ELASTIC-PLASTIC ANALYSIS USING A TRIANGULAR RING ELEMENT IN NASTRAN

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SUMMARY

An elastic-plastic triangular ring element is implemented in NASTRAN computer program. The plane-strain problem of partially-plastic thick-walled cylinder under internal pressure is solved and compared with the earlier finite-difference solution. A very good agreement has been reached. In order to demonstrate its application to more general problems, an overloaded thread problem for the British Standard Buttress is examined. The maximum axial and principal stresses are located and their values are determined as functions of loadings.

INTRODUCTION

The piecewise linear analysis option of the NASTRAN program provides an algorithm for solving nonlinear problems in material plasticity (ref. 1). However, the usefulness of this option is quite limited because only a few elements have been implemented. These include rod, tube, bar elements for one-dimensional problems and plate elements for two-dimensional plane stress problems. In a recent paper (ref. 2), the implementation of a trapezoidal ring element in NASTRAN for elastic-plastic analysis was described and two test problems of rotational symmetry were solved. The first is an infinitely long tube under uniform internal pressure. The NASTRAN results are in excellent agreement with an exact solution based on the finite-difference approach (ref. 3). The second problem is a thick-walled cylinder of finite length loaded over part of its inner surface. The NASTRAN results are in good agreement with another finite-element solution (ref. 4).

In the present paper, two elastic-plastic problems of rotational symmetry are solved using triangular ring elements for the finite element models. The implementation of a triangular ring element in NASTRAN for elastic-plastic analysis follows the same procedures in reference 2 for a trapezoidal ring element. In order to test the accuracy of the implementation, the plane-strain problem of a partially-plastic thick-walled cylinder under internal pressure is solved again and compared with the finite difference solution (ref. 3). The second demonstrative problem in using NASTRAN triangular ring elements is an overloaded thread problem for the British Standard Buttress. A detailed discussion for this problem in the elastic range of loading was reported in reference 5. In the present paper, uniform pressure distribution on the primary bearing surface of the thread is assumed and the load is applied in increments beyond the elastic limit. The maximum axial and principal stresses are located and their values are determined as functions of loadings.

TRIANGULAR RING ELEMENTS

The incremental displacement field employed for the triangular ring element are

$$\Delta u(r,z) = \beta_1 + \beta_2 r + \beta_3 Z \ , \tag{1}$$

$$\Delta w(r,z) = \beta_4 + \beta_5 r + \beta_6 Z \ . \tag{2}$$

The transformation from grid point coordinates to generalized coordinates is

$$\{\beta\} = [\Gamma_{\beta q}] \ \{\Delta q\} \tag{3}$$

where

$$\{q\}^T = \lfloor \Delta u_1, \ \Delta w_1, \ \Delta u_2, \ \Delta w_2, \ \Delta u_3, \ \Delta w_3 \rfloor, \tag{4}$$

$$\{\beta\}^T = \lfloor \beta_1, \ \beta_2, \ \beta_3, \ \beta_4, \ \beta_5, \ \beta_6 \rfloor, \tag{5}$$

and the elements of the inverse of the transformation matrix $[\Gamma_{\beta q}]^{-1}$ are the coefficients of the β's in equations (1) and (2), evaluated at the corners of the element.

The stiffness matrix is formed in the same manner as that for the anisotropic elastic element. The final form referred to grid coordinates is

$$[K] = [\Gamma_{\beta q}]^T \ [\bar{K}] \ [\Gamma_{\beta q}] \ , \tag{6}$$

where

$$[\bar{K}] = 2\pi \int r [B]^T \ [D] \ [B] \ dzdr \ . \tag{7}$$

The $[B]$ matrix is the same as the elastic case, but now it expresses the incremental strains in terms of generalized coordinates

$$\{\Delta\varepsilon\} = [B] \ \{\beta\} \ . \tag{8}$$

The $[D]$ matrix which relates the incremental stresses to the incremental strains, i.e.,

$$\{\Delta\sigma\} = [D] \ \{\Delta\varepsilon\} \ , \tag{9}$$

is the same as that for a trapezoidal ring element presented in reference 2. In developing NASTRAN program for strain-hardening materials, we calculate

$[D]^{-1}$ and obtain its inverse $[D]$ numerically. For ideally plastic materials, this procedure fails and we should calculate $[D]$ directly using the closed form.

NASTRAN IMPLEMENTATION

The implementation of a triangular ring element in NASTRAN for elastic-plastic analysis follows the same procedures for a trapezoidal ring element (ref. 2). Changes were required in the functional modules PLA1, PLA3, and PLA4, which included the writing of seven new subroutines. The changes in PLA1 allows this module to identify the new element as a member of the piecewise linear element set and properly initialize the nonlinear Element Summary and Element Connection Property Tables. Three element stress recovery subroutines were added to PLA3: PSIARG, a driver for stress data recovery; PSRIR1 ans PSRIR2, phase I and II stress recovery routines. Element stiffness calculations in PLA4 require four new subroutines: PKIARG, a driver for nonlinear triangular ring elements in PLA4, PKRIR1 and PKRIR2, stress recovery routines which generate stresses for the computation of the nonlinear material matrix; and PKRIRS, the stiffness matrix generation routine for nonlinear triangular ring elements.

PROGRAM EVALUATION

In order to evaluate the accuracy of the computed code, the plane-strain problem of an elastic-plastic thick-walled cylinder under uniform internal pressure is solved again and compared with the finite-difference solution (ref. 3). The tube of outside radius 2" and inside radius 1" has been divided into 10 equal intervals and each interval consists of 2 triangular ring elements. The material constants are $E = 30 \times 10^6$ psi, $\nu = 0.3$, $\sigma_o = 1.5 \times 10^5$ psi and the effective stress-strain curve is represented by three line segments connecting the four points in the $(\bar{\varepsilon},\bar{\sigma})$ plane, $(\bar{\varepsilon},\bar{\sigma}/\sigma_o) = (0.0,0.0)$, (0.005,1.0), (0.055,1.5), (0.1,1.5). Twenty-three load factors are chosen: P/σ_o = 0.4323, 0.4738, 0.5125, 0.5484, 0.5818, 0.6128, 0.6415, 0.6681, 0.6925, 0.7150, 0.7356, 0.7545, 0.7716, 0.7871, 0.8011, 0.8135, 0.8245, 0.8341, 0.8423, 0.8493, 0.8550, 0.863, 0.87. The numerical results based on the NASTRAN program have been obtained. For this problem, exact solution based on a new finite-difference approach (ref. 3) can be used to assess the accuracy of the NASTRAN code. Some of the results for the displacements and stresses are presented graphically in Figures 1 and 2. The radial displacements at the inside as well as outside surface are shown in Figure 1 as functions of internal pressure. Figure 2 shows the distributions of radial, tangential and axial stress components in a partially-plastic tube when the pressure is $p = 0.7356\ \sigma_o$. As demonstrated in Figures 1 and 2, the NASTRAN results are in very good agreement with those based on the finite-difference approach (ref. 3).

THREAD PROBLEM

As an application of using NASTRAN triangular ring elements for two-dimensional problems, an overloaded thread problem for the British Standard Buttress will be examined. A finite element model is shown in Figure 3. A detailed discussion for this problem in the elastic range of loading was reported in reference 5. In the present paper, uniform pressure distribution (P) on the primary bearing surface of the thread is assumed and the load is applied in ten unequal increments beyond the elastic limit. The elastic constants are E = 25000 Ksi, $\nu = 0.25$, and the effective stress-strain curve is represented by five line segments connecting the six points in the ($\bar{\varepsilon}$-$\bar{\sigma}$) plane, ($\bar{\varepsilon}$-$\bar{\sigma}$ in Ksi) = (0.0, 0.0), (0.0048,120), (0.0122,180), (0.0167,200), (0.0918,210), (0.25,210). The sides A-B and C-D are constrained in the axial direction and the side B-C, in the axial and radial directions.

The elastic problem is solved first and the upper limit of the elastic loading (p*) is found to be 54.65 Ksi. Eleven load factors are chosen as follows: p = 54.65, 60.59, 66.22, 71.54, 76.54, 81.23, 85.61, 89.68, 93.43, 96.87, 100.0 Ksi. The numerical results based on the NASTRAN code have been obtained and the total CPU time is 59 minutes on IBM 360 model 44. The plastic zone at the maximum load is indicated by the shaded area in Figure 3. Some of the stress results are shown graphically in Figures 4 to 6. Of particular interest is the region along side D-E-F. The axial stress (σ_z) and major principal stress (σ_1) in the boundary elements along side D-E-F are shown in Figures 4 and 5, for the first and last load factor, respectively. The maximum fillet stress (σ_1) occurs near E while the maximum axial stress always occurs at D. Finally, the maximum fillet stress and axial stress are plotted as functions of contact pressure in Figure 6.

CONCLUSION

An elastic-plastic triangular ring element has been implemented in NASTRAN computer program. Its accuracy has been evaluated by solving a simpler problem for which exact solution is available. Its application to more general problem has also been demonstrated by solving an overloaded thread problem.

REFERENCES

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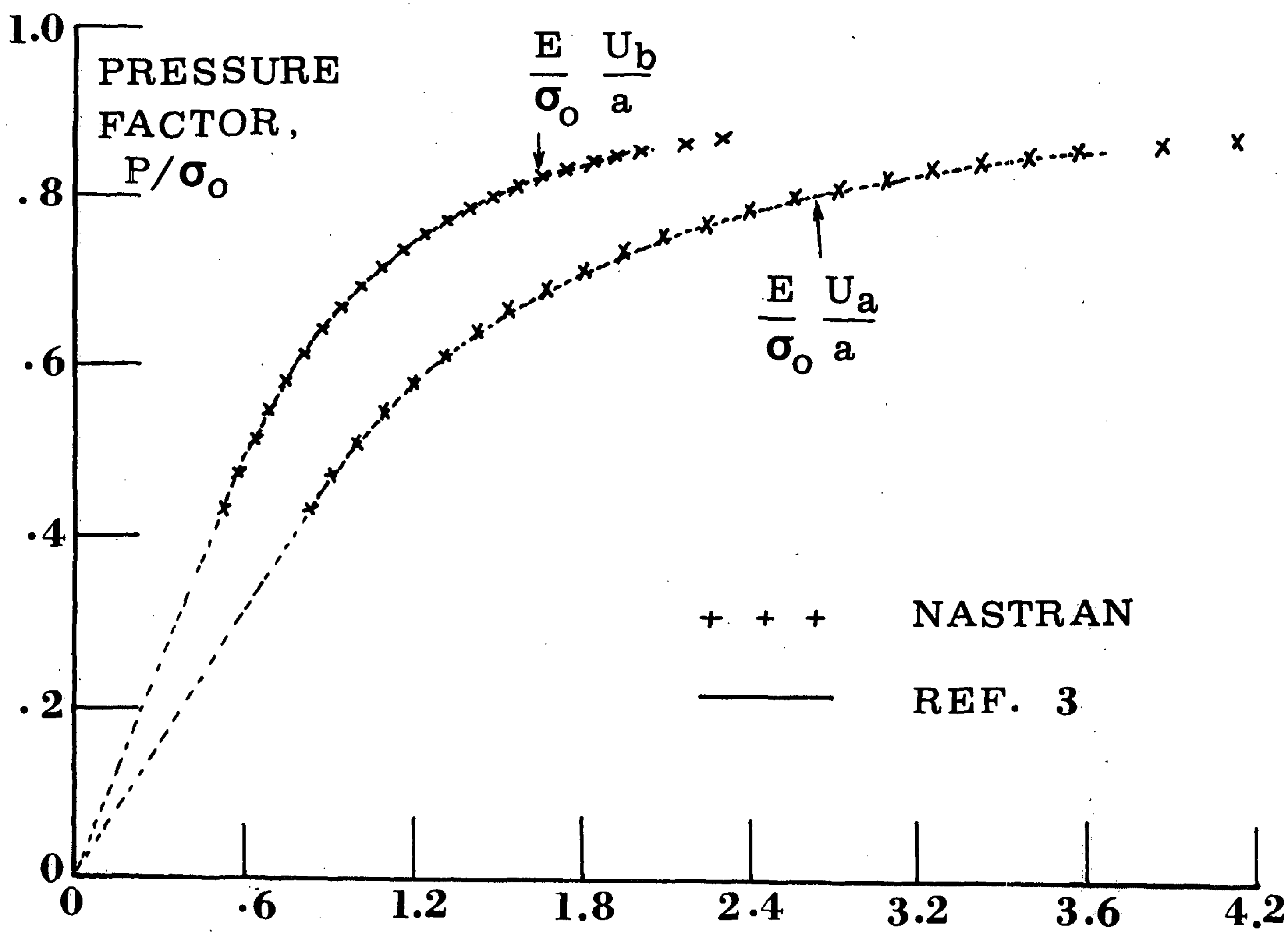

Figure 1. Radial displacement in a pressurized tube.

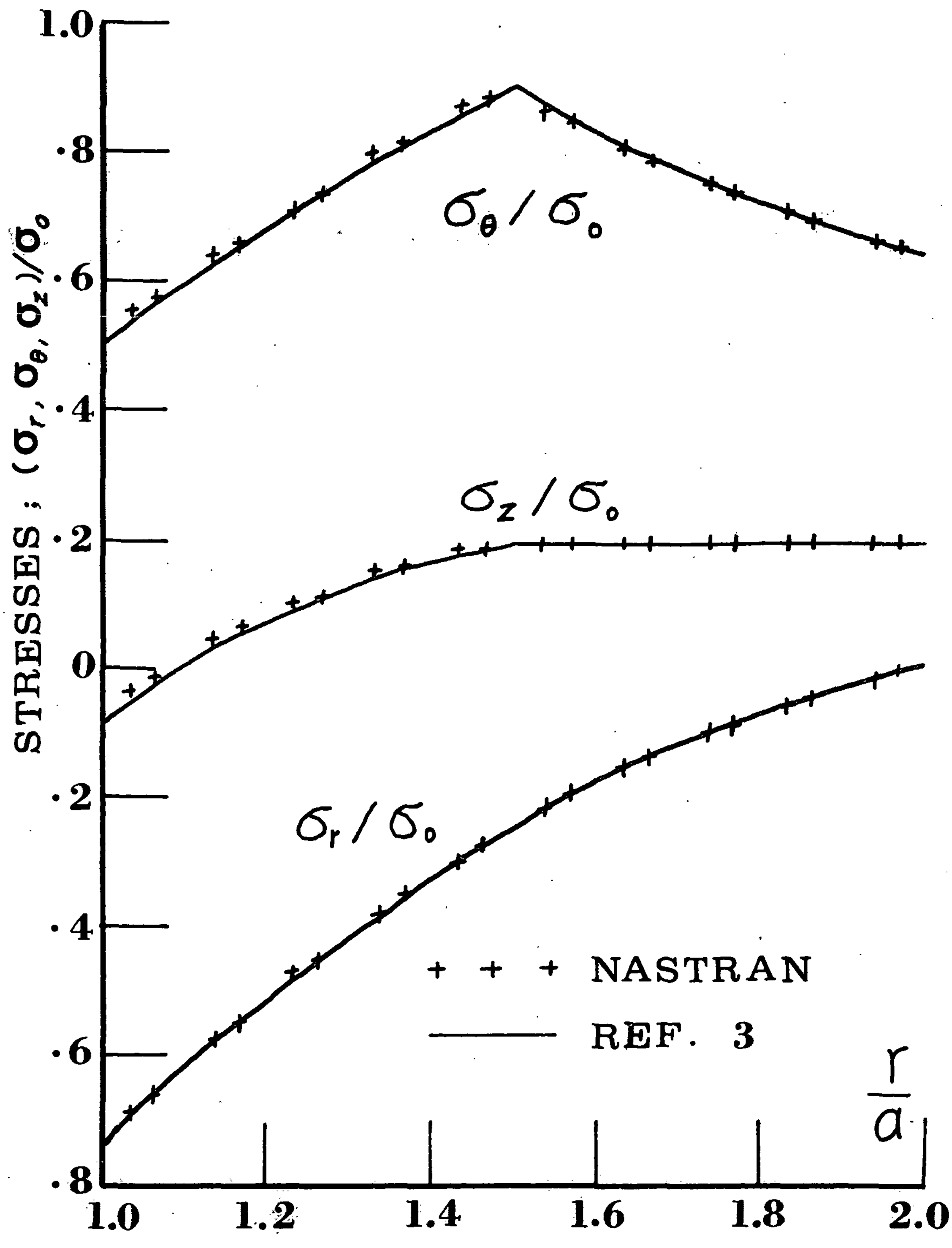

Figure 2. Stresses in a pressurized tube.

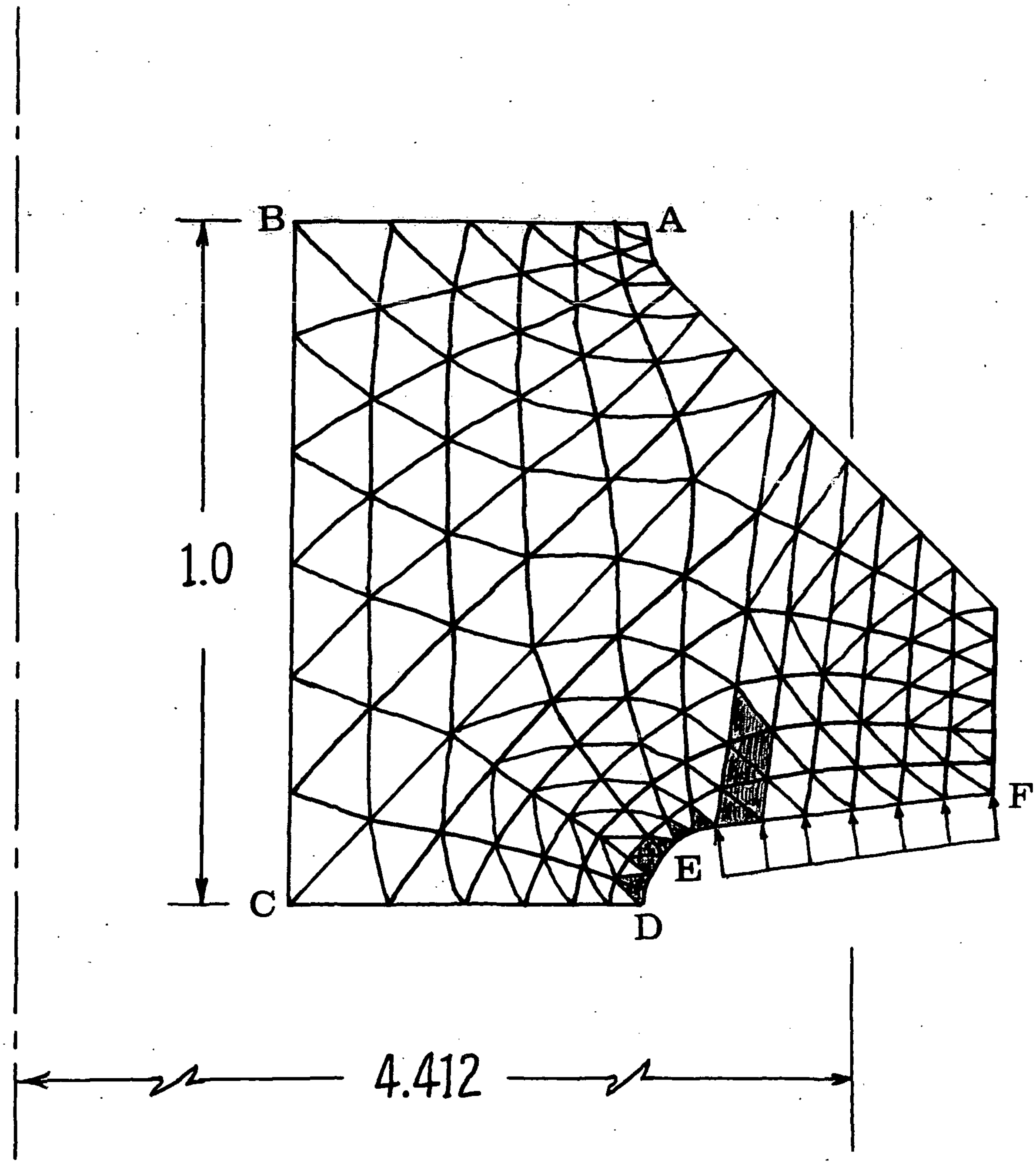

Figure 3. Finite element model for the thread problem.

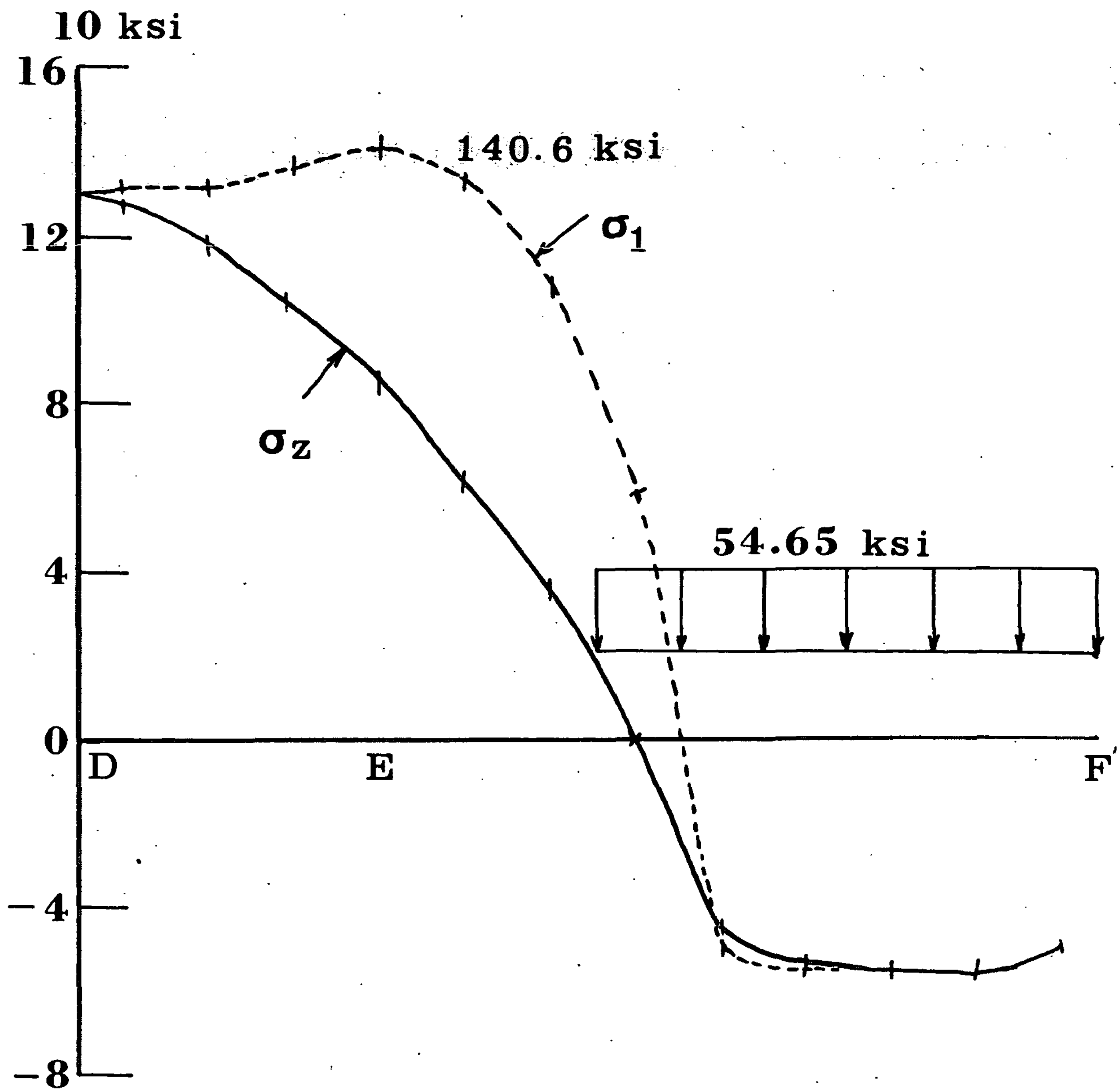

Figure 4. Stresses in boundary elements (side D-E-F) at p = 54.65 Ksi.

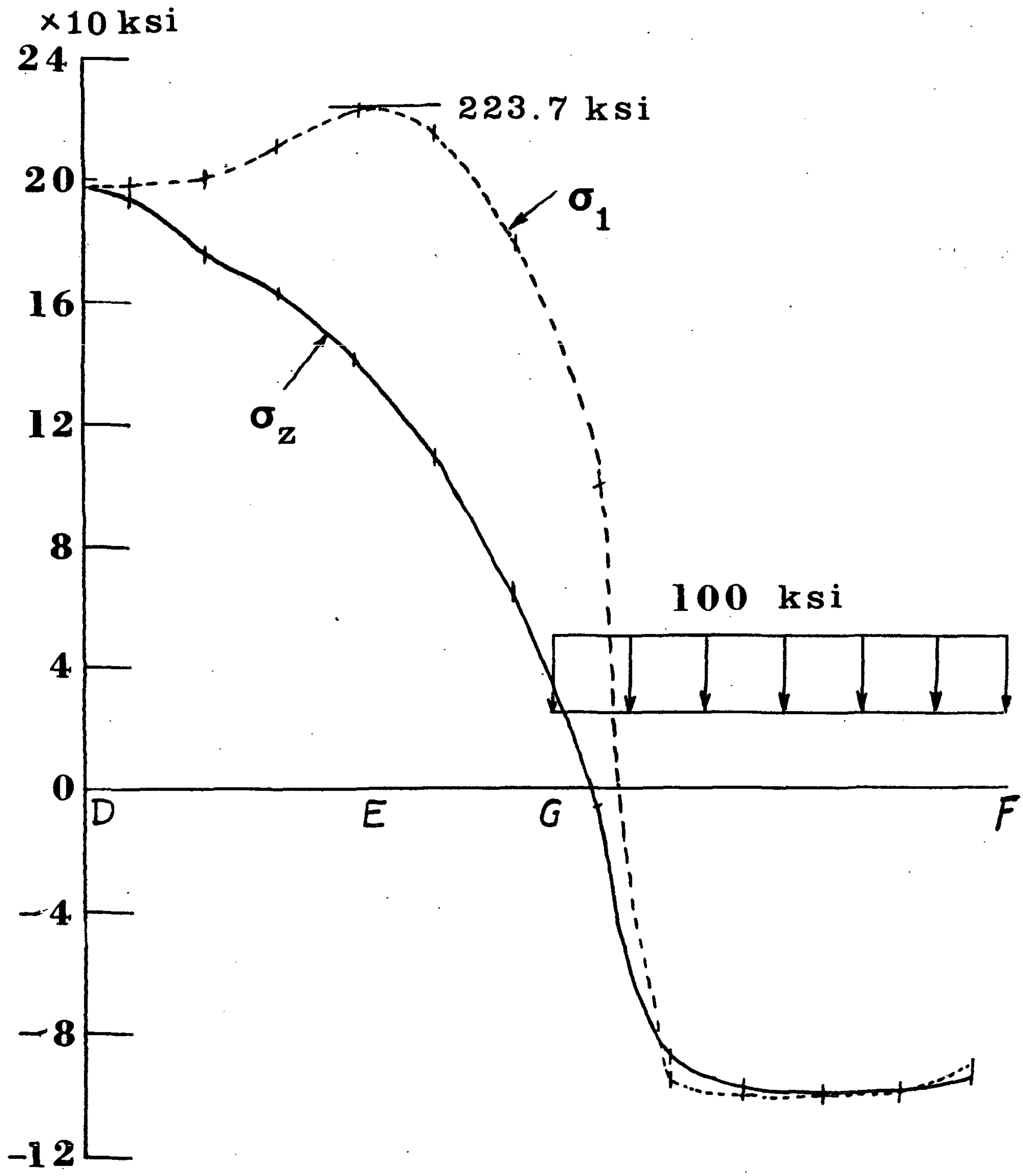

Figure 5. Stresses in boundary elements (side D-E-F) at p = 100 Ksi.

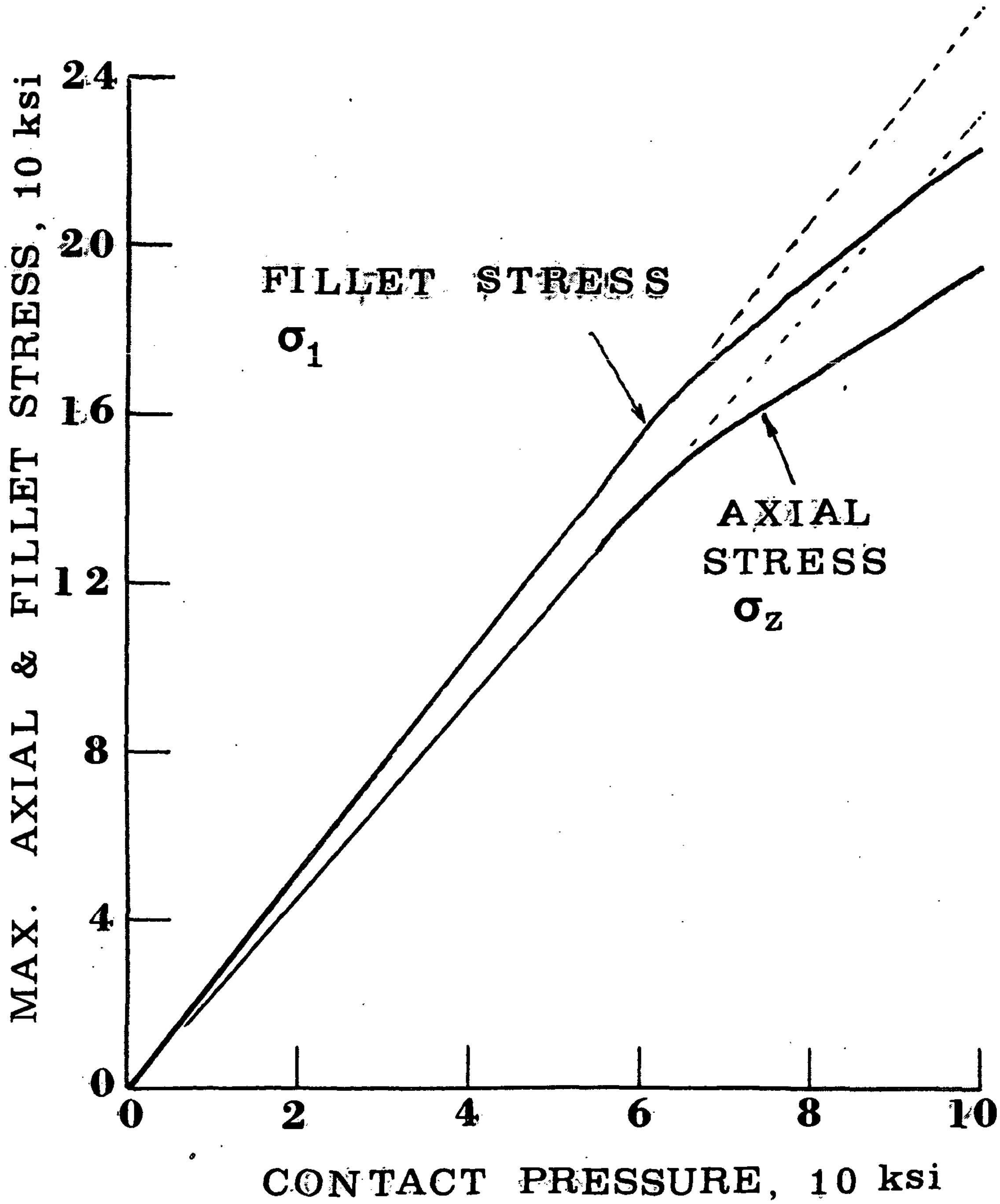

Figure 6. Maximum axial and fillet stresses as functions of contact pressure.