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NASA NSG-7043

Effective Depth of Spectral Line Formation in Planetary Atmospheres

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(NASA-CR-163617) EFFECTIVE DEPTH OF SPECTRAL LINE FORMATION IN PLANETARY ATMOSPHERES (Rice Univ.) 13 p HC A02/MF A01 CSCL 03B

N80-34319

Unclas

G3/91 28968

NOV 1980
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Abstract

The effective level of line formation for spectroscopic absorption lines has long been regarded as a useful parameter for determining average atmospheric values of the quantities involved in line formation. The identity of this parameter has recently been disputed. Here we re-establish the dependence of this parameter on the average depth where photons are absorbed in a semi-infinite atmosphere and show that the mean depths derived by others are similar in nature and behavior.

The concept of a "level of line formation" is scarcely novel in Astrophysics (see the review by Strömgren, 1951, p. 204). Nevertheless, this concept was not introduced into studies of planetary physics until 15 years ago, in a theoretical paper concerned with the hazy atmosphere of Venus (Chamberlain, 1965, p. 1190). Recently, the usefulness and functional behavior of this parameter have been disputed (Wallace and Hunten, 1978; Kattawar, 1979). We contend that for a homogeneous, semi-infinite atmosphere the effective level for line formation is a straightforward matter flowing from the transfer equation.

We define the mean depth as the first moment of the sink function

$$\langle\tau\rangle = \frac{\int_0^\infty S(\tau)\ \tau d\tau}{\int_0^\infty S(\tau)\ d\tau} \tag{1}$$

where

$$S(\tau) = (1 - \tilde{\omega})\ J(\tau,\ \mu_0), \tag{2}$$

$\tilde{\omega}$ is the single scattering albedo, and J is the mean intensity averaged over direction at depth τ. In Chandrasekhar's (1950) first approximation for a semi-infinite atmosphere we have

$$J(\tau,\ \mu_0) = \frac{\tilde{\omega}F}{4}\ (Le^{-k\tau} + \gamma e^{-\tau/\mu_0}) \tag{3}$$

where L and γ are functions of μ_1, μ_0, and $\tilde{\omega}$, πF is the incident flux; $\mu_1 = 1/\sqrt{3}$ and $k = (1 - \tilde{\omega})^{1/2}/\mu_1$. Substitution of (2) and (3) into (1) gives (adapted from Chamberlain, 1965)

$$\langle\tau\rangle = \frac{\mu_1}{(1 - \tilde{\omega})^{1/2}} + \mu_0 - \mu_1. \tag{4}$$

Note that the first term on the right hand side is the asymptotic limit for $\langle\tau\rangle$ for weak lines with a weakly absorbing continuum (i.e., $\tilde{\omega} \approx 1$). However, without loss of generality, τ can represent optical thickness in the continuum as well as a line.

A physical interpretation of this result comes from considerations of a random walk of photons (Chamberlain, 1978, p. 140). Very crudely, the number of scatterings required for a photon to migrate over depth $\tau \gg 1$ is τ^2. With a probability of absorption at each encounter of $(1 - \tilde{\omega})$, we would expect to find a substantial fraction of the photons absorbed only when

$$\tau^2 (1 - \tilde{\omega}) \approx 1$$

$$\text{or } \tau \approx (1 - \tilde{\omega})^{-1/2}.$$

Although the sink function $S(\tau) = (1 - \tilde{\omega}) J(\tau, \mu_0)$ technically depends on μ_0, in the limit of weak absorptions and non-grazing incidence, $\langle\tau\rangle$ is practically independent of μ_0, because the γ or "solar term" (representing primary scattering) in Equation (3) is relatively unimportant at the great depths of $\langle\tau\rangle$.

Wallace and Hunten (1978) derive the equivalent width W, which comes from wavelength integrals of intensity $I_\nu(\tau_\nu = 0; \mu, \mu_0)$, both in the line and continuum, and therefore depends on the viewing geometry. The problem lies in assigning a pressure or depth where the effective pressure-broadening parameter, α_e, can be evaluated. In a real atmosphere α varies with depth, but we suppose there is an effective depth τ_e where the local $\alpha(\tau_e) \equiv \alpha_e$ best

represents the entire composite line. The concept is reasonable, but their evaluation of α_e is suspect. They first of all equate equivalent widths for constant-pressure and varying-pressure models in the limit of a vanishing albedo, $\tilde{\omega} \to 0$, or a single-scattering atmosphere.

As noted above, a μ, μ_0 dependence is then built in; but the limit of single scattering scarcely yields a radiative-transfer (multiple scattering) model. For larger $\tilde{\omega}$ they obtain the numerical coefficient of α_e by matching curves of growth for the two models (constant and varying pressure), but the directional geometry remains the same because it is tied to the case $\tilde{\omega} = 0$. Through a trial-and-error method, they multiply the two curves of growth by different factors which are functions of μ, μ_0, and $\tilde{\omega}$ to achieve overlap and derive:

$$\alpha_e = \frac{\alpha}{\tau_c} \left[\frac{\pi/2}{(1 - \tilde{\omega})^{1/2} \left(\mu^{-1} + \mu_0^{-1}\right)}\right] \tag{5}$$

where α/τ_c is the line width at unit optical depth. It is surprising to see a reflecting layer air mass factor $\eta = \mu^{-1} + \mu_0^{-1}$ in a formula for a scattering layer. Chamberlain (1965) showed that for multiple scattering the μ, μ_0 dependence for weak line equivalent widths, was $(\mu + \mu_0)$. However, if we assume that their method of matching curves of growth is correct, it is easy to suggest an alternative formula with the correct $(\mu + \mu_0)$ variation. The results and figures presented by Wallace and Hunten are for $\mu = \mu_0$; in this case, the factors $(\mu + \mu_0)$ and $1/(\mu^{-1} + \mu_0^{-1})$ are identical in <u>variation</u>. Therefore, since they use arbitrary numerical factors to force their constant and varying-pressure models to give identical curves of growth, we can multiply Equation (5) by $(\mu^{-1} + \mu_0^{-1})(\mu + \mu_0)/4$ which is unity when $\mu = \mu_0$.

Their formula then reads

$$\alpha_e = \frac{\alpha}{\tau_c} \left[\frac{\pi/2}{(1 - \tilde{\omega})^{1/2}} \frac{(\mu + \mu_0)}{4} \right] \tag{6}$$

Not only does this give exactly the same curve of growth as (5), when $\mu = \mu_0$, but it also contains the Chamberlain (1965) μ,μ_0-variation.

Kattawar's (1979) quarrel with Chamberlain's $\langle\tau\rangle$ is conceptual. His own definition of the effective level of line formation is

$$\tau_{eff} = \int_0^\infty \tau N_{se}(\tau)d\tau \Big/ \int_0^\infty N_{se}(\tau)d\tau \tag{7}$$

where $N_{se}(\tau)$ represents the number of photon scatterings occurring at depth τ for light which emerges. In other words, τ_{eff} is the average depth at which an emerging photon has scattered. On the other hand, Chamberlain's $\langle\tau\rangle$ represents the average depth at which photons are absorbed.

Kattawar's principal argument against Chamberlain's $\langle\tau\rangle$ was that as $\tilde{\omega}$ approaches unity, $\langle\tau\rangle$ goes to infinity. This is a result of the fact that as $\tilde{\omega}$ approaches unity the light intensity deep in a homogeneous medium becomes a <u>constant</u> with depth. Obviously, the depth-averaged value of such a constant function will tend to infinity.

However, it is a simple matter to show that Kattawar's τ_{eff}, as defined in Equation (7), has the <u>same</u> asymptotic behavior as $\langle\tau\rangle$. Let $N_s(\tau)$ be the total number of scatterings occurring at depth τ. N_{se} is less than N_s by a

factor $P(\tilde{\omega}, \tau)$ which represents the probability that a photon, which has scattered at depth τ, will eventually random walk out of the atmosphere. Following the lead given by Chandrasekhar (1943) for the random walk of photons with an absorbing screen we obtain (see Appendix)

$$P(\tilde{\omega}, \tau) = \exp\left[-\left(\tau^2 \ln\left(1/\tilde{\omega}^2\right)\right)^{1/2}\right] \tag{8}$$

Multiplying this probability by the mean intensity, $J(\tau, \mu_0)$, gives N_{se} (times a constant) for insertion into the integrands of Equation (7). We then have

$$\tau_{eff} = \frac{[k + m]^{-2} + \frac{\gamma}{L}\left[\mu_0^{-1} + m\right]^{-2}}{[k + m]^{-1} + \frac{\gamma}{L}\left[\mu_0^{-1} + m\right]^{-1}} \tag{9a}$$

where $m = [\ln(1/\tilde{\omega}^2)]^{1/2}$. Note that for $\tilde{\omega} \approx 1$, $\frac{\gamma}{L} = (\mu_1 - \mu_0)/\mu_0$ and $m \approx \mu_1 k(2)^{1/2}$. Since $k \to 0$ as $\tilde{\omega} \to 1$, we have

$$\tau_{eff} = \frac{\mu_1}{\left(1 + \mu_1(2)^{1/2}\right)(1 - \tilde{\omega})^{1/2}} + \mu_0 - \mu_1 \tag{9b}$$

This approaches infinite depth as $\tilde{\omega}$ nears 1 only slightly slower than Chamberlain's $\langle\tau\rangle$ in Equation (4).

This raises the obvious question, if τ_{eff} is so similar to $\langle\tau\rangle$, why has Kattawar argued otherwise? The answer is found in his calculational model. The basic problem is that his model atmosphere is finite. This is not

appropriate for a comparison with $\langle\tau\rangle$, which is derived for a semi-finite homogeneous medium.

The identity of τ_{eff} is further obscured because the top tenuous cloud layers are weighted the same as the deep atmosphere since his integration step size is in kilometers (h_{eff}) and not optical depth.

In Figure 1 we have compared a typical example of Kattawar's radiance distribution with a classic source function. In effect, these are the two weighting functions for the calculations of τ_{eff} (A) and $\langle\tau\rangle$ (B) in Equations (7) and (1) respectively. The effects of the finite atmosphere on τ_{eff} are evident. As $\tilde{\omega}$ approaches unity, τ_{eff} will remain finite. However, with the above defects corrected, Kattawar's τ_{eff} would match $\langle\tau\rangle$ in behavior.

Appendix I

The Escape Probability

In order to calculate the probability of escape from a semi-infinite cloud for a photon which at time $t = 0$ scatters at depth τ, we refer to the classical random walk problem with an absorbing screen as put forth by Chandrasekhar (1943). Then, the probability of escape in precisely n scatterings is given by

$$P(\tilde{\omega}, \tau, n) = \tilde{\omega}^n \frac{\tau}{n} \left(\frac{2}{\pi n}\right)^{1/2} e^{-\tau^2/2n}. \tag{10}$$

Therefore, the total escape probability is

$$P(\tilde{\omega}, \tau) = \frac{\int_1^\infty \tilde{\omega}^n \frac{\tau}{n} \left(\frac{2}{\pi n}\right)^{1/2} e^{-\tau^2/2n}\, dn}{\int_1^\infty \frac{\tau}{n} \left(\frac{2}{\pi n}\right)^{1/2} e^{-\tau^2/2n}\, dn}. \tag{11}$$

Since $P(\tilde{\omega}, \tau, 0) = 0$ (i.e. the probability for escape from depth τ with zero scatterings is negligible) then we can extend the integrals' lower limit to zero. Thus, we have for the numerator

$$I_n \approx \left(\frac{2}{\pi}\right)^{1/2} \tau \int_0^\infty n^{-3/2} e^{-A/n} e^{-mn}\, dn$$

where $A = \tau^2/2$ and $m = -\ln(\tilde{\omega})$. Consider

$$f(A) = \int_0^\infty e^{-A/n - mn} n^{-3/2}\, dn. \tag{12}$$

We see that

$$f(A) = -\frac{d}{dA} \int_0^\infty e^{-A/n - mn} n^{-1/2} \, dn.$$

Letting $x = (n)^{1/2}$, we have

$$f(A) = -2 \frac{d}{dA} \int_0^\infty e^{-A/x^2 - mx^2} \, dx.$$

Now letting $x = (A)^{1/2} y$, we obtain

$$f(A) = -2 \frac{d}{dA} \left[(A)^{1/2} \, g(\gamma) \right] \tag{13}$$

where

$$g(\gamma) = \int_0^\infty e^{-\gamma y^2 - 1/y^2} \, dy \tag{14}$$

and $\gamma = mA$. Now, considering $g(\gamma)$, let $z = 1/y^2$. We then derive

$$g(\gamma) = \int_0^\infty e^{-\gamma/z^2 - z^2} \, dz/z^2$$

so that

$$g(\gamma) = -\frac{d}{d\gamma} \left[\int_0^\infty e^{-\gamma/z^2 - z^2} \, dz \right].$$

Finally making the change of variable $z = (\gamma)^{1/2} y$, we have

$$g(\gamma) = -\frac{d}{d\gamma} \int_0^\infty e^{-1/y^2 - \gamma y^2} \, (\gamma)^{1/2} \, dy.$$

Substituting from Equation (14) then gives

$$g(\gamma) = -\frac{d}{d\gamma}\left[(\gamma)^{1/2}\, g(\gamma)\right]$$

with a solution of

$$g(\gamma) = C\gamma^{-1/2} \exp\left(-2\gamma^{1/2}\right)$$

where C is a constant of integration. Substituting back into Equations (13) and (12) gives

$$P(\varpi, \tau) = \exp\left[-\left(\tau^2 \ln\left(1/\varpi^2\right)\right)^{1/2}\right].$$

Acknowledgements

This work was supported by the Planetary Atmosphere program of the National Aeronautics and Space Administration under NSG-7043 and by the Atmospheric Research Section, National Science Foundation.

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Figure Caption

Figure 1. Optical depth dependence of (A) a radiance distribution for a finite atmosphere (Kattawar, 1979; Figure 2b) and (B) a source function for a semi-infinite homogeneous atmosphere (Chamberlain, 1970).