Supersonic Stall Flutter of High Speed Fans

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INTRODUCTION

Flutter can result in costly (both in time and money) overruns in turbofan-engine development programs. Solving the problem of flutter (at the engine development stage) may well mean major engine redesign and retesting. For this reason, engine manufacturers and government agencies are currently supporting numerous research programs in an attempt to develop flutter prediction systems that can be used to design flutter-free engines. To date, these research programs have identified five regions of the compressor performance map where flutter is generally encountered (Fig. 1).

Of the five regions shown in Figure 1, the supersonic low back-pressure flutter region has been the most thoroughly investigated analytically (refs. 1 to 3). In general, these analyses have considered the flow field through a cascade of two-dimensional airfoils undergoing simple harmonic pitching or plunging. The gas stream was assumed to be an inviscid, nonconducting, perfect gas. Shock waves that originated in the flow field were assumed to be weak so that supersonic small-disturbance theory could be used. Although these assumptions appear to oversimplify the flow conditions encountered by a rotor at the onset of flutter, the flutter boundaries predicted by these analyses correlate well with experimental data.

A recent analysis (ref. 4) has attempted to apply the supersonic, linearized, small-disturbance theory to higher backpressure operating conditions (i.e., region IV of Fig. 1) by including a finite-strength shock wave within the cascade passage. Results from this analysis show that the unsteady motion of the shock wave tends to induce bending flutter. The existence of this flutter mode is documented in Reference 5.

The remaining operating region where supersonic flutter occurs (region V of Fig. 1) lies close to the stall line of a stage. Analyses of this region have not appeared in the open literature. Literature on this subject (e.g., refs. 5 to 7) generally presents experimental data to document the extent of the flutter region. These data provide a very limited base from which an empirical correlation can be derived for predicting the onset of this flutter mode.

The objective of the present analysis is to develop a model for predicting the onset of supersonic stall flutter encountered by rotors that do not have part-span dampers or tip shrouds. Experimental data reveal that the flutter mode is generally the first flexural mode at reduced frequency (based on tip relative velocity and tip semi-chord) of about 0.2. The vibratory pattern around the rotor tends to be very regular: All blades vibrate at the same frequency but are shifted in phase by a positive interblade phase angle of about 20° to 50° (ref. 7). This positive phase shift implies that the vibratory pattern is traveling around the wheel in the direction of rotation (i.e., a forward-traveling wave).

A further characteristic of the flow regime associated with stall flutter is shown by the steady-state pressure distribution in Figure 2. This pressure distribution was produced from measurements taken across a rotor tip while the rotor was operating near the flutter boundary. This figure clearly indicates a detached, leading-edge bow shock wave impinging on the suction surface of the adjacent blade. The large extent of the compression region at the base of the shock wave seems to suggest a separated flow region that would increase in size as the fan operating point moved toward the supersonic stall bending flutter boundary.
The present analysis develops a flutter model for the highly complicated flow field illustrated in figure 2 by using two-dimensional actuator disk theory. The effects of flow separation are included in the model through zero-loss and deviation-angle correlations. For low-speed flows, actuator disk flutter models have been able to predict the onset of a single-degree-of-freedom bending flutter mode (e.g., refs. 8 and 9). The success of the models at low speeds suggests that a compressible actuator disk model might be capable of predicting the onset of bending flutter at supersonic speeds. This hypothesis has been confirmed by a number of calculations. These calculations, however, did require as input the interblade phase angle at the onset of flutter. So that this requirement could be avoided, the actuator disk model was modified to allow for moderate values of interblade phase angle. This modification results in a flutter model that can be developed into a flutter prediction system. The validity of this flutter prediction system is demonstrated by comparing the predicted flutter boundary of a high-speed fan stage with its measured boundary.

**FORMULATION**

The present analysis for stall bending flutter of an isolated rotor models the rotor as a two-dimensional cascade of airfoils. The cascade is defined by the blade-element geometry on a cylindrical surface at a distance $R$ from the axis of rotation. The flow field in the cascade plane is assumed to be two dimensional, compressible, and time-dependent. viscous forces are considered only within the blade channel. The unsteady flow variables associated with the blade flutter motion are assumed to be smaller than their steady-state counterparts. These variables at an instant in time are required to be periodic around the wheel at a period equal to a fractional part of the circumferential distance $d = 2\pi R$.

The motion of the airfoils in the cascade plane is restricted to simple harmonic plunging and edgewise motion at a cyclic frequency $\omega$ (fig. 3). In addition the motion of each airfoil at an instant in time is assumed to be shifted from that of its neighbor by an interblade phase angle $\alpha = 2\pi n/N$, where $n$ is an integer and $N$ is the number of blades in the rotor. In the present analysis the reduced-frequency parameter $k_b$ and the interblade phase angle are assumed to be small. (Reduced frequency $k_b = \omega b/\omega_{\infty}$, where $b$ is the semichord of an airfoil and $\omega_{\infty}$ is the magnitude of the relative inlet velocity in the cascade plane.)

**Field Equations**

The governing linearized equations for the flow field upstream and downstream of the cascade are written with respect to the relative coordinate system. The variables in these equations are non-dimensionalized by the steady inlet speed $\omega_{\infty}$, the circumferential distance around the wheel $d$, the inlet steady-state fluid density $\rho_{\infty}$, and the inlet steady-state fluid temperature $T_{\infty}$. In addition the subscripts $(\omega, \omega_{\infty})$ signify downstream and upstream flow variables.

The linearized continuity equation for the upstream and downstream flow fields is

$$\frac{\partial \tilde{\rho}_{\omega}}{\partial t} + \frac{\delta \tilde{U}_{\omega}}{\partial x_1} = 0 \quad (1)$$

where the tilde denotes a time-dependent variable and the bar signifies a steady-state variable. The variables $\tilde{\rho}_{\omega}$, $\tilde{U}_{\omega}$, and $\tilde{x}_1$ are the density, velocity components, time, and spatial coordinates, respectively. (See fig. 3 for definition of coordinate system.) In addition, the use of repeated indices denotes summation with respect to the repeated index. The linearized momentum equations for the flow fields are

$$\frac{\partial \tilde{U}_{\omega}}{\partial t} + \frac{\partial \tilde{P}_{\omega}}{\partial x_1} = \frac{1}{\rho_{\infty}} \frac{\partial \tilde{p}_{\omega}}{\partial x_1} = 0 \quad (2)$$

where $\tilde{P}_{\omega}$ is the nondimensional pressure. The remaining field equations are the energy equation for an inviscid, nonconducting gas

$$\frac{\partial \tilde{S}_{\omega}}{\partial t} + \frac{\partial \tilde{U}_{\omega} \tilde{S}_{\omega}}{\partial x_1} = 0 \quad (3)$$

and the equation of state

$$\frac{\partial \tilde{p}_{\omega}}{\partial t} + \frac{1}{\rho_{\infty}} \frac{\partial \tilde{p}_{\omega}}{\partial x_1} = 0 \quad (4)$$

The variables appearing in equations (3) and (4) are the entropy $S$; the ratio of specific heats $\gamma$; and the inlet steady-state relative Mach number $M_{\infty}$. The energy equation was nondimensionalized with respect to $\gamma(\gamma - 1)/\gamma R^2 P_{\infty}$.

The present analysis assumes that there are no sources of entropy or vorticity upstream of the cascade. The upstream, unsteady velocity field must therefore be irrotational and hence is equal to the gradient of a potential function $\Phi_{\omega}$, where

$$\underline{U}_{\omega} = \frac{\partial \Phi_{\omega}}{\partial x_1} \quad (5)$$

The unsteady velocity field downstream of the cascade is expressed as

$$\underline{U}_{\omega} = \frac{\partial \Phi_{\omega}}{\partial x_1} = \frac{\partial \Phi_{\omega}}{\partial x_1} + \frac{\partial \Phi_{\omega}}{\partial x_1} \quad (6)$$

where $\frac{\partial \Phi_{\omega}}{\partial x_1}$ represents the irrotational component of the field and $\frac{\partial \Phi_{\omega}}{\partial x_1}$ its rotational component. The source of the rotation field is the vorticity shed by the oscillating airfoils.

The solution to the field equations is obtained by assuming a simple harmonic spatial and temporal dependency for the potential, the rotational velocity fields and the downstream entropy field. The details of the solution procedure can be found in reference 10.

**Boundary Conditions**

It is shown in reference 10 that for boundary conditions must be specified at the actuator disk to relate the unsteady flow field to the cascade deformation and the steady inlet flow conditions.

The first of these boundary conditions requires the flow to be continuous across the deforming disk. The analytical form of this boundary condition is
\( (1 + \frac{e}{\gamma - 1}) u_{n1}^{\text{Re}} = (\frac{p_0}{\gamma} + \frac{p_1}{\gamma}) u_{n1}^{\text{Re}} \) at \( x_1 = 0 \) \( (7) \)

where \( u_{n1}^{\text{Re}} \) represents the normal velocity components relative to the deforming disk. These velocity components to first order (i.e., small cascade-plane deflection) are equal to

\[
\begin{align*}
    u_{n1}^{\text{Re}} & = U_1^{\text{Re}} - \frac{\partial U_1^{\text{Re}}}{\partial x_1} x_1 - \frac{\partial U_1^{\text{Re}}}{\partial x_2} x_2 - \frac{U_1^{\text{Re}}}{2} \frac{\partial^2 U_1^{\text{Re}}}{\partial x_2^2} x_1^2 + \frac{U_1^{\text{Re}}}{2} \frac{\partial^2 U_1^{\text{Re}}}{\partial x_2^2} x_2^2 \\
    u_{n2}^{\text{Re}} & = U_2^{\text{Re}} - \frac{\partial U_2^{\text{Re}}}{\partial x_1} x_1 - \frac{\partial U_2^{\text{Re}}}{\partial x_2} x_2 - \frac{U_2^{\text{Re}}}{2} \frac{\partial^2 U_2^{\text{Re}}}{\partial x_2^2} x_1^2 + \frac{U_2^{\text{Re}}}{2} \frac{\partial^2 U_2^{\text{Re}}}{\partial x_2^2} x_2^2
\end{align*}
\]

where \( h_1 \) is the displacement of the disk from its mean position in the axial \( (x_1) \) direction (fig. 4).

The next boundary condition requires the total enthalpy with respect to the deforming actuator disk to be locally conserved across the disk. The equivalent mathematical statement is

\[
\frac{1}{\gamma - 1} \frac{1}{T_{\text{m}}} \frac{1}{T_{\text{m}}} + \frac{1}{2} U_1^{\text{Re}} + \frac{1}{2} U_2^{\text{Re}} \] at \( x_1 = 0 \) \( (10) \)

where \( T_{\text{m}} \) is the upstream and downstream gas temperature and \( q_{\text{Re}}^{\text{m}} \) is the magnitude of the relative velocity approaching and leaving the disk. The expressions for \( q_{\text{Re}}^{\text{m}} \) are

\[
q_{\text{Re}}^{\text{m}} = \left[ \left( U_1^{\text{Re}} - \frac{\partial U_1^{\text{Re}}}{\partial x_1} x_1 - \frac{\partial U_1^{\text{Re}}}{\partial x_2} x_2 - \frac{U_1^{\text{Re}}}{2} \frac{\partial^2 U_1^{\text{Re}}}{\partial x_2^2} x_1^2 + \frac{U_1^{\text{Re}}}{2} \frac{\partial^2 U_1^{\text{Re}}}{\partial x_2^2} x_2^2 \right)^2 + \left( U_2^{\text{Re}} - \frac{\partial U_2^{\text{Re}}}{\partial x_1} x_1 - \frac{\partial U_2^{\text{Re}}}{\partial x_2} x_2 - \frac{U_2^{\text{Re}}}{2} \frac{\partial^2 U_2^{\text{Re}}}{\partial x_2^2} x_1^2 + \frac{U_2^{\text{Re}}}{2} \frac{\partial^2 U_2^{\text{Re}}}{\partial x_2^2} x_2^2 \right)^2 \right]^{1/2}
\]

The third boundary condition requires that the local inlet and exit flow angles relative to the deforming disk satisfy the deviation angle correlation of the form

\[
\delta = g(a, M, \nu, \nu') \] \( (13) \)

where \( a \) is the local, relative incidence angle, \( M \) is the local, relative inlet Mach number, \( \nu \) is the local solvity of the deformed cascade, and \( \nu' \) is the local stagger angle measured from the normal to the leading-edge plane (fig. 4).

The variable \( \delta \) in this equation is the angle between the chord line and the tangent to the camber line at the leading edge. The corresponding relation for the local deviation angle is

\[
\delta = a' + \alpha - \delta \] \( (14) \)

where \( a' \) is the local, relative exit flow angle measured from the normal to the trailing-edge plane and \( \delta \) is the angle between the chord line and the tangent to the camber line at the trailing edge.

Relationships for the angles, inlet Mach number and cascade solidity in the above equations are derived by expanding these variables along with equations (13), (14) and (15) to first order. The details of this expansion can be found in reference 10.

The last boundary condition specifies the local entropy that is generated as the fluid passes through the cascade. The loss associated with the local entropy rise is defined in terms of a loss coefficient defined as

\[
\frac{\delta}{\nu_{\text{m}}} = \frac{\rho_{\text{Re}}^{\text{m}} - \rho_{\text{Re}}^{\text{m}}}{\frac{1}{2} \rho_{\text{Re}}^{\text{m}}} = \frac{\rho_{\text{Re}}^{\text{m}} - \rho_{\text{Re}}^{\text{m}}}{\frac{1}{2} \rho_{\text{Re}}^{\text{m}}} \] \( (16) \)

The variable \( \frac{\delta}{\nu_{\text{m}}} \) in this equation represents the local total-pressure loss relative to the deforming actuator disk. The entropy rise across the disk is related to this quantity by the equation of state

\[
\rho_{\text{Re}}^{\text{m}} - \rho_{\text{Re}}^{\text{m}} = \rho_{\text{Re}}^{\text{m}} \left( 1 - e^{-\frac{\nu_{\text{m}}^2}{\nu_{\text{m}}} \Delta s} \right)
\]

Thus \( \Delta s \) must be equal to

\[
\Delta s = \frac{1}{\gamma_{\text{m}}} \ln \left( 1 - \frac{1}{2} \frac{\rho_{\text{Re}}^{\text{m}} - \rho_{\text{Re}}^{\text{m}}}{\frac{1}{2} \rho_{\text{Re}}^{\text{m}}} \right)
\] \( (17) \)

The analysis assumes that the local loss coefficient \( x \) is related to the local inlet flow field, as observed with respect to the deforming disk, by a relationship of the form

\[
x = x(a, M, \nu, \nu') \] \( (18) \)

As in equation (13) the parameters appearing in this equation can be related to the unsteady flow field approaching the cascade and the cascade deflection. Upon combining equation (17) with equation (18) the entropy rise across the cascade can be evaluated in terms of these perturbed quantities.

Aerodynamic Force

Having described the procedure for determining the flow field surrounding the cascade, the unsteady forces acting on the cascade and hence its stability can be assessed. The aerodynamic force acting on the cascade is obtained by considering the flow field through a control volume that is fixed to a cascade passage (fig. 3). The present analysis assumes that the relative motion between neighboring airfoils, a measure of which is the interblade phase
angle, is small and that the reduced frequency of this motion is also small. The first assumption implies that the spatial variations of the flow variables across a cascade passage are small and thus can be neglected. The low-frequency assumption suggests that the rate of change of mass and momentum within the control volume is smaller than the net mass and momentum flux across the control volume surface and thus will also be neglected. Based on these considerations the linearized momentum equation for the control volume illustrated in Figure 3 can be written as

\[ \mathbf{F}_1 = (\mathbf{p}_m - \mathbf{p}_w) \mathbf{T} + (\mathbf{p}_m - \mathbf{p}_w) \mathbf{T} \]

\[ + \mathbf{w}^\text{rel}(\mathbf{u}_{1,m} - \mathbf{u}_{1,w}) + \mathbf{w}(\mathbf{u}_{1,m} - \mathbf{u}_{1,w}) \]

\[ \mathbf{F}_2 = (\mathbf{p}_m - \mathbf{p}_w) \mathbf{T} \mathbf{n}_2 \]

\[ + \mathbf{w}^\text{rel}(\mathbf{u}_{2,m} - \mathbf{u}_{2,w}) + \mathbf{w}(\mathbf{u}_{2,m} - \mathbf{u}_{2,w}) \]

where \( \mathbf{F}_1 \) is the force exerted by the airfoils on the control volume, \( T \) is the pitch of the cascade, \( \mathbf{w}^\text{rel} \) is the mass flow exiting the control volume, and \( \mathbf{n}_1 \) is the unit vector normal to the exit plane of the control volume. This equation can be derived by expanding the quasi-steady momentum equations for the control volume shown in Figure 3 to first order in perturbed variables. The details can be found in reference [6].

The work done by the aerodynamic force on an airfoil over a cycle of motion is equal to

\[ W_{\text{Aero}} = -\frac{1}{2} Re \left[ \int_0^\infty \frac{2\pi}{k} (\mathbf{F}_1 \mathbf{T} + \mathbf{F}_2 \mathbf{T} \mathbf{n}_2) \right] \]

(21)

where \( Re [\cdot] \) denotes the real part of the expression and the asterisk superscript signifies the complex conjugate of the variable. The airfoil's motion can be described by

\[ \mathbf{n}_1 = h_1 e^{-ikx_2} e^{ikt} \]

\[ \mathbf{n}_2 = h_2 e^{-ikx_2} e^{ikt} \]

where \( k_2 \), the wave number in the \( x_2 \) direction, is equal to \( k_2 = 2\pi n \). Equation (21) can be integrated to yield

\[ W_{\text{Aero}} = -2\Im(\mathbf{F}_{1,n}) \]

(22)

where \( \Im [\cdot] \) represents the imaginary part of the expression within the brackets. At the onset of flutter the aerodynamic work per cycle is equal to the mechanical energy dissipated over a cycle. If the mechanical dissipative force is proportional to the velocity of the airfoil, the mechanical energy dissipated over a cycle of motion is

\[ W_0 = \frac{C}{2\pi - \phi_0 - \phi} Re \left[ \int_0^{2\pi/k} \left( \frac{\mathbf{w}_1 \mathbf{w}_2^* + \mathbf{w}_2 \mathbf{w}_1^*}{\mathbf{v}_1 \mathbf{v}_2^*} \right) dt \right] \]

(23)

where the damping coefficient \( C \) is defined as

\[ C = \frac{\mathbf{w}_0}{2\pi - \phi_0 - \phi} \]

(24)

The variable \( C \) in this equation is the log decrement of the mechanical damping, and \( \mathbf{w}_0 \) is the mass per unit length of a rotor blade. This variable is defined as

\[ \mathbf{w}_0 = 2\pi k \left( \frac{\mathbf{w}_0}{\mathbf{w}_0^*} \right) \]

(25)

The symbol \( \mathbf{w}_0 \) denotes the minimum value of the function with respect to \( k_2 \). If the available mechanical damping of a rotor blade exceeds this critical level, any small bending motion imparted to the blade will decay in time. Hence the system is stable and flutter will not occur. However, if the onset is greater than the available mechanical damping of a rotor blade, any small bending motion imparted to the blade will cause the blade to flutter.

RESULTS

For the flutter mode under study the amplitude of the motion increases monotonically along the blade span from the node line. For blades that are rigidly fixed at their root (i.e., mechanically constrained), the node line can be assumed to lie outboard of the blade platform (i.e., outboard of the aerodynamic hub of the blade). The node line of the first flexural mode of the rotor to be investigated later in this section is approximately 20 percent of span height outboard of the platform. Thus, the vibratory motion of the blade in the rotor at any spanwise section is

\[ h_1 = -\zeta \sin \omega_1 t \]

(28)

\[ h_2 = \zeta \cos \omega_1 t \]

(29)
The amplitude of the motion at the section and in the blade element is the stagger angle at 20 percent of span.

The aerodynamic work per unit of span at a given radial location depends on the amplitude of the motion of the section and the relative dynamic pressure of the incoming streamline to the section. Thus, both increase with distance from the hub, the total work done by the airstream on a blade is strongly influenced by the unsteady flow field surrounding the tip region. A simple calculation shows that this influence is concentrated in the outer 25 percent of the span. If the outer-casing boundary layer is assumed to influence the flow field over 5 percent of the tip region of a blade, a direct correlation might exist between the flutter boundary of a rotor and a two-dimensional cascade whose geometry and dynamic response coincide with that of the rotor at 85 percent of span (i.e., a geometric mean of 75 and 95 percent of span). The results presented in the remainder of this section are based on this premise.

The objective of the first series of calculations is to establish the influence of reduced frequency \( k_b \) and interblade phase angle on the aerodynamic stability of a cascade of airfoils. For this study a normalized damping parameter defined as

\[
\alpha = \frac{1}{2} \frac{\overline{\rho}}{\rho_{0,\text{Amb}}} \frac{1}{k_b^2} \left[ \sin \left( \frac{2\pi}{20} \right) J_m \left( f_1 e^{-ik_c X_2} e^{-ikt} \right) \right. \\
+ \cos \left( \frac{2\pi}{20} \right) J_m \left( f_2 e^{-ik_c X_2} e^{-ikt} \right) \right]
\]

(30)

is calculated as a function of \( k_b \) and the interblade phase angle. The parameter \( \overline{\rho} \) in this equation is the inlet stagnation density measured in the absolute coordinate system. For \( \alpha \) greater than zero, there is energy being supplied to the cascade. If the mechanical damping of the cascade system is zero, the operating condition would be unstable. For values of \( \alpha \) less than zero the cascade is doing work on the airstream, and hence the system is stable.

Solving equation (30) requires as input information the steady-state inlet flow properties, the geometry of the cascade, and the frequency of oscillation. A set of loss and deviation-angle correlations must also be specified. The present analysis makes use of the correlations derived in reference 10. The steady-state inlet flow conditions are derived from the blade-element data, measured at 85 percent of design speed, of the second stage of the NASA 1450 ft/sec-tip-speed two-stage fan. This blade-element data set is reported in reference 6. The cascade geometry represents the geometry of the second-stage rotor element at 85 percent of span height from the hub.

Figure 5 shows a plot of \( \alpha \) as a function of interblade phase angle and reduced frequency \( k_b \) for the second stage rotor operating at the flutter boundary at 85 percent of design speed. This figure shows that the bending flutter mode is associated with a positive interblade phase angle (i.e., implies that the vibration pattern is traveling around the rotor in the direction of rotation). The figure also shows that increasing the reduced frequency stabilizes the motion at a constant interblade phase angle.

Figure 6 shows a plot of \( \alpha \) as a function of interblade phase angle and percentage of design weight flow along the 85-percent-speed line. For a given interblade phase angle, decreasing the weight flow at constant wheel speed (i.e., increasing the steady-state aerodynamic loading) tends to destabilize the rotor.

The results presented in figures 5 and 6 are also consistent with the observed characteristics of supersonic bending flutter reported in reference 7. A deficiency of the present theory, as shown by the results in figures 5 and 6, is that it predicts the damping, \( \alpha \), to increase monotonically with interblade phase angle. This trend is physically unrealistic, and is a consequence of the small interblade phase angle assumption inherent in actuator disk theory. To overcome this deficiency an additional assumption was incorporated into the theory which limited the value of the aerodynamic damping. The assumption is made to replace \( \alpha \) by \( \sin \alpha \) everywhere in the analysis. This assumption is mathematically consistent with the actuator disk model. Figure 7 shows the results obtained with this assumption incorporated into the analysis. For \( |\alpha| \leq 45^\circ \), the modified analysis yields results that are comparable to the results of the original formulation. In addition the modified analysis predicts a finite maximum value for \( \alpha \) for \( \alpha = 90^\circ \). This maximum value for damping is less than the value computed at \( \alpha = 90^\circ \) using the modified formulation. The experimental results of reference 7 show that this value for \( \alpha \) (i.e., \( \alpha = 90^\circ \)) corresponds rather closely to the measured interblade phase angle of the least stable flutter mode.

An additional validation of the modified analysis is as a flutter prediction system for supersonic stall bending flutter can be established by showing that, for a given rotor, flutter will be observed whenever the predicted maximum value of \( \alpha \) exceeds the normalized structural damping of the rotor assembly. For this demonstration the measured flutter boundary of a scaled model of Fan C of the NASA Quiet Engine Program (ref. 6) is correlated with the predicted boundary. The performance map for the scale model (fig. 8) shows two flutter instability zones. The first zone is torsional flutter that occurs at part speed and lies close to the line. Just above this line lies a zone of bending flutter that appears to extend past 95 percent of design speed. The bending flutter mode of this rotor is analyzed in this paper. Figure 9 shows a plot of the phase shift between the amplitude time history of a rotor blade and the reference blade No. 17 at the 95 percent of design speed flutter point. The data implies the flutter mode is predominantly a 3 nodal diameter forward traveling wave which corresponds to an interblade phase angle of approximately 41°. This value for the interblade phase angle of the flutter mode correlates well with the data of reference 7 and lies within the unstable range predicted by the analysis. Figure 10 shows the calculated maximum normalized aerodynamic damping for this rotor as a function of weight flow for reduced speed 1. The results presented on this curve were calculated using the blade element data recorded at the five operating condition shown on figure 8. The shaded region on figure 10 represents the unstable flutter region. The boundary of this region is estimated by assuming a log decremental damping of the assembly (ref. 3). Hence all operating conditions which have an aerodynamic damp-
The results on Figure 10 clearly show the significant influence of wheel speed and aerodynamic loading on the aerodynamic damping. The calculated aerodynamic damping at 60 and 70 percent wheel speed lie significantly below the instability boundary. No bending flutter was observed at these operating conditions. At 80 percent of design speed bending flutter was observed near the stall region. The calculated aerodynamic damping for the near stall operating point is seen to lie just inside the unstable region. Bending flutter was also observed at 95 percent of design speed near the stall boundary. The calculated damping for this operating point lies well within the unstable region of Figure 10. At the intermediate operating point on the 95 percent speed line the calculated aerodynamic damping falls below the instability boundary. Therefore the predicted flutter boundary at this wheel speed lies between the measured flutter boundary and the intermediate operating point. Transcribing the stability boundary of Figure 10 onto the Fan C performance map yields a theoretical flutter boundary, which is shown as a dashed line on Figure 11. The measured operating conditions at which bending flutter occurred are shown on this figure as solid symbols. The overall agreement between theory and measurement is very good, with a slight overestimation of the extent of the flutter boundary at 95 percent of wheel speed. Below 90 percent of wheel speed the analysis predicts the flutter boundary will bend back into the stall region. This result is confirmed by the experimental measurements.

The results presented in this section clearly show that the current analysis can be used to predict the onset of aerodynamic bending flutter in unshrouded fans. The input variables to the analysis are all derived from fan structural and aerodynamic design variables. The governing equations are simple algebraic equations, and hence are easily programmed. These features provide the designer with a simple and reliable model for predicting the susceptibility of a fan design to supersonic stall bending flutter. In addition it provides him with a tool that can also be used to evaluate proposed fixes to an existing flutter problem.

The engineering approximation introduced to extend the range of validity of actuator disk theory appears reasonable. To develop a correct extension of actuator disk theory to finite interblade phase angle for the flow conditions as they exist at the tip of a rotor at the onset of supersonic stall flutter would require a viscous transonic cascade analysis for finite interblade phase angle. No such analysis appears to be forth coming in the near future. Any extension which is derived from inviscid aerodynamic theory would be an approximation, the result perhaps being no more accurate than the simple extension developed in this analysis.

CONCLUDING REMARKS

In the present investigation an analytical model was developed to predict the onset of supersonic stall bending flutter in high-speed rotors. The stability boundary predicted by the analysis was shown to be in good agreement with the measured boundary of a high speed fan. The analysis also predicted that the flutter mode would be a forward traveling wave, be sensitive to wheel speed and aerodynamic loading. This was confirmed by the experimental measurements. In addition the analysis showed that reduced frequency and dynamic head also play a significant role in establishing the supersonic stall bending flutter boundary of an unshrouded fan.

The analysis provides the designer with a simple and efficient prediction tool which he can use to analyze the susceptibility of a fan design to supersonic stall bending flutter.

APPENDIX A

SYMBOLS

\[ \begin{align*}
  b & \quad \text{airfoil semicord} \\
  c & \quad \text{average blade chord and damping coefficient} \\
  d & \quad \text{circular distance around wheel} \\
  F_1 & \quad \text{aerodynamic force} \\
  h_1 & \quad \text{axial displacement of an airfoil} \\
  h_2 & \quad \text{tangential displacement of an airfoil} \\
  i & \quad \sqrt{2} \\
  K & \quad \text{constant} \\
  k & \quad \text{reduced frequency based on circumferential distance, } \omega d/q_m \\
  k_b & \quad \text{reduced frequency based on semicord, } \omega b/q_m \\
  k_2 & \quad \text{wave number of disturbance along cascade plane} \\
  M & \quad \text{Mach number} \\
  M_b & \quad \text{mass of a rotor blade per unit length} \\
  N & \quad \text{number of blades in rotor} \\
  n & \quad \text{integer} \\
  n_1 & \quad \text{unit vector} \\
  p & \quad \text{total pressure} \\
  p & \quad \text{static pressure} \\
  q & \quad \text{magnitude of velocity field} \\
  R & \quad \text{radial distance} \\
  s & \quad \text{entropy} \\
  T & \quad \text{temperature} \\
  t & \quad \text{time} \\
  U_j & \quad \text{velocity components} \\
  U_n & \quad \text{relative velocity component normal to disk} \\
  U_R & \quad \text{rotational velocity}
\end{align*} \]
\[ W_{\text{Aero}} \] aerodynamic work per cycle of motion  
\[ W_0 \] energy dissipated per cycle of motion by mechanical damping  
\[ w \] mass flow rate  
\[ X_1 \] special coordinates  
\[ \phi \] incidence angle  
\[ \theta \] flow angle  
\[ \gamma \] ratio of specific heats; axial wave number  
\[ \epsilon \] log decrement; deviation angle, eq. (13), eq (15)  
\[ \delta \] metal angle at leading edge  
\[ \delta_1 \] metal angle at trailing edge  
\[ \phi \] stagger angle  
\[ \mu \] solidity  
\[ \xi \] amplitude of blade motion  
\[ \rho \] density  
\[ \sigma \] interblade phase angle  
\[ \tau \] cascade pitch or average thickness of a rotor blade  
\[ \phi \] velocity potential  
\[ \chi \] total-pressure-loss coefficient  
\[ \omega \] frequency of oscillation  

**SUBSCRIPTS**  
\( \text{upstream variable} \)  
\( \text{downstream variable} \)  

**SUPERSCRIPTS**  
\( \text{steady-state variable} \)  
\( \text{time-dependent variable} \)  
\( \text{complex conjugate} \)  
\( \text{relative component} \)  

**REFERENCES**  
Region I
(subsonic/transonic flutter)

Region V
(supersonic stall flutter)

Region IV
(high-backpressure supersonic flutter)

Region III
(low-backpressure supersonic flutter)

Choke boundary

Figure 1. - Compressor performance and stability map.

<table>
<thead>
<tr>
<th>Curve</th>
<th>Static pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>82 (12)</td>
</tr>
<tr>
<td>2</td>
<td>89 (13)</td>
</tr>
<tr>
<td>3</td>
<td>96 (14)</td>
</tr>
<tr>
<td>4</td>
<td>103 (15)</td>
</tr>
<tr>
<td>5</td>
<td>110 (16)</td>
</tr>
<tr>
<td>6</td>
<td>117 (17)</td>
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<tr>
<td>7</td>
<td>124 (18)</td>
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<tr>
<td>8</td>
<td>131 (19)</td>
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<tr>
<td>9</td>
<td>137 (20)</td>
</tr>
<tr>
<td>10</td>
<td>144 (21)</td>
</tr>
</tbody>
</table>

Relative airflow

Figure 2. - Typical tip rotor static-pressure distribution near supersonic stall flutter boundary.
Figure 3. - Cascade plane deformation.

Enlarged view

Figure 4. - Geometry of deformed cascade.
Figure 5. - Calculated, normalized damping as a function of interblade phase angle and reduced frequency for NASA two-stage fan.

Figure 6. - Calculated, normalized damping as a function of interblade phase angle and weight flow for NASA two-stage fan at 85 percent of design wheel speed.
Figure 7. - Original and modified analyses for NASA two-stage fan at 85 percent of design wheel speed and 75.68 percent of design weight flow.

Figure 8. - NASA rotating rig, fan C model.
Figure 9. - Rotating rig blade phase data 95% speed, flexural flutter.
Figure 10. - Maximum normalized damping as a function of weight flow and wheel speed for fan C model.

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Figure 11. - NASA rotating rig, fan C model.