTEM

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A substructure procedure to include the flexibility of the tile in the stress analysis of the shuttle TPS is described. In this procedure, the TPS is divided into substructures of (a) the tile which is modeled by linear finite elements and (b) the SIP which is modeled as a nonlinear continuum. This procedure was applied for loading cases of uniform pressure, uniform moment, and an aerodynamic shock on various tile thicknesses. The ratios of through-the-thickness stresses in the SIP which were calculated using a flexible tile compared to using a rigid tile were found to be less than 1.05 for the cases considered.

INTRODUCTION

The purpose of the study described in this report is to include the effects of tile flexibility in the stress analysis of the shuttle TPS. A finite element model is used to represent the flexibility of the tile and to calculate the distribution of stresses in the tile. Also, the effects of tile flexibility on through-the-thickness stresses ($\sigma_{TTT}$) in the strain isolator pad (SIP) are determined.

An analysis procedure was developed and implemented for a unit width slice or cross-section of the TPS as shown in figure 1. Layers of elements are added or removed on the top of the model to represent different tile thicknesses. Two-dimensional orthotropic membrane elements are used to represent the LI-900 or LI-2200 material and rod elements are used to represent the tile coating. The rod elements representing the coating are located between adjacent joints all along the top surface and down the sides of the tile except for the portion at the bottom to allow tile venting.
These rods are shown in figure 1 by lines of greater width on the outer surface of the tile.

An initial attempt was made to model the SIP with vertical rods whose areas were changed as a function of stress to account for the nonlinear properties of the SIP material. Difficulties with convergence of this procedure along with lengthy computational times that resulted made this approach undesirable.

In a second approach which is described herein, the TPS is divided into substructures of (a) the tile which is modeled by finite elements and (b) the SIP which is modeled as a nonlinear continuum as described in reference 1. The nonlinear SIP analysis program for a rigid tile of reference 1 was modified to include an additional loop which accounts for tile flexibility as described in the next section. This substructure approach is an effective method for considering tile flexibility effects. This approach was implemented and used for the calculation of numerical results presented herein.

ANALYSIS PROCEDURE

The substructure analysis procedure used herein is shown schematically in figure 2 as a three step process. In step 1, a linear, finite element analysis program is used to calculate needed flexibility characteristics of the tile. The SPAR structural analysis system, reference 2, is used herein but the procedure is applicable to use of any finite element program. Columns of a flexibility matrix are generated as displacements resulting from applying unit loads in the vertical direction at joints along the bottom of the tile model which are adjacent to the SIP. This flexibility matrix is used later to calculate tile displacements resulting from forces in the SIP. The displacements of joints in the tile model adjacent to the SIP resulting from the applied loads are also calculated. These displacements from SIP reaction forces and from the applied forces can be superimposed to give the displacement shape of the bottom surface of the tile. To perform the above calculations, statically determinate constraints are required on the tile to prevent rigid body displacements.
The location of these constraints is arbitrary since the reaction forces at the constraints are zero when the SIP forces and applied forces are in equilibrium which is the condition of interest. The tile flexibility matrix and displacements from applied loads from the finite element program are saved for subsequent use by the nonlinear SIP analysis program.

In step 2 of the procedure, a modified version of the nonlinear SIP analysis program is used to calculate a set of equilibrated loads and compatible displacements between the tile and SIP. Initially, the iterative procedure described in reference 1 uses the Newton-Raphson method to converge on the vertical displacement and rotations of a rigid tile for which there is equilibrium between the applied forces and SIP forces. A nonlinear stress-strain relationship is used for integration of the forces over the SIP area. This analysis for a rigid tile is represented in figure 2 as the inner loop of step 2. The effects of tile flexibility are included by the outer loop of step 2. The forces acting at the bottom surface of the tile are calculated by integrating the SIP stresses over the areas associated with each of the joints along the SIP/tile interface. These forces are used with the information calculated in step 1 to calculate a displacement shape at the bottom surface of the tile. This displacement shape is added to the initial substrate imperfection to give an equivalent imperfection to be used in the analysis with a rigid tile. Iteration is required until an equilibrium condition is reached and the resulting through-the-thickness stresses ($\sigma_{TTT}$) in the SIP include the effects of tile flexibility.

The stresses of the tile model are calculated in step 3 using the finite element stress analysis program with equilibrated applied forces and the SIP forces which were calculated in step 2. Use of this substructure procedure is effective for analysis of the TPS, since the nonlinear aspect of the problem is isolated and formulated in terms of the unknown rigid body motions of the tile. Computational time for this procedure is significantly less than if both the tile and SIP were modeled as finite elements and the stiffness matrix updated and resulting equations solved in an iterative manner as in conventional nonlinear finite element analysis programs.
The substructure procedure is implemented for a unit width slice or cross-section of the TPS. The rigid tile motion is described by a vertical displacement and a rotation about an axis which is normal to the cross-section. Two-dimensional, membrane plate elements and rod elements are used for the finite element model. However, the same procedure is applicable for the rigid tile motion described by a normal displacement to a rigid plane and rotations about two perpendicular axes in that plane and with three-dimensional brick elements used for the tile.

NUMERICAL RESULTS

The substructure procedure was used to assess the effects of tile flexibility on the TPS structural analysis. The structural model was a unit width strip from a tile with a horizontal dimension of 15.24 cm (6.00 in.). The SIP had a horizontal dimension of 12.70 cm (5.00 in.) and a thickness of 0.41 cm (0.16 in.). An initial imperfection of the substrate under the SIP was taken to be a full cosine wave with a maximum amplitude of 0.48 mm (0.019 in.) at the center of the SIP and zero amplitude at each edge.

A parameter study was performed using this model for three different applied load distributions and three tile thicknesses. Tile thicknesses of 1.27, 2.54, and 5.08 cm (0.5, 1.0, and 2.0 in.) were analyzed. The three loading cases considered were uniform pressure, uniform moment, and a loading representative of the passage of an aerodynamic shock as shown in figure 3. The pressure distributions shown in the figure were considered to have a nominal maximum value of 36.9 Pa (1.0 psi) and results were calculated for a range of loading multiplication factor $\lambda$ applied to this nominal value. The factor $\lambda$ was varied between 0.0 and 5.0 for the uniform pressure and uniform moment and between 0.0 and 3.0 for the shock loading case.

Results were calculated which gave the distribution of through-the-thickness stresses ($\sigma_{TTT}$) in the SIP material as a function of the load factor $\lambda$ for the three load cases and three tile thicknesses. The maximum values of $\sigma_{TTT}$ for a flexible tile were compared to corresponding values calculated for a rigid tile. The flexible to rigid ratio of these
maximum stresses are presented in figure 4 as a function of load factor for the various load cases and tile thicknesses. The effect of tile flexibility was found to be largest for the 1.27 cm (0.5 in.) thick tile under uniform pressure but the effect was less than 5 percent compared to the rigid tile. Tile flexibility effects were less than 2 percent for all other cases.

Deformations of the tile are caused by loads from the SIP and externally applied loads. These deformations have opposite effects on the SIP stresses. The SIP loads alone produce deformations that tend to relieve the stresses as compared to the rigid tile. This effect is evident where the load factor \( \lambda \) is zero in figure 4 where only the initial imperfection is producing stresses. External forces on the tile overhang beyond the edges of the SIP produce deformations which tend to make the stresses greater than for a rigid tile. The net result is shown to generally be an increase in stress for the flexible tile at higher loads although it is case dependent as illustrated by the exceptions for the 1.27 cm (0.5 in.) thick tile under either uniform moment or shock loading.

These results indicate that consideration of tile flexibility has a small effect on the maximum SIP stresses. If the tile flexibility effects are neglected, the structural analysis procedure shown in figure 2 can be simplified by omitting step 1 and omitting the outer loop of step 2. Then the SIP forces from a nonlinear SIP analysis program using a rigid tile can be applied to the finite element model of the tile along with external applied loads to calculate the tile stress distribution as in step 3.

CONCLUDING REMARKS

A substructure approach was developed and found to be an effective method for considering the effects of tile flexibility in the stress analysis of the shuttle TPS. In this approach, the TPS is divided into substructures of (a) the tile which is modeled by linear finite elements and (b) the SIP which is modeled as a nonlinear continuum. This procedure was applied for loading cases of uniform pressure, uniform moment, and an aerodynamic shock on various tile thicknesses. The ratios of through-the-thickness stresses
in the SIP which were calculated using a flexible tile compared to using a rigid tile were found to be less than 1.05 for the cases considered. If the tile flexibility effects are neglected, the structural analysis procedure is simplified by using SIP forces from a rigid tile, nonlinear analysis program along with the externally applied loads to calculate stresses in the finite element model of the tile.

REFERENCES

Figure 1.- Cross-section of shuttle TPS illustrating finite element modeling of the tiles.
**STEP 1**

**GENERATE:**
- TILE FLEXIBILITY MATRIX, [A]
- DISPLACEMENTS FROM APPLIED LOADS, \( \{u_a\} \)

**STEP 2**

**NEWTON-RAPHSON ANALYSIS WITH NONLINEAR SIP, RIGID TILE, AND INITIAL IMPERFECTION**

EQUILIBRIUM OF APPLIED FORCES AND SIP REACTIONS?

- **NO**
  - CHANGE TILE RIGID DISPLACEMENT AND ROTATION

- **YES**
  - SIP FORCES, \( \{F_{s}\} = \int \sigma_{s} dA \)
  - TILE DISPLACEMENTS FROM SIP FORCES, \( \{u_{s}\} = [A]\{F_{s}\} \)

**STEP 3**

**TILE STRESS ANALYSIS USING EQUILIBRATED APPLIED FORCES AND SIP FORCES**

EQUIVALENT IMPERFECTION
\( \{w_{I}\} = \{w_{0}\} + \{u_{t}\} \)

TILE DISPLACEMENT STATE
\( \{u_{t}\} = \{u_{s}\} + \{u_{a}\} \)

EQUILIBRIUM?

- **NO**
- **YES**

*Figure 2.* Substructure analysis procedure to include tile flexibility effects.
Figure 3.— Applied loading cases.
Figure 4.- Ratio of maximum through-the-thickness SIP stresses ($\sigma_{TTT}$) for flexible tile compared to rigid tile.
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