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OZONE DATA AND MISSION SAMPLING ANALYSIS

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SUMMARY

Techniques have been developed to analyze global data sets of atmospheric constituents and to evaluate mission sampling strategies using these global data sets. Mathematical formulations and computer programs were developed to reduce and model global data fields and to perform statistical analyses of results.

The grouping scheme used to reduce data into a global grid network is shown and data storage methods are discussed. Procedures for modeling these data with spherical harmonic functions and empirical orthogonal functions (EOF) are detailed mathematically and numerical computer solutions are developed. Eigenanalysis techniques in conjunction with these EOF models are illustrated for reducing the dimensionality of large data sets.

The seemingly ever-present "missing data" problem is examined using the sample autocorrelation function. A linear regression technique is demonstrated which generates a "corrected" ozone satellite data set based on Dobson spectrophotometer (land based) measurements.
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I. INTRODUCTION

Defining the temporal and spatial variability of atmospheric constituents requires a sampling strategy and sensing technique that is consistent with the nature of the species being studied. Measurements of the global ozone field have been made for years from the ground\(^1\), from aircraft and balloons\(^2\), and more recently from satellites\(^3\). This information can be examined to determine something about the statistical nature of these data and, generally, the types of sampling schemes that should be considered.

The objective of this sampling study is to evaluate various sampling schemes which are based on the current understanding of the global ozone field and on other mission related constraints. To accomplish this, representative data must be acquired and reduced into a usable form. A model of the global ozone field must be developed. Computer simulated missions can be generated by "measuring" the global ozone field as represented by this model as viewed by selected sampling schemes. How well these sampling missions "recover" the model is determined by statistical analysis techniques which serve in the mission evaluation process.

This report addresses itself not so much to the overall sampling evaluation problem but to the techniques that have been thus far developed and utilized toward that end, especially in the areas of preliminary data manipulation and reduction, model development, computer simulations of sampling missions, and the associated statistical analysis techniques used throughout the work.

Appendix A shows the primary computer programs mentioned during the discussion.
II. PRELIMINARY DATA ANALYSIS - DATA MANIPULATION AND REDUCTION

Global ozone data utilized in this study are primarily from the Backscattered Ultraviolet (BUV) experiment aboard the Nimbus 4 satellite. These data have been received from the National Space Science Data Center in the form of IBM unformatted binary magnetic tapes. To some lesser extent ozone measurements from Dobson spectrophotometers are used. These data will be mentioned later in this report. This section is concerned with the BUV data. Particular items discussed include:

1. Conversion of the data tapes from IBM internal format to NOS-CDC internal format
2. Preliminary data analysis
3. Data grouping
   (a) Global grid system
   (b) Statistical analysis
   (c) Data retrieval technique
1. **IBM Format to CDC Format Conversion**

The BUV ozone data used for this study have been received on magnetic tape written in IBM internal format. In order to generate a NOS compatible set of data tapes the 32 bit IBM words must be unpacked into 60 bit CDC words, and the IBM internal format must be converted to CDC internal format.

A computer program (BUVCOP2) was written to accomplish this task. This program has successfully generated a set of NOS tapes containing global ozone data from April 10, 1970 through May 6, 1977. Table B-1 shows the time coverages and designations of the various magnetic tapes involved in this process. Appendix B discusses the data tape structure and the IBM to NOS-CDC internal format conversion in more detail.

2. **Preliminary Data Analysis**

Once a set of usable data tapes have been acquired, they must be carefully reviewed to ensure that their general format and content are consistent with the user's understanding and that there are no apparent problems with the data. A computer program (BUV3) has been written to look for particular problems associated with the data. These include:

1. **Out of sequence (OOS) data** - data that are chronologically out of order.

2. **Out of range (OOR) data** - a data record containing a latitude or longitude value out of its realistic range (-90° > latitude > 90°, 0° > longitude > 360°), or a measured observable whose value is inconsistent with accompanying user information.
3. Inconsistent local time (ILT) data - since Nimbus 4 is in a Sun-synchronous orbit, the satellite should cross the equator at approximately the same local solar time each orbit. This is the case for both the ascending and descending portions of the orbit. However, only the ascending portion of the orbit is of concern here since the descending portion of the orbit is on the dark side of the Earth and the BUV experiment only works in the sunlight. Local solar times can readily be calculated by the expression,

\[ t_e = t_g + \phi/15^\circ, \]

where \( t_e \) is the local time, \( t_g \) is the Greenwich mean solar time (GMT) of the observation, and \( \phi \) is the longitude of the observation measured eastward from the prime meridian (PM). The analysis program calculates \( t_e \) for observations within 5° of the equator and compares them to the known local crossing time, \( t_k \). If the difference \( t_e - t_k \) is less than some predetermined acceptable \( \Delta t \), agreement in the two is assumed to be good. Due to orbital considerations \( t_k \) may change slightly as a function of time over several years.

4. Repetitive data - two or more data records that occurred at either the same time or same position (the latter being consecutive measurements) but that differ in the values of other parameters.

5. Duplicate data - a data record or records that exactly duplicates another data record.

6. Reversed ground track - a series of data records that show the satellite ground track moving the wrong direction latitudinally. Such problems need to be identified and, where practical, eliminated. If known or suspected problem areas remain, one must be mindful of their potential impact in further analyses.
Table 1 is representative of the information one may expect from the BUV3 analysis program. This particular analysis table is for the third set of BUV data (BUV III). Items such as the number of files, number of records (observations), and mission duration give information useful for future analyses as well as confirming the data tapes' general structure and content. The diagnostics such as the quantity and nature of abnormal ozone values, help in determining what, if any, data editing must be performed. For example, an observation near the equator whose calculated local time, \( t_e \), disagrees with the known local time, \( t_k \), significantly could mean that either the GMT or longitude are incorrect. However, if several observations near the equator for a given orbit show disagreement, the entire orbit is suspect and requires more careful scrutiny. Appendix C describes a linear approximation for calculating local time as a function of latitude that is used between \( \pm 60^\circ \) for this purpose.

3. Data Grouping Scheme

The most recent set of BUV data tapes covers the period from April 10, 1970 through May 6, 1977 and contains 1,034,456 total ozone observations. There are 20 parameters associated with each observation as described in Table 8-2. This amounts to 20,689,120 computer words of data that are contained on these tapes. In order to work with such large quantities of data they must be grouped in a manageable form and stored in such a way as to be easily retrievable.

It was decided to group the data according to a global grid system each element of which would be 5° in latitude by 15° in longitude. This arrangement lends itself nicely to the format of a data array dimensioned 36 x 24 where there are 36 rows representing the 5° latitudinal zones and 24 columns representing the 15° longitudinal sectors. This global grid system is illustrated in Figure 1. The indices shown in the figure follow from the expressions,
\[ \begin{align*}
    i &= \left( \theta/5^\circ \right) + 1, \quad \text{for} \quad 0^\circ \leq \theta < 90^\circ \\
    &\quad \left( \theta/5^\circ \right) + 19, \quad \text{for} \quad -90^\circ < \theta < 0^\circ
\end{align*} \quad (1) \]

and

\[ j = \left( \phi_w/15^\circ \right) + 1, \quad \text{for} \quad 0^\circ \leq \phi_w < 360^\circ, \quad (2) \]

where \( \theta \) is the latitude and \( \phi_w \) is the longitude measured westward from the PM.

As the data are being grouped into this grid, it is convenient to compile a set of elementary statistics describing the data's behavior. Useful quantities that can be readily calculated for a given time period include:

1. Sampling distribution
2. Data means
3. Data variances.

The basis for this analysis is the grid format described. Each observation is placed in a grid block based on its latitude (\( \theta \)) and longitude (\( \phi_w \)) according to equations (1) and (2). The global sampling distribution can readily be determined by counting the accumulation of observations into each block \( (i,j) \). The zonal data distribution is found by summing this result over \( j \) \((1 \leq j \leq 24)\) for each individual zone \((1 \leq i \leq 36)\).

For each grid block the mean ozone value, the mean position of observations, and the mean time of observations are calculated as shown below:

\[ X_{ij} = \sum_{\ell=1}^{k_{ij}} \frac{d_{\ell}}{k_{ij}} \quad (3) \]

where the \( d_{\ell} \) represent the \( \ell \)th data record of either latitude, longitude, time, or ozone value contained in block \((i,j)\); \( k_{ij} \) is the number of observations contained in block \((i,j)\); and \( X_{ij} \) is the block mean for whichever of the above quantities is represented by \( d_{\ell} \).
Zonal means are calculated by

$$X_i = \frac{\sum_{j=1}^{24} (\sum_{i=1}^{k_{ij}} d_{ij})_j}{\sum_{i=1}^{24} k_{ij}},$$

or

$$X_i = \frac{\sum_{j=1}^{24} X_{ij} k_{ij}}{h_i},$$

where

$$h_i = \sum_{j=1}^{24} k_{ij}.$$  

Associated variance calculations follow from

$$\text{VAR}(X) \equiv \langle (x - \langle x \rangle)^2 \rangle.$$  

Then the variance of the data contributing to the grid block mean becomes

$$\sigma^2_{k_{ij}} = \frac{1}{k_{ij}-1} \left[ k_{ij} \sum_{i=1}^{24} d_{ij}^2 - k_{ij} X_{ij}^2 \right],$$

and the variance of the data contributing to the zonal mean becomes

$$\sigma^2_{h_i} = \frac{1}{h_i-1} \left[ \sum_{j=1}^{24} (\sum_{i=1}^{k_{ij}} d_{ij})_j - h_i X_i^2 \right].$$

The subscripts on $\sigma^2$ show the number of ozone observations in the sample being considered.

Finally, a "data mean" and variance are calculated which include all available data from the global grid. The data mean is

$$X = \frac{\sum_{i=1}^{36} \sum_{j=1}^{24} (\sum_{i=1}^{k_{ij}} d_{ij})_j}{\sum_{i=1}^{36} \sum_{j=1}^{24} k_{ij}},$$

and the variance of the data becomes

$$\text{VAR}(X) \equiv \langle (x - \langle x \rangle)^2 \rangle.$$
or
\[
X = \sum_{i=1}^{36} \sum_{j=1}^{24} X_{ij} k_{ij}/M,
\]
where
\[
M = \sum_{i=1}^{36} \sum_{j=1}^{24} k_{ij}.
\]

This "data mean", \(X\), is not referred to as a global mean since the spatial distribution of the BUV data is non-uniform and, therefore, \(X\) is necessarily area biased. In addition, these data do not provide global coverage due to orbit and sensor design. In fact, BUV annual coverage extends only from approximately 80° south latitude to 80° north latitude. Otherwise, the extent to which the global grid is filled depends on the length of the time interval being considered and upon the actual portion of the BUV mission being examined. The latter is due to the fact that the data density per unit time decreases in the later years of the mission.

The variance of the data contributing to the data mean is
\[
\sigma_M^2 = \frac{1}{M-1} \left[ \sum_{i=1}^{36} \sum_{j=1}^{24} k_{ij} \left( \sum_{l=1}^d d_{ij}^l \right)^2 - MX^2 \right].
\]

The computer program OZSTAT2 was written to perform these analysis tasks. Graphics capabilities included in the OZSTAT2 program provide for each case a plot of the zonal means with ±1σ\(X_i\) error bars, a scatter diagram of the ozone distribution as a function of latitude, and histograms of the data sampling distribution as a function of latitude or longitude. Examples of the graphics output are shown in Figures 2 through 4. A listing of this computer program and accompanying subroutines is included in Appendix I.

A means of storing and accessing these reduced data for specified time intervals is required. Typical time periods examined in this study include seasonal (90 days), monthly (30 days), weekly (7 days), and, less frequently, daily intervals. It was, therefore, decided to store this information on a daily basis in such a way that data for larger time intervals can conveniently be generated by accumulating the appropriate daily values.
Specific quantities that must be accessible on a daily basis per grid block are,

1. Sampling Distribution
2. Ozone Mean
3. Average Latitude of Observations
4. Average Longitude of Observations
5. Average Time of Observations
6-9. Variances Associated with Items 2-5 above.

Rather than storing these specific nine pieces of information per grid block, it was decided to save the sums and the sums of the squares of the ozone, time, latitudinal, and longitudinal values along with the sampling distribution from which the required means and variances are readily calculable by

\[
\bar{X}_\varepsilon = \frac{\sum_{ij} X_{ij\varepsilon}}{k_{ij}}
\]  

(14)

and

\[
\sigma^2_{X\varepsilon} = \frac{1}{k_{ij}-1} \left[ \sum_{ij} X_{ij\varepsilon} - (\sum_{ij} X_{ij\varepsilon})^2/k_{ij} \right]
\]  

(15)

where \( \bar{X}_{\varepsilon} \) is the mean, \( \sigma^2_{X_{\varepsilon}} \) is the associated variance, \( \sum_{ij} X_{ij\varepsilon} \) is the sum, and \( \sum_{ij} X_{ij\varepsilon}^2 \) is the sum of the squares of the \( \varepsilon \)th quantity for grid block \((i,j)\).

The \( \varepsilon \)'s signify the following:

\( \varepsilon = 1 \) Ozone,
\( \varepsilon = 2 \) Time,
\( \varepsilon = 3 \) Latitude,
\( \varepsilon = 4 \) Longitude.

The number of samples per block \((i,j)\) is \(k_{ij}\).
It was further decided to store this information on a mass storage random access (MSRA) file, primarily because this approach minimizes the computer storage problem inherent with these large data sets and also because of the convenience associated with utilizing the MSRA file for this kind of storage and retrieval process. A set of subroutines have been designed to access this MSRA file returning to the calling program a data array in the form of the standard 36 x 24 global grid system containing one of the nine quantities mentioned above for a given day or collection of days. These subroutines can be easily incorporated into computer programs requiring these grided ozone data without drastically affecting the program's storage requirement. Details concerning the MSRA file, its creation and its access are contained in Appendix D.

The preliminary data analysis concepts discussed above are beneficial for the following reasons:

1. Setting up a standard grid network as outlined establishes a basis for data analysis and lends itself nicely to making preliminary statistical calculations.

2. The preliminary statistical analysis shows how the data distribution varies as a function of latitude and longitude which helps in the development of mission sampling strategies.

3. Large data sets become more easily manageable when described by a global grid network which can be put into the form of a data array in the computer and saved on MSRA files.
III. STATISTICAL MODELING AND ANALYSIS TECHNIQUES

An essential part of this mission sampling study is the development of models which describe the variability of the global ozone field and the statistical analysis techniques which can be used to evaluate these models and the sampling schemes that they represent. The model primarily used in this work has been the Spherical Harmonic model, though the modeling of data with Empirical Orthogonal Functions has also been investigated and used to some extent throughout the effort. These models and certain statistical analysis techniques have been incorporated into computer programs which will be discussed.

Cases arise where it is desirable to have a completely filled global grid system. The BUV data does not provide this required global coverage. A computer program has been prepared to handle this missing data problem using either a Spherical Harmonic model or a "model" based on autocorrelation functions. The data fill problem is discussed later in this section.

4. Spherical Harmonic Model - Parameter Estimation and Evaluation

The form of the spherical harmonic model chosen for this study is,

$$y(\theta, \phi) = \sum_{m=0}^{M} \sum_{n=0}^{M} \left[ A_{mn} Y_{m}^{n}(\theta, \phi) + D_{mn} Y_{m}^{0}(\theta, \phi) \right] + \epsilon_i$$

(16)

where

$$Y_{m}^{n}(\theta, \phi) = \cos(m\phi) F_{nm}^{S} p_{m}^{n}(\cos \theta),$$

(17)

$$Y_{m}^{0}(\theta, \phi) = \sin(m\phi) F_{nm}^{S} p_{m}^{n}(\cos \theta),$$

(18)

$$F_{nm}^{S} = \begin{cases} 
1, & \text{for } m=0 \\
\frac{2(n-m)!}{(n+m)!} \frac{1}{1/2}, & \text{for } m>0 
\end{cases}$$

(19)
\( P_n^m(\cos \theta) \) are the associated Legendre functions of degree \( n \) and order \( m \), and \( A_{mn} \) and \( D_{mn} \) are the coefficients associated with the functions \( Y_n^m(\theta, \phi) \) and \( Y_n^0(\theta, \phi) \), respectively. \( F_{mn}^S \) is the Adolf Schmidt seminormalization constant. \( \epsilon_i^2 \) is the error associated with the \( i \)th observation at colatitude \( \theta_i \) and longitude \( \phi_i \).

For a given data set coefficients for a spherical harmonic model of specified degree and order are determined by a least squares solution that minimizes the sum of the squares of the residuals \( \epsilon_i^T \epsilon_i \).

Equation (16) above can be rewritten as

\[
y(\theta_i, \phi_i) = \sum_{n=1}^{N} f_n(\theta_i, \phi_i) b_n + \epsilon_i
\]  

(20)

where both the odd and even functions, \( Y_n^0(\theta, \phi) \) and \( Y_n^E(\theta, \phi) \), are included in \( f(\theta, \phi) \), and, similarly, the coefficients \( A_{mn} \) and \( D_{mn} \) are included in \( b \). Some care must be exercised in maintaining the proper ordering of the terms in equation (20). Note that \( N \) in equation (20) is the number of coefficients (and therefore the number of functions) contained in the model and not the degree of the model. Generally, the order and degree of the spherical harmonic models used in this study are equal. If a specified model is of order \( M \) and degree \( M \), then

\[
N = (M + 1)^2.
\]  

(21)

Equation (20) can be written in matrix form as

\[
\underline{Y} = \underline{F} \underline{B} + \underline{E}.
\]  

(22)

The double underline signifies a matrix quantity while a single underline denotes a vector. To minimize the sum of the squares of the residuals the quantity

\[
SS = \underline{E}^T \underline{E} = (\underline{Y} - \underline{F} \underline{B})^T (\underline{Y} - \underline{F} \underline{B})
\]  

(23)
must be differentiated with respect to \( \beta \). This leads to the so-called "normal equations" which can be solved such that

\[
\hat{\beta} = (F^T F)^{-1} F^T Y. \tag{24}
\]

The estimated coefficients contained in the \( \hat{\beta} \) vector are unbiased since

\[
\hat{\beta} = \beta. \tag{6}
\]

Information regarding the sampling can also be gained from equation (24). To that end calculate the covariance of \( \hat{\beta} \) as follows. Rewrite equation (24) as

\[
\hat{\beta} = G Y, \tag{25}
\]

where \( G = (F^T F)^{-1} F^T \) is a function of sampling position only and is therefore treated here as a constant. The covariance matrix for \( \hat{\beta} \) can be found by the law of propagation of errors\(^6\) such that

\[
\text{Covar}(\hat{\beta}) = G \text{Covar}(Y) G^T. \tag{26}
\]

It is assumed that all components of \( Y \) are independent,

\[
\text{Covar}(y_i, y_j) = 0 \quad \text{for} \ i \neq j, \tag{27-a}
\]

and have the same variance,

\[
\text{Var}(y_i) = \sigma^2, \tag{27-b}
\]

so that

\[
\text{Covar}(Y) = \sigma^2 I \tag{27-c}
\]

where \( I \) is the identity matrix.

Then equation (26) may be rewritten as

\[
\text{Covar}(\hat{\beta}) = G \sigma^2 I G^T \tag{28-a}
\]

\[
= [(F^T F)^{-1} F^T] [(F^T F)^{-1} F^T]^T \sigma^2 \tag{28-b}
\]

\[
\text{Covar}(\hat{\beta}) = [F^T F]^{-1} F[(F^T F)^{-1}]^T \sigma^2. \tag{28-c}
\]

Now consider some symmetric matrix \( z \). Since the operations TRANSPOSE(T) and invserse (-1) commute\(^7\),

\[
(z^{-1})^T = (z^T)^{-1}. \tag{29}
\]
But \( \mathbf{Z} \) is also symmetric; therefore,
\[
\mathbf{Z} = \mathbf{Z}^T, \tag{30}
\]
and
\[
(\mathbf{Z}^{-1})^T = \mathbf{Z}^{-1}. \tag{31}
\]
As \( \mathbf{F}^T \mathbf{F} \) is also a symmetric matrix, by applying equation (31), equation (28-c) may be written as
\[
\operatorname{Covar}(\hat{\mathbf{b}}) = (\mathbf{F}^T \mathbf{F})^{-1} \sigma^2, \tag{32}
\]
where the variances associated with the estimated coefficients \( \hat{b}_n \) are the corresponding diagonal elements of the covariance matrix. The off-diagonal elements are, of course, the covariance terms. In this study \( \sigma^2 \) has typically been set equal to one, so that
\[
\operatorname{Covar}(\hat{\mathbf{b}}) = (\mathbf{F}^T \mathbf{F})^{-1}. \tag{33}
\]
This is an important result that statistically describes how well the model can be fitted to the sample space being considered. Recall that this result is independent of the observation vector \( \mathbf{Y} \).

An interesting, if only heuristic, illustration is the case where only one sample position is contained in the sampling scheme for a spherical harmonic model. Consider the product of the observation matrix \( \mathbf{F} \) and its transpose written as
\[
\mathbf{S} = \mathbf{F}^T \mathbf{F} = \\
\begin{bmatrix}
\Sigma_{i=1}^p f_{i1}^2 & \Sigma_{i=1}^p f_{i1} f_{i2} & \cdots & \Sigma_{i=1}^p f_{i1} f_{iN} \\
\Sigma_{i=1}^p f_{i2} f_{i1} & \Sigma_{i=1}^p f_{i2}^2 & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\Sigma_{i=1}^p f_{iN} f_{i1} & \cdots & \Sigma_{i=1}^p f_{iN}^2
\end{bmatrix}. \tag{34}
\]
where $P$ is the number of observations and the function $f$ is the same as it was in equation (20). Then

$$S_{kj} = \sum_{i=1}^{P} f_k(\theta_i, \phi_i) f_j(\theta_i, \phi_i),$$

(35)

or

$$S_{kj} = f_k(\theta_1, \phi_1) f_j(\theta_1, \phi_1) + f_k(\theta_2, \phi_2) f_j(\theta_2, \phi_2) + \ldots + f_k(\theta_p, \phi_p) f_j(\theta_p, \phi_p).$$

(36)

The one sample position occurs at

$$\theta = \theta_1 = \theta_2 = \ldots = \theta_p$$

and

$$\phi = \phi_1 = \phi_2 = \ldots = \phi_p.$$

Equation (36) then becomes

$$S_{kj} = P f_k(\theta, \phi) f_j(\theta, \phi),$$

(37)

and equation (34) becomes

$$\mathbf{S} = \mathbf{F}^T \mathbf{F} = P \begin{bmatrix} f_1^2 & f_1 f_2 & \ldots & f_1 f_N \\ f_2 f_1 & f_2^2 & \ldots & \ldots & f_2 f_N \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ f_N f_1 & \ldots & \ldots & f_N^2 \end{bmatrix}.$$  

(38)

Now if for some matrix $\mathbf{Z}$ all elements of a row (or column) may be obtained from the elements of another row (or column) by multiplication by a constant, that is, if $z_{ij} = (\text{constant}) z_{kj}$ for all $j$ or $z_{ij} = (\text{constant}) z_{i\ell}$ for all $i$, then det $\mathbf{Z} = 0$. 6

Also the inverse of a matrix $\mathbf{Z}$ can be calculated element by element according to
\[
(z^{-1})_{ij} = \frac{\text{cofactor}(z_{ji})}{\det(z)}, 
\]
\(39\)

where \(\det(z)\) is the determinant of the \(z\) matrix.

It can then be seen from equations (38) and (39) that

\[
\det(s) = 0, 
\]
\(40\-a\)

hence the covariance matrix

\[
S^{-1} \to \infty, 
\]
\(40\-b\)

or the variance associated with estimated coefficient values would be infinite.

This result demonstrates the inability of the least squares technique to accurately estimate the required coefficients of a model with \(N\) functions \((N > 1)\) when the sample space consists of only one point. A more general comment that may be inferred from this example is that the variance in the estimated coefficient vector is a function of the sampling distribution and is not necessarily dependent on the number of samplings.

5. **Statistical Analysis of Spherical Harmonic Model**

The data variance \(\sigma_d^2\) of the observations contained in the vector \(Y\) is calculated by

\[
\sigma_d^2 = \frac{1}{(P-1)} \left[ \sum_{i=1}^{P} y_i^2 - \left( \sum_{i=1}^{P} y_i \right)^2 / P \right],
\]
\(41\)

which comes simply from the definition of variance as in equation (7), where \(P\) is, as above, the number of observations.

The RMS residual between the measurement and the spherical harmonic model is,

\[
\text{RMS} = \left[ \frac{1}{P} \overline{E^T E} \right]^{1/2}. 
\]
\(42\)

By equation (23),

\[
\text{RMS} = \left[ \frac{1}{P} (Y - \hat{F} \hat{B})^T (Y - \hat{F} \hat{B}) \right]^{1/2} 
\]
\(43\)
where $\hat{B}$ is the vector of estimated coefficients that minimizes the RMS. Equation (43) can be expanded so that

$$\text{RMS} = \left[ \frac{1}{p} \left( Y^T Y - Y^T \hat{B} + \hat{B}^T F^T Y + F^T F \hat{B} \right) \right]^{1/2}. \quad (44)$$

Note the term

$$Y^T \hat{B} = Y^T (1 \times p) \times F(p \times n) \times \hat{B}(n \times 1)$$

is a scalar. Therefore,

$$Y^T \hat{B} = (Y^T \hat{B})^T = \hat{B}^T F^T Y, \quad (45)$$

and

$$\text{RMS} = \left[ \frac{1}{p} \left( Y^T Y - 2\hat{B}^T F^T Y + \hat{B}^T F^T F \hat{B} \right) \right]^{1/2}. \quad (46)$$

Substitution of equation (24) into the last term of equation (46) leads to

$$\text{RMS} = \left[ \frac{1}{p} \left( Y^T Y - 2\hat{B}^T F^T Y + \hat{B}^T F^T F \hat{B} \right) \right]^{1/2}. \quad (47)$$

In this manipulation it must be remembered that

$$(F^T F)^{-1} = I,$$

where $I$ is the identity matrix.

Equation (47) quickly simplifies to

$$\text{RMS} = \left[ \frac{1}{p} \left( Y^T Y - \hat{B}^T F^T Y \right) \right]^{1/2}, \quad (48-a)$$

or

$$\text{RMS}^2 = \frac{1}{p} \left( Y^T Y - \hat{B}^T F^T Y \right). \quad (48-b)$$

The RMS^2 value above is also known as the error variance, $\sigma_e^2$. This is the portion of the data variance not explained by the model. The model variance is then

$$\sigma^2_m = \sigma^2_d - \sigma^2_e.$$
The ratio
\[ R^2 = \frac{\sigma_m^2}{\sigma_d^2} \]
is often used as a criteria to judge the adequacy of the assumed model where
\[ 0 \leq R^2 \leq 1. \]

\( R^2 \) must approach unity for the model to account for the data variability.

The importance of terms in a given model can be measured by the relative power of their coefficients. Of specific interest is the power of the coefficients of degree \( n \). This quantity is referred to as the degree variance, \( \sigma_n^2 \), and is defined as the average square of the spherical harmonic solution, \( \hat{y}_n(\theta, \phi) \), for degree \( n \), or

\[
\sigma_n^2 = \frac{\int_0^{\pi} \int_0^{2\pi} y_n^2(\theta, \phi) \, da}{\int_0^{\pi} \int_0^{2\pi} \, da}, \tag{49}
\]

where \( da = \sin \theta \, d\theta \, d\phi \), and

\[
\hat{y}_n(\theta, \phi) = \sum_{m=0}^{n} \left[ A_{mn} Y_{mn}^e(\theta, \phi) + D_{mn} Y_{mn}^0(\theta, \phi) \right]. \tag{50}
\]

The \( Y_{mn}^e(\theta, \phi) \) and \( Y_{mn}^0(\theta, \phi) \) are spherical harmonic functions as defined in equations (17) and (18). The \( A_{mn} \) and \( D_{mn} \) are the coefficients associated with \( Y_{mn}^e(\theta, \phi) \) and \( Y_{mn}^0(\theta, \phi) \), respectively.

Letting the notation \( \ell \) denote the double integral \( \int_0^{\pi} \int_0^{2\pi} \), the numerator of equation (49) may be written as

\[
I_1 = \sum_{m=0}^{n} \int_{\theta, \phi} \left[ A_{mn} Y_{mn}^e(\theta, \phi) + D_{mn} Y_{mn}^0(\theta, \phi) \right]^2 \, da, \tag{51}
\]
or

\[
I_1 = \sum_{m=0}^{n} \left( A_{mn}^2 \int_{\theta, \phi} (\gamma_{mn})^2 \, da + 2 A_{mn} D_{mn} \int_{\theta, \phi} \gamma_{mn} \gamma_{0}^{2} \, da + D_{mn}^2 \int_{\theta, \phi} (\gamma_{mn}^0)^2 \, da \right). \tag{52}
\]

These integrals are evaluated in Appendix E so that

\[
I_1 = \frac{4\pi}{2n+1} \sum_{m=0}^{n} \left[ A_{mn}^2 + D_{mn}^2 \delta_{m0}^* \right]. \tag{53}
\]

The denominator of equation (49) is

\[
I_2 = \int_{\theta, \phi} \, da = 4\pi,
\]

such that,

\[
\sigma_n^2 = \sum_{m=0}^{n} \frac{(A_{mn}^2 + D_{mn}^2 \delta_{m0}^*)}{(2n+1)}.
\]

or acknowledging the fact that \( D_{mn} = 0 \) for \( m = 0 \),

\[
\sigma_n^2 = \frac{1}{2n+1} \sum_{m=0}^{n} (A_{mn}^2 + D_{mn}^2). \tag{54-a}
\]

The total power in the model coefficients can be found by summing over the \( M \) degree variances such that

\[
\text{Total Power} = \sum_{n=0}^{M} \sigma_n^2 = \sum_{n=0}^{M} \frac{1}{2n+1} \sum_{m=0}^{n} (A_{mn}^2 + D_{mn}^2). \tag{54-b}
\]

Also of interest is the integral

\[
I_3 = \int_{\theta, \phi} \hat{y}_n(\theta, \phi) \hat{y}_0(\theta, \phi) \, da, \tag{55}
\]

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or

\[
I_3 = \sum_{m=0}^{n} \left[ A_{mn} A_{m\theta,\phi} \int \gamma_m \gamma^e \, da + A_{mn} D_{m\theta,\phi} \int \gamma_m \gamma^0 \, da + 
\right. \\
+ D_{mn} A_{m\theta,\phi} \int \gamma^0 \gamma^e \, da + D_{mn} D_{m\theta,\phi} \int \gamma^0 \gamma^0 \, da \right]. 
\]  

(56)

These integrals are also evaluated in Appendix E, so that

\[ I_3 = 0. \]

Then the degree covariances,

\[
\sigma_m^2 = \frac{\int \hat{y}_n(\theta,\phi) \hat{y}_m(\theta,\phi) \, da}{\int \, da}, 
\]

(57)

are zero.

The contribution of the zonal coefficients to \( \sigma^2_n \) is easily determined from

\[
P_{Zn} = \frac{Z_n}{\sigma^2_n} \times 100\%, 
\]

(58)

where \( P_{Zn} \) is the percentage of the zonal contribution to the degree variance at degree \( n \), and where by equation (54-a) for \( m = 0 \),

\[
Z_n = \frac{A_{0n}^2 + D_{0n}^2}{2n + 1}. 
\]

(59)

is the zonal contribution for degree \( n \).

But \( D_{mn} \) does not exist for \( m = 0 \) since by equation (18) \( \gamma^0_{m=0,n}(\theta,\phi) = 0 \), so

\[
Z_n = \frac{A_{0n}^2}{2n + 1}. 
\]

(60)
Substituting this result along with equation (54-a) into equation (58) gives

$$P_{zn} = \frac{A_{on}^2}{\sum_{m=0}^{n} (A_{mn}^2 + D_{mn}^2)} \times 100\%,$$

(61)

again remembering that $D_{mn} = 0$ when $m = 0$.

This result is useful in determining the relative importance of the zonal contribution to the $n$th degree variance.

The model statistics discussed above, $P_{zn}$, degree variance, and total power, explain the distribution of power in the spherical harmonic model.

Computer program GLSRAN2 performs the various calculations mentioned thus far in this section. Comments concerning the associated Legendre function recurrence relations utilized in GLSRAN2 are given in Appendix F. Specific details concerning file manipulations, calculation methods, and output are elaborated on in Appendix G.

Further statistical analyses are performed utilizing the results of computer program GLSRAN2 mentioned above. These are the zonal and global means and variances as based on the least squares fit to the spherical harmonic model.

First, it is desired to derive an expression for the ozone value as a function of colatitude only which will serve as an estimate of the zonal mean, $\bar{z}(\theta)$. To accomplish this the model estimate as given in equation (16) is integrated with respect to longitude such that

$$\bar{z}(\theta) = \frac{\int_{\phi=0}^{\phi=\pi} \hat{y}(\theta,\phi)d\phi}{\int_{\phi=0}^{\phi=\pi} d\phi}.$$  

(62)

The numerator may be written as

$$\int_{\phi=0}^{2\pi} \hat{y}(\theta,\phi)d\phi = \sum_{m=0}^{M} \sum_{n=m}^{M} P_{mn}^S p_n^m(\cos \theta) \int_{\phi=0}^{2\pi} [\hat{A}_{mn} \cos(m\phi) + \hat{D}_{mn} \sin(m\phi)]d\phi,$$  

(63)
or
\[
\int_{0}^{2\pi} \hat{y}(\theta, \phi) d\phi = 2\pi \sum_{n=0}^{M} \hat{A}_{on} P_n(\cos \theta).
\]

Then
\[
\bar{z}(\theta) = \sum_{n=0}^{M} \hat{A}_{on} P_n(\cos \theta).
\]

The estimated global mean is found from
\[
\bar{g} = \frac{\int \hat{y}(\theta, \phi) da}{\int da}.
\]

The numerator may be written as
\[
\int \hat{y}(\theta, \phi) da = \sum_{m=0}^{M} \sum_{n=m}^{M} F_{mn} \int_{0}^{\pi} P_n^m(\cos \theta) \sin \theta d\theta \int_{0}^{2\pi} [\hat{A}_{mn} \cos(m\phi) + \hat{D}_{mn} \sin(m\phi)] d\theta.
\]

Evaluation of the integration over \( \phi \) gives
\[
\int_{0}^{2\pi} [\hat{A}_{mn} \cos(m\phi) + \hat{D}_{mn} \sin(m\phi)] d\phi = \begin{cases} 0, & \text{for } m \neq 0 \\ 2\pi \hat{A}_{on}, & \text{for } m = 0 \end{cases}.
\]

The integral over \( \theta \) in equation (66) may be written as
\[
\int_{0}^{\pi} P_n^m(\cos \theta) \sin \theta d\theta = \int_{x=-1}^{1} P_n(x) dx
\]

using the substitution \( x = \cos \theta \) and where because of equation (67) there is only reason to evaluate the integral for \( m = 0 \).

From Appendix E
\[
\int_{x=-1}^{1} P_{\xi}(x) P_n(x) dx = \frac{2}{2n+1} \delta_{n\xi};
\]

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then, if \( R = 0 \),

\[
\int_{x=-1}^{1} P_0(x) P_n(x) \, dx = 2\delta_{n0}.
\]  

(69)

Recalling that \( P_0(x) = 1 \), equation (69) may be written as

\[
\int_{x=-1}^{1} P_n(x) \, dx = 2\delta_{n0},
\]  

(70)

and the numerator of equation (65) becomes

\[
\int \hat{y}(\theta, \phi) \, da = \begin{cases} 
4\pi \hat{A}_{00}, & \text{for } m = n = 0 \\
0, & \text{otherwise}
\end{cases}
\]  

(71)

Finally, since \( \int_{\theta, \phi} da = 4\pi \), the estimated global mean is

\[
\bar{g} = \hat{A}_{00},
\]  

(72-a)

the variance of which is

\[
\text{Var}(\bar{g}) = \text{Var}(\hat{A}_{00}),
\]  

(72-b)

where \( \text{Var}(\hat{A}_{00}) \) is calculated by equation (33).

The global mean can also be calculated in terms of area weighted zonal means. Another representation of the variance of the global mean can be estimated from this result in terms of \( \text{Covar}(\hat{B}) \) elements. This technique is developed below.
In terms of zonal means the global mean may be written as

\[ \bar{g}_z = \frac{1}{I} \sum_{i=1}^{I} a_i \bar{z}_i / A \]  

(73)

where \( \bar{z}_i \) is the estimated mean for the \( i \)th zone, the constant weighting factor \( a_i \) is the surface area of the \( i \)th zone, and

\[ A = \sum_{i=1}^{I} a_i \]

is the global surface area. Equation (73) may be rearranged as

\[ A \bar{g}_z = \sum_{i=1}^{I} a_i \bar{z}_i \]  

(74)

and, if the variance is taken of both sides, it becomes

\[ \text{Var}(A \bar{g}_z) = \text{Var}(\sum_{i=1}^{I} a_i \bar{z}_i) \]  

(75)

where \( I \) is the number of zones and the \( \bar{z}_i \) are to be treated as random variables. By the definition of variance (equation 7) the right hand side of equation (75) becomes

\[ \text{Var}( \sum_{i=1}^{I} a_i \bar{z}_i ) = \langle \left( \sum_{i=1}^{I} a_i \bar{z}_i - \langle \sum_{i=1}^{I} a_i \bar{z}_i \rangle \right)^2 \rangle \]

= \[ \langle \left( \sum_{i=1}^{I} a_i \bar{z}_i - \sum_{i=1}^{I} a_i \langle \bar{z}_i \rangle \right)^2 \rangle \]

(76)

\[ \text{Var}( \sum_{i=1}^{I} a_i \bar{z}_i ) = \langle \left( \sum_{i=1}^{I} a_i \bar{z}_i \right)^2 \rangle \]  

(77)

where

\[ Z_i = (\bar{z}_i - \langle \bar{z}_i \rangle) \].
Equation (76) may be expanded such that

\[ \text{Var}(\Sigma_{i=1}^{I} a_i \bar{z}_i) = \langle(a_1 \bar{z}_1)^2 + (a_2 \bar{z}_2)^2 + \ldots \rangle + \langle(a_i \bar{z}_i)^2 + 2a_1 \bar{z}_1 a_2 \bar{z}_2 + \ldots + 2a_{I-1} \bar{z}_{I-1} a_I \bar{z}_I \rangle \]

or

\[ \text{Var}(\Sigma_{i=1}^{I} a_i \bar{z}_i) = \langle \sum_{i=1}^{I} a_i^2 \bar{z}_i^2 + 2 \sum_{j=1}^{I-1} \sum_{k=j+1}^{I} a_j a_k \bar{z}_j \bar{z}_k \rangle. \]  

(78)

Notice that

\[ \langle \bar{z}_i^2 \rangle = \langle(\bar{z}_i - \langle\bar{z}_i\rangle)^2 \rangle = \text{Var}(\bar{z}_i) \]  

(79)

and

\[ \langle \bar{z}_j \bar{z}_k \rangle = (\bar{z}_j - \langle\bar{z}_j\rangle)(\bar{z}_k - \langle\bar{z}_k\rangle) = \text{Covar}(\bar{z}_j, \bar{z}_k); \]  

then equation (78) becomes

\[ \text{Var}(\Sigma_{i=1}^{I} a_i \bar{z}_i) = \sum_{i=1}^{I} a_i^2 \text{Var}(\bar{z}_i) + 2 \sum_{j=1}^{I-1} \sum_{k=j+1}^{I} a_j a_k \text{Covar}(\bar{z}_j, \bar{z}_k). \]  

(80)

The left side of equation (75) is

\[ \text{Var}(\bar{g}_z) = A^2 \text{Var}(\bar{g}_z). \]  

(81)

Equating equations (81) and (82) it is found that

\[ \text{Var}(\bar{g}_z) = \frac{\sum_{i=1}^{I} a_i^2 \text{Var}(\bar{z}_i) + 2 \sum_{j=1}^{I-1} \sum_{k=j+1}^{I} a_j a_k \text{Covar}(\bar{z}_j, \bar{z}_k)}{A^2}. \]  

(82)
Var(\(\overline{z}_i\)) is the variance of the mean for zone \(i\). The colatitude position of zone \(i\) is taken to be at \(\theta_i\) so that from equation (64)

\[
\text{Var}(\overline{z}_i) = \text{Var}\left(\sum_{n=0}^{M} \hat{A}_{on} P_n(\cos \theta_i)\right)
\]

(84)

or

\[
\text{Var}(\overline{z}_i) = \sum_{n=0}^{M} P_n^2 \text{Var}(\hat{A}_{on}) + 2 \sum_{n=0}^{M-1} \sum_{\ell=n+1}^{M} P_n P_{\ell} \text{Cov}(\hat{A}_{on}, \hat{A}_{o\ell}).
\]

(85)

where \(P_n\) is the \(n\)th degree Legendre function evaluated for \(\theta_i\). Equation (85) may be evaluated in an analogous fashion to the technique used in the evaluation of equation (75) where \(\hat{A}_{on}\) is to be treated as the random variable. Then equation (85) may be rewritten by comparison with equation (83) as

\[
\text{Var}(\overline{z}_i) = \sum_{n=0}^{M} P_n^2 \text{Var}(\hat{A}_{on}) + 2 \sum_{n=0}^{M-1} \sum_{\ell=n+1}^{M} P_n P_{\ell} \text{Cov}(\hat{A}_{on}, \hat{A}_{o\ell}).
\]

(86)

In order to complete the evaluation of equation (83) \(\text{Cov}(\overline{z}_j, \overline{z}_k)\) must be written in terms of known quantities. Recall that the covariance is defined as

\[
\text{Cov}(x, y) \equiv \langle (x - \langle x \rangle)(y - \langle y \rangle) \rangle.
\]

(87)

Then substituting

\[
\overline{z}_i = \sum_{n=0}^{M} P_n \hat{A}_{on}
\]

(88)

into the covariance definition, equation (87),

\[
\text{Cov}(\overline{z}_j, \overline{z}_k) = \langle (\sum_{n=0}^{M} P_n \hat{A}_{on} - \langle \sum_{n=0}^{M} P_n \hat{A}_{on} \rangle)(\sum_{n=0}^{M} P_n \hat{A}_{on} - \langle \sum_{n=0}^{M} P_n \hat{A}_{on} \rangle) \rangle
\]

\[
= \langle (\sum_{n=0}^{M} P_n \hat{A}_{on} - \sum_{n=0}^{M} P_n \langle \hat{A}_{on} \rangle)(\sum_{n=0}^{M} P_n \hat{A}_{on} - \sum_{n=0}^{M} P_n \langle \hat{A}_{on} \rangle) \rangle.
\]

(89)
Let

\[ W_n = \hat{A}_{on} - \langle \hat{A}_{on} \rangle, \]  

(90)

then

\[ \text{Covar}(\overline{Z}_j, \overline{Z}_k) = \left\langle \left( \sum_{n=0}^{M} P_{nj} W_n \right) \left( \sum_{n=0}^{M} P_{nk} W_n \right) \right\rangle, \]

and

\[ \text{Covar}(\overline{Z}_j, \overline{Z}_k) = \left\langle \sum_{n=0}^{M} \sum_{\ell=0}^{M} P_{nj} P_{\ell k} W_n W_\ell \right\rangle, \]

or

\[ \text{Covar}(\overline{Z}_j, \overline{Z}_k) = \left\langle \sum_{n=0}^{M} \sum_{\ell=0}^{M} P_{nj} P_{\ell k} \langle W_n W_\ell \rangle \right\rangle. \]  

(91)

However, by equation (90),

\[ \langle W_n W_\ell \rangle = \left\langle (\hat{A}_{on} - \langle \hat{A}_{on} \rangle) (\hat{A}_{o\ell} - \langle \hat{A}_{o\ell} \rangle) \right\rangle = \text{Covar}(\hat{A}_{on}, \hat{A}_{o\ell}). \]  

(92)

Then substituting equation (92) into equation (91) the required covariance becomes

\[ \text{Covar}(\overline{Z}_j, \overline{Z}_k) = \sum_{n=0}^{M} \sum_{\ell=0}^{M} P_{nj} P_{\ell k} \text{Covar}(\hat{A}_{on}, \hat{A}_{o\ell}). \]  

(93)

With the help of a little algebra equation (91) may also be written as

\[ \text{Covar}(\overline{Z}_j, \overline{Z}_k) = \sum_{n=0}^{M} \sum_{n=0}^{M-1} P_{nj} P_{nk} \langle W_n^2 \rangle + \sum_{n=0}^{M-1} \sum_{\ell=n+1}^{M} (P_{nj} P_{\ell k} + P_{nj} P_{n k}) \langle W_n W_\ell \rangle. \]  

(94)

By equation (90)

\[ \langle W_n^2 \rangle = \langle (\hat{A}_{on} - \langle \hat{A}_{on} \rangle)^2 \rangle = \text{Var}(\hat{A}_{on}). \]  

(95)
Substituting equations (92) and (95) into equation (94) gives

\[
\text{Covar}(\bar{z}_j, \bar{z}_k) = \sum_{n=0}^{M} P_{nj} P_{nk} \text{Var}(\hat{A}_{on})
\]

\[
+ \sum_{n=0}^{M} \sum_{\ell=n+1}^{M} (P_{nj} P_{\ell k} + P_{n\ell} P_{nk}) \text{Covar}(\hat{A}_{on}, \hat{A}_{\ell 0}).
\]

This is a reassuring result since it reduces to equation (86) for the zonal variance when \( j = k \).

Computer programs GLOBZON and ZONVAR have been prepared to perform these calculations based on the results of the least squares fit to the spherical harmonic model, specifically the model's zonal coefficients, \( \hat{A}_{on} \), and the zonal elements of the covariance matrix, \( \text{Covar}(\hat{A}_{on}, \hat{A}_{\ell 0}) \). When the model is written in the form of equation (22) these quantities are calculated from equations (24) and (33), respectively.

To summarize these results the mean, \( \bar{z}_i \), for zone \( i \) as found by computer program GLOBZON is

\[
\bar{z}_i = \sum_{n=0}^{M} P_{ni} \hat{A}_{on}
\]

where \( M \) is the degree of the model, \( P_{ni} \) is the \( n \)th degree Legendre function for the colatitude \( \theta_i \) at the center of the zone, and \( \hat{A}_{on} \) is the \( n \)th degree zonal coefficient. This result was shown in equation (64) and further developed and used in equation (84).

The global mean, \( \bar{g} \), was shown by equation (72) simply to be the first spherical harmonic model coefficient, or

\[
\bar{g} = \hat{A}_{00}.
\]
In order to calculate the global variance, $\text{Var}(\bar{g}_z)$, the global mean was written out in terms of area weighted zonal means as shown in equation (73). The global variance was found to be

$$
\text{Var}(\bar{g}_z) = \frac{\sum_{i=1}^{I} a_i^2 \text{Var}(\bar{z}_i) + 2 \sum_{j=1}^{I-1} a_j a_k \text{Covar}(\bar{z}_j, \bar{z}_k)}{A^2}
$$

as shown in equation (83). A favorable comparison of this result with equation (72-b),

$$
\text{Var}(\bar{g}) = \text{Var}(\hat{A}_{oo})
$$

tends to confirm the accuracy of the zonal variance calculation as used in the $\text{Var}(\bar{g}_z)$ calculation. The zonal variance, $\text{Var}(\bar{z}_i)$, is

$$
\text{Var}(\bar{z}_i) = \sum_{n=0}^{M} P_{ni}^2 \text{Var}(\hat{A}_{on}) + 2 \sum_{n=0}^{M-1} \sum_{n=n+1}^{M} P_{ni} P_{kj} \text{Covar}(\hat{A}_{on}, \hat{A}_{o\ell}),
$$

and

$$
\text{Covar}(\bar{z}_j, \bar{z}_k) = \sum_{n=0}^{M} P_{nj} P_{nk} \text{Var}(\hat{A}_{on}) + \sum_{n=0}^{M-1} \sum_{n=n+1}^{M} \left( P_{nj} P_{\ell k} + P_{n\ell} P_{nk} \right) \text{Covar}(\hat{A}_{on}, \hat{A}_{o\ell}).
$$
6. Eigenanalysis - Empirical Orthogonal Functions

The subject of eigenanalysis may best be introduced by means of a simple, if not trivial, illustration. Consider the data shown in the table below.

Table. Data Set as Viewed in the $x_1 - x_2$ Coordinate System.

<table>
<thead>
<tr>
<th>Observation No.</th>
<th>$x_{i1}$</th>
<th>$x_{i2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

The data means and covariance matrix can be calculated as

$$\bar{x}_1 = 2,$$

$$\bar{x}_2 = 2,$$

and

$$\text{COVAR}_x = \begin{pmatrix} 2/3 & 2/3 \\ 2/3 & 2/3 \end{pmatrix},$$

where

$$\text{COVAR}_{ij} = \frac{1}{3} \sum_{k=1}^{3} (x_{ki} - \bar{x}_i)(x_{kj} - \bar{x}_j).$$

The data variance is the sum of the diagonal terms in the covariance matrix or the "trace" of that matrix and is written as

$$\sigma^2 = \text{Tr}(\text{COVAR}) = 4/3.$$
Now with the data mean \( \bar{x}_1 = \bar{x}_2 = 2 \) taken to be the origin of a new coordinate system with axes \( u_1 \) and \( u_2 \), the data in the table above are distributed as shown in the figure below.

Figure. The \( u_1 - u_2 \) Coordinate System shows the "mean centered" data representation.

\[
\begin{align*}
    u_2 \\
    \uparrow
\end{align*}
\]

\[
\begin{array}{c}
    1 \\
    \bullet \\
    \bullet
\end{array}
\]

The origin of this new coordinate system may be thought of as being displaced by some mean vector, \( \mathbf{m} \), where

\[
\mathbf{m} = 2\hat{x}_1 + 2\hat{x}_2. 
\]

(100)

\( \hat{x}_1 \) and \( \hat{x}_2 \) are unit vectors along the \( x_1 \) and \( x_2 \) axes, respectively.

Define another coordinate axis such that it is colinear with the data. Call this axis \( \psi_1 \). The third coordinate system is completed by placing the coordinate axis \( \psi_2 \) through \( u_1 = u_2 = 0 \) and perpendicular to \( \psi_1 \) in the direction shown in Figure 5. The coordinates of the data in the \( \psi_1 - \psi_2 \) coordinate system are tabulated below.
Table. Coordinates of Data in $\psi_1 - \psi_2$ System.

<table>
<thead>
<tr>
<th>Observation No.</th>
<th>$\psi_{11}$</th>
<th>$\psi_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-\sqrt{2}$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$\sqrt{2}$</td>
<td>0</td>
</tr>
</tbody>
</table>

The covariance matrix for the data as represented in this coordinate frame is

$$\text{COVAR} = \begin{pmatrix} 4/3 & 0 \\ 0 & 0 \end{pmatrix}. \quad (101)$$

This analysis is of interest since it shows that the data set initially represented by two coordinate axes, $x_1$ and $x_2$, can be represented, with the proper translation and rotation of these axes, by only one axis, $\psi_1$, as the $\psi_2$ component of all three observations is zero. This result effectively cuts in half the amount of information required to describe this set of data. It follows then that the data variance must all be accounted for along the $\psi_1$ axis as is shown in equation (101) in accordance with equation (99). Because of this, and since the data mean is zero, the variance may be found from the mean of the sum of the squares of the $\psi_{11}$ axis data coordinates, or

$$\sigma^2 = \frac{1}{3} \sum_{i=1}^{3} \psi_{11}^2, \quad (102-a)$$

and

$$\sigma^2 = \frac{4}{3}. \quad (102-b)$$

Also, by equation (101), $\psi_{11}$ and $\psi_{12}, i = 1, 2, 3$, are uncorrelated since

$$\text{COVAR}(\psi_1, \psi_2) = 0.$$

Now consider the case where the data set is in the form of a matrix $\mathbf{X}(M \times N)$. $\mathbf{X}$ may be thought of as containing $M$ measurements of an observable vector dimensioned by $N$ or as $N$ coordinate vectors dimensioned by $M$. The objective is to reduce the number of coordinate vectors required to accurately represent $\mathbf{X}$ and at the same time to keep account of the
data variability explained by this representation. Though more computationally involved, this problem is fundamentally the same as the preceding example. That is, by the proper selection of another coordinate system, the data may be arranged with respect to its coordinate axes so that the data variance is maximized along a smaller number of its coordinate vectors and so that the various coordinate vectors are uncorrelated with each other, i.e.,

\[ \text{COVAR}(\psi_i, \psi_j) = 0, \text{ for } i = 1, 2, \ldots, N \]
and \( i \neq j \).

To this end the covariance matrix describing the data set must be diagonalized (all off diagonal terms are required to be zero). The covariance matrix is

\[ \text{COVAR}(\mathbf{x}) = \frac{1}{M} \mathbf{U}^T \mathbf{U} \]  \hspace{1cm} (103)

where the \( \mathbf{U} \) matrix is defined such that

\[ u_{ij} = x_{ij} - \bar{x}_j, \]  \hspace{1cm} (104-a)

and

\[ \bar{x}_j = \frac{1}{M} \left( \sum_{i=1}^{M} x_{ij} \right) \]  \hspace{1cm} (104-b)

is the data average for the \( j \)th column of \( \mathbf{x} \).

Diagonalizing the covariance matrix defined by equation (103) results in a new covariance matrix statistically describing the data in a new coordinate system or "eigenspace". This covariance matrix is of the form

\[
\Lambda = \begin{bmatrix}
\lambda_1 & 0 & \cdots & 0 \\
0 & \lambda_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \lambda_N
\end{bmatrix}
\]  \hspace{1cm} (105)
All off-diagonal elements are zero. The diagonal elements of $\Lambda$ are eigenvalues, or characteristic values as they are sometimes called. Associated with each eigenvalue is a principal axis, a coordinate axis in eigenspace. Any vector $\psi$, as defined in equation (106) below, that is parallel to a principal axis is called an eigenvector. The eigenvalue equation is

$$[(COVAR) - \lambda I] \psi = 0, \quad (107)$$

which may be rewritten as

$$[(COVAR) - \lambda I] \psi = 0, \quad (107)$$

where $I$ is the identity matrix.

It is necessary to find non-trivial solutions for equation (107), that is, solutions where $\psi \neq 0$. Since equation (107) is a representation of $N$ homogeneous simultaneous equations, it can only be solved if the determinant of the coefficients vanishes, or

$$|COVAR - \lambda I| = 0. \quad (108)$$

This is often referred to as the secular equation. Values for the scalar constant $\lambda$ which come from the solution of the secular equation are the sought eigenvalues. These eigenvalues are arranged in decreasing magnitude along the diagonal of $\Lambda$ in equation (105).

Once the eigenvalues are known, the associated eigenvectors can be found by equation (107). The $N$ eigenvectors that pass through the origin are the coordinate axes in the eigenspace coordinate frame. The coordinates of the data in eigenspace are given by

$$C = U \psi^T \quad (109)$$

where $U$ is defined by equations (104) and $\psi$ is a square matrix containing the $N$ eigenvectors by row. The coordinates of the data in the original coordinate system can be found by

$$\chi = U + a \quad (110-a)$$

where

$$U = C \psi, \quad (110-b)$$
and the matrix $\mathbf{a}$, containing the $N$ column means of $\bar{X}$, is given by

$$
a_{kj} = \frac{1}{M} \left( \sum_{i=1}^{M} x_{ij} \right),
$$

for $k = 1, 2, \ldots, M$.

It will now be of interest to return to the earlier illustrative example solving it from the point of view of an eigenvalue problem as developed above. From the data in the table (Data Set as Viewed in the $x_1 - x_2$ Coordinate System) and with equations (97) and (104) the $\mathbf{U}$ matrix may be written as,

$$
\mathbf{U} = \begin{pmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}.
$$

(111)

This is the "mean centered" or "zero mean" data representation as shown in the figure above. Then by equation (103)

$$
\text{COVAR}(\bar{X}) = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} \end{pmatrix},
$$

(112)

which is in agreement with equation (97-c). The required eigenvalues can be found with a little algebra and equation (108) as follows:

$$
\frac{2}{3} \left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} \end{pmatrix} \right| = 0.
$$

$$
\frac{2}{3} \left| \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} \frac{3}{2 \lambda} & 0 \\ 0 & \frac{3}{2 \lambda} \end{pmatrix} \right| = 0.
$$

(113)
Let
\[ \lambda' = \frac{3}{2} \lambda , \]  
so that
\[ \begin{vmatrix} 1 - \lambda' & 1 \\ 1 & 1 - \lambda' \end{vmatrix} = 0 . \]  

The determinant on the left hand side is readily evaluated giving
\[ \lambda'^2 - 2\lambda' = 0 . \]  
The solution of this quadratic equation is
\[ \lambda'_1 = 2 , \]  
and
\[ \lambda'_2 = 0 , \]
or, by equation (114),
\[ \lambda_1 = 4/3 , \]  
and
\[ \lambda_2 = 0 . \]
Substituting this result into equation (107) yields
\[ \psi_{11} = \psi_{12} \]  
for the first eigenvalue, and
\[ \psi_{21} = -\psi_{22} \]  
for the second eigenvalue. Here \( \psi_{ij} \) is the component of the \( i \)th eigenvector along the \( u_j \) axis.
The eigenvector associated with the first eigenvalue is any vector which has equal components along the $u_1$ and $u_2$ axes. Then the principal axis can be taken as the eigenvector that passes through the origin of the $u_1$ - $u_2$ coordinate system, such that the unit vector along this principal axis is

$$
\hat{e}_1 = \frac{\hat{e}_1 + \hat{e}_2}{\sqrt{2}},
$$

(121)

where $\hat{e}_1$ and $\hat{e}_2$ are unit vectors along the $u_1$ and $u_2$ axes, respectively, and the $1/\sqrt{2}$ is a normalization constant.

Similarly, the unit vector along the second principal axis is

$$
\hat{e}_2 = \frac{\hat{e}_2 - \hat{e}_1}{\sqrt{2}}.
$$

(122)

Equations (121) and (122) can be combined and represented in matrix form as

$$
\psi = \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
$$

(123)

Here the matrix $\psi$ contains in rows the two eigenvectors that represent the principal axes in eigenspace.

It can quickly be shown that $\psi_1$ and $\psi_2$ form an orthonormal set since

$$
\psi_1^T \psi_2 = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}.
$$

From this result it follows that

$$
\psi_1 \cdot \psi_1 = \psi_2 \cdot \psi_2 = 1
$$

demonstrating that $\psi_1$ and $\psi_2$ are normalized and that

$$
\psi_1 \cdot \psi_2 = \psi_2 \cdot \psi_1 = 0
$$
showing that $\psi_1$ and $\psi_2$ are orthogonal to each other.

To digress a bit it is interesting to note that the matrix $\psi$ in equation (123) is, in fact, the transpose of a rotational transformation and can be calculated by the perhaps more conventional technique illustrated in the figure below.

Figure. Coordinate Axes Rotation

In the figure the primed axes, $x'$ and $y'$, have been rotated as shown through an angle $\theta$. They can be represented with respect to the original axes as

$$x' = x \cos \theta + y \sin \theta$$  \hspace{1cm} (124-a)

and

$$y' = y \cos \theta - x \sin \theta$$  \hspace{1cm} (124-b)

or written in matrix form

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}.$$  \hspace{1cm} (125)
Notice that for \( \theta = 45^\circ \) the rotational transformation in equation (125) becomes

\[
\Psi_T = \begin{bmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}.
\]  

(126)

Now, by equations (109), (111), and (123), the coordinates of the original data (Table. Data Set as Viewed in the \( x_1 - x_2 \) Coordinate System) in eigenspace can be calculated as

\[
\mathbf{c} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix},
\]

or

\[
\mathbf{c} = \begin{bmatrix} -\sqrt{2} & 0 \\ 0 & 0 \\ \sqrt{2} & 0 \end{bmatrix}.
\]  

(127)

The original data coordinates can be reconstructed by equations (110) combined as

\[
\mathbf{x} = \mathbf{a} + \mathbf{c} \Psi_T.
\]  

(128)

Substituting equations (97), (123), and (127) into equation (128) gives

\[
\mathbf{X} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -\sqrt{2} & 0 \\ 0 & 0 \\ \sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\]

\[
= \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}
\]  

(129)
However, as previously stated, the $\mathbf{X}$ data matrix should be retrievable by utilizing data along only the $\psi_1$ axis. Equation (129) for only the first eigenvector becomes

$$
\mathbf{X} = \begin{pmatrix}
2 & 2 \\
2 & 2
\end{pmatrix} + \begin{pmatrix}
-\sqrt{2} \\
\sqrt{2}
\end{pmatrix} \begin{pmatrix}
\frac{1}{\sqrt{2}} \\
\frac{-1}{\sqrt{2}}
\end{pmatrix}
$$

(131)

$$
\mathbf{X} = \begin{pmatrix}
2 & 2 \\
2 & 2
\end{pmatrix} + \begin{pmatrix}
-1 & -1 \\
0 & 0
\end{pmatrix} + \begin{pmatrix}
1 & 1 \\
2 & 2
\end{pmatrix}
$$

(132)

To calculate individual elements of $\mathbf{X}$ equation (128) may be written as

$$
x_{ij} = a_j + \sum_{k=1}^{N} C_{ik} \psi_{kj}
$$

(133)

where the $i$ subscript on $a$ has been dropped since the column index, $j$, determines the value of $a$, now treated as a vector.

These eigenanalysis techniques have been used to some extent in the ozone sampling study. Empirical orthogonal functions (EOF) have been used in the development of a global ozone model. These empirical orthogonal functions are the eigenvectors of the covariance matrix associated with a set of gridded ozone data. The coefficients associated with these functions are the coordinate vectors of the gridded ozone data represented in eigenspace as found in the $C$ matrix defined above.
This EOF ozone model will be discussed below according to the following four development stages.

1. Establish an appropriate data grid system.
2. Calculate data base for model.
3. Develop model.
4. Test model.

The EOF model development is based on the assumption that there will be no missing data blocks in the grid system. This assumption eliminates from consideration polar regions where there is no BUV data coverage. A further consideration is whether latitudinal or longitudinal variability is being investigated. For latitudinal variability studies data are arranged as shown in Figure 6-A. For longitudinal variability studies data are arranged as shown in Figure 6-B. The elements of the grids are found from equation (3) where the i and j indices are now defined as in Figures 6-A and B.

Three data arrays constitute the minimum data base requirement for the EOF model. One of these arrays contains the eigenvector matrix, another contains the matrix of coordinate vectors in eigenspace or the coefficient matrix, and the last contains the N column averages of the gridded data (Figures 6-A and B). With one set of these three arrays the EOF model can reconstruct the original data grid for some specified time period. Though the EOF model is time independent, by supplying eigenvector, coefficient, and column average arrays for several time periods a model which is effectively time dependent can be formulated. The EOF model data base as generated during this study of the BUV-I data consists of one such set of arrays per week for 50 weeks.

Data base array sets are calculated by computer program EOFA2. In this program the column averages are found by equation (104-b), the eigenvectors, \( \psi \), defined by equation (107) are computed by subroutine SYMQL\(^5\), and the coefficient matrix is calculated by equation (109). These data base arrays are saved and maintained on a MSRA file such that model data is accessible on a weekly basis.
For purposes of this discussion, it will be assumed that the source data for the EOF model is arranged as in Figure 6-A. The fundamental model representation of an ozone value in grid block (i,j) is

\[ x_{ij} = a_j + \sum_{k=1}^{n} C_{ik} \psi_{kj} \]  

(134)

as shown in equation (133). In equation (134)

\[ 1 \leq n \leq N \]

where \( n \) is the number of eigenvalues to be used by the model and is determined by the percentage of the total variability, \( P(\%) \), to be explained or accounted for. The expression showing the relationship between \( n \) and \( P(\%) \) is given below as

\[ P(\%) = \frac{1}{\sigma^2} \sum_{k=1}^{n} \lambda_k \times 100\% , \]  

(135)

where

\[ \sigma^2 = \sum_{k=1}^{N} \lambda_k . \]  

(136)

Also, as has been demonstrated above (equations 99 and 102), the data variance may be written as

\[ \sigma^2 = \text{Tr}(COVAR) = \frac{1}{M} \sum_{j=1}^{N} \sum_{i=1}^{M} C_{ij}^2 . \]  

(137)

The model's time dependency is incorporated by the proper selection of the \( a \) vector and the \( C \) and \( \psi \) matrices from the MSRA file as discussed above.

The model development thus far makes available only the somewhat limited capability of calculating discrete ozone values associated with grid block (i,j). This capability must be extended so that ozone values for specified positions on the Earth's surface, within the geographic boundary limitations of the model's data base, can be computed. This would result in a model of the form

\[ OZONE = X[\psi(\theta), C(\phi), t, P(\%)]. \]  

(138)

To this end Fourier series representations are calculated for the required eigenvectors and column means as a function of latitude, \( \theta \), and for the required coefficients as a function of longitude, \( \phi \). Appendix H gives a brief development of the Fourier series representation that will be utilized below.
First consider approximating an eigenvector "curve" composed of discrete values. These values are equally spaced along a latitudinal axis and are located at latitude zone centers as shown on the bottom scale of Figure 7. The BUV-I data modeled by this technique generally has good latitudinal coverage, depending on the season, from the latitude zone centered at -77.5° to the zone centered at 77.5°. As can be seen from Figure 7 this corresponds to an eigenvector of 32 discrete components. For the purpose of representing an eigenvector by a Fourier series this figure also shows certain transition scales. The "Fictitious Latitude Scale" simply shows the latitudinal data range where zero degrees corresponds to the gridded data's southern extreme. The "Fourier Scale Range" shows the domain of the periodical Fourier functions which will be used to represent the eigenvector.

Notice that the Fourier scale range extends slightly beyond the discrete data scale. As far as the Fourier scale is concerned there are 33 pieces of data, but due to the periodic nature of the Fourier representation the functional value of the first discrete data point must equal the functional value of the last, or
\[ f(0°) = f(360°). \] (139)

Then over the Fourier scale range there are 32 intervals between the equally spaced data so that
\[ \frac{\text{Fourier Scale Range}}{\text{No. of Intervals over Scale}} = \frac{360°}{32} = 11.25°/\text{interval}. \] (140)

Both latitude scales contain 31 intervals so that the length of either latitude scale in terms of the Fourier scale is
\[ 11.25°/\text{interval} \times 31 \text{ intervals} = 348.75°. \]

Let \( \kappa_1 \) be the conversion factor from the fictitious latitude to the Fourier scale such that
\[ \kappa_1 = \frac{348.75°}{155°} = 2.25. \] (141)
Also let $\theta$ be the actual latitude value, $\theta_2$ be the fictitious latitude value, $\theta_1$ be the Fourier scale value, and $d_\theta$ be the discrete data point number including any fractional part. Then,

\[ \theta_1 = \kappa_1 \theta_2. \]  

But

\[ \theta_2 = \theta + 77.5^\circ, \]  

so

\[ \theta_1 = \kappa_1 (\theta + 77.5^\circ). \]  

This expression shows the relationship between the actual latitude, $\theta$, and the corresponding Fourier angle, $\theta_2$.

The relationship between $d_\theta$ and $\theta_2$ may be written as

\[ \theta_2 = \kappa_2 (d_\theta - 1), \]  

where

\[ \kappa_2 = \frac{155^\circ}{31 \text{ intervals}}. \]  

Then by equation (143)

\[ \theta = \kappa_2 (d_\theta - 1) - 77.5^\circ, \]  

and by equation (144)

\[ \theta_1 = \kappa_1 \kappa_2 (d_\theta - 1), \]  

which gives the Fourier angle in terms of the discrete data point number scale.

From the development in Appendix H the required eigenvector may be approximated by a Fourier series expansion

\[ \psi(\theta) = A_0 + \sum_{\ell=1}^{Q} [A_{\ell} \cos(\ell \kappa_1 (\theta + 77.5^\circ)) + B_{\ell} \sin(\ell \kappa_1 (\theta + 77.5^\circ))] \]  

where $Q = 16$, since $2Q = 32$ is the number of independent discrete pieces of data, and where equation (144) was substituted into equation (H-6) for the Fourier angle.
The procedure for finding an approximation for the mean vector "curve" is quite the same and leads to

$$a(\theta) = E_0 + \sum_{\ell=1}^{Q} [E_\ell \cos(\ell \kappa_1(\theta + 77.5^\circ)) + J_\ell \sin(\ell \kappa_1(\theta + 77.5^\circ))], \quad (150)$$

where only the Fourier coefficients are changed. They are found as outlined in equations (H-4) and (H-5).

The Fourier representation for the coefficients is similar to that above except for certain scaling differences. The BUV-I gridded data ranges on the longitude scale from the block centered on 7.5° to the block centered on 352.5°.

The Fourier angle may be written as

$$\phi_1 = \phi - 7.5^\circ, \quad (151)$$

from which by equation (H-6)

$$C(\phi) = R_0 + \sum_{\ell=1}^{Q} [R_\ell \cos(\ell (\phi - 7.5)) + S_\ell \sin(\ell (\phi - 7.5))], \quad (152)$$

where Q = 12, since 2Q = 24 is the number of discrete independent pieces of data, corresponding in this case to longitudinal sectors, and where the coefficients are found again by equations (H-4) and (H-5) using the already known discrete values of C.

Computer program EAMOD1 was prepared to implement this model and to briefly analyze the results. To summarize the model development above as incorporated into the computer model consider the problem of finding an ozone value for some point on the Earth's surface ($$\theta', \phi'$$) at time t. Further, P'(% of the data variability is to be accounted for.

First, a set of data arrays for the appropriate time period t are accessed from MSRA file. Recall these three data arrays contain the eigenvector matrix, $$\Psi$$, the coefficient matrix, C, and the column average vector, a. The number of eigenvectors required to achieve the specified data variability can be determined from equation (135).
Since eigenvalues are not saved on the MSRA file, elements from the coefficient matrix may be used for this task. As shown above, the kth eigenvalue may be written as

\[ \lambda_k = \frac{1}{M} \sum_{i=1}^{M} C_{ik}, \]  

(153)

and equation (135) becomes

\[ p(\%) = \frac{\sum_{k=1}^{n} \frac{1}{M} \sum_{i=1}^{M} C_{ik}^2}{\sigma^2} \times 100\%. \]  

(154)

Equation (154) is summed iteratively over k until

\[ p(\%) \geq p'(\%) \]  

(155)

at which point \( n = k \) is taken to be the required number of eigenvectors.

The eigenvectors are arranged by row, and the associated coefficients are arranged by column as stored in their respective arrays. Each of the first \( n \) eigenvectors are fitted according to equation (149), and each of the first \( n \) coefficients (column vectors) are fitted according to equation (152). Similarly the mean vector is represented by equation (150). The required ozone value can now be computed by rewritting equation (134) in terms of actual model parameters, instead of the global grid indices \((i,j)\), as

\[ X[\psi(\theta'), C(\phi'), t, p'(\%)] = a_t(\theta') + \sum_{k=1}^{n[p'(\%)]} C_t(\phi')_k \psi_{t(\theta')}_k \]  

(156)

where the notation \( n[p'(\%)] \) signifies that the number of eigenvectors used is a function of the explained data variability and where the subscript \( t \) indicates that the arrays come from the data base for the time interval \( t \).

The ozone modeling technique using empirical orthogonal functions has been implemented and briefly analyzed by computer program EAMOD1. The modeling aspect has been described above. As a quick evaluation of the model's usefulness, selected BUV-I sampling data was used to generate the required model data base. Then the BUV-I ozone values associated with this sampling were compared with the model predictions for those values. Within the latitudinal limits of the model, the errors ranged from 0% to 10%.
7. Data Fill Technique by Autocorrelation Functions

The autocorrelation function typically thought of as being associated with time series analysis has been somewhat modified here and has been engineered into a data fill technique on a spatial basis.

\[
R(k) = E[x_n x_{n+k}] \tag{157}
\]

is the definition of the autocorrelation function where \( E \) is the expectation operator and the set of \( x_i, i = 0, 1, 2, \ldots, N \), is "zero mean" data. The sample autocorrelation function

\[
R_N(k) = \frac{1}{N} \sum_{i=0}^{N-1} x_i x_{i+k} \tag{158}
\]

is the estimate of the autocorrection function, where \( k \), the lag, is representative of time separation.

Consider the case of global spatial distribution instead of time distribution. Let \( k \) represent a lag of 5° latitudinally and \( \ell \) represent a lag of 15° longitudinally. The number of samples with respect to latitude is

\[
N_k = \frac{180°}{5°} = 36, \quad \tag{159}
\]

and with respect to longitude is

\[
N_\ell = \frac{360°}{15°} = 24. \quad \tag{160}
\]

Then by analogy to equation (158)

\[
R(k, \ell) = \frac{1}{N(k, \ell)} \sum_{i=1}^{24} \sum_{j=1}^{18} x_{ij} x_{i+k,j+\ell} \tag{161}
\]

However, in accordance with the latitudinal index convention as shown in Figure 1, equation (161) is written as

\[
R(k, \ell) = \frac{1}{N(k, \ell)} \sum_{i=1}^{19} \sum_{j=1}^{18} x_{ij} x_{i+k,j+\ell} \tag{162}
\]
where

\[ N(k, \ell) = N_k N_\ell - N_d \]

and \( N_d \) is the number of grid blocks containing no data.

Now consider the data block \((i, j)\), shown in the figure below, containing no data.

![Diagram showing a grid block \((i, j)\) surrounded by 48 blocks.]

In a sense the objective is to find a weighted mean of the 48 blocks surrounding \((i, j)\) which will serve as the "fill-in" value for the block \((i, j)\).

In general a weighted mean may be written as

\[ \bar{x} = \frac{\sum_{i=1}^{n} a_i x_i}{\sum_{i=1}^{n} a_i} \] (163)

where \( a_i \) is the weighting factor associated with \( x_i \). Should some \( x_i \) have no value, indicated by \( x_i = 0 \), over the range \( 1 \leq i \leq n \), then equation (163) is written as

\[ \bar{x} = \frac{\sum_{i=1}^{n} a_i x_i}{\sum_{i=1}^{n} \delta_i a_i} \] (164)

where

\[ \delta_i = \begin{cases} 1, & \text{for } x_i \neq 0 \\ 0, & \text{for } x_i = 0 \end{cases} \] (165)
Finding the value for the block \( (i, j) \) is a two-dimensional problem requiring summation over latitude and longitude. Let \( Y_{ij} \) be the required weighted mean. Then by equation (164)

\[
Y_{ij} = \frac{\sum_{k=-3}^{3} \sum_{\ell=-3}^{3} R_{|k|,|\ell|} Y_{i+k,j+\ell}}{\sum_{k=-3}^{3} \sum_{\ell=-3}^{3} R_{|k|,|\ell|} \delta_{i+k,j+\ell}}
\] (166)

where \( R_{|k|,|\ell|} \) is now treated as a weighting factor and from equation (162)

\[
R_{(k,\ell)} = \frac{1}{N(k,\ell)} \sum_{j=1}^{24} \sum_{i=1}^{18} \sum_{i=19}^{36} Y_{ij} Y_{i+|k|,j+|\ell|}.
\] (167)

The technique briefly discussed above is currently being used as implemented in computer program OZFILL1 on two levels, partial fill and complete fill. Using the partial fill technique \( 1/2 \) of the total surrounding 48 data blocks must contain non-zero ozone values (an ozone value of zero implies no data). Also previously filled blocks are not included in this count. The complete fill technique is used without regard to the above restrictions.
IV. BUV CORRECTION TECHNIQUE - DOBSON DATA

Ozone data as measured by the Dobson spectrophotometer have been investigated and analyzed in conjunction with the BUV sampling analysis.\textsuperscript{11} These data were obtained from the World Ozone Data Centre in Ontario, Canada and subsequently have been used to adjust the BUV data as will be briefly explained below.

For a given Dobson station certain BUV measurements are selected based on temporal and spatial considerations in order to calculate a linear least squares fit between the Dobson, \( y_d \), and the BUV, \( y_b \), data. The great circle distance, \( s \), between the Dobson station \((\theta_1, \phi_1)\) and the BUV subsatellite point \((\theta_2, \phi_2)\) is given by

\[
s = R \cos^{-1}(\sin \theta_1 \sin \theta_2 + \cos \theta_1 \cos \theta_2 \cos(\phi_2 - \phi_1)),
\]

(168)

where \( R = 6367.3951 \) kilometers is the average earth radius based on the Clarke spheroid of 1866.\textsuperscript{12} The least squares fit is of the form

\[
y_d = \beta_0 + \beta_1 y_b
\]

(169)

where \( \beta_0 \) and \( \beta_1 \) are the resulting regression coefficients.

A sufficient number of Dobson stations are utilized so that the range in latitudinal coverage is from approximately \( 75^\circ \) to \(-45^\circ \). Both \( \beta_0 \) and \( \beta_1 \) may be fit as a function of latitude, \( \theta \), by the least squares method so that

\[
\beta_0 = \alpha_{01} + \alpha_{02} \cos^2 \theta
\]

(170a)

and

\[
\beta_1 = \alpha_{11} + \alpha_{12} \cos^2 \theta.
\]

(170b)

Then the "corrected" BUV ozone measurements, \( y_c \), as "adjusted" by the Dobson data may be calculated from

\[
y_c = \alpha_{01} + \alpha_{02} \cos^2 \theta + (\alpha_{11} + \alpha_{12} \cos^2 \theta)y_b.
\]

(171)
Table 1. Preliminary Analysis of the BUV-III Data

<table>
<thead>
<tr>
<th>FTN FILE #</th>
<th>1ST DAY</th>
<th>LAST DAY</th>
<th>TOT. DAYS</th>
<th>1ST REC.</th>
<th>LAST REC.</th>
<th>TOTAL REC.</th>
<th>ABNORM. OZ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-999.</td>
<td>-77.</td>
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<tr>
<td>1</td>
<td>199</td>
<td>226</td>
<td>28</td>
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<td>23,591</td>
<td>23,591</td>
<td>6</td>
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<td>2</td>
<td>126</td>
<td>153</td>
<td>28</td>
<td>1</td>
<td>23,592</td>
<td>23,782</td>
<td>0</td>
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<tr>
<td>3</td>
<td>154</td>
<td>182</td>
<td>29</td>
<td>1</td>
<td>47,374</td>
<td>24,770</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>182</td>
<td>210</td>
<td>29</td>
<td>1</td>
<td>72,144</td>
<td>25,166</td>
<td>10</td>
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<td>5</td>
<td>210</td>
<td>238</td>
<td>29</td>
<td>1</td>
<td>97,310</td>
<td>25,451</td>
<td>11</td>
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<tr>
<td>6</td>
<td>238</td>
<td>266</td>
<td>29</td>
<td>1</td>
<td>122,761</td>
<td>24,707</td>
<td>0</td>
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<tr>
<td>7</td>
<td>266</td>
<td>293</td>
<td>28</td>
<td>1</td>
<td>147,468</td>
<td>24,275</td>
<td>17</td>
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<td>8</td>
<td>294</td>
<td>322</td>
<td>29</td>
<td>1</td>
<td>171,743</td>
<td>26,830</td>
<td>86</td>
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<td>9</td>
<td>322</td>
<td>349</td>
<td>28</td>
<td>1</td>
<td>198,573</td>
<td>27,967</td>
<td>0</td>
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<tr>
<td>10</td>
<td>350</td>
<td>364</td>
<td>15</td>
<td>1</td>
<td>226,540</td>
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<td>392</td>
<td>28</td>
<td>1</td>
<td>240,934</td>
<td>23,796</td>
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<tr>
<td>12</td>
<td>393</td>
<td>420</td>
<td>28</td>
<td>1</td>
<td>264,730</td>
<td>19,469</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>421</td>
<td>448</td>
<td>28</td>
<td>1</td>
<td>284,199</td>
<td>18,675</td>
<td>1</td>
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<tr>
<td>14</td>
<td>449</td>
<td>490</td>
<td>42</td>
<td>1</td>
<td>302,874</td>
<td>23,981</td>
<td>13</td>
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<tr>
<td>15</td>
<td>491</td>
<td>518</td>
<td>28</td>
<td>1</td>
<td>326,855</td>
<td>18,797</td>
<td>7</td>
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<tr>
<td>16</td>
<td>519</td>
<td>546</td>
<td>28</td>
<td>1</td>
<td>345,652</td>
<td>21,822</td>
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<tr>
<td>17</td>
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<td>1</td>
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<td>19,505</td>
<td>0</td>
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<tr>
<td>18</td>
<td>575</td>
<td>603</td>
<td>29</td>
<td>1</td>
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<td>19,552</td>
<td>2</td>
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<tr>
<td>19</td>
<td>603</td>
<td>623</td>
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<td>1</td>
<td>406,531</td>
<td>13,714</td>
<td>0</td>
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<tr>
<td>20</td>
<td>631</td>
<td>658</td>
<td>28</td>
<td>1</td>
<td>420,245</td>
<td>20,909</td>
<td>4</td>
</tr>
<tr>
<td>21</td>
<td>659</td>
<td>686</td>
<td>28</td>
<td>1</td>
<td>441,154</td>
<td>20,903</td>
<td>7</td>
</tr>
<tr>
<td>22</td>
<td>687</td>
<td>714</td>
<td>28</td>
<td>1</td>
<td>462,057</td>
<td>21,148</td>
<td>1</td>
</tr>
<tr>
<td>23</td>
<td>715</td>
<td>729</td>
<td>15</td>
<td>1</td>
<td>483,205</td>
<td>12,121</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>730</td>
<td>757</td>
<td>28</td>
<td>1</td>
<td>495,326</td>
<td>25,432</td>
<td>2</td>
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<tr>
<td>25</td>
<td>758</td>
<td>785</td>
<td>28</td>
<td>1</td>
<td>520,758</td>
<td>26,189</td>
<td>32</td>
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<tr>
<td>26</td>
<td>786</td>
<td>813</td>
<td>28</td>
<td>1</td>
<td>546,947</td>
<td>24,969</td>
<td>8</td>
</tr>
<tr>
<td>27</td>
<td>814</td>
<td>841</td>
<td>28</td>
<td>1</td>
<td>571,916</td>
<td>22,974</td>
<td>4</td>
</tr>
<tr>
<td>28</td>
<td>842</td>
<td>855</td>
<td>14</td>
<td>1</td>
<td>594,890</td>
<td>13,085</td>
<td>0</td>
</tr>
</tbody>
</table>

TOTALS: 607,974 219 20,481 30 17

SUMMARY:

- NO. OF FTN FILES: 28
- NO. OF RECORDS: 607,974
- ABNORMAL OZONE
  - OZ = -999: 219
  - OZ = -77: 20,481
  - OTHER: 30

- RECORDS SUCH THAT ABSOLUTE LATITUDE ≤ 5°: 34,698
- BAD CROSSING TIMES (IEQC): 17
- OBSERVATIONS ON EQUATOR: 0
- TOTAL DAYS ON TAPE: 757

NOT INCLUDED IN THE INTERVAL 0.200 ≤ ABSOLUTE OZONE VALUE ≤ 0.650
Figure 1. Global Grid System as Developed in Computer Program OZSTAT2's Data Grouping Scheme
Figure 2. Example of Program OZSTAT2 Graphics Capability
This plot shows BUV Zonal Means for June 22, 1970.
Figure 3. Example of Program OZSTAT2 Graphics Capability

This scatter diagram shows the BUV ozone data distribution for June 22, 1970.
Figure 4. Example of Program OZSTAT2 Graphics Capability

This histogram shows the latitudinal sampling distribution of BUV ozone data for June 22, 1970. Actual Number of Data Points = Normalized Number of Data Points x 103.
Figure 5. Relationship of the Three Coordinate Systems $x_1 - x_2$, $u_1 - u_2$, and $\psi_1 - \psi_2$
Figure 6-A. EOF Model Arrangement for Latitudinal Variability Studies

Figure 6-B. EOF Model Arrangement for Longitudinal Variability Studies
Figure 7. Transition Scales from the Latitude Scale to the Fourier Scale for Eigenvector Representation.
<table>
<thead>
<tr>
<th>Program Name</th>
<th>Purpose</th>
<th>Reference Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. BUVCOP2</td>
<td>Convert magnetic tapes from IBM to NOS-CDC internal format.</td>
<td>IBM Format to CDC Format Conversion (section 1) and APPENDIX B.</td>
</tr>
<tr>
<td>2. BUV3</td>
<td>Preliminary data analysis program.</td>
<td>Preliminary Data Analysis (section 2).</td>
</tr>
<tr>
<td>3. OZSTAT2</td>
<td>Group data into global grid system. Perform elementary statistical calculations. Generate statistical graphics describing global grid grouping.</td>
<td>Data Grouping Scheme (section 3).</td>
</tr>
<tr>
<td>5. GLOBZON</td>
<td>Calculate global and zonal means based on spherical harmonic model coefficients.</td>
<td>Statistical Analysis of Spherical Harmonic Model (section 5).</td>
</tr>
<tr>
<td>6. ZONVAR</td>
<td>Calculate global and zonal variances based on zonal elements of covariance matrix describing spherical harmonic coefficients.</td>
<td>Statistical Analysis of Spherical Harmonic Model (section 5).</td>
</tr>
<tr>
<td>7. EOFA2</td>
<td>Eigenanalysis program calculates data base arrays for EOF model.</td>
<td>Eigenanalysis - Empirical Orthogonal Functions (section 6).</td>
</tr>
<tr>
<td>8. EAMOD1</td>
<td>EOF model and analysis.</td>
<td>Eigenanalysis - Empirical Orthogonal Functions (section 6).</td>
</tr>
<tr>
<td>9. OZFILL1</td>
<td>Implements data fill techniques by autocorrelation and spherical harmonic functions.</td>
<td>Data Fill Technique by Autocorrelation Functions (section 7).</td>
</tr>
</tbody>
</table>
APPENDIX B

To illustrate the IBM to NOS-CDC conversion process, the most recent set of data received will be considered. These data are contained on three IBM 9-track magnetic tapes. The following information comes from documentation received with these data tapes.

1. Tape density  - 1600 BPI
2. Mode    - Binary
3. Parity  - Odd
4. Block (PRU) size  - 8000 bytes
5. Logical record length  - 80 bytes

All three tapes were generated on an IBM 360, and each tape contains 14 files.

With the technique used, one physical record unit (PRU, 8000 bytes) or block of data is buffered into the central processor at a time. This is the equivalent of 2000 IBM words or 1067 CDC words. Figure B-1 illustrates what shall be referred to in the subsequent discussion as a sub-block, that is, 15, 32-bit words arranged as eight packed 60-bit words. Sub-blocks are 480 bits long since this is the smallest common multiple of 32 and 60. The conversion process is accomplished with one sub-block at a time. The procedure as coded in Program BUVCOP2 is described below.

The first block of data is buffered into an array A dimensioned by 1100. Unused storage locations of this array contain the value of zero. The first sub-block (eight words) from A is placed into the array C dimensioned by eight. The 15, 32-bit words in the sub-block are unpacked and arranged into 15 right justified 60-bit words in Subroutine IBMWRDS. These 15, 60-bit words are stored in a temporary array B dimensioned by 15 and subsequently into the first 15 locations of an array D dimensioned by 2200. This process continues until all words in the data block have been stored in D. Subroutine IBMFPC from the READIBM subroutine package can now convert these numbers to CDC internal format floating point numbers which are stored in an array E dimensioned by 2010. In general, the E array contains 2,010 words. That is,
Number of words in E =

Number of sub-blocks x Number of 32 bit words/sub-block,

= 134 sub-blocks x 15 words/sub-blocks

= 2,010 words.

The number of complete 20 word logical records in E is the integer part of 2,010/20 or 100 records. The elements of the E array are finally written 20 words (one logical record) at a time onto an output file which is stored on NOS 9-track tapes. This procedure for converting a block of data from IBM internal format to CDC internal format is shown schematically in Figure B-2.

This process is repeated with the next block until the end of the tape is reached. A listing of Program BUVCOP2 and Subroutine IBMWRDS follows this appendix.

A final comment regarding this conversion concerns the actual storage of data on magnetic tapes. The above technique converts one tape at a time. The program must therefore be run three times since three IBM tapes were received containing these data. The minimum amount of data contained on any one of these three tapes is 278,259 logical records or 5,565,180 words. Since a standard NOS 9-track tape, 2,400 feet in length, will hold only 3,880,421 60-bit words, two of these tapes are required. Three NOS tapes were required to hold the data from the IBM tape with the most data. Tape designations, and associated coverage periods, are shown in Table B-1.

These NOS tapes have been prepared to be read with an unformatted binary READ, one logical record (20 words) at a time. These 20 words are listed in Table B-2. The six of these words stored per record on the condensed tapes, generated to minimize storage and reduce computer time, are indicated with an asterisk (*).
### Table B-1. Magnetic Tape Designations and Their Corresponding Time Coverages

<table>
<thead>
<tr>
<th>Time Period</th>
<th>IBM Reel Designation</th>
<th>NOS TAPE (1) Designation</th>
<th>NOS TAPE (2) Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 10, 1970</td>
<td>30906</td>
<td>NV0738</td>
<td></td>
</tr>
<tr>
<td>- May 6, 1971</td>
<td></td>
<td>NV0739</td>
<td></td>
</tr>
<tr>
<td>May 7, 1971</td>
<td>34037</td>
<td>NN1004</td>
<td>NV0740</td>
</tr>
<tr>
<td>- May 5, 1972</td>
<td></td>
<td>NV0103</td>
<td>NV0104</td>
</tr>
<tr>
<td>May 6, 1972</td>
<td>32701</td>
<td>NV0333</td>
<td></td>
</tr>
<tr>
<td>- May 7, 1977</td>
<td></td>
<td>NV0334</td>
<td>NV0335</td>
</tr>
</tbody>
</table>

1 - Contains 20 words per logical record.
2 - Contains 6 words per logical record - condensed tape.
Table B-2. The Twenty Words that Constitute a Logical Record on the BUV Data Tapes

| Logical Sequence Number | Orbit Number | Year* | Day of Year* | Seconds of Day* | Latitude* | Longitude (westward)* | Solar-Zenith Angle | Monochromator N Values, (312.5 - 339.8)nm | Photometer N Values, (312.5 - 339.8)nm | A Channel Total Ozone Value | B Channel Total Ozone Value | Recommended Reflectivity | Recommended Total Ozone* |

* Designates those six words maintained on condensed data tapes.
Figure B-1. Sub-Block Structure.*

<table>
<thead>
<tr>
<th>Packed 60-bit Word Number</th>
<th>32 BIT WORD ARRANGEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32 bit word</td>
</tr>
<tr>
<td>2</td>
<td>4 bits</td>
</tr>
<tr>
<td>3</td>
<td>8 bits</td>
</tr>
<tr>
<td>4</td>
<td>12 bits</td>
</tr>
<tr>
<td>5</td>
<td>16 bits</td>
</tr>
<tr>
<td>6</td>
<td>20 bits</td>
</tr>
<tr>
<td>7</td>
<td>24 bits</td>
</tr>
<tr>
<td>8</td>
<td>28 bits</td>
</tr>
</tbody>
</table>

* Sub-block contains 480 bits of information. This is the equivalent of 8 60-bit words or 15 32-bit words.
Figure B-2. Schematic Showing the Procedure Used to Convert a Block of IBM to NOS-CDC Internal Format

Buffer in Block of IBM Data

Array A (1100)

1 PRU IBM Data

Array C (8)

Sub-block

15 32-bit words

Array D (2200)

60 bit word no.

Not Filled

1

zeros 32 bit word

2

zeros 32 bit word

Array D

Filled

60 bit words

... ...

2010

Complete 60 bit NOS-CDC word

60 bit word no.

1

2

3

... ...

2010

Array B (15)
PROGRAM BUVCOP2
DIMENSION A(1100)
DIMENSION B(15)
DIMENSION D(2200), E(2200), F(12)
DIMENSION U0(120)

C CONTAINS 8 COL 60 BIT WORDS PACKED WITH IBM 32 BIT WORDS
C B CONTAINS 15 COL 60 BIT WORDS Filled with right justified IBM 32 BIT WORDS
C D CONTAINS 2000 COL 60 BIT WORDS Filled with right justified IBM 32 BIT WORDS
C E CONTAINS 2000 COL 60 BIT WORDS EQUIV. TO THE 32 BIT IBM WORDS

10 NEUF=0
IGNIT=0
1MOD=0
4 CONTINUE
IF (NEUF.GE.14) GO TO 300
DO 2 1BI=1,10000
DO 13 MHL=1,1100
13 A(MHL)=0.
BUFFER IN (1,1) A(1100)
IF (UNIT(11) .EQ. 3) THEN
20 NWORDS=LENGTH(1)
IC=0
IOUT=0
IOUT=0
20 DO 10 IN=1,NWORDS
DO 11 IN=1,IOUT
10 IN=0

11 (IOUT)= I4(IOUT) + 1
CALL IBMMOD(G,8)
DO 12 1O08,1IOUT
IC=IC+1
12 I1O8+10=0(IOUT)
IOUT=IOUT+10
10 CONTINUE
14 CONTINUE
CALL IBMPPC (UI/EC)
COMMENT -- IT IS THE NUMBER OF 60 BIT WORDS CONTAINED IN THE A ARRAY.
C THIS NUMBER IS EQUIVALENT TO THE NUMBER OF 32 BIT WORDS
C IN SOME BLOCK ON THE IBM TAPE.
C 141 IS THE TOTAL NUMBER OF 20 WORD LOGICAL RECORDS/PHYS.
ICINIT=0
C SUBTRACT 1 SINCE RECORD PRIOR TO THE END OF BLOCK CONTAINS
C BAD DATA.

45 C IC1=IC1-1

IC2 IS THE NUMBER OF GOOD NON-EMPTY WORDS IN BLOCK.

46 C IC2=IC2+20

IF (NEOF.EQ.0) PRINT 910, IC1, IC1, IC2

910 FORMAT (150,3(15,3X))

IC1=IC1+11

IREC=IREC+IC1

IV=0

UU 250 III=1,11

UU 200 III=1,20

50

IU=IV+1

UU(III)=IU(IV)

200 CONTINUE

WRITE (2) OUT

IF (IMM.LE.101, ANU, EU, 1) PRINT 905, IMM, UU(13), UU(4),

905 FORMAT (15X,*IREC, *;1D;5X,*YEAR, *DAY, *MMU*;3(2X,F9.3))

250 CONTINUE

2 CONTINUE

3 NEOF=NEOF+1

PRINT 101, NEOF, 1, IMM, UU(13), UU(4), UU(19)/3800.

101 FORMAT (* END FILE SIZE BLOCKS READ = *1D,* IMM = *1D,*

13X,*YEAR,*DAY,*MMU*,3(2X,F9.3))

IMEC=0

GO TO 3

70

300 CONTINUE

PRINT 100, ILEN

900 FORMAT (*END/7110, *ILEN = *18)

STOP

5 PRINT 103

75 103 FORMAT (* PARITY ERROR*)

END

SYMBOLIC REFERENCE MAP (X=1)
SUBROUTINE IBMARDS
DIMENSION A(8), B(15)
INTEGER TAIL, HEAD
IBM=777777777760000000Q8
JBI
1X 10 B, 181, 8
LAST8(1)=1
IF(LAB1+EU,0)=0 TO 1
MASK0Z=(LAB1+1)=1
MASKOZ=(LAB1)=1
AAABMIFI(A(1)], LASTB)
TAIL=Mask.AND. AA
B(J)=BMIFI(AA, AND. IBM, 32)
JOY+1
IF(AI.EQ.0) GO TO 2
IF0759=LAB1=Q
MASKOZ( IF0759) =1
MASKOZ( IF57) =1
HEABAI(1), AND, MARK
B(J)=BMIFI(MEAB, J2=IF57)
JOY+1
10 CONTINUE
21 RETURN
END

SYMBOLIC REFERENCE MAP (IBM)
ENTRY POINTS
3 IBMARDS

VARIABLES | SN | TYPE | DESCRIPTION
--- | --- | --- | ---
0 | A | REAL | AMAY F.P. |
1 | B | REAL | AMAY F.P. |
2 | I | INTEGER | |
72 | IFSI | INTEGER | |
73 | LASTB | INTEGER | |
APPENDIX C - LINEAR APPROXIMATION FOR CALCULATING LOCAL TIME AS A FUNCTION OF LATITUDE

A straight line approximation to the ascending portion of the local time variation for a Sun-synchronous orbit curve from $-60^\circ$ to $+60^\circ$ latitude was calculated and is shown in Figure C-1. The relationship between the local solar time, $t_l$, and the latitude, $\theta$, for the observation was originally estimated to be

$$t_l = \frac{629.45 - \theta}{53.57}.$$  \hspace{1cm} (C-1)

Since, selected BUV-III data, closely corroborated by TRACK2 computer program simulations, have led to what is thought to be a better estimate, that is

$$t_l = \frac{604.54 - \theta}{50.93}.$$  \hspace{1cm} (C-2)

In any case, the error table shown below shows the maximum difference between equations (C-1) and (C-2) to be $0.1780$ hours ($10.68$ minutes) where

$$\Delta t = t_l - t_l.$$  \hspace{1cm} (C-3)

Table. Error Analysis

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\Delta t$ (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$60^\circ$</td>
<td>0.0618</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>0.0909</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>0.1200</td>
</tr>
<tr>
<td>$-30^\circ$</td>
<td>0.1490</td>
</tr>
<tr>
<td>$-60^\circ$</td>
<td>0.1780</td>
</tr>
</tbody>
</table>
Figure C-1. Approximation to the ascending portion of the local time variation of the Nimbus 4 Sun-synchronous orbit curve.
APPENDIX D - STORAGE OF GRIDDED OZONE DATA ON A MASS STORAGE RANDOM ACCESS FILE

A global grid system in the form of an array dimensioned 36 x 24 has been selected to represent the BUV ozone data. Each of the 36 rows corresponds to a 5° latitudinal zone while each of the 24 columns corresponds to a 15° longitudinal sector. Associated with each of the 864 blocks of the global grid are nine values that must be saved and stored such that they will be readily accessible when needed. For each of these values there is a separate array identified by the parameter ISET as shown in the table below.

Table. Global Arrays Saved on Mass Storage Random Access File

<table>
<thead>
<tr>
<th>ISET</th>
<th>Array Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>KK</td>
<td>Sampling Distribution</td>
</tr>
<tr>
<td>2</td>
<td>SUMX</td>
<td>Sum of ozone observations for each block</td>
</tr>
<tr>
<td>3</td>
<td>SUMXSQ</td>
<td>Sum of squares of ozone observations for each block</td>
</tr>
<tr>
<td>4</td>
<td>SUMT</td>
<td>Sum of observation times for each block</td>
</tr>
<tr>
<td>5</td>
<td>SUMTSQ</td>
<td>Sum of squares of the observation times for each block</td>
</tr>
<tr>
<td>6</td>
<td>SUMLT</td>
<td>Sum of the observed latitude for each block</td>
</tr>
<tr>
<td>7</td>
<td>SUMLTSQ</td>
<td>Sum of squares of the observed latitude for each block</td>
</tr>
<tr>
<td>8</td>
<td>SUMLG</td>
<td>Sum of the observed longitude for each block</td>
</tr>
<tr>
<td>9</td>
<td>SUMLGSQ</td>
<td>Sum of squares of the observed longitude for each block</td>
</tr>
</tbody>
</table>

It was decided that these arrays should be accessible on a daily basis for the 392 days beginning April 10, 1970 and ending May 6, 1971 or according to the time convention adopted during this study, NIMDAYS 100-491.
Making use of a mass storage random access (MSRA) file for this purpose is quite suitable. As can be seen, the actual data storage requirement here is

\[ 9 \text{ arrays} \times \frac{864 \text{ words}}{\text{array}} \times 392 \text{ days} = 3,048,192 \text{ words}. \]

However, by specifying a particular array for a given day, or several days, the computer storage requirement is reduced to that needed for only one array plus an INDEX array mentioned below.

This is illustrated in the following figure.

Figure. Mass Storage Random Access File Arrangement of Global Data Arrays

<table>
<thead>
<tr>
<th>MSRA Day No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>390</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>391</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>392</td>
<td>3520</td>
<td>3521</td>
<td>3522</td>
<td>3523</td>
<td>3524</td>
<td>3525</td>
<td>3526</td>
<td>3527</td>
<td>3528</td>
</tr>
</tbody>
</table>

Each of the blocks (1-3528) shown in the figure represent a data array. Let "NDEX" be the number that specifies a particular array, and "IDAY" be the MSRA day number specification. Then

\[ NDEX = 9 \times (IDAY - 1) + \text{ISET}. \]  

(D-1)
Since
\[ \text{IDAY} = \text{NIMDAY} - 99, \]  
expression (D-1) may be written in terms of NIMDAY as
\[ \text{NDEX} = 9 \times (\text{NIMDAY} - 100) + \text{ISET}, \]  
For example, if the array SUMXSQ(ISET = 3) were required for NIMDAY 101, then
\[ \text{NDEX} = 12, \]  
and the 12th array would be accessed from the mass storage file.

The INDEX array mentioned earlier must be present and must be dimensioned by \((A + 1)\) where A is the total number of arrays on the MSRA.

Listings of the subroutines GETDAT1 and GETDAT2 which access the BUV MSRA file follow this appendix.
SUBROUTINE GETDATI (XDATA, NDATA, NIMDAY, ISET)
DIMENSION XDATA(36,24), NDATA(36,24)

COMMENT == MUST "CALL OPENMS (1, INDEX, 3529, 0)" IN MAIN OR CALLING

PROGRAM.

COMMENT == ID(0) 1, 392, CORRESPONDS TO NIMDAY=100, 491, WHERE NIMDAY=100
18 APRIL 10, 1970.

COMMENT == FOR ISET=1, THE DATA DISTRIBUTION ARRAY CORRESPONDING TO
NIMDAY IS RETURNED AS NDATA.

COMMENT == OTHERWISE, ONE OF THE FOLLOWING ARRAYS IS RETURNED IN XDATA.

ISET=2, SUM
ISET=3, SUMXBO
ISET=4, SUMT
ISET=5, SUMBO
ISET=6, SUMLT
ISET=7, SUMLTSW
ISET=8, SUMLG
ISET=9, SUMLGBO

INDEX = (NIMDAY=100)*9 + ISET

IF (ISET.EQ.1) GO TO 75
CALL READMS (1, XDATA, BOB, INDEX)
RETURN

75 CONTINUE
CALL READMS (1, NDATA, BOB, INDEX)
RETURN
END

SYMBOLIC REFERENCE MAP (R=1)

RY POINTS
3 GETDATI
SUBROUTINE GETDATE (A, AVAR, KDATA, IT1, IT2, ISPEC, ICODE)
DIMENSION A (36, 24), AVAR (36, 24), KDATA (36, 24), XDATA (36, 24)

COMMENT == SUBROUTINE GETDATE FINDS THE MEAN, VALUE AND VARIANCE, IF
REQUESTED (SEE "ICODE" BELOW), OVER SOME SPECIFIED TIME
INTERVAL FROM SUMX AND SUMXSQ TYPE DATA WHICH HAS BEEN
STORED IN THE FORM OF A RANDOM ACCESS FILE, ACCESSIBLE
ON LOCAL FILE TAPE.

COMMENT == DEFINITIONS OF FORMAL PARAMETERS.
THE FOLLOWING ARE INPUT PARAMETERS:
IT1 = IS THE FIRST DAY OF THE TIME INTERVAL OVER WHICH
CALCULATIONS ARE MADE,
IT2 = IS THE LAST DAY OF THE TIME INTERVAL OVER WHICH
CALCULATIONS ARE MADE.
ISPEC1 = FIND MEAN OXIGEN VALUES.
ISPEC2 = FIND MEAN TIME OF OBSERVATION VALUES.
ISPEC3 = FIND MEAN LATITUDE.
ISPEC4 = FIND MEAN LONGITUDE.
ICODE0 = FIND ONLY MEAN VALUES.
ICODE1 = FIND VARIANCE ASSOCIATED WITH ABOVE MEAN.

THE FOLLOWING ARE OUTPUT ARRAYS:
A = CONTAINS MEAN VALUES.
AVAR = CONTAINS VARIANCE VALUES (SEE "ICODE" BELOW).
KDATA = CONTAINS DATA DISTRIBUTION

COMMENT == THE FOLLOWING STATEMENT REQUIRES THAT
"PHYSFIND" IN THE LISTED CARD.
IF (IT1, FD, KCOEF1, AND, IT2, EW, KCOEF2) GO TO 20
DO 15 I=1,36
DO 15 J=1,24
KDATA (I, J) = 0
15 CONTINUE
20 CONTINUE
DO 25 I=1,36
DO 25 J=1,24
A(I, J) = AVAR (I, J)
C 25 CONTINUE

C IRET=1BPEC+2
DO 150 INMDAYQIT1, IT2

C COMMENT -- THE FOLLOWING STATEMENT REQUIRED THAT
C "PREBIRED" IN THE IJSET CARD.
DO (IT1, EQ, KCODE=1, AND, IT2, EQ, KCODE=2) GO TO 31
CALL GETDAT1 (XDATA, NDATA, NIMDAYS, 1)
DO 20 I=1,36
DO 20 J=1,24
XDATA(I, J) = XDATA(I, J) + NDATA(I, J)
20 CONTINUE

95 30 CONTINUE

35 CONTINUE
CALL GETDAT1 (XDATA, NDATA, NIMDAYS, IBET)
DO 40 I=1,36
DO 40 J=1,24
A(I, J) = A(I, J) + XDATA(I, J)
40 CONTINUE

C 75 IF (ICODE, FV, 0) GO TO 150
ISET=IBET+1

65 CALL GETDAT1 (XDATA, NDATA, NIMDAYS, IBET)
DO 55 I=1,36
DO 55 J=1,24
AVA(I, J) = A(I, J) + XDATA(I, J)
55 CONTINUE

70 55 CONTINUE

C 150 CONTINUE

C 75 IF (ICODE, FV, 0) GO TO 300

75 DO 175 I=1,36
DO 175 J=1,24
TP (XDATA(I, J), LE, 1) GO TO 165
AVA(I, J) = A(I, J) + A(I, J) * XDATA(I, J) / (XDATA(I, J)**2)
165 CONTINUE

80 165 CONTINUE
AVA(I, J) = 0.

175 CONTINUE

300 CONTINUE
DO 350 I=1,36
SUBROUTINE GETDATE

DO 350 J=1,24
   IF (KDATA(1,J).EQ.0) GO TO 350
   A(1,J)=A(1,J)/KDATA(1,J)
   CONTINUE
   KCODE1=IT1  
   KCODE2=IT2
RETURN
END

SYMBOLIC REFERENCE MAP (Z=1)

ENTRY POINT:
3 GETDATE

VARIABLES : AN TYPE   RELOCATION
0 A : REAL   ARRAY   F.P.
241 I : INTEGER   
243 I : INTEGER   
0 ITI : INTEGER   F.P.
242 J : INTEGER   
240 KCODE2 : INTEGER   
200 KDATA : INTEGER   ARRAY   F.P.
245 XDATA : REAL   ARRAY

EXTERNALS : TYPE   ARGS
GETDATE  4

STATEMENT LABELS
0 19 40 20
0 30 105 35
0 55 150 150
174 175

178 300 212 350

LOOP LABEL  INDEX  FROM-TO  LENGTH  PROPERTIES
25 15  I  35 3A  3H  NOT INHER
32 15  J  36 3A  2B  INSTACK  NOT INHER
41 25  I  40 43  1B  INSTACK  NOT INHER
47 25  J  41 43  3B  INSTACK  NOT INHER
60 140  NIMDAY  46 72  73H  FIX, REFS  NOT INHER
APPENDIX E - ORTHONORMALITY PROPERTY OF SPHERICAL HARMONIC FUNCTIONS

The functions $\psi_k(x)$ for $k = 1, 2, 3, \ldots$, are orthogonal over the interval $(a,b)$ and, therefore, have the property that
\[
\int_a^b \psi_i(x) \psi_j(x) \, dx = 0, \text{ for } i \neq j. \tag{E-1}
\]
If $i = j$, and if
\[
\int_a^b [\psi_i(x)]^2 \, dx = 1, \tag{E-2}
\]
then the functions are also normal, or normalized, and form an orthonormal set of functions over the interval $(a,b)$. Equations (E-1) and (E-2) can be written as
\[
\int_a^b \psi_i(x) \psi_j(x) \, dx = \delta_{ij}, \tag{E-3}
\]
where $\delta_{ij}$, the Kronecker delta, has the property that
\[
\begin{align*}
\delta_{ij} &= 0, \text{ for } i \neq j \\
\delta_{ij} &= 1, \text{ for } i = j
\end{align*} \tag{E-4}
\]
This concept can be expanded to include spherical harmonic functions over the surface of a unit sphere. Let $y(\theta, \phi)$ be a function on the surface of a unit sphere, such that
\[
y(\theta, \phi) = \sum_{m=0}^{M} \sum_{n=m}^{M} [A_{mn} Z_n^m(\theta, \phi) + D_{mn} Z_n^0(\theta, \phi)], \tag{E-5}
\]
where
\[
Z_n^m(\theta, \phi) = \cos(m\phi) P_n^m(\cos \theta), \tag{E-6a}
\]
and
\[
Z_n^0(\theta, \phi) = \sin(m\phi) P_n^0(\cos \theta). \tag{E-6b}
\]
The $P_n^m(\cos \theta)$ are associated Legendre functions.
It can be shown that
\[ \int_{x=-1}^{1} p_n^m(x) \, p_\ell^m(x) \, dx = \frac{(n+m)!}{(n-m)!} \frac{2}{2n+1} \delta_{n \ell} \tag{E-7} \]
from which it follows that
\[ \int_{x=-1}^{1} p_n(x) \, p_\ell(x) = \frac{2}{2n+1} \delta_{n \ell} \tag{E-8} \]
where \( p_n(x) \) and \( p_\ell(x) \) are associated Legendre functions for \( m = 0 \) or simply Legendre functions.

Now consider the following integral equations which must be evaluated:

\[ I_1 = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} e^{Z_{mn}(\theta, \phi)} Z_{k \ell}^0(\theta, \phi) \, da, \tag{E-9} \]
\[ I_2 = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} e^{Z_{mn}(\theta, \phi)} Z_{k \ell}^e(\theta, \phi) \, da, \tag{E-10} \]
\[ I_3 = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} Z_{mn}^0(\theta, \phi) Z_{k \ell}^0(\theta, \phi) \, da. \tag{E-11} \]

The first may be written as
\[ I_1 = \int_{\theta, \phi} p_n^m(\cos \theta) p_\ell^k(\cos \theta) \cos(m\phi) \sin(k\phi) \, da \tag{E-12} \]
where
\[ da = \sin \theta d\theta d\phi \tag{E-13} \]
is the differential surface area of a unit sphere and the notation
\[ \int_{\theta, \phi} \] is equivalent to \[ \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \].
Consider the integration over $\phi$.

\[
\int_{\phi=0}^{2\pi} \cos(m\phi) \sin(k\phi) d\phi = 0 \quad (E-14)
\]

for $m = k$ or $m \neq k$. Substituting this result into equation (E-12) leads to

\[ I_1 = 0 \]

or

\[
\int_{\theta,\phi} Z_{mn}^e(\theta,\phi) Z_{k\ell}^0(\theta,\phi) d\alpha = 0. \quad (E-15)
\]

The next integral can be written as

\[ I_2 = \int_{\theta,\phi} p_n^m(\cos\theta) p_k^k(\cos\theta) \cos(m\phi) \cos(k\phi) d\alpha \quad (E-16) \]

or by (E-13) as

\[ I_2 = \int_{\theta,\phi} p_n^m(\cos\theta) p_k^k(\cos\theta) \sin\theta \cos(m\phi) \cos(k\phi) d\phi d\theta. \quad (E-17) \]

Again integrating over $\phi$ yields

\[
\int_{\phi=0}^{2\pi} \cos(m\phi) \cos(k\phi) d\phi = \begin{cases} 0, & \text{for } m \neq k \\ \pi, & \text{for } m = k \end{cases} \quad (E-18)
\]

and $I_2 = 0$ for $m \neq k$. Otherwise, equation (E-17) becomes

\[ I_2 = \pi \int_{x=-1}^{1} p_n^m(x) p_k^0(x) dx \quad (E-19) \]
where the substitutions $x = \cos \theta$ and $dx = -\sin \theta d\theta$ have been made along with corresponding changes in the limits of integration. Substituting equation (E-7) into equation (E-19) leads to

$$
\pi \int_{x=-1}^{1} P_n^m(x) P_\ell^m(x) dx = \frac{2\pi}{2n+1} \frac{(n+m)!}{(n-m)!} \delta_{n\ell}
$$

(E-20)

for $m \neq 0$.

If $m = k = 0$, the integral in equation (E-18) becomes

$$
\int_{\phi=0}^{2\pi} \cos(m\phi) \cos(k\phi) d\phi = \int_{\phi=0}^{2\pi} d\phi = 2\pi,
$$

(E-21)

and

$$
I_2 = \frac{4\pi}{2n+1} \delta_{n\ell}
$$

(E-22)

for $m = k = 0$. Then the integral in equation (E-10) has been evaluated and can be written as

$$
\int_{\theta,\phi} Z_\ell^m(\theta,\phi) Z_\ell^m(\theta,\phi) da = \begin{cases} 
\frac{4\pi}{2n+1} \delta_{n\ell}, & \text{for } m = k = 0 \\
\frac{2\pi}{2n+1} \frac{(n+m)!}{(n-m)!} \delta_{n\ell} \delta_{mk}, & \text{otherwise}
\end{cases}
$$

(E-23)

The final integral may be written as

$$
I_3 = \int_{\theta,\phi} P_n^m(\cos \theta) P_\ell^k(\cos \phi) \sin(m\phi) \sin(k\phi) da.
$$

(E-24)

By inspection, if $m = 0$, $I_3 = 0$. 

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For \( m \neq 0 \) the integration over \( \phi \) gives
\[
\int_{\phi=0}^{2\pi} \sin(m\phi) \sin(k\phi) d\phi = 0, \text{ for } m \neq k
\]
\[
\pi, \text{ for } m = k.
\] (E-25)

Equation (E-24) then becomes for \( m = k \)
\[
I_3 = \pi \int_{\theta=0}^{\pi} p_n^m(\cos \theta) p_k^m(\cos \theta) \sin \theta d\theta,
\] (E-26)

which as before can be written as
\[
I_3 = \pi \int_{x=-1}^{1} p_n^m(x) p_k^m(x) dx = \frac{2\pi}{2n+1} \frac{(n+m)!}{(n-m)!} \delta_{nk}.
\] (E-27)

Finally, the integral in equation (E-11) is
\[
\int_{\theta, \phi} Z_{mn}^0(\theta, \phi) Z_{k\ell}^0(\theta, \phi) d\theta = \frac{2\pi}{2n+1} \frac{(n+m)!}{(n-m)!} \delta_{nk} \delta_{mk} \delta_{\ell m},
\] (E-28)

where \( \delta_{ab} \) is defined such that
\[
\delta_{*ab} = \begin{cases} 
0, & \text{for } a = b \\
1, & \text{for } a \neq b
\end{cases}
\] (E-29)

Now define
\[
\gamma^e_{mn}(\theta, \phi) = F_{mn}^e Z_{mn}^e(\theta, \phi),
\] (E-30)

and
\[
\gamma^0_{mn}(\theta, \phi) = F_{mn}^0 Z_{mn}^0(\theta, \phi),
\] (E-31)
where

\[
F^s_{mn} = \begin{cases} 
1, \text{ for } m = 0 \\
\left[\frac{2(n-m)!}{(n+m)!}\right]^{1/2}, \text{ for } m > 0
\end{cases}
\]  

(E-32)

It is necessary to evaluate the three integrals

\[
I'_1 = \int_{\phi} Y^{e}_{mn}(\theta, \phi) Y^0_{k\ell}(\theta, \phi) d\alpha, 
\]

(E-33)

\[
I'_2 = \int_{\phi} Y^{e}_{mn}(\theta, \phi) Y^{e}_{k\ell}(\theta, \phi) d\alpha, 
\]

(E-34)

\[
I'_3 = \int_{\phi} Y^0_{mn}(\theta, \phi) Y^0_{k\ell}(\theta, \phi) d\alpha. 
\]

(E-35)

The right-hand sides of equations (E-33) through (E-35) may be written as

\[
\int_{\phi} Y^{e}_{mn}(\theta, \phi) Y^0_{k\ell}(\theta, \phi) d\alpha = F^s_{mn} \int_{\phi} Z^{e}_{mn}(\theta, \phi) Z^0_{k\ell}(\theta, \phi) d\alpha, 
\]

(E-36)

\[
\int_{\phi} Y^{e}_{mn}(\theta, \phi) Y^{e}_{k\ell}(\theta, \phi) d\alpha = F^s_{mn} \int_{\phi} Z^{e}_{mn}(\theta, \phi) Z^{e}_{k\ell}(\theta, \phi) d\alpha, 
\]

(E-37)

\[
\int_{\phi} Y^0_{mn}(\theta, \phi) Y^0_{k\ell}(\theta, \phi) d\alpha = F^s_{mn} \int_{\phi} Z^0_{mn}(\theta, \phi) Z^0_{k\ell}(\theta, \phi) d\alpha, 
\]

(E-38)

Substituting equation (E-15) into equation (E-36) yields

\[
\int_{\phi} Y^{e}_{mn}(\theta, \phi) Y^0_{k\ell} d\alpha = 0. 
\]

(E-39)
Similarly by equations (E-23) and (E-37)

\[
\int_{\theta, \phi} \gamma^e_{mn}(\theta, \phi) \gamma^e_{kl}(\theta, \phi) \, da = \frac{4\pi}{2n+1} \delta_{nk} \delta_{mk}.
\]  (E-40)

Finally, equation (E-38) may be evaluated by equation (E-28) as

\[
\int_{\theta, \phi} \gamma^0_{mn}(\theta, \phi) \gamma^0_{kl}(\theta, \phi) \, da = \frac{4\pi}{2n+1} \delta_{nk} \delta_{mk} \delta^*_{m0}.
\]  (E-41)

The results required for arriving at equation (53) can be found from equations (E-39) through (E-41), respectively. That is,

\[
\int_{\theta, \phi} \gamma^e_{mn}(\theta, \phi) \gamma^0_{mn}(\theta, \phi) \, da = 0,
\]  (E-42)

\[
\int_{\theta, \phi} \left[ \gamma^e_{mn}(\theta, \phi) \right]^2 \, da = \frac{4\pi}{2n+1},
\]  (E-43a)

and

\[
\int_{\theta, \phi} \left[ \gamma^0_{mn}(\theta, \phi) \right]^2 \, da = \frac{4\pi}{2n+1} \delta^*_{m0}.
\]  (E-43b)

Though incidental to this discussion, it should be noted that the functions \( \gamma^e_{mn}(\theta, \phi) \) and \( \gamma^0_{mn}(\theta, \phi) \) are orthogonal over the unit sphere since

\[
\int_{\theta, \phi} \gamma^e_{mn}(\theta, \phi) \gamma^0_{im}(\theta, \phi) \, da = 0
\]  (E-44a)

and

\[
\int_{\theta, \phi} \gamma^0_{mn}(\theta, \phi) \gamma^0_{im}(\theta, \phi) \, da = 0
\]  (E-44b)

for \( n \neq i, m \neq k, \) or both, and

\[
\int_{\theta, \phi} \gamma^e_{mn}(\theta, \phi) \gamma^0_{im}(\theta, \phi) \, da = 0
\]  (E-44c)

in any case.
However, these functions are not normalized over the sphere as can be seen by equations (E-43) but are said to be semi-normalized according to Adolf Schmidt by the constant $F^S$ defined in equation (E-32).
APPENDIX F - RECURRENCE RELATIONS FOR ASSOCIATED LEGENDRE POLYNOMIALS

In the modeling of atmospheric constituents with spherical harmonic functions it is useful to have the capability of calculating the required associated Legendre functions using recurrence relations. Many such relations exist for associated Legendre polynomial functions.

Subroutine LEGNDR4 has been written to calculate the associated Legendre functions up to and including those of some specified order, m, and degree, n, for a given colatitude, $\theta$. This subroutine is listed in Appendix G with the GLSRAN2 program.

If $P_n^m(x)$ is the associated Legendre function of order $m$ and degree $n$, then the first two functions are defined as:

\[ P_0^0(x) = 1, \quad (F-1) \]

and

\[ P_1^0(x) = x, \quad (F-2) \]

where

\[ x = \cos(\theta). \quad (F-3) \]

The functions of higher order and degree are evaluated by two recurrence relations. Consider the recurrence relation:

\[ P_{n+1}^m(x) = \frac{1}{n-m+1} [(2n+1) \times P_n^m(x) - (n+m) \times P_{n-1}^m(x)]. \quad (F-4) \]

This expression is used to calculate zero order ($m=0$) functions of degree $n+1$ from the two preceding zero order terms. Setting $m=0$, equation (F-4) becomes:

\[ P_{n+1}^0(x) = \frac{1}{n+1} [(2n+1) \times P_n^0(x) - nP_{n-1}^0(x)]. \quad (F-5) \]
Equation (F-5) is the first recurrence relation used in subroutine LEGNDR4.

The second recurrence relation used in LEGNDR4 comes from\(^7\)

\[(2n+1)(1-x^2)^{1/2} p_n^m(x) = p_{n+1}^{m+1}(x) - p_{n-1}^{m+1}(x). \quad (F-6)\]

Replacing \(n+1\) with \(n\) and \(m+1\) with \(m\) equation (F-6) may be rewritten as

\[p_n^m(x) = p_{n-2}^m(x) + (2n-1)(1-x^2)^{1/2} p_{n-1}^{m-1}(x). \quad (F-7)\]

Consider the first term on the right-hand-side of equation (F-7). Since the order must be equal to or less than the degree of the function (see equation (16)),

\[m \leq n-2, \quad (F-8a)\]

or

\[n > m+2, \quad (F-8b)\]

and the required recurrence relation for the higher order \((m>0)\) associated Legendre functions becomes,

\[p_n^m(x) = PQ + (2n-1)(1-x^2)^{1/2} p_{n-1}^{m-1}(x) \quad (F-9)\]

where

\[PQ = \begin{cases} 
p_n^m(x), & \text{for } n > m+2 \\
0, & \text{otherwise} 
\end{cases} \quad (F-10)\]

The numerical technique described above as utilized in subroutine LEGNDR4 has been verified up through \(m = n = 12\) on the NOS-CDC computer system at NASA/LaRC.
The primary purposes of the GLSRAN2 program as used in the ozone sampling study are to generate global stratospheric ozone models in terms of surface spherical harmonic functions by performing least squares fits to sets of BUV data and to perform certain statistical analyses as have been outlined in this report ("Spherical Harmonic Model" and "Statistical Analysis of Spherical Harmonic Model"). The spherical harmonic model representation as shown in equation (20) is used by GLSRAN2. The table below shows the relationship between the functions, \( f_i \), as used in this representation and those, \( Y_{mn}^{e} \) and \( Y_{mn}^{o} \), as shown in equation (16).

Table. Relationship Between Spherical Harmonic Function Representations

<table>
<thead>
<tr>
<th>Zonal Functions</th>
<th>Sectoral Functions</th>
<th>Tesseral Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1 = p_0^0 )</td>
<td>( F_{M+2} = Y_{11}^e )</td>
<td>( F_{(3M+1)+1} = Y_{12}^e )</td>
</tr>
<tr>
<td>( F_2 = p_1^0 )</td>
<td>( F_{M+3} = Y_{11}^o )</td>
<td>( F_{(3M+1)+2} = Y_{12}^o )</td>
</tr>
<tr>
<td>( F_3 = p_2^0 )</td>
<td>( F_{M+4} = Y_{22}^e )</td>
<td>( F_{(3M+1)+3} = Y_{13}^e )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( F_{M+5} = Y_{22}^o )</td>
<td>( F_{(3M+1)+4} = Y_{13}^o )</td>
</tr>
<tr>
<td>( F_{M+1} = p_M^0 )</td>
<td>( F_{3M} + Y_{MM}^e )</td>
<td>( F_{(3M+1)+NT-1} = Y_{M-1,M}^e )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( F_{3M+1} = Y_{MM}^o )</td>
<td>( F_{(3M+1)+NT} = Y_{M-1,M}^o )</td>
</tr>
</tbody>
</table>

In the table the functions \( p_i^0 \), for \( i = 0, 1, \ldots, M \), are the zonal associated Legendre functions, or simply Legendre functions. \( M \) is the order and degree of the model.

\[ NT = M(M - 1) \]  

\[(G-1)\]
is the number of tesseral functions. There are \( M + 1 \) zonal functions and \( 2M \) sectoral functions. The number of terms in a model of order and degree \( M \) is
\[
N = (M + 1)^2. \tag{G-2}
\]
Model coefficients are computed according to equation (24) which may be written as
\[
B = S^{-1} R \tag{G-3}
\]
where \( S \), the "information" matrix, is defined by equation (34) and
\[
R = E^T Y. \tag{G-4}
\]
The \( S \) matrix, dimensioned \( N \times N \), is strictly a function of the sampling. As \( S \) is a symmetric matrix only its upper full triangle--the diagonal elements and those above the diagonal--is used in GLSRAN2. This implementation reduces computer time as well as the storage requirement. Since solving for the model coefficients requires that the inverse of \( S \) be computed, these time and storage savings become even more noteworthy.

The upper full triangle of \( S \) is "packed" into a vector. This vector, called \( V \) to avoid confusion, contains
\[
e = \frac{N}{2} (N + 1) \tag{G-5}
\]
elements. The correspondence between \( S \) matrix elements and \( V \) vector elements is given by
\[
V(i) = S(m,n) \tag{G-6a}
\]
where
\[
i = m + n(n - 1)/2. \tag{G-6b}
\]
The GLSRAN2 program is set up to either calculate the \( V \) vector based on input sample data or to access a previously calculated \( V \) vector through a local file. This is also the case for the \( R \) vector though to calculate \( R \) actual ozone observations must also be available.
Once these data are contained on working local files, GLSRAN2 makes available several options regarding which model coefficients or set of coefficients can be computed. The S matrix elements contained on local file are associated with a "master" model. The most obvious option is to compute the N coefficients for this master model. Three other options exist as listed below.

1. Coefficients may be calculated for a model of order L (L < M). To do this the program selects the required "subset" of the packed S matrix elements contained on local file and forms a new set of packed S matrix elements. The same is done for the R vector.

2. Model coefficients may be calculated based on a specified number of independent sampling observations (for example, a certain number of Dobson stations). When this option is selected the program determines the size of the model such that the number of model terms is equal to or less than the number of independent observations and then proceeds to find the S matrix elements required to form the new S matrix for the subset model.

3. Particular model coefficients may be specified according to degree, n, order, m, and whether they are to be associated with an odd (i = 1), $Y^0_{mn}(\theta, \phi)$, or even (i = 0), $Y^e_{mn}(\theta, \phi)$, spherical harmonic function (see equations 17 and 18). Identification of required coefficients by this option follows from the expression:

$$k = \begin{cases} 
  n + 1, & \text{for } m = 0, \\
  (M + 1) + 2m - 1 + 1, & \text{for } m = n, \\
  3M + m(2n - m - 1) + i, & \text{for } m \neq 0 \text{ and } m \neq n.
\end{cases}$$ (G-7)

This technique is illustrated below since the idea is fundamental to the three options as used to determine spherical harmonic function indices or the master S matrix elements required to form the subset S matrix. Assume the S matrix is associated with a master model of degree and order $M = 5$ and that the coefficients specified in the table below are sought.
Table. $y^i_{mn}$ Functional Form Indices with Corresponding $F_k$ Functional Form Indices

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>3</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>18</td>
</tr>
</tbody>
</table>

From the table it can be seen that for a 5th degree spherical harmonic model

$y^e_{00} = F_1$,

$y^e_{03} = F_4$,

$y^e_{22} = F_9$,

and

$y^o_{12} = F_{18}$.

Also in terms of master S matrix elements the subset S matrix for this example is

$$SS = \begin{bmatrix}
S_{11} & S_{14} & S_{19} & S_{1,18} \\
S_{41} & S_{44} & S_{49} & S_{4,18} \\
S_{91} & S_{94} & S_{99} & S_{9,18} \\
S_{18,1} & S_{18,4} & S_{18,9} & S_{18,18}
\end{bmatrix}$$

The following discussion pertains to the input/output (I/O) requirements and capabilities of GLSRAN2. As a complete listing of GLSRAN2 and its subroutines is included in this appendix the discussion is limited to I/O items involving the spherical harmonic model.
Four NAMELIST input lists control the program's operation. These are named below along with their associated parameters.

1. DATA
   (a) NDATA - number of observations in data set.
   (b) MORD - order of master model.

2. JOB
   (a) IDATA = 1 - simulate a data set based on an input sampling scheme and model coefficients.
       = 2 - data set is an input quantity.
   (b) IFUNC = 2 - spherical harmonic model fit to be performed.
   (c) IOPT = 0 - do not calculate S matrix. S matrix is already on local file TAPE4.
       = 1 - calculate S matrix and store it on local file TAPE4.
       = 2 - calculate S matrix, store it on local file TAPE4, and STOP program execution.
   (d) JOPT - same description as IOPT above except that JOPT pertains to the R vector.
   (e) ITAPE = 1
   (f) ICASE - number of cases to be run requiring a new data set.
   (g) JCASE - number of "sub-model" cases to be run per data set.

3. PARAMTR
   (a) BETA - input coefficients used for data simulation.

4. JOB2
   (a) METHOD = 1 - calculate coefficients for specified subset model.
       = 2 - determine number of coefficients to calculate based on specified number of independent observations.
       = 3 - particular coefficients to be calculated are specified.
       = 4 - calculate coefficients for complete master model.
   (b) NFUNC - number of coefficients in subset model.
   (c) MMORD - order of subset model.
   (d) ICODE = 0 - do not compute coefficients.
       = 1 - compute coefficients.
GLSRAN2 uses the FORTRAN variable dimensions source statement preprocessor program PRE. Variables input by this program control the size of GLSRAN2 arrays. These variables are:

1. N - the number of coefficients in the master model.
2. NN - the maximum value of NFUNC for a given run such that NN ≤ N.
3. NV - the number of element in the packed S matrix array such that NV = N(N + 1)/2.

Local files used by GLSRAN2 include:

1. TAPE1 - used for input data that must be rearranged by a user supplied subroutine to meet TAPE2 input file requirements.
2. TAPE2 - standard format input data file read by subroutine REALDAT.
3. TAPE3 - used to store such items as model coefficients and covariance matrix elements for future use.
4. TAPE4 - contains elements of packed master S matrix.
5. TAPE7 - contains master R vector.
PROGRAM GLSRAN2 (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT, 
1TAPE1, TAPE2, TAPE3, TAPE4, TAPE7)
COMMON PI, X1, X2, YVAR, DX, X, ER, IFUNC, YZ1, Z2, Z, MORD, NDATA
DIMENSION F(169), S(14365), R(169), B(169)
DIMENSION INDEX (169)
DIMENSION NAMELIST/OATA/X1, X2, Y1, Y2, YVAR, NDATA, Z1, Z2, MORD
NAMELIST/JOB/IDATA1, IFUNC, JOPT, IOPT, ITAPE, ICASE, JCASE
NAMELIST/PARAM/RETA
NAMELIST/JOB2/METHOD, NFUNC, MMORD, ICODE
DATA NAME/7HS, 7HR, 7HCOVAR, 7HB, 7HCOV, 
7HCOR, 7HZBAR, 7HSY, 7HRY, 7HA, 7HB1, 
7HCORI, 7HRCOVAR, 7HSI, 7HCOVARI /

COMMENT -- GLSRAN2 - PARAMETER LIST.
N IS THE TOTAL NUMBER OF COEFFICIENTS (AND THEREFORE THE NUMBER
OF FUNCTIONS) THAT MAKE UP THE "MASTER" MODEL.
N IS A VARDIM INPUT PARAMETER.
(IF A NEW MASTER MODEL IS NOT BEING CALCULATED, N MAY BE SET
EQUAL TO NN, SEE BELOW).
NFUNC IS THE TOTAL NUMBER OF COEFFICIENTS (AND THEREFORE THE NUMBER
OF FUNCTIONS) THAT MAKE UP THE "SUBSET" MODEL FOR A
PARTICULAR CASE.
NFUNC IS DETERMINED AS FOLLOWS,
METHOD=1, ORDER AND DEGREE, MMORD AND NNDEG, RESPECTIVELY,
ARE BOTH KNOWN FOR THE DESIRED "SUBSET" MODEL.
THEN,
NFUNC=1+NNDEG+MMORD(MMORD+1).
METHOD=2, NFUNC=NUMBER OF INDEPENDENT OBSERVATIONS
TO BE MODELED.
METHOD=3, NFUNC=NUMBER OF COEFFICIENTS SPECIFIED
TO BE MODELED.
METHOD=4, USE ENTIRE "MASTER" MODEL.
NFUNC=N.
NN IS THE MAXIMUM VALUE OF NFUNC DURING A GIVEN RUN,
BUT NOT TO BE LARGER THAN N.
NN IS A VARDIM INPUT PARAMETER.
NV IS THE NUMBER OF ELEMENTS IN THE V-VECTOR (PACKED FORM OF
NV IS A VARDIM INPUT PARAMETER.
NV=N(N+1)/2.
PROGRAM GLSRAN2 74/74  OPT=1  FTN 4.7+485  80/01/29. 14.11.35

C NVV  IS THE NUMBER OF ELEMENTS IN THE VV-VECTOP(PACKED FORM OF
C THE UPPER FULL TRIANGLE OF S-MATRIX, S-MATRIX FOR THE
C "SUBSET" MODEL).
C NVV*NFUNC*(NFUNCTION+1)/2.
C NOTE -- IN GLSRAN2, BOTH VECTORS V AND VV WILL USE THE STORAGE
C SPACE IN THE S ARRAY(ONE AT A TIME). THEREFORE,
C THE S-ARRAY MUST BE DIMENSIONED BY NV(the larger of NV AND
C NVV) WHEN THE V VECTOR IS TO BE CALCULATED.

COMMENT -- GLSRAN2 -- LOCAL FILE REQUIREMENTS.
C TAPE1 -- DATA FOR SUBROUTINE INPUT WHICH IS TO BE REARRANGED
C AND PUT ONTO TAPE2.
C TAPE2 -- LOCAL FILE CONTAINING DATA TO BE READ IN BY
C SUBROUTINE REALDAT.
C TAPE3 -- RESERVED FOR RESULTS SUCH AS CALCULATED MODEL COEFFICIENTS
C SO THAT THEY MAY BE SAVED ON PERMANENT FILE OR
C ON MAGNETIC TAPE SUBSEQUENT TO PROGRAM EXECUTION.
C TAPE4 -- LOCAL FILE TO CONTAIN UPPER FULL TRIANGLE OF S-MATRIX
C WHICH IS STORED THERE IN "PACKED" FORM. THIS DATA MAY
C ALREADY EXIST OR MAY BE CALCULATED IN THE PROGRAM.
C TAPE7 -- LOCAL FILE TO CONTAIN R-MATRIX ASSOCIATED WITH THE SAME
C SAMPLING SCHEME DEFINED BY THE S-MATRIX BEING USED.

C N  =169
NN  =169
NV  =14365
READ (5,DATA)
READ (5,JOB)
READ (5,PARAMTR)
WRITE (6,DATA)
WRITE (6,JOB)
WRITE (6,PARAMTR)

COMMENT -- SET SEED FOR RANF AS PI=ARC COS (-1)
PI=ACOS (-1.
CALL RANSET (PI)

DO 999 LLL=1,ICASE
PRINT 175, LLL

C IF (ITAPE.EQ.1) READ(5,401) NDATA
401 FORMAT (I5)
CALL SECOND(TIME)
PRINT 300, NDATA, TIME
300 FORMAT (1X,*NDATA=*,I8,5X,*TIME=*,F10.3)
COMMENT -- IF IOPT=0, NO NEW V-ARRAY IS REQUIRED.
C IF JOPT=0, NO NEW R-ARRAY IS REQUIRED.
IF (JOPT.EQ.0.AND.IOPT.EQ.0) GO TO 58
C
COMMENT -- INITIALIZE INPUT PARAMETERS TO GLSCOR1.
W=1.
F(1)=1.
C
CALL GLSCOR1 (F,S,R,W,B,Y,N,NV,JOPT,SUMY,YSQSUM,IERR)
C
* *
C
COMMENT -- PROGRAM CHOOSES EITHER TO USE REAL DATA OR TO SIMULATE
ITS OWN DATA SUCH THAT,
C IDATA=1 --- DATA IS SIMULATED
C IDATA=2 --- REAL DATA IS READ IN
C
SUBSEQUENTLY THE REQUIRED MODEL FUNCTIONS ARE CALCULATED.
ICOUNT=0
25 CONTINUE
IF (IDATA.EQ.1) CALL SIMDAT1 (BETA,N,F)
IF (IDATA.EQ.2) CALL REALDAT (F,N)
IF (IFUNC.EQ.999) STOP2
IF (IFUNC.EQ.998) STOP3
C
CALL GLSSUM1 (F,S,R,W,B,Y,N,NV,JOPT,SUMY,YSQSUM,IEPR)
115 ICOUNT=ICOUNT+1
IF (ICOUNT.EQ.NDATA) GO TO 50
GO TO 25
C
50 CONTINUE
IF (JOPT.EQ.0) GO TO 54
REWIND 7
DO 52 I=1,N
WRITE (*) R(I)
52 CONTINUE
IF (JOPT.EQ.2) STOP5
54 CONTINUE
   IF (IOPT.EQ.0) GO TO 58
   REWIND 4

130   DO 56 I=1,NV
   WRITE (4) S(I)
   56 CONTINUE
   IF (IOPT.EQ.2) STOP5
   58 CONTINUE

   C
   COMMENT -- FOLLOWING STATEMENT 58, THE REQUIRED R AND S ARRAYS ARE
   STORED ON LOCAL FILES TAPE7 AND TAPE4, RESPECTIVELY.

140   C
   NOW USING THESE ARRAYS OR SPECIFIED SUBSETS OF THEM
   (SPECIFIED ACCORDING TO THE VALUE OF "METHOD" -SEE TABLE
   BELOW), CALCULATE THE COEFFICIENTS OF THE REQUIRED FUNCTION
   (ACCORDING TO "IFUNC") AND STORE RESULTS IN THE ARRAY "B".
   THE JCASE PARAMETER SPECIFIES THE NUMBER OF CASES TO BE
   RUN WITH CURRENT LOCAL FILE DATA.

145   C
   COMMENT --
   METHOD       DESCRIPTION       NFUNC       MMORD
   *****        **********       *****

150   C
   1       SPECIFY SUBSET MODEL       NUMBER OF
   ORDER     SUBSET MODEL
   FUNCTIONS

155   C
   2       SPECIFY NUMBER OF
   ORDER     INDEPENDENT OBSERVATIONS
   OBSERVATIONS

160   C
   3       SPECIFY PARTICULAR
   ORDER     COEFFICIENTS
   COEFFICIENTS

165   C
   4       USE COMPLETE MASTER
   MODEL DATA AS REQUIRED
   NOTE -- NN IS VARIOUSLY DIMENSIONED BY THE
   PREPROCESSOR PROGRAM, PRE, AS THE
   LARGEST NUMBER OF SUBSET FUNCTIONS
   REQUIRED IN A GIVEN RUN.
DO 950 JJJ=1,JCASE
READ (5,JOB2)
WRITE (6,JOB2)
IF (METHOD.EQ.4) GO TO 60
IF (METHOD.EQ.1) CALL SUBS1 (INDEX,NFUNC,MMORD,N,MORD)
IF (METHOD.EQ.2) CALL SUBS2 (INDEX,NFUNC,MORD)
IF (METHOD.EQ.3) CALL SUBS3 (INDEX,NFUNC,NDEG,MORD)
IF (INDEX(1),EQ.-999) GO TO 60
GO TO 65
60 CONTINUE
NFUNC=N
NFUNC IS SET EQUAL TO N HERE FOR THE CASE OF USING THE FULL
MASTER MODEL WHEN METHOD=2 (IE. INDEX(1)=-999 WAS RETURNED
FROM SUBROUTINE SUBS2).
THEREFORE NFUNC WILL NOT HAVE TO BE DEFINED FOR CASES
WHERE METHOD=4.
DO 62 I=1,N
INDEX(I)=I
62 CONTINUE
INDEX(I)=I
180 CONTINUE
REWIND 4
REWIND 7
KCOUNT=0
JCOUNT=0
DO 70 II=1,NFUNC
I=INDEX(II)
70 CONTINUE
COMMENT -- IF ICODE=0 (ACCORDING TO 'JOB2' NAMELIST INPUT), DO NOT
COMPUTE COEFFICIENTS. THEREFORE, TAPE7 IS NOT REQUIRED.
IF (ICODE.EQ.0) GO TO 67
67 CONTINUE
IF (KCOUNT.GT.INDEX(NFUNC)) GO TO 90
KCOUNT=KCOUNT+1
READ(7) PR
IF (KCOUNT.NE.I) GO TO 66
R(II)=RR
66 CONTINUE
R(II)=RR
DO 70 JJ=1,II
70 CONTINUE
COMMENT -- IVV IS THE INDEX FOR THE VECTOR VV, TO BE STORED IN THE
C S-ARRAY.
IVV=(II*(II-1))/2+JJ
J=INDEX(JJ)
210 CONTINUE
COMMENT -- IV IS THE INDEX FOR THE VECTOR V, NOW CONTAINED ON TAPE4.
PROGRAM GLSRAN2  74/74  OPT=1

IV=(I*(I-1))/2+J

68 CONTINUE
IF (JCOUNT.GT.IV) GO TO 90
JCOUNT=JCOUNT+1

215 READ(4) V
IF (JCOUNT.GE.IV) GO TO 68
S(IVV)=V

70 CONTINUE
COMMENT -- NOW HAVE THE REQUIRED S AND R ARRAYS.

220 DO 71 I=1,IEND
WRITE (3,789) S(I)

71 CONTINUE

C

230 NVV=NFUNC*(NFUNC+1)/2
CALL GLSINV1 (F,S,R,B,Y,NFUNC,NVV,ICODE,SUMY,YSQSUM,IERR)

235 IEND=HMORD+1
IEND=IEND*(IEND+1)/2
DO 71 I=1,IEND
WRITE (3,789) S(I)

71 CONTINUE

C

240 NDEG=MORD
NZM=NDEG+1
NSM=NZM+2*MORD
DO 80 I=1,NFUNC
IF (INDEX(I).LE.NZM) GO TO 74
IF (INDEX(I).LE.NSM) GO TO 75
NOTF=INDEX(I)-NSM
NOTF1=NOTF+1
NTG=O
DO 72 J=1,MORD
LDEG=(NOTF1-NTG)/2+J
IF (LDEG.LE.NDEG) GO TO 73
72 CONTINUE
NTG=NTG+(MORD-J)*2

73 CONTINUE

245 ICPD=J
LORD=MOD(NOTF1,2)
GO TO 76

74 CONTINUE

250 LEO=O
LEO=O
LDEG=INDEX(I)-1
GO TO 76
CONTINUE
NOSF=INDEX(I)-NZM
LDEG=(NOSF+1)/2
LEO=MOD(NOSF+1,2)
LORD=LDEG
CONTINUE
K=I+(I*(I-1))/2
WRITE (6,200) B(I),S(K),SORT(S(K)),I,R(I),INDEX(I),LORD,LDEG,LEO
CONTINUE
GO TO 95
CONTINUE
PRINT 110, KCOUNT,JCOUNT,NFUNC,INDEX(NFUNC),II,I,II,IVV,IV
STOP6
CONTINUE
DO 900 I=1,NFUNC
K=I+(I*(I-1))/2
WRITE(3,789) B(I),S(K)
CONTINUE
VARANDA=(YSOSUM-SUMY*SUMY/NDATA)/(NDATA-1)
BR1=R(1)*B(1)
BR=0.
DO 515 I=1,NFUNC
BR=BR+B(I)*R(I)
CONTINUE
A1=(R(1)*SUMY)/(NDATA-1)
A2=-((SUMY*SUMY)/(NDATA*NDATA-1))
PRINT 1005, NDATA,NFUNC,NNDEG,SUMY,SUMY*SUMY,YSOSUM,BR,AL,A2
1005 FORMAT (*1*,**FUNDAMENTAL STATISTICAL PARAMETERS**/
11X,*TOTAL MEASUREMENTS(NDATA),*1T40,** *,16/
21X,*NUMBER OF MODEL COEFFICIENTS(NFUNC),*4T40,** *,14/
31X,*DEGREE OF MODEL(NDGEG),*4T40,** *,13/
41X,*SUM Y SQUARED(SUMY X SUMY),*4T40,** *,15.8/
51X,*YSOSUM*,*4T40,** *,15.8/
61X,*YSOSUM*,*4T40,** *,15.8/
71X,*EXPSOSUM(BR),*4T40,** *,15.8/
81X,*R(1) X R(1) (BR),*4T40,** *,15.8/
**PROGRAM GLSRAN2**  74/74  OPT=1  

```
295  91X,*A1*,T40,** *,E15.8/
    *1X,*A2*,T40,** *,E15.8)
C
C
VARERR=(YSQSUM-BR)/(NDATA-1)
VARMOD=VARDATA-VARERR
C
MDF=NFUNC=1
XMSH=VARMOD/MDF
IERDF=NDATA-NFUNC
XMSE=VARERR/IERDF
C
COMMENT -- CALCULATE THE DEGREE VARIANCES(AVERAGE SQUARE) OF THE
C SPHERICAL HARMONIC MODEL.
C
COMMENT -- LET THE FIRST FIVE(5) ELEMENTS OF THE ARRAY R CONTAIN
C VARDEG1 THROUGH 5), FOR 5-TH DEGREE MODEL.
DO 525 I=1,MMORD
  F(I)=0.
  R(I)=0.
525 CONTINUE
C
COMMENT -- INN IS THE DEGREE OF THE COEFFICIENT.
INN=I
IJ=I+1
DO 524 J=1,IJ
524 CONTINUE
C
COMMENT -- JMM IS THE ORDER OF THE COEFFICIENT.
JMM=J-1
C
COMMENT -- IF JMM=0, COEFFICIENT IS ZONAL.
C IF JMM=INN, COEFFICIENT IS SECTORAL.
C OTHERWISE THE COEFFICIENT IS TESSERAL.
IF (JMM.EQ.0) GO TO 518
IF (JMM.EQ.INN) GO TO 520
C
COMMENT -- CALCULATE B INDEX FOR TESSERAL COEFFICIENTS.
NT=INN-JMM
C
COMMENT -- JMMM IS THE NUMBER OF PRECEDING ROWS CONTAINING TESSERAL
C FUNCTIONS.
JMMM=JMM-1
IF (JMMM.EQ.0) GO TO 517
DO 516 II=1,JMMM
  NT=NT+(MMORD-II)
516 CONTINUE
517 CONTINUE
```
PROGRAM GLSRAN2  74/74  OPT=1

            KEVEN=3*MMORD+2*NT
            KODD=1+KEVEN
            GO TO 521

340  518  CONTINUE

C  COMMENT — CALCULATE B INDEX FOR ZONAL COEFFICIENTS.
            KEVEN=INN+1
            KODD=-99
            GO TO 521

520  CONTINUE

C  COMMENT — CALCULATE B INDEX FOR SECTORAL COEFFICIENTS.
            KEVEN=MMORD+2*JMM
            KODD=KEVEN+1
            521  CONTINUE

C  COMMENT — CALCULATE THE "SQUARE ROOT" OF THE EVEN AND ODD TERM
C  CONTRIBUTIONS OF THE DEGREE VARIANCES.
            EVEN=B(KEVEN)
            REVEN=R(KEVEN)
            IF (KODD.EQ.-99) GO TO 522
            ODD=0(KODD)
            RODD=R(KODD)

360  CONTINUE
            GO TO 523

523  CONTINUE
            ODD=0.
            RODD=0.

365  CONTINUE
            F(I)=F(I)+EVEN*REVEN+ODD*RODD
            R(I)=R(I)+EVEN*EVEN*ODD*ODD
            PRINT 1001, I,J,KODD,OSS,RODD,KEVEN,EVEN,REVEN,F(I)
            1001  FORMAT (1X,*I**,I3,* J**,I3,
                        1* KODD**,I3,* ODD**,E15.8,
                        2* RODD**,E15.8,* KEVEN**,I3,
                        3* EVEN**,E15.8,* REVEN**,E15.8,
                        4* F(I)**,E15.8)

370  CONTINUE
            BR1=BR1+F(I)

375  CONTINUE
            BR1=BR1+F(I)

C  COMMENT — F(I) CONTAINS VALUES FOR THE MEAN SQUARE DUE TO
C  COEFFICIENTS PER DEGREE.
C  COMMENT — F(I+NNDEG) CONTAINS VALUES OF DEGREES OF FREEDOM
C  FOR THESE MEAN SQUARE CALCULATIONS.
PROGRAM GLSRAN2 74/74  OPT=1  FTN 4.7+485  80/01/29. 14.11.35

380  F(I)=F(I)/(NDATA-1)
     F(I+NNDEG)=2.*INN+1.
     F(I)=F(I)/(I+NNDEG)

COMMENT -- S(I) CONTAINS VALUES OF THE ZONAL POWER PER DEGREE.

S(I)=
R(I)=R(I)/(2*INN+1)

390 525 CONTINUE

PRINT 1002, BR, BR1
1002 FORMAT (1X,*BR-*,F15.8,*BR1-*,E15.8)
    TPOWER=0.
    DO 550 I=1, NNDEG
    TPOWER=TPOWER+R(I)
    550 CONTINUE

PRINT 590, VARDATA, VARMOD, XMSE, IERDF, DATMOD2
1VARMOD/VARDATA, TPOWER, A1, A1+A2, A1+A2

395 590 FORMAT (1X,*PERCENT.*,T49,*DEG. CONTRIB.*,T83,*ACCUMULATE*,T100,*MEAN SQUARE*,T117,*DEG. FREEDOM*/
    *T37,*ZON. POWER*,T49,*DEG. CONTRIB.*,T69,** A2/)
    *1X,*VARDATA= *,E15.8/
    11X,*VARMOD= *,E15.8,T98,E15.8,T117,I5/
    21X,*VARERR= *,E15.8,T98,E15.8,T117,I5/
    31X,*RATIO= *,E15.8/
    41X,*TPOWER= *,E15.8/
    5T43,*A1=*, E15.8,T64,E15.8,T81,E15.8)

400 C

405 ACCUM=A1+A2
    DO 535 I=1, NNDEG
    CAPPA=F(I)*F(I+NNDEG)
    ACCUM=ACCUM+CAPPA
    PRINT 595, I, R(I), S(I), CAPPA, CAPPA+A2, ACCUM, F(I), F(I+NNDEG)

410 595 FORMAT (1X,*VARDEG(*,I2=*,E15.8,T36,F10.5,T47,E15.8,T64,E15.8,T81,E15.8)
    1T81,E15.8,T98,E15.8,T117,F5.0)

COMMENT -- LET R(I) NOW CONTAIN PERCENTAGE POWER.

R(I)=(R(I)/TPOWER)*100.

415 CONTINUE

535 CONTINUE

PRINT 585, I, R(I)
585 FORMAT (1X,I2,5X,*PERCENT. POWER= *,F10.5)

455 CONTINUE

950 CONTINUE

490 CONTINUE

999 CONTINUE
STOP
C
C 100 FORMAT (1X,*Z1= *,E15.8,5X,*Z2= *,E15.8,5X,*Y= *,E15.8,5X,*ER= *, GLSRAN2
1E15.8)
105 FORMAT (T10,*T= *,F15.8)
110 FORMAT (*1*, *STOP6 INDICATES A POTENTIAL RUN-AWAY LOOP SITUATION EGLSRAN2
1XISTS IN LOOP - 7C IN GLSRAN2. */1X,*THE FOLLOWING PARAMETERS ARE PGLSRAN2
PRINTED AS DIAGNOSTIC AIDS -- KCOUNT,JCOUNT,NFUNC,INDEX(NFUNC),II, IGLSRAN2
430 3, JJ, JIVV, IV*/1X,10(15,5x))
175 FORMAT (*1*,///======T25,*BEGIN PRINT FOR CASE NUMBER *,I3///======GLSRAN2
1///)
200 FORMAT (T10,E15.8,T30,E15.8,T50,E15.8,T2,I3,T72,E15.8,T90,I4,T101,GLSRAN2
II2,T110,I2,T120,I1) GLSRAN2
435 201 FORMAT (//T14,*EST. COEF.*,T34,*BETA VAR.*,T53,*ST. DEVIATION*,
1T75,*R-VECTOR*,T90,*INDFV*,T99,*ORDER*,T108,*DEGREE*,T118,
2*EVEN=0*/
3T17,*R *,T34,*COVAR (1,1)*,T53,*SORT(COVAR (1,1))*T90,*APRAY*,T119,GLSRAN2
4*TDD=1*/) GLSRAN2
440 215 FORMAT (///1X, A7)
789 FORMAT(2(5X,E15.8))
END

SYMBOLIC REFERENCE MAP (R=1)

TPY POINTS

502 GLSRAN2

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<tr>
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<td>ACCUM</td>
<td>REAL</td>
<td>20332 A1 REAL</td>
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<td>333</td>
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<td>REAL</td>
<td>55137 B REAL</td>
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<td>BETA</td>
<td>REAL</td>
<td>20331 B REAL</td>
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<td>BR1</td>
<td>REAL</td>
<td>20357 CAPPA REAL</td>
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<td>4</td>
<td>DXY</td>
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<td>351</td>
<td>EVEN</td>
<td>REAL</td>
<td>20360 F REAL</td>
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<tr>
<td>276</td>
<td>I</td>
<td>INTEGER</td>
<td>20256 ICASE INTEGER</td>
</tr>
<tr>
<td>263</td>
<td>ICODE</td>
<td>INTEGER</td>
<td>20275 ICOUNT INTEGER</td>
</tr>
<tr>
<td>252</td>
<td>IDATA</td>
<td>INTEGER</td>
<td>20313 IEND INTEGER</td>
</tr>
</tbody>
</table>
SUBROUTINE GLSCOR1 (F,R,W,B,Y,NV,ICODE,SUMY,YSQSUM,IERR)
DIMENSION F(N),S(NV),R(N),B(N)

C COMMENT -- INITIALIZATION LOOP FOR THE R, B, AND S ARRAYS.
DO 25 I=1,N
   R(I)=0.
   B(I)=0.
25 CONTINUE
DO 35 I=1,NV
   S(I)=0.
35 CONTINUE
   SUMY=0.
   YSQSUM=0.
RETURN

C C C
ENTRY GLSSLM1
C COMMENT -- SUMMATION OF THE R AND S ARRAYS.
IF (ICODE.EQ.0) GO TO 150
DO 125 I=1,N
   R(I)=R(I)+F(I)*W*Y
125 CONTINUE
C C
C COMMENT -- CALCULATE THE SUM OF THE Y*S AND Y SQUARED SUMMED.
   SUMY=SUMY+Y
   YSQSUM=YSQSUM+Y*Y
150 CONTINUE
C C
C COMMENT -- CALCULATE THE UPPER FULL TRIANGLE OF THE S-MATRIX AND STORE
C THE RESULT IN THE ONE-DIMENSIONAL ARRAY, S, DIMENSIONED
C BY NV -- SEE PARAMETER LIST IN GLSRAN2.
   DO 175 J=1,N
      K=J+(J*(J-1))/2
      S(K)=S(K)+F(I)*F(J)*W
175 CONTINUE
RETURN

C C C C C C C C C C C C C
SUBLINE GLSCOR1  74/74  LPT=1  FTN 4.7+485  90/01/23. 19.03.20

C
C      ENTITY GLSCOR1
C
COMMENTS -- FIND INVERSE OF S AND STORE IN S.*
C      FIND ESTIMATION PARAMETERS, b*SR WHERE S IS NOW THE
C      (COVARIANCE MATRIX OF PARAMETRIC ESTIMATION).
        1CP=1
        CALL SPDIHM (N,S,ICP,DEP,ISPE,IEERR)
100     IF (ICP.EQ.0) GO TO 250
        DL 225 I=1,N
        E(I)=0.
        DL 225 J=1,N
        IL=1
        JD=J
        IF (J.LT.J) GO TO 200
        IC=J
        IU=1
200     CONTINUE
        K=I0+(IU*(IU-1))/2
        E(I)=B(I)+I*(K)*K(J)
        225     CONTINUE
250     CONTINUE
        RETURN
        END

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
  3 GLSCOR1  101 GLSCORI  30 GLSSUM1

  VARIABLES   SN   TYPE     RELLOCATION
              NAME   LENGTH      VALUE
  0  R       REAL     ARRAY      F.P.*
  0  F       REAL     ARRAY      F.P.*
  1  ICOR1   INTEGER   ARRAY      F.P.*
  2  J       INTEGER   F.P.*
  3  ISPE    INTEGER   F.P.*
  4  IEERR   INTEGER   F.P.*
  5  ICP     REAL      INTEGER
  6  DEP     INTEGER   INTEGER
  7  IL      INTEGER   INTEGER
  8  JU      INTEGER   INTEGER
  9  B       INTEGER   INTEGER
  10  K       INTEGER   INTEGER
  11  S       REAL      ARRAY      F.P.*
SUBROUTINE SIMDAT1 74/74 OPT=1

SUBROUTINE SIMDAT1 (BETA,N,F)
COMMON PI,X1,X2,YVAK,DX,X,ER,IFUNC,Y,Z1,Z2,Z,MORD
DIMENSION BETA(N),F(N)
CALL SIMDAT2

COMMENT -- SEE REALDAT FOR EXPLANATION OF DATA TO BE SIMULATED AS INPUT.
IF (MORD.NE.0) CALL SIMDAT3
IF (IFUNC.EQ.1) GO TO 25
IF (IFUNC.EQ.2) GO TO 125
IF (IFUNC.EQ.4) GO TO 250
IFUNC=999
RETURN

COMMENT -- CALCULATE THE LINEAR POLYNOMIAL FUNCTIONS OF DEGREE "NP".
25 CONTINUE
CALL POLY (F,BETA,N)
50 CONTINUE
Y=Y+ER
RETURN

COMMENT -- CALCULATE THE FIRST "NP" ZONAL SPHERICAL HARMONIC FUNCTIONS.
125 CONTINUE
CALL SPHAH (F,N)
25 CONTINUE
135 CONTINUE
Y=0.
DO 150 I=1,N
Y=Y+BETA(I)*F(I)
150 CONTINUE
GO TO 50

COMMENT -- END SPHERICAL HARMONIC CALCULATIONS.

COMMENT -- CALCULATE THE "NP" FOURIER FUNCTIONS, OTHER THAN F(1)=1.
35 CONTINUE
250 CONTINUE
CALL F4RNC1 (F,N)
GO TO 135

COMMENT -- END FOURIER CALCULATIONS.

END
SUBROUTINE REALDAT (F,N)
COMMON P1,X1,X2,YVAR,DX,X,ER,IFUNC,Y,Z1,Z2,Z,MORD
DIMENSION F(N)

COMMENT -- EXPLANATION OF INPUT.

IFUNC=1, INDEPENDENT VARIABLE
Z, NONE
Y, DEPENDENT VARIABLE

IFUNC=2, GD-LATITUDE (RADIANS)
LONGITUDE (RADIANS)
DEPENDENT VARIABLE (OZONE)

IFUNC=3, LATITUDE (RADIANS)
NONE
DEPENDENT VARIABLE

IFUNC=4, SCALED FOURIER ANGLE (RADIANS)
NONE
APPROPRIATE DEPENDENT VARIABLE

IF (MORD.EQ.0) GO TO 15
READ (2) X,Z,Y
IF (EOF(2)) 25,50
15 CONTINUE
READ (2) X,Y
IF (EOF(2)) 25,50
25 CONTINUE
IFUNC=99E
RETURN

40 C
10 CONTINUE
IF ("IFUNC.EQ." 67 TO 75
SUBROUTINE REALDAT 74/74 OPT=1

IF (IFUNC.EQ.2) GO TO 150
IF (IFUNC.EQ.3) GO TO 250
IF (IFUNC.EQ.4) GO TO 350
IFUNC=999
RETURN

C

COMMENT -- CALCULATE LINEAR POLYNOMIAL FUNCTIONS THROUGH DEGREE "NP".

75 CONTINUE
DO 125 I=2,N
F(I)=F(I-1)*X
125 CONTINUE

C
RETURN

C

COMMENT -- CALCULATE THE "NP" SPHERICAL HARMONIC FUNCTIONS.

150 CONTINUE
CALL SPHARM (F,N)
RETURN

C

COMMENT -- CALCULATE F(2)=COS(2LAT), WHERE X=LAT.

250 CONTINUE
F(2)=COS(2*X)
RETURN

C

COMMENT -- CALCULATE THE "NP" FOURIER FUNCTIONS, OTHER THAN F(1)=1.

350 CONTINUE
CALL FORFNC1 (F,N)
RETURN

C
END

SYMBOLIC REFERENCE MAP (k=1)
SUBROUTINE POLY (F,A,N)
COMMON PI,X1,X2,YVAR,DX,X,EK,IFUNC,G
DIMENSION F(N),A(N)

IF (N.EQ.0) RETURN
M=1,NP
J=NP+1-M
F(M+1)=F(M)*X
G=A(J)+X*G
CONTINUE
RETURN
END

SYMBOLIC REFERENCE MAP (R=1)

TRY POINTS

<table>
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<th>SN</th>
<th>TYPE</th>
<th>RELLOCATION</th>
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<td>EK</td>
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<td>G</td>
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<tr>
<td>J</td>
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<td>N</td>
<td>0</td>
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STATEMENT LABELS

0 15

LDBS LABEL INDEX FROM-TO LENGTH PROPERTIES

<table>
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<tr>
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<th>INDEX</th>
<th>FROM-TO</th>
<th>LENGTH</th>
<th>PROPERTIES</th>
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<tr>
<td>20</td>
<td>15</td>
<td>7 11</td>
<td>5B</td>
<td>INSTACK</td>
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</tbody>
</table>

INMUN BLOCKS LENGTH

//
SUBROUTINE SPHARM (F,N)
COMMON P1,X1,X2,YVAK,DX,X,ER,IR,UNNC,Y,Z1,Z2,Z,MORD
DIMENSION F(1)
DIMENSION Q(13,13)
*C
COMMENT -- CALCULATE THE SECOND THROUGH THE (NDEG+1) - TH F-FUNCTIONS.
MPLUS=MORD+1
NP=N-1
NDEG=NP-MORD*MPLUS
CALL LEGNDR4 (MPLUS,NDEG,X,Q)
M=0
DO 25 I=1,NDEG
F(I+1)=Q(1,1+1)
25 CONTINUE
*C
COMMENT -- CALCULATE THE 2*MORD SECTORIAL FUNCTIONS.
STORE RESULTS IN THE F ARRAY, ELEMENTS (NDEG+2) THROUGH
(NDEG+1+2*MORD).
NN1=NDEG+3
NN2=2*MORD+NN1-2
M=0
DO 50 I=NN1,NN2,2
M=M+1
FS=SCRT(2./FFAC(M+M))
FH=Q(M+1,M+1)*FS
F(I-1)=FH*CS (M*Z)
F(I)=FH*SIN(M*Z)
50 CONTINUE
IF (MORD.LE.1) RETURN
*C
COMMENT -- CALCULATE THE NUMTE5=N-NH2 TESSERAL FUNCTIONS.
C
SUBROUTINE SPHARM  74/74  CPT=1

45
NN1=NN2+2
NN2=N
M=0
NN=NDEG
DL 75 I=NN1,NN2+2
IF (NN*LT*NDEG) GO TO 70
M=M+1

50
NN=M

70 CONTINUE
NN=NN+1
FS=SQRT(2.*RFAC(NN-M)/RFAC(NN+M))
FN=Q(M+1,NN+1)*FS
F (1-J) = FN*COS (M*Z)
F(1) = FN*SIN(M*Z)

75 CONTINUE
RETURN
END

SYMBOLIC REFERENCE MAP (K=1)

TRY POINTS
3 SPHARM

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>SN</th>
<th>TYPE</th>
<th>ALLOCATION</th>
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<td>7 JFLNC</td>
<td>140</td>
<td>M</td>
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<td>14 MURD</td>
<td>143</td>
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<td>Z2</td>
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<td>11 Z1</td>
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</table>

55
FUNCTION RFAC

1

DOULELE FUNCTION RFAC (NU)
RFAC=1.
IF(NU.LT.2) GO TO 11
DC 9 I=1,NU
5
RFAC=RFAC*I
9 CONTINUE
11 RETURN
END

SYMBOLIC REFERENCE MAP (R=1)

KEY POINTS
5 RFAC

ZABLES SN TYPE RELLOCATION
30 1 INTEGER 0 NO INTEGER F0P0
26 RFAC DOUBLE

ATMENT LABELS
0 9 24 11

OPS LABEL INDEX FROM-TO LENGTH PROPERTIES
16 9 1 4 6 5B INSTACK

ATISTICS
PROGRAM LENGTH 318 25
520000 CM USED
SUBROUTINE FORFNC1 (F,H)
COMMON F1,X1,X2,YVAR,DX,X
DIMENSION F(N)

***** *******

COMMENT -- NP=N-1 FOURIER FUNCTIONS ARE CALCULATED PER
CALL TO SUBROUTINE FORFNC1.
-- THESE NP FUNCTIONS ARE OF THE FORM,
F(2*1)*COS (I*X)

AND
F(2*1+1)= SIN (I*X),
FOR I=1,N
WHERE M=NP/2.

***** *******

M=(N-1)/2
DO 25 I=1,M
F(2*1)*COS(I*X)
F(2*1+1)=SIN(I*X)
25 CONTINUE

RETURN
END

SYMBOLIC REFERENCE MAP (R=1)

TRY POINTS
3 FORFNC1

<table>
<thead>
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<th>SN TYPE</th>
<th>F.P. LOCATION</th>
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<td>F.P.*</td>
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<tr>
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<tr>
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</tbody>
</table>

F.P.* REAL  ARRAY  F.P.*
0           PI     REAL  / / 
1           X1     REAL  / / 
3           YVAR   REAL  / / 
0           M      INTEGER / / 
23          M      INTEGER / /
SUBROUTINE LEGNDR4 (NORD, NDEG, COLAT, P)
DIMENSION P(13,1)
DOUBLE PRECISION Q(13,13)
DOUBLE PRECISION X, SINE

MORD = NORD - 1
Q(1,1) = 1.0
F(1,1) = Q(1,1)
IF (NDEG*EQ.0) RETURN
X = COS(COLAT)
SINE = SQRT(1.0 - X*X)
Q(1,2) = X
F(1,2) = Q(1,2)
IF (NDEG*EQ.1.AND.MORD*EQ.0) RETURN
N = N + 1

50 CONTINUE
IF (MORD*NE.0) GO TO 150
N = N + 1
75 CONTINUE

COMMENT -- CALCULATE ZERO ORDER TERM OF DEGREE N+1 WITH THE TWO
PREVIOUS ZERO ORDER TERMS.
C
Q(1,N+2) = ((2*N+1)*X*Q(1,N+1) - N*Q(1,N))/(N+1)
P(1,N+2) = Q(1,N+2)
IF (MORD*EQ.0.AND.NDEG*EQ.N+1) RETURN
GO TO 50

150 CONTINUE
N = N + 1
M = M + 1

225 CONTINUE
COMMENT -- CALCULATE HIGHER THAN ZERO ORDER TERMS OF DEGREE N.
M = M + 1
IF (X*EQ.1.0 OR X*EQ.-1.0) GO TO 290
Q = 0.0
IF (N*GE.M+2) Q = Q(M+1,N-1)
Q(M+1,N+1) = Q + (2*N-1)*SINE*Q(M,N)
GO TO 300

250 CONTINUE
Q(M+1,N+1) = C
300 CONTINUE
P(M+1,N+1) = Q(M+1,N+1)
IF (M*EQ.MORD.AND.N*EQ.NDEG) RETURN
IF (M*EQ.N*OR.M*EQ.MORD) GO TO 75
GO TO 225
SUBROUTINE LEGNR4  74/74  GPT=1  FTN 4.7+485  80/01/23. 19.03.2C

RD NR.  SEVERITY DETAILS  DIAGNOSIS OF PROBLEM
  12  1  P  ARRAY REFERENCE OUTSIDE DIMENSION BOUNDS.

SYMBOLIC REFERENCE MAP (R=1)

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<th>NTGY POINTS</th>
<th>3  LEGNR4</th>
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<td>P</td>
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<td>157</td>
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<tr>
<td>155</td>
<td>X</td>
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</table>

EXTERNALS  TYPE  ARGS
  REAL  1 LIBRARY  DSQRT DOUBLE  1 LIBRARY

STATEMENT LABELS
  35  50  40  75  76  150
  101  225  132  230  136  300

STATISTICS
  PROGRAM LENGTH  767B  495
  52000B CM USED
SUBROUTINE SUBS1 (INDEX,NN,MMORD,N,MORD)
DIMENSION INDEX (NN)

COMMENT -- DEFINE REQUIRED PARAMETERS.
  NNDEG=NN-NNMORD*(MMORD+1)-1
  NSZON=NNDEG+1
  NIZ=NSZON
  ISS=1+NIZ
  NSSSEC=2*MORD
  NIS=NZS+NSSSEC
  ITS=1+NSS
  NTS=S=NN-(NSZON+NSSSEC)
  NTS=NSS+NSTES
  NDEG=N-MORE*(MORD+1)-1
  NNZON=NDEG+1
  NIZ=NIZON
  NMSEC=2*MORD
  NSK=NSM+NMSEC

COMMENT -- CALCULATE THE ELEMENTS OF THE INDEX ARRAY.
  DO 50 KZON=1,NSZ
  INDEX(KZON)=KZON
  CONTINUE
  DO 100 KSEC=ISS,NSS
  K=K+1
  INDEX(KSEC)=K
  CONTINUE

IF (NSTES.EQ.0) GO TO 200

COMMENT -- IROWS IS THE TOTAL NUMBER OF ROWS OF TESSERAL FUNCTIONS
  IN THE SUBSET MODEL.
  IROWS=MMORD-1

COMMENT -- LEFT IS THE NUMBER OF 'MASTER' SPHERICAL HARMONIC TESSERAL
  FUNCTIONS REMAINING IN ROW IROW.
  LEFT=0
  KTES=NSS
  DO 175 IROW=1,IROWS
  K=K+LEFT

COMMENT -- NOMTTR IS THE NUMBER OF 'MASTER' MODEL TESSERALS IN ROW IROW.
  NOMTTR=(MORD-IROW)*2

COMMENT -- NOMTTR IS THE NUMBER OF 'SUBSET' MODEL TESSERALS IN ROW IROW.
  NOMTTR=(MORD-IROW)*2
  DO 150 KSTEP=1,NOMTTR
SUBROUTINE SUB1 74/74 OPT=1

KTES=KTES+1
K=K+1
INDEX(KTES)=K

150 CONTINUE
LEFT=NGMTN-NOMMTR
175 CONTINUE
200 CONTINUE
RETURN
END

SYMBOLIC REFERENCE MAP (R=1)

TRY POINTS
3 SUB1

VARIABLES SN TYPE RELLOCATION
0 INDEX INTEGER ARRAY INDEX 125 IROW INTEGER
122 IKOWS INTEGER 134 ISS INTEGER
107 ITS INTEGER 120 K INTEGER
121 KSEC INTEGER 130 KSTFP INTEGER
124 KTES INTEGER 117 KZON INTEGER
123 LEFT INTEGER 0 MMORD INTEGER
0 MORD INTEGER 0 N INTEGER
112 NŒG INTEGER 115 NMSEC INTEGER
113 NMZON INTEGER 0 NN INTEGER
101 NŒGE INTEGER 127 NOMMTR INTEGER
126 NUMLR INTEGER 116 NSM INTEGER
106 NS INTEGER 105 NSSE INTEGER
110 NSTES INTEGER 102 NSZUN INTEGER
111 NTS INTEGER 114 NZM INTEGER
103 NŽ INTEGER

STATEMENT LABELS
0 50 0 100 0 150
0 175 1CC 200
SUBROUTINE SUBS2 (INDEX,M,MORD)
COMMENT -- GIVEN A NUMBER M, OF INDEPENDENT OBSERVATIONS, SUBROUTINE
SUBS2 CALCULATES THE ARRAY INDEX WHICH CONNECTS THE INDICES
OF THE S-MATRICES(UPPER FULL TRIANGLES ONLY) OF THE MASTER
MODEL(ORDER=MORD) AND THE SUBSET MODEL(SIZE TO BE
DETERMINED IN THIS SUBROUTINE BASED ON M).

COMMENT -- NOTE M MUST BE .GE. 1.

DIMENSION INDEX(M)
NMSEC=2*MORD
IMAX=MORD+1

COMMENT -- NMSEC IS THE MAXIMUM NUMBER OF SECTORAL FUNCTIONS IN THE
MASTER MODEL.
IMAX IS THE MAXIMUM NUMBER OF ZONAL FUNCTIONS IN THE
MASTER MODEL -- REFERRED TO AS NMZON IN SUBROUTINE SUB51.
IMAXSQ=IMAX*IMAX

COMMENT -- IMAXSQ IS THE MAXIMUM NUMBER OF FUNCTIONS AVAILABLE IN THE
MASTER MODEL.

IF (M-IMAXSQ) .LT. 225,250
15 CONTINUE
D2 25 I=2,IMAX
IF (I*1.0,G7,0) GO TO 50
25 CONTINUE

COMMENT -- THIS LOOP SHOULD NOT FINISH NORMALLY. IF IT DOES A
DIAGNOSTIC WILL BE PRINTED AND EXECUTION STOPPED.
PRINT 925
925 FORMAT (*1*,*EXECUTION STOPPED IN SUBROUTINE SUBS2*)
STOP

C
C * * * * * * * * * * * * * * *
C

50 CONTINUE
NISSEC=2*(I-2)
NISZON=I-1

COMMENT -- NISSEC AND NISZON ARE THE NUMBER OF SECTORAL AND ZONAL
FUNCTIONS, RESPECTIVELY, IN THE INITIAL SUBSET MODEL OF
ORDER= I-2.
MDIFF =M-NISZON*NISZON

COMMENT -- MDIFF IS THE DIFFERENCE BETWEEN M AND THE NUMBER OF FUNCTIONS
IN A MODEL OF ORDER=I-2, WHICH CONTAINS (I-1)*(I-1) FUNCTIONS
MSEC=MDIFF/2
MZON=MD1FF-MSEC
MTES=0.

45 COMMENT -- MSSEC AND MZON ARE THE NUMBER OF EXTRA SECTORAL AND ZONAL
FUNCTIONS, RESPECTIVELY. MTES IS THE NUMBER OF EXTRA
TESSERAL FUNCTIONS REQUIRED, IF ANY.

46 ITEST=NISSC+MSEC-NMSEC
47 IF (I TEST) 75,75,70

50 CONTINUE

50 COMMENT -- MSSEC+NISSC IS LARGER THAN NMSEC, THE MAXIMUM NUMBER OF
SECTORAL FUNCTIONS AVAILABLE IN THE MASTER MODEL.

52 MSEC=NMSEC-NISSC
53 MTES=I TEST

55 CONTINUE

55 ITEST=MZON+NISSC-IMAX
56 IF (I TEST) 85,85,80

60 CONTINUE

60 MZONE=IMAX-NISSC
65 MTES=MTES+I TEST

65 CONTINUE

65 COMMENT -- THE CALCULATION OF MTES IS NOT ACTUALLY REQUIRED IN ORDER
TO FIND NTESTS -- HOWEVER, AS A DEBUGGING AID IT IS A
USEFUL PARAMETER.

66 N.ZS=NISSC+MZONE
67 I SS=N.ZS+1
68 N.SS=N.ZS+NISSC+MSEC
69 I TS=N.SS+1
70 NSSEC=N.SS-N.ZS
71 MTESTS=M-NSSEC-N.ZS
72 NTS=N.SS+NTESTS

B.
C

C

C

C

C

75 DC 125 KZONE=1,N.ZS
76 INDEX(KZONE)=KZONE
125 CONTINUE!
175 IF (NSSEC(I T,1)) GO TO 155

80 K=IMAX
80 DC 150 KSEC=ISS*N.SS
85 K=K+1
86 INDEX(KSEC)=K
150 CONTINUE

SUBROUTINE SUBS2

85 CONTINUE
If (NSTES.LT.1) GO TO 180

COMMENT -- IPWS IS THE TOTAL NUMBER OF ROWS OF TESSERAL FUNCTIONS
C
IN THE SUBSET MODEL.

IROWS=1-2

90 C
NOTE -- RECALL THAT NISZON=I-1
If (NISZON*NISZON.EQ.M) IROWS=IROWS-1
K=3*MODD+1

COMMENT -- LEFT IS THE NUMBER OF 'MASTER' SPHERICAL HARMONIC TESSERAL
C
FUNCTIONS REMAINING IN ROW IROW.

95 LEFT=0
KTES=SS
G0 175 IRGW=1,IROWS
K=K+LEFT

COMMENT -- NUMTR IS THE NUMBER OF 'MASTER' MODEL TESSERALS IN ROW IROW.
C
NUMMTX IS THE NUMBER OF 'SUBSET' MODEL TESSERALS IN ROW. IROW.

100 NUMMTX=(MOKL-IROW)*2
NUMMTX=(NISZON-IPOW)*2
If (NISZON*NISZON.EQ.M) NUMTR=NUMMTX-2
D0 160 KSTEP=1,NUMTR
K=K+1
INDEX(KTES)=K

105 CONTINUE
If (KTES.GT.IM) GO TO 180
K=K+1
CONTINUE

110 LEFT=NUMTR-NOMTR
175 CONTINUE
160 CONTINUE
RETURN
C

115 CONTINUE
PRINT 950, K
950 FORMAT (/1X,M,M, I4, *, *), IS THE TOTAL NUMBER OF FUNCTIONS AVAIL-
ABLE IN THE SPECIFIED MASTER MODEL.*/1X, * THEREFORE THE UPPER FULL
2TH DIAGONAL OF THE MASTER MODEL WILL BE READ DIRECTLY FROM TAPE AND
3USED IN THE FOLLOWING CALCULATIONS. ()*
G0 TO 300
250 CONTINUE
PRINT 975, M, IMAXSQ
IS GREATER THAN THE NUMBER OF FUNCTIONS, *I4, *, CONTAINED IN THE S
PECIFIED MASTER MODEL. */1X, * THEREFORE, THE UPPER FULL TRIANGLE OF
3 THE MASTER MODEL WILL BE READ DIRECTLY FROM TAPE AND USED IN THE FOLLOWING CALCULATIONS.

300 CONTINUE

130 INDEX(1) = -999
RETURN
END

SYMBOLIC REFERENCE MAP (R=1)

TRY POINTS
3 SUBS2

ARIALES   SN TYPE  RELLOCATION
114     I    INTEGER  112 IMAX INTEGER  ARRAYS
113 IMAXSQ INTEGER 0 INDEX INTEGER
141 IROW INTEGER 136 IROWS INTEGER
125 ISS INTEGER 123 ITEST INTEGER
127 ITS INTEGER 134 K INTEGER
135 KSEC INTEGER 144 KSTEP INTEGER
140 KTES INTEGER 133 Kzon INTEGER
137 LEFT INTEGER 0 M INTEGER
117 MLIFF INTEGER C MURD INTEGER
120 MSEC INTEGER 122 MTES INTEGER
121 MZON INTEGER 115 NISESC INTEGER
116 NISZON INTEGER 111 NMSEC INTEGER
143 NOMMTX INTEGER 142 NOMTP INTEGER
126 NSS INTEGER 130 NSSFC INTEGER

RD NR. SEVERITY DETAILS DIAGNOSIS OF PROBLEM
SUBROUTINE SUBS3 (INDEX,NFUNC,NDEG,MORD)

COMMENT -- PARTICULAR COEFFICIENTS OF INTEREST ARE SPECIFIED AS CARD

C    INPUT TO BE READ FROM THIS SUBROUTINE. THE NUMBER OF
C    FUNCTIONS, NFUNC, TO BE READ IS A FORMAL PARAMETER OF THIS
C
SUBROUTINE AND DEFINES THE NUMBER OF ELEMENTS CONTAINED
C    IN THE ARRAY INDEX.

DIMENSION INDEX(NFUNC)

DO 100 I=1,NFUNC

100 READ (5,915) LORD,LDEG,LEO

COMMENT --

C    LORD -- ORDER OF FUNCTION.
C    LDEG -- DEGREE OF FUNCTION.
C    LEO -- =0, FUNCTION IS EVEN.
C    =1, FUNCTION IS ODD.

IF (LORD.EQ.0) GO TO 25
IF (LORD.EQ.LDEG) GO TO 50

K=(NDEG+1)+2*MORD+LEO+LORD*(2*LDEG-LORD-1)-1
GO TO 75

25 CONTINUE

K=LDEG+1
GO TO 75

30 CONTINUE

K=(NDEG+1)+2*LORD-1+LEO
75 CONTINUE

PRINT 925,1,LORD,LDEG,LEO,K
INDEX(1)=K
100 CONTINUE
RETURN

915 FORMAT (12,12,11)
925 FORMAT (1X,*FUNCTION NUMBER *,*14,* HAS BEEN SPECIFIED AS ORDER= *,

112,** DEGREE= *,*12,* AND LEO= *,*11,* K= *,*74)
END

SYMBOLIC REFERENCE MAP (R=1)

NTKY POINTS
3 SUBS3
SUBROUTINE PRSYM1 (S,NV,NFUNC,ICODE)

COMMENT -- S IS A VECTOR WHICH CONTAINS THE UPPER FULL TRIANGLE OF
   SOME SYMMETRIC MATRIX Z.  S IS PACKED AS FOLLOWS,

   K=0

   DD 20 J=1,NFUNC
   DD 10 I=1,J
   K=K+1
   S(K)=Z(I,J)

   10 CONTINUE

WHERE NFUNC IS THE ORDER OF THE MATRIX Z.

PRSYM1 WILL EITHER,

A. PRINT THE VECTOR S AS THE UPPER FULL TRIANGLE OF Z,
   ICODE=0, OK,

B. PRINT THE CORRELATION FORM OF Z(SAME FORMAT AS ABOVE)
   FOR ICODE=1.

NFUNC AS DEFINED IN GLSRAN2 IS THE NUMBER OF COLUMNS IN THE
  Z-MATRIX.

MCOL = NUMBER OF COLUMNS/LINE OF PRINT AND MUST BE IN
  AGREEMENT WITH FORMAT STATEMENT 900 AND 950.

THE ARRAY A DIMENSIONED AS A(MCOL) IS USED FOR PRINTING
  SUCH THAT S WILL NOT BE DESTROYED WHEN ICODE=1.

Z IS DIVIDED INTO IR SECTIONS FOR PRINTING.

JIDX = J INDEX OF FIRST COLUMN
  FOR A PARTICULAR SECTION, KSEC.
JNDX = J INDEX OF LAST COLUMN
  FOR A PARTICULAR SECTION, KSEC.
JIDX ALSO = I INDEX OF LAST ROW
  FOR A PARTICULAR SECTION, KSEC.
KSEC = NUMBER OF SECTION BEING PRINTED.

DIMENSION S(NV),A(12)
MCOL=12
JIDX=0
IKATIO=NFUNC/MCOL
IP=IKATIO+1
ICIFF=NFUNC-IKATIO*MCOL
IF (IDIFF .NE. 0) GO TO 25
45
IF = IRATIO
IDIFF = MCOL
25 CONTINUE
DL 150 KSEC = 1, IR
JLX = JNDX + 1
JNX = JNDX + MCOL
IF (KSEC .EQ. IR) JNDX = (JNDX - MCOL) + IDIFF
JEND = JNDX = (KSEC - 1) * MCOL
PRINT 950, (J, J = JIDX, JNDX)
50
DL 100 I = 1, JNDX
DL 30 J = JIDX, JNDX
55
J1 = J - (KSEC - 1) * MCOL
A(J1) = 0.
30 CONTINUE
IF (I.LT.IIDX) JIDX = JIDX + 1
DO 50 J = JIDX, JNDX
60
J1 = J - (KSEC - 1) * MCOL
K1 = (J*(J-1))/2 + 1
IF (ICUDE .EQ. 0) GO TO 35
K2 = (J*(J-1))/2
65
A(J1) - S(K)/SQRK(S(K1)*S(K2))
GO TO 50
35 CONTINUE
A(J1) = S(K)
50 CONTINUE
PRINT 900, I, (A(J1), J1 = 1, JEND)
70
100 CONTINUE
RETURN
75
FORMAT (T4, I3, 12(E10.3))
950
FORMAT (/T11, 12(I3, 7X) /)
END
Appendix H - FOURIER SERIES REPRESENTATION OF A DISCRETE DATA SET

The Fourier series approximation takes the form

\[ g(x) = A_0 + \sum_{\ell=1}^{q} [A_{\ell} \cos(\ell x) + B_{\ell} \sin(\ell x)], \quad (H-1) \]

where \( g(x) \) is periodic over \( 2\pi \) and \( q \leq Q \). If now \( f(x) \) is a function for \( 2Q + 1 \) discrete equally spaced values of \( x \) over the same period as \( g(x) \) above, a set of Fourier coefficients may be found that satisfies equation (H-1) by using the least-squares criterion. Let \( \varepsilon_r \) be the error associated with the \( r \)th value of \( x \), then

\[ \varepsilon_r = f(x_r) - g(x_r), \]

and

\[ \varepsilon_r^2 = [f(x_r) - A_0 - \sum_{\ell=1}^{q} (A_{\ell} \cos(\ell x_r) + B_{\ell} \sin(\ell x_r))]^2. \quad (H-2) \]

The least-squares technique as discussed earlier in this report requires that

\[ \sum_r [f(x_r) - A_0 - \sum_{\ell=1}^{q} (A_{\ell} \cos(\ell x_r) + B_{\ell} \sin(\ell x_r))]^2 \]

be minimized. This leads to

\[ A_0 = \frac{1}{2Q} \sum_{r=-Q+1}^{Q} f(x_r), \quad (H-3a) \]

\[ A_{\ell} = \frac{1}{Q} \sum_{r=-Q+1}^{Q} f(x_r) \cos(\ell x_r), \quad (H-3b) \]

for \( \ell \neq 0, Q \),

\[ A_Q = \frac{1}{2Q} \sum_{r=-Q+1}^{Q} f(x_r) \cos(Q x_r), \quad (H-3c) \]
and

\[ B_{Q} = \frac{1}{Q} \sum_{r=-Q+1}^{Q} f(x_r) \sin(lx_r). \]  

Equations (H-3) may be rewritten as

\[ A_0 = \frac{1}{Q} \left[ \frac{1}{2} H_0 + H_1 + H_2 + \ldots + H_{Q-1} + H_Q \right], \]  

\[ A_{Q} = \frac{1}{Q} \left[ \frac{1}{2} H_0 - H_1 + H_2 - \ldots + (-1)^{Q-1} \right] H_{Q-1}, \]  

\[ A_{Q} = \frac{1}{Q} \left[ \frac{1}{2} H_0 - H_1 + H_2 - \ldots + \right] \]  

\[ B_{Q} = \frac{2}{Q} \left[ G_1 \sin(lx_1) + G_2 \sin(lx_2) + \ldots + G_{Q-1} \sin(lx_{Q-1}) \right], \]  

where

\[ H(x) = \frac{1}{2} \left[ f(x) + f(-x) \right], \]  

\[ G(x) = \frac{1}{2} \left[ f(x) - f(-x) \right]. \]  

Equations (H-4) and (H-5) then give the required Fourier coefficients which will satisfy equation (H-1).

Since \( f(x) \) is periodic over \( 2\pi \),

\[ f(x) = f(-x), \]  

so that there are \( 2Q \) independent pieces of data. Then, as the objective of this Fourier series representation is data interpolation, rather than say "smoothing", the \( q \) in equation (H-1) takes on the value \( Q \) so that all \( 2Q \) possible terms are used. Equation (H-1) may now be written as

\[ f(x) = A_0 + \sum_{\ell=1}^{Q} \left[ A_{\ell} \cos(\ell x) + B_{\ell} \sin(\ell x) \right]. \]
APPENDIX I - THE OZSTAT2 PROGRAM

This appendix contains a listing of the OZSTAT2 program and its subroutines.

The OZSTAT2 program has three basic capabilities:

a. Data Grouping
b. Statistical Analysis
c. Computer Graphics

The program is designed to read the BUV data formatted as described in Appendix B and to group the data into a global grid system. Based on this grid system means and variances are calculated for individual grid blocks, latitudinal zones, and for that part of the grid system that contains data. These calculations are described in more detail in section 3.

Graphics capabilities included in the OZSTAT2 program provide for each case a plot of the zonal means with ±1 sigma error bars, a scatter diagram of the ozone distribution as a function of latitude, and histograms of the data sampling distribution as a function of latitude or longitude.
PROGRAM OZSTAT2

COMMON/ DD/ DATE

DIMENSION A(4), K(5), ALAT(100), AOZ(100), DATE(2)

DIMENSION SUMXQ(36, 24), KK(36, 24), GP(36, 24), SUMX(36, 24)

DIMENSION KCOUNT(36, 24), SUMGPSQ(36, 24), VARGP(36, 24)

DIMENSION ILAT(36), RLAT(38), RKK(36, 24), R LATBND(38), NAME(36)

DIMENSION RLNBK(26), RK(26)

DIMENSION VARLATB(36), AVGLATB(36), RAT(36), STDEVP(36), STDEVN(36)

DIMENSION XDATA(6)

DIMENSION SUMT(36, 24), SUMTSQ(36, 24), SUMLT(36, 24), S UMLG(36, 24)

DATA NAME/'3HI-1, 3HI-2, 3HI-3, 3HI-4, 3HI-5, 3HI-6, 3HI-7, 3HI-8, 3HI-9, 4HOZSTAT2'

II = 10, 4HI = 11, 4HI = 12, 4HI = 13, 4HI = 14, 4HI = 15, 4HI = 16, 4HI = 17, 4HI = 18/

SET UP PLOT VECTOR FILE

SUBROUTINE PSEUDO (FN), FN = FILENAME, CAN BE FOUND IN SECTION 1.4.1 OF THE GRAPHICS MANUAL

IF LEROY IS NOT SPECIFIED, LIQUID INK PEN, BALL POINT PEN IS AUTOMATICALLY CALLED, IF REQUIRED, BY DEFAULT.

LEROY IS ONLY USED FOR THE CALCOMP POSTPROCESSOR.

FIRST FRAME MUST CONTAIN AT LEAST FIVE PLOT VECTORS WHEN USING CALCOMP.

CALL PSEUDO(6LMYSAV1)

CALL LEROY

DO 10 I = 1, 6

CALL CALPLT (0., 0, -3)

* * * * * * * *

INITIALIZE PARAMETERS FOR SUBROUTINE SELECT OPTIONS

IF JSELECT = 0 DO GRID BLOCKS ONLY

IF JSELECT = 1 DO GRID POINTS ONLY

IF JSELECT = 2 DO BOTH

JSELECT = 1

JSELECT = 2

MSELECT = 0, SKIPS BOTH PLOT ROUTINES

MSELECT = 1, CALL AVARPLT ONLY

MSELECT = 2, CALL HISTPLT ONLY

MSELECT = 3, CALLS BOTH

MSELECT = 0

MSELECT = 1

MSELECT = 2

MSELECT = 3
C  *  *  *  *  *  *  *  *

C INITIALIZE PARAMETERS FOR SELECTING GRID BLOCK SIZE

45 C ISIZE IS THE LATITUDE DIMENSION IN DEGREES
C JSIZE IS THE LONGITUDE DIMENSION IN DEGREES
C ISIZE=5
C JSIZE=15
C NLAT IS THE NUMBER OF ISIZE LATITUDE ZONES
C NLONG IS THE NUMBER OF JSIZE LONGITUDE BANDS
C DIMENSION STATEMENTS MUST BE ADJUSTED FOR EACH RUN ACCORDING
C TO NLAT AND NLONG.
C NLAT=180/ISIZE
C NLONG=360/JSIZE
C PI=ACOS(-1.)
C NCALC IS THE NUMBER OF TIME PERIODS OVER WHICH CALCULATIONS
C WILL BE EXECUTED.
C NCALC=8
C NCALC=1

60 C DO 200 L=1,NCALC
C READ (5,225) K4,NDAY,DATE
C FROM TIME INTERVAL NCALC+1 MUST BE GREATER THAN NDAY
C FROM TIME INTERVAL NCALC
C ************************************************************
C ********** TOTAL OZONE DATA IS AVERAGED. MEAN AND VARIANCE ARE PUT INTO
C PARTICULAR ELEMENTS OF AN NLAT X NLONG GRID SYSTEM. ***
C ******************************************************
C 70 C IN THE FOLLOWING STATEMENTS, CERTAIN PARAMETERS ARE INITIALIZED
C IJ IS USED AT STATEMENT 57
C IJ=-1
C IEOF IS USED AT STATEMENT 21. IEOF=1 INDICATES THAT THE
C END OF FILE HAS BEEN REACHED.
C IEOF=0
C IDAY=K4
C IISUM=0
C GSUM=0.
C DO 15 I=1,NLAT
C DO 15 J=1,NLONG
C KK(I,J)=0
C SUMXSQ(I,J)=0.
C SUMX(I,J)=0.
C SUMT(I,J)=0.

C OCT79
PROGRAM OZSTAT2 74/74 OPT=1

CONTINUE

85 SUMTSQ(I,J)=0.
SUML(I,J)=0.
SUMLG(I,J)=0.
SUMLTSQ(I,J)=0.
SUMLGO(I,J)=0.

90 15 CONTINUE
**********

C IF(MSELECT.EQ.0).OR.(MSELECT.EQ.2)) GO TO 18
C CALL AVARPLT TO CONSTRUCT AXES AND LABELS FOR SCATTER DIAGRAM,
C MEAN CURVE AND STANDARD DEVIATION PLOT. DEFINE M AS ANYTHING.
M=100
C CALL AVARPLT (VARLATB,AVGLATB,ALAT,AOZ,RAT,STDEVP,STDEVN,NLAT)

18 CONTINUE
DO 35 M=1,100

20 READ(1) (XDATA(J),J=1,6)
IF (EOF(1)) 21,22

21 1EOF=1
GO TO 23

35 CONTINUE

K(4)=XDATA(2)
K(4)=(XDATA(1)-1970.)*365.+K(4)
10R. XDATA(1).EQ.1976.) K(4)=K(4)+1
IF (XDATA(1).EQ.1977.) K(4)=K(4)+2
RK5=XDATA(3)/3600.
K(5)=RK5
A(1)=(RK5-K(5))*60.
A(2)=XDATA(4)
A(3)=XDATA(5)
A(4)=ABS(XDATA(6))
IF (A(4).EQ.999. OR. A(4).EQ.77.) GO TO 20
K(4) IS THE DAY NUMBER **********

115 IF (A(4).LT.0.200. OR. A(4).GT.0.65) PRINT 905, XDATA

C OCT79 8
OCT79 9
OCT79 10
OCT79 11
OCT79 12
OCT79 82
OCT79 83
OCT79 84
OCT79 85
OCT79 86
OCT79 87
OCT79 88
OCT79 89
OCT79 90
OCT79 91
OCT79 13
OCT79 93
OCT79 21
OCT79 94
OCT79 95
OCT79 96
OCT79 14
OCT79 15
OCT79 16
OCT79 17
OCT79 18
OCT79 19
OCT79 20
OCT79 21
OCT79 22
OCT79 23
OCT79 24
OCT79 25
OCT79 97
OCT79 26
OCT79 27
OCT79 98
OCT79 99
OCT79 100
OCT79 101
OCT79 102
OCT79 103
OCT79 104

PRINT 905, XDATA

905 FORMAT (*0*,T10,6(E15.8,5X))

120 IF (K(4).LT.IDAY) GO TO 20
IF (K(4).LE.NDAY) GO TO 30
BACKSPACE 1

23 IF (M.EQ.1) GO TO 25
IF((MSELECT.EQ.0).OR.(MSELECT.EQ.2)) GO TO 25

125 CALL SCAT (VARLATB,AVGLATB,ALAT,AOZ,M-1,RAT,STDEVP,STDEVN,NLAT)

25 IF (IEOF.EQ.1) GO TO 75
GO TO 50
C
A(2) IS THE LATITUDE
C
30 ALAT(M)=A(2)
130 AOZ(M)=A(4)
K4=K(4)
C *******************************************************************************
C DATA RECORDS FOR WHICH THE LATITUDE = 0 DEGREES WILL BE ASSIGNED
C TO THE northern hemisphere. However, it is believed that no
C SUCH RECORDS OCCUR BETWEEN DAYS 101 AND 465.
C *******************************************************************************
C LAT=ABS(A(2))
I=LAT/ISIZE+1
IF (A(2).LT.0) I=I+NLAT/2
140 C
A(3) IS THE LONGITUDE
LONG=A(3)
J=LONG/JSIZE+1
KK(I,J)=KK(I,J)+1
XSQ=A(4)*A(4)
SUMXSQ(I,J)=SUMXSQ(I,J)+XSQ
SUMX(I,J)=SUMX(I,J)+A(4)
TH=K(4)+XDATA(3)/86400.
TMSQ=TM*TM
SUMTSQ(I,J)=SUMTSQ(I,J)+TMSQ
SUMT(I,J)=SUMT(I,J)+TH
SUMLT(I,J)=SUMLT(I,J)+A(2)
SUMLG(I,J)=SUMLG(I,J)+A(3)
SUMLTSQ(I,J)=SUMLTSQ(I,J)+A(2)*A(2)
SUMLGSQ(I,J)=SUMLGSQ(I,J)+A(3)*A(3)
150 CONTINUE
IF((MSLECT.EQ.0).OR.(MSLECT.EQ.2)) GO TO 18
CALL SCAT (VARLATB,AVGLATB,ALAT,AOZ,100,RAT,STDEVST,STDEVT,NLAT)
155 GO TO 18
C *******************************************************************************
C *********************************************************************************************
160 C
BEGIN STATISTICS CALCULATIONS FOR GRID SYSTEMS.
C 50 IF (JSELECT.EQ.1) GO TO 97
PRINT 115, NLAT,ISIZE,JSIZE,NLAT/2,NLAT/2+1,NLAT,NLONG
PRINT 116, DATE
165 PRINT 117, L
GVAR=0.
KBLK=0
DO 65 I=1,NLAT
PROGRAM OZSTAT2 74/74 DPT=1

170     SUM=0.
        ISUM=0.
        SSQLATB=0.
        TSUM=0.
        TSQLATB=0.
        PRINT 120

175     C

        ****************************************************
        DO 55 J=1,NLONG
        IF (KK(I,J).EQ.0) GO TO 51
        AX=SUMX(I,J)/KK(I,J)
        AT=SUMT(I,J)/KK(I,J)
        AVLAT=SUMLAT(I,J)/KK(I,J)
        AVLONG=SUMLONG(I,J)/KK(I,J)
        IF (KK(I,J).EQ.0.1) GO TO 52
        VARX=(SUMXSQ(I,J)-KK(I,J)*AX*AX)/(KK(I,J)-1.)
        VART=(SUMTSQ(I,J)-KK(I,J)*AT*AT)/(KK(I,J)-1.)
        SUM=SUM+SUMX(I,J)
        TSUM=TSUM+SUMT(I,J)
        GSUH=GSUH+SUMX(I,J)
        ISUM=ISUM+KK(I,J)
        SSQLATB=SSQLATB+SUMXSQ(I,J)
        TSQLATB=TSQLATB+SUMTSQ(I,J)
        IF (KK(I,J).EQ.0.1) GO TO 55

185     GO TO 53

51     AX=0.
        AT=0.
        AVLAT=0.
        AVLONG=0.

190     52     VARX=0.
        VART=0.
        CONTINUE

195     C SUM IS ACCUMULATIVE OZONE CONTENT/LATITUDE BAND
        OZSTAT2 144
        SUM=SUM+SUMX(I,J)
        OZSTAT2 145
        TSUM=TSUM+SUMT(I,J)
        OZSTAT2 146
        GSUH=GSUH+SUMX(I,J)
        OCT79 39
        ISUM=ISUM+KK(I,J)
        OCT79 40
        SSQLATB=SSQLATB+SUMXSQ(I,J)
        OCT79 41
        TSQLATB=TSQLATB+SUMTSQ(I,J)
        OCT79 42
        PRINT 100, AX, VARX, KK(I,J), I, J, AVLAT, AVLONG, AT, VART
        OCT79 43
        IF (KK(I,J).EQ.0.1) GO TO 55

200     KBLK=KBLK+1

205     WRITE (10) (90.-AVLAT)*PI/180., AVLONG*PI/180., AX*1000.

210     C * * * * * * * * * * * *
PROGRAM OZSTAT2 74/74 OPT=1

* GRID BLOCK DATA FOR NCALC WEEKS IS STORED IN ZDATA.
* NUMBER OF DATA POINTS/GRID BLOCK FOR NCALC WEEKS
* IS STORED IN NDATA.

* ************
* IISUM IS THE TOTAL NUMBER OF OZONE RECORDS USED
* IN THESE CALCULATIONS.
* IISUM=IISUM+ISUM
* ************

C IF (I.GE.NLAT/2+1) GO TO 57
ILAT(I+NLAT/2)-IISUM
GO TO 58
C
C IJ IS INITIALIZED AS -1.
IJ=IJ+2
ILAT(I-IJ)-IISUM
C
C ************
* IF (IISUM.EQ.0) GO TO 60
AX=SUM/IISUM
AT=TSUM/IISUM
GVAR=GVAR+SQSLATB
IF (IISUM.EQ.1) GO TO 61
VAX=(SQSLATB-AX*AX*IISUM)/(IISUM-1.)
VART=(TSQSLATB-AT*AT*IISUM)/(IISUM-1.)
GO TO 64

C
C
C
C 60 AX=0.
AT=0.

C 61 VAX=0.
VART=0.

C 64 VARLATB(I)=VAX
AVGLATB(I)=AX

C
C
C
C 65 PRINT 110, I, AX, IISUM, VAX, AT, VART
PROGRAM DZSTAT1  74/74  OPT=1  FTN 4.7+485  80/01/23. 14:08:48

C *  *  *  *  *  *  *  *  *          OCT79  73
C WRITE(10) KK          OCT79  74
C WRITE(10) XLAT          OCT79  75
C WRITE(10) XLONG          OCT79  76
C WRITE(10) XOZ          OCT79  77
C *  *  *  *  *  *  *  *  *          OCT79  78
C *  *  *  *  *  *  *  *  *          OCT79  79

260 C***********************************************************************
C
PRINT 125, IISUM, IDAY, K4          OCT79  80
GAV=GSUM/IISUM          OCT79  81
PRINT 130, GAV          OCT79  82

265 GVAR=(GVAR-GAV*GAV/IISUM)/(IISUM-1.)          OCT79  83
PRINT 135, GVAR, SQRT(GVAR)          OCT79  84

135 FORMAT (1X,*GLOBAL VARIANCE= *,E15.8/
11X,*STANDARD DEVIATION= *,E15.8)          OCT79  85
PRINT 925, KBLK          OCT79  86

270 925 FORMAT (*0**,A TOTAL OF *,I6**, BLOCKS ARE FILLED,*
C ***********************************************************************
C
IF (MSELECT.EQ.0) GO TO 71          OCT79  88
IF (MSELECT.EQ.2) GO TO 70          OCT79  89
CALL ASTD (VARLATB, AVGLATB, ALAT, ADZ, M, RAT, STDEVPS, STDEVN, NLAT)          OCT79  90
IF (MSELECT.EQ.1) GO TO 71          OCT79  91

70 CALL HISTPT (ILAT, KK, NLAT, NLONG, RLAT, RKK, RLAGTH, RLNGBK, RK, NAME,
1NLAT+2, NLONG+2)          OCT79  92
71 CONTINUE          OCT79  93

C***********************************************************************
C
IF (JSELECT.EQ.2) GO TO 97          OCT79  94
GO TO 200          OCT79  95

75 PRINT 145, K(4)
IF (K(4).EQ.173) GO TO 50
PRINT 160          OCT79  98

285 STOP 1          OCT79  99

97 CALL GRID (SUMX, SUMXSQ, KK, GP, NLAT, NLONG, KCOUNT, SUMGPSQ, VARGP)          OCT79  100
C***********************************************************************
C
C***********************************************************************
C
C***********************************************************************
C
290 CONTINUE          OCT79  103

200 CONTINUE          OCT79  104
CALL NFRAME          OCT79  105
CALL CALPLT (0., 0., 999)
STOP          OCT79  106

OCT79  107
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OCT79  237
**PROGRAM OZSTAT2**

**74/74**  **OPT=1**  **FTN 4.7+485**  **80/01/23. 14.08.48**

### 100 FORMAT (1X,2(E16.8,5X),I3,5X,2(I2,5X),T70,F7.3,T80,F8.3,T105,F8.3,OCT79)  87
1T119,E15.8) OCT79  88

110 FORMAT (*O*,*THE AVERAGE OZONE DENSITY FOR THE LATITUDE BAND*/IX,*OZSTAT2  239
ICORRESPONDING TO */I2,* IS */E16.8,* THIS CALCULATION IS BASED OZSTAT2  240
2DN */I6,* RECORDS OF DATA*/IX,* THE VARIANCE OF THE MEAN IS */E16.8OZSTAT2  241
1/IX,*TIME AVERAGE= */F8.3,10X,*TIME VARIANCE= */E15.8) OCT79  89

115 FORMAT (*I*,*IN THE FOLLOWING */,I2,* TABLES, OZONE DENSITY HAS BEEN OZSTAT2  243
1N AVERAGED BY GRID BLOCKS */,IX,* THESE BLOCKS REPRESENT AN AREA ON OZSTAT2  244
2 THE EARTH'S SURFACE THAT IS */,I2,* DEGREES LATITUDE BY */,I2,* DEG0ZSTAT2  245
3EES LONGITUDE */,IX,* LATITUDE BANDS CORRESPONDING TO LATITUDE INDOZSTAT2  246
4CES 1 THROUGH */,I2,* ARE IN THE NORTHERN HEMISPHERE */,IX,* WHILE LAT0ZSTAT2  247
5ITUDE INDICES */,I2,* THROUGH */,I2,* ARE IN THE SOUTHERN HEMISPHERE OZSTAT2  248
6. LONGITUDE INDICES (1- */,I2,* RANGE FROM */,IX,* THE GREENWICH HERIOZSTAT2  249
7DIA WESTWARD THROUGH 360 DEGREES.*) OZSTAT2  250

116 FORMAT (1X,*THESE CALCULATIONS ARE FOR THE TIME PERIOD */,A10,A4) OZSTAT2  251
117 FORMAT (11X,*AND ARE FOR CASE NUMBER */,I2) OCT79  90

120 FORMAT (*O*,T8,*,MEAN*,T27,*,VARIANCE*,T43,*,K(I,J)*,T53,*,I*,T60,*,J*,OZSTAT2  252
1T70,*,MEAN LATITUDE*,T88,*,MEAN LONGITUDE*,T105,*,AVERAGE TIME*,  OZ79  91
2T119,*,VARIANCE*) OCT79  92

125 FORMAT (*O*,I7,* RECORDS OF DATA ARE USED IN THE ABOVE CALCULATION OZSTAT2  254
15* */,IX,* THIS DATA INCLUDES RECORDS FROM DAYS */,I4,* THROUGH */,I4,*,OCT79  93
2 INCLUSIVE.*) OZSTAT2  256

130 FORMAT (*O*,*THE GLOBAL OZONE LAYER THICKNESS AVERAGE FOR THIS TIMOZSTAT2  257
1E PERIOD IS */,E15.8) OZSTAT2  258

145 FORMAT (*O*,*REACHED END OF FILE. DAY NUMBER = */,I5) OCT79  94

160 FORMAT (////*,REACHED EOF PRIOR TO LAST DAY ON FILE*) OZSTAT2  260
225 FORMAT (2I4,A10,A4) OCT79  95

END

**ID NR. SEVERITY DETAILS DIAGNOSIS OF PROBLEM**

13 I NAME DATA VARIABLE LIST EXCEEDS ITEM LIST, EXCESS VARIABLES NOT INITIALIZED.

**SYMBOLIC REFERENCE MAP (R=1)**
SUBROUTINE GRID

COMMON/DO/DATE
DIMENSION S(NLAT,NLONG),K(NLAT,NLONG),GP(NLAT,NLONG),KCOUNT(NLAT,NGRID)
DIMENSION DATE (2)
M=NLAT/2-1
M1=NLAT/2+1
M2=NLAT/2
M3=NLAT/2+2

PRINT 105, NLAT,M2,M1,M3,NLAT,NLONG
PRINT 110, DATE

C GRID POINTS IN THE NORTHERN HEMISPHERE ARE CALCULATED BELOW
DO 25 I=1,M
DO 25 J=1,NLONG
L=J+1
IF (L.EQ.NLONG+1) L=1
N=I+1
SUMGPSQ (I,J)=SS(I,J)+SS(I,L)+SS(N,J)+SS(N,L)
KCOUNT(I,J)=K(I,J)+K(I,L)+K(N,J)+K(N,L)
IF (KCOUNT(I,J).EQ.0) GO TO 20
IF (KCOUNT(I,J).EQ.1) GO TO 21
VARGP (I,J)=(SUMGPSQ(I,J)-KCOUNT(I,J)*GP(I,J)*GP(I,J))/(KCOUNT(I,J)-1.)
1) GO TO 25
20 GP(I,J)=0.
21 VARGP (I,J) = 0.
25 CONTINUE

C GRID POINTS ALONG THE EQUATOR ARE CALCULATED BELOW
DO 30 J=1,NLONG
L=J+1
IF (L.EQ.NLONG+1) L=1
SUMGPSQ(M1,J)=SS(1,J)+SS(1,L)+SS(M1,J)+SS(M1,L)
KCOUNT (M1,J)=K(1,J)+K(1,L)+K(M1,J)+K(M1,L)
IF (KCOUNT(M1,J).EQ.0) GO TO 27
GP(M1,J)=(S(1,J)+S(1,L)+S(M1,J)+S(M1,L))/KCOUNT(M1,J)
IF (KCOUNT(M1,J).EQ.1) GO TO 28
VARGP (M1,J)=(SUMGPSQ(M1,J)-KCOUNT(M1,J)*GP(M1,J)*GP(M1,J))/(KCOUNT(M1,J)-1.)
IT(M1,J)=1.
30 CONTINUE
SUBROUTINE GRID

27 GP (M1, J) = 0.
28 VARGP (M1, J) = 0.
29 GP (M2, J) = 0.
30 CONTINUE

C ************************************************************************************
31 GRID POINTS IN THE SOUTHERN HEMISPHERE ARE CALCULATED BELOW
32 DO 35 I = M3, NLAT
33 DO 35 J = 1, NLONG
34 L = J + 1
35 IF (L .EQ. NLONG + 1) L = 1
36 N = I - 1
37 SUMGPSQ (I, J) = SS(I, J) + SS(I, L) + SS(N, J) + SS(N, L)
38 KCOUNT(I, J) = K(N, J) + K(N, L) + K(I, J) + K(I, L)
39 IF (KCOUNT(I, J) .EQ. 0) GO TO 33
41 IF (KCOUNT(I, J) .EQ. 1) GO TO 34
42 VARGP (I, J) = (SUMGPSQ(I, J) - KCOUNT(I, J) * GP(I, J) * GP(I, J)) / (KCOUNT(I, J) - 1)**2
43 GO TO 35
44 33 GP (I, J) = 0.
45 VARGP (I, J) = 0.
46 CONTINUE

C ************************************************************************************
47 KC IS A COUNTER FOR THE TOTAL NUMBER OF DATA POINTS USED IN THESE
48 CALCULATIONS
49 OZDEN IS AN ACCUMULATIVE SUM OF TOTAL OZONE DENSITY, SUMMED IN A
50 PARTICULAR LATITUDE BAND.
51 KC1 IS A COUNTER FOR THE NUMBER OF DATA POINTS USED IN THE
52 CALCULATIONS FOR A PARTICULAR LATITUDE BAND.
53 ************************************************************************************
54 KC = 0
55 DO 55 I = 1, NLAT
56 OZDEN = 0.
57 KC1 = 0
58 PRINT 100
59 DO 50 J = 1, NLONG
60 KC = KC + KCOUNT(I, J)
61 KC1 = KC1 + KCOUNT(I, J)
62 OZDEN = OZDEN + GP(I, J) * KCOUNT(I, J)
63 50 PRINT 125, GP(I, J), KCOUNT (I, J), I, J, VARGP (I, J)
64 55 OZDEN = OZDEN + GP(I, J) * KCOUNT(I, J)
SUBROUTINE GRID 74/74 OPT=1

AVDEN=OZDEN/KC1
PRINT 150, I,AVDEN,KC1
PRINT 175,KC
100 FORMAT (*0*,T4,*MEAN OZONE DENSITY*,T27,*VARIANCE*,T43,*KCOUNT*,T5GRID
13,*I*,T60,*J*)
105 FORMAT (*1**,IN THE FOLLOWING *,I2,** TABLES, OZONE DENSITY HAS BEEN
1IN AVERAGED BY GRID POINTS. LATITUDE INDICES 1 THROUGH *,I2** GRID
2/1X,*CORRESPOND TO THE NORTHERN HEMISPHERE, LATITUDE INDEX *,I2,* GRID
3** CORRESPONDS TO THE EQUATOR, AND LATITUDE INDICES *,I2,** THROUGH *,GRID
412/1X,*CORRESPOND TO THE SOUTHERN HEMISPHERE. LONGITUDE INDICES AGRID
5RE SIMILAR TO THOSE ABOVE WITH J= *,I2,** CORRESPONDING TO LONGITUDGRID
6E**0*/1X,*DEGREES OR THE MERIDIAN THROUGH GREENWICH*)
110 FORMAT (1X,*THESE CALCULATIONS ARE FOR THE TIME PERIOD *,A10,A4) GRID
125 FORMAT (1X,E16.8,T44,I4,T53,12,T60,I2,T25,E16.8) GRID
150 FORMAT (*0*,*THE AVERAGE OZONE DENSITY CORRESPONDING TO LATITUDE IGRID
1INDEX *,I2,* IS *,E16.8,**. THIS MEAN IS BASED ON *,I6,* DATA POINTGRID
100
175 FORMAT (*0*,I7,** DATA RECORDS ARE USED IN THE ABOVE CALCULATIONS.*GRID
1)
RETURN
END

SYMBOLIC REFERENCE MAP (R=1)

TRY POINTS
3 GRID

RIABLES SN TYPE RELLOCATION
500 AVDEN REAL
0 GP REAL ARRAY F.P.
472 J INTEGER
475 KC INTEGER
477 KC1 INTEGER
465 M INTEGER
467 M2 INTEGER
474 N INTEGER
0 NLONG INTEGER F.P.
0 S REAL ARRAY F.P.
0 DATE REAL ARRAY DD
SUBROUTINE HISTPLT (ILAT, KK, NLAT, NLONG, RLAT, RK, RLATBND, RLNGBK, RK, HISTPLT)

COMMON/DD/DATE

DIMENSION ILAT(NLAT), RLAT(K), KK(NLAT, NLONG), RK(NLAT, NLONG)

DIMENSION RLATBND(K), RLNGBK(L), RK(L), NAME(NLAT)

DIMENSION DAT(2)

K1 = K - 1 $ L1 = L - 1

ICOUNT = 0 $ M = 17

NLAT1 = 0

RLATBND(K1) = 0.

RLATBND(K) = 1.

RLAT(K1) = 0.

RLAT(K) = 1.

RLNGBK(L1) = 0.

RLNGBK(L) = 2.

RK(L1) = 0.

RK(L) = 0.1

ILAT CONTAINS THE NUMBER OF DATA POINTS/LATITUDE BAND

KK CONTAINS THE NUMBER OF DATA POINTS/GRID BLOCK

FIND MAX VALUES OF ILAT AND KK, ILATMAX AND KKMAX, RESPECTIVELY

ILATMAX = 0 $ KKMAX = 0

DO 15 I = 1, NLAT

IF (ILAT(I) .GT. ILATMAX) ILATMAX = ILAT(I)

15 CONTINUE

PRINT 101

PRINT 100, ILAT

PRINT 103, DATE

PRINT 104

PRINT 106, DATE

IF (NLAT .LE. 18) GO TO 21

ICOUNT = ICOUNT + 1

M = M + 18

NLAT1 = NLAT1 + 18

PRINT 105, (I = I, NLAT1)

DO 20 J = 1, NLONG

PRINT 110, J, (KK(I, J), I = M, NLAT1)

20 IF (NLAT - ICOUNT .GE. 18) GO TO 19

M = M + 18

IF (M .LT. 0) M = 1
SUBROUTINE HISTPLT 74/74 OPT-1

PRINT 105, (I, I=M, NLAT)
DO 22 J=1, NLONG
22 PRINT 110, J, (KK(I, J), I=M, NLAT)
C FIND NORMALIZED VALUES OF ILAT AND KK
DO 25 I=1, NLAT
C RLAT IS NORMALIZED VALUE OF ILAT
RLAT(I)=ILAT(I)/FLOAT(ILATMAX)
DO 25 J=1, NLONG
C RKK IS NORMALIZED VALUE OF KK
RKK(I, J)=KK(I, J)/FLOAT(KKMAX)
CONTINUE
PRINT 125, ILATMAX, KKMAX
PRINT 126
DO 26 I=1, NLAT
PRINT 127, RLAT(I)
PRINT 103, DATE
PRINT 128
PRINT 106, DATE
ICOUNT=0
M=-17
NLAT1=0
IF (NLAT.LE.18) GO TO 31
29 ICOUNT=ICOUNT+1
M=M+18
NLAT1=NLAT1+18
PRINT 105, (I, I=M, NLAT1)
DO 30 J=1, NLONG
70 PRINT 135, J, (RKK(I, J), I=M, NLAT1)
IF (NLAT-ICOUNT*18.GT.18) GO TO 29
M=M+18
31 IF (M.LT.0) M=1
PRINT 105, (I, I=M, NLAT)
DO 32 J=1, NLONG
32 PRINT 135, J, (RKK(I, J), I=M, NLAT)
XL=9.
YL=5.
C FOR PROPER SCALING MULTIPLY RLAT(I) BY THE LENGTH OF THE Y-AXIS
80 DO 35 I=1, NLAT
RLAT(I)=RLAT(I)*YL
35 RLATBND(I)=(XL/NLAT)/2.+*(I-1)*XL/NLAT
DO 36 J=1, NLONG
36 RLNGBK(J)=J
85 C DX AND DY ARE X AND Y AXES SCALE FACTORS
  DX=180./XL
  DY=1./YL
C T(X OR Y) = FREQUENCY OF TIC MARKS/SCALE FACTOR, WHERE SCALE
C FACTOR(DX OR DY) IS A CONVERSION FACTOR BETWEEN AXES UNITS
90 C AND REAL LENGTH
  TX=-10./DX
  TY=-1./DY
C THE INPUT PARAMETERS REQUIRED FOR SUBROUTINE BARPLT HAVE BEEN
C CALCULATED.
95 C * * * * * * *
C PLOT HISTOGRAMS
C
C DATA DISTRIBUTION PER LATITUDE ZONE
CALL NFRAME
CALL AXES(0.,0.,0.,XL,-90.,DX, TX,0.,14HLATITUDE (DEG),10,-14)
CALL AXES(0.,0.,90.,YL,0.,DY,TY,0.,32HNORMALIZED NUMBER OF DATA POINTS)
CALL NOTATE (2.,5.,600.,15.,30HNUMBER OF DATA POINTS/LAT BAND,0.,30)
CALL NOTATE (2.,5.,75.,15.,32HCHAPTER INCLUDES DATA FOR DAYS,0.,32)
100 C WBAR IS THE BAR WIDTH WHICH IS ISIZE(SEE OZSTAT PARAMETER LIST)
C DEGREES WIDE.
105 WBAR=(180./NLAT)/DX
CALL BARPLT (RLATBND,RLAT,NLAT,1.,1.,WBAR,0)
CALL NOTATE (2.,5.,600.,15.,30HNUMBER OF DATA POINTS/LAT BAND,0.,30)
CALL NOTATE (2.,5.,75.,15.,32HCHAPTER INCLUDES DATA FOR DAYS,0.,32)
110 CALL NOTATE (2.,5.,50.,15.,DATE,0.,14)
ISELECT =0
IF (ISELECT.EQ.0) GO TO 90
C
C DATA DISTRIBUTION PER GRID BLOCK
DO 50 I=1,NLAT
DO 45 J=1,NLONG
45 RK(J)=RKK(I,NLONG+1-J)
115 C NO MODIFICATIONS FOR VARIABLE BLOCK SIZE BELOW THIS POINT.
C
C CALL NFRAME
CALL AXES (0.,0.,0.,18.,0.,2.,TX,0.,33HLONGITUDE INDICES FOR GRID)
CALL AXES(0.,0.,90.,YL,0.,DY,TY,0.,32HNORMALIZED NUMBER OF DATA POINTS)
120 C
125
SUBROUTINE HISTPLT 74/74 OPT-1  
FTN 4.7+485 80/01/23. 14.08.48

110 FORMAT (1X,*1AT-MAX*,*I5)  
115 FORMAT (I2,4X,18(I4,3X))  
120 FORMAT (I2,4X,18(I4,3X))  
125 FORMAT (*140, *NORMALIZED NUMBER OF DATA POINTS/LATITUDE BAND*)  
126 FORMAT (I2,4X,18(I4,3X))  
130 FORMAT (*140, *NORMALIZED NUMBER OF DATA POINTS/GRID BLOCK*)  
131 FORMAT (I2,4X,18(I4,3X))  
132 FORMAT (*140, *NORMALIZED NUMBER OF DATA POINTS/LATITUDE BAND*)  
136 FORMAT (*140, *NORMALIZED NUMBER OF DATA POINTS/GRID BLOCK*)  
140 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
141 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
142 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
143 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
144 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
145 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
146 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
147 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
148 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
149 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
150 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
151 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)  
152 FORMAT (*140, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH TO NORTH ARE*)

SYMBOLIC REFERENCE MAP (R=1)

TRY POINTS
3 HISTPLT

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>SN</th>
<th>TYPE</th>
<th>RELOCATION</th>
</tr>
</thead>
<tbody>
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<td>DATE</td>
<td>REAL</td>
<td>ARRAY DD</td>
</tr>
<tr>
<td>141</td>
<td>DY</td>
<td>REAL</td>
<td></td>
</tr>
<tr>
<td>127</td>
<td>ICOUNT</td>
<td>INTEGER</td>
<td></td>
</tr>
<tr>
<td>132</td>
<td>ILATMAX</td>
<td>INTEGER</td>
<td></td>
</tr>
<tr>
<td>135</td>
<td>J</td>
<td>INTEGER</td>
<td></td>
</tr>
</tbody>
</table>
SUBROUTINE AVARPLT (V,A,U,T,M,RAT,STDEV,STDEVN,NLAT)
COMMON/DD/D
DIMENSION V(NLAT),A(NLAT),RAT(NLAT),STDEV(NLAT),STDEVN(NLAT)
DIMENSION U(M),T(M)
DIMENSION D(2)

V AND A ARE THE VARIANCE AND AVERAGE, RESPECTIVELY, OF THE OZONE DENSITY/LATITUDE BAND
U AND T ARE THE LATITUDE AND OZONE DENSITY INFORMATION/DATA RECORD
USED TO PLOT THE SCATTER DIAGRAM.
M IS THE DIMENSION OF U AND T.
STDEV IS THE STANDARD DEVIATION + MEAN, PROPERLY SCALED TO PLOT
STDEVN IS THE STANDARD DEVIATION - MEAN, PROPERLY SCALED TO PLOT
SUBROUTINE CALPLT (X,Y,IPEN) IS LOCATED IN SECTION 1.4.3 OF THE GRAPHICS MANUAL
THE GRAPHICS MANUAL
SUBROUTINE PNTPLT (X,Y,ISYM,IS) CAN BE FOUND IN SECTION 1.4.70 OF THE GRAPHICS MANUAL
SUBROUTINE PSEUDO (FN), FN - FILENAME, CAN BE FOUND IN SECTION 1.4.1 OF THE GRAPHICS MANUAL
INITIALIZE PARAMETERS
XL=8.
YL=6.0
DX=180./XL
YMAX=0.65
YMIN=0.15
DY=(YMAX-YMIN)/YL
TX=-10./DX $ TY=-1./DY
XT=ABS(TX)
UMAX=0.
UMIN=0.
TMAX=-1.
TMIN=1.

CONSTRUCT PLOT LABELS AND AXES.
CALL NFRAME
CALL NOTATE (2.,5.00,.15,15HSCATTER DIAGRAM,O.,IS)
CALL NOTATE (2.,4.75,.15,36HINCLUDES MEAN AND STANDARD DEVIATION,O,AVARPLT
1.,36)
SUBROUTINE AVARPLT 74/74 OPT=1
FTN 4.7+485  80/01/23. 14.08.48

CALL NOTATE (2.00, 4.50, .15, .12) AVARPLT 39
CALL NOTATE (2.00, 4.25, .15, .35) AVARPLT 40
CALL NOTATE (2.00, 4.00, .15, .14) AVARPLT 41
CALL NOTATE (2.00, 3.75, .15, .14) AVARPLT 42
CALL NOTATE (2.00, 3.50, .15, .14) AVARPLT 43
CALL NOTATE (2.00, 3.25, .15, .14) AVARPLT 44
CALL NOTATE (2.00, 3.00, .15, .14) AVARPLT 45
CALL NOTATE (2.00, 2.75, .15, .14) AVARPLT 46
CALL NOTATE (2.00, 2.50, .15, .14) AVARPLT 47
CALL NOTATE (2.00, 2.25, .15, .14) AVARPLT 48
CALL NOTATE (2.00, 2.00, .15, .14) AVARPLT 49
CALL NOTATE (2.00, 1.75, .15, .14) AVARPLT 50
CALL NOTATE (2.00, 1.50, .15, .14) AVARPLT 51
RETURN

C******************************************************************************
ENTRY SCAT
C PLOT SCATTER DIAGRAM - ALSO FIND MAXIMUM AND MINIMUM LATITUDES AVARPLT 52
DO 30 I=1, M AVARPLT 53
C FIND MAXIMUM AND MINIMUM LATITUDE VALUES *************** AVARPLT 54
IF (U(I).GT.UMAX) UMAX=U(I) AVARPLT 55
IF (U(I).LT.UMIN) UMIN=U(I) AVARPLT 56

COMMENT -- FIND MAXIMUM AND MINIMUM OZONE VALUES *************** OCT79 9
IF (T(I).GT.TMAX) TMAX=T(I) AVARPLT 57
IF (T(I).LT.TMIN) TMIN=T(I) AVARPLT 58

C******************************************************************************
ENTRY ASTD
X=U(I)+90./DX AVARPLT 59
Y=T(I)-YMIN)/DY AVARPLT 60

30 CALL PNTPLT(X, Y, -21, 1) AVARPLT 61
RETURN

C******************************************************************************
ENTRY NLAT1=NLAT/2 AVARPLT 62
NLAT2=NLAT1+1 AVARPLT 63
DO 35 I=1, NLAT AVARPLT 64
STDEV(I)=(A(I)+SORT(V(I)))/DY AVARPLT 65
STDEVN(I)=(A(I)-SORT(V(I)))/DY OCT79 13
STDEV(I)=STDEV(I)-YMIN/DY OCT79 14
STDEVN(I)=STDEVN(I)-YMIN/DY OCT79 15
CONTINUE OCT79 16
DO 40 I=1, NLAT1 AVARPLT 66
J=I+NLAT1 AVARPLT 67
RETURN

C******************************************************************************
C RAT IS THE SCALED DATA MATRIX FOR SPACING PLOTTED AVERAGES AND
C STANDARD DEVIATION ALONG THE X - AXIS
RAT(I)=XS/2.+(NLAT/2-1+I)*XS
RAT(J)=XS/2.+(NLAT-J)*XS
40 CONTINUE
C
C COMMENT -- FIND LATITUDE INDEXES, IMAX AND IMIN, CORRESPONDING TO
C UMAX AND UMIN. THEN, CALCULATE AN ADJUSTED VALUE OF
C RAT(IMAX) AND RAT(IMIN) SUCH THAT THE EXTREME MEANS WILL
C BE PLOTTED IN THE END LATITUDE ZONES HALF-WAY BETWEEN THE
C ZONE'S BEGINNING AND THE EXTREMUM LATITUDE VALUES.
C
IMAX=UMAX/(180/NLAT)
LMAX=IMAX*(180/NLAT)
IMAX=IMAX+1
C
C COMMENT -- RATMAX IS THE HALF-WAY POINT FOR THE EXTREME MAXIMUM
C LATITUDE ZONE.
RATMAX=(UMAX-LMAX)/(2.*DX)
RAT(IMAX)=RAT(IMAX)-XS/2.
RAT(IMAX)=RAT(IMAX)+RATMAX
IMIN=UMIN/(180/NLAT)
LMIN=IMIN*(180/NLAT)
IMIN=(NLAT/2+1)-IMIN
105 IMIN=(NLAT/2+1)-IMIN
C
C COMMENT -- RATMIN IS THE HALF-WAY POINT FOR THE EXTREME MINIMUM
C LATITUDE ZONE.
RATMIN=(UMIN-LMIN)/(2.*DX)
RAT(IMIN)=RAT(IMIN)+XS/2.
RAT(IMIN)=RAT(IMIN)+RATMIN
C
C ************ ************ ************ ************ ************ ************
C
110 DO 41 I=1,NLAT
C A NOW BECOMES THE PROPERLY SCALED AVERAGE TO BE PLOTTED
C ALONG THE Y-AXIS
A(I)=(A(I)-YMIN)/DY
C
C ************ ************ ************ ************ ************ ************
C
C NOW PLOT MEANS AND DRAW IN CONNECTING "CURVE"
C
115 IF (A(I).EQ.-YMIN/DY) GO TO 42
CALL PNTPLT (RAT(I),A(I),-11,2)
41 CONTINUE
C
C ************ ************ ************ ************ ************ ************
C
C CALL PNTPLT (RAT(I),A(I),-11,2)
42 CONTINUE
C
C ************ ************ ************ ************ ************ ************
C
C CALL PNTPLT (RAT(I),A(I),-11,2)
SUBROUTINE AVARPLT  74/74  OPT=1  FTN 4.7+485  80/01/23. 14.08.48

45 CONTINUE
CALL CALPLT (RAT(1),A(1),3)
DO 50 I=NLAT2,IMIN
130 IF (A(I).EQ.-YMIN/DY.AND.I.EQ.NLAT2) GO TO 46
IF (A(I).EQ.-YMIN/DY.OR.A(I-1).EQ.-YMIN/DY.AND.I.GT.NLAT2) GOTO 46
CALL CALPLT (RAT(I),A(I),2)
46 CONTINUE
IF (A(I).EQ.-YMIN/DY) GO TO 50
CALL PNTPLT (RAT(I),A(I),-11,2)
50 CONTINUE
PLOT STANDARD DEVIATION
DO 55 I=1,IMIN
140 IF ((I.GT.IMAX).AND.(I.LT.NLAT2)) GO TO 55
IF (A(I).EQ.-YMIN/DY) GO TO 55
CALL CALPLT ((RAT(I)-0.06),STDEVN(I),3)
CALL CALPLT ((RAT(I)+0.06),STDEVN(I),2)
CALL CALPLT (RAT(I),STDEVN(I),3)
CALL CALPLT (RAT(I),STDEVNP(I),2)
CALL CALPLT ((RAT(I)-0.06),STDEVNP(I),3)
CALL CALPLT ((RAT(I)+0.06),STDEVNP(I),2)
55 CONTINUE
PLOT MEANS AND STANDARD DEVIATIONS AS A SEPERATE FRAME.
CALL NFNAME
CALL NOTATE (2.00,5.00,.15,27HMEAN AND STANDARD DEVIATION,0.,27) AVARPLT 128
CALL NOTATE (2.00,4.75,.15,42HDATA TAKEN FROM NIMBUS IV BUV MEASURAVARPLT 129
1EMENTS,0.,42) AVARPLT 130
CALL NOTATE (2.00,4.50,.15,35HTHIS DIAGRAM INCLUDES DATA FOR DAYS,AVARPLT 131
10.,35) AVARPLT 132
CALL NOTATE (2.00,4.25,.15,D,0.,14)
CALL AXES (0.,0.0,XL,-90.,DX,TX,0.,14HLATITUDE (DEG),10.,14) AVARPLT 134
CALL AXES (0.,0.90.,YL,-15.,DY,TY,0.,20HTOTAL OZONE (ATM-CM),10,AVARPLT 120)
CALL CALPLT (0.,YL,3)
CALL CALPLT (XL,YL,2)
CALL CALPLT (XL,0.,2)
CALL CALPLT (0.,0.,3)
C NOW PLOT MEANS AND DRAW IN CONNECTING "CURVE"
160 CALL PNTPLT (RAT(1),A(1),-22,2)
C
556 CONTINUE
DO 56 I=2,IMAX
165 IF (A(I-1).EQ.-YMIN/DY) GO TO 556
CALL PNTPLT (RAT(I),A(I),-22,2)
556 CONTINUE
DO 56 I=2,IMAX
165 IF (A(I-1).EQ.-YMIN/DY) GO TO 557
CALL PNTPLT (RAT(I),A(I),-22,2)
557 CONTINUE

SUBROUTINE AVARPLT 74/74 OPT-1

CALL CALPLT (RAT(I),A(I),2)

557 CONTINUE

IF (A(I).EQ.-YMIN/DY) GO TO 56
CALL PNTPLT (RAT(I),A(I),-22,2)

56 CONTINUE

CALL CALPLT (RAT(I),A(I),3)

DO 57 INLAT,ZIMIN

IF (A(I).EQ.-YMIN/DY.AND.I.EQ.NLAT2) GO TO 558

IF (A(I).EQ.-YMIN/DY.AND.I.GT.NLAT2) GO TO 56
CALL CALPLT (RAT(I),A(I),2)

558 CONTINUE

IF (A(I).EQ.-YMIN/DY) GO TO 57
CALL PNTPLT (RAT(I),A(I),-22,2)

57 CONTINUE

CALL PLOT STANDARD DEVIATION

DO 58 INLAT,IMIN

IF ((I.GT.IMAX).AND.(I.LT.NLAT2)) GO TO 58
IF (A(I).EQ.-YMIN/DY) GO TO 58
CALL CALPLT ((RAT(I)-0.06),STDEVN(I),3)
CALL CALPLT ((RAT(I)+0.06),STDEVN(I),2)
CALL CALPLT (RAT(I),STDEVN(I),3)
CALL CALPLT (RAT(I),STDEVN(I),2)
CALL CALPLT (RAT(I)-0.06),STDEVV(I),3)
CALL CALPLT (RAT(I)+0.06),STDEVV(I),2)

58 CONTINUE

PRINT 110

PRINT 115, D

DO 60 INLAT

60 PRINT 100, I,RAT(I),A(I),V(I),STDEVV(I),STDEVN(I)

PRINT 105, M,UMAX,UMIN

PRINT 120,TMIN,TMAX

RETURN

120 FORMAT (1X,*TMIN= *,E15.8,*TMAX= *,E15.8)

RETURN

100 FORMAT (*0*,*I= *,E15.8,*RAT= *,F6.2,*A= *,F6.2,*V= *,E11.4)

14X,STDEVV= *,F6.2,4X,STDEVN= *,F6.2)

105 FORMAT (*0*,*M= *,I6,5X,UMAX= *,F7.3,5X,UMIN= *,F7.3)

205 FORMAT (*1*,*X-AXIS SCALE -RAT-,AVERAGES,VARIANCES,AND STANDARD DEAVARPLT

1VIATIONS USED IN AVARPLT*)

110 FORMAT (*0*,*FOR THE TIME PERIOD A10,A4)
REFERENCES


REFERENCES
(continued)


A methodology has been developed to analyze discrete data obtained from the global distribution of ozone. Statistical analysis techniques are applied to describe the distribution of data variance in terms of empirical orthogonal functions and components of spherical harmonic models. The effects of uneven data distribution and missing data are considered. Data fill based on the autocorrelation structure of the data is described. Computer coding of the analysis techniques is included.
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