Final Report
Propeller Propulsion Integration Phase I

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ABSTRACT

This report summarizes the propeller propulsion integration (PPI) study conducted under this grant. The work is continuing under the Lewis Research Center direction. A bibliography has been compiled of all readily available sources of propeller analytical and experimental studies conducted during the 1930 through 1960 period. A propeller test stand was developed for the measurement of thrust and torque characteristics of full scale general aviation propellers and installed in the LaRC 30 x 60 foot full scale wind tunnel.

A tunnel entry was made during the January through February 1980 period. Several propellers were tested, but unforeseen difficulties with the shaft thrust-torque balance severely degraded the data quality.
I. INTRODUCTION

The Propeller Propulsion Integration (PPI) research program, initiated in April 1977, under this Grant NSG 1402, was established to help the general aviation industry design propeller propulsion systems. There has always been concern about the lack of definitive experimental data and of useable analytical methods to define the interactions between propeller and the airframe.

The escalating cost of fuel has placed increased emphasis upon the fuel efficiency of general aviation aircraft. The increasing level of sophistication of the panel methods for the analysis of flow about wings and bodies has made possible the prospect of being able to model the complex propeller/airframe interaction problem. Thus, the PPI program was initiated to carry out a set of experiments which would establish a data base for the definition of interference effects and for the validation of analytical methods. After each wind tunnel experiment, comparisons are to be made between theory and experiment.

To carry out these objectives the PPI overall research program can be summarized in the following major steps.


2. Define General Aviation Isolated Propeller Characteristics (Phase I).

3. Define General Aviation Propeller/Nacelle Interference Characteristics (Phase II).
4. Define General Aviation Propeller/Nacelle/Wing/Fuselage Interference Characteristics (Phase III).

5. Develop Analytical Propulsion Integration Methods for General Aviation Aircraft Design (Phase I through Phase IV).

Phase Zero was a review of the state-of-the-art in terms of current design practice and a determination of industry design requirements and recommendations for program emphasis. A detailed account of the discussions with industry design teams is reported in Reference 1.

The industry state-of-the-art design process is best represented as table look-up methods. One or more catalogues, such as the Hamilton Standard "Red Book" or the "Gray Charts," which list isolated propeller performance characteristics in terms of geometric parameters, are used to select propeller candidates. Performance flight test measurements are then used to make the final choice. In most cases, little account is taken, either during the airframe design stage or during propeller selection, of the interference between propeller and airframe upon the predicted installed propulsive efficiencies. This is due to the absence of suitable design data and practical analytical tools.

A comprehensive review of the literature has been undertaken also as a part of Phase Zero and is included in this report as Appendix A. Over one thousand reports and papers have been identified which relate to propeller design or selection, but, few of these consider the mutual influence of propeller and airframe. None considered the geometry peculiar to contemporary flat engine designs with asymmetric blockage-area distribution around the propeller shaft. Some insight can, however, be gained by analysis of the available data. Figure 1, which was obtained by a plot
of Reference 2 data, shows clearly the influence of an afterbody on apparent propeller efficiency, which becomes greater than 1 with a blockage ratio, \((a/D)\), over 0.5. The net efficiency, which is directly proportional to the available thrust power decreased dramatically so that the net thrust available for propelling the airplane is greatly and adversely affected by a blunt afterbody even though apparent propeller efficiency is over 100%. Figure 2 shows data for a simple streamline afterbody shape and the effect is less pronounced. The influence of thrust line displacement and thrust line angle relative to a wing chord is shown in Figure 3. The data shows a maximum variance of 10 percentage points for a 15% displacement of the thrust line and 5° thrust line angle. Thus, the thrust line location and angle are also quite important in determining the net efficiency of the propulsion system. All of these effects upon propulsive efficiency need to be explored further through experiment and analysis.

The goal of the PPI research program is to develop such design data and analytical tools. This goal is to be attained through a combined program of wind tunnel investigations in the NASA Langley 30' x 60' Full Scale Wind Tunnel and the development of appropriate analytical design methods. Where possible, specific tests or analyses are to be performed to bring into use results generated from previous investigations during the 1930's and 1940's involving primarily radial engine configurations.

In the Phase I program, a Propeller Test Stand (PTS), for use in the full scale tunnel, was designed and fabricated. The test stand is shown in Figure 4 and the installation in the full scale tunnel is shown in Figure 5. The Propeller Test Stand is capable of testing full general
aviation propellers using a variable frequency electric drive. The propellers can be operated over a 10 degree angle of attack range. A thrust/torque balance is used to measure shaft thrust. The PTS is attached to the wind tunnel balance to measure total forces. The first tunnel entry was made during the period January through early March 1980. The goal of this entry was to develop the isolated propeller baseline data for future airframe interference tests. A family of 13 test propellers were selected and arrangements were made to acquire the propellers at no cost to the program. The propellers and first entry test plan are described in Reference 3. Comparisons were to be made between the measured data and a current analytical propeller model.

This report summarizes the first wind tunnel entry for Phase I of the PPI study. At this point the management of the PPI investigation was transferred to the Lewis Research Center under Grant NAG-3-56. The LeRC Grant was supposed to conduct Phase II of the PPI study, but due to unforeseen shaft balance problems, the isolated propeller tests will also be repeated using an abbreviated test program. Phase II, is designed to explore propeller/nacelle interference effects. The PTS developed during Phase I will be used for this experiment. Two nacelle shapes are to be considered; a single engine nacelle, Figure 6, and an axisymmetric body, Figure 7. The shaft thrust is to be measured along with body pressure distribution and wake surveys. A critical survey of analysis programs available for the analysis of propeller performance in a nonuniform flow field and of the interaction of a propeller slipstream upon an airframe will be made. Comparisons will be made between the analytical methods and data obtained from the PPI wind tunnel experiments and other sources.
Phase III, is the final experimental step in the development of a more complete understanding of the propeller/nacelle/wing/fuselage interference problem. It is contemplated that the PTS will be utilized as shown in Figures 8, 9, and 10, for this experiment. This test stand will be quite flexible and capable of a wide range of configurations.

Phase IV is contemplated as an attempt to optimize the propeller/airframe configuration for overall aircraft efficiency. In this case, the analytical methods will be used to define a configuration (Ref. 4), and the experiment conducted for verification.

This report describes the details of the propeller test stand and examines the data obtained in the first entry in the LaRC 30 x 60 foot full scale wind tunnel. The test stand is capable of a wide range of propeller experiments as outlined in the PPI overview. Further development of the prop shaft thrust-torque balance is required to full exploit the concept.
2. PROPELLER TEST STAND DEVELOPMENT

2.1 Test Stand Design

In Chapter I the three propeller test programs defined for the PPI investigation were described. A study was undertaken to determine the best configuration for a propeller test stand which would allow these three study segments to be conducted using a single drive motor support configuration. Other design considerations were.

1. Utilize two GFE 266 horsepower variable frequency electric motors connected in tandem as the propeller drive motors.

2. Propeller angle of attack range -10 to +20 degree.

3. Motor support structure must minimize interference with propeller flow field.

4. System must be capable of being mounted within a Piper Chieftain nacelle.

5. PTS must be mounted on the 30 x 60 foot wind tunnel force balance to measure total forces.

Figure 4 shows the configuration which was developed to satisfy the specifications outlined above. To minimize propeller interference effects, to minimize propeller vibration levels, and to ease angle of attack change mechanism design difficulties, a steel cantilever beam structure was chosen.

The propeller test stand includes six pieces of structural hardware. These are the motor case, the motor case cradle, the mast, the mast fairing, nacelle, and the sector fairing. The motor case, cradle, nacelle, and sector fairing carry the aerodynamic loads on the propeller,
2. PROPELLER TEST STAND DEVELOPMENT

2.1 Test Stand Design

In Chapter I the three propeller test programs defined for the FPI investigation were described. A study was undertaken to determine the best configuration for a propeller test stand which would allow these three study segments to be conducted using a single drive motor support configuration. Other design considerations were:

1. Utilize two GFE 200 horsepower variable frequency electric motors connected in tandem as the propeller drive motors.

2. Propeller angle of attack range -10° to +20° degree.

3. Motor support structure must minimize interference with propeller test field.

4. System must be capable of being mounted within a Piper Chieftain nacelle.

5. TTS must be mounted on the 30 x 60 foot wind tunnel force balance to measure total forces.

Figure 5 shows the configuration which was developed to satisfy the specifications outlined above. To minimize propeller interference effects, to minimize propeller vibration levels, and to ease angle of attack change mechanism design difficulties, a steel cantilever beam structure was chosen.

The propeller test stand includes six pieces of structural hardware. These are the motor case, the motor case cradle, the mast, the mast fairing, nacelle, and the sector fairing. The motor case, cradle, nacelle, and sector fairing carry the aerodynamic loads on the propeller,
nacelle, and sector fairing through the mast to the wind tunnel balance system. The aerodynamic loads on the mast are shielded from the wind tunnel balance by the mast fairing which is cantilevered from the tunnel floor independently of the balance system.

Figure 11 shows the steel structure of the PTS. An electric jack-screw was used to vary the angle of attack (AOA) while the tunnel was operating. To ensure a fail-safe design a counterweight was added to the structure so that the motor system would pitch up to preclude the propeller from striking the support column if the jack-screw should fail.

The maximum design torque and thrust developed by the propellers are 600 lbf thrust and 4200 in lbf torque. At a speed of 500 RPM and an angle of attack of 12 degrees maximum harmonic variations of 180 lbf in thrust and 6360 in lbf in yaw moment are estimated. The structure must safely support the static loads and must not be excited to vibrate by the harmonic loads. To accomplish this it was decided to design the mast strong enough to support the static loads but flexible enough so that the lowest exciting frequency of 17HZ (500 RPM for a two blade propeller) would be well above the natural frequency of the system.

The natural frequencies of bending and torsion were found to be 3.5HZ, 6.18HZ, and 6.7HZ. These were calculated assuming a rigid support (the balance system is not rigid so the frequencies are actually lower than those calculated), no aerodynamic or structural damping, and the mass of the mast was neglected. The mass on the end of the mast is about seven times the mast mass so one would expect little influence on the natural frequencies due to the mast mass. However, a lumped mass analysis including the mast mass was made to confirm this assumption, and
it produced the same frequencies as above. Complete details of the
static and dynamic structural analysis of the PTS are given in
Appendix B.

Structural details of the support column of the PTS is given in
Figure 12. The structure was conventional welded steel plate construc-
tion. Especial care was taken to weld the structure in steps to mini-
mize warping. The details of the motor support structure is given in
Figure 13. The drive motors were encased in a 0.5 in. steel tube, thus
the structure was not required to align the motors, but rather transfer
the motor weight onto the support column with a minimum distortion.
Also the motor support structure was constrained to minimize the cross-
sectional area of the nacelle. Also the structure was originally required
to fit inside a Piper Chieftain nacelle. This design requirement was
followed for the counterweight design.

The details of the nacelle structure are given in Figure 14. The
nacelle was configured to minimize the interference with the propeller
flow. The nacelle was constructed using standard aluminum structural
practice. The tail cone was constructed of fiberglass to achieve the
desired shape. The nacelle was attached to the motor support structure
at only two points to allow the installation of a nacelle force balance
at a later time. The upper half of the nacelle structure carries all
of the loads with the lower half divided into two parts for ease of
assembly and access.

Figure 15 shows the fairing constructed to shield the support
column from the tunnel flow. The fairing was constructed of aluminum
in two parts to allow easy erection and access to the jackscrew motor,
power cables, and instrumentation lines. A two segment fairing was designed to ensure the intersection between the nacelle and the support column remained a low drag configuration over the -10 to +20 degree AOA range. The constraint was for the fairing to clear the support column when the propeller was at +20 degree and yet fill the gap when the propeller AOA was -10 degrees. The sector fairing is attached to the motor support structure, thus the forces on the sector fairing are measured by the wind tunnel external balance. The sector fairing was constructed of 0.125 soft aluminum plate and contoured using a segmented welded approach.

2.2 Structural Vibration Analysis Summary

This section summarizes the vibration analysis of the PTS given in Appendix B. The initial study showed that the stresses in the structure were well below the allowable except the bolts which attach the PTS to the wind tunnel balance frame. These bolts do not have sufficient strength to withstand the loads induced on them for the case of the loss of a propeller at speed. The remainder of the structure can withstand this condition.

The mast used to support the propeller test drive motors must be cantilevered from the balance table and offer minimum wind resistance. It was also desirable that the mast be tapered to minimize mast thickness at the motor attachment location. A mast height of about fourteen feet was required to place the propellers at the centerline of the tunnel.

The maximum anticipated loads expected for the most extreme test cases were 600 lbf thrust and 290 ft. lbf torque steady loads. The
weight of the motors and structure when added to the applied propeller loads gave a loading which was not severe for a design with even a modest cross-section. Thus it was decided to choose a design based on stiffness criterion rather than strength, since there was no over-riding reason for minimizing the weight or size of the beam. The approach was to design a beam with natural frequencies well below the minimum expected harmonic excitations. This approach allows dynamic amplitudes somewhat greater than static deflections, but the static deflections are small due to the smallness of the loads. The applied loads shown in Figure B2 produced the bending moments, torque, and axial load distributions for the analysis.

The mast deflections and rotations under the assumed loads were computed using Castigliano’s Theorem. Using Castigliano’s method the strain energy was first calculated from which the deflections and rotations were found as derivatives of the strain energy. A matrix formulation of the deflections and rotations in terms of the applied loads was made. If the mass of the mast is neglected and the equations of motion for the motor case – motor cradle – propeller and counterweight system are formulated, the natural frequencies of vibration can be found. Solving the free vibration equations for the five natural frequencies and mode shape gives the following table.
<table>
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<th>MODE</th>
<th>DESCRIPTION</th>
<th>FREQUENCY (Hz)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<td>6.77</td>
<td>0</td>
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<td></td>
<td>fore and aft</td>
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<td>6.18</td>
<td></td>
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<td>2</td>
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<td>1.865</td>
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<td></td>
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<td></td>
<td>39.0</td>
<td></td>
<td>.0366</td>
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<td></td>
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<tr>
<td>4</td>
<td>rolling mode</td>
<td></td>
<td>59.0</td>
<td></td>
<td>0</td>
<td>3.357</td>
<td>.1</td>
</tr>
<tr>
<td>5</td>
<td>yaw mode</td>
<td></td>
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Each mode has been normalized to a maximum rotation of .1 radian. The fore and aft bending mode and the yaw mode are uncoupled from the lateral bending, pitching, and rolling modes.

For a forced vibration analysis, propeller loads must be converted to an equivalent force system at the center of gravity of the motor assembly. At a speed of 500 RPM there is a harmonic thrust force of 180 lbf and a harmonic yaw moment of 6360 in-lbf with frequency twice the rotational speed for a two blade propeller. This condition occurs at an angle of attack of 12 degrees and is the lowest frequency (17 Hz) excitation expected other than an unbalance in the propeller shaft. 17 Hz is well above the bending and torsion frequencies but is below the rolling and yawing frequencies. The maximum dynamic stress is at the base of the mast and is 336 psi which is very small. At the lowest propeller frequency expected, the vibrational modes of the mast are not excited. The magnitude of the exciting loads are also low which helps account for the low dynamic stresses.

The worst case of failure would be to lose a propeller blade while in operation. The maximum stress induced by this condition is 45,000 psi which is greater than the yield stress but less than the ultimate stress.
It is possible that the mast would hold together until the motors could be stopped. The critical component is the mast holddown bolts which would probably fail.

2.3 Wind Tunnel Installation

The PTS was installed in the LaRC 30 x 60 foot full scale wind tunnel as shown in Figure 5. The steel support column was attached directly to the wind tunnel balance frame. The fairing for the support column was attached to the floor plane. The electric motors were driven by a variable frequency master generator set. The nacelle angle of attack was controlled through the jackscrew and sensed by an inclinometer installed on the motor support frame.

2.4 Instrumentation Description

The test was conducted using the LaRC full scale wind tunnel data acquisition system. The propeller thrust-torque balance output was transmitted through a slip ring to data lines installed in the stationary structure. The wind tunnel balance forces were recorded and proved to be the primary source of thrust data. The drive motor currents were monitored, but since the motor torque versus current relationship was not known, torque could not be determined from this source. Vibration accelerometers were installed on the motor support structure near the propeller plane to monitor the vibration levels at the propeller thrust-torque balance. An automatic shut down system was installed to prevent divergence.
3. ENTRY I WIND TUNNEL TEST

3.1 Test Program

A total of fifteen different propellers had originally been selected to test in the NASA Langley Full Scale Wind Tunnel. These propellers are listed in Table 1. The actual test program included seven of these propellers and consisted of 163 runs (each run being a test of a particular propeller at a particular blade angle, angle of attack and tunnel speed with the propeller speed variable during the run). Of these 163 runs only about half yielded useful information concerning propeller performance; the remaining runs were judged unacceptable due to problems which will be discussed shortly. Table 2 lists the propellers, tunnel speeds and blade angles for which possibly useful data was obtained.

The large number of unacceptable runs was the result of several equipment related problems which were encountered during the tests. Approximately 60 runs were initially required to sort out the instrumentation and obtain what was considered to be "good" data. Just as this status had finally been reached the propeller shaft bent during a run. After the shaft was repaired the program essentially started over again. A further 100 runs were made but these were plagued by drift in the output of the thrust-torque balance. This was first observed as a change in the zero tunnel speed-zero propeller speed balance readings (or "zeroes") before and after a run. At least 20 runs made after the shaft was repaired are unacceptable because of a large change in the balance zeroes. Many more runs are of questionable use for this
same reason. There were also situations when vibration of the propeller-afterbody unit became excessive and forced a run to be stopped before the desired range of propeller speeds could be obtained. Finally, some data which initially looked acceptable turned out to yield meaningless results due to a mismatch in the size of the thrust-torque balance. That is, for lightly loaded or small propellers the loads generated were too small to be accurately measured by the balance (which was designed for 1200 lbs. maximum thrust). Therefore the results for the Yankee propellers (configurations 7 and 8), for example, are not valid although the measurements themselves were not subject to excess vibration or zero shift. As a consequence of these various difficulties only a limited quantity of reliable data has been taken and only three propellers can be thought of as being reasonably well-documented (configurations 1, 4 and 10).

Further comments on the thrust-torque balance drift are appropriate. Several tests were made to establish the nature of the drift. Here the propeller and tunnel speeds were set at fixed values and the thrust-torque balance output was monitored. An example of these tests is shown in Figure 16 where the thrust and torque of the Hartzell 2-bladed propeller are plotted versus time. The continual decrease in measured output with time is quite clear. There were also instances when the test engineers observed sudden jumps in the balance output although these were not documented. The reasons for these changes is still unexplained. The drift could, at times, be minimized or eliminated by running the propeller for 10 to 30 minutes prior to taking data for a
given run. This warmup procedure was used in the latter stages of the test program with apparently some success.

3.2 Results

The measured quantities include the thrust, $T$, and torque, $Q$, acting on the propeller as obtained from the thrust-torque balance, the total force acting on the propeller-afterbody combination as measured by the tunnel scales, the propeller blade angle, $\beta$, the propeller rpm, $N$, and the free stream air velocity, $V_\infty$. Measurements of the afterbody drag with no propeller along with a correction to the drag to account for the propeller slipstream permitted the tunnel scales data to be used to provide a second and independent measurement of the propeller thrust. From these measurements the advance ratio $J$, thrust coefficient $C_T$, torque coefficient $C_Q$, power coefficient $C_p$ and efficiency $\eta_p$ are determined. Figures 17 through 22 present $C_T$, $C_p$ and $\eta_p$ for the McCauley 3-bladed prop (configuration 10) at $\beta = 16^\circ$, $28^\circ$ and $40^\circ$ and for the basic Hartzell 2-bladed prop (configuration 1) at the same blade angles. The open symbols refer to data entirely from the thrust-torque balance; the filled symbols are for scales measured thrust.

The thrust coefficients of the McCauley propeller (Figure 17) form reasonable curves with fairly small scatter and close agreement in the two measurement methods. The scatter and disagreement become greater at large values of $J$ as the loading goes to zero. Here the noise and accuracy of the balances is the same magnitude as the thrust so the poorer behavior might be expected. The significance of the fact that
at $\beta = 16^\circ$ the scale $C_T$ is slightly below the thrust-torque balance $C_T$ while at $\beta = 40^\circ$ the situation is reversed is not quite clear. The power coefficients (Figure 18) (which could only be determined from the thrust-torque balance) also form fairly smooth curves with little scatter.

That problems may still exist becomes more apparent when the efficiencies (Figure 19) are examined. The scatter is now greater (though the plotting scale makes the scatter appear worse than it is) and the disagreement between the two thrust measuring techniques is increased, especially for $\beta = 40^\circ$. More serious, however, are the highly suspicious magnitudes of the peak efficiencies which approach, and even exceed, $\eta_p = 1$. Whether the thrust is being overestimated, the torque underestimated or both is not yet certain.

The results for the Hartzell propeller are not as good, especially in terms of agreement between the two thrust measurement methods, as the McCauley results. Figure 20 shows that there are substantial differences in $C_T$ as determined by the two methods for the entire range of $\beta$ investigated, with the thrust-torque balance yielding consistently smaller coefficients. There is also increased scatter in the $C_T$ curves for this propeller. The plots for $C_p$, on the other hand, (Figure 21) are reasonably smooth with small scatter. The scatter and disagreement of the $C_T$ data are also reflected in the efficiency curves in Figure 22. One noticeable aspect of the efficiencies, however, is that, except for two obviously erroneous points, the maximum efficiencies are considerably smaller and more reasonable than the McCauley values.
4. CONCLUSIONS

The propeller test stand proved to be structurally sound and exhibited the predicted supercritical structural modes. The unresolvable thrust-torque balance drift problems precluded a successful test of a range of full scale general aviation propellers. The following recommendations are made.

1. Find the source of drift in the thrust-torque balance.
2. Measure the electric motor torque-current relationship experimentally to allow an independent measurement of propeller torque.
References


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### Table II

**Propellers Tested**

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Figure 1. Nacelle Blockage Effects (NACA Report 177, 1923)
Figure 2. Nacelle Blockage Effects - Partial Streamlining
(NACA Report 177, 1923)
Figure 3. Wing Position Effects (A.R.C. R & M 2374, 1950)
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Figure 15. Details of Aluminum Support Column Wing Shield Fairing.
Thrust and Torque Time History
Hartzell Basic 2 Blade
Configuration 1
β=24°, Tunnel RPM=170
Propeller RPM=1500

Figure 16. Time History of Shaft Balance Thrust and Torque Outputs.
Figure 17. Thrust Coefficient Versus Advance Ratio for the McCauley 3 Blade Propeller.

Balance Scales

McCauley 3 Blade Configuration 10

J - Advance Ratio

C^ - Thrust Coefficient
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Figure 19a. Propulsion Efficiency Versus Advance Ratio for the McCauley 3 Blade Propeller at a Blade Angle of 16°.
Figure 19b. Propulsion Efficiency Versus Advance Ratio for the McCauley 3 Blade Propeller at a Blade Angle of 28°.
Figure 19c. Propulsion Efficiency Versus Advance "Ratio for the McCauley 3 Blade Propeller at a Blade Angle of 40°."

McCauley 3 Blade Configuration 10, β = 40°
Figure 20. Thrust Coefficient Versus Advance Ratio for the Hartnell Two Blade Propeller.
Hartzell Basic 2 Blade Configuration 1

Figure 21. Power Coefficient Versus Advance Ratio for the Hartzell 2 Blade Propeller.
Figure 22a. Propeller Efficiency Versus Advance Ratio for the Hartzell 2 Blade Propeller at a Blade Angle of 16°.
Hartzell Basic 2 Blade
Configuration 1, \( \beta = 28^\circ \)

Figure 22b. Propeller Efficiency Versus Advance Ratio for the Hartzell 2 Blade Propeller at a Blade Angle of 28°.
Figure 22c. Propeller Efficiency Versus Advance Ratio for the Hartzell 2 Blade Propeller at a Blade Angle of 40°.
APPENDIX A

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by

Stan Miley


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APPENDIX B

Structural Integrity Report for Propeller Test Stand
for Langley Full Scale Wind Tunnel

by

John C. McWhorter
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NOTICE

THIS DOCUMENT HAS BEEN REPRODUCED FROM MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE.
\( \bar{x}, \bar{y} \) \hspace{1cm} \text{Components of} \ \bar{r}_G \\
M \hspace{1cm} \text{Mass of Motor Assembly} \\
W \hspace{1cm} \text{Weight of Motor Assembly} \\
A_{Gx}, A_{Gy}, A_{Gz} \hspace{1cm} \text{Acceleration Components of Center of Gravity of Motor Assembly} \\
\ddot{\theta}_x, \ddot{\theta}_y, \ddot{\theta}_z \hspace{1cm} \text{Angular Acceleration Components of Motor Assembly} \\
I_{Mx}, I_{My}, I_{Mz} \hspace{1cm} \text{Moments of Inertia about Body Centroidal Axes} \\
I_{Gx}, I_{Gy}, I_{Gz} \hspace{1cm} \text{Moments of Inertia of Motor Assembly about} \ x^1, y^1, z^1 \text{ Axes} \\
A, B, C, D, E \hspace{1cm} \text{Amplitudes of Motion of Top of Mast} \\
\omega \hspace{1cm} \text{Frequency} \\
t \hspace{1cm} \text{Time} \\
F_0 \hspace{1cm} \text{Magnitude of Exciting Force}
The propeller test project includes six pieces of structural hardware (see drawings). These are the motor case, the motor case cradle, the mast, the mast fairing, nacelle, and the sector fairing. The motor case, cradle, nacelle, and sector fairing carry the aerodynamic load on the propeller, nacelle, and sector fairing through the mast to the balance system. The aerodynamic loads on the mast are shielded from the balance by the mast fairing which is cantilevered from the tunnel floor independently of the balance system.

Aerodynamic Loads

The maximum torque and thrust developed by the propellers is 600 lbf thrust and 4200 in lbf torque. At a speed of 500 RPM and an angle of attack of 12 degrees maximum harmonic variations of 180 lbf in thrust and 6360 in lbf in yaw moment are experienced. The structure must safely support the static loads and must not be excited to vibrate by the harmonic loads. To accomplish this it was decided to design the mast strong enough to support the static loads but flexible enough so that the lowest exciting frequency of 17HZ (500 RPM for a two blade propeller) would be well above the natural frequency of the system. The natural frequencies of bending and torsion were 3.5HZ, 6.18HZ, and 6.2HZ. These were calculated assuming a rigid support (the balance system is not rigid so the frequencies are actually lower than those calculated), no aerodynamic or structural damping, and the mass of the mast was neglected. The mass on the end of the mast is about seven times the mast
mass so one would expect little influence on the natural frequencies due to the mast mass. However, a lumped mass analysis including the mast mass was made to confirm this assumption, and it produced the same frequencies as above.

The aerodynamic load on the nacelle at 20 degrees angle of attack is about 400 lbf of lift normal to the nacelle. The sector fairing is parallel to the flow and has only small shear loads on it. The mast fairing would have a lift load of about 200 lbf at one degree yaw angle (angle of attack) at a tunnel speed of 100 MPH.

Analysis of Nacelle

The nacelle is a cylindrical shell stiffened with rings attached to the cradle at four points so that it approximates a simply supported beam with a distributed load of 400 lbf total over a span of eight feet.

\[
M = 4800 \text{ in lbf}
\]
\[
I = \frac{1}{2} J = \frac{1}{2} r^2 A = \frac{1}{2} r^2 2\pi r t = \pi r^3 t
\]
\[
\sigma = \frac{M r}{I} = \frac{M r}{\pi r^2 t} = \frac{4800}{\pi (10^2)(.05)}
\]
\[
\sigma = 305 \text{ psi}
\]
\[
\sigma_{cr} = C_b E \left( \frac{L}{r} \right) = .16 \left( 10^7 \right) \left( \frac{1}{200} \right) = 8000 \text{ psi}
\]

Thus the actual stress 305 psi is about \(\frac{1}{26}\) th of the bending stress which would cause buckling of the cylinder.

Analysis of Mast Fairing

The mast fairing is a rigid shell structure stiffened with ribs at two foot intervals. The chord is five feet nine inches and the span is approximately fourteen feet. At a tunnel speed of 100 MPH and an angle of incidence of one degree, the symmetric airfoil would generate 210 lbf of lift located
conservatively at mid span. This lift would produce a root moment of 17,640 in lbf to be reacted by the front and rear spars (neglect bending strength of skin except over spar caps - conservative assumption). The moment of inertia is 84 in\(^4\) and

\[
\sigma = \frac{Mc}{I} = \frac{17,640}{84} = 1470 \text{ psi}
\]

which is well below both the tensile yield and compressive crippling stress for the spar. Rather massive steel hold down fittings are attached to the base of each spar cap by epoxy and by rivets. These fittings allow the fairing to be bolted to the floor of the tunnel to form a cantilever beam which surrounds the mast with a clearance of one inch on all sides. The fairing deflection under a distributed 200 lbf lift load is less than .03 inches so there should be no interference between the mast and mast fairing.

Analysis of Mast

The mast used to support the propeller test drive motors must be cantilevered from the balance table and offer minimum wind resistance. It was also desirable that the mast be tapered to minimize mast thickness at the motor attachment location. Other design considerations were ease of construction, economy of construction, static response to propeller loads, dynamic response to harmonic loads induced by the propellers, and stress levels at critical points due to propeller loads. A mast height of about fourteen feet was required to place the propellers at the centerline of the tunnel.

The maximum anticipated loads expected for the most extreme test cases were 600 lbf thrust and 290 ft lbf torque steady loads. The weight of the motors and structure when added to the applied propeller loads gave a loading which was not severe for a design with even a modest cross-section. Thus it
was decided to choose a design based on stiffness criterion rather than strength, since there was no over-riding reason for minimizing the weight or size of the beam. To withstand dynamic loads two approaches were considered. First the beam could be made stiff enough so its natural frequency was well above the frequencies of all harmonic loads. This would have required a massive cross-section. The second approach was to design a beam with natural frequencies well below the minimum expected harmonic excitations. This approach allows dynamic amplitudes somewhat greater than static deflections, but the static deflections are small due to the smallness of the loads.

Several cross-sections were analyzed with 0.5 inch and 0.375 inch steel plate being considered for structural material. The final design dimensions will be used to explain the analysis procedure used to determine the frequencies of vibration and the stresses in the mast.

Dimensions of Mast and Internal Loads

The mast is a tapered box beam stiffened with bulkheads (Figure 1). It has base dimensions of 20 inches by 11 inches and a top 20 inches by 3.6 inches. Dimensions of 20 inches by 3.5 inches were used in the analysis and later changed to 20 inches by 3.6 inches to produce an integer number for the taper ratio. This produced negligible changes in the stresses and frequencies. A plate thickness of 0.375 inches was used.

Equations for the variation in the moments of inertia, torsion constant, and area are given on the following page:
\[ I_x = 1240 - 3.66Z \] ............................ (1a)

\[ I_y = 490.8 - 5.037Z + 0.01457Z^2 - 0.00000814Z^3 \] ............................ (1b)

\[ J = \frac{155,729 - 1540Z + 3.807Z^2}{157.333 - 0.2703Z} \] ............................ (1c)

\[ A = 22.6875 - 0.03801Z \] ............................ (1d)

where \( I_x, I_y, \) and \( J \) have units of in\(^4\), area has units of in\(^2\), and \( Z \) is in inches.
The applied loads shown in Figure 2 produce the following bending moments, torque, and axial load distributions.

\[ M_z = M_{za} \]  
\[ M_y = M_{ya} + P_z L - P_z Z \]  
\[ M_x = -M_{xa} + P_z L - P_z Z \]  
\[ P_z = -P_{za} - 817.7 + 6.307Z - 0.05282Z^2 \]

where moments are in inch pounds, forces in pounds, and \( Z \) and \( L \) in inches.

**Deflection Analysis of Mast**

The mast deflections under the above loads can be computed by fundamental beam theory or by Castigliano's Theorem. Using Castigliano's method we first calculate the strain energy from

\[ U = \frac{1}{2} \int_0^L \left[ \frac{M_x^2}{I_x} + \frac{M_y^2}{I_y} + \frac{E z^2}{G J}\right]dZ \]

from which we find deflections

\[ \delta_x = \frac{\partial U}{\partial P_x} \]  
\[ \delta_y = \frac{\partial U}{\partial P_y} \]  
\[ \theta_x = \frac{\partial U}{\partial M_{xa}} \]  
\[ \theta_y = \frac{\partial U}{\partial M_{ya}} \]  
\[ \theta_z = \frac{\partial U}{\partial M_{za}} \]
Substituting Equations 2 into Equations 4 and taking the respective derivatives gives

\[ \delta_x = \frac{1}{E} \int_0^L \frac{1}{y} \left( M \frac{y}{x} + P L - P x \right) (L - Z) dZ \]  
\[ \delta_y = \frac{1}{E} \int_0^L \frac{1}{x} \left( -M \frac{y}{x} + P y \right) (L - Z) dZ \]  
\[ \theta_x = \frac{1}{E} \int_0^L \frac{1}{y} \left( M \frac{y}{x} - P L + P x \right) dZ \]  
\[ \theta_y = \frac{1}{E} \int_0^L \frac{1}{x} \left( M \frac{y}{x} + P L - P x \right) dZ \]  
\[ \theta_z = \frac{1}{G} \int_0^L \frac{M z}{J} dZ \]

Now substitute the moments of inertia, Equation 1, and integrate. The expression for \( \delta_x \) will be used as an example of the procedure and the results for the other deflection given without details of algebra.

\[ \delta_x = \frac{1}{E} \int_0^L \frac{1}{490.8 - 5.037Z + .014,57Z^2 - .000,008,14Z^3} \left( \frac{M}{x} + P L \right) (L - Z) dZ \]

Normalize the Z coordinate by letting \( Z = \bar{Z}L \) and \( dZ = Ld\bar{Z} \).

\[ \delta_x = \frac{1}{E} \int_0^1 \frac{1}{490.8 - 5.037L\bar{Z} + .014,57L^2\bar{Z}^2 - .000,008,14L^3\bar{Z}^3} \left( \frac{M}{x} + P L \right) Ld\bar{Z} \]

\( L = 148 \) inches

\[ \delta_x = \frac{1}{E} \int_0^1 \frac{1}{490.8 - 745.48\bar{Z} + 319.14\bar{Z}^2 - 26.38\bar{Z}^3} \left( \frac{M}{x} + 2P L \right) \bar{Z} \]  
\[ \delta_x = \frac{1}{E} \int_0^1 \frac{1}{490.8 - 745.48\bar{Z} + 319.14\bar{Z}^2 - 26.38\bar{Z}^3} \left( \frac{M}{x} + 2P L \right) d\bar{Z} \]

Now factor the denominator into the form \((1 + a\bar{Z})(1 + b\bar{Z})(1 + c\bar{Z})\)

\[ \delta_x = \frac{1}{E} \int_0^1 \frac{1}{490.8} \left( \frac{M}{x} + 2P L \right) \frac{1}{(1 - .108\bar{Z})(1 - .7053\bar{Z})^2} \]
\[
\delta_x = \frac{L^2}{490.8E} \left[ (M_{ya} + P_x L) \int_0^1 \frac{dZ}{(1.0 - 0.108Z) (1.0 - 0.7053Z)^2} + \\
- (M_{ya} + 2P_x L) \int_0^1 \frac{Z dZ}{(1.0 - 0.108Z) (1.0 - 0.7053Z)^2} + \\
+ P_x L \int_0^1 \frac{Z^2 dZ}{(1.0 - 0.108Z) (1.0 - 0.7053Z)^2} \right]
\]

These expressions can be integrated by reference to a book of integral tables. More algebraic manipulation yields

\[
\delta_x = \frac{1}{E} [3780 P_x + 48.9 M_{ya}] \quad \ldots \ldots \ldots \ldots \ldots (6a)
\]

Similar work for \( \delta_y, \theta_x, \theta_y \) and \( \theta_z \) yields

\[
\delta_y = \frac{1}{E} [986.8 P_y - 10.5 M_{xa}] \quad \ldots \ldots \ldots \ldots \ldots (6b)
\]

\[
\theta_x = \frac{1}{E} [-10.5 P_y + 0.1568 M_{xa}] \quad \ldots \ldots \ldots \ldots \ldots (6c)
\]

\[
\theta_y = \frac{1}{E} [48.87 P_x + 1.107 M_{ya}] \quad \ldots \ldots \ldots \ldots \ldots (6d)
\]

\[
\theta_z = \frac{1}{E} [1.1428 M_{za}] \quad \ldots \ldots \ldots \ldots \ldots (6e)
\]

In matrix form the results for the 20 x 11 mast made of .375 inch thick plate are

\[
\begin{bmatrix}
\delta_x \\
\delta_y \\
\theta_x \\
\theta_y \\
\theta_z
\end{bmatrix} = \frac{1}{E} \begin{bmatrix}
3780 & 0 & 0 & 48.87 & 0 \\
0 & 986.8 & -10.5 & 0 & 0 \\
0 & -10.5 & 0.1568 & 0 & 0 \\
48.87 & 0 & 0 & 1.107 & 0 \\
0 & 0 & 0 & 0 & 1.1428
\end{bmatrix} \begin{bmatrix}
P_x \\
P_y \\
M_{xa} \\
M_{ya} \\
M_{za}
\end{bmatrix}
\]

Similar results for a 23 x 16.25 mast made of .5 inch thick plate are

B11
Dynamic Analysis of Mast

If we neglect the mass of the mast and write the equations of motion for the motor case - motor cradle - propeller and counterweight, the natural frequencies of vibration can be found. Let \( \mathbf{r} \) be a position vector from the center of gravity of the motor-propeller assembly to the end of the mast (station 148 inches). Assume the motor-propeller assembly and that portion of the mast above station 148 inches to be rigid. Inertial and mass properties of the assembly can be found in Appendix A.
\[ P_G = \frac{G}{\eta + zk} \]

\[ \Sigma F_x = MA_{Gx} \]
\[ -P_x = M(\delta + y \theta_z) \] \hspace{1cm} (8a)

\[ \Sigma F_y = MA_{Gy} \]
\[ -P_y = M(\delta_y - Z \theta_x) \] \hspace{1cm} (8b)

\[ \Sigma F_z = MA_{Gz} \]
\[ -P_z -W = -M y \theta_x \] \hspace{1cm} (8c)

\[ \Sigma M_{Gx} = I_{Gx} \theta_x \]
\[ -M_{xa} - P \frac{y}{z^2} - P \frac{y}{Z} = I_{Gx} \theta_x \] \hspace{1cm} (8d)

\[ \Sigma M_{Gy} = I_{Gy} \theta_y \]
\[ -M_{ya} + P \frac{y}{Z} = I_{Gy} \theta_y \] \hspace{1cm} (8e)

\[ \Sigma M_{Gz} = I_{Gz} \theta_z \]
\[ -M_{za} + P \frac{y}{Z} = I_{Gz} \theta_z \] \hspace{1cm} (8f)

Now invert matrix (7a) to get

\[
\begin{bmatrix}
P_x \\
P_y \\
M_{xa} = 10^6 x \\
M_{ya} \\
M_{za}
\end{bmatrix} =
\begin{bmatrix}
.01849 & 0 & 0 & -.8162 & 0 \\
0 & .10575 & 7.0818 & 0 & 0 \\
0 & 7.0818 & 665.554 & 0 & 0 \\
.8162 & 0 & 0 & 63.13 & 0 \\
0 & 0 & 0 & 26.25 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_x \\
\delta_y \\
\theta_x \\
\theta_y \\
\theta_z
\end{bmatrix}
\]

Assume simple harmonic motion
\[ \delta_x = A \sin \omega t \]
\[ \delta_y = B \sin \omega t \]
\[ \theta_x = C \sin \omega t \] \hspace{1cm} (10)
\[ \theta_y = D \sin \omega t \]
\[ \theta_z = E \sin \omega t \]
and substitute equations (9) and (10) into equations (8) and simplify to get

\[
(18,490 - 12.31w^2)A - 816,200D - 151.413w^2E = 0
\]

\[
(105,750 - 12.31w^2)B + (7,081,800 + 215.37w^2)C = 0
\]

\[
8,933,483 B + (789,556,318 - 20,243.4w^2)C = 0
\]

\[
-1,139,960 A + (77,422,000 - 570w^2)D = 0
\]

\[
-18,490 A + 816,200 D + (2,134,146 - 1479w^2)E = 0
\]

Solving these free vibration equations for the five natural frequencies and mode shape gives:

<table>
<thead>
<tr>
<th>MODE</th>
<th>DESCRIPTION</th>
<th>FREQUENCY (HZ)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>lateral bending</td>
<td>3.5</td>
<td>6.77</td>
<td>0</td>
<td>0</td>
<td>.1</td>
<td>.031</td>
</tr>
<tr>
<td>2</td>
<td>fore and aft bending</td>
<td>6.18</td>
<td>0</td>
<td>-8.5</td>
<td>.1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>torsion or pitching mode</td>
<td>6.20</td>
<td>1.865</td>
<td>0</td>
<td>0</td>
<td>.02766</td>
<td>-.1</td>
</tr>
<tr>
<td>4</td>
<td>rolling mode</td>
<td>39.0</td>
<td>.0536</td>
<td>0</td>
<td>0</td>
<td>.1</td>
<td>.0004</td>
</tr>
<tr>
<td>5</td>
<td>yaw mode</td>
<td>59.0</td>
<td>0</td>
<td>3.357</td>
<td>.1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Each mode has been normalized to a maximum rotation of .1 radian.

The fore and aft bending mode and the yaw mode are uncoupled from the lateral bending, pitching, and rolling modes.

For a forced vibration analysis, propeller loads must be converted to an equivalent force system at the center of gravity of the motor assembly. The equations of motion of the motor assembly (11) would be modified by including the magnitude of the harmonic applied loads on the right hand side of the equations and interpreting \( \omega \) as the frequency of the applied loads and \( A, B, C, D, \) and \( E \) as the amplitudes of the resulting forced motion. These five simultaneous equations can be solved for the amplitudes of forced motion from which the bending moments and twisting moment distribution can be computed via equations (9) and (2).
Calculation of Maximum Static Stresses

The maximum static loads are 600 lbf thrust and 4200 in lbf torque.

At zero angle of attack the thrust force lies 2.49 inches above the center of gravity of the motor-prop assembly. The equivalent force system at the center of gravity for this worst static condition would then be:

\[ P_y^G = -600 \text{ lbf}, M_y^G = -4200 \text{ in lbf}, M_x^G = 2.5(600) = 1500 \text{ in lbf}. \]

Put these static loads on the right hand side of equation (11) and set \( \omega = 0 \):

\[
\begin{align*}
18,490 A - 816,200 D &= 0 \\
105,750 B + 7,081,800 C &= -600 \\
8,933,483, B + 789,556,318 C &= 1500 \\
-1,139,960 A + 77,422,000 D &= -4200 \\
-18,490 A + 816,200 D + 2,134,146 E &= 0
\end{align*}
\]

Solving yields: \( A = -.006841 \text{ in} \)

\( B = -.02394 \text{ in} \)

\( C = .0002728 \text{ radian} \)

\( D = -.000155 \text{ radian} \)

\( E = 0.0 \text{ rad} \)

Insert these deflections into equations (9) to get equivalent loads on top of mast:

\[
\begin{bmatrix}
P_x \\
P_y \\
M_{xa} \\
M_{ya} \\
M_{za}
\end{bmatrix} = 10^6 \times 
\begin{bmatrix}
0.01849 & C & 0 & -0.8162 & 0 \\
0 & 0.10575 & 7.0818 & 0 & 0 \\
0 & 7.0818 & 665.554 & 0 & 0 \\
-0.8162 & 0 & 0 & 63.13 & 0 \\
0 & 0 & 0 & 0 & 26.25
\end{bmatrix} \begin{bmatrix}
-68.41 \\
-239.4 \\
2.728 \times 10^6 \\
-1.55 \\
0
\end{bmatrix}
\]}
P_x = -0.02 lbf (0)
P_y = -599.74 lbf (-600)
M_{xa} = 12,024.8 in lbf (-12,000)
M_{ya} = -4201.5 in lbf (-4200)
M_{za} = 0 (0)

These results could have been determined by reducing the propeller loads to an equivalent force system at the top of mast. The values above in parenthesis indicate results obtained by statics. This provides a partial check on the equations (11) and (9).

Now substitute the loads into equations (2).

\[ M_x = -100,800 + 600Z \]
\[ M_y = -4200 \]
\[ M_z = 0 \]

Normal stress in the mast is given by:

\[ \sigma = \frac{M_y}{I_y} x + \frac{M_x}{I_x} y - \frac{W}{A} \]

Moments of inertia given by equations (1) when inserted into (12) gives

\[ \sigma = \left(\frac{-4200}{490.8}\right) x + \left(\frac{-100,800}{1240}\right) y - \left(\frac{5600}{22.68}\right) \text{ for } Z = 0. \]

\[ \sigma = 8.5575x - 81.290y - 247.0 \]

@x = -5.5 in and y = 10.0 in

\[ \sigma = -1107 \text{ psi (compression)} \]

@x = 5.5 in, y = -10.0 in

\[ \sigma = 613 \text{ psi (tension)} \]

On the next page is a table for properties, moments, and stress at the quarter points of the mast.
The mast is constructed of standard structure steel plate with a yield stress of 36,000 psi. This gives an allowable stress of 12,000 psi and the maximum static stresses are well within this value.

The critical buckling stress can be calculated for the steel plate at the bottom of the mast assuming unrestrained edges (conservative).

\[
\sigma_{CR} = \frac{K\pi^2E}{12(1-\mu^2)} \left( \frac{t}{b} \right)^2
\]

\[
\sigma_{CR} = \frac{(4)(\pi^2)(30 \times 10^6)}{10.92 \times 10^6} \left( \frac{3.75}{20} \right)^2
\]

\[
\sigma_{CR} = 38,000 \text{ psi (conservative)}
\]

This stress is above the yield point so the plate would buckle inelastically. The allowable stress remains 12,000 psi.

The recommended working stress for various welds of low carbon steel is 16,000 psi for static loads and 8,000 psi for dynamic loads. Stress concentration factors up to 2 should be used for certain butt joints with sharp corners. The edges of the mast welded to the base plate were beveled to eliminate the sharp corners. Even using the stress concentration factor and the working stress for dynamic loads, an allowable stress of 4000 psi is obtained which is well above the tensile stress of 613 psi and the compressive tension.
stress of 1107 psi on the base weld. Thus the mast is well within the allowable stress limits for static loads.

Calculation of Maximum Dynamic Stresses

At a speed of 500 RPM there is a harmonic thrust force of 180 lbf and a harmonic yaw moment of 6360 in lbf with frequency twice the rotational speed for a two blade propeller. This condition occurs at an angle of attack of 12 degrees and is the lowest frequency (17Hz) excitation expected other than an unbalance in the propeller shaft. 17Hz is well above the bending and torsion frequencies but is below the rolling and yawing frequencies.

These load produce an equivalent force system at the center of gravity of $P_{yG} = -176.1$ lbf, $P_{zG} = 37.4$ lbf, $M_{xG} = 450$ in lb, and $M_{zG} = 6360$ in lb. The moments of inertia and center of gravity of the motor - propeller assembly are not changed significantly by a rotation of 12 degrees. Put the exciting forces and moments on the right hand side of equation (11) and inserting $\omega = 16.66$Hz = 104.7 radians/sec gives:

$$-116,493 A - 816,200 D - 1,660,296 E = 0$$
$$-29,233 B + 9,443,400 C = -176.1$$
$$8,933,483 B + 567,580,000 C = 450$$
$$-1,139,960 A + 71,172,000 D = 0$$
$$-18,490 A + 816,200 D - 14,083,600 E = 6360$$

Solution of these equations for the dynamic displacements gives:

$A = 581.56 \times 10^{-5}$ inches
$B = 103.22 \times 10^{-5}$ inches
$C = -1.5454 \times 10^{-5}$ radians
$D = 9.315 \times 10^{-5}$ radians
$E = -45.383 \times 10^{-5}$ radians
These displacements produce equivalent loads on the top of the mast of

\[ P_x = 31.5 \text{ lbf}, \]
\[ P_y = -0.3 \text{ lbf}, \]
\[ M_{xa} = -2976 \text{ in lbf}, \]
\[ M_{ya} = 1134 \text{ in lbf}, \]
\[ M_{za} = 11,913 \text{ in lbf}, \]

which produce moments at the base of the beam of

\[ M_x = 2932 \text{ in lbf} \]
\[ M_y = 5796 \text{ in lbf} \]
\[ M_z = 11,913 \text{ in lbf}. \]

The maximum dynamic stress is

\[
\sigma = \frac{5796}{490.8} x + \frac{2932}{1240} y - \frac{5600}{22.68} \text{ at the base of the mast.}
\]

\[ \sigma_{\text{max}} = -159 \text{ psi} \]
\[ \sigma_{\text{min}} = -336 \text{ psi} \]

At the lowest frequency expected, the vibrational modes of the mast are not excited. The magnitude of the exciting loads are also low which helps account for the low dynamic stresses.

The absolutely worst case of failure would be to lose a propeller blade at low speed. This rotating unbalance would produce exciting loads at the center of gravity of

\[ P_{xG} = -F_o \cos \omega t \]
\[ P_{zG} = F_o \sin \omega t \]
\[ M_{xG} = -94.7 F_o \cos \omega t \]
\[ M_{yG} = -2.5 F_o \cos \omega t \]
\[ M_{zG} = -94.7 F_o \cos \omega t \]
At $\omega = 500$ RPM or 52.36 rad/sec, $F_0 = m\omega^2$ where $m$ is the propeller blade mass and $e$ is the centroidal distance of the blade from the propeller shaft. $F_0 = 5000$ lbf for $m = 1.82$ ft-slugs. For $\omega = 52.36$ rad/sec equations (11) become:

\begin{align*}
-15,231 A - 816,200 D - 415,110 E &= P_x(t) \\
72,029 B + 7,672,252 C &= P_y(t) \\
8,933,483 B + 734,057,000 C &= M_{xa}(t) \\
-1, 139,960 A + 75,859,000 D &= M_{ya}(t) \\
-18,490 A + 816,200 D - 1,920,600 E &= M_{za}(t)
\end{align*}

Now solve for the dynamic amplitudes resulting from $F = F_1 + F_2$ where

\[
F_1 = \begin{bmatrix}
-473,500 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
sin 52t \quad \text{and} \quad F_2 = \begin{bmatrix}
-5000 \text{ lbf} \\
0 \\
-12,500 \text{ in lbf} \\
0 \\
-473,500 \text{ in lbf} \\
\end{bmatrix} \cos 52t
\]

For $F_1$

\begin{align*}
72.03 B + 7672.25 C &= 0 \\
8933.5 B + 734,057 C &= -473.5 \\
A &= 0 \\
B &= -0.2319 \text{ inches} \\
C &= 0.002177 \text{ radians} \\
D &= 0 \\
E &= 0
\end{align*}

For $F_2$

\begin{align*}
+15.23 A + 81612 D + 415.1 E &= +5 \\
72.03 B + 7,672.25 C &= 0 \\
8933.5 B + 734,057 C &= 0
\end{align*}
-1140 A + 75,859 D = -12.5
-18.49 A + 816.2 D - 1920.6 E = -473.5
A = -3.72
B = 0
C = 0
D = -.056
E = .2585

Now combining equations (2) and (9)

\[
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix} = 10^6 \begin{bmatrix}
0 & 8.569 & 382.55 & 0 & 0 \\
1.92 & 0 & 0 & -57.668 & 0 \\
0 & 0 & 0 & 0 & 26.25
\end{bmatrix} \begin{bmatrix}
\delta_x \\
\delta_y \\
\theta_x \\
\theta_y \\
\theta_z
\end{bmatrix}
\]

For \( F_1 \)

\[
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix} = 10^6 \begin{bmatrix}
0 & 8.569 & 382.55 & 0 & 0 \\
1.92 & 0 & 0 & -57.668 & 0 \\
0 & 0 & 0 & 0 & 26.25
\end{bmatrix} \begin{bmatrix}
0 \\
-.2319 \\
.002177 \sin 52t \\
0 \\
0
\end{bmatrix}
\]

For \( F_2 \)

\[
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix} = 10^6 \begin{bmatrix}
0 & 8.569 & 382.55 & 0 & 0 \\
1.92 & 0 & 0 & -57.668 & 0 \\
0 & 0 & 0 & 0 & 26.25
\end{bmatrix} \begin{bmatrix}
-3.72 \\
0 \\
0 \cos 52t \\
0 \\
-.056 \\
.2585
\end{bmatrix}
\]

\[
M_x = 1.154 \times 10^6 \sin 52t
\]

\[
M_y = -3.913 \times 10^6 \cos 52t
\]

\[
M_z = 6.785 \times 10^6 \cos 52t
\]
\[
\sigma = \frac{3.913 \times 10^6 \cos 52t}{490.8} x + \frac{-1.154 \times 10^6 \sin 52t}{1240} y - 247 + \frac{5000 \sin 52t}{22.68}
\]

\[
\sigma = -7973 \times \cos 52t - 930.6 \times \sin 52t - 247 + 220.5 \sin 52t
\]

Consider three points on the cross-section located at

- \(x = 0, y = -10\) \(\text{pt a}\)
- \(x = 5.5, y = 0\) \(\text{pt c}\)
- \(x = 5.5, y = 10\) \(\text{pt b}\)

\[
\sigma_a = 9306 \sin 52t - 247 + 220 \sin 52t
\]

\[
\sigma_a = 9526 \sin 52t - 247
\]

\[
\sigma_c = -43,852 \cos 52t - 247 + 220 \sin 52t
\]

\[
\sigma_b = -43,852 \cos 52t + 9306 \sin 52t - 247 + 220 \sin 52t
\]

\[
\sigma_b = -43,852 \cos 52t + 9526 \sin 52t - 247
\]

\[
\sigma_b = 44,875 \sin (52t - 1.36) - 247
\]

<table>
<thead>
<tr>
<th>point</th>
<th>max tensile stress</th>
<th>max comp. stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>9279</td>
<td>9773</td>
</tr>
<tr>
<td>b</td>
<td>44,628</td>
<td>45,122</td>
</tr>
<tr>
<td>c</td>
<td>43,605</td>
<td>44,099</td>
</tr>
</tbody>
</table>

These stresses are greater than the yield stress but less than the ultimate stress. It is possible that the mast would hold together until the motors could be stopped.

**Calculation of Hold Down Bolt Stresses**

The mast is connected to the balance system by four 3/4 inch 16NF bolts three inches long with a recommended yield strength of 100,000 psi (Figure 3). These bolts are subjected to essentially the loads at the base.
of the mast, specifically $M_x = 2932$ in lb, $M_y = 5707$ in lb, and $M_z = 11,913$ in lb for the maximum dynamic loads at 178Z. For the bolts $I_x = 0.3724 (9.5)^2 \times 4 = 134.4$ in$^4$ where 0.3724 is the area at the root of the bolt threads. $I_y = 0.3724 (6.8)^2 \times 4 = 69$ in$^4$.

$$\sigma = \frac{M_x}{I_x} + \frac{M_y}{I_y}$$

$$\sigma = \frac{2932}{134.3} (9.5) + \frac{5795}{69} (6.8)$$

$$\sigma = 778.3 \text{ psi}$$

$M_z = 2V \times d$ where $V$ is the shear force on a bolt and $d$ is the diagonal distance between the bolts. The four bolts form two couples $V \times d$ which resist $M_z$.

$$V = \frac{M_z}{2d} = \frac{11,913}{2(23.86)} = 252 \text{ lbf}$$

$$\tau = \frac{V}{A} = \frac{252}{0.3724} \text{ psi}$$

$$\tau = 677 \text{ psi}$$

These dynamic stresses are well within the allowable stress for the bolt material which is $\tau = \frac{1}{6} \sigma = \frac{1}{6} (100,000) = 16,667$ psi.

The maximum static loads are $M_x = 100,800$ in lbs and $M_y = 4200$ in lbs.

$$\sigma = \frac{M_x}{I_x} + \frac{M_y}{I_y}$$

$$\sigma = \frac{100,800}{134.4} (9.5) + \frac{4200}{69} (6.8)$$

$$\sigma = 7539 \text{ psi}$$
Resisting the bending moments by a couple does not take into account the area in bearing which is much greater than the bolt area. Thus the above stresses are conservative. For bolted joints carrying moments it is desirable that the bolts be torqued to provide a bolt pre-load which is at least equal to 1.25 M divided by the section modulus of the contact area times the contact area.

\[ A = 16.75 (23.5) = 394 \text{ in}^2 \]

\[ S_x = \frac{1}{12} \frac{(16.75)(23.5)^3}{23.5/2} = 1542 \text{ in}^3 \]

\[ S_y = \frac{1}{12} \frac{(23.5)(16.75)^3}{16.75/2} = 1099 \text{ in}^3 \]

\[ T_{\text{PRE}} = 4T = \frac{1.25 MA}{S_x} = \frac{1.25 (100,800)(394)}{1542} = 32,194 \text{ lb} \]

\[ T = 8049 \text{ lb} \]

\[ T_{\text{PRE}} = 4T = \frac{1.25 MA}{S_y} = \frac{1.25 (4200)(394)}{1099} = 1882 \text{ lb} \]

\[ T = 471 \text{ lb} \]

Thus a bolt pre-load of 8049 lb per bolt is necessary to keep the joint in compression. This is a stress of 21,614 psi, well less than 50,000 psi. A torque of 100 ft lbf on the bolts would be required to induce a load of 8049 lbf. This value is obtained from Torque = .2d T\(^*\) = \( (.2) \left( \frac{3}{4} \right) \left( \frac{1}{12} \right) \times 8049 \) ft lbf.

If a propeller blade was lost, the dynamic loads induced would be sufficient to fail the hold down bolts although the rest of the structure (mast) would remain intact.

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*Ibid., page 204.*
Appendix A

Inertial Properties of Motor - Cradle - Counterweight

<table>
<thead>
<tr>
<th>ITEM</th>
<th>Weight</th>
<th>X'</th>
<th>Y'</th>
<th>WY'</th>
<th>WZ'</th>
<th>WY'</th>
<th>WZ'</th>
<th>IM&lt;sub&gt;xc&lt;/sub&gt;</th>
<th>IM&lt;sub&gt;zc&lt;/sub&gt;</th>
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</thead>
<tbody>
<tr>
<td>Motor</td>
<td>2000</td>
<td>-40</td>
<td>20</td>
<td>-80</td>
<td>40</td>
<td>3200</td>
<td>800</td>
<td>35</td>
<td>1350</td>
</tr>
<tr>
<td>Prop</td>
<td>100</td>
<td>-107</td>
<td>20</td>
<td>-10.7</td>
<td>2</td>
<td>1145</td>
<td>40</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>Counterweight</td>
<td>1700</td>
<td>24</td>
<td>18</td>
<td>40.8</td>
<td>30.6</td>
<td>979</td>
<td>551</td>
<td>20</td>
<td>90</td>
</tr>
<tr>
<td>Actuator</td>
<td>60</td>
<td>24</td>
<td>5</td>
<td>1.44</td>
<td>.3</td>
<td>34.6</td>
<td>1.5</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Top of Mast</td>
<td>150</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>.45</td>
<td>0</td>
<td>1.35</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>Channel</td>
<td>312</td>
<td>-17</td>
<td>14</td>
<td>-5.3</td>
<td>4.37</td>
<td>90.17</td>
<td>61</td>
<td>5</td>
<td>410</td>
</tr>
<tr>
<td>Square Bars</td>
<td>120</td>
<td>6</td>
<td>16</td>
<td>.72</td>
<td>1.92</td>
<td>4.3</td>
<td>31</td>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>Thick Channel</td>
<td>60</td>
<td>-25</td>
<td>12</td>
<td>-1.5</td>
<td>.72</td>
<td>37.5</td>
<td>8.6</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>Hubs &amp; Balance</td>
<td>40</td>
<td>-98</td>
<td>20</td>
<td>-3.92</td>
<td>.8</td>
<td>384</td>
<td>16</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Misc.</td>
<td>200</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>20</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Pivot</td>
<td>8</td>
<td>-12</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
\bar{Y} = \frac{-58,460}{4750} = 12.3
\]

\[
\bar{Z} = \frac{83,160}{4750} = 17.5
\]

\[
\sum W_{\text{xc}} = IM_{\text{xc}}
\]

\[
\sum W_{\text{zc}} = IM_{\text{zc}}
\]

\[
IM_{\text{x}} = \Sigma WY'^{2} + \Sigma WZ'^{2} + IM_{\text{xc}}
\]

\[
IM_{\text{x}} = (5.875 + 1.53 + 1.996)10^{6} \text{ lb in}^{2}
\]

\[
IM_{\text{x}} = 9.401 \times 10^{6} \text{ lb in}^{2}
\]

\[
IM_{Gx} = 9.401 \times 10^{6} - 4750 (12.3^{2} + 17.5^{2})
\]

\[
IM_{Gx} = 7.23 \times 10^{6} \text{ lb in}^{2}
\]

\[
IM_{Gx} = 18,725
\]
\[ IM_y = \sum w_z^2 + IM_{yc} \]
\[ IM_y = (1.53 + .122) \times 10^6 \text{ lb in}^2 = 1.634 \times 10^6 \text{ lb in}^2 \]
\[ IM_{cy} = 1.654 \times 10^6 - 4750 (17.5)^2 = .20 \times 10^6 \text{ lb in}^2 \]
\[ IM_{cy} = 511 \text{ sec}^2 \cdot \text{ in} \]
\[ IM_z = \sum w_y^2 + IM_{zc} \]
\[ IM_z = 5.875 \times 10^6 + 1.996 \times 10^6 = 7.871 \times 10^6 \text{ lb in}^2 \]
\[ IM_{gz} = 7.871 \times 10^6 - 4750 (12.3)^2 \]
\[ IM_{gz} = 18,529 \text{ sec}^2 \cdot \text{ in} \]

These moments of inertia about the centroidal \( x_G, y_G, z_G \) axes differ slightly from the ones used in the calculations due to slight changes in the design made after the computations were completed. The variations are small enough that the results are essentially unaffected.
Figure B2. Loads on Mast
Figure B3. Hold Down Bolt Locations