N O T I C E

THIS DOCUMENT HAS BEEN REPRODUCED FROM MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE

JPL PUBLICATION 80-95

On The Power Spectral Density of Quadrature Modulated Signals

T. Y. Yan

(JPL-Pub-80-95) ON THE POWER SPECTRAL DENSITY OF QUADRATURE MODULATED SIGNALS (Jet Propulsion Lab.) 34 p HC A03/MF A01 CSCL 17B

N81-16341

Unclas
G3/32 41232

FEB 1 '81
RECEIVED
NASA STI FACILITY
ACCESS DEPT.

January 15, 1981

National Aeronautics and
Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

JPL PUBLICATION 80-95

On The Power Spectral Density of Quadrature Modulated Signals

T. Y. Yan

January 15, 1981

National Aeronautics and
Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

CONTENTS

I. INTRODUCTION 1-1

II. THE BASEBAND MODEL 2-1

III. THE EFFECT OF BANDPASS HARDLIMITING 3-1

IV. NUMERICAL EXAMPLES 4-1

V. SUMMARY AND CONCLUSION 5-1

REFERENCES 6-1

ACKNOWLEDGEMENT 6-1

APPENDIXES

A. GENERALIZED SPECTRAL DENSITY OF NONSTATIONARY PERIODIC PROCESSES A-1

B. POWER SPECTRAL DENSITY OF BASEBAND NONOVERLAPPING CORRELATED PHASE MODULATED SIGNALS B-1

Figures

4-1. Power Spectral Density for Offset QPSK With and Without Bandpass Hardlimiting 4-8

4-2. Power Spectral Densities for MSK With and Without Bandpass Hardlimiting 4-9

4-3. Power Spectral Densities for Offset Triangular Keying With and Without Bandpass Hardlimiting 4-10

4-4. Power Spectral Density for Mixed Keying With and Without Bandpass Hardlimiting 4-11

PRECEDING PAGE BLANK NOT FILMED

ABSTRACT

The conventional (no-offset) quadriphase modulation technique suffers from the fact that hardlimiting will restore the frequency sidelobes removed by proper filtering. Thus offset keyed quadriphase modulation techniques are often proposed for satellite communication with bandpass hardlimiting. In this report, a unified theory is developed which is capable of describing the power spectral density before and after the hardlimiting process. Using the in-phase and the quadrature-phase channel with arbitrary pulse shaping, analytical results are established for generalized quadriphase modulation. In particular MSK, QPSK, or the recently introduced overlapped raised-cosine keying all fall into this general category. It is shown that for a linear communication channel the power spectral density of the modulated signal remains unchanged regardless of the offset delay. Furthermore, if the in-phase and the quadrature-phase channel have identical pulse shapes without offset, the spectrum after bandpass hardlimiting will be identical to that of the conventional QPSK modulation. Numerical Examples are given for various modulation techniques. A case of different pulse shapes in the in-phase and the quadrature-phase channel is also considered.

I. Introduction

Quadrature modulated signals are generally represented by the in-phase and the quadrature-phase components. The fact that the offset keying technique will ease the carrier timing recovery and have low sidelobe regeneration is well known[1,2]. MSK and Offset QPSK are typical examples utilizing this technique. In fact, MSK can be viewed as a special case of offset QPSK with sinusoidal pulse weighting [3]. In this report we develop the theory for evaluating the power spectral density of a generalized quadrature modulated signal with arbitrary pulse shaping in the in-phase and the quadrature-phase channels. The theory can be applied to compute the power spectral density when the quadriphase signal is passed through a band-pass hardlimiter.

Consider a complex analytic function representation of the quadrature signal $v(t) = \mathrm{Re}[e^{j\omega_c t} x(t)]$ where $x(t)$ is the baseband digitally modulated signal. Prabhu and Rowe[4] show that for phase modulated signals the power spectral density of $v(t)$ can be determined primarily by the power spectral density of $x(t)$. It has been shown[4] that except for some degenerate cases, the baseband model is sufficient for the spectral analysis. Hence only the baseband signal model is studied throughout the remainder of this report.

This report is organized in three remaining sections. In section II, the baseband modulator is described. The mean function and the autocorrelation function of the process $x(t)$ is evaluated. Through the use of the cyclic property of the autocorrelation function, the generalized power spectral density of this cyclic process is defined, and the average power spectral density is computed. In section III, the case of bandpass hardlimiting is considered. The bandpass hardlimiter will result in a correlation between successive symbols. Consequently the power spectral density after the bandpass hardlimiter can only be evaluated

for some classes of examples. In section IV, numerical examples are given to compare the spectrum before and after the hardlimiting process. MSK, offset QPSK and triangular pulse keying are considered in this section. Furthermore, an example is given where different pulse shaping is used in the in-phase and the quadrature-phase channels.

II. The Baseband Model

Consider a random process $x(t)$, $-\infty < t < \infty$, defined by

$$x(t) = \sum_{k=-\infty}^{\infty} q_{S_k}(t - kT) \qquad -\infty < t < \infty \tag{2-1}$$

where $\{S_k\}_{-\infty}^{\infty}$ is a stationary random sequence which takes on values $\{1,2,\cdots, M\}$. For each $S_k = \ell$, a complex signaling waveform $q_\ell(t)$ is generated. This composite signal $x(t)$ is then passed through a linear filter with impulse response $h(t)$ giving an output $y(t)$

where

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau$$

$$= \sum_{k=-\infty}^{\infty} g_{S_k}(t-kT),$$

and

$$g_{S_k}(t) = \int_{-\infty}^{\infty} h(t-\tau)q_{S_k}(\tau)d\tau \tag{2-2}$$

In general $y(t)$ is a nonstationary random process. It will be seen that the expected value of $y(t)$ is periodic and a generalized power spectral density function is defined. Let $Pr(S_k = \ell) = P_\ell$ be the marginal probability of S_k. Then

$$\eta(t) \triangleq E\{y(t)\} = \sum_{\ell=1}^{M} \sum_{k=-\infty}^{\infty} P_\ell g_\ell(t-kT)$$

$$= \eta(t+T) \tag{2-3}$$

The Fourier series expansion of $\eta(t)$ is written as

$$\eta(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \quad -\infty < t < \infty \quad , \omega_0 = \frac{2\pi}{T}$$

and

$$c_n = \frac{1}{T} \int_0^T \eta(t) e^{-jn\omega_0 t} \, dt \tag{2-4}$$

Hence c_n defines the discrete frequency component of $\eta(t)$ and is referred to as the line spectrum. Since the process $y(t)$ is nonstationary, the autocorrelation function is defined in the ensemble average sense. Let $R_y(t,\tau)$ be the autocorrelation function of the process $y(t)$. Then

$$R_y(t,\tau) = E\left\{[y(t) - \eta(t)]\,[y^*(\tau) - \eta^*(\tau)]\right\}$$

$$= E\left\{y(t)y^*(\tau)\right\} - \eta(t)\eta^*(\tau)$$

and

$$E\{y(t)y^*(\tau)\} = \sum_{k=-\infty}^{\infty} \sum_{k'=-\infty}^{\infty} E\left\{g_{S_k}(t-kT) g^*_{S_{k'}}(\tau-k'T)\right\} \tag{2-5}$$

where

$$E\left\{g_{S_k}(t-kT)g_{S_{k'}}^{*}(\tau-k'T)\right\} = \begin{cases} \sum_{\ell=1}^{M} p_\ell g_\ell(t-kT)g_\ell^{*}(\tau-kT); & k=k' \\ \sum_{\ell=1}^{M} \sum_{\ell'=1}^{M} P_{\ell,\ell',k-k'}\, g_\ell(t-kT)g_{\ell'}^{*}(\tau-k'T); & k\neq k' \end{cases}$$

and

$$P_{\ell,\ell',k-k'} = \Pr\left[S_k = \ell,\ S_{k'} = \ell'\right] \quad \forall \quad k\neq k'$$

Consequently

$$R_y(t+T,\ \tau+T) = R_y(t,\tau) \tag{2-6}$$

It is shown in appendix A that the generalized spectral density of $y(t)$ can be written as

$$S_{y,k}(\omega) = \frac{1}{T}\int_0^T \int_{-\infty}^{\infty} R_y(t+\tau,\ t)\, e^{-j\omega\tau}\, e^{-jk\omega_0(t+\tau)}\, d\tau\, dt$$

The zeroth term is generally referred to as the average power spectral density of the process $y(t)$. Hence

$$S_y(\omega) \triangleq S_{y,o}(\omega) = \frac{1}{T}\int_0^T \int_{-\infty}^{\infty} R_y(t+\tau,\ t)\, e^{-j\omega\tau}\, d\tau\, dt \tag{2-7}$$

Equation (2-7), combined with (2-5) and (2-3), can be used to evaluate the power spectral density of the process y(t). For quadrature modulated signal, we consider the case that

$$q_{S_k}(t) = a_k f(t) + jb_k g(t) \tag{2-8}$$

where a_k's and b_k's are independent identically distributed random variables with zero mean and variances σ_a^2 and σ_b^2 respectively. The f(t) and g(t) are real valued time limited functions which may last longer than a period T. From (2-2),

$$E\left\{y(t)\, y^*(\tau)\right\} = \sigma_a^2 \sum_{k=-\infty}^{\infty} \tilde{f}(t-kT)\, \tilde{f}(\tau-kT) + \sigma_b^2 \sum_{k=-\infty}^{\infty} \tilde{g}(t-kT)\, \tilde{g}(\tau-kT)$$

where

$$\tilde{f}(t) = \int_{-\infty}^{\infty} h(t-\tau)\, f(\tau)\, d\tau$$

and

$$\tilde{g}(t) = \int_{-\infty}^{\infty} h(t-\tau)\, g(\tau)\, d\tau$$

Also

$$\eta(t) = 0$$

and

$$S_y(\omega) = \frac{1}{T} \int_0^T \int_{-\infty}^{\infty} R_y(t+\tau,\ t)\ e^{-j\omega\tau}\ d\tau\ dt$$

$$= \frac{1}{T}\ \sigma_a^{\ 2}\ |\tilde{F}(\omega)|^2 + \frac{1}{T}\ \sigma_b^{\ 2}\ |\tilde{G}(\omega)|^2 \qquad (2\text{-}9)$$

where

$$\tilde{F}(\omega) = \int_{-\infty}^{\infty} \tilde{f}(t)\ e^{-j\omega t}\ dt$$

$$\tilde{G}(\omega) = \int_{-\infty}^{\infty} \tilde{g}(t)\ e^{-j\omega t}\ dt$$

The power spectral density given by (2-9) consists of two components. The first term is the in-phase spectrum and the second term is the quadrature-phase spectrum. This additive property of the spectral density evaluation enables us to define any in-phase and quadrature-phase symbol weightings.

III. The Effect of Bandpass Hardlimiting

The baseband model discussed in the last section can be applied to evaluate the power spectral density of a quadriphase signal after bandpass hardlimiting. From (2-8), we have

$$q_{S_k}(t) = a_k\, f(t) + jb_k\, g(t)$$

and

$$g_{S_k}(t) = a_k\, \tilde{f}(t) + jb_k\, \tilde{g}(t) \tag{3-1}$$

The bandpass hardlimiting model can be written as

$$z(t) = H\,[y(t)] = e^{j\phi(t)}$$

where

$$\phi(t) = \arg\,[y(t)]$$

Since $\tilde{f}(t)$ and $\tilde{g}(t)$ may last longer than a period of T, the nonlinearity introduced by the bandpass hardlimiter will result in a correlation between the successive symbol periods. Let $\tilde{f}(t)$ and $\tilde{g}(t)$ be nonzero over a period of LT. Then we can write

$$\phi(t) = \sum_{k=-\infty}^{\infty} \psi_{\alpha_k}(t - kT) \tag{3-2}$$

where $\{\alpha_k\}_{-\infty}^{\infty}$ is a stationary random sequence which takes on values $\{1, 2, \ldots, 2^{2L}\}$. Relating (3-2) to (3-3) we obtain

$$\psi_{\alpha_k}(t-kT) = \tan^{-1}\left[\frac{\sum_{\ell=0}^{L-1} b_{k-\ell}\, \tilde{g}[t-(k-\ell)T]}{\sum_{\ell=0}^{L-1} a_{k-\ell}\, \tilde{f}[t-(k-\ell)T]}\right] \tag{3-3}$$

Prabhu[5] has evaluated the power spectral density for baseband correlated phase modulated signals. For the case that the successive $\tilde{f}(t)$ and $\tilde{g}(t)$ do not overlap (L=1), and that the a_k's and b_k's are equiprobable independent binary random variables, then from (3-2)

$$\underline{z}(\underline{t}) = \sum_{k=-\infty}^{\infty} a_k\, \hat{f}(t-kT) + \underline{j} \sum_{k=-\infty}^{\infty} b_k\, \hat{g}(t-kT)$$

where

$$\hat{f}(t) = \frac{\tilde{f}(t)}{\sqrt{\tilde{f}^2(t) + \tilde{g}^2(t)}} \tag{3-4}$$

and

$$\hat{g}(t) = \frac{\tilde{g}(t)}{\sqrt{\tilde{f}^2(t) + \tilde{g}^2(t)}}$$

From (2-1) and (2-9), the power spectral density of the output from the bandpass hardlimiter $S_z(\omega)$ is given by

$$S_z(\omega) = \frac{1}{T}\left\{|\hat{F}(\omega)|^2 + |\hat{G}(\omega)|^2\right\}$$

where

$$\hat{F}(\omega) = \int_{-\infty}^{\infty} \hat{f}(t)\, e^{-j\omega t}\, dt$$

$$\hat{G}(\omega) = \int_{-\infty}^{\infty} \hat{g}(t)\, e^{-j\omega t}\, dt \tag{3-5}$$

Appendix B provides an alternative approach for evaluating the power spectral density of baseband nonoverlapping phase modulated signals. Setting L=1 in (3-3), we obtain

$$\psi_{\alpha_k}(t-kT) = \tan^{-1}\left(\frac{b_k\, \tilde{g}(t-kT)}{a_k\, \tilde{f}(t-kT)}\right) .$$

Since a_k's and b_k's are equiprobable independent binary random variables, $\psi_{\alpha_k}(t)$ takes the following four possible forms;

$$\psi_{\alpha_k}(t) = \begin{cases} \pm \tan^{-1}\left(\frac{\tilde{g}(t)}{\tilde{f}(t)}\right) \\ \pi \pm \tan^{-1}\left(\frac{\tilde{g}(t)}{\tilde{f}(t)}\right) \end{cases} \tag{3-6}$$

where α_k's are equiprobable independent random variables which take on values $\{1, 2, 3, 4\}$. Substituting (3-6) in (B-7) and using

$$P_{\ell,m,k} = P_\ell P_m = \frac{1}{4}\ \forall\ \ell \text{ and } m,$$

we obtain the same result as given by (3-5).

IV. Numerical Examples

Example 1. We consider the case that $g(t) = f(t - \frac{T}{2})$. Consequently

$$|G(\omega)| = |F(\omega)| \tag{4-1}$$

and

$$S_y(\omega) = \frac{\sigma_a^2 + \sigma_b^2}{T} |\tilde{F}(\omega)|^2 = \frac{\sigma_a^2 + \sigma_b^2}{T} |F(\omega)\ H(\omega)|^2$$

However, the nonlinearity introduced by bandpass hardlimiting restricts the evaluation of (3-5) to some special cases. Suppose that the signal y(t) is unfiltered and f(t) is nonzero for a duration less than or equal to T. Then

$$|H(\omega)| = 1$$

and

$$\hat{F}(\omega) = \int_{-\infty}^{\infty} \frac{f(t)}{\sqrt{f^2(t) + f^2(t - \frac{T}{2})}} e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \frac{f(t + \frac{T}{2})}{\sqrt{f^2(t + \frac{T}{2}) + f^2(t)}} e^{-j\omega[t + \frac{T}{2}]}\, dt$$

If f(t) is also an even function

$$|\hat{F}(\omega)| = \int_{-\infty}^{\infty} \frac{g(t)}{\sqrt{f^2(t - \frac{T}{2}) + f^2(t)}} e^{j\omega t}\, dt \quad = |\hat{G}(\omega)|$$

Hence

$$S_z(\omega) = \frac{2}{T} \left|\hat{F}(\omega)\right|^2 \tag{4-2}$$

For the Offset QPSK

$$f(t) = \begin{cases} 1 & |t| \leq \frac{T}{2} \\ 0 & |t| > \frac{T}{2} \end{cases}$$

Then

$$S_y(\omega) = \frac{2}{T} \left| \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-j\omega t}\, dt \right|^2$$

$$= 2T \operatorname{sinc}^2 \left(\frac{\omega T}{2}\right)$$

and

$$S_Z(\omega) = \frac{2}{T} \left|\hat{F}(\omega)\right|^2$$

$$= \frac{2}{T} \left| \int_{-\frac{T}{2}}^{0} e^{-j\omega t}\, dt + \int_{0}^{\frac{T}{2}} \frac{1}{\sqrt{2}} e^{-j\omega t}\, dt \right|^2$$

$$= T \left[\frac{3}{4} + \frac{1}{\sqrt{2}} \cos \frac{\omega T}{2} \right] \operatorname{sinc}^2 \left(\frac{\omega T}{4} \right)$$

For the MSK

$$f(t) = \begin{cases} \cos \frac{\pi t}{T} & |t| \leq \frac{T}{2} \\ 0 & |t| \geq \frac{T}{2} \end{cases}$$

Then

$$S_y(\omega) = \frac{2}{T} \left| \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos \frac{\pi t}{T}\, e^{-j\omega t}\, dt \right|^2$$

$$= \frac{8\, \pi^2\, T}{(\pi^2 - \omega^2\, T^2)^2} \cos^2 \frac{\omega T}{2}$$

and

$$S_Z(\omega) = \frac{2}{T} \left| \int_{-\frac{T}{2}}^{0} e^{-j\omega t}\, dt + \int_{0}^{\frac{T}{2}} \cos \frac{\pi t}{T}\, e^{-j\omega t}\, dt \right|^2$$

$$= 2T \left\{ \frac{1}{4} \operatorname{sinc}^2 \left(\frac{\omega T}{4}\right) + \frac{\pi^2 + \omega^2 T^2 - 2\pi\omega t \sin \frac{\omega T}{2}}{(\pi^2 - \omega^2 T^2)^2} \right.$$

$$\left. + \frac{2\omega T \left(1 - \cos \frac{\omega T}{2}\right) + 2\pi \left(\sin \omega T - \sin \frac{\omega T}{2}\right)}{\omega T \left(\pi^2 - \omega^2 T^2\right)} \right\}$$

Lastly for triangular pulse

$$f(t) = \begin{cases} 1 - \frac{2}{T} |t| & |t| \leq \frac{T}{2} \\ 0 & |t| \geq \frac{T}{2} \end{cases}$$

then

$$S_y(\omega) = \frac{T}{2} \operatorname{sinc}^4 \left(\frac{\omega T}{4}\right)$$

and

$$S_z(\omega) = \frac{2}{T} \left| \int_{-\frac{T}{2}}^{0} e^{-j\omega t}\, dt + \int_{0}^{\frac{T}{2}} \frac{1 - \frac{2}{T}t}{\sqrt{1 - \frac{4}{T}t + \frac{8t^2}{T^2}}}\, e^{-j\omega t}\, dt \right|^2$$

and

$$S_Z(\omega) = \frac{2}{T} |\hat{F}(\omega)|^2$$

$$= \frac{2}{T} \left| \int_{-\frac{T}{2}}^{0} e^{-j\omega t}\, dt + \int_{0}^{\frac{T}{2}} \frac{1}{\sqrt{2}} e^{-j\omega t}\, dt \right|^2$$

$$= T \left[\frac{3}{4} + \frac{1}{\sqrt{2}} \cos \frac{\omega T}{2} \right] \operatorname{sinc}^2 \left(\frac{\omega T}{4} \right)$$

For the MSK

$$f(t) = \begin{cases} \cos \frac{\pi t}{T} & |t| \leq \frac{T}{2} \\ 0 & |t| \geq \frac{T}{2} \end{cases}$$

Then

$$S_y(\omega) = \frac{2}{T} \left| \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos \frac{\pi t}{T} e^{-j\omega t}\, dt \right|^2$$

$$= \frac{8 \pi^2 T}{(\pi^2 - \omega^2 T^2)^2} \cos^2 \frac{\omega T}{2}$$

and

$$S_Z(\omega) = \frac{2}{T} \left| \int_{-\frac{T}{2}}^{0} e^{-j\omega t}\, dt + \int_{0}^{\frac{T}{2}} \cos \frac{\pi t}{T}\, e^{-j\omega t}\, dt \right|^2$$

$$= 2T \left\{ \frac{1}{4} \operatorname{sinc}^2 \left(\frac{\omega T}{4}\right) + \frac{\pi^2 + \omega^2 T^2 - 2\pi\omega t \sin \frac{\omega T}{2}}{(\pi^2 - \omega^2 T^2)^2} \right.$$

$$\left. + \frac{2\omega T \left(1 - \cos \frac{\omega T}{2}\right) + 2\pi \left(\sin \omega T - \sin \frac{\omega T}{2}\right)}{\omega T \left(\pi^2 - \omega^2 T^2\right)} \right\}$$

Lastly for triangular pulse

$$f(t) = \begin{cases} 1 - \frac{2}{T} |t| & |t| \leq \frac{T}{2} \\ 0 & |t| \geq \frac{T}{2} \end{cases}$$

then

$$S_y(\omega) = \frac{T}{2} \operatorname{sinc}^4 \left(\frac{\omega T}{4}\right)$$

and

$$S_z(\omega) = \frac{2}{T} \left| \int_{-\frac{T}{2}}^{0} e^{-j\omega t}\, dt + \int_{0}^{\frac{T}{2}} \frac{1 - \frac{2}{T}t}{\sqrt{1 - \frac{4}{T}t + \frac{8t^2}{T^2}}}\, e^{-j\omega t}\, dt \right|^2$$

Example 2. Suppose $g(t) = f(t)$. Then

$$S_y(\omega) = \frac{\sigma_a^2 + \sigma_b^2}{T} |F(\omega)\ H(\omega)|^2 \tag{4-3}$$

which is identical to equation (4-1). Hence the power spectral density of quadrature modulated signals generated by (2-8) is independent of the timing offset in the in-phase and the quadrature-phase channel before the bandpass hardlimiting. Let us consider the same condition that the signal $y(t)$ is unfiltered and $f(t)$ is nonzero for a duration less than or equal to T. Then

$$\hat{f}(t) = \begin{cases} \frac{1}{\sqrt{2}} & |t| \leq \frac{T}{2} \\ 0 & |t| > \frac{T}{2} \end{cases}$$

and

$$S_z(\omega) = \frac{2}{T}\ |\hat{F}(\omega)|^2 = T\ \mathrm{sinc}^2\ \frac{\omega T}{2}$$

That is, no matter how the symbols are weighted before transmission, the bandpass hardlimiter will restore all sidelobes so that the output resembles the output of conventional QPSK. This undesirable feature makes the offsetting technique attractive for nonlinear bandpass channels.

We can see from example 1 and 2 that although the offsetting technique does not change the power spectral density of $y(t)$, the bandpass hardlimiter does create differences between the two techniques. Figures 4-1, 4-2 and 4-3 show the cases for offset QPSK, MSK, and triangular keying unfiltered before

and after the bandpass hardlimiter. Clearly the theoretical sidelobe regeneration of MSK is smaller than that of offset QPSK but is similar to that of the triangular keying technique.

Example 3. We consider a mixed keying case where the in phase and the quadrature phase have different symbol weightings. Let

$$f(t) = \begin{cases} \cos \frac{\pi t}{T} & |t| \leq \frac{T}{2} \\ 0 & |t| \geq \frac{T}{2} \end{cases}$$

and

$$g(t) = \begin{cases} 1 & |t| \leq \frac{T}{2} \\ 0 & |t| > \frac{T}{2} \end{cases}$$

be the pulses used in equation (2-4). Then

$$S_y(\omega) = T \left\{ \frac{4\pi^2}{(\pi^2 - \omega^2 T^2)^2} \cos^2 \frac{\omega T}{2} + \operatorname{sinc}^2 \left(\frac{\omega T}{2} \right) \right\}$$

which is independent of timing offset in the in-phase and the quadrature-phase channels. However, the power spectral density $S_z(\omega)$ in (3-5) after the bandpass hardlimiting will not be the same. From (3-5)

$$S_z(\omega) = \frac{1}{T}\left|\int_{-\frac{T}{2}}^{0} e^{-j\omega t}\,dt + \int_{0}^{\frac{T}{2}} \frac{\cos\frac{\pi t}{T}}{\sqrt{1+\cos^2\frac{\pi t}{T}}}\, e^{-j\omega t}\,dt\right|^2$$

$$+\frac{1}{T}\left|\int_{0}^{\frac{T}{2}} \frac{1}{\sqrt{1+\cos^2\frac{\pi t}{T}}}\, e^{-j\omega t}\,dt + \int_{\frac{T}{2}}^{T} e^{-j\omega t}\,dt\right|^2$$

when f(t) and g(t)have a timing offset T/2, but

$$S_z(\omega) = \frac{1}{T}\left|\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{\cos\frac{\pi t}{T}}{\sqrt{1+\cos^2\frac{\pi t}{T}}}\, e^{-j\omega t}\,dt\right|^2$$

$$+\frac{1}{T}\left|\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{1}{\sqrt{1+\cos^2\frac{\pi t}{T}}}\, e^{-j\omega t}\,dt\right|^2$$

when there is no timing offset. Figure 4-4 shows the power spectral density for both cases.

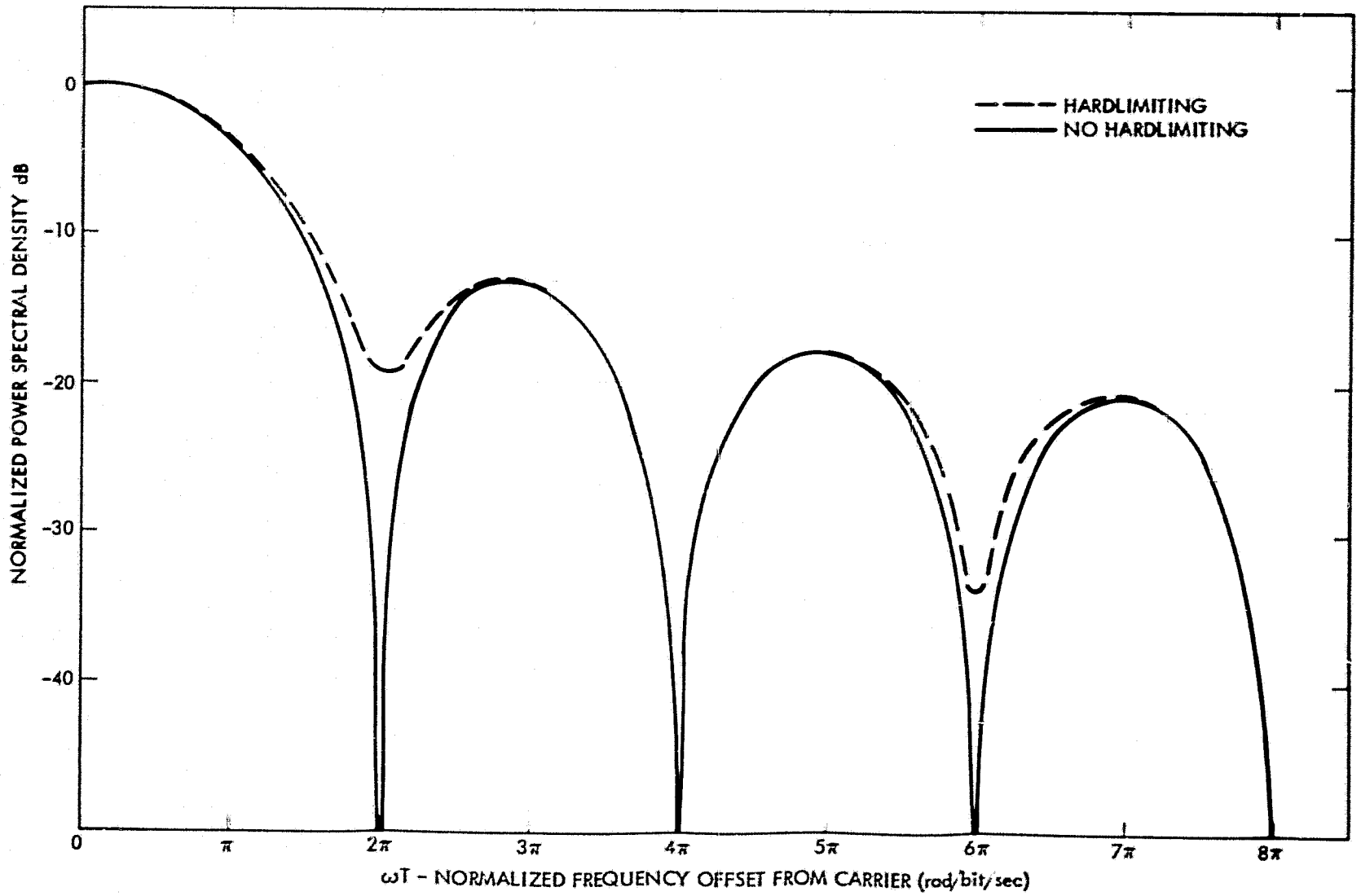

Figure 4-1. Power Spectral Density for Offset QPSK With and Without Bandpass Hardlimiting

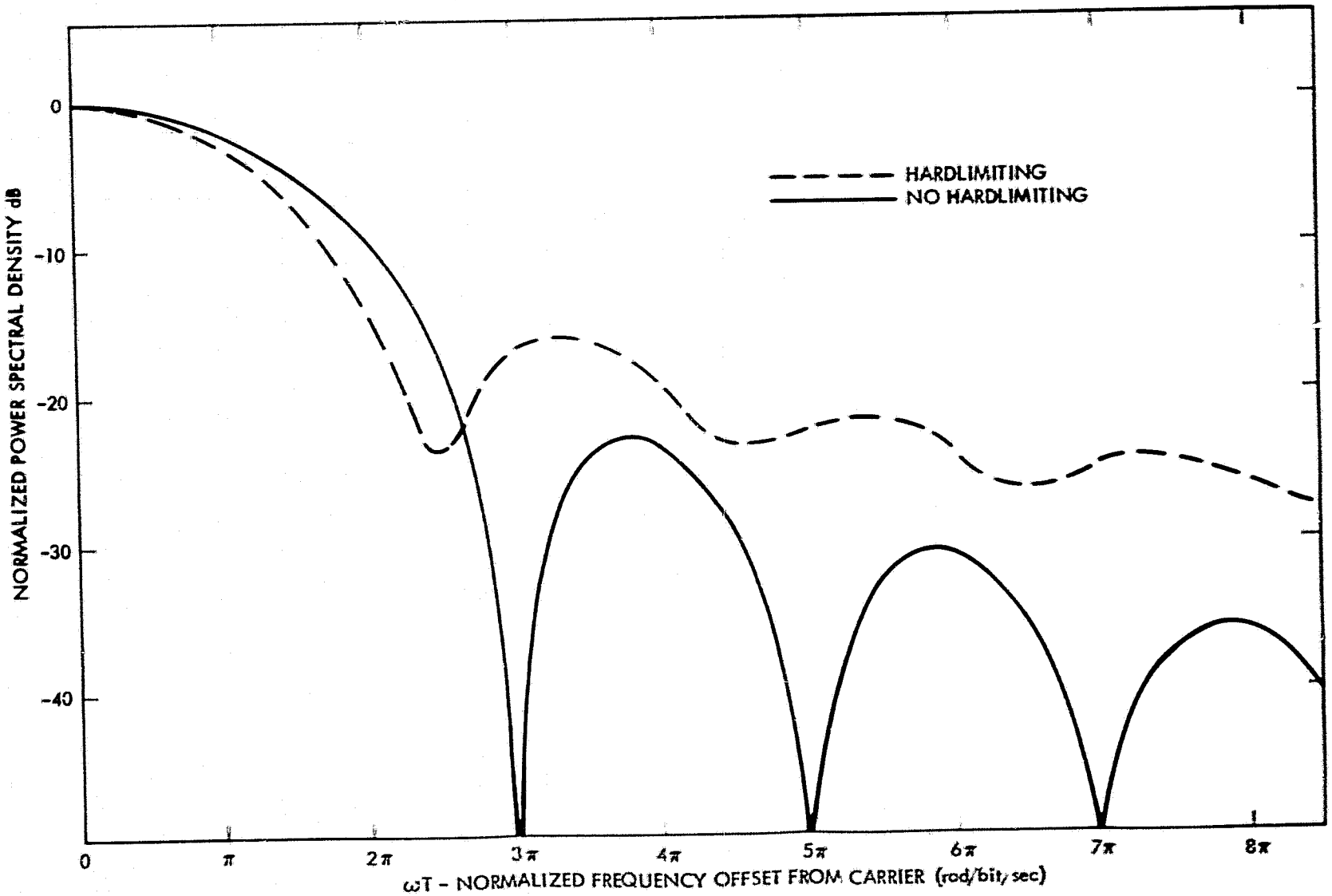

Figure 4-2. Power Spectral Densities for MSK With and Without Bandpass Hardlimiting

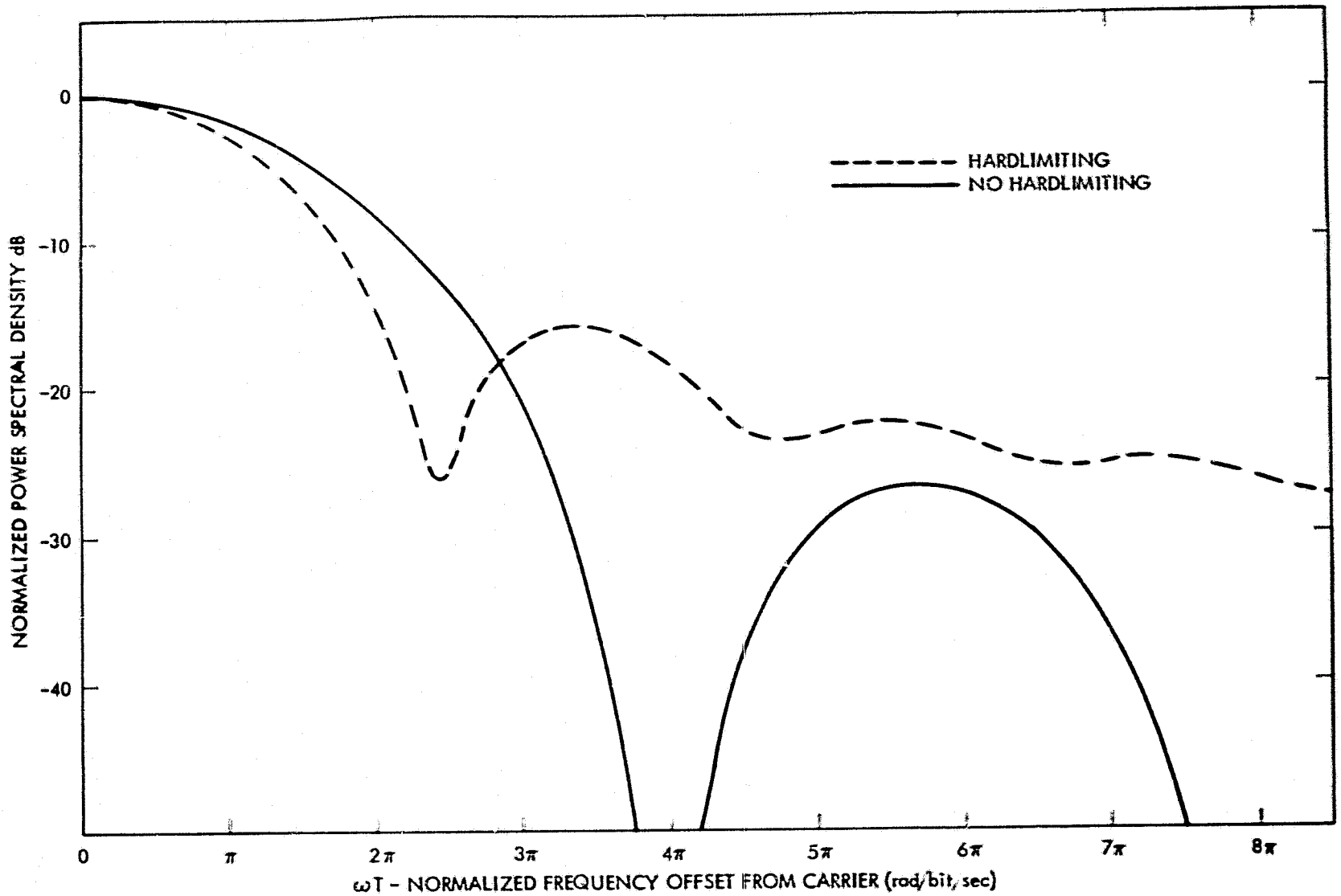

Figure 4-3. Power Spectral Densities for Offset Triangular Keying With and Without Bandpass Hardlimiting

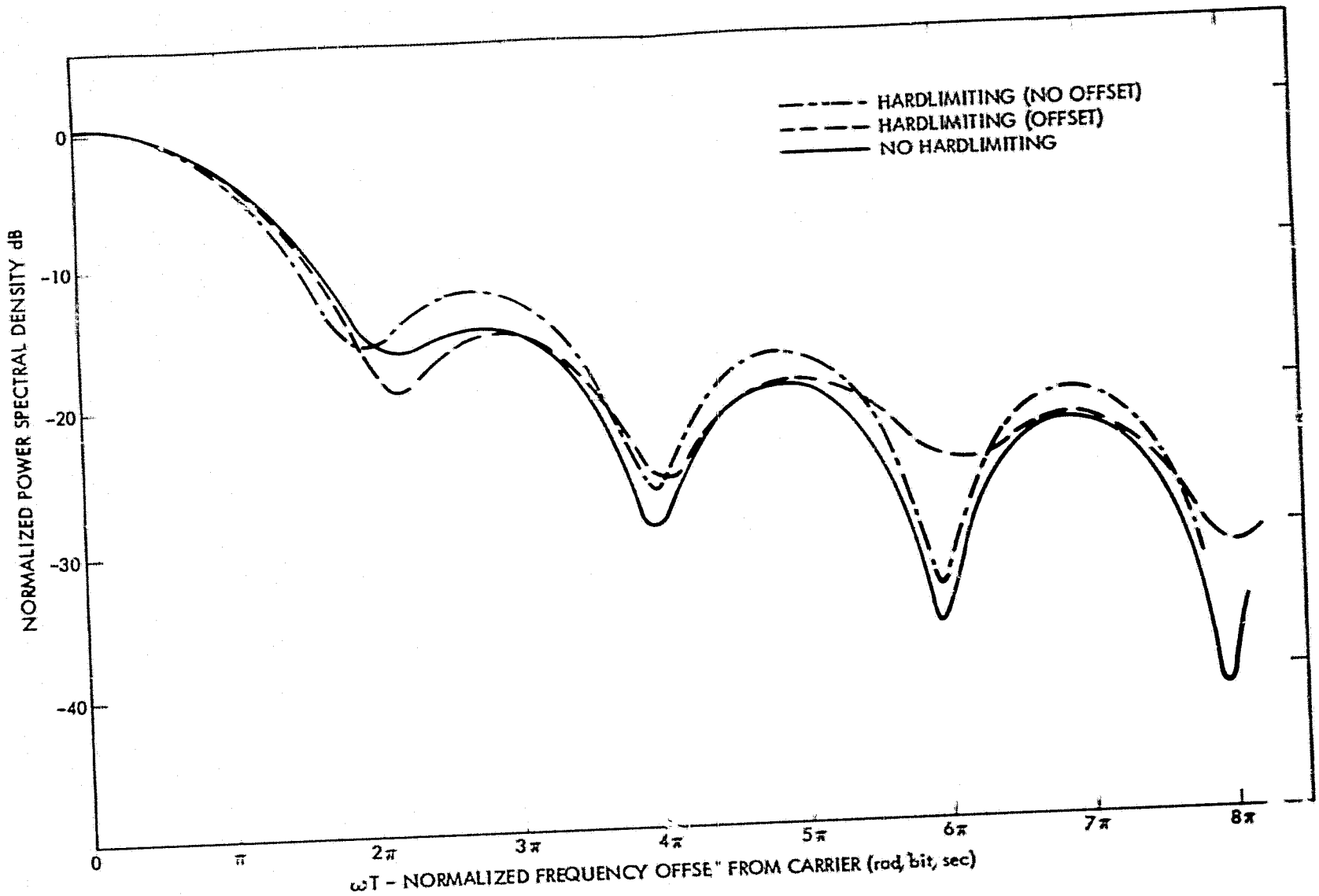

Figure 4-4. Power Spectral Density for Mixed Keying With and Without Bandpass Hardlimiting

V. Summary and Conclusion

A generalized theory for evaluating the power spectral density of quadrature modulated signals is developed. Arbitrary pulse shaping is allowed in the in-phase and the quadrature-phase channels to model various kinds of modulation techniques. In particular, the theory is applied to investigate the effect of bandpass hardlimiting on quadriphase unfiltered signals. Numerical examples show the wide range of applications for which this model can be used. It has been shown that the power spectral density of the modulated signal is independent of the timing offset in the in-phase and the quadrature-phase channels. Furthermore, if the in-phase and the quadrature-phase channels have identical pulse shapes without offset, the spectrum after bandpass hardlimiting is identical to that of the conventional QPSK modulation.

In general, the offset keying technique used in the nonlinear bandpass channel performs better than conventional pulse keying. Depending on the bandwidth availability of the communication channel and the required data transmission rate, various keying techniques can be developed to meet the specific environment. The theory described in this report provides yet another tool for understanding the effects of bandpass hardlimiting encountered in the nonlinear satellite channels.

References

1. Rhodes, S. A., "Effects of Hardlimiting on Bandlimited Transmissions with Conventional and Offset QPSK Modulation", Proceeding NTC, 1972.

2. Austin, M. C., and Chang, M. V., "Quadrature Overlapped Raised-Cosine Modulation", Proceeding NTC, 1980.

3. Gronemeyer, S. A., and McBride, A. L., "MSK and Offset QPSK Modulation", IEEE Transactions of Communication, vol. COM-24, No. 8, Aug. 1976 (pp. 809-820).

4. Prabhu, V. K., and Rowe, H. E., "Spectra of Digital Phase Modulation by Matrix Methods", Bell System Technical Journal, May-June 1974 (pp. 899-935).

5. Prabhu, V. K., "PSK-Type Modulation with Overlapping Baseband Pulses," IEEE Transaction on Communication, vol. COM-25, No. 9, Sept. 1977.

Acknowledgement

The author wishes to thank Dr. James Lesh for his review and helpful discussions as well as special thanks to Mr. Pil Lee for his numerical computations.

APPENDIX A

GENERALIZED SPECTRAL DENSITY OF NONSTATIONARY PERIODIC PROCESSES

Let $\eta(t)$ and $R(t,\tau)$ be the mean function and the auto correlation function of a nonstationary process $y(t)$ which satisfies

$$\eta(t) = \eta(t+T)$$

and

$$R(t,\tau) = R(t+T,\ \tau+T) \tag{A-1}$$

for some period T. Also let $S(\omega,\omega')$ be the two dimensional Fourier transform of $R(t,\tau)$. Then from (A-1)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(t+T,\ \tau+T)\ e^{-j(\omega t-\omega'\tau)}\ dt\ d\tau$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(t',\tau')\ e^{-j(\omega t'-\omega T-\omega'\tau'+\omega' T)} dt'\ d\tau'$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(t,\tau)\ e^{-j(\omega t-\omega'\tau)}\ dt\ d\tau = S(\omega,\omega')$$

Since (A-1) must be satisfied for all ω,ω' from $-\infty$ to $+\infty$, the following conditions on $S(\omega,\omega')$ hold

$$S(\omega,\omega') = \begin{cases} 0 & \omega-\omega' \neq k\omega_o \\ S(\omega,\omega') \sum_{k=-\infty}^{\infty} \delta(\omega-\omega' - k\omega_o) & \text{For all other } \omega,\omega' \end{cases} \tag{A-2}$$

where $\omega_o = 2\pi/T$.

Hence

$$R(t,\tau) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} S(\omega' + k\omega_o, \omega')\, e^{j(\omega'+k\omega_o)t}\, e^{-j\omega'\tau}\, d\omega'$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{jk\omega_o t} \int_{-\infty}^{\infty} S(\omega' + k\omega_o, \omega')\, e^{j\omega'(t-\tau)}\, d\omega'$$

then

$$R(t+\tau, t) = \sum_{k=-\infty}^{\infty} \gamma_k(\tau)\, e^{jk\omega_o t} \tag{A-3}$$

where

$$\gamma_k(\tau) = \frac{1}{2\pi} e^{jk\omega_o \tau} \int_{-\infty}^{\infty} S(\omega' + k\omega_o, \omega')\, e^{j\omega'\tau}\, d\omega' \tag{A-4}$$

and

$$S(\omega' + k\omega_0, \omega') = \int_{-\infty}^{\infty} e^{-jk\omega_0\tau} \gamma_k(\tau)\, e^{-j\omega'\tau}\, d\tau \tag{A-5}$$

From (A-3), $R(t+\tau, t)$ is a periodic function in t with τ as a moving parameter. Therefore

$$\gamma_k(\tau) = \frac{1}{T} \int_0^T R(t+\tau, t)\, e^{-jk\omega_0 t}\, dt \qquad -\infty < \tau < \infty \tag{A-6}$$

The generalized spectral density of y(t) can be defined by (A-5) and (A-6) so that

$$S_k(\omega') \triangleq S(\omega' + k\omega_0, \omega')$$

$$= \frac{1}{T} \int_0^T \int_{-\infty}^{\infty} R(t+\tau, t)\, e^{-j\omega'\tau}\, e^{-jk\omega_0(t+\tau)}\, d\tau\, dt \tag{A-7}$$

APPENDIX B

POWER SPECTRAL DENSITY OF BASEBAND NONOVERLAPPING CORRELATED PHASE MODULATED SIGNALS

Consider a baseband signal $v(t)$, $-\infty < t < \infty$ defined by

$$v(t) = e^{j\phi(t)} \tag{B-1}$$

where

$$\phi(t) = \sum_{k=-\infty}^{\infty} \psi_{\alpha_k}(t-kT), \quad -\infty < t < \infty$$

The $\{\alpha_k\}_{-\infty}^{\infty}$ is a stationary random sequence which takes on values $\{1,2,\cdots, M\}$. For each $\alpha_k = \ell$, a phase-signaling waveform $\psi_\ell(t)$ is generated. Clearly the process $v(t)$ satisfies the following conditions.

$$E\{v(t)\} = E\{v(t+T)\}$$

and

$$R_v(t, \tau) = R_v(t+T, \tau+T) \tag{B-2}$$

where

$$R(t, \tau) = E\{v(t)v^*(\tau)\} - E\{v(t)\}E\{v^*(\tau)\}$$

Then

$$E\{v(t)\} = \sum_{\ell=1}^{M} \sum_{k=-\infty}^{\infty} P_\ell \, e^{j\psi_\ell(t-kT)} \tag{B-3}$$

and

$$E\{v(t)\ v^*(\tau)\} = \sum_{k=-\infty}^{\infty} \sum_{k'=-\infty}^{\infty} E\left\{e^{j\psi_\ell(t-kT)-j\psi_\ell(\tau-k'T)}\right\} \tag{B-4}$$

where

$$E\left\{e^{j\psi_\ell(t-kT)-j\psi_\ell(\tau-k'T)}\right\} = \begin{cases} \sum_{\ell=1}^{M} P_\ell\ e^{j\psi_\ell(t-kT)-j\psi_\ell(\tau-kT)} & ;\ k=k' \\ \sum_{\ell=1}^{M} \sum_{\ell'=1}^{M} P_{\ell,\ell',k-k'}\ e^{j\psi_\ell(t-kT)-j\psi_\ell(\tau-k'T)} & ;\ k\neq k' \end{cases}$$

$$P_\ell = P_r\ [\alpha_k = \ell]$$

and

$$P_{\ell,\ell',k,k'} = P_r\ [\alpha_k = \ell,\ \alpha_{k'} = \ell']$$

From (A-7), the generalized power spectral density of v(t) is given by

$$S_n(\omega) = \frac{1}{T} \int_0^T \int_{-\infty}^{\infty} R\ (t+\tau,\ t)\ e^{-j\omega\tau}\ e^{-jn\omega_o(t+\tau)}\ dt\ d\tau \tag{B-5}$$

Let D be the domain of integration in (B-5). Then k' equals zero in (B-4) for all t, τ that lie in D. Hence

$$E\{v(t+\tau)\ v^*(t)\} = \sum_{\ell=1}^{M} P_\ell\ e^{j\psi_\ell(t+\tau)-j\psi_\ell(t)}$$

$$+ \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} P_{\ell,\ell',k}\ e^{j\psi_\ell(t+\tau-kT)-j\psi_{\ell'}(t)} \qquad \text{(B-6)}$$

Using (B-6), (B-3) and (B-2) in (B-5), we obtain

$$\bar{S}_n(\omega) = \frac{1}{T}\sum_{\ell=1}^{M} P_\ell\ \Psi_\ell\ (\omega + n\omega_o)\ \Psi_\ell^*(\omega) - \frac{1}{T}\sum_{\ell=1}^{M}\sum_{m=1}^{M} P_\ell\ P_m\ \Psi_\ell\ (\omega + n\omega_o)\ \Psi_m^*(\omega)$$

$$+ \frac{1}{T}\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty}\sum_{\ell=1}^{M}\sum_{m=1}^{M} (P_{\ell,m,k} - P_\ell\ P_m)\ e^{-jkT(\omega+n\omega_o)}\ \Psi_\ell\ (\omega + n\omega_o)\ \Psi_m^*(\omega) \qquad \text{(B-7)}$$

where

$$\Psi_\ell(\omega) = \int_{-\infty}^{\infty} e^{j\psi_\ell(t)}\ e^{-j\omega t}\ dt$$