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SIMULATION AND VISUALIZATION OF FACE SEAL MOTION STABILITY
BY MEANS OF COMPUTER GENERATED MOVIES

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A computer aided design method for mechanical face seals is described. Based on computer simulation, the actual motion of the flexibly mounted element of the seal can be visualized. This is achieved by solving the equations of motion of this element, calculating the displacements in its various degrees of freedom vs. time, and displaying the transient behavior in the form of a motion picture. Incorporating such method in the design phase allows one to detect instabilities and to correct undesirable behavior of the seal. A theoretical background is presented. Details of the motion display technique are described, and the usefulness of the method is demonstrated by an example of a noncontacting conical face seal.

NOMENCLATURE

$C_0$  
equilibrium center-line clearance
$D$  
simulated center-line clearance
$F$  
dimensionless axial force
$I$  
dimensionless moment of inertia
$K^*$  
support stiffness
$M$  
dimensionless moment
$m$  
dimensionless mass
$m^*$  
ring mass
$R$  
simulated seal radius
$r_g$  
ring radius of gyration
A mechanical face seal consists of two discs one of which is attached to the shaft and rotates with it while the other one is stationary. Usually one of the two discs is flexibly supported to allow self alignment and tracking of the mating disc. In figure 1 a face seal with fixed rotor and flexibly mounted stator is shown. The stator can move axially and tilt about two orthogonal diameters while its circumferential rotation is prevented by anti-rotation locks, it can also move radially in two perpendicular directions. Thus, the flexibly mounted stator has 5 degrees of freedom. The motion of the flexibly mounted element is controlled by the forces acting upon it as well as its mass inertia, and stiffness and damping properties of the dynamic system. This motion can be quite complex and difficult to visualize from simple, two dimensional plots of time variations in the various degrees of freedom.
A well designed seal is one in which the motion of the flexibly mounted element is stable. Stable motion means steady tracking of the fixed element by the flexibly mounted one. The optimum design is one that results in perfect tracking where the two discs remain parallel with a constant separation at all times. As was mentioned above many factors affect the dynamic behaviour of the seal and hence the motion of the flexibly mounted element. The seal can be either stable or unstable depending on the compatibility of its operating conditions and design parameters. Indeed, various sources of seal instabilities were observed and reported in the literature, for example, references [1-6].

Many seal failures could be avoided if the designer had an easy and fast means by which his design could be checked. It is the objective of this paper to describe a method, based on computer simulation, by which the motion of the flexibly mounted element of the seal can be visualized. A noncontacting conical face seal was selected for the purpose of demonstrating the method. Nevertheless, this method is suitable for any other configuration as long as the fluid film pressure distribution and the dynamic properties of the seal system are known. Basically the computer simulation is based on a solution of the equations of motion of the flexibly mounted element. Accelerations, velocities and displacements in the various degrees of freedom are calculated and the resultant transient behaviour of the seal is displayed in the form of a motion picture. Observation of the seal motion allows one to detect instabilities or any other undesirable behaviour. It further allows the designer to correct such undesirable situations by changing his design parameters and observing the resulting effect on the seal's dynamic behavior.
THEORETICAL BACKGROUND

The seal model is shown in figure 2. The seal seat is the fixed rotor rotating at a speed \( \omega \) about the \( z \) axis of an inertial reference system \( xyz \). The seal ring is the flexibly mounted stator. It can move axially along the \( z \) axis and tilt about the \( x \) and \( y \) axes of the inertial reference system \( xyz \). These motions are the three major degrees of freedom of the seal ring. Usually the ring has two more degrees of freedom consisting of radial displacements along the \( x \) and \( y \) axes. However, these are much more restricted than the previous three major displacements and will not be treated here. Thus, the ring displacements are \( Z^* \) along the \( z \) axis, and \( \alpha^*_x \) and \( \alpha^*_y \) about the \( x \) and \( y \) axes, respectively. The resultant orientation of the seal ring in the inertial reference \( xyz \) can be described by a rotating coordinate system \( 123 \). In this system axis 3 is perpendicular to the plane of the stator, axis 1 remains in the \( xy \) plane, and axis 2 passes through the point of maximum clearance. Thus, the coordinate system \( 123 \) has a nutation \( \gamma^* \) and a precession \( \psi \) as shown in figure 2. It should be noted that while the ring itself is prevented from any circumferential rotation by the seal's anti-rotation locks, the system \( 123 \) is free to rotate with respect to the ring while the plane \( 12 \) remains always in the plane of the ring.

An axial force \( F_z^* \) and tilting moments \( M_x^* \) and \( M_y^* \) are acting upon the stator causing its motion in the three major degrees of freedom. The system of force and moments is contributed by the pressure in the fluid film between stator and rotor, by the balance pressure on the back of the stator, and by various elements of the flexible support, for example, mechanical springs and secondary seal. Reference [6] describes in detail the system of forces and moments for the model of a coned seal with perfectly aligned
rotor. The motion of this coned model will be shown in the following
discussion but, as was mentioned in the introduction, any other seal model
can be treated along the same line of approach.

The general dimensionless equations of motion of the flexibly mounted
stator are

\[
\begin{align*}
F_z &= m\ddot{z} \\
M_x &= I\ddot{\alpha}_x \\
M_y &= I\ddot{\alpha}_y
\end{align*}
\]

Knowing the force \( F_z \) and moments \( M_x \) and \( M_y \) for a given position
of the stator relative to the rotor, enables one to calculate the accelerations
\( \ddot{z} \), \( \ddot{\alpha}_x \), and \( \ddot{\alpha}_y \) from equations (1) to (3). A time integration
routine can then be used to find the velocities \( \dot{z} \), \( \dot{\alpha}_x \), \( \dot{\alpha}_y \), and the new
displacements \( z \), \( \alpha_x \), and \( \alpha_y \). From the new relative position between
stator and rotor a new system of force \( F_z \) and moments \( M_x \) and \( M_y \)
is found and the whole process repeats itself.

The pressure contributed by the fluid film between stator and rotor is
found by solving the Reynolds equation taking into account hydrostatic,
hydrodynamic, and squeeze film effects [6]. Cavitation in the fluid film is
also considered by summing up the hydrodynamic, squeeze, and hydrostatic
pressures at each point in the sealing area and checking the total against a
selected value of a cavitation pressure. If the total pressure at a point
is less than the cavitation value it is replaced by the selected cavitation
pressure prior to integration.

Since the actual clearance in mechanical face seals is very small (of
the order of few micrometers) the angles \( \alpha_x^* \), \( \alpha_y^* \) and \( \gamma^* \) are also very
small and, hence, can be treated as vector. From figure 3 we have

\[
\alpha_x^* = \gamma^* \cos \psi
\]
\[ a_y^* = \gamma^* \sin \psi \]  

(5)

Hence, the dimensionless nutation \( \gamma \) and the precession angle \( \psi \) can be found from equations (4) and (5) in the form

\[ \gamma = \left( a_x^2 + a_y^2 \right)^{1/2} \]  

(6)

\[ \psi = \tan^{-1} \left( \frac{a_y}{a_x} \right) \]  

(7)

These two time dependent angles along with the axial displacement \( Z \) allow a three dimensional representation of the stator motion. Using computerized graphics this motion can be either watched on-line, or filmed frame by frame at each time step to provide a computer generated movie.

**SEAL MOTION DISPLAY**

The rotating seal seat and the flexibly mounted seal ring are represented in the movies by two circles, one above the other. The lower circle represents the rotating seat and the upper circle represents the ring. The inertial coordinate system \( xyz \) with its origin at the center of the lower circle, as shown in figure 4, is used as a reference to describe the relative motion between the seal seat and the seal ring. The rotation of the seat is represented by a moving symbol which is drawn on the lower circle. This symbol represents the position of a fixed point on the seat as the seat rotates at a constant shaft speed.

Both circles are drawn with an arbitrary radius \( R \) representing the outer radius of the seal. The origins of the two circles, designated by two different symbols, are located on the \( Z \) axis. The design clearance is represented by an arbitrary distance \( D \) between the fixed origin of the lower circle and another fixed symbol on the \( Z \) axis.
The equations for the bottom circle in the $xyz$ coordinate system are the equations of a circle, namely,

$$x = R \cos \phi \quad \quad y = R \sin \phi \quad \quad z = 0$$

where $0 < \phi < 2\pi$ and is measured from the $x$ axis. The equations for the upper circle, representing the seal ring, are more complex and are obtained by transformations through the precession angle $\psi$ and the nutation angle $\gamma^*$. The procedure is as follows:

Consider first a rotation of the $xyz$ system of figure 2 by an angle $\psi$ about the $z$ axis. The new system designated $\bar{x}\bar{y}\bar{z}$ has its $\bar{x}$ and $\bar{z}$ axes coincide with axes 1 and $z$, respectively, of figure 2. Any point in the $\bar{x}\bar{y}\bar{z}$ coordinate system can be transformed into the $xyz$ system by

$$x = \bar{x} \cos \psi - \bar{y} \sin \psi \quad (8)$$
$$y = \bar{x} \sin \psi + \bar{y} \cos \psi \quad (9)$$
$$z = \bar{z} \quad (10)$$

The coordinate system $\bar{x}\bar{y}\bar{z}$ is now rotated by an angle $\gamma^*$ about the $\bar{x}$ axis. The new system is designated $x^*y^*z^*$ and its axes coincide with the coordinate system 123 of figure 2. The transformation from the coordinate system $x^*y^*z^*$ into the system $\bar{x}\bar{y}\bar{z}$ is

$$\bar{x} = x^* \quad (11)$$
$$\bar{y} = y^* \cos \gamma^* - z^* \sin \gamma^* \quad (12)$$
$$\bar{z} = y^* \sin \gamma^* + z^* \cos \gamma^* \quad (13)$$
The equations of a circle in the \( x^*y^*z^* \) coordinate system are

\[
\begin{align*}
    x^* &= R \cos \phi \\
    y^* &= R \sin \phi \\
    z^* &= 0
\end{align*}
\]

However, since the seal ring is prevented from rotation, and the coordinate system \( x^*y^*z^* \) has been rotated by an angle \( \psi \) with respect to the inertial system \( xyz \), any point on the upper circle has to be rotated back by an angle \( (-\psi) \) about the \( z^* \) axis. Thus, each point \((R,\phi)\) on the upper circle will have, after the rotation, the coordinates

\[
\begin{align*}
    x^* &= R \cos \phi \cos \psi + R \sin \phi \sin \psi \\
    y^* &= -R \cos \phi \sin \psi + R \sin \phi \cos \psi \\
    z^* &= 0
\end{align*}
\]

Using equations (11) to (16) in equations (8) to (10), recalling that the origin of the upper circle in figure 4 is a distance \( D(1 + Z) \) above the origin of the lower circle, the equations of the upper circle in the \( xyz \) coordinate system of figure 4 are

\[
\begin{align*}
x &= (R \cos \phi \cos \psi + R \sin \phi \sin \psi) \cos \psi \\
    &\quad + (R \cos \phi \sin \psi - R \sin \phi \cos \psi) \cos \gamma' \sin \psi \\
y &= (R \cos \phi \cos \psi + R \sin \phi \sin \psi) \sin \psi \\
    &\quad - (R \cos \phi \sin \psi - R \sin \phi \cos \psi) \cos \gamma' \cos \psi \\
z &= D(1 + Z) - (R \cos \phi \sin \psi - R \sin \phi \cos \psi) \sin \gamma'
\end{align*}
\]

where \( \gamma' \) is a scaled angle of nutation related to \( \gamma^* \) by

\[
\frac{R \sin \gamma'}{D} = \frac{R' \sin \gamma^*}{C_0}
\]

Since the angle \( \gamma^* \) is very small we have \( \sin \gamma^* = \gamma^* \) and by using the dimensionless angle of nutation, \( \gamma \), of equation (6), equation (20) becomes

\[
\gamma' = \sin^{-1} \left( \gamma \frac{D}{R} \right)
\]
Because the physical angle $\gamma^*$ and the design clearance $C_0$ are very small, $D$ and $R$ are chosen so that differences in the axial separation and in the angle of nutation appear in the movie to be relatively large. Thus, making it easier to visualize the seal motion. Care must be taken to choose $R$ and $D$ so that $\gamma D/R$ is not greater than one. Typical values used were $R = 2$, $D = 1$.

The $x$, $y$, and $z$ coordinates defining the upper and lower circles of figure 4 have to be transformed to a $\xi$, $\eta$ coordinate system so that the circles can be projected onto a two dimensional frame as shown in figure 5. The computer program uses the three spherical coordinate parameters $R_p$, $\theta_p$, and $\phi_p$ of figure 5 to define the three dimensional perspective of the two circles. The circles are then viewed from a point in space defined by these three parameters where $R_p$ is the distance from the origin of the $xyz$ coordinate system to the viewer's eye. In the present movies $R_p$ was chosen to be 15, or 7.5 times the radius of the circle. The angles $\theta_p$ and $\phi_p$ were chosen to be both $45^0$.

The displacements $L_i$, $a_{x_i}$, and $a_{y_i}$ corresponding to times $t_i$, where $t_i = t_{i-1} + \Delta t$, $i = 1, 2, \ldots n$, were obtained from a solution of equations (1) to (3). The values of times and corresponding displacements were stored on a tape which was later used for the plotting program. Various values of the time step $\Delta t$ were examined, and it was found that fifty time steps per one shaft revolution clearly reveal the seal motion. A varying number of points was tried to define the circumference of each circle. It was found that 40 circumferential points connected by straight lines are enough to obtain smooth circles.
The movie plotting program was run on an IBM 360 computer, and it took 10 minutes of computer time to generate 1000 frames of film which correspond to 20 shaft revolutions. Each frame appeared on the display screen for 0.09 seconds.

Three examples consisting of portions of three computer generated movies are shown in figures 6 to 8. The frame sequence shown in these examples is 5 frames per one shaft revolution. The number of shaft revolutions counted from the instance of an initial disturbance given to the upper circle is shown on the lower left corner of each frame. The three examples correspond to the three basic modes of operation that were found in reference [6] namely, unstable, critically stable and stable modes. In all three cases shown in figures 6 to 8 the seal radius ratio is $r_i/r_o = 0.9$ and the coning is $\beta*r_o/C_o = 10$. All three cases have the same pressure parameter $(r_m/r_sp)^2(p_o - p_i)r_o^2/K*C_o = 90.25$, but they differ in their speed parameter values. The speed parameter values $(r_g/r_sp)^2 m*\omega^2/K*$, are 32, 21.95, and 8 for the cases shown in figures 6 to 8, respectively. Figure 9, which is taken from reference [6] shows stability maps for seals of radius ratio $r_i/r_o = 0.9$. From this figure it is clear that the seal of figure 6 is unstable, the seal of figure 7 is critically stable, and the seal of figure 8 is stable. Indeed, in figure 6 a small initial disturbance is developed into a catastrophic failure, and rubbing contact occurs after about 20 shaft revolutions. The development of stator motion is shown in figure 6 for the 1st, 17th, 18th, and 19th revolutions. At the beginning the nutation angle increases very slowly but towards the 18th revolution this increase becomes quite significant and is followed by a substantial increase in the center-line clearance.
In the example of the critically stable seal shown in figure 7, the initial disturbance results in some angular and axial vibrations which are damped very rapidly. After about two shaft revolutions the stator assumes a constant nutation angle and from there on it wobbles at a constant frequency of half the shaft speed. Although the two seal faces do not contact, and hence no rubbing-caused failure occurs, the seal may still fail due to an excessive leakage caused by the large relative tilt between its mating faces, and the increase in the center-line clearance which are visible in figure 7.

Figure 8 shows an example of a stable seal. Here a fairly large initial disturbance causes axial and angular vibrations which are damped out very rapidly. After about two shaft revolutions the seal faces became parallel and the designed center-line clearance is maintained.

CONCLUDING REMARKS

The simulation and visualization of noncontacting face seal motion stability by means of computer generated movies is described. The technique is based on solving the equations of motion of the flexibly mounted seal ring and displaying the relative position of the seal ring and seal seat at subsequent time steps. Using computer graphics a perspective view of the seal is obtained on a display screen thus enabling observation of the seal motion.

This simulation and visualization technique allows the designer to detect instabilities or any other undesirable behaviour of the seal. Thus, the designer can correct undesirable situations by changing the design parameters and observing their effect on the seal dynamics. It is believed that using such techniques can save time and money spent on testing, and can result in more reliable seal designs.
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REFERENCES


Figure 1. - Schematic of a radial face seal.

Figure 2. - Seal model and coordinates systems.
Figure 3. - Orientation of rotating coordinate system 123 in inertial reference xyz.

Figure 4. - Coordinate system xyz for the upper and lower circles.
Figure 5. - Two dimensional perspective transformation and viewing position of the $xyz$ coordinate system.

Figure 6. - Simulated seal motion at indicated no. of revolutions, unstable case.
Figure 7. - Simulated seal motion at indicated no. of revolutions, critically stable case.

Figure 8. - Simulated seal motion at indicated no. of revolutions, stable case.
Figure 9 - Stability maps for seals of radius ratio $R_1 = 0.9$. 

$\frac{1}{r} \frac{1}{2} (r^2 - r_0^2) \beta r_0 C_0$
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Seal dynamics

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