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THEORY OF TIME-DEPENDENT RUPTURE IN THE EARTH
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ABSTRACT

Using the concepts of fracture mechanics, we develop a theory of the earthquake mechanism which includes the phenomenon of subcritical crack growth. The theory specifically predicts the following phenomena: slow earthquakes, multiple events, delayed multiple events (doublets), postseismic rupture growth and afterslip, foreshocks, and aftershocks. The theory also predicts that there must be a nucleation stage prior to an earthquake, and suggests a physical mechanism by which one earthquake may 'trigger' another.

These predictions are obtained by combining two fundamental concepts. The first is that

\[ k = C \Delta \tau X \]

and the second, that

\[ k = K_0 \left( \frac{X}{V_0} \right)^{1/n} \]

where \( k \) is the stress intensity factor, \( \Delta \tau \) stress drop, \( X \) rupture length, \( X \) rupture velocity, \( C \) a geometrical factor, and \( K_0 \), \( V_0 \) and \( n \) are material constants. The first is a fundamental result of fracture mechanics; the second describes stress...
corrosion cracking, a well established physical process that results in subcritical crack growth.

We investigate in detail two phenomena of special interest and which are not predicted by ordinary fracture mechanics: nucleation and delayed multiple events. In the first case we find that all earthquakes must be preceded by quasistatic slip over a portion of their rupture surfaces, but it may be difficult to detect in practice. In the second case we studied two pairs of delayed multiple events that were separated by the same 'barrier' in order to calculate $n$. We find that the stress corrosion index, $n = 24$. 
GLOSSARY OF SYMBOLS

C = geometric constant - 1 for a two-dimensional crack
    = 20/7\(\pi\sqrt{2}\) for a circular shear crack

k = stress intensity factor
k_D = dynamic stress intensity factor
k_s = static stress intensity factor
K_o = stress corrosion limit
K_c = modulus of cohesion in the presence of corroder
K*_c = modulus of cohesion under corroder-free conditions
K_1 = a fixed arbitrary point on the (k-\(\dot{X}\)) curve
L = barrier width
\(\ell_1, \ell_2\) = lengths of rupture zones of two events in a pair of delayed multiple events
m = frequency of aftershock occurrence
n = stress corrosion index
t = time since occurrence of main shock
t_D = delay time between a pair of multiple events
t_R = maximum of rise times of individual events in a sequence of multiple events
t_f = time to failure (instability) or nucleation time of an earthquake
t' = t - \(\Delta t\)
u = slip on crack plane
V_o = crack growth velocity at the stress corrosion limit
\( V_1 = X \) - coordinate corresponding to \( K_1 \)

\( W_0 \) = a material property

\( x \) = crack length (for two-dimensional cracks) and crack radius (for circular shear crack)

\( \dot{x} \) = crack edge velocity

\( X_0 \) = initial \( x \)

\( X_f \) = \( x \) at time \( t \) before failure

\( \beta \) = an elastic wave velocity

\( \Delta t \) = static stress-drop

\( \Delta t \) = short time before instability
INTRODUCTION

In recent years, the developments of linear elastic fracture mechanics have been applied to an important problem in geophysics: development of a dynamic model of earthquakes [Kostrov, 1966; Richards, 1978; Andrews, 1976; Fossum and Freund, 1975; Das and Aki, 1977a, b; Freund, 1979]. The central concept of fracture mechanics, which has its roots in the Griffith energy balance [Lawn and Wilshaw, 1975], is that for a crack in an elastic medium, no propagation takes place until the stress intensity factor at the crack tip, k, reaches a value $K_c$, a property of the medium. $K_c$ is called the "modulus of cohesion" [Kostrov et al., 1969]. When $k ≥ K_c$, the Griffith instability arises and the crack propagates dynamically with a velocity limited by an elastic wave velocity.

Most oxides and silicates, however, exhibit more complicated behavior due to environmental effects. For these materials, the crack will propagate when $k ≥ K_o$, where $K_o < K_c$, at a velocity $X$ which is a well defined function of $k$. This propagation is stable and quasistatic and is referred to as subcritical crack growth. (We shall use the terms "stable", "quasi-static" and "subcritical" to mean propagation at velocities much less than the sonic velocities of the medium). This behavior results from stress induced corrosion at the crack tip, the principal corrodent for the present application being $H_2O$. This behavior has been firmly established in the laboratory for Mode I (tensile) cracks in a wide variety of materials including silicates and silicate glasses [see, e.g., Scholz, 1968a, 1972a; Martin, 1972; Wiederhorn and Bolz, 1970; Atkinson, 1979; Lawn and Wilshaw, 1975; Knott, 1973].

The form of the relationship between $k$ and $X$ does not vary significantly with the material; only the parameters in the law vary. As an example we show in
Figure 1 data on subcritical crack growth in quartz [after Atkinson, 1979]. The empirical relationship is found to take the form,

\[ \dot{X} = k^n \tag{1} \]

or

\[ X = e^k \tag{2} \]

Since \( n \), the stress corrosion index, is large (12.5 in Fig. 1), 1 and 2 are nearly indistinguishable.

Since a shallow focus earthquake is a shear crack growing in silicate in an aqueous environment, eqn. 1 or 2 should be used as a complete description of the fracture process. Since an earthquake is a mixed Mode II and III shear crack and data are only available for the Mode I case, this involves an assumption: that the form of the law (but not necessarily the parameters) does not depend on mode (Atkinson’s method may actually put the crack into mixed Mode I and III, an unsupported statement in Evans [1972] being the only argument to the contrary). Later we shall determine \( n \) for an earthquake and show that it is in remarkable agreement with Atkinson’s results. Although we cannot prove this assumption, it seems entirely reasonable since when one considers the physical mechanism of stress corrosion there seems to be no physico-chemical reason why the process should depend on mode. The indirect evidence in support of this is that rock exhibits dilatant creep and static fatigue, both processes that result from stress corrosion, in compression and under high confining pressure [Scholz, 1968a; Kranz and Scholz, 1977; Kranz, 1980]. We shall also assume that there is a lower limit, \( K_o \), such that when \( k < K_o \) no crack growth occurs. There is only limited data to
support this [Wiederhorn and Bolz, 1970; Evans, 1972] but it is well founded in stress corrosion theory. In any case, our results are not critically dependent upon this assumption of a stress corrosion limit, because if it does not exist, we can simply define \( K_0 \) as a value of \( k \) below which the crack velocity is vanishingly small and can be neglected.

It should also be pointed out that in using eqns. (1) or (2) and the fracture mechanics approach, we are implicitly assuming that the most important forces governing the propagation of the rupture are the forces at the crack tip and that the friction that acts on the crack behind the crack tip plays no role on the motion of the crack tip. This is clearly an approximation, but the complete problem cannot be handled until the full energy balance, as discussed by Kostrov [1974] can be solved. For the present time, we will have to justify this assumption with the success we have, with our present approach, in predicting the observations.

In this paper, then, we consider an earthquake as a shear crack that propagates according to a law given by (1) up to the time when the propagation becomes dynamic. This theory predicts a variety of phenomena, all of which have been observed for earthquakes. The phenomena which arise quite naturally in the theory are slow earthquakes, multiple events, delayed multiple events (doublets), aftershocks, foreshocks and postseismic rupture extension and afterslip. The model also contains specific predictions about the earthquake nucleation process. All of these phenomena we now see simply as different facets of the same phenomenon.

In order to underscore the underlying simplicity of this model, we will first qualitatively describe how the various phenomena noted above arise, and then follow that with a quantitative model which we will apply to several examples.
regardless of mode. If \( k \geq K_o \) we immediately see, combining (3) with (1) that the crack will accelerate to a catastrophe (this is a stronger catastrophe than a Malthusian one since the latter is a simple exponential), the relevant equations for which are given in a later section. This catastrophe occurs when the crack becomes critical and propagates dynamically. We shall show later that this occurs at a well defined value of \( k \), which we will call \( K_c \). Note that \( K_c \) is different from \( K_c^* \) the value of \( k \) at which instability occurs in the absence of stress corrosion. \( K_c^* \) is a material property that for these materials can be measured only in a corrodent-free environment, e.g., a high vacuum. \( K_c^* \) on the other hand, is not an independent material property, it depends on \( n \). We do not assume a value of \( K_c^* \) in our problem, we calculate a value of \( K_c^* \). If \( K_c \leq K_c^* \), then we need not consider \( K_c^* \) at all. If \( K_c^* < K_c \) then the instability will occur earlier than we calculate, but otherwise our results will be unchanged. It is most likely, in fact, that \( K_c = K_c^* \), since as the crack-edge velocity approaches sonic velocities, the crack is propagating too fast for the corrodent to diffuse to the crack-edge and a vacuum exists at the tip of the crack [Wiederhorn, 1967]. It may not seem obvious at first that this will occur for a shear crack, but for a topographically rough surface it is unlikely that shear can take place without some dilation. The velocity, \( V_o \), at \( K_o \) is also important and will be discussed in a later section.
The stress intensity factor in (3) is simply the applied stress at infinity (or in the case of a crack with friction, the stress drop) multiplied by some geometrical factors. On the other hand, the fracture criterion (1) depends on the material properties \( K_0 \) and \( n \). Since a fault is not a homogeneous surface, both the applied stress and hence \( k \), and the material properties \( K_0 \) and \( n \) will be functions of position on the fault plane. Since a fault plane consists of two surfaces in contact, and the topography of surfaces is known to be Brownian [Sayles and Thomas, 1978] or fractally Brownian [Mandelbrot, 1977], we should expect that the heterogeneity of applied stresses and material properties is at least as random as the topography and that this heterogeneity exist at all scales. If we compare a fault plane with a mathematically flat plane, it is one of the properties of Brownian surfaces that the standard deviation from a flat plane increases as \( x^{1/2} \), where \( x \) is a scale length of the section of the Brownian surface that is sampled. We should therefore consider \( K, n, \) and \( K_0 \) as random variables on the fault plane and the rupture process a stochastic growth process [see, also, the discussion in Scholz, 1988b]. Then the sampling dimension \( x \) becomes the earthquake radius or length dimension, and the consequence of the fault being a Brownian surface is that the larger the earthquake becomes, the larger will be the wavelength and intensity of heterogeneities that it encounters. The only reason smaller earthquakes appear simpler than larger ones is due to the fact that we observe earthquakes with band-limited instruments and because the higher frequency waves radiated by the smaller heterogeneities are more strongly attenuated in propagating from the source to the instrument.

For the purpose of the remainder of this discussion, however, we will consider only gross heterogeneities which we will call barriers, after the usage of Das and Aki [1977b]. These are regions on the fault plane that are particularly resistant to slip either because of low applied stress or exceptionally high strength properties. The term barrier, then, refers to a heterogeneity of sufficient size that its effects on the rupture propagation can be observed instrumentally.
We are now ready to discuss the broader implications of the model. We will do so by discussing as scenarios the various possible phenomena that can result due to spatial and temporal variations in \( k, K_o \), and \( n \) on the fault. These scenarios are illustrated in Figures 2 and 3.

**Nucleation.** The loading that is implied in the elastic rebound theory ([Reid, 1910]) is a tectonic process consisting of a steady increase in the applied stress, at a very slow rate, such stress being released by the earthquake. This tectonic loading process is equivalent to a steady increase in \( k \) for a potentially growing crack. It is usually thought that no motion occurs until \( k = K_o \), when the earthquake initiates. With a fracture process such as described by (1), however, this is impossible. Instead, propagation of the crack begins when \( k = K_o \), and it quasistatically accelerates up to sonic velocity at \( k = K_o \) (Figure 2). It is thus fundamental to this model that an earthquake be preceded by some precursory slip. We call this stage the nucleation phase. The size of the nucleation region and the time scale of the process depend only on \( n \) and \( K_o \) and their spatial distributions on the fault. Because of its importance to earthquake prediction, we will discuss this quantitatively later. See Smith and Wyss [1968], Sacks [1978], Sacks et al. [1980, 1981], and Kanamori and Cipar [1974] for possible observational examples.

**Slow earthquakes.** Although the catastrophe implicit in (3) and (1) is very strong, there is a finite probability, because of the heterogeneity of the fault, that the rupture will propagate into regions in which \( k < K_o \) during the nucleation phase and stop. What results is a slow earthquake ([Kanamori, 1972; Kanamori and Stewart, 1979; Sacks et al., 1978]). We expect, of course, that this is an uncommon phenomenon but worthy of study because it yields a minimum estimate of the moment in the nucleation phase.

**Foreshocks.** During the nucleation phase, we can also expect it to be likely that \( k \geq K_o \) for small regions of the fault within the nucleation region. These
regions will grow dynamically, only to stop by running into adjoining regions where
\( k < K_c \). When this happens, a foreshock results. Because of the accelerating
nature of the nucleation process, the probability for foreshocks to occur increases
very rapidly as the time approaches the time of the main shock. Note also that
foreshocks are not an intrinsic part of the nucleation process so that they are not
required to fit any regular pattern nor are all earthquakes required to have fore-
shocks. All three of these predicted properties of foreshocks mentioned above are
confirmed in the observations. This suggests that the study of foreshocks may
provide information concerning the spatial and temporal development of the
nucleation phase.

Stopping. In nucleation, only two possibilities can occur: a slow earthquake
or a 'normal' earthquake (Figure 2). In the stopping process, however, more
possibilities can occur (Figure 3). When a rupture is propagating dynamically, the
stress-intensity factor at its tip is a dynamic one, \( k_D \). (All k's up to now were
static stress-intensity factors.) The rupture will stop propagating at a point on its
perimeter when \( k_D < K_e \), but slip will continue within the rupture perimeter as
the displacement field tends to static equilibrium. After static equilibrium has
been reached there will be a static stress-intensity factor \( k_s \) at the crack tip,
where \( k_s > k_D \) [see Achenbach, 1973, eqn. 5.7]. It is this extra complication that
produces the additional phenomena.

For simplicity we will assume that the rupture stops at a barrier where
\( k_D < K_c \) (a barrier is indicated by a sawtooth in Figure 3). What happens next
depends on the properties of the barrier. If:

\[ k_s < K_c \]

The trivial case results. The earthquake simply stops and the
barrier becomes the end of the rupture (Figure 3a).

\[ k_s > K_c > k_D \]

The barrier is breached before slip stops within the perimeter and a multiple event occurs, with a delay time \( t_D < t_R \), the rise time
This is the type of multiple event first described by Wyss and Brune [1985] and modelled by Das and Aki [1977b]. It is worth remarking that because heterogeneity exists at all scales, earthquakes are by their very nature infinitely multiple. It is simply that our data allow us to describe only the gross heterogeneities.

\[ K_0 < k_s < K_c \] This is an interesting case because, unlike those discussed above, it cannot be explained on the basis of ordinary fracture mechanics. In this case the rupture subcritically propagates through the barrier, going critical when it breaches the barrier after a time delay \( t_D \gg t_R \). This results in an earthquake occurring just adjacent to a previous earthquake and a short time after it. We call this process a delayed multiple event (Figure 3c). That this type of event occurs has been very well documented for the Nankai trough of western Japan by Ando [1975]. The most prominent events discussed there are the Ansei I and II (1854) events, adjacent earthquakes with a time delay of 32 hours, and the Tonankai (1944) and Nankaido (1946) events, separated by two years. We shall model these two pairs below, and calculate \( n \) and \( K_0 \) from then. This type of multiple event is common in some regions [Sykes, 1971; McCann, 1980; Lay and Kanamori, 1980].

There are two differences between these two types of multiple events. The first is the delay time, the other is more subtle. Denote by \( t_1 \) the length of the rupture just as the barrier is encountered, and \( t_2 \) as the distance the rupture propagates after breaching the barrier. In the ordinary multiple event, the region \( t_1 \) has not come to static equilibrium when the barrier breaks, so that the region in \( t_1 \) continues to slip as the rupture propagates to its final dimension \( t_1 + t_2 \), and the source parameters are those of a single earthquake of rupture dimension \( t_1 + t_2 \). In the case of the delayed multiple event, however, the region \( t_1 \) comes to static equilibrium before the barrier is breached and the static frictional
strength is re-established, and increases with time of static contact [Scholz and Engelder, 1976; Dieterich, 1978]. In this case, when the barrier is breached, only the new region $l_2$ slips, and the result is two earthquakes with source parameters appropriate for rupture dimensions $l_1$ and $l_2$, respectively.

It is of course not necessary that the barrier be breached in this case. If the barrier were to, say, increase in strength with distance faster than $\sqrt{x}$, the rupture may grow into the barrier for some distance and then stop, when $k < K_0$, as in Figure 3d. If this occurs we will observe postseismic rupture growth. A number of cases of this have been documented, the most prominent being the 1946 Nankaido earthquake [Ando, 1975], in which half the rupture area failed quasistatically. (It is interesting that the region that ruptured quasistatically also had no aftershocks. Our model would predict fewer aftershocks, but not a complete absence of aftershocks. The data should be re-examined to specifically address this question.)

If the original rupture area slips quasistatically as the rupture grows quasistatically, we would observe afterslip. This appears to be a fairly common phenomenon, but of secondary importance, since it normally contains less than 5% of the moment of the main shock [Scholz, 1972b]. Indeed, the Nankaido case is probably an exceptional one, because rupture zones of large earthquakes are commonly observed to nearly abut [Sykes, 1971]. Thus postseismic rupture growth usually appears not to extend the rupture more than a small percentage of its original length, though exceptional cases may occur and thus be of interest to those who study seismic gaps.

**Aftershocks.** Slip of a heterogeneous fault during an earthquake will only on average tend to the static slip distribution expected for the homogeneous case. On a local scale, small patches may slip less (or more) than surrounding regions and thus be dynamically loaded, rather than unloaded, during the earthquake. Note that the barrier of Das and Aki [1977b] is an extreme case of these patches, i.e.,
one that does not slip at all. Since the earthquake occurs dynamically, this loading is very rapid, and the $k$ for these regions can take any value $k < K_c$. Thus the initial conditions are set at the time of the main shock. Any patch for which $K_o < k < K_c$ will grow quasistatically to failure with a time delay that can be calculated from (1) (see Figure 3a). Thus aftershocks are predicted by the model, and should have the following characteristics:

a. Since large earthquakes can be expected to be heterogeneous, the occurrence of aftershocks, unlike foreshocks, should be nearly ubiquitous.

b. Aftershocks should be distributed all over the plane of rupture, not necessarily uniformly, and it is likely that a concentration of them will occur near the ends of the rupture, where the large scale stress concentration exists.

c. If $k$ is distributed randomly between $K_o$ and $K_c$ for the population of patches, then the frequency of failure of these regions, $m$, will be a function of time, $t$, after the main shock and will take the form,

$$ m = \frac{1}{t} \quad (4) $$

The derivation of (4) is given in Scholz [1988b] and follows from a static fatigue law of the type found by Mould and Southwick [1959]. This law was shown to be derivable from eqns. 1 or 2, by Wiederhorn and Bolz [1970].

d. Aftershocks are a second order effect relative to the main shock since they result only from deviations from the mean, so the sum of their moments should be only a small fraction of the moment of the main shock. This is one way to distinguish them from delayed multiple events.

One other property of aftershocks which is not directly implied by the model but seems likely is that aftershocks within the perimeter of the rupture zone will statistically tend to occur in isolated patches and hence these aftershocks will not
tend to have aftershock sequences of their own. Aftershocks that occur on the perimeter, and which therefore extend the perimeter, may, on the other hand, produce aftershocks. This effect was observed by Page (1968) for the 1964 Alaska earthquake.

The properties of aftershocks are sufficiently well known that it is not necessary to cite particular examples to state that the observations bear out the above predictions of the model. The prediction of the Omori Law (eqn. 4) is particularly important. In order to see how consistently aftershock sequences obey this law and the other above predictions, one should consult the compendia of Utsu [1969, 1970, 1971, 1972].

Deep earthquakes. In the above discussions we have tacitly been concerned only with tectonic earthquakes of shallow focus, as defined in the usual way. Deep focus earthquakes, on the other hand, do not conform to some of the predictions of the model. Most prominently, they do not have aftershock sequences. This may result either because the corroden responsible for this behavior, free \( H_2O \), is not present at those depths or simply because the mechanism of deep earthquakes is not the type of rupture process that we are discussing.

In the above we have taken a time dependent fracture criterion, which is well established experimentally, and applied it to the earthquake process. Since we know that \( K_c > K_o \) always, \( k_s > k_D \) always and \( k_D < K_c \) when the crack stops propagating dynamically, every possible relative condition between the \( k's \) and the \( K's \) has been considered in the above discussion. We found that in so doing, the theory predicts all of the many facets of rupture in the earth, many of which had no prior explanation and further, that no phenomena have been observed that are not predicted. We thus now have a physical basis for understanding these phenomena. We can see why foreshock sequences may be variable, whereas
aftershock sequences are very regular. We have a physical explanation for the Omori Law, and we can see that since delayed multiple events are a special case and slow earthquakes, an even more special case, that these phenomena should be uncommon. The success of the theory in predicting the observations demonstrates that rupture in the earth obeys a law similar to that observed in the laboratory to result from stress corrosion. It does not, of course, prove that the causative mechanism is necessarily the same. What must be emphasized is that subcritical crack growth, governed by a rate equation of a type similar to eqn. (1) or (2), must play an important role in the earthquake mechanism, although the underlying mechanism(s) cannot with any surety be identified, and in principle may be unidentifiable.
MATHEMATICAL ANALYSIS

We develop here a very simple theoretical approach to calculate properties of stress-corrosion cracking. The stress-intensity factor \( k \) (defined by \( \sigma = k/\sqrt{r} \), where \( \sigma = \) stress at the crack tip and \( r = \) distance from crack tip) for a two-dimensional plane crack (of any mode) and for a circular plane shear crack, in an infinite homogeneous medium which is linearly elastic everywhere off the crack plane, is given by

\[
k = C \Delta \tau \sqrt{X}
\]

(5)

where \( C \) is a geometric constant, \( \Delta \tau \) is the static stress drop and \( X \) is the crack length for a two-dimensional crack and is the radius for a circular crack. \( C \) is equal to 1 for a two-dimensional shear or tensile crack [Lawson and Wilshaw, 1975]. For a circular shear crack, there are two modes (viz. Modes II and III) of propagation at a point along the crack edge, each with its own stress-intensity factor. Taking \( k \) to be the square-root of the sum of the squares of these two stress-intensity factors, \( C \) is given by \( 20/7 \pi \sqrt{2} \) [Sih, 1973]. \( \Delta \tau \) refers to a tensile or a shear stress component depending on the mode of crack propagation.

From experimental results on stress-corrosion cracking [Atkinson, 1979] we find

\[
k = K_o \left( \frac{X}{V_o} \right)^{1/n}
\]

(6a)

or
where \( K_0 \) and \( V_0 \) are the values of \( k \) and \( \dot{X} \) at the initiation of quasistatic crack growth at time \( t = 0 \), and \( n \) is a dimensionless quantity called the 'stress-corrosion' index. \( K_0 \) was discussed in detail in the introduction. The time \( t = 0 \) may actually refer to the time at which the crack velocity is no longer vanishingly small. \( W_0 \) and \( n \) are the material properties of the medium. Combining (5) and (6a),

\[
\dot{X} = V_0 \left[ \frac{C \Delta T \sqrt{X}}{K_0} \right]^n
\]

If \( \dot{X} \) is independent of time, then

\[
\dot{X} = V_0 (\sqrt{\frac{X}{X_0}})^n
\]

Integrating,

\[
X = \left[ \frac{2-n}{2} - \frac{n-2}{2} \frac{V_0 t}{X_0^{n/2}} \right]^{\frac{2}{2-n}}
\]

The free parameters of this equation are \( X_0 \), \( V_0 \) and \( n \). The value of \( n \) (\( n \) is always \( > 2 \)) at this point is simply taken from Atkinson [1979] but later we shall determine it. Note that Figure 1 shows that \( n \) is independent of temperature and humidity but \( (K_0, V_0) \) are not, so that in the earth we expect to find \( n \) to be close to Atkinson's result but \( (K_0, V_0) \) to be very different.

Using (8), we can find the time to failure (defined as the time from which the crack can be detected to start growing subcritically to when it reaches instability) simply by setting the quantity in square brackets equal to zero, since this is where \( \dot{X} \) goes to infinity, since \( n > 2 \). This is the 'nucleation time' of an earthquake. Time to failure is then given by
and the crack size $X_f$ at time $\Delta t$ before failure can be determined by substituting $t' = t_f - \Delta t$ into (8). The velocity at time $\Delta t$ before failure can then be found from (7) by substituting $X_f$ for $X$. Note that as long as $\Delta t$ is independent of time, it does not enter into equations (7), (8), and (9).

At this point let us point out an advantage of using (9) to determine time to failure $t_f$ over the usual way of finding $t_f$ [Evans, 1972]. $t_f$ here depends only on the initial conditions and $n$ and not on the final conditions. If we now assume a value for $\Delta t$, we can also calculate $K_c$ just before failure. By following the above method, we have forced the crack to grow in accordance with a given $k - \dot{X}$ relationship (namely, equation (6)) until it reaches instability. The above method combines the two reinforcing effects of the stress-corrosion instability with the geometric instability. Note that if the stress-corrosion index $n$ and the point $(K_o, V_o)$ of the $(k-\dot{X})$ curve at which the instability occurs are known, the subcritical rupture process cannot be determined, but if $n$ and $(K_o, V_o)$ are known, the total rupture process to instability can be completely determined. In this sense, $K_o$ is a more fundamental property of the material than $K_c$ for cases when stress-corrosion cracking occurs.

The formulation developed above is applicable to the case when the crack does reach instability. If the crack does not reach instability but propagates subcritically through a region of length $L$, say, then we need a minor modification of the method described above. As an example, let us consider the case of a pair of delayed multiple events, with the rupture length of the first rupture being $X_o$ and with the barrier length between the two rupture zones being the length $L$. Then, combining equations (5) and (6b), we get the delay time between the multiple events to be
The unknown parameters in this equation are \( n, W_0, \Delta \tau, \) and \( L. \) If we have at least two sets of delayed multiple events across the same barrier (so that \( L, W_0, C, \Delta \tau, n \) are the same) but the rupture lengths of the first event of each set are different and the delay times for each pair are different, then we shall get a second equation like (10) with different values of \( t_1, \) say \( t_2 \) and a different \( X_0, \) say \( X_0'. \) Dividing the two equations, we get

\[
\frac{t_1}{t_2} = \frac{X_0^{2-n} - (X_0 + L)^{2-n}}{X_0'^{2-n} - (X_0' + L)^{2-n}} \tag{11}
\]

from which \( n \) can be obtained provided we assume a value for \( L. \) Once \( n \) is found, if we assume a \( \Delta \tau \) and take \( K_0 \) to be the stress-intensity factor due to a rupture of length \( X_0, \) we can also determine \( V_0 \) from (10). In the above we used equation (1) together with (5) to derive the mathematical formulation of the problem. We also derived similar relations using (2) and (5) and our results of the later sections were found to be virtually the same, and so this case is not separately discussed.

We point out here that this method is valid only up to the point when the crack-edge velocity approaches the sonic wavespeeds of the medium. Once the crack propagates with velocities comparable to sonic, the problem becomes a dynamic problem and has to be treated as such.

In the next section, we shall use the method developed here to model in detail some of the scenarios described earlier.
THEORETICAL MODELLING OF NUCLEATION PHASE

In our calculations using the method of the previous section, we shall assume that the strength of the material along the zone through which the rupture actually proceeds to be constant, i.e., \( W_0 \) and \( n \) are constants. Clearly, this will not be true in reality but since we do not know the details of this variation of strength, we do not think it meaningful at this time to complicate the model by introducing these variations. Our method, however, can be applied with minor modifications to the case of variable \( W_0 \) and \( n \). In that case the fault plane can be divided into segments of constant \( W_0 \) and \( n \) (concentric ones for circular cracks) and our method can be applied individually to each section until instability is reached. It will generally be intuitively very clear what would happen if \( K_0 \) were larger than the one chosen for the calculations.

We model the nucleation phase of the earthquake as a subcritical extension of a circular plane shear crack. The cases we consider are shown in Table 1. \( X_0, V_0 \) and \( n \) are the input parameters. The values of \( V_0 \) chosen were made consistent with the given \( n \)’s, a stress drop \( \Delta \tau \) of 100 bars, and a point \((K_1, V_1)\) on the \((k-X)\) curve given by \( K_1 = 10^{1.45} \) bar \( \sqrt{\text{cm}} \); \( V_1 = 0.1 \times 10^{-5} \) cm/sec for cases (i) and (ii), by \( K_1 = 10^{1.85} \) bar \( \sqrt{\text{cm}} \); \( V_1 = 0.1 \times 10^{-5} \) cm/sec for cases (iii) and (iv); and by \( K_1 = 10^{3.905} \) bar \( \sqrt{\text{cm}} \); \( V_1 = 0.1 \times 10^{-5} \) cm/sec for case (v). (However, we could also have chosen \( V_0 \) arbitrarily.)

The values of \( K_1 \) \([1 \text{ MNm}^{-3/2} = 10^2 \text{ bar } \sqrt{\text{cm}}]\) for cases (i) - (iv) are within the range shown in our Figure 1. The different cases represent different materials and the time to failure will tell us how strong they are. Using equations (8) and (9), the time to failure and the crack-radius \( X_f \) just before failure can be obtained. Let us take \( X_f \) to be the radius 1 second before failure. The results are shown in the last two columns of Table 1.
To study how the crack approaches instability in each case, we plot $X$ vs. time from equation (8) for approximately the last 100 seconds before failure. The results are shown in Figures 4 and 6. The shape of the curves agree with those obtained in the laboratory by Wiederhorn [1967; Figure 2], and, more interestingly, with the dilatometer records of Sacks et al. [1981] reproduced in Figure 7. The curve of strain as a function of time just prior to an earthquake was found by these authors to be exactly of the form shown in Figures 4 and 6. For case (v), the velocity at the last few time steps is clearly comparable to sonic velocities of the medium so the values of $t_f$ and $X_f$ in this case are not exact. The crack radius increases very slowly until the last few time steps when it increases very rapidly. It is only during this last phase that any precursory strain change may be large enough to be detectable. However, other precursory phenomena may result indirectly from the quasistatic rupture growth during the nucleation phase.

In Figures 5 and 8 we plot the $k$-$\dot{X}$ curves along which the crack extends. We plot the point $(K_o, \dot{X}_o)$, $(K_1, \dot{X}_1)$ and the last three points at the last three seconds prior to failure on each curve. If $K_c$ is the stress-intensity factor one second prior to failure, $K_c$ is given by the topmost point on each $(k$-$\dot{X})$ curve. We have assumed $K_c \leq K_{c*}$ in our calculations here. If $K_c > K_{c*}$, the instability will occur earlier than we calculated, and $X_f$ will be even smaller and the strain-change less detectable.

Thus our model of the nucleation phase of an earthquake implies that there must always be precursory slip and explains why it is seldom detected. We have assumed that only one crack is involved in the nucleation process. In reality there may be many small cracks growing subcritically and if enough cracks are involved in this process, the resulting strain change may be large enough to be measurable.

It is clear from Table 1 and Figures 4 and 6 that the size of the region over which the nucleation process occurs prior to the instability as well as the duration
of the nucleation process are highly dependent on the values of the variables assumed for the calculation. Therefore we have no idea at present as to either the spatial or temporal scale of the phenomenon, but can only make predictions as to its form. The scale can only be determined from observations such as those shown in Figure 7.
THEORETICAL MODELLING OF DELAYED MULTIPLE EVENTS

To search for this type of phenomenon we must look for two earthquakes with nearly adjacent rupture zones that occurred with a time delay $t_D$ much longer than the rise time of an individual event but much shorter than the recurrence time, i.e., $10^1 \text{ sec} << t_D << 10^{10} \text{ sec}$. Furthermore, the second event must be initiated in the region adjacent to the rupture zone of the first event, and propagate away from it. The second event of the pair is 'triggered' by the first and our theory suggests a physical mechanism by which one earthquake may 'trigger' another.

The Kii Peninsula barrier. Using an historical record that dates from 684 A.D., Ando [1975] has shown that this phenomenon has repeatedly occurred in large earthquakes along the Nankai trough in southwest Japan. He found that the plate boundary could be divided into four segments, $A, B, C, D$, that either rupture singly, in adjacent pairs, or all together in a single great earthquake (Figure 9). We show these regions in Figure 10 and note that Ando's observation indicates that the barriers between these regions are persistent, identifiable features. This is particularly true of the barrier of the tip of the Kii Peninsula, between regions $C$ and $B$. Since 684 A.D., this barrier has been one end of the rupture zone of nine great earthquakes.

Ando also showed that the Kii Peninsula barrier has ruptured with a time delay four times during the historic record, producing four pairs of delayed multiple events. The most reliable record dates from 1707, when all four regions ruptured in a single event (Figure 9), thus resetting the initial conditions in all four regions at the same time. The next event was in 1854, when the Ansei I event
ruptured DC, followed 32 hours later by the rupture of BA in the Ansel II event. The next event was the Tonankai event of 1944, which ruptured C, followed two years later by the Nankaido earthquake, which ruptured B dynamically and A quasistatically. Ando was able to show clearly that the Nankaido earthquake initiated near the BC boundary and propagated to the west, as predicted.

These two sets of delayed multiple events (1854 and 1944-46) ruptured the same barrier quasistatically and this provides us with a unique opportunity for calculating n for this barrier. The required model parameters for quantitative modelling of these earthquake pairs are taken from Ando, and shown below using our previous notation.

<table>
<thead>
<tr>
<th>Ansel I-II pair</th>
<th>Tonankai - Nankaido pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>t_1 = 32 hours</td>
<td>t_2 = 750 days</td>
</tr>
<tr>
<td>X_0 = 210 km</td>
<td>X'_0 = 133 km</td>
</tr>
</tbody>
</table>

Let us solve the problem using the various values of L shown in Table 2. For each case, the calculated n is shown in the second column.

We see that the value of n is virtually independent of L. More importantly, n differs from the value found by Atkinson only by a factor of 1 or 2. This is a remarkable agreement considering the simplicity of our model and the fact that we model a Mode II crack while Atkinson's crack propagated mainly in Mode I. If we assume δτ = 100 bars, and take K_o to be the stress-intensity factor due to a rupture of length 230 km, we can determine V_o from (10).

For the multiple event to occur, the barrier must have a strength such that K_o ≤ k_s. For the cases studied here, X_o = 100 km and Δτ = 100 bars, so that K_o ≤ 10^3 bar km^{1/2}. The specific fracture energy is given by G = K^2/4u [Lawn and Wilshaw, 1975], so that G = 10^{11} erg/cm^2. This value, similar to that obtained by
Aki [1979] for the Fort Tejon earthquake, is very high, implying kilobars of shear stress in the vicinity of the crack tip, and a specific surface energy much higher than that of any known material, therefore care should be taken in interpreting its meaning. It seems reasonable to suggest that this energy is dissipated in inelastic deformation within some volume surrounding the crack tip. This is a natural extension of the 'end zone' concept of Barenblatt [1959] discussed by Aki [1979] in his treatment of the subject. A pronounced change in the dip of the Benioff zone associated with the Nankai trough occurs beneath the Kii Peninsula, hence it is likely that this barrier is caused by a jog in the rupture plane (K. Mogi, personal communication, 1980). We envision that the energy required to stop the rupture is dissipated in the volume behind the jog.

Our estimate of $K_0$ for the Kii Peninsula barrier is several orders of magnitude higher than that assumed in our discussion of the nucleation problem. We would therefore not expect nucleation to occur within such an anomalous region, as excessive time (or stress) would be required. Variation in $K_0$ or $K_\sigma$ by orders of magnitude was also found by Aki [1979]. This suggests to us that these variations are due to geometrical complexities of the fault zone, rather than changes in the properties of the fault zone materials which are unlikely to vary so widely. It is interesting that, as discussed earlier with respect to heterogeneity, such geometric irregularity is expected to increase as $X^{1/2}$, which is exactly the manner in which $k$ scales (at constant $\alpha$). This suggests that the probability for an earthquake to stop will be scale independent.

An Example and Counterexample from the Aleutians

We now turn to several other examples of delayed multiple events that illustrate other aspects of the phenomenon. In Figure 11 we show the space time
sequence of large earthquakes in the Aleutian arc [from Sykes et al., 1980]. We see one clear example of a delayed multiple event: the 1957 Central Aleutians earthquake was followed 7 years later by the Rat Island earthquake, which initiated at the end of the 1957 rupture zone and propagated to the west [Sykes, 1971]. The barrier in this case, and in the case of most regions in the Aleutians where the rupture zones of large earthquakes terminate, has strong expression in the bathymetry [Spence, 1977].

It might, on first inspection, also appear from Figure 11 that the November 10, 1938 earthquake and the 1964 Alaskan earthquake are another example of a delayed multiple event pair. This is not the case, however, because the 1964 event initiated in the northeast part of its rupture zone and propagated to the southwest, i.e., towards the rupture zone of the earlier event. Therefore the close occurrence in time of these two contiguous events is a coincidence.

A Second Type of Aftershock Sequence

We note one additional interesting example from the Aleutians. The Rat Island earthquake of 1965 was followed by a sequence of large normal faulting earthquakes in the adjacent outer wall of the trench [Stauder, 1968]. This type of earthquake results from flexure of the outer wall and rise resulting from subduction [Sykes, 1971]. The sudden subduction produced by the Rat Island earthquake would be expected to 'trigger' earthquakes in the outer wall because it would result in a sudden increase in the flexure and it would also result in a sudden reduction in the horizontal compressive stress in the outer rise. Thus $k$ would be increased for normal faults in the outer rise as a result of the Rat Island earthquake, and for any region in which $K_0 < k < K_c$, failure will occur after a time delay. These events are therefore aftershocks of the Rat Island earthquake,
but of a different type than discussed earlier. The largest event in this sequence was an event of $M_s = 7.5$ which occurred 57 days after the Rat Island earthquake. This is thus a minimum estimate of the nucleation time for this earthquake. (It is a minimum estimate because our earlier definition of nucleation time was that it is the time to the instability from the initial condition $k = K_o$.)

**Complex Events**

In the above example, the difference between aftershocks and delayed multiple events is less distinct than in the usual case. The delayed multiple events discussed above are cases in which the two events occurred on the same fault. Cases of multiple events in which the two events occurred on different faults have also been observed. Delayed multiple events occur commonly in the Solomon Islands, and some of these occur near the sharp corner of the trench near New Britain and go around the corner [Lay and Kanamori, 1980; McCann, 1980]. Thus the first event has a N-S strike, the second E-W, or vice versa. Lay and Kanamori [1980] argue that this indicates that the slab is continuous around the corner, but this need not be so. The two Gazli earthquakes of 1976 occurred on two conjugate thrust faults with a time delay of 39 days, and certainly in this case an argument that they occurred on the same fault would be a forced one [Kristy et al., 1980]. There does not need to be a delay in such complex events, however. The 1927 Tango earthquake ruptured two orthogonal, conjugate, strike-slip faults in SW Japan. It is not known, however, whether this was a normal multiple event in which both faults ruptured dynamically or if one fault ruptured dynamically, the other quasistatically. We certainly know that it is possible for the first event to rupture dynamically and the second quasistatically on different faults, since this phenomenon was observed to follow the 1968 Borrego Mountain earthquake in
California [Allen et al., 1972]. The earthquake, which was on the San Jacinto fault, triggered quasistatic slip on the nearby Superstition Hills, Imperial, and San Andreas faults. Although we are not presently prepared to model these complex cases quantitatively, we believe they result from the same physical mechanism as discussed above.

**The Effect of the Phase of the Loading Cycle**

The loading cycle for a given region is the time required for the tectonic process to replenish the stress dropped in the previous earthquake on the same section of fault plane. The period of this cycle is the recurrence time for earthquakes in that place. It is clear from the above that the condition for the occurrence of multiple events and delayed multiple events is that the two regions on either side of the barrier must be nearly in phase in their loading cycles. We might expect, then, to see adjoining regions going in and out of phase in a cycle much longer than the loading cycle. Indeed, two such phase cycles appear in Ando's history (Figure 10). We begin in 887 A.D., when an ABCD event occurred, setting all regions into phase. This was followed by two delayed multiple events: the 1096 A.D. - 1099 A.D. pair and the 1360 A.D. -1361 A.D. pair. Then the 1498 A.D. CD event occurred, with no corresponding BA event, suggesting that BA was too far out of phase to be ruptured as a delayed event. The BA region ruptured 107 years later in 1605 A.D. The ABCD event of 1707 again put the regions in phase. Next follows the Ansei I and II pair, and the Tonankai-Nankaido pair. The regions seem to be getting out of phase again because D did not rupture in the most recent sequence. This region (Suruga Bay and the Tokai district) will either rupture on its own, which has not been observed to happen before, or skip this cycle and rupture in the next ABCD or CD event.
The reason we expect such phase cycles to occur is that two competing effects are at work. The first is the rupture process itself, which simply by it producing rupture on all possible regions in a given event, has a smoothing effect on heterogeneity which results in high phase correlation of adjoining regions. On the other hand, spatial variation of the tectonic process, such as variation in the slip vector magnitude and direction along a plate boundary, and heterogeneity of the geometry and material properties of the fault zone have a roughening effect. The first effect produces strong clustering in a space-time sense, the second randomness. What results, and is observed, is weak clustering. This is why space-time diagrams, such as those of Kelleher et al. [1970] seldom show significant or clearly obvious trends.
CONCLUSIONS

We have presented a detailed discussion of subcritical crack growth and its application to time-dependent rupture in the earth. Using very basic concepts of fracture mechanics and results obtained from laboratory experiments, we found that our theory predicts various scenarios for time-dependent rupture in the earth. We have presented several examples to show that all of the predicted phenomena are actually observed in the earth. The theory does not contain any predictions other than those observed to occur, nor do the observations indicate that any phenomena occur other than those predicted. Our theory explains why some of these phenomena are more common than others. Using a simple theoretical development, we model two cases in detail, the nucleation stage before an earthquake and delayed multiple events. For the nucleation problem we show that all earthquakes must have precursory slip but the resulting strain changes may be much too small to be detectable. For delayed multiple events occurring along the Nankai trough, we found two pairs of delayed multiple events that were separated by the same barrier. We uniquely determined an estimate of the stress corrosion index for the barrier between these events. We thus obtained estimates of the material properties of barriers in the earth. Our theory also suggests a physical mechanism by which an earthquake can 'trigger' another earthquake on an adjacent zone of the same fault or on a different fault in its vicinity.
Acknowledgments. We would like to thank Paul Richards and Lynn Sykes for critically reviewing the manuscript. Selwyn Sacks allowed us to reproduce Figure 7 prior to its publication and we are grateful to him for this. This work was supported by National Aeronautics and Space Administration grant NGR 33-008-146, and National Science Foundation, Division of Earth Sciences, grants EAR-79-01810 and EAR-80-07426. Lamont-Doherty Geological Observatory Contribution No. 0000.
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<table>
<thead>
<tr>
<th>Stress-Corrosion Index, n</th>
<th>Initial Crack Radius, X₀</th>
<th>Initial Crack-Edge Velocity, V₀, cm/sec</th>
<th>Time to Failure, tᵢ</th>
<th>Crack Radius 1 Second Before Failure, Xᵢ</th>
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<tr>
<td>12.5</td>
<td>0.1 cm</td>
<td>$0.169 \times 10^{-7}$</td>
<td>1125906 sec = 31.0 days</td>
<td>1.76 cm</td>
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<tr>
<td>20.0</td>
<td>0.1 cm</td>
<td>$0.146 \times 10^{-8}$</td>
<td>7593035 sec = 20.9 days</td>
<td>0.63 cm</td>
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<td>12.5</td>
<td>0.5 cm</td>
<td>$0.395 \times 10^{-8}$</td>
<td>24094025 sec = 67.9 days</td>
<td>14.18 cm</td>
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<td>20.0</td>
<td>0.5 cm</td>
<td>$0.143 \times 10^{-9}$</td>
<td>388763365 sec = 12.3 yr</td>
<td>5.69 cm</td>
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<tr>
<td>12.5</td>
<td>1.0 km</td>
<td>0.1099</td>
<td>173382 sec = 2.0 days</td>
<td>1.4 km</td>
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<td>Stress-corrosion Index (n)</td>
<td></td>
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<td>----------------------</td>
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<td></td>
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<tr>
<td>1.</td>
<td>23.2</td>
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<tr>
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</tr>
<tr>
<td>50.</td>
<td>24.8</td>
<td></td>
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</tbody>
</table>
FIGURE CAPTIONS

Figure 1. Experimental data obtained by Atkinson [1979] for single crystal quartz showing relationship between stress-intensity factor $k$ and crack tip velocity for subcritical rupture for Mode I cracks.

Figure 2. Schematic diagram showing the nucleation phase of an earthquake and the resulting phenomena. $X$ denotes the distance along the crack, $u$ the corresponding slip, and $B$ an elastic wave velocity. Stippling denotes subcritical rupture and hatching denotes dynamic rupture. The sawtooth in case A indicates the presence of a barrier, i.e., a region in which rupture is relatively inhibited. The conditions on $k$ in the two cases are indicated. A slow earthquake arises when the rupture propagates into regions in which $k < K_o$. In the more likely case, an instability arises and a 'normal' earthquake occurs.

Figure 3. Schematic diagram using the same symbolism as Figure 2 showing how earthquakes stop and the resulting phenomena. The earthquake stops at a barrier when $k_D < K_c$ for the barrier. As the slip between the barriers tends to its static value, $k$ increases to $k_s$. If the barrier has uniform strength, then either $k_s < K_c$, $K_c < k_s < K_s$, or $k_s > K_c$ and we have cases A, B, or C. If the barrier increases in strength with distance, case D occurs. Case E shows the aftershock that occurs as a result of non-uniform slip on the fault.
Figure 4. Diagrams showing the crack-radius as a function of time during the nucleation phase. The star denotes initial crack size.

Figure 5. Stress intensity factor \( k \) as a function of crack edge velocity \( X \) for the cases shown in Figure 4. The last three points on each curve denote the values of \( k \) and \( X \) three seconds prior to failure. The last point is a very good estimate of \( K_c \), the modulus of cohesion for stress corrosion cracking.

Figure 6. Same as Figure 4, but showing case (v).

Figure 7. Plot of strain vs. time recorded with a dilatometer due to a nearby small earthquake in the Matsushiro region. The event 74/07/24 is preceded by a slow strain change which accelerates into a 'normal' earthquake. The earthquake onset is shown by the arrow. Compare the form of the precursor with Figures 4 and 6 [after Sacks et al., 1981].

Figure 8. Same as Figure 5, but showing one case (v). \( X \) is in km/hr. In this case, the \( X \) at the last few points is comparable to the sonic velocity of the medium, so that the quasistatic rupture theory is no longer valid and the estimate of \( K_c \) is not very reliable.

Figure 9. Schematic representation of earthquakes along the Nankai trough, western Japan, since 684 A.D. (modified from Ando [1975]). Since 1707 A.D., further details are included. The rupture lengths of the first event of an event pair and the delay times between pairs are
shown. The arrows indicate that the second rupture initiated from the end where the first rupture had been arrested by the barrier.

Figure 10. Bathymetric map off the southeast coast of Japan showing the segments A,B,C,D along the Nankai trough. The barriers between the segments may have some weak correlation with the bathymetry.

Figure 11. Map of rupture zones and the space time sequence of large earthquakes along the Aleutian arc [after Davies et al., 1980 and Sykes et al., 1980]. We make specific reference to the adjacent 1957 and 1965 events, which are delayed multiple events, and the 1938 and 1964 pair, which are not.
Nucleation

Figure 2

$K_0 < k < K_c$

$\dot{x} \ll \beta$

$\dot{x} = \beta$

$K > K_c$

SLOW EARTHQUAKE

NORMAL EARTHQUAKE

$K < K_0$

$\dot{x} \ll \beta$
II Stopping

\[ k_s < k_0 \]  
EARTHQUAKE STOPS

\[ k_D < k_C \]

A

\[ k_0 < k_C \quad k_s > k_C \]  
MULTIPLE EVENT \( t_D \leq t_R \)

\[ k_0 < k_s < k_C \]  
DELAYED MULTIPLE EVENT \( t_D \gg t_R \)

B

\[ 2-3 \quad k_0 < k_s < k_C \quad k_s < k_0 \]  
POSTSEISMIC RUPTURE GROWTH

C

D

E

AFTERSHOCKS  
\( m \propto \frac{1}{t} \)
Figure 5

CRACK-EDGE VELOCITY (km/hr)

STRESS-INTENSITY FACTOR k (bar/√m)

n = 20.0
n = 12.5

Values:
- \( \Delta r = 1 \text{ cm} \)
- \( x = 3 \text{ cm} \)
- \( x = 1.5 \text{ cm} \)
- \( x = 1.25 \text{ cm} \)
- \( x = 0.8 \text{ cm} \)
- \( x = 0.5 \text{ cm} \)
$X_0 = 1 \text{ km} \quad V_0 = 0.1099 \text{ cm/sec}$

$n = 12.5$

$\ast$—Denotes initial crack size at time $t = 0$
Figure 9